The Kaldor-Verdoorn Law under alternative conceptualizations of the labor market

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Abstract

The Kaldorian interpretation of Verdoorn's Law implies that it is possible to make unambiguous inferences about the returns to scale or demand induced technical change parameters from the Verdoorn coefficient. This interpretation has been widely used in macroeconomic models. I challenge the Kaldorian interpretation of Verdoorn's Law by providing an explicit analysis of what the Verdoorn coefficient signifies in terms of the underlying production conditions and labor market behavior. In the extant literature, the simplifying assumption that an exogenously given wage is equal to the marginal product of labor has been used in the model to derive the Kaldor-Verdoorn (KV) coefficients. Departing from this assumption, I demonstrate that when efficiency wages and a labor-discipline model are used to derive the KV coefficients, the Kaldorian interpretation of Verdoorn's Law does not hold. I also demonstrate that when efficiency wages are used to derive the KV coefficients, an economy without returns to scale or induced technical change can generate KV coefficients in the $(0, 1)$ interval.

1 Introduction

The Kaldor-Verdoorn (KV) Law refers to a positive, but less than one-for-one relationship between productivity growth and output growth in the industrial sector, with causality running from the latter to the former. This law was originally due to Verdoorn (1949). Due to Kaldor's (1966) emphasis on this law, this became the Kaldor-Verdoorn Law.

The Kaldorian interpretation of Verdoorn's Law implies that it is possible to make unambiguous inferences about the returns to scale parameter from the KV coefficient. This interpretation of the law has been widely used in heterodox macroeconomic models.

Ros (2013) and Basu and Budhiraja (2021) have challenged the validity of the Kaldorian interpretation. They demonstrate that the KV coefficient depends on many parameters: the returns to scale parameter; the profit share; the elasticity of factor substitution and the elasticity of labor supply. This implies that the KV coefficient cannot be used, on its own, to make unambiguous inferences about the returns to scale parameter. Thus, in general, the Kaldorian interpretation of Verdoorn's Law does not hold.

But, they demonstrate two special cases where the Kaldorian interpretation holds. The first case is that of Leontief production technology (where the elasticity of factor substitution is zero). The second case is that of Cobb-Douglas production technology (where the elasticity of factor substitution is one); coupled with the economy being on its steady state. Moreover, in this case, the KV coefficient is determined entirely by the parameters from the production side of the economy, but does not take into account the parameters from the labor market.

However, there is limited empirical evidence to support the two special cases where the Kaldorian interpretation of Verdoorn's Law holds. The extant empirical literature on the elasticity of factor substitution suggests that existing estimates of the elasticity of factor substitution usually lie between 0.3 and 1 Gechert, Havránek, Havránková, and Kolcunova (2019), Knoblach, Roessler, and Zwerschke (2020). Thus, empirical evidence to support Leontief production technology and Cobb-Douglas production technology is lacking; which also implies that empirical evidence to support the cases in which the Kaldorian interpretation of Verdoorn's Law holds is also lacking.

Moreover, arguably, the labor market used to derive the KV Law is under-theorized. In the original Verdoorn model, presented in Verdoorn (1949), Verdoorn discussed two distinct linear equations which captured the KV effect. In one equation, the KV effect was determined entirely by the conditions of the product market, while in another equation, the KV effect was determined entirely by the conditions of the labor market. Moreover, in the second special case of Cobb-Douglas production technology where the Kaldorian interpretation of Verdoorn's law holds, the KV coefficient is determined entirely by the parameters from the production side of the economy; and does not take into account parameters from the labor market.

Ros (2013) and Basu and Budhiraja (2021), building on Rowthorn (1979) and Bairam (1987), integrate the analysis of production and the labor market, to derive explicit reduced form expressions of the KV coefficient, determined jointly by production and technology; and labor market behavior. They use a neoclassical labor market model, where firms equate the exogenous real wage to the marginal product of labor on the demand side; combined with a constant labor supply elasticity assumption to derive the Kaldor-Verdoorn Law.

But the extant monopsony literature suggests that markdowns are positive, which challenges the validity of the assumption that an exogenously given wage is equal to the marginal product of labor.

The weakness in the theoretical underpinning, and the deficiency of empirical evidence in favor of the Kaldorian interpretation of Verdoorn's Law; coupled with the under-theorization of the labor market in the model used to derive the KV law, provides the motivation for this essay.

In this essay, I develop an analog of the KV coefficient under alternative labor market conceptualizations. The motivation for this is to examine whether the Kaldorian interpretation of Verdoorn's Law holds under alternative labor market conceptualizations; and concomitantly to address the under-theorization of the labor market in the model used to derive the KV coefficient.

I begin by considering the implications of using a simple efficiency wage model, and contrast this with the implications of using neoclassical assumptions regarding labor market behavior. I then consider the implications of using Bowles (2004) labor discipline model, to characterize labor market behavior in the model used to derive the KV law.

2 The Kaldor-Verdoorn model with efficiency wages

2.1 Production and technology

The following notation is used: B, Q, K, E, W and L denote technology, output, capital stock, effort, wages and employment respectively. Lower case letters denote growth rates of the corresponding variables. For instance, $w = (1/W)(dW/dt)$, denotes the growth rate of wages, and so on. The following parameters are used: the elasticity of factor substitution ($\sigma = 1/(1-\psi) \geq 0$); the profit share, $0 < \pi < 1$; increasing returns to scale parameter $\mu \geq 0$; the demand-induced technical change parameter, $\xi \geq 0$; the elasticity of labor supply, $-\infty < \eta < \infty$; and the Kaldor-Verdoorn coefficient, $0 \leq a_1 \leq 1$.

Production conditions of a perfectly competitive firm in the industrial sector are captured by the following CES production function:

$$Q = B[aK^{\psi} + (1-a)(E(W)L)^{\psi}]^{\frac{1}{\psi}} \tag{1}$$

To integrate efficiency wage considerations in the KV model, the labor input is replaced by the effective labor input, $\tilde{L}$, which is the product of the number of identical workers hired, L, and effort, $E \in (0,1)$, i.e., $\tilde{L} = E(W)L$. Effort may be understood as the fraction of the hour in which the individual is “working”, as opposed to “not working”. In this simple formulation of the efficiency wage model, the absolute value of the real consumption wage is the only determinant of the effort function.

