# A two-sector productivity model

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**Abstract**

We present a Post-Keynesian and Kaleckian two-sector productivity model in a short and a medium run specification. The short run features non-linear consumption, investment, import and export functions. We assume capitalists choose the sector with the highest return for their investment and reflect that in the investment function. The sectors produce different goods with different levels of labour productivity. We introduce a term for relative sector size which allows for the income distribution affecting structural change via spending patterns. More affluent consumers have a higher tendency to spend on goods produced in the sector with higher value added and hence higher labour productivity. Low-income households are more likely to spending on essential goods which are produced in the low-productivity sector. In the medium run specification, the focus shifts to the growth rates of output and productivity. Similar to Naastepad (2006), each sector has a productivity and a demand regime which can either be profit-led or wage-led. The combination of different regimes yields different scenarios for productivity growth.

## Introduction

Productivity has been a focus of economic policy researchers and practitioners for some time now, most definitely because it has been growing rather slowly across high-income economies for decades. As a result, trying to find out why has contributed to productivity research growing rapidly. In the European Union, countries are recommended to form national productivity boards and issue regular updates and reports (Conseil National de Productivité, 2019) while the United Kingdom founded its own *Productivity Institute* which derives both its name and purpose from researching the British "extreme" version of the productivity puzzle (Van Ark and Venables, 2020). Mainstream economic literature offers several explanations as to why that could be, including low within-firm productivity growth, a lack of investment, a productivity or Solow paradox[1], reallocation issues, an innovation slowdown, and others (Riley and Bondibene, 2016; Manyika et al., 2017; Conseil National de Productivité, 2019; CompNet, 2020; Cuadrado, Moral-Benito, and Solera, 2020; Van Ark, De Vries, and Erumban, 2020). Interestingly, there are many supply-side oriented, mainstream contributions whereas Post-Keynesian contributions focus more on other aspects like gender inequality and the resulting demand effects (Onaran, Oyvat, and Fotopoulou, 2019; Onaran, Oyvat, and Fotopoulou, 2022, which also feature more than one sector).

Traditionally, many Post-Keynesian contributions follow Bhaduri and Marglin (1990) and their groundbreaking model which distinguishes between an exhilarationist (or profit-led) and a stagnationist (or wage-led) regime. Basically, for exhilarationist regimes, output grows if the share of income going to capital, or the profit share, increases. The opposite is true for stagnationist regimes: output grows if the share of income going to wages, i.e. the wage share, rises. The key to this categorisation is investment; in other words, an economy's regime can react either more (exhilarationist) or less (stagnationist) sensitively to profitability. Just like Blecker (1989) and Dutt (1984) in a neo-/post-Keynesian tradition, Bhaduri and Marglin (1990) explicitly treat labour productivity as exogenous and constant although the multiplicative inverse of labour productivity is included into the mark-up price equation. While labour productivity is exogenous in this strand of the literature and the determination not explained within the models, it is still considered of utmost importance by their very authors since it determines the "size of the pie that the contending parties struggle over" (Marglin, 2017, p. 367) even though marginal productivity does not necessarily determine wages as neoclassical microeconomics would suggest.

Since then, several contributions have tried to endogenise productivity in various ways. Starting with Bhaduri and Marglin's (1990) and Blecker's (1989) exogenous productivity framework, both Hein and Tarassow (2010) and Naastepad (2006) argue that labour productivity should not be exogenous on an aggregate level and create models with endogenised productivity growth.

[1] The notion of a Solow paradox goes back to a book review by Robert Solow published in the *New York Times Book Review*. The idea is that increased use of computers does not necessarily translate into higher productivity: "You can see the computer age everywhere but in the productivity statistics." Source: http://www.standupeconomist.com/pdf/misc/solow-computer-productivity.pdf

Following the neo-Kaldorian tradition of the export-led cumulative causation (ELCC) models mostly going back to Setterfield and Cornwall (2002), Naastepad (2006) assumes a demand-determined rate of output growth and a so-called "reverse" Say's Law (Setterfield and Cornwall, 2002, p. 70). ELCC models often use Kaldor-Verdoorn channels in their productivity regimes (PR) with productivity growth being a linear function of the output growth rate (ibid.). In contrast to their counterpart, balance-of-payments-constrained (BPCG) growth models, productivity growth and the Verdoorn coefficient directly impact the equilibrium growth rate of output. Empirically, BPCG models are very suitable for representing long-run developments while ELCC are more useful descriptions of short- to medium-run growth (Blecker, 2013).

To our knowledge, there are hardly any Post-Keynesian models with more than one sector which also allow for considering productivity and/or structural change. While there are models with multiple sectors using a more orthodox theoretical background with different focal points (Ireland and Schuh, 2008; Buera, Kaboski, and Shin, 2011, e.g.), they do not focus on the impact of functional inequality on productivity or the supply side more generally. This is why we create a Post-Keynesian and Kaleckian inspired two-sector model with a focus on productivity, mainly following the framework created by Naastepad's (2006) and Hein and Tarassow's (2010) models. Nevertheless, we will deviate from their approaches: not only are we introducing two sectors with different levels of labour productivity, but we also implicitly use a more classical, and hence less traditionally Post-Keynesian, production function in addition to the very Keynesian expenditure approach. Instead of focusing on wage and profit *shares* only, we incorporate nominal wage levels and profit rates to have a closer look at the profitability argument and the distinction between exhilarationist and stagnationist regimes. For clarity, we distinguish between workers who earn wages and capitalists who make profit. We exclude cases where workers can also earn minor profits or capitalists receive wages in addition to their profits for simplicity (Pasinetti, 1962).

Our short-run model features variable sector sizes: this means that income distribution influences the size of sectors since groups with different income levels make different consumption choices and that in turn impacts capitalists' investment choices and net exports. However, we assume that there are equilibrium sector sizes which is why we move away from this concept for the medium run, assuming that structural change is a short-run, aggregate demand-driven phenomenon. Analytically, the medium run is simplified as much as possible and closely resembles Naastepad's (2006) example, but we add intersectoral interactions. The medium run version aims to explore different scenarios and combinations of wage-led and profit-led regimes for productivity and demand.

The model primarily addresses manufacturing dynamics and neglects perspectives on services since the productivity discussion for services, especially care work and services of the "other economy" (Donath, 2000) is a disputed issue. Productivity increases in services are not always desirable and often lead to a loss in quality or a fundamental change in the nature of the service. As our aim is to offer a Post-Keynesian and Kaleckian inspired view on the productivity puzzle of high-income countries which by definition is about the lack of productivity growth, we choose to focus on the manufacturing side of the debate.

We start off with the short run and the components of output and then move on to incorporating productivity and giving shocks to wages in sector I. We then explain the transition towards the medium run and explore different scenarios of combinations between wage-led and profit-led regimes for productivity and demand.

## The short run

First and foremost, we assume a traditional Keynesian demand-side/expenditure definition of output where $Y$ stands for output, $C$ for consumption, $I$ for investment, $G$ for government expenditure – which we choose to neglect in this model as we assume they are exogenous –, $X$ for (gross) exports and $M$ for imports:

$$Y = C + I + G + (X - M) = \Omega Y + (1 - \Omega)Y \tag{1}$$

The left side of equation 1 is just a simple, traditional aggregate demand function whereas the right side represents the spending pattern in the economy, i.e. on which products the left-hand side demand is spent: $0 \leq \Omega \leq 1$ is the share of output $Y$ spent on sector-I-goods and is a function of the wage levels in the two sectors: $\Omega = \Omega(w_1, w_2)$. Put differently, expenditure in sector I is defined as $Y_1 = \Omega Y$ whereas expenditure in sector II is defined as $Y_2 = (1 - \Omega)Y$. Essentially, $\Omega(w_1, w_2)$, i.e. the relative size of sector I with respect to sector II and the overall economy, depends on wages in both sectors. The sum of $[\Omega + (1 - \Omega)]Y = Y_1 + Y_2$, i.e. the two domestic sectors in the economy. This refers to the domestic demand: imports reduce domestic demand and exports increase it. Export demand is also either spent sector I or sector II and accordingly increases or decreases $\Omega$.

This distinction results from specific spending patterns for different levels of income. Poorer households spend more on essential goods while more affluent households have to allocate less of their income towards essential goods. Throughout this paper, we will assume that sector I is the sector with lower productivity and thus lower wages overall. In other words, we assume that sector I produces essential goods while sector II manufactures higher value added goods like investment goods, but also more durable consumer goods or luxury goods. If the composition of income changes such that wages for low-income households and workers increase, this will boost demand for sector I relatively more and thus positively affect relative sector size $\Omega$. Remember that $\Omega Y = Y_1$, i.e. sector I expands as a result. In turn profit incomes, of which a greater share is spent on non-essential sector-II-goods, will decline. Higher profits will relatively strengthen sector II, i.e. negatively affect $\Omega$. While lower paid sector-I workers and workers more generally will also strive to buy sector-II-goods with their additional income, the effect on consumption in sector II will always be greater when better-off households and capitalists have more disposable income. For investment and net exports, the argument follows from this logic via wage levels and depends on whether we are assuming a profit-led or wage-led regime.

To denote the expenditure of each sector, we use $\Omega_{ji}(w_i)$ for $i = 1, 2$ where the subscript $j = w, \pi$ denotes the source of income and tells us whether we talk about the expenditure patterns of wages or profits. The same logic as above applies to expenditure on sector II where $(1 - \Omega_{ji})$ for $j = w, \pi$ and $i = 1, 2$ denotes expenditure on sector-II-goods. For instance, $(1 - \Omega_{w2})(1 - \pi_2)$ is the share of wages earned in sector II spent on sector-II-goods.

