# Household debt and gender: the effects of monetary policy on consumption inequality.

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**Abstract**

We introduce a Post-Keynesian Stock-Flow Consistent (PK-SFC) model in which the household sector is divided into rentiers, middle-income earners, and low-income workers. In this model, we assume that low-income workers are the only category in the household sector for which earned income is not enough to finance their standards of living. As a result, their consumption has to be partially financed by loans. We derive a supermultiplier model in which credit-financed consumption drives growth, allowing for the rate of capacity utilization to adjust to a normal level. We find an analytical solution to the short-run equilibrium of our model, as well as the steady-state rate of growth of our economy. Finally, we employ a numerical illustration of the model using parameter values based on the US economy and we consider the effects

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of interest rates on household income. We propose that the effort of Central Banks to target inflation is detrimental to both low-income and middle-class workers.

# 1 Introduction

Household debt in the United States has shown a consistent trend for at least the last two decades: the financial burden of debt of most families has increased, as shown in figure 1 below. The causes of this trend have been known for some time. Liberalization of labor markets, suppression of government benefits, as well as price increases in health and education are some of the neoliberal policies that drove the deterioration of financial security.

The trend these policies created evolved in tandem with disparities in income distribution. If most families have been incurring debt, others (albeit a minority) have been enjoying extremely high levels of income and wealth accumulation. According to Parker (2014), the top 5% of households increased both their share of wealth and income in the last 20 years, while other socioeconomic groups saw their income and wealth decline in the same period.

The share of credit card debt held by American households poses a significant challenge to the sustainability of economic conditions. As shown in figure 2 below, in 2019, the lower the income percentile to which a household belonged, the higher the share of credit card debt to its total debt, according to the Federal Reserve Board's Survey of Consumer Finance.

In this context, the literature on household debt dynamics has begun to research what monetary authorities have accomplished when they pursue a contractionary monetary policy. For instance, we know that, generally, household spending decreases as the interest rate increases. However, responses vary by race and gender depending on the level of wealth and income, job security, and access to mortgage refinancing options. Puig (2022) argued that the larger share of spending reduction among households headed by whites pertains to the consumption of durable goods, whereas among Black families, everyday goods and services represent the larger share of expenditure reduction.

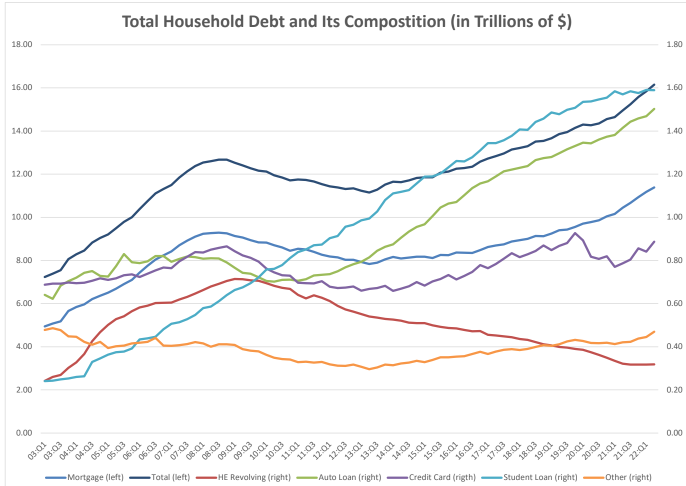


Figure 1: Total household debt. Source: Federal Reserve Bank of New York.

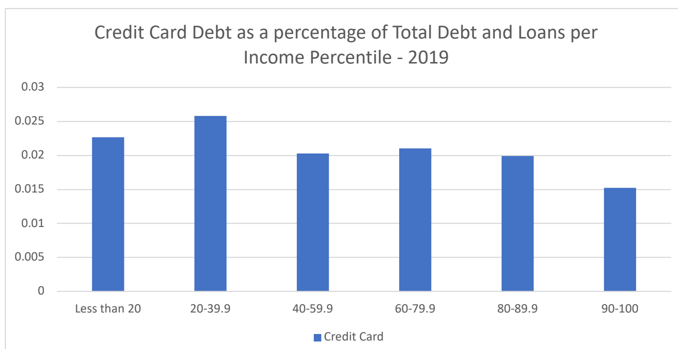


Figure 2: The share of credit card debt held by American households in 2019. Source: Federal Reserve Board

This paper incorporates two main findings in this literature. The first finding shows that middle-income households incur debt to consume durable goods or invest in real estate and that a significant part of their loans is actually held in mortgages. The second finding states that low-income households, which finance their consumption of non-durable goods through credit card debt and personal loans, are primarily headed by females, especially non-white females.