There are two advantages of using the CES specification discussed in Basu and Budhiraja (2021), which are also relevant here. First, a characterization of production conditions of a perfectly competitive firm implies that any scale effects that operate would have to be external to the firm, since perfect competition is incompatible with internal economies, a point stressed by Sraffa (2005). In this context, a CES specification allows scale effects that operate outside the firm to be separated from scale effects that operate at the level of the firm. This is because each firm takes the technology parameter, B, as given. Thus, using the CES specification ensures that there are constant returns to scale at the firm-level, and any scale effects which operate at the aggregate level are external to the firm. Thus, the technical progress parameter in the KV model presented here represents economies of scale which are external to a firm.

Another advantage of using this specification is that once the KV coefficient is derived using a CES specification, it can be used to derive the KV coefficient associated with commonly specified functional forms of production functions which are nested within the CES specification, such as the Cobb-Douglas and Leontief production technology, as special cases.

Since the goal is to derive a relationship between output growth and productivity growth (Q/L), I divide both sides of (1) by L; then take growth rates; and finally use the Solow condition[1], $\frac{E'(W)W}{E(W)} = 1$, to obtain:

$$p = b + \frac{aC^{\psi}c + (1-a)E(W)^{\psi}w}{aC^{\psi} + (1-a)E(W)^{\psi}} \tag{2}$$

Now, since $Q = f(K, \tilde{L})$ is linearly homogeneous, from the Euler's Theorem it follows that:

$$K\frac{\partial Q}{\partial K} + \tilde{L}\frac{\partial Q}{\partial \tilde{L}} \equiv Q \tag{3}$$

Using Euler's Theorem, it follows that the expressions for the profit share, π, and wage share, $1-\pi$, are:

$$\pi \equiv \frac{K}{Q}\frac{\partial Q}{\partial K} = \frac{aC^{\psi}}{aC^{\psi} + (1-a)E(W)^{\psi}}, \tag{4}$$

$$1 - \pi \equiv \frac{\tilde{L}}{Q}\frac{\partial Q}{\partial \tilde{L}} = \frac{(1-a)E(W)^{\psi}}{aC^{\psi} + (1-a)E(W)^{\psi}} \tag{5}$$

Substituting (4) and (5) in (2); and using $c = k - l$, yields:

$$p = b + \pi(k - l) + (1 - \pi)w, \tag{6}$$

Equation (6) shows that the growth rate of labor productivity is equal to the growth rate of technology, captured by b, and capital deepening, $k-l$, mediated by distribution, captured by π, and the growth rate of wages, also mediated by distribution, captured by the wage share, $1 - \pi$.

The validity of the separation of technical change from capital deepening or factor substitution is a difficult issue. Rosenberg (1976), among several others, has argued that this is partly because new capital embodies technical change, and capital deepening entails costly innovative effort. While acknowledging these critiques, rather than taking an a priori stance against such a distinction, I argue that the value of σ which represents capital deepening is ultimately an empirical question. Empirical studies show that the value of σ varies depending on the stage of a country's development and the overwhelming evidence is that it's value lies between 0 and 1 Gechert et al. (2019), Knoblach et al. (2020). Moreover, although capital deepening may embody technical change, unlike pure technical change, (6) shows that capital deepening is mediated by distribution.

Kaldor (1961) himself was explicitly opposed to dichotomizing technical change from capital accumulation. This belief is captured by the following proposition: "The rate of shift of the production function due to the changing

[1] For the derivation of the Solow condition, see the appendix.

state of 'knowledge' . . . .depends upon the rate of accumulation of capital itself. Since improved knowledge is, largely if not entirely, infused into the economy through the introduction of new equipment, the rate of shift of the curve will depend on the speed of movement along the curve, which makes any attempt to isolate one from the other nonsensical"Kaldor (1961).

It may seem unwarranted to use a CES production function to investigate the theoretical basis of the K-V Law, which allows a distinction between technical progress from capital deepening to be made; when Kaldor himself was opposed to such a dichotomy. To justify my theoretical approach, I turn to McCombie and Spreafico (2015), who argue that Kaldor put forward two versions of the technical progress function to remove this dichotomy, but he did not really succeed. This is because the linear version of the technical progress function, put forward in Kaldor (1957) and Kaldor (1961) can be integrated into the Cobb-Douglas production function. Moreover, the steady-state growth conditions are the same as Solow's neoclassical growth model. Thus, any inherent problems with the Cobb-Douglas production function (including the separation of technical progress from capital accumulation) also apply to Kaldor's technical progress function. Thus, although Kaldor's technical progress function provides a theoretical underpinning for the K-V Law, it does not succeed in removing the dichotomy between capital accumulation and technical change. Thus, they argue that the technical progress function did not succeed in giving effect to the relationships Kaldor had in mind. Further, McCombie and Spreafico (2015) argue that the second version of the technical progress function, in Kaldor and Mirrlees (1971) is not markedly different from the former, because although it cannot be integrated into a Cobb-Douglas production function, its steady state growth conditions are the same as Solow's growth model; and concomitantly, the rate of productivity growth is independent of the investment-output ratio.

Moreover, McCombie and Spreafico (2015) have argued that the technical progress function arguably provides a theoretical underpinning of the Verdoorn Law, and Kaldor probably considered Verdoorn's Law as an empirical counterpart to his technical progress function. However, Verdoorn's Law reflects Kaldor's shift of emphasis from the impossibility of this dichotomy to the importance of economies of scale and the resulting cumulative causation nature of growth. The linear version of the technical progress function, if derived from a Cobb-Douglas production function, reflects constant returns to scale, and the non-linear version has decreasing returns to the capital-labor ratio. On the other hand, Verdoorn's law allows for increasing returns to scale.