Since all $\Omega_{ji}$ represent shares of income spent on domestic goods, all wages of sector $i$ are spent either on sector I or sector II, *or* on imported goods *or* saved. The latter two are summarised as the negative of the propensity to consume (MPC) domestically, $MPC = (1 - c_{ji})$ for $j = w, \pi$ and $i = 1, 2$. In other words this is the share of a specific type of income (wages or profits) from sector I or II that is either spent on imports or saved. For the dynamic of the model, it does not matter whether income is spent on imports or saved for the short run; it only impacts or economy when it is spent on goods from either domestic sector.

Figure 1 represents the flows between sectors from the consumption side and illustrates the direction of flows with respect to the marginal propensities to consume for households.[2] For each type of income in each sector, there are four expenditure sources: wages and profits for both domestic sectors and exports. Each type of income, wages and profits for both sectors, respectively, saves or spends on either imports, goods from sector I or sector II. Savings and imports reduce the overall expenditure on the two domestic sectors. The flows, or arrows in Figure 1, toward a sector represent demand for its goods: demand for each sector comes from those earning wages in sector I and II, capitalists of both sectors and exports. Please note that we neglect government expenditure in the model. Changing profits or wages in any sector has consequences for both sectors and the coefficients attached to the arrows reflect the size and mechanism of this process.

### Consumption

Importantly, we define two types of consumption which are characterised by their source of income: consumption out of wages $C_w$ and consumption out of profits $C_\pi$, both of which sum up to aggregate consumption $C$.

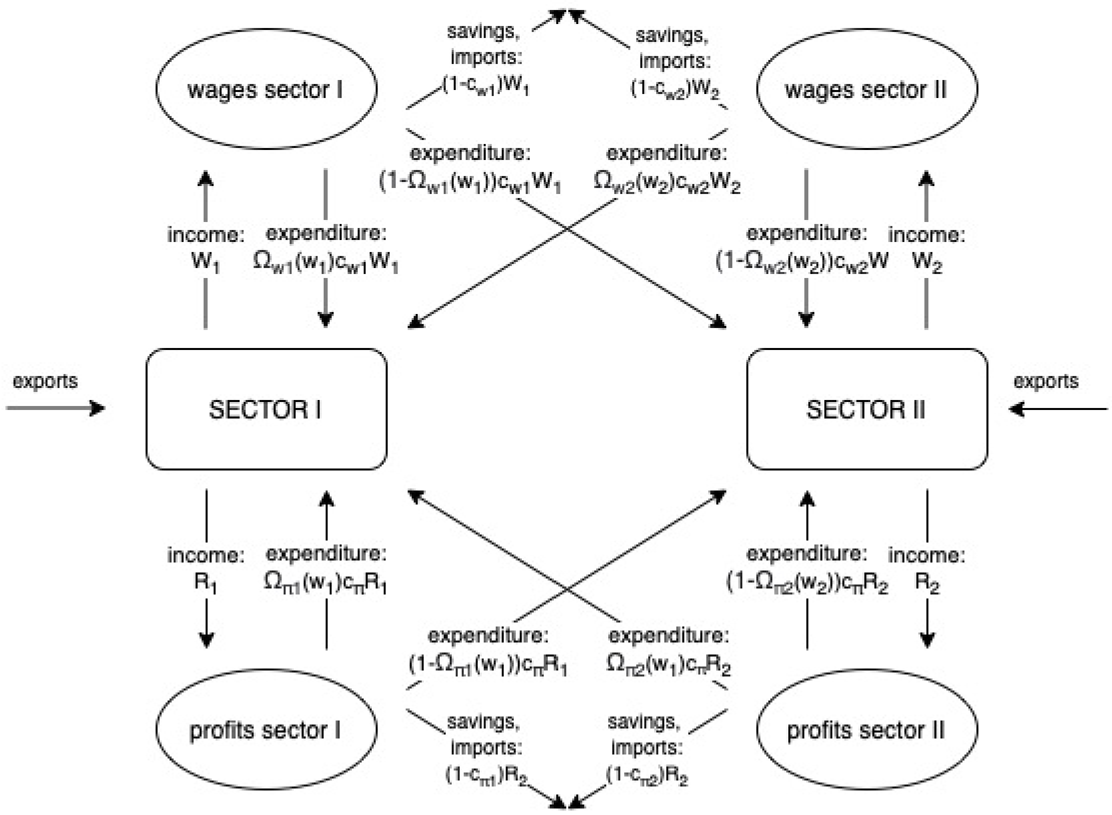
We assume a log-form consumption function for consumption in domestic goods and services following the example of Onaran and Obst (2016), but adding different propensities to consume for wage earners from the two sectors:

$$\log C = c_0 + c_{w1} \log W_1 + c_{w2} \log W_2 + c_\pi \log R \tag{2}$$

where $c_0$ is the autonomous level of consumption, $c_{w1}$ is the domestic rate of consumption out of wages in sector I, $W_1$, $c_{w2}$ is the domestic rate of consumption out of sector II wages, $W_2$, and $c_\pi$ is the share of overall profits, $R$, consumed domestically. Consumption out of wages is characterised by output that is neither profits ($\pi_1$ and $\pi_2$, respectively) nor saved nor spent on imports in both sectors. We assume a lower marginal propensity to consume (MPC) for the higher paying sector in line with common theoretical (Keynes, 1997) and empirical findings (Canbary and

[2]This ignores autonomous consumption $c_0$ because it can be assumed that there are no income effects for autonomous consumption and no changes in sector size as a result.

Figure 1: Intersectoral flows from wages ($W_i$) and profits ($R_i$) of the two sectors $i = 1, 2$ to both respective sectors via consumption or spending patterns. Each type of expenditure is a share of total income that is not saved, i.e. the propensity to consume out of .




Grant, 2019; Onaran, Stockhammer, and Grafl, 2011). However, the marginal propensities to consume out of profits do not depend on the sector since we assume that capitalists will have a low propensity to consume regardless of sector due to their higher level of wealth and the declining marginal propensity to consume for increasing incomes (Fisher et al., 2020; Onaran, Oyvat, and Fotopoulou, 2019).

### Investment

Each sector has its own investment function, depending on the sector's output, profit rate and the differential to the profit rate in the other sector. In our framework, capitalists take into account the difference in profit rates since they have no inherent preferences for a sector. Instead, they tend to invest where they hope to achieve the highest profit rate $r_i$ for $i = 1, 2$ even though they might consider sunk costs, transaction costs, emotional attachments (e.g. family businesses) or other factors like entry barriers keeping them from investing in the other sector. All of this is reflected in the exponents $a_3$ and $b_3$ for sectors I and II, respectively. Including the profit rate differentials ($r_1 - r_2$, $r_2 - r_1$, $r_1 - r_1^f$ and $r_2 - r_2^f$) aims to reflect key insights from the profit equalisation literature (Dutt, 1988; Zacharias, 2001; Kuroki, 1986; Marx, 1981). Although there is no need for assuming perfect competition in any of the two sectors, we do not adopt a specific monopoly framework either (Dutt, 1988).

$$I_1 = Y_1^{a_1} r_1^{a_2} (r_1 - r_2 + 1)^{a_3} (r_1 - r_1^f + 1)^{a_4} \tag{3}$$

and

$$I_2 = Y_2^{b_1} r_2^{b_2} (r_2 - r_1 + 1)^{b_3} (r_2 - r_2^f + 1)^{b_4} \tag{4}$$

Consequently, aggregate investment is the sum of investment in both sectors:

$$I = I_1 + I_2 = Y_1^{a_1} r_1^{a_2} (r_1 - r_2 + 1)^{a_3} (r_1 - r_1^f + 1)^{a_4} + Y_2^{b_1} r_2^{b_2} (r_2 - r_1 + 1)^{b_3} (r_2 - r_1^f + 1)^{b_4} \tag{5}$$

where $Y_i$ for $i = 1, 2$ are sectoral outputs and $a_m$ and $b_m$ for $m = 1, 2, 3, 4$ are coefficients. We can see that investment is driven by output in the respective sector, the profit rate in the respective sector $r_i, i = 1, 2$ and the difference in profit rates between sectors, but also compared to the same sector abroad: $r_i^f, i = 1, 2$ is the profit rate for the respective foreign sector. In order to avoid negative bases for the investment function, +1 is added to the profit rate differentials. We chose to incorporate the profit rate instead of the profit share into the investment function because capitalists will intuitively decide to invest based on their (expected) return on capital, regardless of capital's overall share in this sector. Expectations will be based on current profit rates. The difference in profit rates is important to incorporate since capitalists are willing to switch to the other sector ($a_3 < 0, b_3 < 0$), or even abroad ($a_4 < 0, b_4 < 0$), if it is more profitable. However, they are not indifferent to the sector they invest in, i.e. there will be some investment in the less profitable sector due to sunk costs, contracts, emotional ties, tied-up capital that is more profitable to keep rather than write off.

The profit rate for the two sectors $i = 1, 2$ can be expressed such that it is equal to the profit share divided by the capital-output-ratio:

$$r_i = \frac{R_i}{K_i} = \frac{\pi_i}{K_i / Y_i} \tag{6}$$

since, by definition, the profit share is $\pi_i = \frac{R_i}{Y_i}$, i.e. the share of profits in overall income.

While it may seem that there are no spillover effects since equations 3 and 4 do not contain each other and are connected solely via profit rates, we will see later that they are indeed connected by increasing or decreasing productivity via said profit rates.

