Heterogeneity in unemployment expectations across working households and cyclical fluctuations in macroeconomic activity

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Abstract: In view of the extensive survey evidence on persistent heterogeneity in unemployment expectations across workers, it is a reasonable premise that the perceived risk of job loss and the resulting consumption demand behavior are similarly heterogeneous across workers. Based on such a premise, the paper shows that the evidence of positive correlation between pessimistic unemployment expectations and actual unemployment which has been found empirically arises in a demand-led macrodynamic model augmented with heterogeneous unemployment expectations across workers. The frequency distribution of such expectations is endogenously time-varying as driven by an evolutionary dynamics of expectation formation based on Keynes' (1937) 'practical theory of the future'. As a result, such frequency distribution (and thus the level of macroeconomic activity) is prone to experience self-sustaining cyclical fluctuations over time, which is in keeping with the empirical evidence.

Keywords: Unemployment expectations; demand-led macrodynamic; unemployment rate.

JEL Codes: C60; D84; E24; E70.

1. Introduction

A robust feature of household micro data for several countries is the considerable variation in the marginal propensity to consume across income levels. There is also extensive survey evidence for the U.S. and European countries on persistent and time-varying heterogeneity in unemployment expectations across households. These expectations are typically elicited and reported as ranging from pessimism to optimism, also including neutral households who expect unemployment to remain about the same (see Figures 1 and 2). This paper conceives as analytically plausible to interpret these pieces of evidence as causally interrelated in an evolutionary microdynamic of unemployment expectation switching by working households, which is then embedded in an aggregate demand-led macrodynamic of capacity utilization and output growth. There is also evidence based on U.S. and European surveys showing that households' unemployment expectations are an important driver of actual unemployment, with an increase in pessimism (optimism) leading to a rise (fall) in actual unemployment. Against this backdrop, the ambition of this paper is to suggest a plausible candidate explanation (which of course need not be the only one) for these motivating pieces of evidence by abstracting from several other potential sources of alternative explanations. In fact, the model we set forth down the road succeeds in replicating qualitatively the cyclical fluctuations in the heterogeneity in unemployment expectations across working households and in the unemployment rate shown in Figures 1 and 2, respectively.

In the model developed in this paper, an individual firm is unable to perfectly observe what unemployment expectation a given household member holds in her capacity as a worker or, for that matter, as a consumer. Consequently, although working households have heterogeneous propensities to consume depending on their unemployment expectations for the next future, with to more optimism corresponding higher propensity to consume, they all face the same price level when spending their wage income on consumption. Meanwhile, since firms are unable to perfectly observe whether an individual worker holds a pessimistic unemployment expectation (and could possibly provide relatively more work intensity by having a higher expected cost of job loss) or an optimistic unemployment expectation (and hence could possibly deliver relatively less work intensity by having a lower expected cost of job), all workers are compensated with the same real wage. However, the resulting heterogeneity in the expected cost of job loss across working households, despite translating into heterogeneity in their propensity to consume, does not translate into heterogeneity in their intensity of work, so that labor productivity (and hence the real unit labor cost and the corresponding markup) is uniform across firms.

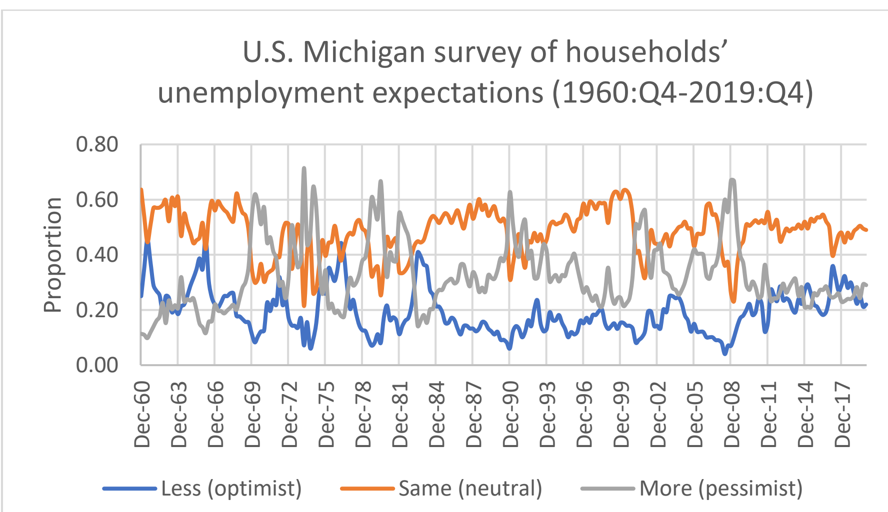

Figure 1. Time-varying proportions of unemployment expectations in the US (see footnote 1).

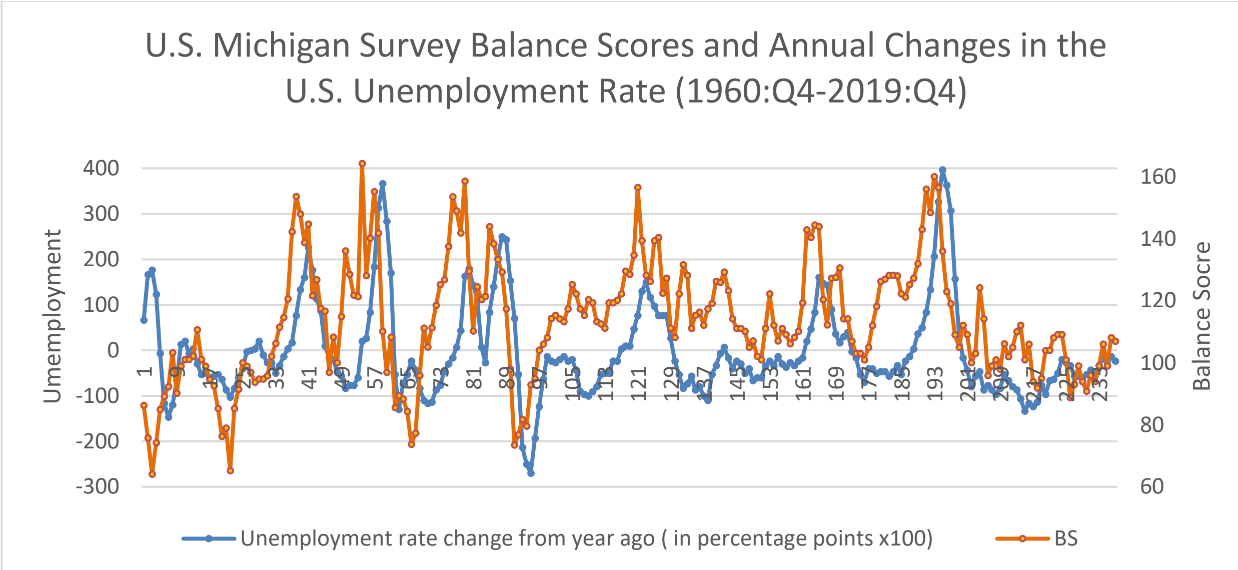

Figure 2. Unemployment expectations and changes in the unemployment rate in the U.S (see footnote 1).

According to Keynes (1936, 1937), people's propensity to consume is influenced by many factors such as the distribution of income and their expectations about the future. In effect, individuals may refrain from spending out of their incomes in order to build up a reserve against unforeseen contingencies. Several authors (e.g., Jappelli and Pistaferri, 2010, 2014; Carroll et al., 2014; Carroll et al., 2017) provide evidence that there is considerable variation in the marginal propensity to consume across income levels, with the propensity to consume being higher for lower-income households. Meanwhile, Carroll and Dunn (1997) find that higher unemployment expectations predict lower consumer spending, even when controlling for the movement in consumption that could be attributed to predictable movements in labor earnings. They consistently find that the lagged value of the U.S. Michigan survey index of households' unemployment expectations (used as a preferred measure of uncertainty) has an influence on spending decisions. In fact, unemployment expectations have been consistently shown to be significant drivers of the consumption/saving behavior of households (Carroll et al., 2012, 2014, 2019). Unemployment expectations are relevant because typically the most drastic fluctuations in a household's income are those associated with spells of unemployment, so that unemployment expectations can be interpreted as a proxy for zero-income probability (Carroll, 1992). In fact, Christelis et al. (2015) use survey data for U.S. households and find that loss of employment reduces spending by 10%.