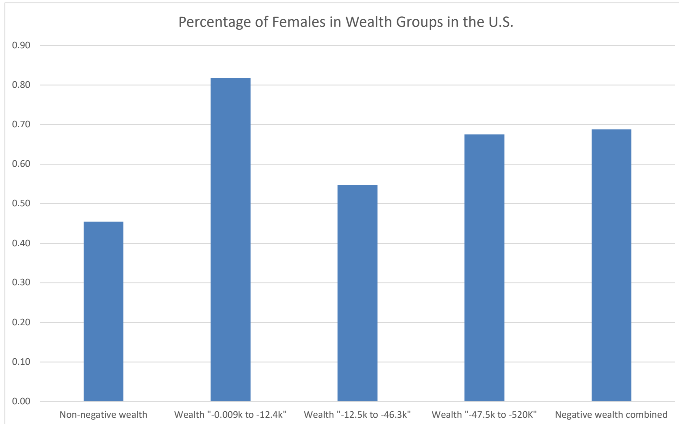


Figure 3: Female participation in wealth ranges in the US. Source: Federal Reserve Bank of New York.

This second finding has also unveiled gender and racial dynamics involved in the differentiation described above. More precisely, as shown in recent results of the consumer expenditure survey (see Puig (2022) for more details on this), we can see that households headed by men or whites are more likely to be mortgagors or homeowners, while those headed by women or blacks are more likely to be renters. Confirming the trend of increasing inequality, women represent a significant share of different wealth ranges, as figure 3 illustrates. In fact, women represent the largest share of all the negative-wealth ranges, and they are 80% of those with wealth between $-0.009K and $-12.4K. Despite representing a majority of the population, women persistently face disadvantageous conditions in the housing market. They constitute the majority of renters, but the minority of homeowners with mortgages, for instance (see figure 4).

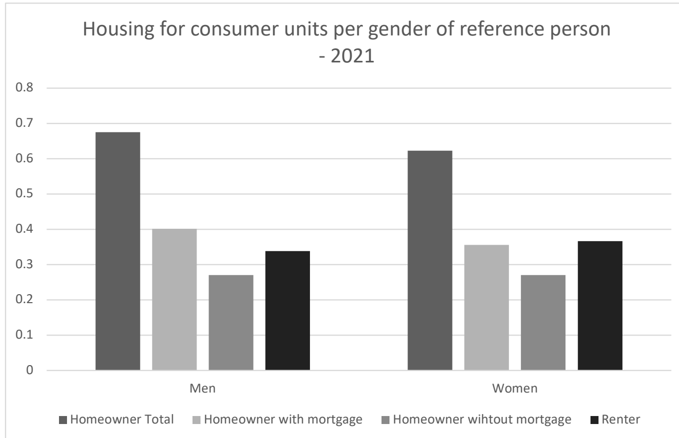


Figure 4: Male and female participation in the housing market in 2021. Source: Bureau of Labor Statistics

Given these trends of debt accumulation observed across different income groups, Bartscher et al. (2021) emphasizes that the effect of monetary policy might not be the same for everyone, especially if we focus on the asset price effect of interest rate.[1] Additionally, as suggested by Barba and Pivetti (2009) rising household debt can be at least partially explained by "efforts of low and middle-income classes to maintain, as long as possible, their relative standards of consumption in the face of persistent changes in income distribution in favor of households with higher income" (Barba and Pivetti, 2009, p. 122). Moreover, the gap between targeted consumption and household wages is further increased by the degree to which social provisions have been reduced and replaced by private consumption, as argued in Lapavitsas (2012).[2]

Based on these observations, and drawing on Szymborska (2022), we develop a Post-Keynesian Stock-Flow Consistent model (PK-SFC) where we divide the household sector into three, low-income workers, middle-class workers, and rentiers. Additionally, following Freitas and Serrano (2015) and Avritzer and Brochier (2022), we assume a type of supermultiplier model where low-income household credit financed

[1] See Góes (2021) for more details on the transmission mechanisms between monetary policy and consumption.

