Government Debt, Interest Rates, and Income Distribution

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Abstract

We look at two stylized macroeconomic facts of the past 50 years in developed economies and analyze a possible nexus between the two: The rise in income inequality and the increase in government debt-to-GDP ratios. We ask whether, and under what conditions, rising public debt might have an impact on the inequality of income distribution. Our approach is based on the view that the question of the distributional effects of government debt cannot be addressed in a partial-analytic way, but requires a macro framework broad enough to include factors such as growth and interest rate dynamics. The theoretical Post-Keynesian model we use, comprises workers receiving wage and interest income and capitalists receiving income from profits and interest. It serves to analyze how an increase in government debt leads to changes in income distribution through its expansionary effects. The paper shows that the distributional effects depend on the interplay between government spending, real interest rates, debt-to-capital ratios, income, investment, consumption and growth. We find that the distributional effects of government debt are ambiguous, as they depend on several parameters, the real interest rate being the most important.

Keywords: Government debt, income distribution, growth, real interest rates, Post-Keynesian model

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1. Introduction

The development of advanced economies over the past four to five decades has been characterized, among other things, by a rise in income inequality and by an increase in the ratio of public debt to GDP. This paper analyzes some important relationships between these trends. Among other things, we examine the question of whether and under what circumstances an increase in the public debt to GDP ratio increases the inequality of income distribution. A widespread view in this context, which we challenge here, is that higher government debt generally increases income inequality because wealthier classes are more likely to hold government bonds and therefore benefit from interest income (Anselmann and Krämer 2016). Although questions of public debt are inextricably linked to questions of distribution, the linkages between public debt and income distribution have been analyzed only to a minor extent in the economic literature. Early contributions such as Miller (1950) or Cohen (1951) based their reasoning on the mechanism Gandenberger (1970) referred to as the *transfer approach*. According to the transfer approach, government debt leads to a redistribution of income from taxpayers to recipients of federal interest payments. Since government securities are usually distributed more progressively than taxes, a rise in government debt is assumed to transfer income from the bottom and middle to the top of the income distribution. However, the transfer approach is often criticized for its basic simple link between state interest payments and tax rates which is too narrow a view accounting for the complexities linking public debt and income distribution (Anselmann and Krämer 2016).

Besides these complexities, an empirical examination of the distributional effects of government debt ownership is further complicated as data on the distribution of government bonds as well as the distribution of tax payments within the population is difficult to obtain (Hager 2013). Moreover, as Arbogast (2020) shows for the case of Italy, a breakdown of the indirect holding structure of government bonds is complicated as the majority of government bonds is held by different financial institutions. But above all, it is not the formal incidence underlying the idea of the transfer approach that is decisive for the distributional effects, but rather how the recipients of the interest payments and the taxpayers react in their savings and consumption decisions and what effects this has on macroeconomic variables such as on investments and growth (Oberhauser 2008). Hence, model-theoretic studies such as Schlicht (2006) or, based on the Post-Keynesian tradition, Pasinetti (1989) and You and Dutt (1996), analyzed the distributional effects of government debt ownership by incorporating growth and interest dynamics into a total-analytic framework.

However, the changing macroeconomic environment of today highlights the need for an update of the assumptions and considerations made by these contributions. Building on these contributions, our theoretical model takes account of major recent macroeconomic trends.
These trends include, among others, a decline of growth rates of advanced economies over the past decades. Inspired by this weak economic performance, current contributions, such as Summers (2014, 2015) and Rachel and Summers (2019), refer to Hansen’s theory of secular stagnation (Hansen 1939). Hansen formulated his theory in a time of prolonged depressing economic performance in the United States which he described as “the essence of secular stagnation – sick recoveries which die in their infancy and depressions which feed on themselves and leave a hard and seemingly immovable core of unemployment” (ibid., p. 4). Summers (2014, 2015) relates the ongoing decline in growth performance in major advanced economies to increased saving and decreased investment activities leading to a stagnation in economic activity. He argues that although the decline in investment activity is in part a consequence of tighter financial regulation measures due to the high debt levels during the financial crisis years, it is mainly driven by structural developments such as technological changes and a decline in population growth leading to a reduction in investment demand.

Connected to these developments is a decline in long-term real interest rates. Eggertsson et al. (2019) refer to several reasons being responsible for this trend. Among them are a reduction in loan demand as a consequence of a decline in population growth. Their explanation includes a declining productivity growth which leads to an increase in saving and a tightening of borrowing constraints due to deteriorating growth expectations. Further factors include a fall in the price of investment goods reducing the demand for savings to finance investment and increasing the stock of savings (see also Eichengreen, 2015). Another driver of declining real interest rates is a rise in the propensity to save of private households. This is due on the one hand, to changing demographics as argued by von Weizsäcker and Krämer (2021). They point to a rise in life expectancy which induces households to a shift in their intertemporal consumption decisions leading to a rise in saving propensities to accommodate future consumption. Another argument put forward by Mian et al. (2021), relates the rise in saving to an increase in incomes earned by top income earners who tend to have a higher propensity to save out of income.

In the present article, we analyze the implications of the mentioned trends for the nexus between government debt and income distribution. We show that the real rate of interest and economic growth are important determinants of this nexus. Our model, which extents a model presented by You and Dutt (1996), analyzes how a decrease in investment activity leads to a decline in output and capital accumulation resulting in a rise in the long-run debt-to-capital ratio. Moreover, an increase in saving has a negative impact on output and the rate of capital accumulation which can be offset by an increase in the long-run debt-to-capital ratio as this raises consumption expenditures of workers and capitalists. However, this offsetting effect occurs only as long as the real rate of interest is positive.

The article is structured as follows: Section 2 provides some empirical insights regarding the
long-run developments of factor shares and debt-to-capital ratios. Section 3 gives an overview of the relevant literature. The theoretical model is introduced in section 4. Section 5 continues with an analysis of the consequences of a decrease in private investment activity, a fall in the real rate of interest and a rise in government spending for the nexus between government debt and income inequality. Finally, section 6 concludes.

2. Empirical Trends

In this section, we provide a descriptive empirical analysis to shed light on trends regarding the development of government debt and inequality over the past century.