There is correlation evidence based on the major U.S. and European households surveys which was shown econometrically to indicate that the unemployment expectations held by workers are an important driver of actual unemployment, in that a rise (fall) in pessimism (optimism) leads to an increase in actual unemployment (Leduc and Sill, 2013; Girardi, 2014; Lehmann and Weyh, 2016). In this regard, the main qualitatively result coming out of our heterogeneous expectations-augmented demand-led macroeconomic model is precisely that more pessimistic average unemployment expectations will raise actual unemployment. Girardi (2014) employs a dynamic multivariate setup in a (pseudo) real-time context and finds robust evidence that shifts in households' unemployment expectations as shown in a major European Union survey are an exogenous source of fluctuations in the actual unemployment rate in the euro area. On average across the entire sample of survey respondents, it is found that a negative shock that lowers households' unemployment expectations leads to a fall in the current unemployment rate. Dickerson and Green (2012) use two nationally representative panels from Germany and Australia that regularly survey workers' employment expectations and estimate a model with fixed effects intended to control for personality traits. The authors offer several interesting findings. First, although there is persistent heterogeneity in these expectations, on average, the expectations of job loss robustly predict the probability of subsequent actual job loss. Second, workers' perceptions of re-employment probabilities if the job loss materializes are also robust predictors of their actual outcomes. Third, although these expectations have such predictive power, there are nonetheless significant biases: on average, workers overestimate the probability of losing their job, and they underestimate the probability of job replacement. Blanchflower and Bryson (2021) use panel data for 29 European countries over 439 months between January 1985 and July 2021 to predict changes in the unemployment rate 12 months in advance based on individuals' fears of unemployment as estimated in a major European Union survey. The authors find that individuals' fears of unemployment predict subsequent changes in unemployment 12 months later in the presence of country and year fixed effects and lagged unemployment.

Meanwhile, Leduc and Sill (2013) take unemployment expectations collected by means of different surveys from U.S. households and professional forecasters, and introduce them into an otherwise conventional vector autoregression (VAR) econometric framework. The main empirical finding is that changes in unemployment expectations do contain quantitatively important information that helps account for drivers of fluctuations in actual unemployment. Indeed, a variance decomposition exercise employing unemployment expectations held by either households or professional forecasts shows that the contribution of exogenous shocks to unemployment expectations for the variance of actual unemployment is substantial. On impact, the estimated VARs robustly indicate that a negative innovation that reduces the expected rate of unemployment by 1 percentage point leads to a decrease in the current rate of unemployment of a similar magnitude.

Leduc and Sill (2013) suggest that their estimated result that a decrease in expected unemployment by workers leads to a fall in actual unemployment rate squares well with the predictions of a standard labor matching model. The intuition would be that less pessimistic unemployment expectations by workers raise the marginal benefit of a match and lead to a fall in actual unemployment as more vacancies are posted. Alternatively, we show that a positive relationship between pessimistic unemployment expectations on the part of workers and actual unemployment arises in a novel heterogeneous expectations-augmented aggregate demand-led macrodynamic model in which the frequency distribution of unemployment expectations across workers is endogenously time-varying as governed by evolutionary dynamics of expectation switching. In our model, cyclical fluctuations are not a consequence of continuing exogenous shocks of different types, but they instead result from evolutionary dynamics of unemployment expectation switching which produces recurrent boom-bust phenomena.

2. Structure of the model and temporary equilibrium

The economy is closed and features no government activities, producing only one good for both investment and consumption purposes. Output production is carried out by imperfectly-competitive firms that combine capital and labor through a fixed-coefficient technology:

$$(1) \qquad X = min\left\{K\nu, L/a\right\},$$

where X is the output level, K is the stock of capital, L is the employment level, ν is the full-capacity output to capital ratio, which is an exogenously fixed technological parameter, and a is the labor to output ratio (or the inverse of labor productivity). Although workers' unemployment expectations are heterogeneous, their labor productivity is homogeneous and they receive the same real wage, V. For simplicity, the technical coefficient ν is normalized to one, and we measure the rate of capital capacity utilization, σ, by the output to capital ratio, X / K.

The economy is composed of two social classes, firm-owner capitalists and workers, who earn profits and wages, respectively. The functional division of aggregate income is given by:

(2) $$X = VL + R,$$

where R is the level of aggregate profits. From (1) and (2), the share of profits in aggregate income, $\pi \in (0,1) \subset \mathbb{R}$, is given by:

(3) $$\pi = 1 - Va.$$

Firms sell their output production in oligopolistic markets. The price level of the single good is set as a constant markup factor, $z > 1$, over nominal unit labor costs measured as the nominal wage bill per unit of output, WL / X, where W is the nominal wage, which implies that the real wage is given by $V = 1/az$. We further assume that the labor to output ratio is constant, so that the profit share in (3) is also constant. Thus, the model is cast in real terms and the functional distribution of aggregate income remains constant throughout, given the focus of the paper on other issues.

Firms produce (and hire labor and utilize their installed capital stock) according to aggregate effective demand. As we model only the case featuring excess productive capacity (in labor and capital), the level of employment is determined by output production:

(4) $$L = aX.$$

Workers' propensity to consume varies negatively with their unemployment expectations. There is a continuum of workers (labor is always in excess supply), but we group together three types of employed workers as regards their unemployment expectations and respective consumption propensities, so that these three types can be seen as three average levels. Therefore, there is heterogeneity not only across types, but also within types. More precisely, $c_{w,\tau}$ is the expected (average) propensity to consume of a worker of type $\tau = n, o, p$ (which stands for neutral, optimistic and pessimistic, respectively). Meanwhile, $u_{\tau}^{e} \in (0,1) \subset \mathbb{R}$ is the expected unemployment rate by a worker of type $\tau = n, o, p$. This taxonomy is mostly based on (but not expected to be fully representative of) the U.S. Michigan Survey of Consumers, in which households are monthly asked: "How about people out of work during the coming 12 months — do you think that there will be more unemployment than now, about the same, or less?"[1]

In accordance with the robust empirical evidence from survey data on persistent heterogeneity in unemployment expectations across workers reported earlier, where such expectations range cardinally from more optimistic to more pessimistic ones, we assume the following well-defined ordering for the expected consumption propensities of workers of type $\tau = o, n, p$:

(5) $$0 < c_{w,p} < c_{w,n} < c_{w,o} \leq 1.$$

[1] In addition to 'less', 'more' and 'same', possible answers include 'don't know' and 'no answer', where the latter two usually comprise a very small percentage of all the answers. On the basis of the distribution of all these answers, a measure dubbed balance score is calculated as the percentage of households who thought the unemployment rate would increase minus the percentage who thought it would fall, plus 100 (see https://data.sca.isr.umich.edu/). In Figures 1 and 2 above, the proportion of each type of unemployment expectation and the balance score are calculated ignoring the categories 'don't know' and 'no answer' and using quarterly averages of the monthly data.

Admittedly, no matter how pessimistic about the prospects of remaining employed workers become, they simply cannot afford to save much of their wage income. In fact, it is quite reasonable to assume that even the most pessimistic workers will save less than firm-owner capitalists, whose saving behavior is assumed to be homogeneous. Thus, we further assume that $c_c \in (0, c_{w,p}) \subset \mathbb{R}$ is capitalists' propensity to consume.

For concreteness, for a given propensity to consume of optimistic workers $c_{w,o} \in (0,1] \subset \mathbb{R}$, we assume the following specific form for the well-defined ordering for the consumption propensities out of wage income of neutral and pessimistic workers, respectively:

(6) $$c_{w,n} = \alpha c_{w,o}$$

and

(7) $$c_{w,p} = \alpha c_{w,n},$$

where $\alpha \in (0,1) \subset \mathbb{R}$ is a parametric constant. Thus, it follows from (6) and (7) that $c_{w,p} = \alpha^2 c_{w,o}$. Note that the specification in (6) and (7) will allow us to differentiate a deepening (intensive margin) from a widening (extensive margin) of the effects of workers' unemployment expectations on their consumption behavior and thereby the level of economic activity in such an aggregate demand-led economy. In fact, given the frequency distribution of unemployment expectations across workers, a fall in the proportionality parameter α is equivalent to an increase (fall) in workers' average unemployment pessimism (optimism) along the intensive margin.[2] And, given the proportionality parameter α, a rise in the proportion of pessimistic workers is equivalent to an increase (fall) in workers' average unemployment pessimism (optimism) along the extensive margin.

