The Taylor rule as a stabilizing device in Foley's model of supply-side liquidity/profit-rate

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Abstract

In the original Keynes-Metzler-Goodwin (KGM) platform, the money supply features as an exogenous variable. It is unsettling insofar as there is considerable empirical evidence that most Central Banks pay little attention to monetary aggregates when conducting monetary policy. To cope with this reality, Chiarella et al. (2005) assumed that a Central Bank sets the interest rate following a Taylor rule and then adjusts the money supply, thus providing a more fully-fledged view of the monetary side of the economy. Following this rationale, the present paper aims at updating Foley's (1987) model by introducing the Taylor rule to determine the interest rate to replace the money supply as an exogenous variable. An advantage of our approach has to do with parsimony. While the KGMT consists of a 6-dimension dynamic system, the Taylor-Foley alternative yields a 4-dimension system, providing a stabilising mechanism for liquidity-profit-rate cycles.

Keywords: Liquidity cycles, interest rate, Taylor rule.

JEL classification: E32, E64.

1. Introduction

Since Goodwin (1967), the study of distributive cycles in capitalist economies continues to be a constant presence in the post-Keynesian research agenda [see, e.g., Barbosa-Filho and Taylor (2006), Keen (2013), Skott (1989), and Tavani and Zamparelli (2017)]. His growth-cycle model blends aspects of the Harrod-Domar growth set-up with a Phillips curve. Profits squeezed out of the workforce's income are invested, thus determining the pace of capital accumulation. Such formulation is a pioneer theory of economic fluctuations whereby the economic variables interact with each other in a cyclical and endogenous way.

Peter Flaschel engaged in a major research program on (disequilibrium) macroeconomic analysis that resulted in a massive series of papers and books with several other contributors [see, e.g., Chiarella and Flaschel (2000), Chiarella et al. (2005), Flaschel (2009), Flaschel and Krolzig (2006), and Franke et al. (2006)]. They extended Goodwin's distributive cycle model in several directions, introducing both effective demand and monetary aspects. Building on the Keynesian IS-LM framework, they delivered a baseline platform known as the Keynes-Metzler-Goodwin (KGM) system. Within this framework, the IS curve was disregarded as a temporary equilibrium relationship with the disequilibrium in the goods market buffed by the correspondingly induced changes in inventories. However, the LM curve was maintained unaltered, and the money supply features as an exogenous variable.

Foley's (1987, p. 364) model of liquidity-profit rate also belongs to "the tradition of nonlinear business cycle models studied by Goodwin (1982) and Hicks (1950)". He emphasised money as a link among enterprises, with the economy's growth adjusting to the growth rate of money through changes in the rate of capital outlays [see Foley (1987, p. 372)]. Such a framework focuses on the dynamics of a capitalist economy populated by profit-seeking firms and shares some features of the KGM system. That is perhaps the reason why, in time, it cached Flaschel's attention [see Araujo et al. (2020) and Moreira et al. (2021)]. Foley's contribution has renewed the interest in studying how financial actors and institutions might generate economic fluctuations [see the literature review in Nikolaidi and Stockhammer (2017)].

In the first attempts to extend the Foley model, Araujo et al. (2020) and Moreira et al. (2021) introduced income distribution via a money-wage Phillips curve with a perfect spill-over of price inflation on wage inflation to the model. Arguably, by ignoring the dynamics of wage and price, the original framework overlooks an important dimension of the monetary side, namely the connection between prices and output, and in this vein, the dynamics of supply-side liquidity cannot be taken into full account in the original model. In this extended framework, the equilibrium is shown to be unstable, meaning that the integration of the liquidity profit rate cycle of Foley with the Goodwin employment rate profitability requires a monetary rule.

The rule adopted by Araujo et al. (2020) consisted in adding to the exogenous money supply growth an additional term that compares the steady-state profit rate r_0 with the actual one, namely r . If $r > r_0$ then the monetary authority increases the money supply to avoid a liquidity shortage. In the symmetrical case, namely $r < r_0$, the monetary authority decreases the money supply to avoid a liquidity surplus. This simple rule was able to stabilize the model, but the money supply is a variable directly controlled by the monetary authority. The point is that there is considerable empirical evidence that most Central Banks pay little attention to monetary aggregates when conducting monetary policy.

Therefore, the Foley and KMG models cannot be taken as appropriate descriptions of modern economies. Still, at least for the KMG framework, the necessary adjustments were accomplished by Chiarella et al. (2005). They showed how the Central Bank sets the interest rate following a Taylor rule and then adjusts the money supply to determine investment and aggregate demand. The resulting system received the label Keynes-Metzler-Goodwin-Taylor (KMGT).

The present paper aims at updating Foley's model along similar lines by considering the Taylor rule to determine the interest rate in an endogenous way. Hence, in the present paper, interest rates replace the money supply as a dynamic variable, offering a more fully-fledged view of the monetary side of the economy. We assume the Taylor rule is conveyed by an interest rate reaction function that depends on deviations of current inflation from its target and the output gap as captured by the capacity utilisation rate. Our proposal is justified by the importance of the Foley model for the disequilibrium literature as one of the seminal contributions to formalise initial insights into Minsky's reasoning. Another advantage of our approach over the KGMT has to do with parsimony. While the latter consists of a 6-dimension dynamic system, the Taylor-Foley alternative yields a 4-dimension system[1].