Figure 1 displays the level of the public debt to GDP ratio for a selection of 24 advanced economies from the year 1900 to 2011.\textsuperscript{1} The data reveals a sharp increase in public debt during World War I. After a short period of fiscal consolidation, public debt peaked at 60% of GDP in the Great Depression of the 1930s. While public debt-to-GDP began to elevate during World War II reaching a peak at 73%, this trend reverted at the end of World War II and public debt decreased to about 19% of GDP in the middle of the 1960s reflecting a substantial acceleration of economic growth through a rise in investment activities and negative real interest rates (Abbas et al. 2011; Eichengreen and Vazquez 2000). However, triggered by oil price shocks, public debt levels began a steady rise since the mid 1970s reaching a peak at 67% of GDP in 1996, followed by a shorter period of fiscal consolidation from the end of the 1990s until the beginning of the financial crisis, in which primary balances improved (Eichengreen, El-Ganainy, et al. 2021). This increase in debt burdens was accompanied by increasing real interest rates in advanced economies government debt as well as a decline in growth performance leading to rising unemployment rates and decreasing tax incomes. The recent rise in public debt levels is closely linked to the financial crisis period in which primary balances deteriorated due to declining public revenues and the issuance of stimulus packages (Abbas et al. 2011; Reinhart and Rogoff 2011).

\textsuperscript{1}A complete country list can be found in Appendix A.
Figure 1: Median public debt-to-GDP ratio in percent for 24 advanced economies. Data are from Mauro et al. (2015).

Figure 2 shows wage shares for a selection of advanced economies from the year 1900 to 2018. Whereas wage shares in France, Great Britain, Germany, Italy and Sweden declined in the early years of the 20th century reaching a low at the beginning of the first World War, they increased during the Great Depression in Germany, Great Britain, the United States, Italy and Sweden. Followed by some fluctuations until the beginning of the second World War, wage shares began to rise during World War II. In the aftermath of World War II, they followed different trajectories. Obviously, from the mid 1970s to 1980s wage shares declined steadily until the beginning of the 2000s.
Our empirical observations mesh with the literature, which highlights a steady decline in the wage share of income, an income source gaining in relevance if one moves from the top to the bottom of the income distribution (Piketty and Saez 2007). This literature points to several factors held responsible for the decline in the wage share. Those include a decline in the bargaining power of workers, a rise in market capitalization of firms as well as an increased importance of finance related services (Dünhaupt 2017; Kohler et al. 2019; Pariboni and Tridico 2019), a rise in trade and financial globalization (Jayadev 2007; Stockhammer 2013) and an increasing automation (Bergholt et al. 2022).

Empirical evidence not only points to a decline in the wage share over the past decades, but also to a rise in the capital share of income (Bengtsson and Waldenström 2018). Figure 3 depicts the development of capital shares for a selection of advanced economies from 1900 to 2018. The development of capital shares can be seen as the mirror image of the development of wage shares. Whereas capital shares in France, Great Britain, Germany, Italy and Sweden increased in the beginning of the 20th century, they decreased during the Great Depression of the 1920s in Germany, Great Britain, the United States, Italy and Sweden. During the second World War, capital shares declined again. From the mid 1970s and the beginning of the 1980s,

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For a recent overview of factors that have contributed to the fall in the wage share, see Krämer et al. (2023)
capital shares steadily increased until the beginning of the 2000s. In addition to the factors held responsible for declining wage shares, Piketty and Zucman (2014) point to a rise in the elasticity of substitution between labor and capital as a cause for the increase in capital shares since the 1970s.

Figure 3: Capital shares in percent for a selection of advanced economies. Data are from Bengtsson and Waldenström (2018).

3. Government Debt and Income Distribution

3.1. Empirical Studies

Only a limited number of empirical studies trying to measure the redistributive effects of government debt exist. This can be attributed to the difficulty of obtaining appropriate and satisfactory data on the distribution of government bonds as well as the distribution of tax payments within the population (Hager 2013). Despite these difficulties, studies attempt to measure the redistributive effects of government debt came to divergent conclusions. In the following, we provide a brief overview of the empirical evidence presented so far.

A high concentration of private sector government debt holdings is found by Arbogast (2020). He shows that in the period from 1998-2016 the share of government bonds held by the top 10% households of the wealth distribution amounted to approximately 49% in Italy. The top 1% of Italian households accounted for 15%.

Other studies measured the redistributive effects of government debt by comparing the income
from interest payments with the rate at which different income groups were taxed, a reasoning known as the transfer approach (Gandenberger 1970). Michl (1991) finds a regressive tendency of federal interest payments. He shows that the federal interest payments benefitting the top 1% of the income distribution amounted to 22.5%-33.3%. This income group was taxed at a rate between 11.2%-14.6%, however. This progressivity in the tax system is confirmed by Kurz and Rall (1984) who found no redistributive effects of public debt to the detriment of lower income classes in Germany for the year 1982.

So far, the presented empirical studies based their conclusions on different data sources as well as methodological approaches and provided evidence based on single year observations. Hence, they do not exploit the time series dimension by examining the development of government debt ownership over time (Hager 2016). Hager (2013) tried to overcome these empirical limitations and provided a study of the development of government debt ownership in the United States over the past decades. The data revealed that the share of government debt held by the top 1% of the income distribution has increased since the 1970s. While the interest income of the top 1% increased steadily their tax payments remained almost constant, however, pointing to a redistribution of income from the bottom to the top through federal interest payments. Salti (2015) used cross-country panel data spanning the years 1990-2007 to examine the redistributive effects of government debt by means of fixed effects panel regression methods. Her study revealed a positive relationship between the level of domestic public debt and personal income inequality measured by the Gini coefficient. This result is robust to several specifications and when controlling for factors such as the share of external debt servicing, inflation, political stability and several types of government expenditures. Thus, according to her findings, a rise in the domestic share of public debt leads to a higher level of income inequality.

To summarize, the presented empirical evidence points to a concentration of government bond holdings at the top of the income distribution. This, together with the progressivity in tax systems, favors a redistribution of income from the bottom to the top of the income distribution. This is supported by time series evidence revealing a positive relationship between the level of domestic public debt and personal income inequality.