The proportions of optimistic, neutral and pessimistic workers are denoted, respectively, by $\theta \in [0,1] \subset \mathbb{R}$, $\eta \in [0,1] \subset \mathbb{R}$ and $\rho \in [0,1] \subset \mathbb{R}$, such that $\theta + \rho + \eta = 1$. The expected (average) propensity to consume out of wage income across types of working households can then be defined as $c_w = \theta c_{w,o} + \eta c_{w,n} + \rho c_{w,p}$, which, considering that $\eta = 1 - \theta - \rho$, can be re-written as a function of the frequency distribution of working households' types as follows:

(8) $$c_w = c_{w,o}\Phi(\theta, \rho, \alpha),$$

where

(9) $$\Phi(\theta, \rho, \alpha) \equiv \alpha + (1-\alpha)\theta - \alpha(1-\alpha)\rho.$$

From now on we will simplify matters by assuming that optimistic working households spend in consumption all of their wage income, that is, $c_{w,o} = 1$, so that $c_w = \Phi(\theta, \rho, \alpha)$. Thus, it follows from (8) and (9) that for any $(\eta, \theta, \rho) \in \Delta \equiv \left\{(\eta, \theta, \rho) \in \mathbb{R}^3_+ : \eta + \theta + \rho = 1\right\}$ we have:

(10) $$\frac{\partial c_w}{\partial \theta} = \frac{\partial \Phi(\theta, \rho, \alpha)}{\partial \theta} = 1 - \alpha > 0,$$

[2] Given the qualitative nature of the respective question posed in the U.S. Michigan and EU surveys of unemployment expectations by households, as described in the preceding footnote, a sufficiently more precise empirically based specification of such an effect along the intensive margin is not available, thus the simplified (but qualitatively representative) specification in (6) and (7).

(11) $$\frac{\partial c_w}{\partial \rho} = \frac{\partial \Phi(\theta,\rho,\alpha)}{\partial \rho} = -\alpha(1-\alpha) < 0,$$

and

(12) $$\frac{\partial c_w}{\partial \alpha} = \frac{\partial \Phi(\theta,\rho,\alpha)}{\partial \alpha} = 1-\theta-\rho+2\alpha\rho = \eta+2\alpha\rho > 0.$$

Therefore, (10) and (11) show that a rise in workers' unemployment optimism (pessimism) along the extensive margin raises (lowers) workers' average propensity to consume, whereas (12) shows that a fall (rise) in workers' unemployment pessimism (optimism) along the intensive margin raises workers' average propensity to consume. Besides, given that (10) and (11) imply that (9) is a strictly increasing (decreasing) linear function of the proportion of optimistic (pessimistic) workers, we have that $\Phi(1,0,\alpha)=1$ is the maximum value and $\Phi(0,1,\alpha)=\alpha^2<1$ is the minimum value of the function in (9), for a given α. Thus, we have that $c_w=\Phi(\theta,\rho,\alpha)\in[\alpha^2,1]\subset\mathbb{R}_{++}$ for any $(\eta,\theta,\rho)\in\Delta$.

As a result, recalling that $c_{w,o}=1$, aggregate saving, which is composed of savings by workers, S_w, and capitalists, S_c, is given by:

(13) $$S = S_w + S_c = [1-\Phi(\theta,\rho,\alpha)]VL + (1-c_c)(X-VL),$$

where we used (2) and (8). Normalizing (13) by the capital stock, we have:

(14) $$g^S \equiv \frac{S}{K} = [1-\Phi(\theta,\rho,\alpha)]\frac{VL}{X}\frac{X}{K} + (1-c_c)\left(\frac{X}{K}-\frac{VL}{X}\frac{X}{K}\right) = \left\{[1-\Phi(\theta,\rho,\alpha)](1-\pi)+(1-c_c)\pi\right\}\sigma,$$

where we used (3) and recall that $\sigma = X/K$ is the rate of capital capacity utilization.

Firms make capital accumulation plans described by the following desired investment function (expressed as a proportion of the capital stock):

(15) $$g^I \equiv \frac{I}{K} = \gamma + \delta\sigma,$$

where $\gamma\in\mathbb{R}_{++}$ and $\delta\in\mathbb{R}_{++}$ are parametric constants. We draw on Rowthorn (1982) and Dutt (1984) in making the desired rate of capital accumulation by firms to depend positively on the rate of capital capacity utilization due to accelerator-type effects. Hence, the specification in (15) implies that the functional distribution of income between wages and profits (which is anyway constant over time) impacts directly on aggregate effective demand only through the consumption demand by workers and capitalists. As explored later, an exogenous change in the functional distribution of income will nonetheless also impact indirectly on aggregate effective demand through the capital capacity utilization effect on firms' desired investment function in (15).

Reasonably, not all variables of the model vary at the same time, and some variables are taken to be predetermined at any given moment in time. Recalling that the real wage and the labor input (which are both homogeneous across workers) have already been assumed to remain constant throughout, we further assume that the capital stock, K, the labor supply, N, and the frequency distribution of unemployment expectations in the population of workers, (η,θ,ρ), are all predetermined at any given moment in time. The assumed existence of excess productive capacity in capital and labor implies that aggregate output adjusts at any given moment in time to remove any excess aggregate demand or supply in the economy. Thus, in such a temporary equilibrium configuration, aggregate saving is equal to aggregate investment. Solving for the resulting temporary equilibrium value of the capital capacity utilization using the goods market equilibrium condition given by $g^S = g^I$, we obtain:

$$(16) \qquad \sigma^* = \frac{\gamma}{[1-\Phi(\theta,\rho,\alpha)](1-\pi)+(1-c_c)\pi-\delta} \equiv \sigma(\theta,\rho,\alpha,\gamma,\delta,c_c,\pi).$$

Assuming for innocuous simplicity that the capital stock does not depreciate, the temporary equilibrium value of the output growth rate can be obtained by substituting (16) into (15), which yields:

$$(17) \qquad g^* = \gamma\left(1+\frac{\delta}{[1-c_{w,o}\Phi(\theta,\rho,\alpha)](1-\pi)+(1-c_c)\pi-\delta}\right) \equiv g(\theta,\rho,\alpha,\gamma,\delta,c_c,\pi).$$

We suppose that the temporary equilibrium values in (16) and (17) are stable at any given moment in time by assuming that the aggregate saving (as a proportion of the capital stock) in (14) is more responsive than the desired capital accumulation in (15) to changes in the rate of capital capacity utilization, which in turn requires that the denominator of the expression in (16) is positive. This is the standard Keynesian stability condition typically assumed in aggregate demand-led models like the one set forth in this paper. In the present model, the satisfaction of this condition also ensures that the temporary equilibrium value of capital capacity utilization in (16) is strictly positive. Note that the response of the aggregate saving in (14) to a change in capital capacity utilization crucially depends on the frequency distribution of unemployment expectations across workers, given that workers' average saving propensity depends on that frequency distribution. As we have assumed earlier that optimistic workers spend in consumption of all their wage income, it follows that for the Keynesian stability condition given by $[1-\Phi(\theta,\rho,\alpha)](1-\pi)+(1-c_c)\pi > \delta$ to be satisfied it is sufficient to assume that:

$$(18) \qquad [1-\Phi(1,0,\alpha)](1-\pi)+(1-c_c)\pi = (1-c_c)\pi > \delta,$$

which corresponds to a situation in which all workers hold optimistic unemployment expectations (and hence workers' average saving propensity is at its lower bound of zero) and only firm-owner capitalists save.

Regarding comparative statics, it can be easily algebraically verified (and intuitively explained by the aggregate demand-led nature of the model) that the temporary equilibrium values of the rates of capital capacity utilization and output growth in (16) and (17) vary positively with the parameters of the desired investment function in (15). These temporary equilibrium values also intuitively vary positively with the average propensity to consume of workers in (8), recalling that we have assumed that optimistic workers spend in consumption of all their wage income, so that $c_{w,o} = 1$:

$$(19) \qquad \frac{\partial\sigma^*}{\partial\Phi} = \frac{\partial\sigma^*}{\partial c_w} = \frac{(1-\pi)\sigma^*}{[1-\Phi(\theta,\rho,\alpha)](1-\pi)+(1-c_c)\pi-\delta} > 0 \text{ and } \frac{\partial g^*}{\partial\Phi} = \gamma\frac{\partial\sigma^*}{\partial\Phi} > 0.$$

Meanwhile, the temporary equilibrium values of the rates of capital capacity utilization and output growth in (16) and (17) vary negatively with the profit share in income:

$$(20) \qquad \frac{\partial\sigma^*}{\partial\pi} = \frac{[c_c-\Phi(\theta,\rho,\alpha)]\sigma^*}{[1-\Phi(\theta,\rho,\alpha)](1-\pi)+(1-c_c)\pi-\delta} < 0 \text{ and } \frac{\partial g^*}{\partial\pi} = \gamma\frac{\partial\sigma^*}{\partial\pi} < 0.$$

The numerator of the first derivative in (20) is strictly negative given that our earlier assumption that even the most pessimistic workers cannot afford to have a higher saving propensity than firm-owner capitalists, so that $c_{w,p} > c_c$, implies that $0 < c_c < \alpha^2 \le \Phi(\theta,\rho,\alpha) \le 1$. The substance of this result is that an exogenous redistribution of income from workers to capitalists reduces aggregate consumption and thereby aggregate demand even when the average propensity to consume of workers is at its lower bound of $\Phi(0,1,\alpha) = \alpha^2 < 1$, which corresponds to an extreme situation with all workers holding pessimistic unemployment expectations. Besides, such a functional redistribution of income is aggregate demand-reducing also by negatively affecting firms' investment spending through the capital capacity utilization effect on their desired capital accumulation.