Besides Kaldor's technical progress function, other attempts to investigate the theoretical basis of the K-V Law include Bairam (1987) - who used a Cobb-Douglas production function which is limited in scope. Others include and Ros (2001) and Ros (2013) - who used the CES production function, which is a substantial improvement over Bairam (1987). Other attempts to investigate the theoretical basis of the KV Law can be attributed to Storm, Naastepad, et al. (2012), and Basu and Budhiraja (2021), who used a CES production function as well. Following this strand of literature, I draw out the implications of using this strand of literature to investigate the theoretical basis of the K-V Law. It is true that these frameworks assume the separation of technical progress from capital accumulation, but I am unaware of existing frameworks which avoid this dichotomy, and can be used to investigate the theoretical basis of the K-V Law (with the exception of Kaldor's technical progress function, which does not succeed).

Now, to complete the description of production, following Basu and Budhiraja (2021), I consider two alternative characterizations of technological progress.

The first specification of technology is called 'Rosenstein-Rodan' technology, as conceived by Rosenstein-Rodan (1943). It captures technological external economies which generate aggregate increasing returns to scale. This is specified by the technology parameter, B, in equation (1), as an increasing function of the economy-wide average size of the capital stock: $B = \tilde{K}^{\mu}$. Here, $\tilde{K}$ is the average capital stock in the industry, and μ captures the increasing returns to scale parameter generated by technological external economies of scale. In equilibrium, all firms have the same industry-average capital stock, i.e., $\tilde{K} = K$, so that $B = K^{\mu}$. This yields: $b = \mu k$.

The second characterization is called 'Kaldor technology'. It captures "demand induced technical change", based on Kaldor's (1966). In this specification, the growth rate of the technology parameter, B in (1) is proportional to the growth rate of output, i.e., $b = \xi q$, where ξ is the demand induced technical progress parameter.

Now, I find an expression for (6) under different characterizations of technology.

Substituting the Rosenstein-Rodan characterization of technology, $b = \mu k$, into (6), yields:

$$p = (\mu + \pi)k - \pi l + w(1 - \pi) \tag{7}$$

Turning to the Kaldor characterization of technology, substituting $b = \xi q$, in (6), yields:

$$p = \frac{\pi}{1-\xi}k + \frac{\xi - \pi}{1-\xi}l + \frac{(1-\pi)}{1-\xi}w \tag{8}$$

2.2 The Labor Market

The labor demand curve can be obtained from the profit maximizing behaviour of capitalist firms. The profit function of a representative firm is given by:

$$R = B[aK^{\psi} + (1-a)(E(W)L)^{\psi}]^{\frac{1}{\psi}} - WL \tag{9}$$

Using the first order condition with respect to the choice variable, L, yields:

$$W = B(1-a)[aC^{\psi} + (1-a)E(W)^{\psi}]^{\frac{1-\psi}{\psi}} E(W)^{\psi} \tag{10}$$

Now, taking growth rates of (10), using the Solow condition; and using $c = k - l$, yields:

$$w = \frac{b}{\pi\sigma} + (k - l) \tag{11}$$

On the supply-side of the labor market, assuming constant elasticity of labor supply implies that the labor supply function in levels is: $LE(W) = AW^{\eta}$[2]. Taking growth rates, and using the Solow condition, yields the labor supply equation:

$$l = (\eta - 1)w \tag{12}$$

When the labor market is in equilibrium, the growth rate of labor demand is equal to the growth rate of labor supply. When technology is of the Rosenstein-Rodan variety, the labor market equilibrium condition is given by:

$$l = \left(1 - \frac{1}{\eta}\right)\left(\frac{\mu}{\pi\sigma} + 1\right)k \tag{13}$$

When technology is of the Kaldor variety, the labor market equilibrium condition is given by:

$$l\left[\frac{\eta}{\eta - 1}\right] = \frac{\xi Q}{\pi\sigma} + k \tag{14}$$

2.3 Deriving the Kaldor-Verdoorn Coefficients

Explicit expressions for the Kaldor-Verdoorn coefficients can be derived by bringing together the characterization of production conditions, and labor market equilibrium.
In the case of Rosenstein-Rodan technology, using the labor market equilibrium condition captured by (13), the production conditions captured by (7), and using $l = q - p$, yields:

$$a_1 = \frac{\pi\eta\mu(\sigma - 1) + \pi(\mu + \pi\sigma)}{(\eta - 1)(\mu + \pi\sigma) + \pi\eta\mu(\sigma - 1) + \pi(\mu + \pi\sigma)} \tag{15}$$

In the case of Kaldor technology, using the labor market equilibrium captured by (15), and the technological constraints captured by (8), yields:

$$a_1 = \frac{\xi(1 - \sigma)(1 - \eta) + \pi\sigma}{\sigma(\eta + \pi - 1)} \tag{16}$$

The value of the KV coefficient in (15) and (16) lies in the $(0, 1)$ interval if the numerator and denominator in each equation are of the same sign, and the absolute value of the numerator is less than the absolute value of the denominator.

[2] Although the paper assumes that the growth of labor supply responds positively to the growth of the real wage, special cases (including elasticities of zero and infinity) are considered.
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Empirically, the KV coefficient is derived from a regression of productivity growth on output growth. From a developmental perspective, it is ideal if the KV coefficient lies in the $(0, 1)$ interval. This ensures that the growth rate of output or demand has a positive effect on the growth rate on technological change, captured by productivity growth. Moreover, this also ensures that the effect of output growth on productivity growth is less than one-for-one, which implies that output growth is accompanied by a positive growth rate of employment. If the KV coefficient is greater than unity, this implies that the effect of output growth on productivity growth is more than one-for-one; implying negative employment growth. If, on the other hand, the KV coefficient is negative, although employment growth would be positive; this would imply that labor is reallocated to a sector with falling productivity. Both these scenarios are undesirable from a developmental perspective.

2.4 Special Cases

This section considers some special cases related to different conceptualizations of the labor market, and production conditions; under Rodenstein-Rodan and Kaldor conceptualizations of technology.

2.4.1 Leontief Production Function ($\sigma = 0$)

A Leontief specification of technology implies that the elasticity of factor substitution is zero. In the case of Rosenstein-Rodan technology, substituting $\sigma = 0$ in (15), yields:

$$a_1 = \frac{-\pi}{1 - \pi} \tag{17}$$

In this case, the KV coefficient is negative, which implies that output growth leads to a fall in productivity growth in the industrial sector. Although this implies that employment growth is positive, this is attenuated by the fact that labor is absorbed in a sector with falling productivity.