3.2. Theoretical Studies

As already noted, data limitations complicate an empirical examination of the distributional effects of government debt. Consequently, a number of studies focus on a theoretical analysis of the relationship between government debt and income distribution. These studies provide a broader picture of the link between government debt and income distribution as they incorporate factors such as economic growth, interest rates and financial globalization into their analytic framework. In the following, we provide a brief review of this theoretical literature.
The theoretical model by You and Dutt (1996) is based on the assumption that economic activity is determined by aggregate demand rather than by the supply of resources. Additionally, investment activities in the corporate sector depend on expectations about the future of the economy reflecting the animal spirits hypothesis. Further assumptions include that money supply is determined by aggregate demand and that workers and capitalists differ in their propensities to save out of wage and non-wage income. By incorporating distributional effects into their model, they show that a rise in the debt-to-capital ratio increases income inequality. This effect is partly attributable to a rise in the interest income of capitalists due to their holdings of government bonds. However, it is to a further extent attributable to the rise in interest income of capitalists in excess of the rise in wage income of workers due to the positive impact of expansionary fiscal policy on economic activity (ibid.).

The model by Pasinetti (1989), is comprised of workers and capitalists, both receiving profits and saving a fraction of their disposable income. However, only workers earn wages. In the model, equilibrium growth at full employment is sustained through investment activities which are financed by profits. Hence, when the government decides to vary the tax rate, this classifies profits as prerogative to wages. When government spending is financed by an increase in taxes, workers and capitalists set aside part of their savings in order to service a future rise in tax rates. The reduction in the savings of capitalists is compensated by an increase in profits. By contrast, the disposable income of workers is not corrected by the reduced amount of savings leading to a greater inequality in income distribution. Furthermore, if additional government spending is financed by an increase in public debt, each individual taxpayer serves as debtor but only the fraction of the population holding government bonds will receive interest income. This contributes to the increase in income inequality.

Azzimonti et al. (2014) provide a theoretical model where they relate the level of government debt to income inequality and the degree of financial openness. In this model, the equilibrium interest rate is below the intertemporal discount rate. This enables entrepreneurs to use public debt in order to smooth consumption. On the other hand, workers use public debt as a substitute for private debt. However, this is possible only until a certain threshold of public debt is reached. Above that limit the equilibrium interest rate will rise. In this model, government debt increases as a response to increasing financial integration and rising inequality. This is based on two mechanisms. First, financial integration reduces the interest rate elasticity of public debt inducing governments to increase borrowing. Second, the decreased interest rate elasticity due to financial integration eases the possibility of entrepreneurs to insure against increases in income risk associated with a more unequal distribution of income.

While the above mentioned studies focus mainly on income distribution, Schlicht (2006) is concerned with the distribution of wealth. His study illustrates the equilibrium dynamics be-
tween public debt, government spending and the rate of taxation under the assumptions of full employment, fixed production and a fixed interest rate. Schlicht refers to two policy scenarios in which equilibrium dynamics are determined either by variations in government spending or variations in the tax rate. In the first case, government spending is fixed and a reduction in the tax rate induces an increase in government debt. This increase in government debt leads to an increase in income and wealth, an increase in private consumption expenditures and a subsequent rise in aggregate demand. In the second case, the tax rate is fixed and the increase in aggregate demand is brought about by an increase in government expenditures financed by an increase in government debt. This increase in government debt leads to a rise in income and wealth. A relaxation of the assumptions of a fixed level of government spending or a fixed rate of taxation offers important insights into the role of public debt in equilibrium. Consequently, the relative levels of private consumption and public spending are mediated by the level of government debt. Increasing the level of private consumption in equilibrium increases government debt and decreases government spending and the tax rate (Schlicht 2006).

To conclude, the mentioned literature focuses on the functional income distribution. In the studies by You and Dutt (1996) and Pasinetti (1989) only a part of the population (the capitalists) holds government bonds. In their model, a rise in interest income and a subsequent change in the distribution of income result in an increase of the level of government debt. By contrast, in the model by Azzimonti et al. (2014) both entrepreneurs and workers hold government bonds and changes in the distribution of income associated with changes in income risk lead to changes in government debt. The increase in government debt is supported by a higher level of financial integration as this leads to a decline in the interest rate elasticity of public debt. In the model by Schlicht (2006), the distribution of income and wealth is determined by the level of government debt which has an intermediating role between private consumption and public spending.

4. Theoretical Model
The Post-Keynesian model developed by You and Dutt (1996) provides us with a basic framework to analyze the relationship between government debt and functional income distribution in a broader setting, which takes interest and growth dynamics into account. In the following section, we build on this framework and - in constrast to You and Dutt (ibid.) - introduce the real instead of the nominal interest rate. Moreover, we extend their model by assuming that not only capitalists, but also workers save a fraction of their disposable income and can hold government bonds. To deepen the analysis, we will distinguish three cases to study the links between government debt and (functional) income distribution. The three cases differ as follows: First, we look at a decline in economic growth induced by a decrease in investment activity and a rise in savings. Second, we study a fall in the real interest rate. Third, we analyze the effects of a rise
in government spending.

4.1. The Model in the Short-run

We assume a closed economy in which aggregate demand determines output and there are no supply constraints regarding labor and production capacity. Consumption demand is given by the following equation

\[ C = (1 - s_w)(1 - \tau_w)[(1 - \mu)Y + (1 - \phi)(rD/p)] + (1 - s_c)(1 - \tau_c)[(\mu Y + \phi(rD/p)] \] (1)

where \( s_w \) denotes the saving rate of workers, \( s_c \) the saving rate of capitalists and \( Y \) national income. Thus, both capitalists and workers save a part of their disposable income. Following Kaldor (1955), we assume \( s_w < s_c \) and hence, that saving out of profits by capitalists exceeds saving out of wages by workers. \( \tau_w \) is the income tax rate for income from wages and \( \tau_c \) the income tax rate for income from profits. \( \mu \) denotes the share of profits in net national income and \( 1 - \mu \) the share of wages in net national income. \( \phi \) denotes the relative share of government debt held by capitalists which we assume to be larger than \( (1 - \phi) \), the relative share of government debt held by workers.\(^3\) The real rate of interest is defined as \( r = i - \pi \) where \( i \) is the nominal interest rate and \( \pi \) the rate of inflation. \( D \) determines the nominal stock of government debt and \( p \) the price level. The profit share, the saving rates, the share of government bonds held by capitalists, the nominal interest rate, the price level and the inflation rate are exogenously given.