### Export and import

Coming back to the components of output as defined in equation 1, we define the export function as follows:

$$\log X = \beta_0 + \beta_1 \log Y_f + \beta_2 \log \left[ \frac{z_1 P(w_1, w_2)}{\epsilon z_1^f P_f} \right] + \beta_3 \log \left[ \frac{z_2 P(w_1, w_2)}{\epsilon z_2^f P_f} \right] \tag{7}$$

where $z_i$ are domestic unit labour costs and $z_i^f$ for $i = 1, 2$ are unit labour costs abroad, both for the two sectors respectively; $Y_f$ is foreign GDP, $P_f$ is foreign price level and $P(w_1, w_2)$ is the domestic price level which is a function of wages in both sectors (but not specified) $\epsilon$ is the exchange rate. Domestic unit labour costs are a function of prices, real wages and productivity $\lambda_i$ in the respective sector $i = 1, 2$:

$$z_i = \frac{w_i}{\lambda_i} \tag{8}$$

Similarly, the import function is defined as:

$$\log M = \alpha_0 + \alpha_1 logY + \alpha_2 log\Big[\frac{\frac{w_1}{\lambda_1}P(w_1, w_2)}{\epsilon z_1^f P_f}\Big] + \alpha_3 log\Big[\frac{\frac{w_2}{\lambda_2}P(w_1, w_2)}{\epsilon z_2^f P_f}\Big] \tag{9}$$

Coming back to equation 1, we now combine the consumption, investment, import and export functions and end up with the short-run case:

$$\begin{aligned}
Y =& C + I + (X - M) = \\
=& c_0 + c_{w1} \log W_1 + c_{w2} \log W_2 + c_\pi \log (R_1 + R_2) \\
&+ Y_1^{a_1} r_1^{a_2} (r_1 - r_2 + 1)^{a_3} (r_1 - r_1^f + 1)^{a_4} + Y_2^{b_1} r_2^{b_2} (r_2 - r_1 + 1)^{b_3} (r_2 - r_2^f + 1)^{b_4} \\
&+ \beta_0 + \beta_1 \log Y_f + \beta_2 \log \Big[\frac{z_1 P(w_1, w_2)}{\epsilon z_1^f P_f}\Big] + \beta_3 \log \Big[\frac{z_2 P(w_1, w_2)}{\epsilon z_2^f P_f}\Big] \\
&- \alpha_0 - \alpha_1 logY - \alpha_2 log\Big[\frac{\frac{w_1}{\lambda_1}P(w_1, w_2)}{\epsilon z_1^f P_f}\Big] - \alpha_3 log\Big[\frac{\frac{w_2}{\lambda_2}P(w_1, w_2)}{\epsilon z_2^f P_f}\Big]
\end{aligned} \tag{10}$$

As we can see, price level and inflation are only important for imports and exports. This is mainly because we do not explicitly talk about prices and thus neglect the distributive forces of inflation since we also use nominal wages. For an open economy framework, we try to indirectly include a real exchange rate channel.

## Productivity

Let us now turn to productivity. In general, we define labour productivity as $\lambda = \frac{Y}{L}$, i.e. the ratio of output to employment, and $\lambda_i$ as the productivity of the the sectors according to the respective subscript $i = 1, 2$. Therefore, output can be expressed as productivity times employment $Y_i = \lambda_i L_i$ and labour input as

$$L_i = L_i(Y_i, \lambda_i) = \frac{Y_i}{\lambda_i} \tag{11}$$

which serves as our employment function for sectors $i = 1, 2$. By definition, the larger output in a sector, the higher employment for given level of productivity. However, the higher labour productivity, the less labour is employed. We do not make any specific assumptions about the segmentation of the labour market or whether workers can or cannot move sectors; however, if sector I is the sector with lower value added, lower overall productivity and lower wages, it might also require a lower level of skills. This does not prevent any workers from moving sectors or obtaining a different skill set in the medium run, e.g., just as a prior investment does not prevent capitalists from investing in the other sector although there may be certain considerations or attachments like class barriers, racist institutions and employers or gender biases.

Using our employment function from above and coming back to equation 6, we can also infer the relationship between the profit rate and productivity when the employment function is incorporated as a function of productivity:

$$r_i = \frac{L_i}{K_i}(\lambda_i - w_i) = \frac{Y_i}{\lambda_i K_i}(\lambda_i - w_i) = \frac{Y_i}{K_i} - \frac{w_i Y_i}{\lambda_i K_i} \tag{12}$$

Very simply, the profit shares in the respective sectors can be expressed as

$$\pi_i = 1 - \frac{w_i}{\lambda_i} \tag{13}$$

Next, we want to specify our productivity function. We vaguely follow Naastepad's (2006) intuition for a productivity regime, but use logs instead of a linear productivity function:

$$\log \lambda_1 = q_{EX,1} + q_{KV,1} \log Y_1 + q_{MB,1} \log w_1 \tag{14}$$

$$\log \lambda_2 = q_{EX,2} + q_{KV,2} \log Y_2 + q_{MB,2} \log w_2 \tag{15}$$

where $q_{EX,i}$ represents exogenous factors such as major technological improvements, and $q_{KV,i}$ and $q_{MB,i}$ for $i = 1, 2$ can be described as Kaldor-Verdoorn coefficient and Marx-biased technical change coefficients, respectively. The Kaldor-Verdoorn effect, Verdoorn's law or the Verdoorn channel is largely based on Verdoorn (2002). The effect relies on a "statistical relationship between the long-run rate of growth of labour productivity and the rate of growth of output" (McCombie, Pugno, and Soro, 2002, p. 1). In essence, the effect presumes a positive correlation and connection between the growth of labour productivity and output growth in industrial production due to economies of scale, rationalisation and mechanisation (Verdoorn, 2002). As McCombie, Pugno, and Soro (2002) argue, Verdoorn's reasoning became widely known and more popular after the interpretation of the late Nicholas Kaldor in his Cambridge inaugural lecture (1966) who also coined the term *Verdoorn's Law* (ibid., p. 2).

Marx-biased technical change is essentially a special case of technical change which, unlike Harrod-neutral or Hicks-neutral technical change, is labour-saving but capital-using. If real wages rise with labour productivity, i.e., the wage share is constant (or slowly declining), this "can (but need not necessarily) lead to a falling rate of profit." (Foley and Marquetti, 1997, p. 5). The close association between Marx and falling profit rates seems to have been the inspiration for the label of the effect (Tavani and Zamparelli, 2021).

Combining both these effects, an increase in wages in sector I affects productivity as follows:

$$\lambda'_{1,w_1} = \frac{\partial \lambda_1}{\partial w_1}\Big|_{Y=\bar{Y}} = \Big[\frac{q_{KV,1}\Omega'}{\Omega} + \frac{q_{MB,1}}{w_1}\Big]\lambda_1 \tag{16}$$

Equation 16 shows us that the Kaldor-Verdoorn channel functions through the change in relative size of sector I, denoted by $\Omega'$, for now since overall output $Y$ is kept constant in the short run. The Kaldor-Verdoorn channel can be reinforced or weakened by the size effect; in other words, an increase in sector-I wages can boost or diminish the sector size. This depends on whether $\Omega'/\Omega$ is greater than or smaller than 1 which is determined by the spending pattern of both sector-I wages and profits. For the Marx-biased technical change channel, we can clearly see that a higher wage rate reduces $\lambda'_{1,w_1}$, ceteris paribus. Due to the log-form of the productivity function, we multiply the derivative by the original productivity rate, $\lambda_1$.

$$\lambda'_{2,w_1} = \frac{\partial \lambda_2}{\partial w_1}\Big|_{Y=\bar{Y}} = -\Big[\frac{q_{KV,2}\Omega'}{1-\Omega}\Big]\lambda_2 \tag{17}$$

For sector-II productivity, there is no Marx-biased technical change effect since we are looking at a wage increase in sector I only. The Kaldor-Verdoorn effect is, again, ambiguous. Depending on the size effect and the size of the Kaldor-Verdoorn coefficient, productivity in sector II can both grow for large Kaldor-Verdoorn coefficients and decline if the size effect is substantial, i.e., when the relative size of sector I increases a lot due to the spending pattern of sector-I wage incomes.

Since productivity is now specified and the employment function is defined as in equation 11, employment in sector I reacts as follows when wages increase:

$$\frac{\partial L_1}{\partial w_1}\Big|_{Y=\bar{Y}} = \frac{\Omega' Y(1-q_{KV,1})}{\lambda_1} + \frac{q_{MB,1}\Omega Y}{\lambda_1 w_1} \tag{18}$$

The effects are rather obvious: higher productivity in the short run reduces employment for fixed output. The more important the Kaldor-Verdoorn and Marx-biased technical changes coefficients, the smaller $\frac{\partial L_1}{\partial w_1}$. Positive size effects for sector I, i.e. $\Omega' > \Omega$, affect sector-1 employment positively c.p. as we would expect for a (relatively) growing sector.

$$\frac{\partial L_2}{\partial w_1}\Big|_{Y=\bar{Y}} = \frac{q_{KV,2}\Omega' Y - \Omega Y}{\lambda_2} \tag{19}$$

For sector-II employment, the picture is not quite a mirror image of sector I: While greater productivity in the sector decreases employment as described above, a positive size effect for sector I, i.e., higher $\Omega'$, reduces employment on sector II since the sector loses relative size. The more important the Kaldor-Verdoorn channel, the smaller the effect on sector-II employment c.p. There is no Marx-biased technical change in the derivative since we assume an exogenous wage increase in sector I only.