Hence, while in the KMGT the stability analysis can only be performed using numerical methods our 4D extended Foley's model allows an analytical approach using the Routh-Horwitz criteria[2]. We also study the existence of limit cycles in this model. Our findings confirm the existence of a stable equilibrium that cannot be destabilized into a limit cycle, which can be seen as evidence that Taylor's rule provides a stabilising mechanism for liquidity-profit-rate cycles. These findings are somewhat following what was reported related to the KGMT: "What was important for us was not so much the

[1] This reduction follows from our focus on profit-seeking firms, leaving aside the role of households. Furthermore, inventory plays no role in our formalisation, improving tractability.

[2] As pointed out by Chiarella et al. (2005, p. 427), "a stability condition (…) which we were previously able to prove analytically and which we feel should similarly hold true in KMGT, can now be addressed only in numerical ways."

existence of periodic orbits, because the circumstances under which they emerge can be rather special (…)" [Chiarella et al. (2005, p. 444)]. This paper is organized as follows: in the next section, we advance the model, in section 3, we analyse the equilibrium and its stability, examining the existence of cycles. Section 4 concludes.

2. The Taylor Rule in the extended Foley Model

In Foley's (1987) model money supply grows at an exogenous constant rate and is transferred as a subsidy to firms. Araujo et al. (2020) studied how the dynamics of Foley's (1987) model may be affected by the introduction of a money-wage Phillips curve with a perfect spill-over of price inflation on wage inflation. The upshot is a model with endogenous price and wage dynamics with unstable equilibrium, meaning that the integration of the Foley liquidity/profit rate cycle with the Goodwin cycle requires state intervention to stabilize the economy. With this approach, they have shown that Foley's initial insight is consistent with flexible prices and wages only in the presence of a rule for money supply. Hence, after a money supply rule for the economy is introduced, the system becomes stable.

But within such a setup, the central bank is assumed to pursue a very simple monetary supply rule for the stabilization of the real-financial market. According to that rule, the money supply grows at a target rate μ if the interest rate is equal to the steady state interest rate r_0, namely, $r = r_0$. The case in which $r > r_0$ would require that the monetary authority increase the money supply to avoid a liquidity shortage. In the symmetrical case, namely $r < r_0$, the monetary authority decreases the money supply to avoid a liquidity surplus. Although of interest, such a rule contrasts with the empirical evidence that most Central Banks pay little attention to monetary aggregates when conducting monetary policy in modern economies. Chiarella et al. (2005) for instance, assume that Central Bank sets the interest rate following a Taylor rule and then adjusts the money supply. Hence, in what follows we aim at updating Foley's model along similar lines by replacing the money supply rule adopted by Araujo et al. (2020) with a Taylor rule to determine the interest rate. Following this approach, we offer a more fully-fledged view of the monetary side of the economy.

A key variable considered in the model is firms' debt, denoted by D, which equals M, the monetary assets of the firms, plus its holdings of other enterprise debt, denoted by F, that is $D = M + F$. Foley assumes that the variation of debt per unit of capital, namely $\frac{\dot{D}}{K}$, is a function of the difference between profit and interest rate, namely $r - i$. He also assumes that the difference between the profit and interest ratio depends only on the ratio $m = \frac{M}{K}$, namely $r - i = p(m)$. Moreira et al. (2021) have introduced a slight change considering that the interest rate is a function of the amount of money per unit of capital according to:

$$\frac{\dot{D}}{K} = B[r - i(m)] \quad \text{with } B'[.] > 0, i'(.) < 0 \tag{1}$$

In this case, $B[p(r,m)] = B[r - i(m)]$. But here, following Chiarella et al. (2005), we introduce another key innovation assuming that the nominal interest rate is not determined by m, but is given by a Taylor rule according to:

$$i(u,\pi) = i_o + (\pi - \pi^0) + \alpha_\pi(\pi - \pi^0) + \alpha_u(u - 1) \tag{2}$$

where u is the rate of capacity utilization, π is the inflation rate, π^0 is the inflation target rate, i_o is the steady state nominal interest rate, and $\alpha_\pi > 0$ and $\alpha_u > 0$ stands for policy coefficients that measure the responsiveness of the interest rate to the deviations of inflation and utilization, respectively. The last term on the right-hand side of equation (2) is a proxy for the output gap. With this approach, we reverse the direction of causality[3] insofar as now the

[3] For Foley (1987, p. 365) money supply grows at the constant rate and is transferred as a subsidy to firms. Besides, "the higher is m, the lower will be the interest rate relative to the profit rate, since

central bank sets the level of i and then adjusts M. This change follows straight from the approach adopted by Chiarella et al. (2005, p. 373) to extend the KGM approach to consider the Taylor rule: "In the traditional mode, M (or $m = M/pK$, for that matter) is one of the state variables since it is needed to (co)determine the interest rate, which in turn determines investment and so aggregate demand". With this change, and considering real balances, money, and loans per unit of capital, namely $d = \frac{D}{K} = \frac{M}{K} + \frac{F}{K} = m + f$, we can rewrite equation (1) to obtain the first differential equation of our system, namely:

$$d' = B[r - i(u, \pi)] - rd \quad \text{with } B'[.] > 0 \tag{3}$$

In the original model, Foley (1987) does not make considerations about income distribution. To tackle this issue, Araujo et al. (2020) made the classical assumption that all wages are consumed, and all profits are invested. The output, Y, is by assumption either consumed or invested in this Marxian supply-side model. Denoting the wage share by v, we can write the profit rate as:

$$r = \frac{(1-v)Y}{K} \tag{4}$$

where Y is effective output and K is the stock of capital. Besides, they introduced the dynamics of income distribution through a money wage Phillips curve with a perfect spill-over of price inflation on wage inflation. Here, while we keep the Phillips curve, we will adopt an alternative specification which focuses on the rate of capacity utilization and not on income distribution given by:

$$\pi' = \beta_\pi(u - 1) \tag{5}$$

As pointed out by Chiarella et al. (2005, p. 384), equation (5) "assumes an accelerationist relationship, which is to say that the rate of inflation is