We further assume that \( i \) is fixed and hence, that the nominal interest rate may have reached a lower bound or is fixed at a certain level (Schlicht 2006).

The investment function is defined as

\[ I = [\alpha_0 + \alpha_1 u + \alpha_2 (1 - \tau_c) \mu]K \] (2)

where the rate of capacity utilization is defined as \( u = \frac{Y}{K} \), with \( K \) as the stock of capital, and \( \alpha_0, \alpha_1 \) and \( \alpha_2 \) being positive and constant parameters. Investment demand is determined by animal spirits of firms, represented by the parameter \( \alpha_0 \), as well as expectations of firms regarding the future profit share \( \mu \) and capacity utilization \( u \). In other words, when firms are optimistic, i.e. they expect a higher rate of capacity utilization or a higher profit share, investment demand increases. The strength with which optimistic expectations, changes in the rate of capacity utilization and changes in the profit share influence investment decisions depends on

\(^3\)Ederer et al. (2021) show for a selection of Euro Area economies that the absolute amount of government bond holdings by the top vigintile of the wealth distribution exceeds the amount held by the bottom vigintiles of the wealth distribution. A study by Arbogast (2020) reveals that the share of government debt held by the top 10% households of the Italian wealth distribution amounts to 49% for the years 1998 to 2016.
the parameters $\alpha_0, \alpha_1$ and $\alpha_2$ (Lavoie 2014).

Real government expenditure $G$ is given by

$$G = \beta K$$  \hspace{1cm} (3)

where $\beta$ is a constant parameter with $0 \leq \beta \leq 1$. Hence, government spending is modeled as a constant fraction of the capital stock (You and Dutt 1996).

Using $Y = C + I + G$ and substituting equations (1) - (3) we obtain the short-run equilibrium national income $(Y)$

$$Y = (1 - s_w)(1 - \tau_w)(1 - \mu)Y + (1 - \phi)(r D/p) + (1 - s_c)(1 - \tau_c)(\mu Y + \phi(r D/p)] + [\alpha_0 + \alpha_1 u + \alpha_2(1 - \tau_c)\mu]K + \beta K$$  \hspace{1cm} (4)

We proceed by dividing both sides of equation 4 by $K$ and solve for $u$

$$u(\sigma, \mu) = \gamma(\mu)[(1 - s_w)(1 - \tau_w)(1 - \phi)r \sigma + (1 - s_c)(1 - \tau_c)\phi r \sigma + \alpha_0 + \alpha_2(1 - \tau_c)\mu + \beta]  \hspace{1cm} (5)$$

where $\gamma(\mu) = (\tau_w + s_w - s_w \tau_w + \theta \mu - \alpha_1)^{-1}$, with $\theta = (1 - s_w)(1 - \tau_w) - (1 - s_c)(1 - \tau_c)$, is the output multiplier, $\sigma = \frac{D}{pK}$ and $\gamma, \theta > 0$.

In order to determine the impact of the real stock of government debt on the rate of capacity utilization, we take the partial derivative of $u(\sigma, \mu)$ with respect to $\sigma$.

$$\frac{\partial u}{\partial \sigma} = \gamma(\mu)[(1 - s_w)(1 - \tau_w)(1 - \phi)r + (1 - s_c)(1 - \tau_c)\phi r] > 0  \hspace{1cm} (6)$$

Equation 6 shows a positive impact of the stock of government debt $\sigma$ on the rate of capacity utilization provided that $r > 0$. This comes about through a rise in interest income of workers and capitalists and a resulting increase in consumption expenditures. However, in case that $r = 0$ and $i$ is positive, the interest income of workers and capitalists is compensated by the positive rate of inflation. By contrast, if $i$ has reached the zero lower bound, a zero real interest rate means that the inflation rate is also zero. In either case, workers derive their income solely from wages whereas capitalists derive their income solely from profits and there is no impact of $\sigma$ on the rate of capacity utilization through changes in the interest income of workers and capitalists.

On the other hand, in case that $r < 0$ the interest income is negative leading to a reduction in consumption demand and hence, amplified by the multiplier effect, a negative impact on the rate of capacity utilization. The reduction in the rate of capacity utilization has a negative impact on
the rate of capital accumulation and through the reduction in the disposable income of workers a negative impact on workers consumption demand.

The effect of changes in the saving rates on capacity utilization is obtained by taking the partial derivative with respect to $s_w$ and $s_c$ respectively.

\[
\frac{\partial u}{\partial s_w} = \gamma(\mu)[-(1 - \tau_w)((1 - \mu)u + (1 - \phi)(rD/p))] < 0 \quad (7)
\]

\[
\frac{\partial u}{\partial s_c} = \gamma(\mu)[-(1 - \tau_c)(\mu u + \phi(rD/p))] < 0 \quad (8)
\]

noticing that

\[
\frac{\partial (C/K)}{\partial s_w} = -(1 - \tau_w)((1 - \mu)u + (1 - \phi)(rD/p)) \quad (9)
\]

\[
\frac{\partial (C/K)}{\partial s_c} = -(1 - \tau_c)[\mu u + \phi(rD/p)] \quad (10)
\]

Equations 7 and 8 show that a rise in the propensity to save of workers and capitalists reduces capacity utilization through a reduction in their respective disposable incomes resulting in a decrease in consumption expenditures. As can be seen from equation 13 below, a reduction in capacity utilization has a negative impact on the rate of accumulation. Additionally, taking the partial derivative of the profit rate $\nu = (1 - \tau_c)\mu u$ with respect to $u$ we obtain

\[
\frac{\partial \nu}{\partial u} = \mu - \tau_c \mu > 0 \quad (11)
\]

and recognize that a decrease in the rate of capacity utilization has a negative impact on the rate of profit. This leads us to conclude that the paradox of saving, according to which increases in saving propensities lead to a reduction in consumption demand with negative effects for the rates of capacity utilization and capital accumulation (Hein 2014), holds in our model as well.