Considering the effects of a wage increase in sector I on productivity, we now want to look at what happens to the profit rate in the same sector:

$$\begin{aligned} \frac{\partial r_1}{\partial w_1}\Big|_{Y=\bar{Y}} &= \frac{\Omega' Y(\lambda_1^2 - \lambda_1 w_1) + \Omega Y(\lambda'_{1,w1} w_1 - \lambda_1)}{\lambda_1^2 K_1} \\ &= \frac{\Omega' Y[\lambda_1 - w_1 + q_{KV,1} w_1] - \Omega Y[q_{MB,1} - 1]}{\lambda_1 K_1} \end{aligned} \tag{20}$$

We can clearly see that the size effect of sector I, represented by $\Omega$ and $\Omega'$, or more precisely, the difference in size effects plays an important role, albeit ambiguous due to its interconnection with productivity levels. For higher levels of productivity, the effect of a wage increase on the profit rate will be smaller. Similarly, for higher levels of capital stock in sector I, the effect of an increase in wages in sector I on profits is smaller. Alternatively, we could argue that in more capital-intensive or even more automated sectors, changes in wage levels are less important to the profit rate.

$$\frac{\partial r_2}{\partial w_1}\Big|_{Y=\bar{Y}} = Y\Big[\frac{\Omega'\lambda_2(1-\lambda_2) - w_2\lambda'_{2,w1}(1-\Omega)}{\lambda_2^2 K_2}\Big] = \Omega' Y\Big[\frac{w_2 - \lambda_2 - q_{KV,2}w_2}{\lambda_2 K_2}\Big] \tag{21}$$

For profit shares, the picture is similar: for higher levels of productivity, the impact of wage increases is smaller for both sectors due to the squared productivity featuring in the denominator of the derivative.

$$\frac{\partial \pi_1}{\partial w_1} = \frac{w_1\lambda'_{1,w1} - \lambda_1}{\lambda_1^2} \tag{22}$$

$$\frac{\partial \pi_2}{\partial w_1} = \frac{w_2\lambda'_{2,w1}}{\lambda_2^2} \tag{23}$$

### An increase in wages in sector I

When looking at the bigger picture, this remains relevant since many of the signs, i.e. the direction of an effect of sector I wage increases, follow from equations 16-23:

$$\begin{aligned}\frac{\partial Y}{\partial w_1}\Big|_{Y=\bar{Y}} = C\Big[\frac{\partial C_{w1}}{\partial w_1}\Big|_{Y=\bar{Y}} - \frac{\partial C_{w2}}{\partial w_1}\Big|_{Y=\bar{Y}} - \frac{\partial C_{\pi}}{\partial w_1}\Big|_{Y=\bar{Y}}\Big] \pm \frac{\partial I_1}{\partial w_1}\Big|_{Y=\bar{Y}} \pm \frac{\partial I_2}{\partial w_1}\Big|_{Y=\bar{Y}} \\ \pm \frac{\partial (X-M)_1}{\partial w_1}\Big|_{Y=\bar{Y}} \pm \frac{\partial (X-M)_2}{\partial w_1}\Big|_{Y=\bar{Y}}\end{aligned} \tag{24}$$

where $C_{w1} = c_{w1}\log W_1, C_{w2} = c_{w2}\log W_2$ and $C_\pi = c_\pi \log(R_1 + R_2)$. The effects on investment and net exports are ambiguous and will be explored below.

First, we focus on consumption in more detail and find a positive effect for the consumption out of wages in sector I, a negative effect for consumption out of sector-II-wages due to the size effect (via $\Omega$) and a negative effect for consumption out of profits overall (recall that $\pi$ is the profit share):

$$\begin{aligned}\frac{\partial C}{\partial w_1}\Big|_{Y=\bar{Y}} = C\Big[c_{w1}\Big(\frac{1}{w_1} + \frac{\Omega'}{\Omega} - \frac{\lambda'_{1,w1}}{\lambda_1}\Big) - c_{w2}\Big(\frac{\Omega'}{1-\Omega} + \frac{\lambda'_{2,w1}}{\lambda_2}\Big) \\ + c_\pi\Big(\frac{w_1(\Omega\lambda'_{1,w1} - \Omega'\lambda_1) - \Omega\lambda_1}{\lambda_1^2\pi} + \frac{w_2([1-\Omega]\lambda'_{2,w1} + \Omega'\lambda_2)}{\lambda_2^2\pi}\Big)\Big]\end{aligned} \tag{25}$$

A logarithmic function, the right hand side is multiplied by $C$, the original/previous level of consumption. In all three components of consumption, productivity is the determining factor. As we would expect, there is a positive consumption effect for sector I. Noticeably, the Kaldor-Verdoorn and the Marx-biased technical change coefficients are subtracted from the level of productivity, the former for both sectors and the latter for sector I only. Both coefficients are positive and thus increase productivity. Recall that productivity, by the definition, is $\lambda_i = Y_i/L_i$

for both $i = 1, 2$. In the short run, output is constant; when productivity increases, labour input, and hence labour income (the total wage bill $W_i$ for $i = 1, 2$), declines. If either coefficient increases c.p., this will diminish the positive consumption effect of a wage increase. The opposite is of course true for consumption out of profits: the bigger the productivity effects, the less the negative impact on consumption out of profits. For consumption out of wages, the size effect from the productivity function $\Omega'/\Omega$ (and $(1-\Omega')/(1-\Omega)$) is relevant again which is determined by the spending patterns of wage incomes (see above). Indirectly, spending patterns are decisive for whether a sector is wage- or profit-led. Using equations 20 and 21, an increase in sector-1-wages has the following effect on investment:

$$\begin{aligned}\frac{\partial I}{\partial w_1}\Big|_{Y=\bar{Y}} = {} & \frac{a_1 I_1 \Omega'}{\Omega} + \frac{a_2 I_1}{r_1}\frac{\partial r_1}{\partial w_1}\Big|_{Y=\bar{Y}} + \frac{a_3 I_1}{(r_1 - r_2 + 1)}\Big[\frac{\partial r_1}{\partial w_1}\Big|_{Y=\bar{Y}} - \frac{\partial r_2}{\partial w_1}\Big|_{Y=\bar{Y}}\Big] \\ & + \frac{a_4 I_1}{(r_1 - r_1^f + 1)}\frac{\partial r_1}{\partial w_1}\Big|_{Y=\bar{Y}} - \frac{b_1 I_2 \Omega'}{1-\Omega} + \frac{b_2 I_2}{r_2}\frac{\partial r_2}{\partial w_1}\Big|_{Y=\bar{Y}} \\ & + \frac{b_3 I_2}{(r_2 - r_1 + 1)}\Big[\frac{\partial r_2}{\partial w_1}\Big|_{Y=\bar{Y}} - \frac{\partial r_1}{\partial w_1}\Big|_{Y=\bar{Y}}\Big] + \frac{a_4 I_2}{(r_2 - r_2^f + 1)}\frac{\partial r_2}{\partial w_1}\Big|_{Y=\bar{Y}}\end{aligned} \tag{26}$$

Again, we can see that the most important effects work via sector size, whether directly as in the very first term, or indirectly via the derivative of the profit rates or their difference (equations 20 and 21). Recall that size effects are determined by spending patterns and give us a hint whether a sector is wage- or profit-led. More specifically, sector I can be considered profit-led if $\frac{\Omega'}{\Omega} < 1$ and wage-led if $\frac{\Omega'}{\Omega} > 1$ in the first term of equation 26.

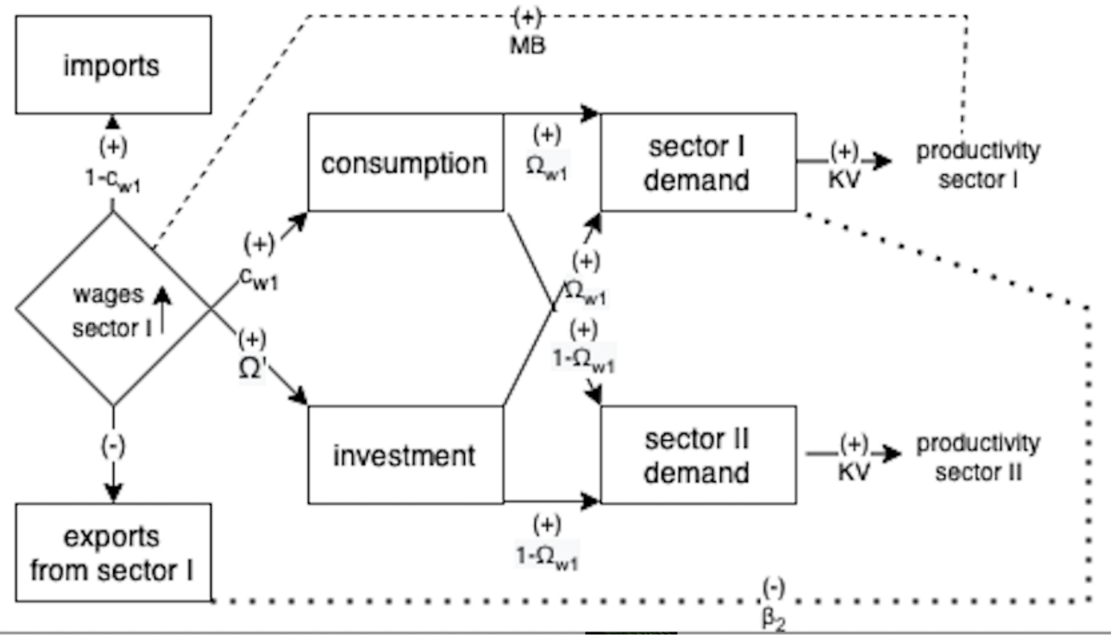
For the last part of our output function, we turn to net exports. The effect on increasing wages in sector I mainly depends on the relationship of parameters $\beta_i$ and $\alpha_i$ for $i = 2, 3$ and the relative size of the imports and exports. In addition, there are sectoral size effects. For instance a higher Kaldor-Verdoorn effect or positive size effect for sector I occurs, then net exports of sector I decline and might lead to a positive effect on net exports in sector II due to the size effect.

$$\begin{aligned}\frac{\partial (X-M)}{\partial w_1}\Big|_{Y=\bar{Y}} &= (\beta_2 X - \alpha_2 M)\Big[\frac{P_1'}{P} - \frac{\lambda_{1,w1}'}{\lambda_1} + \frac{1}{w_1}\Big] + (\beta_3 X - \alpha_3 M)\Big[\frac{P_1'}{P} - \frac{\lambda_{2,w1}'}{\lambda_2}\Big] \\ &= (\beta_2 X - \alpha_2 M)\Big[\frac{P_1'}{P} - \frac{q_{KV,1}\Omega'}{\Omega} + \frac{1 - q_{MB,1}}{w_1}\Big] + (\beta_3 X - \alpha_3 M)\Big[\frac{P_1'}{P} + \frac{q_{KV,2}\Omega'}{1-\Omega}\Big]\end{aligned} \tag{27}$$

where $P_1' = \frac{\partial P(w_1, w_2)}{\partial w_1} > 0$ and the $P = P(w_1, w_2)$.