Recall that our specification of the ordering of the consumption propensities out of wage income in (6) and (7) allows us to differentiate a deepening (intensive margin) from a widening (extensive margin) of the effects of workers' unemployment expectations on the level of economic activity. Intuitively, a rise in the proportion of optimistic workers is equivalent to an increase in workers' average unemployment optimism along the extensive margin, which raises the temporary equilibrium values of the rates of capital capacity utilization and output growth by lifting workers' average propensity to consume (per (10)):

(21) $$\frac{\partial\sigma^*}{\partial\theta} = \frac{\partial\sigma^*}{\partial\Phi}\frac{\partial\Phi}{\partial\theta} = \frac{\sigma^*(1-\pi)(1-\alpha)}{[1-\Phi(\theta,\rho,\alpha)](1-\pi)+(1-c_c)\pi-\delta} > 0 .$$

Analogously, a rise in the proportion of pessimistic workers is equivalent to an increase in workers' average unemployment pessimism along the extensive margin, which lowers the temporary equilibrium values of capital capacity utilization and output growth by reducing workers' average propensity to consume (per (11)):

(22) $$\frac{\partial\sigma^*}{\partial\rho} = \frac{\partial\sigma^*}{\partial\Phi}\frac{\partial\Phi}{\partial\rho} = \frac{-\sigma^*(1-\pi)\alpha(1-\alpha)}{[1-\Phi(\theta,\rho,\alpha)](1-\pi)+(1-c_c)\pi-\delta} < 0 .$$

Meanwhile, a rise in the proportionality parameter α is equivalent to a decrease in workers' average unemployment pessimism along the intensive margin, which raises the temporary equilibrium values of the rates of capital capacity utilization and output growth by lifting workers' average propensity to consume (per (12)):

(23) $$\frac{\partial\sigma^*}{\partial\alpha} = \frac{\partial\sigma^*}{\partial\Phi}\frac{\partial\Phi}{\partial\alpha} = \frac{\sigma^*(1-\pi)(\eta+2\alpha\rho)}{[1-\Phi(\theta,\rho,\alpha)](1-\pi)+(1-c_c)\pi-\delta} > 0 .$$

The unemployment rate is another variable of interest measuring the level of economic activity, it being related to the rate of capital capacity utilization as follows:

(24) $$u = 1-\frac{L}{N} = 1-\frac{L}{X}\frac{X}{K}\frac{K}{N} = 1-a\sigma k ,$$

where k is the capital to labor supply ratio. Therefore, given that the labor to output ratio a remains constant throughout and the capital to labor supply ratio is predetermined at any given moment in time, we can use the temporary equilibrium value of the rate of capital capacity utilization in (16) to express the temporary equilibrium value of the unemployment rate as follows:

(25) $$u^* = 1-ak\sigma^* \equiv u(a,k,\theta,\rho,\alpha,\gamma,\delta,c_c,\pi) .$$

Therefore, the temporary equilibrium values of the rates of capital capacity utilization and unemployment are inversely related, so that the respective comparative statics for the unemployment rate in the temporary equilibrium can be straightforwardly inferred from (19)-(23).

Moreover, note the asymmetry involved in (21) and (22), in that $\partial\sigma^* / \partial\theta > \left|\partial\sigma^* / \partial\rho\right|$, which implies that $\partial g^* / \partial\theta > \left|\partial g^* / \partial\rho\right|$ and $\left|\partial u^* / \partial\theta\right| > \partial u^* / \partial\rho$. Thus, a higher ρ accompanied by a less than proportionally higher θ may still imply higher instead of lower rates of capital capacity utilization and output growth, and a lower instead of higher unemployment rate.

3. Unemployment expectations across workers as driven by evolutionary dynamics

The dynamics of the economy is driven by changes in the labor supply, N, the aggregate stock of capital, K, and the frequency distribution of unemployment expectations in the population of workers, (η, θ, ρ), which are predetermined at any given moment in time. While the capital stock varies positively over time as determined by the aggregate demand-driven output growth rate in (17), we assume that the growth rate of the labor supply is such that the existing labor surplus is continually replenished to an extent sufficing to avoid that labor ever becomes a constraint to capital accumulation and output growth. More specifically, we innocuously simplify matters by assuming that the labor supply grows endogenously at the same rate as capital accumulation, so that the capital to labor supply ratio k remains constant over time. Using the well-known Harrodian terminology, the aggregate demand-driven warranted growth rate in (17) is unique, stable (per (18)) and equal to the natural growth rate (which is equal to the growth rate of the labor supply, as labor productivity is constant) through the endogenous adjustment of the latter. Therefore, the dynamics of the economy is crucially driven by changes in the frequency distribution of unemployment expectations across workers, as explored in what follows.

3.1 Evolutionary expectation dynamics

Working households are assumed to form unemployment expectations constrained by their inescapable uncertain knowledge about the future. As a result, workers revise (and possibly switch) their unemployment expectations for the relevant future under conditions of bounded rationality. In this context, heterogeneity in working households' views about unemployment even in the proximate future cannot be considered to reflect ignorance or irrationality on the part of them, but should instead be seen as reflecting their reasonably different perceptions and beliefs with respect to an uncertain future.

The evolutionary revision protocol driving the dynamics of the frequency distribution of unemployment expectations in the population of workers draws to some extent on Keynes' (1937) suggestion of a practical theory of the future. According to Keynes, since decision-making is constrained by the inescapable uncertain knowledge about the future, economic agents have devised for the purpose several strategies. One is to suppose that the present is a useful guide to the future and mostly disregard the possibility of future changes about which they simply know nothing. Another is to presume that the existing state of opinion as expressed in prices and the existing output (and hence employment, we would add) is based on a correct summing up of future prospects, and thus can be taken as such unless and until something new and relevant arises. Yet another strategy involves economic agents, aware as they are that their own individual judgment is insufficient, attempting to conform with the behavior of the majority or the average which is possibly better informed.

Our specification of workers' unemployment expectations as evolutionary expectation dynamics incorporates several features of Keynes' (1937) appealing suggestion of a practical theory of the future. First, the possibility of decision-makers adopting different strategies to deal with the inevitable uncertainty about even the proximate future indicates that heterogeneity in workers' unemployment expectations is potentially persistent despite being endogenously time-varying through periodic revision. Second, workers may reasonably conceive of the present as a reliable guide to the relevant future and disregard the possibility of a change in unemployment in the proximate future. Third, and relatedly, the existing state of opinion as expressed in the current unemployment may be seen by workers as reflecting a correct summing up of unemployment prospects in the proximate future. Fourth, the state of opinion, as more broadly reflected in the current popularity of optimistic, pessimistic and neutral unemployment expectations across workers, plays in and of itself a key role in the revision protocol of workers attempting to conform with the behavior of the majority or the average which is possibly better informed about the proximate future. Fifth, it is sensible for a worker forming unemployment expectation for the proximate future to take into special account the respective expectation of the majority, in that the latter may become a self-fulfilling expectation by crucially affecting aggregate spending and hence the level of economic activity. In a nutshell, it is reasonable that the state of opinion about the unemployment prospects in the proximate future and the current unemployment, both in and of itself and as a perceived reflection one of the other, participate as key drivers in our specification of workers' unemployment expectations as following evolutionary revision dynamics.

While the current unemployment is an easily accessible public information when workers are revising their unemployment expectations, we assume that the state of opinion about the unemployment prospects in the proximate future is conventionally inferred by workers from a survey measure similar to that computed with data collected in the U.S. Michigan survey. As intimated earlier, this survey measure is defined as a balance score equal to the percentage of households who thought the unemployment rate would increase minus the percentage who thought it would fall, plus 100.[3] Therefore, in terms of our modelling framework, the related U.S. Michigan survey measure can be expressed as $100(\rho-\theta+1)$, which range from 0 to 200. In order to better attend our modelling purposes, we rescale such a measure of the state of opinion about the unemployment prospects in the proximate future as:

(26) $$B=(\rho-\theta+1)/2,$$

which range from 0 to 1. Therefore, while in the U.S. Michigan survey the baseline with $\rho=\theta$ is 100, in our alternative specification it is $1/2$.