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with a higher m the enterprises are liquid and would have a high supply of loanable funds." Hence, the causality runs from m to i. whereas here we assume that it runs from i to m.

given at any point in time and shifts up (down) when utilization is above (below) normal". As we assume here that income distribution is given and exogenous in this model, we obtain from eq. (4) after some algebraic manipulation:

$$\hat{r} = \hat{Y} - \hat{K} \tag{6}$$

As in Foley (1987), we assume that the output increases following an aggregate output-expansion function:

$$\frac{\dot{Y}}{Y} = A(r, d) \text{ with } A_d > 0, A_r > 0 \tag{7}$$

Hence, from expressions (6) and (7), and since $K' = rK$, we obtain the third differential equation of the model, namely:

$$r' = r[A(r, d) - r] \tag{8}$$

The possibility of under-capacity utilization of the capital stock is another novelty of our modelling concerning Foley (1987) and Araujo et al. (2019). Following Chiarella et al. (2005, p. 389), we assume that capacity utilization varies over time as an exclusive function of the real interest rate, which is given by a dynamic version of the IS relation according to:

$$u' = \phi_u[i(u, \pi) - \pi] \text{ with } \phi'_u < 0 \tag{9}$$

Eq. (9) conveys what was reported by Chiarella et al. (2005, p. 389): "In almost all models of the Keynes–Phillips–Taylor approach, the output gap is determined by a reduced-form equation of what is viewed as a dynamic IS relationship". Consider now the system formed by equations (3), (5), (8) and (9). This is a non-linear dynamical system in four variables: (d, r, u, π) that we study in the next section.

3. The equilibria: existence, and local stability.

To find closed-form solutions for the dynamical system formed by equations (3), (5), (8) and (9), we assume the following functional forms for the required functions:

$$B(r-i) = b_1(r-i) \tag{10}$$

$$A(r,d) = a_1 r + a_2 d \tag{11}$$

$$\phi_u(i-\pi) = -\phi_1(i-\pi) \tag{12}$$

The choice of linear specifications rests on the fact that we aim at focusing on the non-linearities that accrue from the interactions of the endogenous variables. Hence, considering the Taylor rule, given by eq. (2), the system formed by equations (3), (5), (8) and (9) in a steady state gives rise to the following non-zero equilibrium point $P^*(d^*, r^*, u^*, \pi^*)$, where:

$$u^* = 1 \tag{13}$$

$$\pi^* = \frac{\pi^0(\alpha_\pi + 1) - i_0}{\alpha_\pi} \tag{14}$$

$$d^* = \frac{a_2 b_1 \phi_1 \alpha_\pi + \sqrt{\Delta}}{2\phi_1 \alpha_\pi a_2} \tag{15}$$

$$r^* = \frac{a_2 b_1 \phi_1 \alpha_\pi + \sqrt{\Delta}}{2\phi_1 \alpha_\pi (1 - a_1)} \tag{16}$$

where $\Delta = -4(1-a_1)\phi_1^2 a_2 \alpha_\pi b_1[\pi^o(1+\alpha_\pi) - i_o] + a_2^2 b_1^2 \phi_1^2 \alpha_\pi^2$. To avoid a complex solution without economic meaning we need that $\Delta \geq 0$. If $a_1 < 1$ and $\alpha_\pi < \frac{i_o - \pi^0}{\pi^0}$ we guarantee not only that $\Delta > 0$ but also that $r^* > 0$.

There exists an alternative solution in which $r^* = \frac{1}{2\phi_1 \alpha_\pi (a_1 - 1)}(-a_2 b_1 \phi_1 \alpha_\pi + \sqrt{\Delta}) > 0$, where: $\Delta = a_2^2 b_1^2 \phi_1^2 \alpha_\pi^2 + 4a_1 a_2 b_1 \pi^o \phi_1^2 \alpha_\pi(\alpha_\pi + 1) - 4a_1 a_2 b_1 i_o \phi_1^2 \alpha_\pi + 4a_2 b_1 \pi^o \phi_1^2 \alpha_\pi^2 - 4a_2 b_1 \pi^0 \phi_1^2 \alpha_\pi + 4a_2 b_1 i_0 \phi_1^2 \alpha_\pi > 0$, but the conditions to guarantee non-negativity of $r^*$and d^* are harder to establish. The Jacobian matrix due to the linearization of (3), (5), (8) and (9) around the equilibrium $P^*(d^*, r^*, u^*, \pi^*) \epsilon R_+^4$ is given by: $J_{P^*(d^*,r^*,u^*,\pi^*)} = (f_{ij})(P^*) = \frac{\partial f_i}{\partial x_j}(P^*)$, where:

$f_{11}(P^*) = -r^*, f_{12}(P^*) = b_1 - d^* = b_1 - \left(\frac{r^*(1-a_1)}{a_2}\right),\ f_{13}(P^*) = -b_1\alpha_u,$

$f_{14}(P^*) = -b_1(1+\alpha_\pi);\ f_{21}(P^*) = a_2 r^*, f_{22}(P^*) = (a_1 - 1)r^*, f_{23}(P^*) = 0, f_{24}(P^*) = 0; f_{31}(P^*) = 0, f_{32}(P^*) = 0, f_{33}(P^*) = -\phi_1\alpha_u,\ f_{34}(P^*) = -\phi_1\alpha_\pi;$ $f_{41}(P^*) = 0, f_{42}(P^*) = 0, f_{43}(P^*) = \beta_\pi, f_{44}(P^*) = 0.$