As we can see, the effect of an increase in sector-I-wages depends on the overall prior relationship between exports and imports in each sector. By design, inflation – or more precisely, the price level increase with respect to changes in sector-I-wages – plays a role in both sectors (see equations 7 and 9). The negative effects of $\frac{\lambda_{i,w1}'}{\lambda_i}, i = 1, 2$ might seem counter-intuitive at first: we would probably expect an increase in productivity to affect net exports positively. A closer look reveals that the relative size effect $\frac{\Omega'}{\Omega}$ is at fault for sector I: an increase in sector-size (i.e. a wage-led sector I) reinforces the negative effect of rising wages on net exports; if sector I is profit-led and shrinks as a result of increasing wages, i.e. $\frac{\Omega'}{\Omega} < 1$, the negative effect on net exports of sector I is smaller. For sector II, the story is more straight forward as we can also see in equation 21, the derivative of the profit rate in sector II with regard to wages in sector I.

Figure 2: Effects of an increase in sector-I-wages: the wage-led case.




More generally, an increase in wages in sector I will affect imports, exports, consumption, investment in both sectors and also directly impact productivity in sector I . In the case presented in Figure 2, sector I is wage-led, i.e. the effect of a wage increase on investment is positive and $\frac{\Omega'}{\Omega} > 1$. The effect on productivity is direct, via the Marx-biased technical change component, and indirect via increased demand (Kaldor-Verdoorn).

### An increase in productivity in sector I

As per our specificaiton in equations 14 and 15, an increase in productivity is a result of either an expanding sector (Kaldor-Verdoorn) or an increase in wages (Marx-biased technical change). However, there may also be unspecified increases in labour productivity which are represented as $q_{EX,1}$ and exogenous to our model. In equation 28, we can see how an increase in productivity affects output. Again, the derivative consists of the sum of derivatives for consumption, investment, import and export as shown in equation 24:

$$\frac{\partial Y}{\partial \lambda_1}|_{Y=\bar{Y}} = \frac{(c_\pi - c_{w1})C - \alpha_2 M + \beta_2 X}{\lambda_1} + \frac{w_1 Y_1}{\lambda_1^2 K_1}\left[\frac{a_2 I_1}{r_1} + \frac{a_3 I_1}{(r_1 - r_2 + 1)} + \frac{a_4 I_1}{(r_1 - r_1^f + 1)} - \frac{b_3 I_2}{(r_2 - r_1 + 1)}\right] \tag{28}$$

An exogenous productivity shock in sector I has a positive effect on consumption out of profits, but a negative effect on consumption out of wages in sector I, primarily because of the decreased labour input required (see equation 11). Imports decrease and exports increase because of lower domestic unit labour cost ceteris paribus. For investment, there is a positive effect for sector I, but a negative effect for sector II since productivity in the former improves compared to the latter and the profit rate differential in the investment function reflects that capitalists consider such changes for their investment allocation decision. For higher levels of labour productivity, the effects are smaller overall as $\lambda_1$ and $\lambda_1^2$ are in the denominators.

## The medium run

Let us now turn to the medium run. We want to look at what happens to our short run model when we look at it in terms of changes over time and growth rates aka changes over levels. Crucially, that is

$$\hat{Y}_i = \frac{\dot{Y}_i}{Y_i} \tag{29}$$

for $i = 1, 2$ as well as without subscripts $\hat{Y} = \frac{\dot{Y}}{Y}$ when referring to the growth rate of total output.

But how do we get from the short run to the medium run and output growth rates? Following the intuition by Naastepad (2006), we reduce our medium run model to two equations: an output growth regime and a productivity growth regime. Unlike Naastepad (ibid.), our model has two sectors and we create regimes for each sector accordingly, i.e. two regimes for each sector. Essentially, we identify components that are not determined elsewhere in the model. This means that we reduce our regimes to the components that ultimately determine them. By doing so, we aim to omit indirect factors for growth rate regimes if these factors are in turn determined by a component already represented elsewhere in the regime. For example, if the profit rate can be expressed in other terms as suggested by equation 12, we drop $r_i, i = 1, 2$ and dissect it into the terms it is determined by to simplify our model, that would be $Y_i, K_i, w_i$, and $\lambda_i$. We can see that profit rates fundamentally depend on wages and productivity given that labour input is implicitly determined by productivity. For profit rates, we also have to consider capital which we will turn to below. Foreign profit rates, foreign output and price levels, foreign productivity and wages as well as exchange rates are all considered exogenous to our framework but will not determine medium run growth rates (although they might be very important for levels!).

In equation 10, the total wage bills in each sector are determined by wage rates and labour input: $W_i = w_i L_i$ for $i = 1, 2$. We still treat wages as determined by bargaining power and institutions, both of which are exogenous to the model. Wages therefore are not determined by anything represented elsewhere and thus have to be considered in the corresponding growth regime. While endogenising wage growth would certainly bee an interesting exercise given that labour productivity growth can be referred to in wage bargaining processes, we assume that ultimately wage growth is determined by exogenous factors such as institutional arrangements,

Table 1: Medium run coefficients and the variables they indirectly affect, including equivalent short run coefficients

| MR coefficient (sector I) | MR coefficient (sector II) | implies effects to | equivalent SR coefficients |
|---|---|---|---|
| $\eta_{1,1}$ | $\eta_{2,1}$ | $r_1$, $P(w_1, w_2)$, $z_1$, $\Omega$ | $c_{w1}$, $c_\pi$, $a_2$, $a_3$,$a_4$, $b_3$, $\alpha_2$, $\alpha_3$, $\beta_2$, $\beta_3$ |
| $\eta_{1,2}$ | $\eta_{2,2}$ | $r_2$, $P(w_1, w_2)$, $z_2$, $\Omega$ | $c_{w2}$, $c_\pi$, $a_3$, $b_2$, $b_3$, $b_4$, $\alpha_2$, $\alpha_3$, $\beta_2$, $\beta_3$ |
| $\eta_{1,3}$ | $\eta_{2,3}$ | $r_1$, $L_1$, $z_1$ | $c_{w1}$,$c_\pi$,$a_2$, $a_3$,$a_4$, $b_3$, $\alpha_2$, $+\beta_2$ |
| $\eta_{1,4}$ | $\eta_{2,4}$ | $r_2$, $L_2$, $z_2$ | $c_{w2}$, $c_\pi$, $a_3$, $b_2$, $b_3$, $b_4$,$\alpha_3$, $\beta_3$ |
| $\theta_{1,1}$ | / | $\Omega$ | $c_{w1}$, $c_\pi$, $a_1$, $\alpha_1$ |
| $\theta_{1,2}$ | / | $r_1$, $P(w_1, w_2)$, $z_1$, $\Omega$ | $c_{w1}$, $c_\pi$ $a_2$, $a_3$,$a_4$, $b_3$, $\alpha_2$, $\alpha_3$, $\beta_2$, $\beta_3$ |
| $\theta_{2,1}$ | / | $\Omega$ | $c_{w2}$, $c_\pi$, $b_1$, $\alpha_1$ |
| $\theta_{2,2}$ | / | $r_2$, $P(w_1, w_2)$, $z_2$, $\Omega$ | $c_{w2}$,$c_\pi$, $a_3$, $b_2$, $b_3$, $b_4$, $\alpha_2$, $\alpha_3$, $\beta_2$, $\beta_3$ |

power, union density etc.

The next factor of the consumption part of output ($W_i = w_i L_i$ ) is labour input $L_i$. The labour input required depends on the the technique and technology used, i.e., the necessary labour required for a certain production level ($Y_i$). Effectively, this is just another way of simply referring to labour productivity ($\lambda_i$) if we recall that they are inversely related (equation 11). The growth rate of productivity therefore already represents the growth of labour input by design. By determining the relative size of the two sectors via $\Omega = \frac{Y_1}{Y}$ and $1 - \Omega = \frac{Y_2}{Y}$, the other sector is indirectly involved in the output of each sector due to overall output being inversely related to $\Omega$. There are no more levels for output in the medium run as there were in the short run, therefore structural change is represented by diverging growth rates of output in sectors I and II. The last remaining variable determined within the system is the domestic price level which – as clearly pointed out in equation 10 – depends on wages in both sectors. So will the growth rate in $P(w_1, w_2)$, $\hat{P}(\hat{w}_1, \hat{w}_2)$.