In order to evaluate the consequences of redistribution for the rate of capacity utilization, we take the partial derivative of $u$ with respect to the profit share $\mu$

\[
\frac{\partial u}{\partial \mu} = [\alpha_2(1 - \tau_c) - \theta u]\gamma(\mu) \quad (12)
\]

where $\frac{\partial (I/K)}{\partial \mu} = \alpha_2(1 - \tau_c)$ and $\frac{\partial (C/K)}{\partial \mu} = -\theta u$. From equation 12 we notice that a rise in the profit share $\mu$ has a positive effect on capacity utilization if its positive impact on investment exceeds its negative impact on consumption. In this case, demand is profit-led. The positive impact of the profit share increases with the weight it is given in the investment function by the parameter $\alpha_2$. However, it decreases with $\tau_c$ as increases in the income tax
rate lead to a reduction in the disposable income of capitalists dampening the positive impact of increases in the profit share on capacity utilization. On the contrary, if the negative impact on consumption exceeds the positive impact on investment demand, a rise in the profit share has a negative impact on the rate of capacity utilization and demand is wage-led. This effect is dampened by an increasing saving rate of workers $s_w$ or an increasing income tax rate $\tau_w$, however.

The rate of capital accumulation is defined as follows

$$g(\sigma, \mu) = \frac{I}{K} = \alpha_0 + \alpha_1 u(\sigma, \mu) + \alpha_2 (1 - \tau_c) \mu$$  \hspace{1cm} (13)

We take the partial derivative of 13 with respect to $\sigma$

$$\frac{\partial g}{\partial \sigma} = \alpha_1 \frac{\partial u}{\partial \sigma}(\mu) > 0$$  \hspace{1cm} (14)

and notice a positive relationship between the rate of capital accumulation and the debt-to-capital ratio. Hence, as shown by equation 6, provided that $r > 0$, a rise in the debt-to-capital ratio induces a rise in the rate of capacity utilization through an increase in the disposable income of workers and capitalists. This leads to a rise in investment demand and a resulting increase in the rate of accumulation $g$. However, when $r = 0$, this channel breaks down implying that a rise in $\sigma$ has no impact on the rate of accumulation through an increase in the disposable income of workers and capitalists. When $r < 0$, a rise in the debt-to-capital ratio has a negative impact on the rate of accumulation as the reduction in the disposable income of workers and capitalists leads to a decrease in the rate of capacity utilization resulting in a reduction in investment demand.

Taking the partial derivative of $g$ with respect to the profit share $\mu$ we obtain

$$\frac{\partial g}{\partial \mu} = \alpha_1 \frac{\partial u}{\partial \mu} + \alpha_2 (1 - \tau_c)$$  \hspace{1cm} (15)

noticing that in the case of profit-led demand a rise in the profit share has a positive impact on the rate of accumulation. Hence, growth is profit-led. By contrast, in the case of wage-led demand, a rise in the profit share has a negative impact on the rate of accumulation and growth is wage-led. Both effects are determined by the elasticity of accumulation with respect to changes in demand $\alpha_1$. In case that the second term of equation 15 is greater than the negative impact of $\mu$ on $u$ the higher elasticity of accumulation with respect to changes in the profit share $\alpha_2$ results in profit-led growth.

Income between workers and capitalists is distributed as follows
\[ \vartheta(\sigma, \mu) = \frac{(1 - \tau_c)(\mu u + \phi r \sigma)}{(1 - \tau_w)(1 - \mu)u + (1 - \phi)r \sigma} \] (16)

which can be reduced to

\[ \vartheta(\sigma, \mu) = \frac{\rho(\mu + \phi r \sigma / u)}{(1 - \mu) + (1 - \phi)r \sigma} \] (17)

where \( \rho = \frac{(1 - \tau_c)}{(1 - \tau_w)} \). Equation 17 reveals a positive relationship between changes in the interest income of capitalists as well as changes in profits and income inequality. By contrast, the relationship between the rate of capacity utilization and income inequality is negative. Additionally, changes in the interest income of workers are negatively related to changes in the distribution of income. Further, a rise in the rate of inflation \( \pi \) reduces the real interest rate leading to a reduction in interest income of workers and capitalists. The effect on income distribution depends on the relative shares of government debt held by workers and capitalists. As we assume that \( \phi > (1 - \phi) \), changes in the interest income of capitalists have a stronger impact on the distribution of income than changes in the interest income of workers. Hence, the negative impact of a decline in the interest income of capitalists is stronger than the positive impact of a decline in the interest income of workers resulting, overall, in a decreasing income inequality.\(^4\)

Taking the partial derivative of \( v \) with respect to \( \mu \) we obtain

\[ \frac{\partial v}{\partial \mu} = \frac{[\rho - \rho(\phi r \sigma / u^2)(\partial u / \partial \mu)]}{(1 - \mu) + (1 - \phi)r \sigma} \] (18)

and notice that in the case of profit-led demand, a rise in the profit share may lead to lower income inequality. However, this necessitates that the interest income of capitalists as a share of output is large. In the case of wage-led demand, a rise in the profit share exerts a negative impact on the rate of capacity utilization. This leads to higher income inequality.

Following You and Dutt (1996), we assume that the debt-to-capital ratio is fixed in the short-run. In the next section, we relax this assumption and analyze the impact of the long-run debt-to-capital ratio on the distribution of income.

### 4.2. The Model in the Long-run

In this section we focus our theoretical analysis on the long-run dynamics of the debt-to-capital ratio. The time path of the stock of government debt is defined as

\(^4\)We assume that the importance of capital income as a source of income increases when one moves from the bottom to the top of the personal income distribution. By contrast, labor income is a more important source of income for lower income classes. Hence, increases in the capital share are associated with a stronger concentration of income at the top of the personal income distribution (Bengtsson and Waldenström 2018).
\[
\frac{dD}{dt} = p(G - T) + rD
\]  

(19)

where \( T = \tau_w[(1 - \mu)Y + (1 - \phi)rD/p] + \tau_c(\mu Y + \phi rD/p) \) is the real income from income taxes on worker’s and capitalist’s income. The differentiation of \( \sigma \) with respect to time results in

\[
\frac{d\sigma}{dt} = \left[ \beta - \tau^* u(\sigma) - \tau_w(1 - \phi)r\sigma - \tau_c\phi r\sigma \right] + \left[ r - g(\sigma) \right]\sigma
\]

(20)

where \( \tau^* = \tau_w(1 - \mu) + \tau_c\mu \) defines the average income tax rate. Equation 20 shows that the dynamics of the debt-to-capital ratio are determined by the primary deficit and the difference between the real interest rate on government debt and the growth rate of the economy, \( r - g \).