In this case, the characteristic equation of the Jacobian matrix can be written as follows: $\lambda^4 + b_1\lambda^3 + b_2\lambda^2 + b_3\lambda + b_4 = 0$, where: $b_1 = -\text{trace}\, J(P^*) = -(f_{11} + f_{22} + f_{33} + f_{44})$,

$b_2 =$ sum of the second-order minor principals of $J(P^*)$

$$= \begin{vmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{vmatrix} + \begin{vmatrix} f_{11} & f_{13} \\ f_{31} & f_{33} \end{vmatrix} + \begin{vmatrix} f_{11} & f_{14} \\ f_{41} & f_{44} \end{vmatrix} + \begin{vmatrix} f_{22} & f_{23} \\ f_{32} & f_{33} \end{vmatrix} + \begin{vmatrix} f_{22} & f_{24} \\ f_{42} & f_{44} \end{vmatrix} + \begin{vmatrix} f_{33} & f_{34} \\ f_{43} & f_{44} \end{vmatrix},$$

$b_3 = -$ sum of the third-order minor principals of $J(P^*)$

$$= -\left(\begin{vmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{vmatrix} + \begin{vmatrix} f_{11} & f_{12} & f_{14} \\ f_{21} & f_{22} & f_{24} \\ f_{41} & f_{42} & f_{44} \end{vmatrix} + \begin{vmatrix} f_{11} & f_{13} & f_{14} \\ f_{31} & f_{33} & f_{34} \\ f_{41} & f_{43} & f_{44} \end{vmatrix} + \begin{vmatrix} f_{22} & f_{23} & f_{24} \\ f_{32} & f_{33} & f_{34} \\ f_{42} & f_{43} & f_{44} \end{vmatrix}\right),$$

$b_4 = det\, J(P^*).$

Lemma 1. The sums of minors of order one (trace), two, three and four (determinant) of the Jacobian matrix are respectively given by

$$b_1 = \left(\frac{a_2 b_1 \phi_1 \alpha_\pi + \sqrt{\Delta}}{2\phi_1\alpha_\pi}\right)\left(\frac{2-a_1}{1-a_1}\right) + \phi_1\alpha_u$$

$$b_2 = \frac{(a_2 b_1 \phi_1 \alpha_\pi + \sqrt{\Delta})}{2\phi_1\alpha_\pi(1-a_1)}\left[\frac{\sqrt{\Delta}}{\phi_1\alpha_\pi} + \phi_1\alpha_u + (1-a_1)\phi_1\alpha_u\right] + \phi_1\alpha_\pi\beta_\pi;$$

$$b_3 = \frac{(a_2 b_1 \phi_1 \alpha_\pi + \sqrt{\Delta})}{2\phi_1\alpha_\pi(1-a_1)}\left[\sqrt{\Delta}\frac{\alpha_u}{\alpha_\pi} + \phi_1\alpha_\pi\beta_\pi + (1-a_1)\phi_1\alpha_\pi\beta_\pi\right]$$

$$b_4 = \frac{\beta_\pi\sqrt{\Delta}(a_2 b_1 \phi_1 \alpha_\pi + \sqrt{\Delta})}{2\phi_1\alpha_\pi(1-a_1)}$$

Proof. From an inspection of the Jacobian matrix and using the Routh-Hurwitz criteria, one obtains:

$$b_1 = r^* + (1 - a_1)r^* + \phi_1 \alpha_u;$$

$$b_2 = r^* \frac{\sqrt{\Delta}}{\phi_1 \alpha_\pi} + r^* \phi_1 \alpha_u - r^*(a_1 - 1)\phi_1 \alpha_u + \phi_1 \alpha_\pi \beta_\pi;$$

$$b_3 = r^* \sqrt{\Delta} \frac{\alpha_u}{\alpha_\pi} + r^* \phi_1 \alpha_\pi \beta_\pi - r^*(a_1 - 1)\phi_1 \alpha_\pi \beta_\pi$$

$$b_4 = r^* \beta_\pi \sqrt{\Delta}$$

By inserting the steady-state values of $P^*(d^*, r^*, u^*, \pi^*)$ in the above expressions and after some algebraic manipulations we obtain the lemma. Q.E.D.

Lemma 2. Necessary and sufficient conditions for all the roots of the characteristic polynomial to have negative real parts are $R_1: b_1 > 0, b_3 > 0, b_4 > 0,$ and $R_2: \phi(P^*) = b_1 b_2 b_3 - b_3^2 - b_1^2 b_4 > 0$. If the above conditions are satisfied then the non-zero equilibrium P^* will be locally stable.

The proof of Lemma 2 follows from an inspection of the Jacobian matrix of linearization in the neighbourhood of $P^*(d^*, r^*, u^*, \pi^*)$ and using the Routh-Hurwitz criteria.

Proposition 1: If $4 - 2\sqrt{3} < a_1 < 1$ and $\phi_1 \alpha_u^2 < 4\beta_\pi \alpha_\pi$ the equilibrium solution is given by eqs. (13) to (16) is locally and asymptotically stable.