Given that workers can hold either optimistic or pessimistic or neutral unemployment expectations for the proximate future, we express the dynamics of the frequency distribution of unemployment expectations across them with the following system of differential equations in the proportions of optimistic and pessimistic workers, respectively:

(27) $$\dot{\theta}=(1-\theta-\rho)\left(\frac{1}{1+e^{B}}\right)+\rho\left(\frac{ak\sigma^{*}}{1+e^{B}}\right)-\theta\left(\frac{1}{1+e^{B}}\right)-\theta\left(\frac{1-ak\sigma^{*}}{1+e^{B}}\right),$$

and

[3] See https://data.sca.isr.umich.edu/data-archive/mine.php for complete time series monthly data for these percentages back to 1978. The main survey of unemployment expectations in the European Union countries similarly monthly asks households how do they expect the number of people unemployed in the country to change over the next 12 months. Answers include 'increase sharply' (PP), 'increase slightly' (P), 'remain the same' (E), 'fall slightly' (M), 'fall sharply' (MM), and 'don't know' (DK), where the latter usually comprises a very small percentage of all the answers. The composite indicator for the EU is built from surveys conducted by national institutes in the Member States and the candidate countries (https://ec.europa.eu/info/sites/info/files/bcs_user_guide_en_0.pdf). The balance score measure for the EU survey is given by $(0.5P+PP)-(0.5M+MM)$.

(28) $\dot{\rho} = (1-\theta-\rho)\left(\frac{1}{1+e^{-B}}\right) + \theta\left(\frac{1-ak\sigma^*}{1+e^{-B}}\right) - \rho\left(\frac{1}{1+e^{B}}\right) - \rho\left(\frac{ak\sigma^*}{1+e^{B}}\right)$,

where σ^* is the temporary equilibrium value of the capital capacity utilization in (16), which depends on the frequency distribution of unemployment expectations across workers. The state space of the dynamical system in (27)-(28) is given by $\Theta = \{(\theta,\rho) \in \mathbb{R}^2_+ : (\eta,\theta,\rho) \in \Delta\}$.

The substance of the terms in the RHS of the expression in (26) is the following. One potential source of inflow to the subpopulation of optimistic workers is the subpopulation of workers who expected that the unemployment rate would remain the same, whose proportion is given by $1-\theta-\rho$. We assume that the propensity (or proclivity) of neutral workers to switch to optimism reasonably varies negatively with the pessimism of the state of opinion as inferred from the balance score in (26). More precisely, the propensity (or proclivity) of neutral workers to switch to optimism is a decreasing logistic function of the balance score B. Therefore, the first term in the RHS of the expression in (27), given by $(1-\theta-\rho)\left(\frac{1}{1+e^{B}}\right) \in [0,1) \subset \mathbb{R}$, is the measure of the inflow of neutral workers to the subpopulation of optimistic workers. Another potential source of inflow to the subpopulation of optimistic workers is the subpopulation of pessimistic workers, who expected that the unemployment rate would rise. We assume that the propensity of pessimistic workers to switch to optimism varies negatively (as described by a logistic function) with the pessimism of the state of opinion as expressed in the balance score in (26), and positively with the current employment rate given by $ak\sigma^*$ (per (25)). Considering (25) and the assumption that $0 < u^* < 1$, it follows that $0 < ak\sigma^* < 1$. Therefore, the second term in the RHS of the expression in (27), which is given by $\rho\left(\frac{ak\sigma^*}{1+e^{B}}\right) \in [0,1) \subset \mathbb{R}$, is the measure of the inflow of pessimistic workers to the subpopulation of optimistic workers.

Meanwhile, the third and fourth terms in the RHS of the expression in (27) denote the outflow from the subpopulation of optimistic workers to the subpopulations of neutral and pessimistic workers, respectively. While the propensity (or proclivity) of optimistic workers to switch to neutrality varies positively with the pessimism of the state of opinion as deduced from the balance score measure given by B, the propensity of optimistic workers to switch to pessimism varies positively with both the pessimism of the state of opinion as inferred from the balance score given by B, and the current unemployment rate given by $1-ak\sigma^*$ in (25). Therefore, the propensity to switch to a neighboring unemployment expectation type (for instance, from optimistic to neutral or vice versa) depends only on the state of opinion as expressed in the balance score B, while the propensities to the two farthest expectation switches (from optimism to pessimism or vice versa) depend on the current unemployment rate in addition to the state of opinion as deduced from the balance score measure B.

The substance of the terms in the RHS of the expression in (28) is analogous. One potential source of inflow to the subpopulation of pessimistic workers is the subpopulation of neutral workers, whose proportion is given by $1-\theta-\rho$. We suppose that the propensity (or proclivity) of neutral workers to switch to unemployment pessimism plausibly varies positively (in a logistic manner) with the pessimism of the state of opinion as inferred from the balance score in (26). Therefore, the first term in the RHS of the expression in (28) is the measure of the inflow of neutral workers to the subpopulation of pessimistic workers. Another potential source of inflow to the subpopulation of pessimistic workers is the subpopulation of optimistic workers, who expected that the unemployment rate would fall. We suppose that the propensity of optimistic workers to turn pessimistic varies positively with the pessimism of the state of opinion as expressed in the balance score measure given by B, and negatively with the current employment rate given by $ak\sigma^*$ (per (25)). Therefore, the second term in the RHS of the expression in (28) is the measure of the inflow of optimistic workers to the subpopulation of pessimistic workers.

Meanwhile, the third and fourth terms in the RHS of the expression in (28) denote the outflow from the subpopulation of pessimistic workers to the subpopulations of neutral and optimistic workers, respectively. While the propensity (or proclivity) of pessimistic workers to switch to neutrality varies negatively (as described by a logistic function) with the pessimism of the state of opinion as deduced from the balance score B in (26), the propensity of pessimistic workers to become optimistic varies negatively with both the pessimism of the state of opinion as inferred from the balance score given by B, and the current unemployment rate given by $1-ak\sigma^*$ in (25). Therefore, as in the dynamics in (27), the propensity to switch to a neighboring unemployment expectation type (for instance, from pessimistic to neutral or vice versa) depends only on the state of opinion as inferred from the balance score B, while the propensities to the two farthest expectation switches (from pessimism to optimism or vice versa) depends on both the state of opinion as inferred from B and the current unemployment rate.

Notice that all the six different types of propensities to switch in (27)-(28) are strictly positive but smaller than one (given that $0 < ak\sigma^* < 1$) and endogenously time-varying (as they depend on the frequency distribution of unemployment expectations across workers). In some sense, this behavior involves a kind of 'spontaneous urge to switch expectation rather than not to switch', analogous to Keynes' notion of animal spirits ("a spontaneous urge to action rather than inaction") on the part of decision makers facing an uncertain future. However, note that the specification of the propensities to switch in (27)-(28) incorporates a bias for "neighboring switches", or even a "neutral-reverting" proclivity. This is evident in the case of the propensity (or proclivity) of neutral workers to switch to either pessimism or optimism, which are the two possible unemployment expectation switches available to neutral workers. In fact, note in (27) that the propensity (or proclivity) of neutral workers to switch to optimism is greater than the propensity (or proclivity) of pessimistic workers to make the same switch, given that $\left(\frac{1}{1+e^B}\right) > \left(\frac{ak\sigma^*}{1+e^B}\right)$, recall that $0 < ak\sigma^* < 1$. It is also the case that the propensity (or proclivity) of optimistic workers to switch to neutrality is greater than the propensity (or proclivity) of these workers to switch to pessimism, as $\left(\frac{1}{1+e^{-B}}\right) > \left(\frac{1-ak\sigma^*}{1+e^{-B}}\right)$.