Now, in the case of Kaldor technology, substituting $\sigma = 0$ in (16), yields: $a_1 \rightarrow \infty$. This suggests that the effect of output growth on productivity growth is more than one-for-one, implying negative employment growth. This too is a perverse scenario from a developmental perspective.

These results suggest that when efficiency wages are integrated into the KV model, the Kaldorian interpretation of Verdoorn's Law no longer holds with Leontief production technology. Thus, the KV coefficient cannot be used to draw unambiguous inferences about the increasing returns to scale parameter. Rather, in the case of Rosenstein-Rodan production technology, the KV coefficient can be used to draw unambiguous inferences about the profit share parameter. This is in contrast to the result reported in Ros (2013) and Basu and Budhiraja (2021), wherein Leontief production technology was a benchmark case where the Kaldorian interpretation of Verdoorn's Law was valid.

2.4.2 Cobb-Douglas Production Function($\sigma = 1$)

A Cobb-Douglas specification of technology implies that the elasticity of factor substitution is one. Substituting $\sigma = 1$ in (15), yields:

$$a_1 = \frac{\pi}{\eta + \pi - 1} \tag{18}$$

Now, substituting $\sigma = 1$ in (16), yields the same expression for the KV coefficient:

$$a_1 = \frac{\pi}{\eta + \pi - 1} \tag{19}$$

Since in the case of Cobb-Douglas production technology, the profit share is constant, this result suggests that the KV coefficient is entirely determined by the labor supply elasticity. Moreover, both these results suggest that an important restriction which the elasticity of labor supply, η, must satisfy for the Kaldor-Verdoorn coefficient to lie in the $(0, 1)$ interval is that it must be greater than unity. This is a departure from the neoclassical case reported in Basu and Budhiraja (2021), where η must lie in the $(-1, 0)$ so that the KV coefficient lies in the $(0, 1)$ interval.

Basu and Budhiraja (2021) note that there is wide variation in the existing estimates of labor supply elasticity (η), ranging from -0.03 and 4. Thus, based on existing empirical evidence about the elasticity of labor supply, η, in this case, it is plausible for the KV coefficient to lie in the $(0, 1)$ interval if $\eta > 1$.

Thus, in the case of Cobb-Douglas production technology, when efficiency wage considerations are integrated into the KV model, although the response of productivity growth to output growth continues to be mediated entirely by the labor supply elasticity, the parameter values which the labor supply elasticity must satisfy for the KV coefficient to lie in the $(0, 1)$ interval change.

A special case which has been used extensively to characterize the Kaldorian interpretation of Verdoorn's Law combines Cobb-Douglas production technology with steady-state analysis. Since this special case does not take into account the parameters from the labor market, this interpretation remains valid when efficiency wages are integrated into the KV model.

Cobb-Douglas production technology implies that the profit share, π, is constant, and is given by say, $\pi = a$. If the economy is on a steady-state growth path, then the capital-output ratio is constant, and hence $q = k$. In the case of Rosenstein-Rodan technology, (7) characterizes production. Using $q = k$, replacing l with $q - p$ and using $\pi = a$, yields $a_1 = \frac{\partial P}{\partial Q} = \frac{\mu}{1-a}$.

Similarly, in the case of Kaldor production technology, $a_1 = \frac{\partial P}{\partial Q} = \frac{2a-\xi}{a+1-2\xi}$. Thus, under both characterizations of technology, it is possible to make unambiguous inferences about the technological progress parameters using the KV coefficient.

2.4.3 Labor Constrained Economy($\eta \to 0$)

If the economy is constrained by the supply of labor, the elasticity of labor supply is zero. To compute the KV coefficient, dividing the numerator and denominator in (15) by η, and taking the limit of the expression as $\eta \to 0$ yields a negative KV coefficient:

$$a_1 = \frac{-\pi}{1-\pi} \tag{20}$$

On the other hand, taking the limit of the expression as $\eta \to 0$ in (16), yields:

$$a_1 = \frac{\xi(1-\sigma) + \pi\sigma}{\sigma(\pi - 1)} \tag{21}$$

In this case, the KV coefficient is likely to be negative, since $\xi \geq 0$, $0 < \pi < 1$ and empirical evidence suggests that the elasticity of factor substitution, σ, is likely to be less than unity. These results suggest that in a labor constrained economy with efficiency wages, the KV coefficient is negative, which is inimical to structural change.

This result is in contrast to the result reported for a labor constrained economy in Basu and Budhiraja (2021), where the K-V coefficient was unity under both conceptualizations of technology. In that case, since the labor supply was by assumption fixed in a labor constrained economy, there was a one-for-one change in productivity growth, in response to output growth; with no change in employment growth. However, when efficiency wages are integrated into the KV model, the KV coefficient is negative. Although this suggests that employment growth is positive, this also implies that labor is reallocated to a sector with falling productivity; which is inimical to structural change.

2.4.4 Labor Surplus Economy($\eta \to \infty$)

If the economy has surplus labor in the Lewisian sense, then the labor supply elasticity is infinite. Thus, taking the limit of the expression as $\eta \to \infty$ in (15), yields:

$$a_1 = \frac{\pi\mu(\sigma - 1)}{(\mu + \pi\sigma) + \pi\mu(\sigma - 1)} \tag{22}$$

This suggests that an important restriction which σ must satisfy for the KV coefficient to lie in the $(0, 1)$ interval is that it must be greater than unity. On the other hand, if σ is less than unity, which is the more plausible case as suggested by empirical evidence, $(\sigma - 1)$ is negative; $0 < \pi < 1$, and $\mu \geq 0$, so from (22), it follows that the condition $|\pi\mu(\sigma - 1)| > |\mu + \pi\sigma|$ is required to ensure $a_1 > 0$.