Proof. The eigenvalues are the roots of the characteristic equation, that is:

$$\lambda_{1,2} = -\frac{1}{2}\phi_1 \alpha_u \pm \frac{1}{2}\sqrt{\phi_1^2 \alpha_u^2 - 4\beta_\pi \phi_1 \alpha_\pi};$$

$$\lambda_{3,4} = r^*(\frac{1}{2}a_1 - 1) \pm \frac{1}{2}\sqrt{(\mathrm{r}^*)^2 a_1^2 - 4d^* r^* a_2 + 4r^* a_2 b_1}.$$

If $\phi_1 \alpha_u^2 < 4\beta_\pi \alpha_\pi$ implies that $\lambda_{1,2}$ are complex numbers, with the real part negative. The condition for $\lambda_{3,4}$ to be complex numbers is that $(\mathrm{r}^*)^2 a_1^2 -$

$4d^*r^*a_2 + 4r^*a_2b_1 < 0$. After some algebraic manipulation, we show that this condition is equivalent to: $a_2b_1\phi_1[a_1^2 - 8a_1 + 4] + (a_1^2 - 1)\sqrt{\Delta} < 0$. This condition is satisfied if $4 - 2\sqrt{3} < a_1 < 1$. If these conditions hold, the real part of $\lambda_{3,4}$ is also negative. Q.E.D.

This result is akin to what Chiarella et al. (2005, p. 444) obtained Concerning the KMGT model: "The central result we obtained was that local asymptotic stability prevails if a number of reaction coefficients are sufficiently low (given that a few additional, but unproblematic inequalities are satisfied)". Let us now study the possibility of the existence of a limit cycle in the system (3), (6), (7) and (8) by using the Hopf bifurcation analysis.

Lemma 3. The polynomial equation $\lambda^4 + b_1\lambda^3 + b_2\lambda^2 + b_3\lambda + b_4 = 0$ has no pair of purely imaginary roots and two roots with negative real parts. Hence, the Hopf-Andronov-Poincaré bifurcation will not be performed on the system of equations (3), (6), (7) and (8).

Proof. The eigenvalues are the roots of the characteristic equation, that is:

$$\lambda_{1,2} = -\frac{1}{2}\phi_1\alpha_u \pm \frac{1}{2}\sqrt{\phi_1^2\alpha_u^2 - 4\beta_\pi\phi_1\alpha_\pi};$$

$$\lambda_{3,4} = r^*(\frac{1}{2}a_1 - 1) \pm \frac{1}{2}\sqrt{(\mathrm{r}^*)^2a_1^2 - 4d^*r^*a_2 + 4r^*a_2b_1}.$$

From (1.2), it follows that $\left(\frac{1}{2}a_1 - 1\right) < 0$. Hence, there is no pair of purely imaginary roots as follows:

(a) If $A = \phi_1^2\alpha_u^2 - 4\beta_\pi\phi_1\alpha_\pi > 0$ and $B = (r^*)^2a_1^2 - 4d^*r^*a_2 + 4r^*a_2b_1 > 0$, the eigenvalues of the Jacobian matrix $\lambda_{1,2}$ and $\lambda_{3,4}$ are real roots. Hence, the equilibrium P^* is stable or unstable and there is no pair of purely imaginary roots. Hence, the Hopf-Andronov-Poincaré bifurcation will not be performed on the system of equations (1.1).

(b) If $A = \phi_1^2\alpha_u^2 - 4\beta_\pi\phi_1\alpha_\pi < 0$ or $B = (\mathrm{r}^*)^2a_1^2 - 4d^*r^*a_2 + 4r^*a_2b_1 > 0$, then $\lambda_{3,4}$ are real roots, and $\lambda_{1,2}$ has two imaginary roots with negative real parts, because $-\frac{1}{2}\phi_1\alpha_u < 0$. Thus, following Theorem 1 (H2)(Asada and Yoshida, 2003), the model (1.1) has no Hopf bifurcation.

(c) If $A = \phi_1^2\alpha_u^2 - 4\beta_\pi\phi_1\alpha_\pi > 0$ or $B = (r^*)^2 a_1^2 - 4d^* r^* a_2 + 4r^* a_2 b_1 < 0$, then $\lambda_{1,2}$ are real roots, and $\lambda_{3,4}$ has two imaginary roots with negative real parts, because $r^*(\frac{1}{2}a_1 - 1) < 0$. Thus, following (H2) in Theorem 1 (Asada and Yoshida, 2003), the model (1.1) has no Hopf bifurcation.

Chiarella et al. (2005, p. 444) "What was important for us was not so much the existence of periodic orbits, because the circumstances under which they emerge can be rather special, but a certain tendency for cyclical behaviour in general." Next, we shall consider numerical simulation results to verify the local asymptotically stability of the equilibrium point of the system (1,1). Let $b_1 = 0.1, i_o = 1, \pi^0 = 0.1, \alpha_\pi = \alpha_u = 0.1,\ a_1 = 0.4, a_2 = 0.2, \beta_\pi = 0.8, \phi_1 = 0.11$, in system(1.1). The equilibrium point:

$$P^* = (0.4000249995, 0.3000083332, 1, 0.1909090909)$$

is locally asymptotically stable with eigenvalues

$$\lambda_{1,2} = -0.240006666550000 \pm 0.120007499777670\, i,$$

And $\lambda_{3,4} = -0.00550000000000000 \pm 0.0936469433564171\, i$. The conditions of Routh-Hurwitz in Lemma 2 are satisfied, that is:
$b_1 = 0.4910133331 > 0, b_2 = 0.08578513832 > 0, b_3 = 0.005016172331 > 0, b_4 = 0.0006336440000 > 0$, and $\phi(P^*) = b_1 b_2 b_3 - b_4 b_1^2 - b_3^2 = 0.00003335962815 > 0$. The asymptotic solutions of $d(t), r(t), u(t)\ and\ \pi(t)$ are illustrated in Figs. $1 - 8$.