Considering the definition of the balance score in (26), after some algebraic manipulations the dynamical system in (27) and (28) conveniently simplifies to:

(27a) $$\dot{\theta} = \theta\left[-1+\theta+\frac{2(1-\theta)-\rho}{1+e^B}+ak\sigma^*\rho\right],$$

and

(28a) $$\dot{\rho} = \rho\left[1-\rho-\frac{2(1-\rho)-\theta}{1+e^B}-ak\sigma^*\theta\right].$$

As shown in Appendix 1, the evolutionary expectation dynamics in (27a)-(28a) have three monomorphic evolutionary equilibria. These evolutionary equilibria feature survival of a single type of unemployment expectation by working households in each, which is clearly at odds with the empirical evidence of long persistence of heterogeneity in unemployment expectations across working households. However, we also show in Appendix 1 that under certain conditions there is a unique polymorphic evolutionary equilibrium featuring the coexistence of the three types of unemployment expectations across working households. In effect, there is a unique polymorphic evolutionary equilibrium (θ^*, ρ^*) with $\rho^* = \theta^*$ and $\theta^* \in (0,1/2) \subset \mathbb{R}$ if the following condition regarding the rate of capital capacity utilization is satisfied:

(29) $$\sigma(0,1,\alpha,\gamma,\delta,c_c,\pi) > \frac{e}{ak(e+\sqrt{e})}.$$

The condition in (29) is empirically plausible. In fact, if $ak \cong 1$ we have $\frac{e}{ak(e+\sqrt{e})} \cong 0.62$. As shown in Appendix 2, this polymorphic evolutionary equilibrium is locally unstable, and it is shown in Appendix 3 that there is a limit cycle around it. Under quite general conditions, the frequency distribution of unemployment expectations across workers and thus the rates of capacity utilization, unemployment and output growth, all experience self-sustaining cyclical fluctuations over time. Hence the model succeeds in replicating qualitatively the cyclical fluctuations shown in Figures 1 and 2.

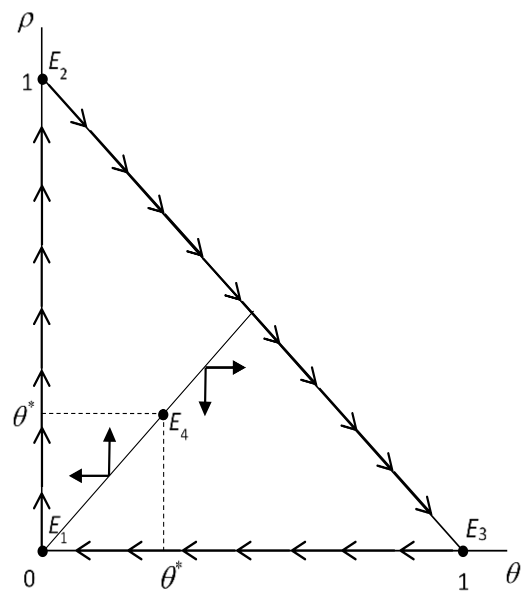

Figure 3. Cyclical fluctuations in the distribution of unemployment expectations across working households.

4. Conclusions

There is robust survey evidence on the non-negligibility and persistence of heterogeneity in the workers' unemployment expectations. There is also extensive empirical evidence that such survey-based unemployment expectations on the part of workers and actual unemployment are positively correlated. Building on these motivating pieces of empirical evidence, we set forth an aggregate demand-led macrodynamic model augmented with heterogeneity in unemployment expectations across working households.

Our modelling exercise has indicated that more pessimistic unemployment expectations across workers (in the extensive or intensive margin) raises actual unemployment. Therefore, the model set forth in this paper yields the positive correlation between unemployment expectations across workers and actual unemployment as it has been consistently found, for instance, with data from the U.S. Michigan survey. Moreover, the frequency distribution of unemployment expectations across working households is endogenously time-varying as driven by evolutionary dynamics of expectation revision, as a result of which this frequency distribution (and therefore the level of economic activity) is prone to experience self-sustaining cyclical fluctuations over time. Thus, the model set forth in this paper succeeds in replicating qualitatively the empirically documented evidence on cyclical fluctuations both in the frequency distribution of unemplyment expectations across working households and in the rate of unemployment shown in Figures 1 and 2, respectively.

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Appendix 1: Existence of multiple evolutionary equilibria

(*i*) *The dynamical system in (27a)–(28a) features three monomorphic evolutionary equilibria.*

It is straightforward to verify that $\dot{\theta}=\dot{\rho}=0$ at $(\theta,\rho)=(0,0)$, that is, the state with all workers expecting that the unemployment rate will remain the same ($\eta=1$) in the proximate future (deemed as relevant for current consumption decisions) is an equilibrium of the evolutionary expectation dynamics in (27a)-(28a). Meanwhile, if $\theta=0$, it follows that $\dot{\theta}=0$ and $\dot{\rho}=\rho(1-\rho)\left[\frac{e^{(1+\rho)/2}-1}{1+e^{(1+\rho)/2}}\right]$. If $\rho>0$, then $e^{(1+\rho)/2}>1$ and, consequently, we obtain $\dot{\rho}=0$ if, and only if, $\rho=1$. Therefore, the state featuring all workers expecting that the unemployment rate will rise ($\rho=1$) in the relevant proximate future is also an equilibrium of the evolutionary expectation dynamics in (27a)-(28a). And if $\rho=0$, it follows that $\dot{\theta}=\theta(1-\theta)\left[\frac{1-e^{(1-\theta)/2}}{1+e^{(1-\theta)/2}}\right]$ and $\dot{\rho}=0$. Assuming that $\theta>0$, we obtain $\dot{\theta}=0$ if, and only if, $\theta=1$. Consequently, the state with all workers expecting that the unemployment rate will fall in the relevant proximate future is also an equilibrium of the evolutionary expectation dynamics in (22a-23a).

(*ii*) *Existence and uniqueness of a polymorphic evolutionary equilibrium in the interior of the state space*

Let us show that there is a unique polymorphic evolutionary equilibrium $(\theta^*,\rho^*)\in\Theta$ if the empirically plausible condition in (29) holds. Suppose that $0<\theta<1$ and $0<\rho<1$. Therefore, based on the dynamical system in (27a)-(28a), we have $\dot{\theta}=\dot{\rho}=0$ if the following conditions are satisfied:

(A-1.1) $$\sigma^*=\frac{1-\theta-\frac{2(1-\theta)-\rho}{1+e^{(1+\rho-\theta)/2}}}{ak\rho}=\frac{1-\rho-\frac{2(1-\rho)-\theta}{1+e^{(1+\rho-\theta)/2}}}{ak\theta}.$$

The second equality in (A-1.1) is satisfied for a state $(\theta,\rho)\in\Theta$ in which either $\rho=\theta$ for all $\theta\in(0,1/2]\subset\mathbb{R}$ or $\rho=1-\theta$ for all $\theta\in(0,1]\subset\mathbb{R}$.

Assume that $\rho=\theta$ for all $\theta\in(0,1/2]\subset\mathbb{R}$. Thus, considering the temporary equilibrium value of capital capacity utilization in (16) and the evolutionary equilibrium condition in (A-1.1), there exists a polymorphic evolutionary equilibrium $(\theta^*,\rho^*)\in\Theta$ with $\rho^*=\theta^*\in(0,1/2)\subset\mathbb{R}$ if, and only if, the following condition is satisfied:

(A-1.2) $$\sigma(\theta^*,\theta^*,\alpha,\gamma,c_c,\pi)=\frac{1}{ak\theta^*}\left(1-\theta^*-\frac{2-3\theta^*}{1+\sqrt{e}}\right).$$

Let $\phi(\theta)\equiv\sigma(\theta,\theta,\alpha,\gamma,c_c,\pi)-\frac{1}{ak\theta}\left(1-\theta-\frac{2-3\theta}{1+\sqrt{e}}\right)$. We will demonstrate that there is a unique $\theta^*\in(0,1/2)\subset\mathbb{R}$ such that $\phi(\theta^*)=0$ if the condition in (29) holds, given the vector of parameters $(\alpha,\gamma,\delta,c_c,\pi)$.

For any $\alpha \in (0,1) \subset \mathbb{R}$ and $(\theta,\rho) \in \Theta$ the function $\Phi(\theta,\rho,\alpha)$ in (9) is continuous. Given such a continuity and the assumption in (18), the temporary equilibrium capital capacity utilization in (16) is also a continuous function. Consequently, $\sigma(0,0,\alpha,\gamma,\delta,c_c,\pi) = \dfrac{\gamma}{(1-\alpha)(1-\pi)+(1-c_c)\pi-\delta}$ and $\sigma(1/2,1/2,\alpha,\gamma,\delta,c_c,\pi) = \dfrac{\gamma}{[(1-\alpha^2)/2](1-\pi)+(1-c_c)\pi-\delta}$ are both well defined. We can then conclude that:

(A-1.3) $$\lim_{\theta\to 0^+}\phi(\theta) = \lim_{\theta\to 0^+}\sigma(\cdot) - \lim_{\theta\to 0^+}\frac{1}{ak\theta}\left(1-\theta-\frac{2-3\theta}{1+\sqrt{e}}\right) = \sigma(0,0,\alpha,\gamma,\delta,c_c,\pi) - \infty = -\infty .$$

Given (A-1.3), it follows that for every real $A<0$ there is some $\varepsilon \in (0,1) \subset \mathbb{R}$ such that, for all $\theta \in (0,\varepsilon) \subset \mathbb{R}$ we have $\phi(\theta) < A$. Hence, we can take $\theta = \varepsilon/2$ such that $\phi(\varepsilon/2) < A < 0$.