Further, substituting $\eta \to \infty$ in (16), yields:

$$a_1 = \xi\left(1 - \frac{1}{\sigma}\right) \tag{23}$$

This suggests an important restriction which σ must satisfy for a_1 to lie in the $(0, 1)$ interval is that it must be greater than unity. However, in this case, since $\xi \geq 0$ and empirical evidence suggests that σ lies in the $(0, 1)$ interval, the

KV coefficient is likely to be negative in this case. As discussed before, this is inimical to structural change.

This is in contrast to the result reported in Basu and Budhiraja (2021), where σ must be less than unity for the KV coefficient to lie in the $(0, 1)$ interval.

2.4.5 Absence of scale effects($\mu \to 0$)

In the case of Rosenstein-Rodan technology, the absence of scale effects can be captured by $\mu = 0$. Substituting $\mu \to 0$ in (15), yields:

$$a_1 = \frac{\pi}{\eta + \pi - 1} \tag{24}$$

This result is similar to (18) and (19). However, in this case, since we are not assuming Cobb-Douglas production technology, the profit share is not constant. Thus, the KV coefficient depends on the labor supply elasticity and the profit share parameters. Here, η must be greater than unity for the Kaldor-Verdoorn coefficient to lie in the $(0, 1)$ interval. As noted earlier, empirical evidence suggests that this is plausible. Moreover, this result is similar to Basu and Budhiraja (2021), in that it suggests the KV effect exists even in the absence of scale effects.

Now, substituting $\xi \to 0$ in (16), yields the same result:

$$a_1 = \frac{\pi}{\eta + \pi - 1} \tag{25}$$

3 Integrating the Kaldor-Verdoorn model with a labor discipline model

Now, an alternative formulation of the labor market characterized by a labor discipline model is considered. The particular formulation considered here, follows Bowles (2004), and the labor market and the concomitant employment relationship is a variant of what Bowles has termed the effort regulation or labor discipline model based on contingent renewal; also termed as the Marx-Coase-Simon model of employment relationships. [3] In this formulation, wages relative to some fallback position, along with monitoring inputs are determinants of the effort function. This formulation of the labor market is a departure from the earlier formulation where the absolute value of the real consumption wages is the only determinant of effort.

[3] Although Bowles's labor discipline or effort regulation model has a set-up similar to a model with efficiency wages, he argues that the term 'efficiency wages' is misleading because the characteristics of the market equilibrium are such that: 1) There is involuntary unemployment, because labor markets do not clear. 2) The resulting exchange (E*,W*) is Pareto inefficient.

3.1 Production and Technology

Similarly to the case of a KV model which integrates a simple efficiency wage set-up, the production conditions are characterized by the following Constant Elasticity of Substitution (CES) production function:

$$Q = B[aK^{\psi} + (1-a)(E(W, M; Z)L)^{\psi}]^{\frac{1}{\psi}} \tag{26}$$

To integrate the labor discipline model in the KV model, the labor input is replaced by the effective labor input, $\tilde{L}$, which is the product of the number of identical workers hired, L, and effort, $E \in (0, 1)$, i.e., $\tilde{L} = E(W, M; Z)L$. Wages relative to the worker's fallback position, Z, along with monitoring inputs, M, are determinants of the effort function. This formulation of the effective labor input is a departure from the earlier case where the absolute value of the real consumption wages is the only determinant of effort function.

Here, the distinction between production process presented here from its Walrasian analogue can be found in the cost function associated with the production function. Central to this distinction is Marx's distinction between work ("labor") and labor time ("labor power"). In the Marxian analysis of the production process, it is labor time, not work itself which is purchased. Thus, the cost of labor cannot be expressed as a market determined wage rate multiplied by the number of labor hours hired. To express the cost function in the same terms as the production function, Bowles (1985) uses a labor extraction function or what is called an effort function in Bowles (2004)- representing the amount of labor done per hour of labor hired as a function of the instruments used to elicit work from workers. Thus, the $\tilde{L} = E(W, M; Z)L = l^{*}lp$, where l^{*} is the labor extraction function and lp represents labor power.

Dividing both sides of (26) by L, taking growth rates; and using the Solow equation with an augmented effort function, $\frac{E_W(W+M)}{E} = \frac{E_M(W+M)}{E} = 1$, yields: [4],

$$p = b + \frac{aC^{\psi}c + (1-a)E(W, M; Z)^{\psi}\left[\frac{W}{W+M}w + \frac{M}{W+M}m\right]}{aC^{\psi} + (1-a)E(W, M; Z)^{\psi}} \tag{27}$$

Using Euler's Theorem, it follows that the expressions for the profit share, π, and wage share, $1-\pi$, are:

$$\pi \equiv \frac{K}{Q}\frac{\partial Q}{\partial K} = \frac{aC^{\psi}}{aC^{\psi} + (1-a)E(W, M; Z)^{\psi}}, \tag{28}$$

$$1 - \pi \equiv \frac{\tilde{L}}{Q}\frac{\partial Q}{\partial \tilde{L}} = \frac{(1-a)E(W, M; Z)^{\psi}}{aC^{\psi} + (1-a)E(W, M; Z)^{\psi}} \tag{29}$$

Substituting the expressions for the profit share and the wage share into (27), and using $c = k - l$ yields:

[4] For it's derivation, see the Appendix.

$$p = b + \pi(k - l) + (1 - \pi)\left[\frac{W}{W+M}w + \frac{M}{W+M}m\right], \tag{30}$$

(30) shows that the growth rate of labor productivity, p, is equal to the growth rate of technology, captured by b, and capital deepening, $k-l$, mediated by distribution, captured by π; and the sum of the growth rates of wages, w, weighted by the share of wages in total expenditure on labor, $W/(W+M)$, and the growth rate of expenditure on monitoring inputs, m, weighted by the share of expenditure on monitoring inputs in total expenditure on labor, $M/(W+M)$. This too is mediated by distribution, captured by the wage share, $1-\pi$.

Using the Rosenstein-Rodan specification of the technological progress parameter, $b = \mu k$ in (30), yields:

$$p = (\mu + \pi)k - \pi l + (1 - \pi)\left[\frac{W}{W+M}w + \frac{M}{W+M}m\right] \tag{31}$$

Using the Kaldor specification of the technological progress parameter, $b = \xi q$ in (30), and using $q = p + l$ yields:

$$p = \frac{\pi}{1-\xi}k + \frac{(\xi - \pi)}{1-\xi}l + \frac{1-\pi}{1-\xi}\left[\frac{W}{W+M}w + \frac{M}{W+M}m\right] \tag{32}$$

3.2 The labor market

The labor demand function can be obtained from the profit maximizing behavior of capitalist firms.