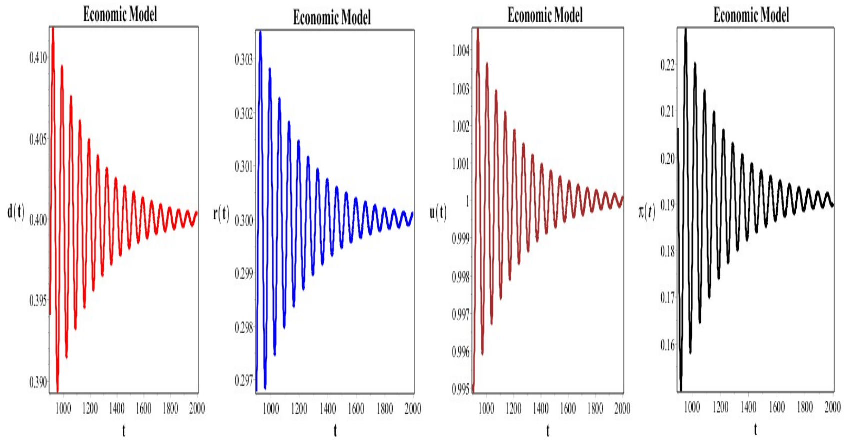

Figure 1. The graph of $d, r, u,$ and π versus t for the set of initial conditions $d(0) = 0.4$, $r\ (0) = 0.3, u(0) = 1$, $\pi(0) = 0.12$, where $t = 900...2000$. The steady state $P^*$is locally asymptotically stable.

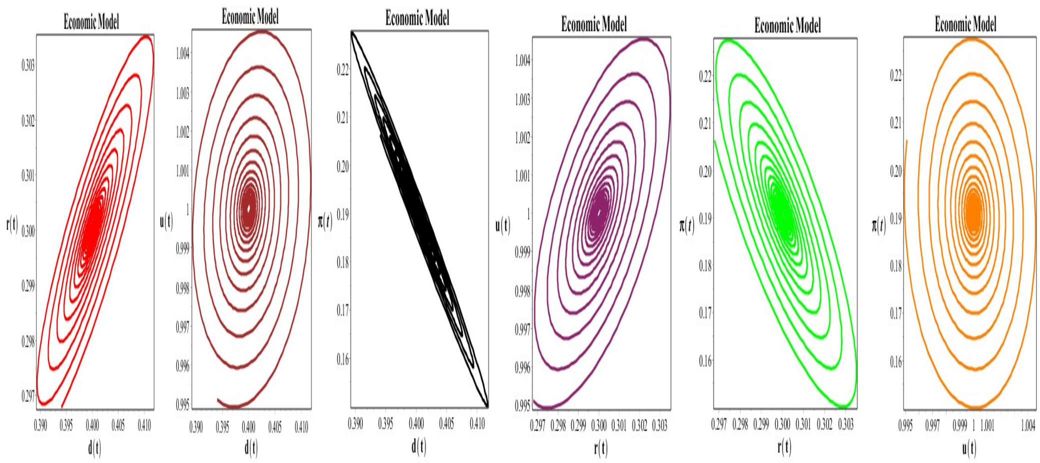

Figure 2. The graph of (d, r), (d, u), (d, π), and (r, u), (r, π) and (u, π) for the set of initial conditions $d(0) = 0.4$, $r\ (0) = 0.3, u(0) = 1$, $\pi(0) = 0.12$, where $t = 900...2000$. The steady state $P^*$is locally asymptotically stable

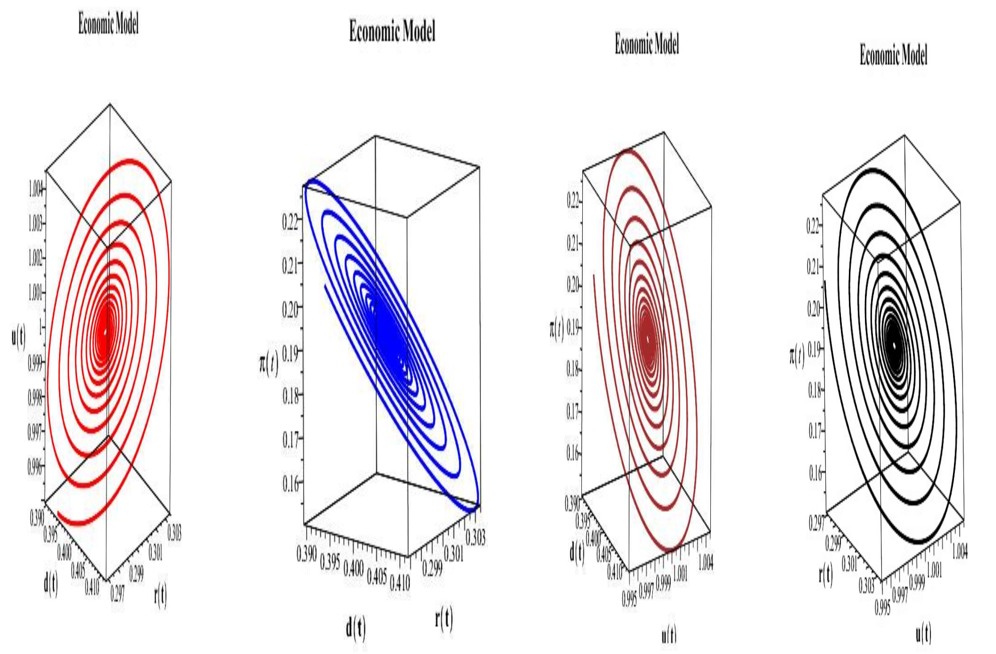

Figure 3. The graph of (d, r, u), (d, r, π), (d, u, π), and (r, u, π) for the set of initial conditions $d(0) = 0.4$, $r\ (0) = 0.3$, $u(0) = 1$, $\pi(0) = 0.12$, where $t = 900...2000$. The steady state $P^*$is locally asymptotically stable.