As mentioned above, we want to consider growth rates for the medium run. Considering the growth rate of output for both sectors $i = 1, 2$, $\hat{Y}_i$, we can see that it basically depends on growth in wage rates and growth in productivity in both sectors, as detailed above for levels. Recall that both growth in wages and productivity are relevant due to the structural change effect via the relative size of sectors, $\Omega$ and $1 - \Omega$. Since the size of $\Omega$ depends on output and can thus be determined by other factors, it is only represented indirectly in the medium run output regimes:

$$\hat{Y}_1 = \eta_{1,1}\hat{w}_1 + \eta_{1,2}\hat{w}_2 + \eta_{1,3}\hat{\lambda}_1 + \eta_{1,4}\hat{\lambda}_2 \quad (30)$$

$$\hat{Y}_2 = \eta_{2,1}\hat{w}_1 + \eta_{2,2}\hat{w}_2 + \eta_{2,3}\hat{\lambda}_1 + \eta_{2,4}\hat{\lambda}_2 \quad (31)$$

Coefficients $\eta_{i,1}$ show all the different ways wage growth ultimately affects output growth via sector-I-consumption, investment via profit rates (see equation 12), price levels/inflation ($P(w_1, w_2)$ or $\hat{P}(w_1, w_2)$) and export/import demand. The size of the coefficient depends on how important these factors are in the overall economy. If output growth is more sensitive to profitability, the coefficient will be larger, c.p. Overall, $\eta_{i,1}$ shows how reactive output growth is to a change in wage growth. Coefficients $\eta_{i,2}$ do the same for sector-II wage growth. Coefficients $\eta_{i,3}$ and $\eta_{i,4}$ sums up the overall influence of productivity growth on output growth, including via profit rates, exports and imports and employment. The more important profit rates are, e.g. in the investment functions (coefficients $a_2, b_2$ in equations 3 and 4), the bigger $\eta_{i,3}$ and $\eta_{i,4}$, respectively.[3]

Returning to total profits in the consumption part and profit rates in the investment part of equation 10, we are left with capital which – as is customary in most medium and long run definitions – is no longer fixed. Let us reconsider equations 14 and 15 where the Kaldor-Verdoorn channel and Marx-biased technical change determine productivity. For the short run, the stock of capital does not matter as it assumed to be fixed anyway. In the medium run, however, the growth in capital $\hat{K}_i = \frac{\dot{K}_i}{K_i}$ or capital deepening does have an impact (Dieppe, Francis, and Kindberg-Hanlon, 2021). We thus slightly amend our short-run productivity function for the medium run, add a capital deepening component $\tilde{q}_{CD,i}$ and get

$$\hat{\lambda}_i = \tilde{q}_{KV,i}\hat{Y}_i + \tilde{q}_{CD,i}\frac{\hat{K}_i}{L_i} + \tilde{q}_{MB,i}\hat{w}_i$$

for $i = 1, 2$ where the tilde over coefficients simply represents the medium-run equivalent to the short-run coefficients from equations 14 and 15. Put simply, the change in capital is equal to investment: $\dot{K}_i = I_i(Y_i, r_1, r_2, r_i^f)$ and as we said before, profit rates essentially depend on wages and productivity in the relevant sector (apart from capital), therefore determined by something else. Applying the above principle, we drop profit rates and focus on the determinants of profit rates instead. Investment in sector $i$, and thus change in capital, is thus determined by the wage rate, output and productivity as well as the profit rate abroad, $\dot{K}_i = I_i(Y_i, w_i, \lambda_i, r_i^f)$. We also drop $r_i^f$ as it is exogenous and does not affect the overall growth trajectory of output or labour productivity, although it could be important for exogenous changes. Since all factors determining capital deepening are now already represented, we can create a comprehensive productivity regime and reduce it to:

$$\hat{\lambda}_1 = \theta_{1,0} + \theta_{1,1}\hat{Y}_1 + \theta_{1,2}\hat{w}_1 \tag{32}$$

and

$$\hat{\lambda}_2 = \theta_{2,0} + \theta_{2,1}\hat{Y}_2 + \theta_{2,2}\hat{w}_2 \tag{33}$$

where $\theta_{i,0}$ are exogenous factors for productivity growth (think Solow residuals and exogenous factors), $\theta_{i,1}$ represents an extended Kaldor-Verdoorn channel (representing factors discussed

[3] Higher productivity is associated with higher profit rates as demonstrated by equation 12.

above) and $\theta_{i,2}$ is an extended Marx-biased technical change effect for $i = 1, 2$. Intersectoral productivity effects or spillover effects work via the extended Kaldor-Verdoorn channel (see equations 30 and 31). So while the size effect of the short run model is less obvious here, intersectoral dynamics are still a pertinent feature of the medium run model. Capital movement across sectors is also indirectly part of the productivity regime via output growth which features crosssectoral effects.

All coefficients established in equation 10 are indirectly incorporated in the medium run regimes, i.e. if they are bigger or smaller in the short run, this will have an effect on the medium run coefficients, too. For example, a change in the propensity to consume out of sector-I-wages will affect $\eta_{1,1}$,$\eta_{1,3}$,$\theta_{1,1}$ and $\theta_{1,2}$ in the medium run regimes as we can see in Table 1 where all coefficients in the short run version are put into context with their medium run counterparts. They are, of course, not perfect equivalents and Table 1 only reflects the general medium-run-coefficient that short-run-coefficients are incorporated into.

The only main feature not explicitly reflected in the medium run are the profit rate differentials from the investment function (equations 3 and 4). We implicitly assume that capitalists find their preferred sectors or investment allocations over time as a way of finding an equilibrium medium run path for investment.

Solving equations 30, 31, 32 and 33, we get that output growth in sector I is:

$$\begin{aligned}\hat{Y}_1 = & \frac{([\eta_{1,1} + \eta_{1,3}\theta_{1,2}][1 - \eta_{2,4}\theta_{2,1}] + \eta_{1,4}\theta_{2,1}[\eta_{2,1} + \eta_{2,3}\theta_{1,2}])\hat{w}_1 +}{[1 - \eta_{1,3}\theta_{1,1}][1 - \eta_{2,4}\theta_{2,1}] - \eta_{1,4}\eta_{2,3}\theta_{1,1}\theta_{2,1}} \\ & + \frac{(\eta_{1,2}[1 - \eta_{2,4}\theta_{2,1}] + \eta_{1,4}[\eta_{2,2}\theta_{2,1} + \theta_{2,2}])\hat{w}_2 + [\eta_{1,3}\theta_{1,0} + \eta_{1,4}\theta_{2,0}][1 - \eta_{24}\theta_{21}] + \eta_{2,3}\theta_{1,0} + \eta_{2,4}\theta_{2,0}}{[1 - \eta_{1,3}\theta_{1,1}][1 - \eta_{2,4}\theta_{2,1}] - \eta_{1,4}\eta_{2,3}\theta_{1,1}\theta_{2,1}}\end{aligned} \tag{34}$$

While equation 34 might look confusing at first sight due the many coefficients involved, it is, in fact, a simple linear function that depends on both the output regimes and the productivity regimes since both are incorporated. The same is true for the productivity regime:

$$\begin{aligned}\hat{\lambda}_1 = & \frac{([\eta_{1,1}\theta_{1,1} + \theta_{1,2}][1 - \eta_{2,4}\theta_{2,1}] + \eta_{1,4}\eta_{2,1}\theta_{1,1}\theta_{2,1})\hat{w}_1}{[1 - \eta_{1,3}\theta_{1,1}][1 - \eta_{2,4}\theta_{2,1}] - \eta_{1,4}\eta_{2,3}\theta_{1,1}\theta_{2,1}} \\ & + \frac{(\eta_{1,2}\theta_{1,1}[1 - \eta_{2,4}\theta_{2,1}] + \eta_{1,4}\theta_{1,1}[\eta_{2,2}\theta_{2,1} + \theta_{2,2}])\hat{w}_2 + \eta_{1,4}\theta_{1,1}\theta_{2,0} + \theta_{1,0}[1 - \eta_{2,4}\theta_{2,1}]}{[1 - \eta_{1,3}\theta_{1,1}][1 - \eta_{2,4}\theta_{2,1}] - \eta_{1,4}\eta_{2,3}\theta_{1,1}\theta_{2,1}}\end{aligned} \tag{35}$$

Note that the denominators are equal and only feature coefficients being connected to either sector-I or sector-II productivity growth in equations 30 and 31 ($\eta_{1,3}, \eta_{1,4}, \eta_{2,3}, \eta_{2,4}$) or the adapted Kaldor-Verdoorn channels from equations 32 and 33 ($\theta_{1,1}, \theta_{2,1}$). This is also true for both productivity and demand regime in sector II:

$$\begin{aligned}\hat{Y}_2 = & \frac{\eta_{2,3}[\eta_{1,4}\theta_{1,1}\theta_{2,0} + \theta_{1,0}] + \eta_{2,4}\theta_{2,0}[1 - \eta_{1,3}\theta_{1,1}] + [\eta_{2,1}(1 - \eta_{1,3}\theta_{1,1}) + \eta_{2,3}(\eta_{1,1}\theta_{1,1} + \theta_{1,2}]\hat{w}_1}{[1 - \eta_{1,3}\theta_{1,1}][1 - \eta_{2,4}\theta_{2,1}] - \eta_{1,4}\eta_{2,3}\theta_{1,1}\theta_{2,1}} \\ & + \frac{([\eta_{2,2} + \eta_{2,4}\theta_{2,2}][1 - \eta_{1,3}\theta_{1,1}] + \eta_{2,3}\theta_{1,1}[\eta_{1,2} + \eta_{1,4}\theta_{2,2}])\hat{w}_2}{[1 - \eta_{1,3}\theta_{1,1}][1 - \eta_{2,4}\theta_{2,1}] - \eta_{1,4}\eta_{2,3}\theta_{1,1}\theta_{2,1}}\end{aligned} \tag{36}$$