Given (18) and (19), we know that $\sigma(\theta,\rho,\alpha,\gamma,\delta,c_c,\pi)$ is a strictly increasing (decreasing) function of θ (ρ). Thus, we know that $\sigma(0,1,\alpha,\gamma,\delta,c_c,\pi)$ is the lowest possible value that the rate of capital capacity utilization can take in the temporary equilibrium, which implies that $\sigma(1/2,1/2,\alpha,\gamma,\delta,c_c,\pi) \geq \sigma(0,1,\alpha,\gamma,\delta,c_c,\pi)$. Based on such inequality and the condition in (29), we can conclude that:

(A-1.4) $$\phi(1/2) = \sigma(1/2,1/2,\alpha,\gamma,\delta,c_c,\pi) - \frac{e}{ak(e+\sqrt{e})} > 0 .$$

Considering that ϕ is continuous in the interval $[\varepsilon/2,1/2] \subset [0,1] \subset \mathbb{R}$, $\phi(\varepsilon/2) < 0$ for some $\varepsilon \in (0,1) \subset \mathbb{R}$, and (per (29)) $\phi(1/2) > 0$, we can then apply the intermediate value theorem and conclude that there is some $\theta^* \in (0,1/2) \subset \mathbb{R}$ such that $\phi(\theta^*) = 0$.

Moreover, given the functions in (5) and (12), we obtain the following derivative of the function ϕ evaluated at any $\theta \in (0,1/2] \subset \mathbb{R}$:

(A-1.5) $$\frac{\partial\phi(\theta)}{\partial\theta} = \frac{\sigma(\theta,\theta,\alpha,\gamma,\delta,c_c,\pi)(1-\pi)(1-\alpha)^2}{[1-\Phi(\theta,\theta,\alpha)](1-\pi)+(1-c_c)\pi-\delta} + \frac{e^{1/2}-1}{ak(1+e^{1/2})} > 0 .$$

Hence, since the derivative in (A-1.5) is continuous in the interval $[\delta/2,1/2] \subset [0,1] \subset \mathbb{R}$, there is a unique $\theta^* \in (0,1/2) \subset \mathbb{R}$ such that $\phi(\theta^*) = 0$.

Now suppose that $\rho = 1-\theta$ for all $\theta \in (0,1) \subset \mathbb{R}$. Let us show that there is no evolutionary equilibrium in such locus. Given the temporary equilibrium value of capital capacity utilization in (12) and the evolutionary equilibrium condition in (A-1.1), there would exist a polymorphic evolutionary equilibrium $(\theta^*,\rho^*) \in \Theta$ with $\rho^* = 1-\theta^* \in (0,1) \subset \mathbb{R}$ if the following condition was satisfied:

(A-1.6) $$\sigma(\theta^*,1-\theta^*,\alpha,\gamma,c_c,\pi) = \frac{1}{ak}\left(\frac{e^{1-\theta^*}}{1+e^{1-\theta^*}}\right).$$

Given the partial derivatives in (18) and (19), we can determine the sign of the directional derivative along the ray $\rho = 1-\theta$ for all $\theta \in [0,1] \subset \mathbb{R}$ as follows:

(A-1.7) $$\left.\frac{d\sigma(\theta,\rho,\alpha,\gamma,\delta,c_c,\pi)}{d\theta}\right|_{\rho=1-\theta} = \frac{\partial\sigma(\theta,1-\theta,\alpha,\gamma,\delta,c_c,\pi)}{\partial\theta} - \frac{\partial\sigma(\theta,1-\theta,\alpha,\gamma,\delta,c_c,\pi)}{\partial\rho} > 0 .$$

Let $\varphi(\theta) = \sigma(\theta,1-\theta,\alpha,\gamma,\delta,c_c,\pi) - \frac{1}{ak}\left(\frac{e^{1-\theta}}{1+e^{1-\theta}}\right)$. It follows from the assumption in (29) that $\varphi(0) = \sigma(0,1,\alpha,\gamma,\delta,c_c,\pi) - \frac{1}{ak}\left(\frac{\sqrt{e}}{1+\sqrt{e}}\right) > 0$. Since φ is a continuous and strictly increasing function at any $\theta \in [0,1] \subset \mathbb{R}$, it follows that $\varphi(\theta) > 0$ for any $\theta \in [0,1] \subset \mathbb{R}$. Thus, there is no $\theta^* \in [0,1] \subset \mathbb{R}$ such that $\varphi(\theta^*) = 0$ and, hence there is no equilibrium along the ray $\rho = 1-\theta$ for all $\theta \in [0,1] \subset \mathbb{R}$.

Appendix 2: Local stability analysis

(*i*) *The evolutionary equilibrium* $(0,0)$ *of the dynamical system in (27a)-(28a) is locally saddle-point unstable.*

Consider the Jacobian matrix of the linearization around the evolutionary equilibrium $(0,0) \in \Theta$ of the system in (27a)-(28a), which is given by:

(A-2.1) $$J(0,0) = \left[\begin{array}{c|c} -1+\frac{2}{1+\sqrt{e}} & 0 \\ \hline 0 & 1-\frac{2}{1+\sqrt{e}} \end{array}\right],$$

with eigenvalues $\alpha_1 = -1+\frac{2}{1+\sqrt{e}}$ and $\alpha_2 = 1-\frac{2}{1+\sqrt{e}}$. As $\alpha_1 < 0$ and $\alpha_2 > 0$, the equilibrium $(0,0)$ of the system in (26a)-(27a) is locally saddle-point unstable. The economically relevant segment of the stable arm of such saddle is given by $\left\{(\theta,\rho) \in \mathbb{R}^2_+ : 0 < \theta < 1, \rho = 0\right\}$, given that for any point in this set we have $\dot{\theta} = \theta(1-\theta)\left[\frac{1-e^{(1-\theta)/2}}{1+e^{(1-\theta)/2}}\right] < 0$ and $\rho = 0$. The respective unstable arm, meanwhile, is given by $\left\{(\theta,\rho) \in \mathbb{R}^2_+ : \theta = 0, 0 < \rho < 1\right\}$, where $\dot{\theta} = 0$ and $\dot{\rho} = \rho(1-\rho)\left[\frac{e^{(1+\rho)/2}-1}{1+e^{(1+\rho)/2}}\right] > 0$.

(*ii*) *The evolutionary equilibrium* $(0,1)$ *of the dynamical system in (27a)-(28a) is locally saddle-point unstable.*

Consider the Jacobian matrix of the linearization around the evolutionary equilibrium $(0,1) \in \Theta$ of the system in (27a)-(28a), which is given by:

(A-2.2) $$J(0,1)=\left[\begin{array}{c|c} -1+\frac{1}{1+e}+ak\sigma(0,1,\alpha,\gamma,\delta,c_c,\pi) & 0 \\ \hline \frac{1}{1+e}-ak\sigma(0,1,\alpha,\gamma,\delta,c_c,\pi) & -1+\frac{2}{1+e} \end{array}\right],$$

and whose eigenvalues are $\alpha_1=\frac{1-e}{1+e}$ and $\alpha_2=\frac{(1+e)ak\sigma(0,1,\alpha,\gamma,\delta,c_c,\pi)-e}{1+e}$. Therefore, it is straightforward to verify that $\alpha_1<0$. Since $\frac{\sqrt{e}}{1+\sqrt{e}}=\frac{e}{1+\sqrt{e}}>\frac{e}{1+e}$, it follows from the condition in (24) that $\alpha_2>0$, so the evolutionary equilibrium $(0,1)$ of the system in (26a)-(27a) is locally saddle-point unstable. The stable arm of this saddle equilibrium is given by $\left\{(\theta,\rho)\in\mathbb{R}_+^2:\theta=0,0<\rho<1\right\}$, as for any point in such set we have $\dot{\theta}=\theta(1-\theta)\left[\frac{1-e^{(1-\theta)/2}}{1+e^{(1-\theta)/2}}\right]<0$. The respective unstable arm is given by $\left\{(\theta,\rho)\in\mathbb{R}_+^2:0<\theta<1,\theta+\rho=1\right\}$, where $\dot{\theta}=\theta(1-\theta)ak\varphi(\theta)=-\dot{\rho}>0$ and $\varphi(\theta)=\sigma(\theta,1-\theta,\alpha,\gamma,\delta,c_c,\pi)-\frac{1}{ak}\left(\frac{e^{1-\theta}}{1+e^{1-\theta}}\right)$. As it has been demonstrated in the end of Appendix 1, we have $\varphi(\theta)>0$ for any $\theta\in[0,1]\subset\mathbb{R}$ from the condition (28). Therefore, we can conclude that $\dot{\theta}=\theta(1-\theta)ak\varphi(\theta)=-\dot{\rho}>0$ for any point in the set $\left\{(\theta,\rho)\in\mathbb{R}_+^2:0<\theta<1,\theta+\rho=1\right\}$.