If we assume that Bowley’s law holds, that is that the profit share is approximately constant in the long-run, \( \mu \) is fixed and \( u \) and \( g \) are solely determined by the debt-to-capital ratio \( \sigma \) (Krämer 2011; You and Dutt 1996).

In order to determine the impact of the real interest rate on the dynamics of the debt-to-capital ratio we have to differentiate between three cases. First, when \( r < g \) and \( r \) is negative the disposable income of workers and capitalists decreases as the inflation rate \( \pi \) exceeds the nominal interest rate \( i \) leading to a negative interest income. The decrease in the interest income of workers and capitalists reduces the tax income of the government which increases the debt-to-capital ratio. Hence, \( \tau_w(1 - \phi)r\sigma \) and \( \tau_c\phi r\sigma \) turn positive. However, a negative real interest rate \( r \) has also a negative impact on the debt-to-capital ratio as the absolute interest payments of the government decrease. This is supported by the negative impact of the rate of capital accumulation on the debt-to-capital ratio. However, even as \( \sigma \) reaches zero, the assumption, as mentioned below, that \( \beta - \tau^*u(0) > 0 \) implies that a primary deficit remains and the government maintains a minimum level of government spending resulting in a positive long-run equilibrium level of the debt-to-capital ratio. Second, when \( r > 0 \) but \( r < g \) workers and capitalists interest income is positive resulting in tax income for the government. This has a negative impact on the debt-to-capital ratio. The positive interest payments on government debt exert a positive impact on \( \frac{du}{dt} \) which is more than compensated by the negative impact of the rate of capital accumulation, however. Third, when \( r = 0 \) the real interest rate has no impact on the debt-to-capital ratio implying that the government receives no income from taxation on interest income of workers and capitalists.

Next, we follow You and Dutt (1996) and define

\[
u(\sigma) = \gamma[\alpha_0 + \alpha_2(1 - \tau_c)\mu + \beta] + \sigma \frac{\partial u}{\partial \sigma} = u(0) + \sigma \frac{\partial u}{\partial \sigma} \quad (21)\]

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\[ g(\sigma) = \left[ \alpha_0 + \alpha_1 u(0) + \alpha_2(1 - \tau_c)\mu \right] + \alpha_1 \frac{\partial u}{\partial \sigma} = g(0) + \alpha_1 \sigma \frac{\partial u}{\partial \sigma}. \]  

(22)

Substituting equation 21 and equation 22 into equation 20 and rearranging, we obtain the equation for the development of the debt-to-capital ratio over time

\[ \frac{d\sigma}{dt} = a_1 \sigma^2 + a_2 \sigma + a_3 \]  

(23)

where \( a_1 = -\alpha_1 \frac{\partial u}{\partial \sigma}, a_2 = [(1 - \tau_c \phi - \tau_w(1 - \phi)]r - g(0) - \frac{\partial u}{\partial \sigma} \tau^* \) and \( a_3 = \beta - \tau^* u(0) \).

Using \( \sigma_{1,2} = \frac{-a_2 \pm \sqrt{a_2^2 - 4a_1a_3}}{2a_1} \) we calculate the long-run equilibrium obtaining two solutions. As \( a_1 < 0 \), the positive and stable long-run equilibrium is

\[ \sigma^* = \frac{-a_2 - \sqrt{a_2^2 - 4a_1a_3}}{2a_1} \]  

(24)

The stability of the equilibrium is determined by \( a_2^2 - 4a_1a_3 > 0 \). \( ^5 \) We follow You and Dutt (1996) and assume that \( a_3 = \beta - \tau^* u(0) > 0 \). Hence, when the debt-to-capital ratio \( \sigma \) is zero, a primary deficit remains implying that the government sustains a minimum level of government spending and the equilibrium \( \sigma^* \) would be positive.

From the stability condition of the long-run equilibrium \( \sigma^* \)

\[ \frac{\partial^2 (d\sigma/dt)}{\partial \sigma^2} = 2a_1 = 2(-\alpha_1 \frac{\partial u}{\partial \sigma}) < 0 \]  

(25)

it follows that two effects determine the stability condition. First, the positive impact of the stock of government debt on the rate of capacity utilization. Second, the product of the multiplier and the accelerator. However, in order for this to hold, the capacity constraint must not be binding (ibid.).

Substituting \( \sigma^* \) into equation 22 we obtain the rate of accumulation in the long-run equilibrium

\[ g^* = g(\sigma^*) = (1 + \alpha_1 \gamma)\left[ \alpha_0 + \alpha_2(1 - \tau_c)\mu \right] + \alpha_2 \gamma \beta + \alpha_1 \sigma^* \frac{\partial u}{\partial \sigma}. \]  

(26)

The long-run income inequality ratio is obtained by substituting \( \sigma^* \) into equation 17

\[ \vartheta^* = \vartheta(\sigma^*) = \rho \frac{[\mu + \phi r \sigma^* / u(\sigma^*)]}{(1 - \mu) + (1 - \phi) r \sigma}. \]  

(27)

where \( u^* = u(\sigma^*) = \gamma [\alpha_0 + \alpha_2(1 - \tau_c)\mu + \beta] + \sigma^* \frac{\partial u}{\partial \sigma}. \)

\( ^5 \)For a discussion of the stationarity conditions see You and Dutt (1996, p. 341).
Now we calculate the impact of the debt-to-capital ratio on income inequality by taking the partial derivative of $\vartheta$ with respect to $\sigma$.

\[
\frac{\partial \vartheta}{\partial \sigma} = \frac{\rho r \phi (u - \sigma \frac{\partial u}{\partial \sigma})}{((1 - \mu) + (1 - \phi) r) u^2} = \frac{\rho r \phi u(0)}{((1 - \mu) + (1 - \phi) r) u^2} > 0.
\] (28)