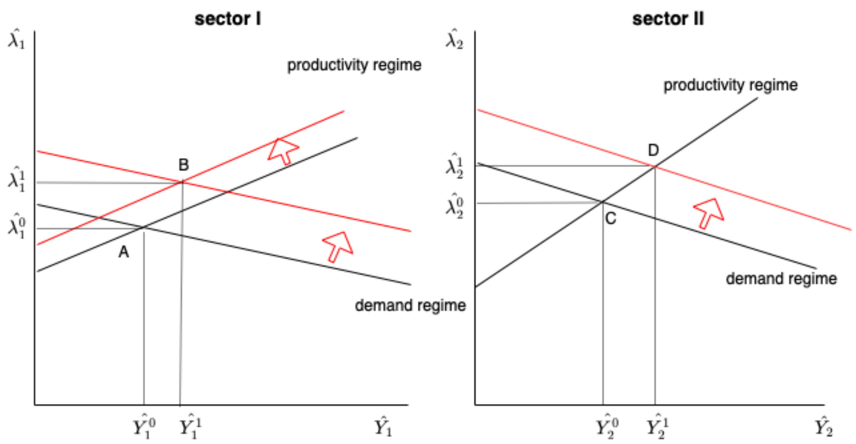
$$\hat{\lambda}_2 = \frac{(\eta_{2,1}\theta_{2,1}[1-\eta_{1,3}\theta_{1,1}] + \eta_{2,3}\theta_{2,1}[\eta_{1,1}\theta_{1,1} + \theta_{1,2}])\hat{w}_1}{[1-\eta_{1,3}\theta_{1,1}][1-\eta_{2,4}\theta_{2,1}] - \eta_{1,4}\eta_{2,3}\theta_{1,1}\theta_{2,1}} + \frac{([\eta_{2,2}\theta_{2,1} + \theta_{2,2}][1-\eta_{1,3}\theta_{1,1}] + \eta_{1,2}\eta_{2,3}\theta_{1,1}\theta_{2,1})\hat{w}_2 + \theta_{2,0}[1-\eta_{1,3}\theta_{1,1}] + \eta_{1,4}\eta_{2,3}\theta_{1,1}\theta_{2,1}}{[1-\eta_{1,3}\theta_{1,1}][1-\eta_{2,4}\theta_{2,1}] - \eta_{1,4}\eta_{2,3}\theta_{1,1}\theta_{2,1}} \tag{37}$$

Table 2: Effects of an increase in the sector-1 wage rate on output and productivity in both sectors. Following Naastepad (2006), combinations signified as $\emptyset$ are not possible.

| | wage-led productivity growth | | profit-led productivity growth | |
|---|---|---|---|---|
| | sector I | sector II | sector I | sector II |
| wage-led demand | $\hat{Y}_1 \uparrow \rightarrow \hat{\lambda}_1 \uparrow$ $\hat{\lambda}_1 \uparrow$ | $\hat{Y}_2 \uparrow \rightarrow \hat{\lambda}_2 \uparrow$ | $\emptyset$ | $\emptyset$ |
| profit-led demand | $\hat{Y}_1 \downarrow \rightarrow \hat{\lambda}_1 \downarrow$ $\hat{\lambda}_1 \uparrow$ | $\hat{Y}_2 \downarrow \rightarrow \hat{\lambda}_2 \downarrow$ | $\hat{Y}_1 \downarrow \rightarrow \hat{\lambda}_1 \downarrow$ $\hat{\lambda}_1 \downarrow$ | $\hat{Y}_2 \downarrow \rightarrow \hat{\lambda}_2 \downarrow$ |

Each of these for regimes from equations 30, 31, 32 and 33 can now either be wage-led or profit-led, i.e. react positively to an increase in wages or not. The direction of this change depends on the relative size of the coefficients stemming from equations 30, 31, 32 and 33. The ultimate effect of a change in wages in either sector depends on whether the output a sector tends to be more or less sensitive to its own wages or that of the other sector. This distinction resembles the categories of wage-led and profit-led demand in one-sector-models of post-Keynesian literature following the Bhaduri and Marglin (1990) tradition. Similar to one of these models, Naastepad (2006), we identify two different growth regimes in our medium-run model: a demand and a productivity regime, both of which can be wage- or profit-led.

Figure 3: Strictly wage-led case, i.e. wage-led demand regimes in both sectors and wage-led productivity regimes. The productivity regime of sector II is quite insensitive to wages in sector I since the only effect is indirect via output.




If wages in either sector increase, the effect on output and productivity depends on whether the sector is more sensitive to changes in wages or profits. However, output also affects productivity as per equations 32 and 33. Table 2 illustrates a simplified version of what happens when wages in sector I increase: in sector I itself, there is a direct effect on both output and productivity and an indirect effect on productivity via output whereas the effect in sector II concerns output directly and productivity indirectly.

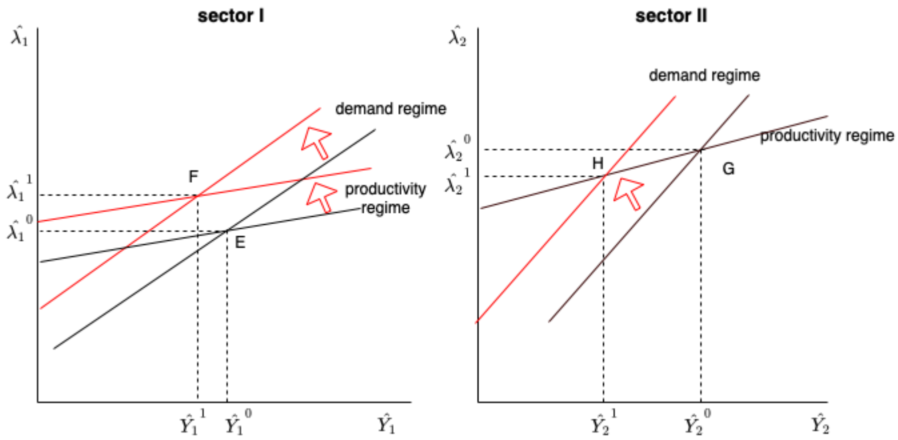
When demand regimes in both sectors are wage-led, we call the economy strictly wage-led. An economy is strictly profit-led if both demand regimes are profit-led. If demand regimes are not the same, an economy is of mixed regime.

For sector I, a sector-I wage increase leads to both an increase in output growth and a direct *and* indirect increase in productivity growth. The reinforcing effects lead to a type of upward spiral. With Figure 3, we follow Naastepad's (2006) notion of regimes and show the changes in a productivity growth and output growth space. We can see a strictly wage-led economy, i.e. that a wage increase in a wage-led sector I positively affects both the demand and productivity regimes by changing the intercept: we move from point A to point B with the productivity regime shifting upwards or towards the left and the demand regime shifting upwards or towards the right. Output grows faster due to growing demand since wage growth affects output growth positively both directly and indirectly via productivity growth. Overall, the demand effect on each sector as well as the sector-size effect or intersectoral demand effect are positive ($\Omega$ in the short run), but the magnitude of these shifts also depends on intersectoral relations and spillover effects.

For the former this implies that a wage increase leads to a higher level of productivity growth for a given level of output growth which intuitively makes sense considering the Marx-biased technical change channel discussed in detail earlier in this paper. This would bring the wage share down and hence reduce growth as we have a wage-led regime: rising wages increase productivity but also suppress the wage share. An increase in wages leads to both higher output and productivity growth for a given level of productivity and output growth, respectively. This is also relatively intuitive for a wage-led regime since wage growth affects output growth both directly and indirectly via productivity growth. Sector II is only affected at the demand regime since the productivity regime does not react to wage increases directly. There, a higher level of wages in sector I leads to higher productivity and output growth for given levels of output and productivity growth, respectively. The higher level of output growth can be attributed to additional demand due to the wage increase and the higher level of productivity growth is a result of capitalists moving away from sector I to sector II (capital movement). Again, a move from point C to point D seems quite intuitive for a wage-led sector although we should keep in mind that the overall extent of this increase depends on the intersectoral demand effect $\Omega$ and overall demand.