The profit function of a representative firm is given by:

$$R = B[aK^{\psi} + (1-a)(E(W,M;Z)L)^{\psi}]^{\frac{1}{\psi}} - (W+M)L \tag{33}$$

Using the first order condition with respect to the choice variable, L, yields:

$$W + M = B(1-a)[aC^{\psi} + (1-a)E(W,M;Z)^{\psi}]^{\frac{1-\psi}{\psi}}E(W,M;Z)^{\psi} \tag{34}$$

Taking growth rates of (34), and using the Solow condition, yields the labor demand function:

$$\frac{W}{W+M}w + \frac{M}{W+M}m = \frac{b}{\pi\sigma} + (k - l) \tag{35}$$

On the supply side of the labor market, assuming a constant elasticity of labor supply implies:

$$LE(W,M;Z) = AW^{\eta} \tag{36}$$

Here, A is a constant; $LE(W,M;Z)$ may be understood as the effective labor supply; and η is the elasticity of effective labor supply with respect to the real wage.

Taking growth rates; and using the Solow condition, yields the labor supply function:

$$\frac{W}{W+M}w + \frac{M}{W+M}m = \eta w - l \tag{37}$$

When the labor market is in equilibrium, the growth rate of labor demand is equal to the growth rate of labor supply. When technology is of the Rosenstein-Rodan variety, the labor market equilibrium condition is given by:

$$w = k\left[\frac{1}{\eta} + \frac{\mu}{\eta\pi\sigma}\right] \tag{38}$$

When technology is of the Kaldor variety, the labor market equilibrium condition is given by:

$$w = \frac{\xi q}{\eta\pi\sigma} + \frac{k}{\eta} \tag{39}$$

3.3 Deriving the Kaldor-Verdoorn coefficients

Explicit expressions for the Kaldor-Verdoorn coefficients can be derived by bringing together the characterization of production conditions and labor market equilibrium.
In the case of Rosenstein-Rodan technology, using the labor market equilibrium condition characterized by (38) and the technological constraints captured by (31); and using $l = q - p$, yields the K-V coefficient, a_1:

$$a_1 = \frac{-\pi}{1-\pi} \tag{40}$$

In this case, the KV coefficient is negative, which implies that output growth leads to a fall in productivity growth in the industrial sector. As emphasised earlier, a negative KV coefficient is inimical to structural change. This result also suggests that unambiguous inferences about the profit share parameter can be drawn from the K-V coefficient.

However, in this case, although the production and labor market side were brought together, since l does not appear in the characterization of labor market equilibrium, one cannot use $l = q-p$. Thus, only the underlying parameters from the production side end up playing a role in determining the K-V coefficient.

In the case of Kaldor technology, using the labor market equilibrium condition characterized by (39) and the technological constraints captured by (32); and using $l = q - p$, yields:

$$a_1 = \frac{\xi - \pi}{1-\pi} + \frac{\xi}{\eta\pi\sigma}\frac{W}{W+M} \tag{41}$$

3.4 Special cases

In the case of Rosenstein-Rodan technology, since the KV coefficient in (40) cannot be simplified further under different conceptualizations of the labor market and production conditions, only the special cases associated with the Kaldor specification of technology are considered.

3.4.1 Leontief Production Technology

Substituting $\sigma = 0$ in (41), yields $a_1 \to \infty$. This suggests that the effect of output growth on productivity growth is more than one-for-one, implying negative employment growth. Such a scenario is perverse from the developmental perspective. This result is the same as Leontief production technology with efficiency wages presented in section 2.4.1, under both conceptualizations of technology.

This result suggests that when the labor discipline model is integrated into the KV model, the Kaldorian interpretation of Verdoorn's Law no longer holds with Leontief production technology. Thus, the KV coefficient cannot be used to draw unambiguous inferences about the increasing returns to scale parameter. This is in contrast to the result reported in Ros (2013) and Basu and Budhiraja (2021), where Leontief production technology was a benchmark case where the Kaldorian interpretation of Verdoorn's Law was valid.

3.4.2 Cobb-Douglas Production Function

Substituting $\sigma = 1$ in (41), yields:

$$a_1 = \frac{\xi - \pi}{1 - \pi} + \frac{\xi}{\eta\pi}\frac{W}{W + M} \tag{42}$$

In this case, the K-V coefficient depends on the profit share (which is constant here under Cobb-Douglas production technology), the labor supply elasticity, the demand-induced technical progress parameter, and the share of wages in total expenditure on labor. This is in contrast to the neoclassical labor market case reported in Basu and Budhiraja (2021) and the simple efficiency wage case with Cobb-Douglas production technology considered in 2.4.2, where the response of productivity growth to output growth was entirely mediated by labor supply elasticity.

The Kaldorian interpretation of Verdoorn's Law remains valid in the special case of Cobb-Douglas production technology, combined with steady state. In the case of Rosenstein-Rodan technology, (31) characterizes production. Using $q = k$, replacing l with $q - p$; and using $\pi = a$, yields $a_1 = \frac{\partial P}{\partial Q} = \frac{\mu}{1-\pi}$.

Similarly, in the case of Kaldor production technology, $a_1 = \frac{\partial P}{\partial Q} = \frac{\xi}{1-\pi}$. Since the profit share is constant in the case of Cobb-Douglas production technology, the KV coefficient is entirely determined by the returns to scale or endogenous technical change parameter.

3.4.3 Labor Constrained Economy

Substituting $\eta \to 0$ in (41), yields $a_1 \to \infty$. This result is in contrast to the analogue of this case reported in Basu and Budhiraja (2021), where the KV coefficient was one; and the analogue of this case with simple efficiency wages reported in 2.4.3, where the KV coefficient was negative. None of these scenarios are desirable from a developmental perspective.