Equation 28 reveals a positive impact of the debt-to-capital ratio $\sigma$ on the distribution of income $\vartheta$. Hence, a rise in $\sigma$ increases the interest income of capitalists which leads to a rise in income inequality as long as $r > 0$. However, the rate of capacity utilization increases as well which has a negative impact on $\vartheta$. Under the condition that $r > 0$, equation 28 is positive. Hence, the increase in the interest income of capitalists is bigger than the rise in the rate of capacity utilization and the increase in the interest income of workers leading to an increase in income inequality. When $r = 0$, workers and capitalists do not have any interest income from government debt and the distribution of income is solely determined by the profit rate, the rate of capacity utilization and the relative tax rates. By contrast, a real interest rate $r < 0$ implies that the rate of inflation exceeds the nominal interest rate resulting in negative interest income for workers and capitalists and consequently, in a reduction in consumption demand. This has a negative impact on the rate of capacity utilization. The consequences for income distribution depend on the relative magnitudes of the mentioned effects.

5. Case Studies

In this section, we analyze the dynamics between changes in the debt-to-capital ratio, income inequality and growth in three different cases.

5.1. A Decrease in Private Investment

We start by analyzing the implications of a decrease in the autonomous investment component for the dynamics between income distribution, government debt and growth. That is, we analyze the implications of a change in the autonomous part of the investment function which does not depend on capacity utilization or past realized profits. We consider a change in the autonomous investment component $\alpha_0$ to reflect a change in capitalists expectations regarding future sales prospects leading to fluctuations in aggregate demand (Lavoie 2014, p. 279).

In order to analyze the consequences of pessimistic expectations of firms associated with a decline in the autonomous investment component $\alpha_0$ in our model, we totally differentiate equation 23 and rearrange equation 24 to $(2a_1\sigma^* + a_2) = -\sqrt{a_2^2 - 4a_1a_3}$ to obtain

\[
d\sigma^* = \frac{[(da_1)\sigma^* \sqrt{a_2^2 - 4a_1a_3} + (da_2)\sigma^* + (da_3)]}{a_2^2 - 4a_1a_3}
\] (29)

Using $\partial a_1/\partial \alpha_0 = 0$, $\partial a_2/\partial \alpha_0 = -(1 + \alpha_1 \gamma) < 0$, and $\partial a_3/\partial \alpha_0 = -\tau^* \gamma < 0$, we notice
that a fall in the autonomous investment component $\alpha_0$ leads to a rise in the long-run debt-to-capital ratio $\sigma^*$. Considering that the fall in autonomous investment has a negative impact on the rate of capacity utilization leading to a decline in the wage income and using

$$d\vartheta^* = \rho \varrho r [u(0)(d\sigma^*) + \gamma \sigma^*(-d\alpha_0)] > 0$$

we recognize that the rise in the long-run debt-to-capital ratio $\sigma^*$ increases the interest income of workers and capitalists provided that $r > 0$. However, as income from wages declines income inequality increases. As can be seen in the next section, evidence points to a questioning of this line of reasoning, as it illustrates a decline in the real rate of interest over the past decades (Rachel and Summers 2019; Summers 2014; Weizsäcker and Krämer 2021). Totally differentiating equation 22, we obtain

$$dg^* = -(1 + \alpha_1 \gamma)(-d\alpha_0) + \alpha_1 \frac{\partial u}{\partial \sigma}(d\sigma^*)$$

which shows us the impact of changes in the debt-to-capital ratio as well as autonomous investment on the long-run rate of accumulation. Bearing in mind that $\alpha_1 \frac{\partial u}{\partial \sigma}$ is small, we see that for the long-run equilibrium rate of accumulation to increase, the rise in the debt-to-capital ratio $\sigma^*$ must be large in order to offset the negative impact of the decrease in autonomous investment (You and Dutt 1996).

We can summarize the implications of a negative change in autonomous investment as follows: A decline in the autonomous investment component induces a decline in the rate of capacity utilization as well as the rate of capital accumulation and a rise in the long-run debt-to-capital ratio which leads to an increase in income inequality as long as $r$ is positive and, compared to workers, capitalists hold a larger share of government bonds. Under the condition that the rise in the long-run debt-to-capital ratio is large, the long-run rate of accumulation increases.

5.2. A Fall in the Real Interest Rate

We now use our theoretical model to continue with an analysis of the consequences of a fall in the real rate of interest for income distribution and growth. In the short-run, as can be seen from equation 1, a fall in the real interest rate leads to a decline in the disposable income of workers and capitalists through a decline in interest income from government debt. From equation 5, we see that this leads to a decline in the rate of capacity utilization through a decrease in consumption expenditures. This decline in the rate of capacity utilization induces a decline in investment activities and a decrease in the rate of accumulation. Additionally, as can be seen from equation 11, the rate of profit decreases. In the long-run, the reduction in consumption demand and the decline in investment activities reduce the rate of capacity utilization which induces a rise in
leading to an increase in the long-run debt-to-capital ratio $\sigma^*$. This offsets the negative effect on output and the rate of accumulation leading to an increase in $u^*$ and $g^*$. Provided that $r < g$, from equation 20, we see that the debt-to-capital $\sigma$ decreases over time. The long-run inequality ratio $u^*$ decreases. This comes about through two effects. First, the decline in the real interest rate reduces the interest incomes of both workers and capitalists. However, as the decline in the interest income of capitalists exceeds the decline in the interest income of workers, income inequality decreases. Second, the long-run increase in the debt-to-capital ratio has a positive impact on the rate of capacity utilization. The resulting rise in income of workers has a negative impact on income inequality.

To sum up, a fall in the real interest rate leads to a decline in the interest incomes of workers and capitalists. As a consequence, consumption expenditures decrease, which has a negative impact on output and consequently, a negative impact on investment activities. However, this is offset by the increase in the long-run debt-to-capital ratio which increases the rate of capacity utilization with positive effects on income from wages and negative effects for income inequality.