(*iii*) *The evolutionary equilibrium* $(1,0)$ *of the dynamical system in (27a)-(28a) is locally unstable.*

Consider the Jacobian matrix of the linearization around the equilibrium $(1,0)\in\Theta$ of the system in (22a)-(23a), which is given by:

(A-2.3) $$J(1,0)=\left[\begin{array}{c|c} 0 & -\frac{1}{2}+ak\sigma(1,0,\alpha,\gamma,\delta,c_c,\pi) \\ \hline 0 & \frac{1}{2}-ak\sigma(1,0,\alpha,\gamma,\delta,c_c,\pi) \end{array}\right],$$

and whose eigenvalues are $\alpha_1=\frac{1}{2}-ak\sigma(1,0,\alpha,\gamma,c_c,\pi)$ and $\alpha_2=0$. Since the latter eigenvalue is equal to zero, the Hartman-Grobman theorem does not apply. Hence we cannot use the linearization around the equilibrium $(1,0)\in\Theta$ to deduce its local stability properties.

We will then apply the Chetaev's instability theorem (Khalil, 2002, p. 125, Theorem 4.3). Let $x=\theta-1$ and $y=\rho-0$ be the deviations of state (θ,ρ) from the evolutionary equilibrium given by $(1,0)$. We can re-write the dynamical system in (27a)-(28a) in terms of these deviation variables as follows:

(A-2.4) $$\dot{x}=(x+1)\left[x+ak\sigma(x+1,y)y-\frac{2x+y}{1+e^{(y-x)/2}}\right],$$

and

(A-2.5) $\dot{y} = y\left[1 - y - ak\sigma(x+1, y)(x+1) + \dfrac{x+2y-1}{1+e^{(y-x)/2}}\right].$

Thus, the equilibrium $(0,0)$ of the system in (A-2.4)-(A-2.5) corresponds to the equilibrium $(1,0)$ of the system in (27a)-(28a).

Consider the following function:

(A-2.6) $V(x, y) = \dfrac{(x+y)^2}{2}$.

Let us show that the function in (A-2.6) is a Chetaev function in the open set defined by:

(A-2.7) $G = \{(x, y) \in \mathbb{R}^2 : -1/2 < x < 0, 0 < y < 1/2, x + y < 0\}$.

Note that $V(x, y) > 0$ for all $(x, y) \in G$, since $x + y < 0$ in G, and $V(0,0) = 0$. Therefore, (A-2.6) can take positive values arbitrarily close to 0. That is, the stationary equilibrium $(0,0)$ is on the boundary of the open set G.

Based on (A-2.4), (A-2.5), and (A-2.6), we obtain:

(A-2.8) $\dot{V}(x, y) = \dfrac{\partial V(x, y)}{\partial x}\dot{x} + \dfrac{\partial V(x, y)}{\partial y}\dot{y} = \dfrac{\left(e^{y/2} - e^{x/2}\right)(x+y)^2 + (1+x-y)}{e^{y/2} + e^{x/2}}$.

The denominator in (A-2.8) is strictly positive. Thus, the sign of $\dot{V}(x, y)$ in (A-2.8) depends on the sign of the numerator. Since for any point $(x, y) \in G$ we have $x < 0 < y$, it follows that $\left(e^{y/2} - e^{x/2}\right)(x+y)^2 > 0$ for all $(x, y) \in G$. Also, at any point $(x, y) \in G$ we have $x + y < 0$ such that $-y > x$. Since for any point $(x, y) \in G$ we also have $-1/2 < x < 0$, it follows that $1 + x > 0$. Using the inequalities $-y > x$ and $1 + x > 0$, we can have that $1 + x - y > 1 + 2x > 0$ for all $(x, y) \in G$. It follows that $\dot{V}(x, y) > 0$ for any point $(x, y) \in G$ such that $\sqrt{x^2 + y^2} < r$, for a given ray $r \in (0, 1/2] \subset \mathbb{R}$. Thus, the function in (A-2.6) is a Chetaev function. And by the Chetaev's instability theorem we can conclude that the equilibrium (0,0) is a repulsor in G and, hence the equilibrium $(1,0)$ of the system in (27a)-(28a) is locally unstable.

Appendix 3: Occurrence of endogenous fluctuations in economic activity

Let $\text{int}\,\Delta \equiv \left\{(\eta, \theta, \rho) \in \Delta : \eta > 0, \theta > 0, \rho > 0\right\}$ be the interior of the simplex represented by $\Delta \equiv \left\{(\eta, \theta, \rho) \in \mathbb{R}^3_+ : \eta + \theta + \rho = 1\right\}$ and $\text{int}\,\Theta = \{(\theta, \rho) \in \mathbb{R}^2_+ : (\eta, \theta, \rho) \in \text{int}\,\Delta\}$ be the interior of the state space Θ (or projection of the simplex on the first quadrant in the plane). Firstly, we will show that the boundary of the state space, given by $\text{bd}\Theta = \Theta \setminus \text{int}\,\Theta$, is positively invariant. In Appendix 1, we show that the points $(0,0)$, $(1,0)$ and $(0,1)$ in that boundary are (monomorphic) evolutionary equilibria. Thus, if the system in (27a)-(28a) starts at one of these states it remains there. Meanwhile, in Appendix 2 we demonstrate that $\dot{\theta} < 0$ and $\dot{\rho} = 0$ at any point in the subset of the state space given by $\Theta_1 \equiv \left\{(\theta, \rho) \in \mathbb{R}^2_+ : 0 < \theta < 1, \rho = 0\right\}$ and $\dot{\theta} = 0$ and $\dot{\rho} > 0$ at any point in the subset of the state space given by $\Theta_2 \equiv \left\{(\theta,\rho) \in \mathbb{R}_+^2 : \theta = 0, 0 < \rho < 1\right\}$. Thus, if the system in (27a)-(28-a) starts at any point in one of these two subsets of the state space, it remains there. Finally, it is straightforward to verify that $\dot{\theta} = \theta(1-\theta)\left[\frac{-e^{1-\theta}}{1+e^{1-\theta}} + ak\sigma^*\right] = -\dot{\rho}$ at any point in the subset of the state space given by $\Theta_3 \equiv \left\{(\theta,\rho) \in \Theta : 0 < \theta < 1, \theta + \rho = 1\right\}$, so that if the system in (27a)-(28-a) starts in any point located in this subset of the state space, it also remains there. Since $\text{bd}\Theta = \left\{(0,0),(0,1),(1,0)\right\} \cup \Theta_1 \cup \Theta_2 \cup \Theta_3$, it follows that this set is positively invariant.

Based on the set of conditions under which the initial value problem of a system of differential equations has a unique solution (see Theorem 1 in Hirsch and Smale, 1974, Ch. 8), we know that the system in (27a)-(28a) has a unique solution for any given initial condition $(\theta_0, \rho_0) \in \Theta$. Thus, any two solutions of this system with two different initial conditions cannot cross each other.

Considering the positive invariance of the boundary of the state space, which is given by $\text{bd}\Theta$, and the uniqueness theorem above, it follows that a solution starting at any point $(\theta,\rho) \in \text{int}\,\Theta$ remains there, as any point $(\theta,\rho) \in \text{bd}\Theta$ belongs to a solution the respective orbit of which is a proper (or strict) subset of $\text{bd}\Theta$.

As shown in Appendix 2, the polymorphic equilibrium (θ^*, θ^*) is locally unstable. Hence, if the system in (27a)-(28a) starts at any initial condition $(\theta_0, \rho_0) \neq (\theta^*, \theta^*)$, with $(\theta_0, \rho_0) \in \text{int}\,\Theta$, it will not either converge to (θ^*, θ^*) or reach the boundary represented by $\text{bd}\Theta$. Thus, based on the Poincaré-Bendixson Theorem (see Hirsch and Smale, 1974, Ch. 11), we can conclude that there is a limit cycle in the subset of the interior of the state space represented by $\text{int}\,\Theta \setminus \{(\theta^*, \theta^*)\}$.