[2] These ideas have also been emphasized in theoretical works by Ryoo (2016), Setterfield and Kim (2016, 2017, 2020) and Dutt (2005, 2006), as well as empirical work, such as Stockhammer and Wildauer (2016).

consumption is the semi-autonomous component of demand that drives growth.[3] We evaluate the changes in consumption in each of these sectors vis-à-vis one another to trace the different responses to movements in the interest rates.

This paper is composed of three sections besides this introduction and a conclusion. Section two provides a detailed description of each sector in our economy as well as their assumed behavior. Section three presents the short-run as well as the long-run steady-state dynamics of our model. Once we account for the steady state short- and long-run dynamics, we present our measures for income and consumption inequality. Finally, section four concludes with a numerical illustration of the model and the results of contractionary monetary policy in income and consumption inequality. We conclude that efforts to target inflation have largely contributed to the further deterioration of consumption inequality between rentiers and the other two household sectors: low-income workers and the middle class.

# 2 Model presentation

In line with the canonical texts in the Post-Keynesian literature, we present in table 1 below the transaction flow matrix of our economy. It is composed of three sectors: households, firms, and banks. As announced earlier, the household sector is divided into three, but we will expand on them in the following section. While the sectors are represented in the matrix columns, in the rows we have the flows of transactions for each asset in the economy. In line with a core principle of stock-flow consistency, the sum of each column and row adds to zero.

Table 2 illustrates the balance sheet matrix of our economy. The sectors in the economy are illustrated in the columns, as in the transaction flow matrix, but the rows represent the assets of the economy.

[3] See Pariboni (2016) and Fagundes (2017) for more details on supermultiplier growth model in which household credit financed consumption is the exogenous component of demand driving growth. Also, see Fiebiger and Lavoie (2017) for more details on semi-autonomous components of demand.

Table 1: Transaction flow matrix

| | Working Class | Middle Class | Rentiers Class | Firms | | Commercial Banks | | Total |
|---|---|---|---|---|---|---|---|---|
| | | | | Current | Capital | Current | Capital | |
| Consumption | $-C^w$ | $-C^m$ | $-C^r$ | $+C$ | | | | 0 |
| Non-residential Investment | | | | $+I$ | $-I$ | | | 0 |
| Residential Investment | | $-\dot{H}^m$ | $-\dot{H}^r$ | $+\dot{H}$ | | | | 0 |
| [GDP] | | | | [ Y ] | | | | |
| Wages | $+W^w$ | $+W^m$ | | $-W$ | | | | 0 |
| Firms Dividends | | | $+FD$ | $-(FD+FU)$ | $+FU$ | | | 0 |
| Banks Dividends | | | $+FB$ | | | $-FB$ | | 0 |
| Interest on deposits | $+r_d D^w$ | $+r_d D^m$ | $+r_d D^r$ | | $+r_d D^f$ | $-r_d D$ | | 0 |
| Interest on loans | $-r_w L^w$ | $-r_m L^m$ | | | $-r_f L^f$ | $+r_w L^w + r_m L^m$ | | 0 |
| | | | | | | $+r_f L^f$ | | 0 |
| Rent on housing | $-R$ | | $+R$ | | | | | 0 |
| $\Sigma$ | $Sav_w$ | $Sav_m$ | $Sav_r$ | 0 | $-I+FU$ | 0 | | 0 |
| $\Delta$ Deposits | $-\dot{D}^w$ | $-\dot{D}^m$ | $-\dot{D}^r$ | | $-\dot{D}^f$ | | $+\dot{D}$ | 0 |
| Change in Loans | $+\dot{L}^w$ | $+\dot{L}^m$ | | | $+\dot{L}^f$ | | $-\dot{L}$ | 0 |
| Change in Equities | | | $-p_e\dot{E}$ | | $+p_e\dot{E}$ | | | 0 |