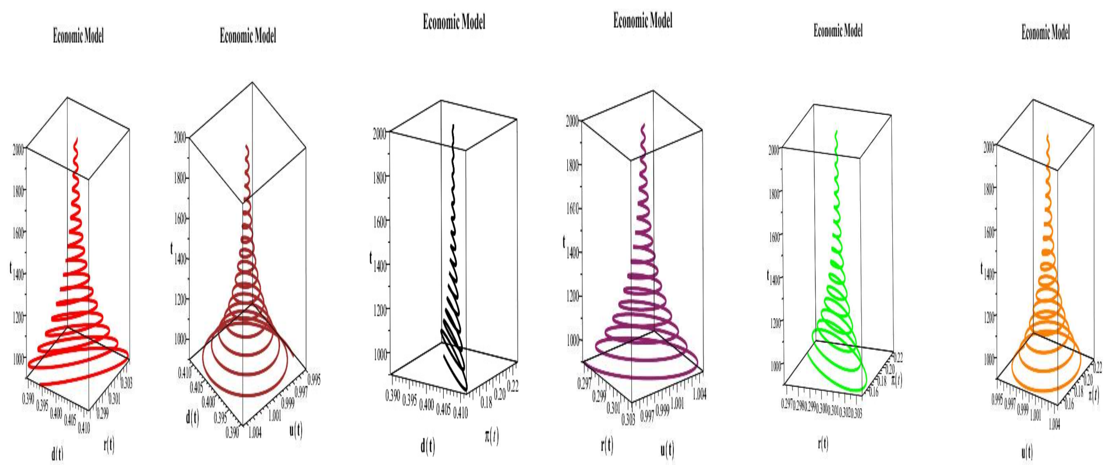

Figure 4. The graphs of (d, r, t), (d, u, t), (d, π, t), (r, u, t), (r, π, t) and (u, π, t) for the set of initial conditions $d(0) = 0.4$, $r\ (0) = 0.3$, $u(0) = 1$, $\pi(0) = 0.12$, where $t = 900...2000$. The steady state $P^*$is locally asymptotically stable.

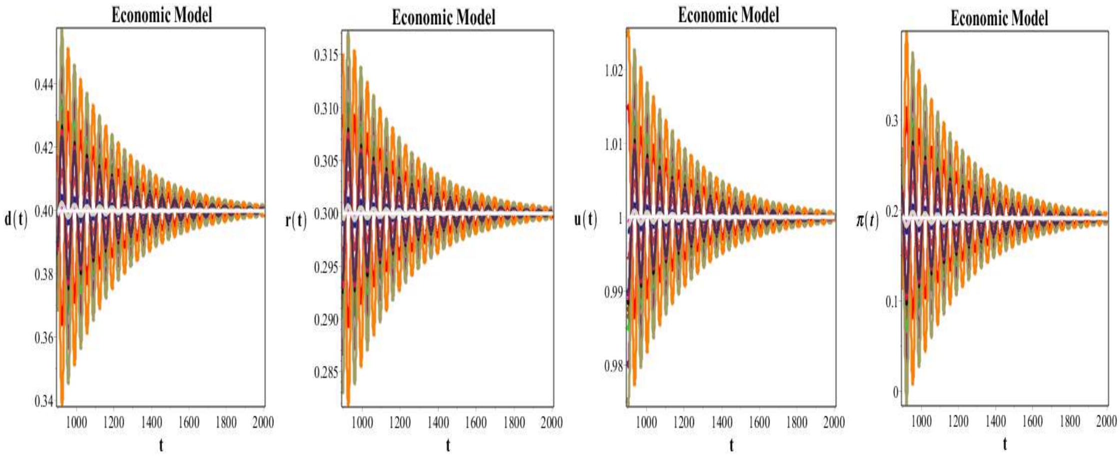

Figure 5. Time series solutions in the planes d, r, u, and π versus t, with 25 initial conditions $\left(d(0), r\ (0), u(0), \pi(0)\right)$, where $t = 900...2000$. The steady state $P^*$is locally asymptotically stable.

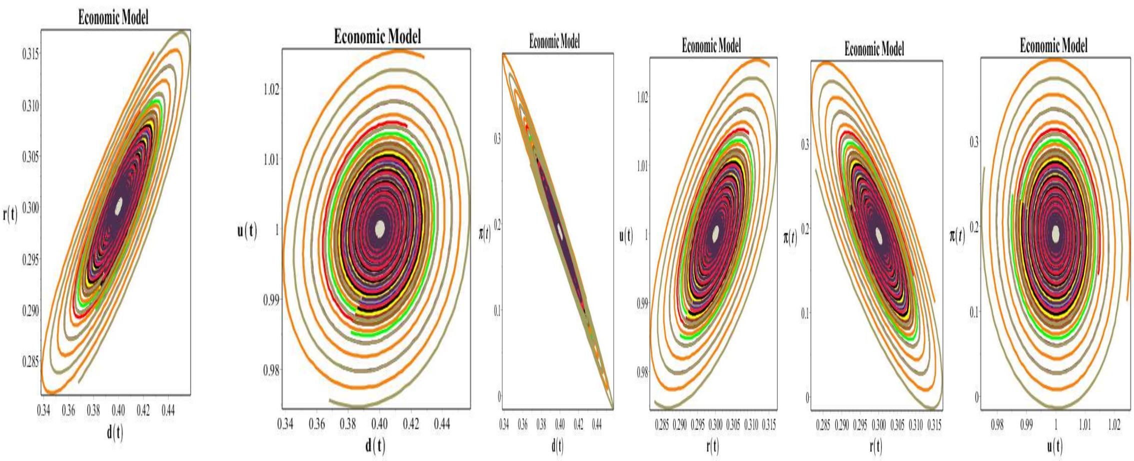

Figure 6. Several solutions projection in the planes $(d, r), (d, u), (d, \pi)$, $(r, u), (r, \pi)$ and (u, π), with 25 initial conditions $(d(0), r\ (0), u(0), \pi(0))$, where $t = 900...2000$. The steady state $P^*$is locally asymptotically stable