### 5.3. A Rise in Government Spending

A fiscal expansion by the government through a rise in government spending increases $\beta$. We assume that this rise is caused by an increase in the long-run debt-to-capital ratio $\sigma^*$ leading to an increase in the rate of capacity utilization and through its positive impact on investment, to a rise in the long-run rate of accumulation $g^*$.

Substituting equation 6 into equation 26 and totally differentiating, we obtain

$$dg^* = \alpha_1\gamma[d\beta + [(1 - s_w)(1 - \tau_w)(1 - \phi)r + (1 - s_c)(1 - \tau_c)\phi r](d\sigma^*)]$$

Equation 32 reveals two effects determining the change in the long-run equilibrium rate of accumulation. First, changes in government spending $\beta$. Second, changes in the interest income of workers and capitalists and resulting changes in consumption demand. Consequently, a rise in government spending has a positive impact on the long-run rate of accumulation. However, the impact of consumption demand by workers and capitalists depends on the real rate of interest. At a positive real interest rate $r$ the rise in government debt increases the interest income of workers and capitalists having a positive impact on $g^*$. When $r = 0$ consumption demand has no impact on the long-run rate of accumulation. In case that $r < 0$, workers and capitalists reduce consumption, which has a negative effect on the long-run rate of accumulation through a reduction in the rate of capacity utilization and a resulting decrease in investment demand. In this case, the rise in government spending must be large enough to offset the negative impact of a decline in consumption demand on the rate of capital accumulation.
We proceed by analyzing the impact of a rise in the long-run debt-to-capital ratio on the long-run distribution of income. Differentiating $u(\sigma^*)$ with respect to $\beta$ we obtain

$$\frac{du(\sigma^*)}{d\beta} = \gamma + \frac{\partial u (d\sigma^*)}{\partial \sigma (d\beta)}$$

(T33)

Totally differentiating equation 27 and substituting equation 21 we the following equation

$$dv^* = \rho \phi r [u(0)(d\sigma^*) - \gamma \sigma^*(d\beta)] \left[ (1 - \mu) + (1 - \phi) \rho \sigma^* r (d\sigma^*) \right] u^2$$

(T34)

which shows us that a rising $\beta$ decreases income inequality. This comes about through the increase in $u(\sigma^*)$ caused by the rise in $\beta$ and a resulting increase in income from wages through the increase in the rate of capacity utilization. On the contrary, a rising long-run debt-to-capital ratio leads to an increase in income inequality. However, this holds as long as $r > 0$. In case that $r < 0$, a rising $\beta$ increases income inequality. By contrast, a rise in the long-run debt-to-capital ratio reduces income inequality as both, the interest income of workers and capitalists decreases with the latter effect having a stronger impact. However, the rise in the long-run rate of capacity utilization, due to an increase in $\beta$, has a positive as well as a negative effect on income inequality. Whether the long-run distribution of income becomes more equal or unequal depends on the relative strength of the effects.

In short, an increase in government spending financed by a rise in the long-run debt-to-capital ratio has a positive impact on the long-run rate of capacity utilization. The impact on the long-run rate of accumulation depends on the real rate of interest, however. At a positive real interest rate, the long-run rate of accumulation increases. By contrast, a negative real interest rate reduces the long-run rate of accumulation through a reduction in consumption demand. This can be offset by an extensive rise in government spending, however. The consequences for income inequality depend on the real rate of interest as well. As long as $r > 0$ holds, income inequality decreases. At a negative real interest rate, the distribution of income is determined by the relative strength of the negative impact of a decrease in interest income of workers and capitalists and the positive impact of a long-run rise in the rate of capacity utilization.

6. Conclusions

In the present article, we analyze a possible nexus between the recent resurge in income inequality and the rise in government debt-to-GDP ratios in major advanced economies. Further, we shed light on the implications of a declining investment activity, a fall in real interest rates and a rise in government spending financed by an increase in government debt for the possible nexus between income inequality and rising government debt-to-GDP ratios.

Specifically, we find that a decrease in investment activity has a negative impact on the rate of
capacity utilization as well as the rate of capital accumulation. This induces a rise in the long-run debt-to-capital ratio. At a positive real interest rate, income inequality increases as, according to our assumption, compared to workers, capitalists hold a larger share of government bonds. Provided that the increase in the long-run debt-to-capital ratio is large, the rate of accumulation increases.

Further, a fall in the real rate of interest has a negative impact on the rate of capacity utilization through a decline in the interest incomes of workers and capitalists and a resulting decrease in consumption expenditures. This leads to a decrease in the rate of capital accumulation. The decline in the rate of capacity utilization is offset by an increase in the long-run debt-to-capital ratio which has a negative impact on income inequality.

Additionally, an increase in government spending which comes about through an increase in the long-run debt-to-capital ratio has a positive impact on the long-run rate of capacity utilization. The long-run rate of accumulation increases at a positive real interest rate. However, at a negative real rate of interest, the long-run rate of accumulation decreases. This decrease is offset by an extensive increase in government spending, however. The consequences for income distribution depend on the strength of the mentioned effects.

The research reported here indicates that the question of the distributional effects of public debt requires a framework broad enough to include factors such as growth and interest dynamics. Besides methodological issues, an analysis based solely on a comparison of interest incomes from government bonds and tax rates would provide only a limited picture of the distributional effects of government debt. Thus, we offer a framework to account for the complexities linking public debt and income distribution.

Our findings provide evidence for the intermediating role of the real interest rate for the nexus between government debt and income distribution. At negative real interest rates, increases in public debt have, ceteris paribus, a positive impact on income inequality. At low real interest rates, raising the level of government spending through an increase in public debt has only slight redistributional effects. However, this alters when real interest rates increase and the rise in the absolute interest income of capitalists is higher than the rise in the interest income of workers which results in higher income inequality.

However, we do not claim generality of our results. Beyond the present analysis, additional impact factors such as inflation dynamics have important implications for the nexus between government debt and income distribution. This points to the necessity of additional research to gain insights into the mediating role of these factors.

Additionally, the model we introduced in this article provides a theoretical framework for the analysis of the distributional effects of government debt without giving insights into the magnitudes of the mentioned effects. This requires a quantification of the theoretical model.
which is left to future research, however.

References


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## Appendix

### A. Country List

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