Table 2: Balance sheet matrix

| | Low Inc. Workers | High Inc. Workers | Rentiers | Firms | Banks | Total |
|---|---|---|---|---|---|---|
| Deposits | $+D^w$ | $+D^m$ | $+D^r$ | | $-D$ | 0 |
| Equities | | | $+Ep_e$ | $-Ep_e$ | | 0 |
| Loans | $-L^w$ | $-L^m$ | | $-L^f$ | $+L$ | 0 |
| Houses | | $H^m$ | $H^r$ | | | $H$ |
| Firm's Capital | | | | $K$ | | $K$ |
| Total | $V^w$ | $V^m$ | $V^r$ | $K - ep_e$ | 0 | $V$ |

Based on these matrices, we can see that the short-run equilibrium of our model is given by:

$$Y_t = C_t^w + C_t^m + C_t^r + I_t + \dot{H}_t \tag{1}$$

Where $Y_t$ is output at time $t$, $C_t^w$ is the consumption of low-income workers at time $t$, $C_t^m$ is the consumption of middle-class workers at time $t$, $C_t^r$ is the consumption of the rentiers at time $t$, $I_t$ is non-residential investment at time $t$, and $\dot{H}_t$ is residential investment at time $t$, which refers to the acquisition of real estate property.

We now proceed to define the elements that determine the consumption of each of the household sectors presented hitherto.

## 2.1 Household Sector

The following tripartite form comprises the household sector. The low-income class of workers, who can only afford to rent their dwellings, do not own houses, and incur credit card debt; the middle-income class, which can afford to own houses, but only through mortgages; and the rentiers, the third type of household that actually can buy houses using its own earnings, and that also owns commercial banks and firms. On one hand, low-income and middle-income workers receive wages. The rentier class, on the other hand, receives profits from firms and commercial banks. In the following subsection, we detail the differences between each of these groups.

### 2.1.1 Working Class

Income distribution between households is given by:

$$W = W^w + W^m = (1 - \pi)Y \tag{2}$$

Where $\pi$ is the profit share, while $W^w$ and $W^m$ are the total amounts of wage bills paid to low-income and middle-income households, respectively. Accordingly, we assume that:

$$W^w = w^w * N^w \tag{3}$$

The total wage bill of low-income working households is determined by a given hourly wage rate, $w^w$, multiplied by the total amount of hours worked, $N^w$. The upper case letters indicate the class to which a given variable references. We assume a similar description for $W^m$, the middle-class workers' wage bill:

$$W^m = w^m * N^m \tag{4}$$

Drawing on Setterfield and Kim (2020), we assume that $\alpha_1$ is the ratio of middle-class to low-class workers' wages. We take into account that $w^m = \alpha_1 w^w$, with $\alpha_1 > 1$, and that $N^m = \alpha_2 N^w$, with $0 < \alpha_2 < 1$, where $\alpha_2$ is the ratio of middle-income to low-income total hours worked. In other words, as the model presents two types of working households, the working class, and the middle class, the latter tends to receive higher remuneration for each hour worked; thus $\alpha_1 > 1$. However, the middle class also tends to work fewer hours and it constitutes a smaller portion of society than the working class. As it contributes a lower amount of labor hours to output, we conclude that $0 < \alpha_2 < 1$. Given these assumptions we derive both the working class and the middle-class wages as a function of income, $Y$, as illustrated below[4]:

$$\begin{aligned} W^w &= \frac{(1-\pi)Y}{(\alpha+1)} \\ W^m &= \frac{\alpha(1-\pi)Y}{(\alpha+1)} \end{aligned} \tag{5}$$

Where $\alpha = \alpha_1\alpha_2 > 0$.