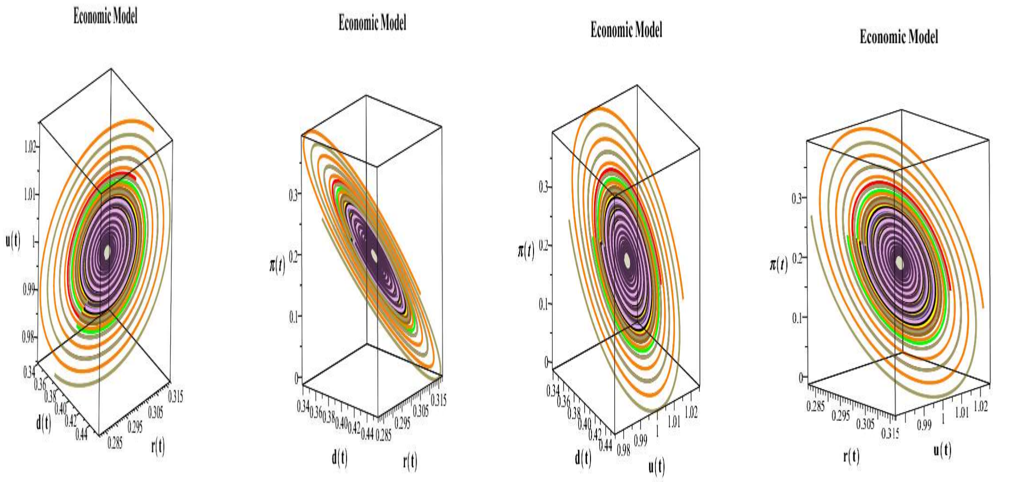

Figure 7. The graphs of (d, r, u), (d, r, π), (d, u, π), and (r, u, π), with 25 initial conditions $(d(0), r(0), r(0), \pi(0))$, where $t = 900...2000$. The steady state $P^*$is locally asymptotically stable.

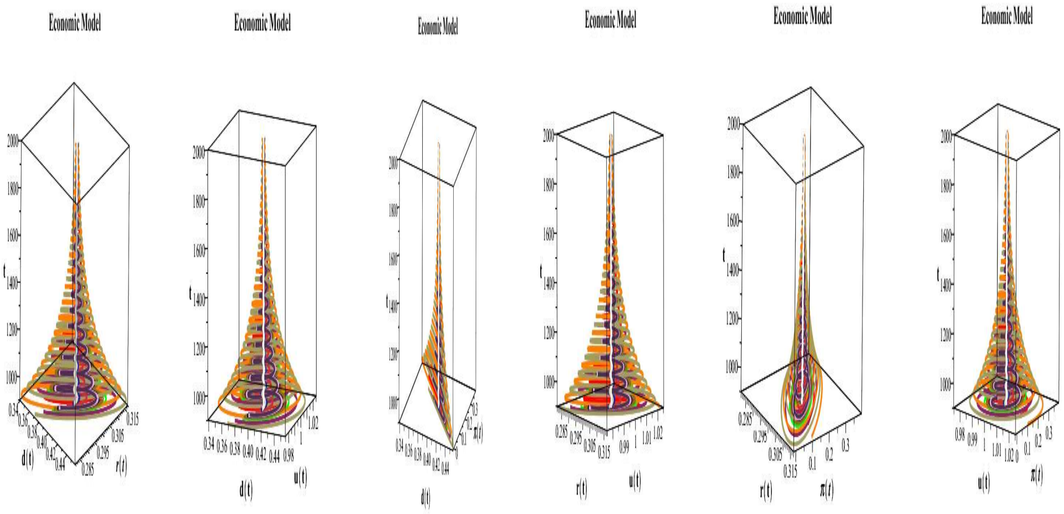

Figure 8. The graphs of $(d, r, t), (d, u, t), (d, \pi, t), (r, u, t)$, (r, π, t) and (u, π, t), with 25 initial conditions $(d(0), r(0), r(0), \pi(0))$, where $t = 900...2000$. The steady state $P^*$is locally asymptotically stable

4. Concluding Remarks

In this paper, we have introduced the Taylor rule in an extended version of the Foley model advanced by Araujo et al. (2020). We aim to make the model more inclusive of the fact that central banks nowadays focus more on a target nominal interest rate than on the money stock. To accomplish such a task we proceed to some changes in the model such as considering explicitly the rate of capacity utilization as one of the endogenous variables insofar as it is used as a measure of the output gap that enters the Taylor rule. Whereas Araujo et al. (2020) had found unstable equilibria in the extended Foley's model, in which the money supply was completely exogenous, our 4-dimensional dynamic system has a stable solution. Hence, we conclude that with an accelarionist Philips curve and a dynamic IS, the central bank is required to pursue an active policy concerning the nominal interest rate; in this vein, we can see the Taylor rule as stabilizing device for the economy. Besides, we show that the system does not meet the conditions of the Hopf bifurcation theorem, which shows it can be destabilized into a limit cycle.

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