When demand regimes in both sectors are profit-led, i.e. in a strictly profit-led economy like Figure 4, the opposite is true, and there is a depressing effect on both productivity and output growth where the former is again affected directly and indirectly. Although higher wages lead to an overall lower level of productivity growth – from point E to point F –, paradoxically, output growth increases a little bit even though we have a strictly profit-led economy. This is caused by higher profit rates due to Marx-biased technical change which will increase productivity

Figure 4: Wage increase in a strictly profit-led economy, i.e. profit-led demand regimes in both sectors. Both productivity and demand regimes shift upwards in sector I, resulting in overall lower output growth but higher productivity growth. The sector-II-demand regime shifts to the left, but the productivity regimes in sector II remains largely unaffected due to there being no direct effect of sector-I-wages. The result are lower levels of output and productivity growth/



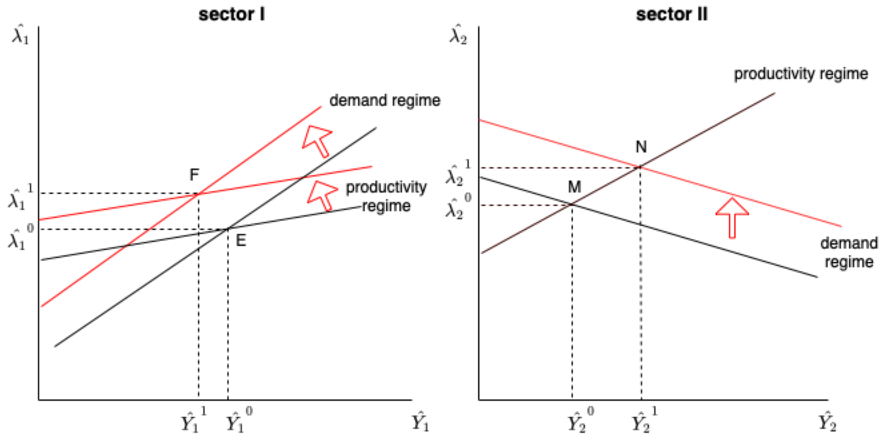

growth via investment in labour-saving and more capital-intensive techniques in the medium run and thus push output growth indirectly. The decreased productivity growth, however is due to the depressed Kaldor-Verdoorn channel which again has to do with the more traditional exhilarationist profitability narrative (Bhaduri and Marglin, 1990). We can also see that the productivity regime in sector II does not have a very strong relation to wages in sector I. As we can see in equation 33, $w_1$ is not part of the regime directly and thus also does not have the paradoxical higher growth in output due to Marx-biased technical change. However, wages in sector I will have an indirect effect via output growth in sector II, $Y_2$ due to equation 31.

The scenarios are less straightforward for mixed-regime economies where sector I has a profit-led and sector II a wage-led demand regime as in Figure 5. What happens in sector I is identical to Figure 4, but intersectoral dynamics are more interesting in this case since the effect on sector II is the opposite: moving from point M to point N, we can see that both productivity growth and output growth increase. Note that the output growth increase is likely to be larger than in sector I since there are counteracting effects at play in sector I. In sector II, however, output growth benefits from capital movement from the profit-led sector I, i.e. capitalists there are more inclined to leave the sector with decreased profitability. Additionally, the expenditure patterns favour sector-II consumption because sector II is more likely to produce more durable or affluent products. Productivity growth also increases via the Kaldor-Verdoorn mechanism.

## Conclusion

In conclusion, we presented a Post-Keynesian and Kaleckian-inspired two-sector productivity model in both a short run and a medium run specification. The short run specification allows for intersectoral effects and structural change via the changing relative size of the two sectors due

Figure 5: Sector-I-wage increase in a mixed-regime economy. While the profit-led demand regime in sector I leads to higher productivity growth and lower output growth, sector II is wage-led and thus benefits from a wage increase in sector I. Again, productivity regime is not directly affected.




to the impact of the expenditure choices. Instead of the traditional wage and profit shares, we focused on nominal wage levels and profit rates. This was particularly useful for the investment function which features profit rate differentials between sectors, but also between a sector and its foreign equivalent. The consumption function is in log-log form allowing for non-linearity in the function. The short run productivity specification includes both a Kaldor-Verdoorn effect via economies of scale and a Marx-biased technical change effect. The latter is important because capitalists might change technique or opt for a more labour-saving technology if wages rise too much. The Marx-biased technical change directly affects productivity whereas the Kaldor-Verdoorn channel is more indirect as it depends on the change in demand.

Our main insight is that expenditure patterns determine the overall intersectoral relations via demand effects and also capital movement in the medium run. For the medium run, we focused on growth rates instead of levels and simplified our framework drastically. Following Naastepad's (2006) example, we established a productivity and demand regime for each sector. The demand regimes include both productivity growth and output growth of the respective other sector which allows for intersectoral effects again. We found that different combinations of wage-led and productivity-led regimes lead to different effects on productivity growth specifically. If a sector is strictly wage-led, both the productivity and demand regimes depend positively on wages. However, if the demand regime is profit-led and the productivity regime is wage-led, the relation of productivity growth to wages could still be negative. This is because both output growth and wage growth matter in the specification of the productivity regime and the coefficient for the former could be relatively large and thus leading to this seemingly paradoxical result.

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## Appendix

### Short run

Equation 13 is derived by starting from a basic income equation adapted to express total profits for $i = 1, 2$:

$$R_i = r_i K_i = Y_i - w_i L_i$$

Dividing both sides by $L_i$ leaves us with

$$\frac{R_i}{L_i} = \frac{Y_i}{L_i} - w_i$$

and allows us to expand the left term by $Y_i/Y_i$. Dividing by $Y_i/L_i = \lambda_i$ allows us to see the original equation:

$$\frac{R_i}{Y_i} = \pi_i = 1 - \frac{w_i}{\lambda_i}$$

### Medium run

#### The productivity growth–output growth space

For the relationship between output growth and productivity growth, we can see that

$$\begin{aligned}\hat{Y}_1 = &\Big[\frac{\eta_{2,1}\theta_{2,1} + \eta_{1,2}}{1 - \eta_{1,4}\theta_{2,1}} + \eta_{1,1} + \eta_{1,4}\eta_{2,1}\theta_{2,1}\Big]\hat{w}_1 + \Big[\frac{\eta_{1,2} + \theta_{2,1}(\eta_{2,2} - \eta_{1,2}\eta_{1,4})}{1 - \eta_{1,4}\theta_{2,1}} \\ &+\theta_{2,2} + \eta_{1,4}\eta_{2,2}\theta_{2,1}\Big]\hat{w}_2 + \Big[\frac{\eta_{2,3}\theta_{2,1}}{1 - \eta_{1,4}\theta_{2,1}} + \eta_{1,3} + \eta_{1,4}\eta_{2,2}\theta_{2,1}\Big]\hat{\lambda}_1 + \frac{\theta_{2,0}}{1 - \eta_{1,4}\theta_{2,1}} + \eta_{1,4}\theta_{2,0}\end{aligned} \tag{38}$$

When deriving an output growth regime in an output growth–productivity growth space for sector I, we first use equation 34 and substitute the productivity growth variables $\hat{\lambda}_i$ for $i = 1, 2$ with equations 32 and 33. Further substituting $\hat{Y}_2$ into 33 and solving for $\hat{\lambda}_2$ leaves us with

$$\begin{aligned}\hat{Y}_1 = &\Big[\frac{\eta_{2,1}\theta_{2,1} + \eta_{1,2}}{1 - \eta_{1,4}\theta_{2,1}} + \eta_{1,1} + \eta_{1,4}\eta_{2,1}\theta_{2,1}\Big]\hat{w}_1 + \Big[\frac{\eta_{1,2} + \theta_{2,1}(\eta_{2,2} - \eta_{1,2}\eta_{1,4})}{1 - \eta_{1,4}\theta_{2,1}} \\ &+\theta_{2,2} + \eta_{1,4}\eta_{2,2}\theta_{2,1}\Big]\hat{w}_2 + \Big[\frac{\eta_{2,3}\theta_{2,1}}{1 - \eta_{1,4}\theta_{2,1}} + \eta_{1,3} + \eta_{1,4}\eta_{2,2}\theta_{2,1}\Big]\hat{\lambda}_1 + \frac{\theta_{2,0}}{1 - \eta_{1,4}\theta_{2,1}} + \eta_{1,4}\theta_{2,0}\end{aligned} \tag{39}$$

With $\hat{w}_1$ expressed by rearranging equation 32, we can eliminate $\hat{w}_1$ altogether, arriving at

$$\begin{aligned}\hat{Y}_1 = &\frac{\eta_{1,4}\theta_{1,2}\theta_{2,0}(1 - \eta_{1,4}\theta_{2,1})}{(\theta_{1,1} + \theta_{1,2})(1 - \eta_{1,4}\theta_{2,1})} + \Big[\frac{\eta_{1,2} + (\eta_{2,2} - \eta_{1,2}\eta_{1,4})\theta_{2,1}}{1 - \eta_{1,4}\theta_{2,1}} + \theta_{2,2} + \eta_{1,4}\eta_{2,2}\theta_{2,1}\Big]\frac{\theta_{1,2}}{\theta_{1,1} + \theta_{1,2}}\hat{w}_2 \\ &+ \frac{\eta_{1,1} + \eta_{1,2} + \eta_{1,3}\theta_{1,2} + \theta_{2,1}[\eta_{1,1} + \eta_{1,4}(\eta_{2,1} - \eta_{1,4}\theta_{2,1} - 1) + \theta_{1,2}(+\eta_{2,3} + \eta_{1,4}[\eta_{2,2} - \eta_{1,3} - \eta_{1,4}\eta_{2,2}\theta_{2,1}])]}{(1 - \eta_{1,4}\theta_{2,1})(\theta_{1,1} + \theta_{1,2})}\hat{\lambda}_1\end{aligned} \tag{40}$$

As Naastepad (2006) already pointed out, we have a wage-led regime here when $\frac{\partial \hat{Y_1}}{\partial \hat{\lambda_1} < 0}$, i.e. effectively when the long term before $\hat{\lambda_1}$ is negative. Notice that the Kaldor-Verdoorn influenced coefficient $\theta_{1,1}$ (see equation 32) only features in the denominator, suggesting that the bigger this coefficient, the closer the overall term is to zero. The Marx-biased technical change related coefficient $\theta_{1,2}$, however, features on both sides of the ratio with its effect depending on the relative size of other coefficients: $\eta_{1,3} + \theta_{2,1}[\eta_{2.3} + \eta_{1,4}(\eta_{2,2} - \eta_{1,3} - \eta_{1,4}\eta_{2,2}\theta_{2,1})] \lesseqgtr 0$. If this term is positive, we have a wage-led demand regime.