3.4.4 Labor Surplus Economy

Substituting $\eta \to \infty$ in (41), yields:

$$a_1 = \frac{\xi - \pi}{1 - \pi} \tag{43}$$

This suggests that in a labor surplus economy, the K-V coefficient depends on the profit share and the demand-induced technical progress parameter. For a_1 to lie in the (0,1) interval, the condition that $0 < \pi < \xi < 1$ must be satisfied.

3.4.5 Absence of scale effects

Substituting $\xi \to 0$ in (41), yields:

$$a_1 = \frac{-\pi}{1 - \pi} \tag{44}$$

In this case, the KV coefficient is negative, which implies that output growth leads to a fall in productivity growth in the industrial sector. As emphasised earlier, this is inimical to structural change. This result also suggest that in the absence of scale effects, the K-V coefficient can be used to draw unambiguous inferences about the profit share parameter.

4 An alternative conceptualization of labor market equilibrium

In this section, I depart from the simplifying assumption of a constant elasticity of labor supply on the supply side. Instead, following Bowles (2004), a worker's best response function is used to represent the supply side of the labor market.

Suppose that all workers receive the following utility in each period:

$$U(W, E) = W - \frac{f/W}{1 - E} \tag{45}$$

with $U_W \geq 0$ and $U_E \leq 0$.

Here, W is the wage rate, $E \in (0, 1)$ is the worker's effort; and f is an exogenous wage norm, which may be understood as the "fair wage".

Capitalists and workers cannot write a contract in terms of E, because effort is not verifiable. Capitalists do not renew the wage contract with workers with probability $t = n(M)(1-E)$; based on the probability $n(M)$ that the employer will observe effort. When workers are terminated, they receive a fallback position equal to Z.

Assuming that workers do not discount the future, and the stationarity assumption holds; the present value of expected utility over an infinite horizon is given by:

$$V = U(W,E) + (1-t)V + tZ \tag{46}$$

i.e.,

$$V = W - \frac{f/W}{1-E} + (1-t)V + tZ \tag{47}$$

Using $t = n(M)(1-E)$, yields the present value of expected utility:

$$V(E,W) = \frac{W}{n(M)(1-E)} - \frac{f/W}{n(M)(1-E)^2} + Z \tag{48}$$

Now, the worker's best response function can be found by using the first order condition associated with maximizing the present value of expected utility with respect to effort.

i.e.,

$$V_E = \frac{W}{n(M)(1-E)^2} - \frac{2f/W}{n(M)(1-E)^3} = 0 \tag{49}$$

This gives the worker's best response function:

$$E = 1 - \frac{2f}{W^2} \tag{50}$$

Now, the profit function of a representative firm is given by:

$$R = B[aK^{\psi} + (1-a)(E(W,M;Z)L)^{\psi}]^{\frac{1}{\psi}} - (W+M)L \tag{51}$$

Using the Solow condition which falls out of the profit maximizing behavior of a representative firm, $E_W = \frac{E}{W}$, yields:

$$E_W = \frac{4f}{W^3} = \frac{E}{W} \tag{52}$$

This gives the following characterization of the labor market equilibrium:

$$E = \frac{4f}{W^2} \tag{53}$$

Now, bringing together the characterization of the production and labor market equilibrium, explicit expressions for the Kaldor-Verdoorn coefficient can be derived.

In the case of Rosenstein-Rodan technology, using the labor market equilibrium condition characterized by (53) and the technological constraints captured by (31); and using $l = q - p$, yields the following KV coefficient:

$$a_1 = \frac{-\pi}{1-\pi} \tag{54}$$

In this formulation, the K-V coefficient is negative. As discussed earlier, this is inimical to structural change. This result also suggests that unambiguous inferences about the profit share parameter can be drawn from the K-V coefficient.

In the case of Kaldor technology, using the labor market equilibrium condition characterized by (53) and the technological constraints captured by (32); and using $l = q - p$, yields the following Kaldor-Verdoorn coefficient:

$$a_1 = \frac{\xi - \pi}{1-\pi} \tag{55}$$

This result suggests that the K-V coefficient depends on the profit share and the demand-induced technical progress parameter. For a_1 to lie in the (0,1) interval, the condition that $0 < \pi < \xi < 1$ must be satisfied.

The only special case which can be examined here is the absence of scale effects, i.e., when the demand induced technical progress parameter, $\xi \to 0$. In this case, the K-V coefficient is given by:

$$a_1 = \frac{-\pi}{1-\pi} \tag{56}$$

In this case, the K-V coefficient is negative, which is inimical to structural change, as discussed earlier. This result also suggests that in the absence of scale effects, the K-V coefficient can be used to draw unambiguous inferences about the profit share parameter.

5 Conclusion

The K-V Law has been widely used in heterodox macroeconomic models; and it is often evoked to represent increasing returns and endogenous technical change. However, its theoretical underpinnings in terms of the underlying assumptions related to the production and labor market side of the economy it represents, have not been well established.

Building on Ros (2001) and Basu and Budhiraja (2021), I integrate the analysis of the labor market and the production side of the economy to derive explicit expressions for the KV coefficient. I depart from the neoclassical assumption that an exogenously given wage is equal to the marginal product of labor; by integrating efficiency wage considerations and a labor discipline model with the K-V model. Similarly to Ros (2001) and Basu and Budhiraja (2021), I find that the KV coefficient is jointly determined by the elasticity of factor substitution, the labor supply elasticity, the profit share parameter, the returns to scale or induced technical change parameters. Moreover, when the KV model is integrated with the labor discipline model, the KV coefficient additionally depends on the share of wages in total expenditure on labor. Thus, the KV coefficient cannot be used for unambiguous inferences about the returns to scale or demand induced technical change parameters; thus challenging the widespread use of the Kaldorian interpretation of Verdoorn's Law.