We define income and consumption for working households as follows. Income is given by:

$$Y_t^w = W_t^w + r_d D_t^w - r_w L_t^w - R_t \tag{6}$$

Where $Y_t^w$ is the household income, $W_t^w$ is the low-income workers' wage at time $t$, $r_d$ is the interest on deposits, $D_t^w$ is the deposit by low-income workers at time $t$, $r_w$ is the interest on loans, $L_t^w$ is the workers' loan at time $t$, and $R_t$ is rent at time $t$.

Following Szymborska (2022) and Teixeira and Petrini (2021), we assume that $r_w = r_{CB}(1 + s_0)$, where $r_{CB}$ is the federal funds rate established by the central bank, and $s_0$ is the spread between the funds rate and the rate imposed by commercial banks on the debt of working households. Finally, for simplicity, we assume that $r_d = 0$, such that banks are not paying interest on deposits.

Consumption is given by:

$$C_t^w = Y_t^w + \dot{L}_t^w \tag{7}$$

Where $\dot{L}_t^w$ is the credit-financed consumption of working households. We also assume that:

[4]The details of the derivation can be found in the appendix

$$R_t = rW_t^w \tag{8}$$

In other words, rent at time $t$ is determined by the share of rent payment on low-income wages, $r$, and low-income wages at time $t$. Finally, drawing on Avritzer and Brochier (2022), we assume the following relations:

$$g_z = \varphi_0 - \varphi_1 \frac{r_w L_t^w}{W_t^w} \tag{9}$$

Where $g_z$ is the rate at which the autonomous component of demand, $Z$, grows, with $Z_t$ being defined as the change in working households loans, $Z = \dot{L}_t^w$, such that at any time $t$, we will have that $Z_t = Z_0 e^{g_z t}$. Furthermore, $\varphi_0$ is an autonomous factor that positively influences the demand for credit, and $\varphi_1$ is a measure of the sensitivity of household credit demand to debt and credit conditions, as determined by the interest rate on loans.

The household consumption behavior described in equations 7 and 9 was first suggested in Avritzer and Brochier (2022). In their model, they assume the consumption of the working class to be partially financed by its income, $Y^w$, and partially financed by credit, represented by the new loans they incur, $\dot{L}^w$. Furthermore, it is also assumed that those households will make decisions on how much more loans to take based on a consumption target, which is income inelastic, as it is path-dependent and historically determined. This is the reasoning behind assuming an exogenous component of growth for $g_z$, which is given by $\varphi_0$. However, they also assume that there is a limit to how much debt low-income households are comfortable owning. This idea implies that households respond to high debt burdens (i.e.: $r_w \frac{L_t^w}{W_t^w}$) by adjusting their loan growth rate. We articulate these dynamics in equation 9 above.

### 2.1.2 Middle class

In our model, the middle class also earns wages. This remuneration was defined by equation 5 in the previous section. Additionally, we assume that the middle class consumes exclusively out of their own income, $Y_t^m$, and incurs debt only in order to buy houses. Thus, we assume that the middle-class income is given by:

$$Y_t^m = W_t^m + r_d D_t^m - r_m L_t^m \tag{10}$$

Where $r_d = 0$ once again for simplicity, and $r_m$ is the mortgage rate that the middle class pays. Following Szymborska (2022) and Teixeira and Petrini (2021), we also assume that the mortgage rate is given by, $r_m = r_{CB}(1 + s_1)$, i.e. the federal funds rate, $r_{CB}$, added by a spread specific to the mortgages market, $s_1$. In this context, middle-class consumption is given by:

$$C_t^m = \alpha_1^m Y_t^m \tag{11}$$

Finally, we assume that middle-class housing depends on how many loans (in this case mortgages) middle-income workers are able to secure, which in turn will be determined by their wages[5]:

$$\dot{H}^m = \dot{L}^m = \eta_0 W^m \tag{12}$$

Where $\eta_0$ is the middle-class mortgage as a share of its wage.

Since the focus of this paper is the debt of low-income working households, we have simplified our model by assuming an endogenous residential investment, similar to what is suggested in Zezza (2008). We are aware of the extensive literature in the supermultiplier model that explores the possibility of residential investment as the exogenous component of demand driving growth in the United States (See Teixeira and Petrini (2021) and Summa et al. (2022) for instance on this). However, adding another autonomous growth component in our analysis would significantly complicate the model without adding much detail into the sector that we are actually interested in, which is the low-income household sector.