Moreover, I find that when efficiency wage considerations or a labor discipline model are integrated with the KV model, Leontief production technology no longer presents a benchmark case where the Kaldorian interpretation of Verdoorn's Law holds. This is in contrast to the results in Ros (2001) and Basu and Budhiraja (2021), where the Leontief production technology case was presented as a benchmark case, where unambiguous inferences about the IRS parameters or demand induced technical change parameters could be derived from the KV coefficient. However, this result was puzzling because there is no empirical evidence in favor of Leontief production technology (which requires that the elasticity of subsitution is zero). Instead, the overwhelming empirical evidence suggests that the value of the elasticity of factor substitution varies depending on the stage of a country's development, and it lies between 0 and 1 Gechert et al. (2019), Knoblach et al. (2020). This suggests a need to be explicit about the underlying assumptions about production and the labor market underlying the KV coefficient.

The only case where the Kaldorian interpretation remains valid is the restrictive case of Cobb-Douglas production, combined with the assumption that the economy is on its steady state growth path. However, when one moves away from this restrictive case towards a more general analysis, the Kaldorian interpretation of Verdoorn's Law does not hold under any underlying conceptualization of the labor market used here, and production conditions.

The investigation of the other special cases also yields important results. For instance, when efficiency wages are integrated into the KV model, in the absence of scale effects, a KV coefficient which lies in the $(0, 1)$ interval can exist, provided that the elasticity of labor supply is greater than unity; which is empirically quite plausible. Thus, a KV effect exists even though returns to scale are ruled out by assumption. On the other hand, when a labor discipline model is integrated with a KV model, the KV coefficient becomes negative. Moreover, when efficiency wages and a labor discipline model are integrated with the KV model, the value of the KV coefficient is significantly altered in the context of Cobb-Douglas production technology, a labor constrained and a surplus labor economy.

6 Appendix

6.1 Derivation of the Solow Condition with a simply efficiency wage model

The profit function of a representative firm is given by:

$$R = B[aK^{\psi} + (1-a)(E(W)L)^{\psi}]^{\frac{1}{\psi}} - WL \tag{A.1}$$

The first order conditions with respect to the choice variables, L and W, of the firm are given by:

$$\frac{\partial R}{\partial L} = B\frac{1}{\psi}[aK^{\psi} + (1-a)(E(W)L)^{\psi}]^{\frac{1-\psi}{\psi}}(1-a)\psi(E(W)L)^{\psi-1}E(W) - W = 0 \tag{A.2}$$

The above first order condition is analogous to the familiar profit maximizing condition that the wage is equal to the marginal product of labor. But with effort endogenous, Bowles states an analogous condition as requiring that the marginal productivity of *effort* be equal to the cost of a unit of *effort*.

$$\frac{\partial R}{\partial W} = B\frac{1}{\psi}[aK^{\psi} + (1-a)(E(W)L)^{\psi}]^{\frac{1-\psi}{\psi}}(1-a)\psi(E(W)L)^{\psi-1}E'(W)L - L = 0 \tag{A.3}$$

From equation (A.2), it follows that:

$$B[aK^{\psi} + (1-a)(E(W)L)^{\psi}]^{\frac{1-\psi}{\psi}}(1-a) = \frac{W}{(E(W)L)^{\psi-1}E(W)} \tag{A.4}$$

Substituting (A.4) in (A.3),

$$\frac{W}{(E(W)L)^{\psi-1}E(W)}(E(W)L)^{\psi-1}E'(W)L = L \tag{A.5}$$

From this follows the so-called 'Solow condition',

$$\frac{E'(W)W}{E(W)} = 1 \tag{A.6}$$

The Solow condition requires that the marginal impact of variations in wages on effort be equal to the average level of effort per dollar of expenditure on labor.

Equation (A.6) may be restated as requiring the marginal productivity of labor time (evaluated at the levels determined by the Solow Condition), be equal to the cost of a unit of labor.

6.2 Derivation of the Solow condition with a labor discipline model

The profit function of a representative firm is given by:

$$R = B[aK^{\psi} + (1-a)(E(W,M;Z)L)^{\psi}]^{\frac{1}{\psi}} - (W+M)L \tag{B.1}$$

The first order conditions with respect to the choice variables, L, W, and M of the firm are given by:

$$\frac{\partial R}{\partial L} = B\frac{1}{\psi}[aK^{\psi}+(1-a)(E(W,M;Z)L)^{\psi}]^{\frac{1-\psi}{\psi}}(1-a)\psi(E(W,M;Z)L)^{\psi-1}E(W,M;Z)-(W+M) = 0 \tag{B.2}$$

$$\frac{\partial R}{\partial W} = B\frac{1}{\psi}[aK^{\psi}+(1-a)(E(W,M;Z)L)^{\psi}]^{\frac{1-\psi}{\psi}}(1-a)\psi(E(W,M;Z)L)^{\psi-1}E_W(W,M;Z)L-L = 0 \tag{B.3}$$

$$\frac{\partial R}{\partial M} = B\frac{1}{\psi}[aK^{\psi}+(1-a)(E(W,M;Z)L)^{\psi}]^{\frac{1-\psi}{\psi}}(1-a)\psi(E(W,M;Z)L)^{\psi-1}E_M(W,M;Z)L-L = 0 \tag{B.4}$$

Let $A = B\frac{1}{\psi}[aK^{\psi}+(1-a)(E(W,M;Z)L)^{\psi}]^{\frac{1-\psi}{\psi}}(1-a)\psi(E(W,M;Z)L)^{\psi-1}$

From (B.2), and using the definition of A, it follows that,

$$A = \frac{W+M}{E} \tag{B.5}$$

The above first order condition is analogous to the familiar profit maximizing condition that the wage is equal to the marginal product of labor. But with effort endogenous, Bowles states an analogous condition as requiring that the marginal productivity of *effort* be equal to the cost of a unit of *effort* (including the cost of monitoring).

Combining (B.3) and (B.4); and using (B.5), yields:

$$E_W = E_M = \frac{1}{A} = \frac{E}{W+M} \tag{B.6}$$

(B.6) is the so-called Solow condition, extended to account for monitoring inputs. It requires that the marginal impact of variations in both wages and monitoring inputs on effort be equal to the average level of effort per dollar of expenditure on labor.

Equation (B.6) may be expresses equivalently as: $AE^* = W^* + M^*$; which may be restated as requiring the marginal productivity of labor time (evaluated at the levels determined by the Solow Condition), be equal to the cost of a unit of labor.

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