[5]As a matter of practice, banks grant mortgages to these households based on how much money these workers earn. It is this money that will allow the households to afford to buy a house

### 2.1.3 Rentier's Households

As stated early, and illustrated in the transaction matrix, the class of rentiers earns profits from the ownership of banks and firms. Its income is given by:

$$Y_t^r = FD_t + FB_t + r_d D_t^r + R_t \tag{13}$$

Where $FD_t$ is the dividends provided by firms at time $t$, and $FB_t$ is the dividends generated by banks at time $t$.

Rentiers consume a constant fraction, $\alpha_1^r$, of these profits:

$$C_t^r = \alpha_1^r Y_t^r \tag{14}$$

They also allocate a fixed part, $\eta_1$, of these earnings to buy houses, such that:

$$\dot{H}^r = \eta_1 Y_t^r \tag{15}$$

## 2.2 Firms

We define total firms' profits, $F$, as follows:

$$F = Y - W \tag{16}$$

Undistributed profits, FU, are the ones that stay in the firms, as capital:

$$FU = F - FD = (1 - \mu)F \tag{17}$$

Distributed profits, FD, are sent to rentiers, as owners of firms.

Along the lines of Caverzasi and Godin (2015), we assume that:

$$\dot{E} = \frac{\epsilon(I - FU)}{p_e} \tag{18}$$

Where $\dot{E}$ is the change in equities, and $p_e$ is their price.

Still in line with Caverzasi and Godin (2015), we assume that changes in loans incurred by firms are given by:

$$\dot{L}^f = I - FU - p_e\dot{E} - r_d D^f + r_f L^f \tag{19}$$

Here, the interest rate attached to the loans, $r_f$, equals the federal funds rate, $r_{CB}$. In other words, firms are not being charged a premium over the federal funds rate target. Thus:

$$\dot{L}^f = (1 - \epsilon)(I - FU) - r_d D^f + r_f L^f \tag{20}$$

Additionally, residential investment is given by:

$$\dot{H}_t = \dot{H}_t^m + \dot{H}_t^r \tag{21}$$

With $\dot{H}_t^m$ and $\dot{H}_t^r$ as defined above by equations 12 and 15. Finally, following Freitas and Serrano (2015), we assume that the investment function follows the usual supermultiplier conditions:

$$I = hY \tag{22}$$

With the marginal propensity to invest, $h$, given by:

$$\dot{h} = \gamma h(u - u_n) \tag{23}$$

This last equation guarantees the adjustment of the rate of capacity utilization, $u_t$, to its normal level, $u_n$, so that an issue of harrodian instability does not arise. See Lavoie (2016), Hein et al. (2012), Setterfield and David Avritzer (2020), Skott (2010), Gahn (2020) and Haluska et al. (2021) for more details on this debate.

## 2.3 Banks

Banks' profits come from the difference between how much interest they charge on loans and how much they pay in interest on deposits (savings):

$$FB = r_w L^w + r_m L^m + r_f L^f - r_d D \tag{24}$$

In this scenario, we also assume that banks create deposits as they grant loans to other economic actors:

$$\Delta D = \Delta L \tag{25}$$

Since we have assumed that $r_d = 0$, $r_w = r_{CB}(1 + s_0)$, $r_m = r_{CB}(1 + s_1)$ and $r_f = r_{CB}$, we have that:

$$FB = r_{CB}(1 + s_0)L^w + r_{CB}(1 + s_1)L^m + r_{CB}L^f \tag{26}$$

The dividends banks pay are a function of the loans granted to low-income households, middle-class households, and firms, with their respective interest rates, as described above.

# 3 Model solution

In what follows, we present both the short-run and the long-run solutions to the model.

## 3.1 Short run equilibrium

To derive the short-run equilibrium to the model, we make the simplifying assumption that banks do not pay interest on deposits. Thus, $r_d = 0$, such that, if we replace equations 7, 11, 14, 21 and 22 in 1, we obtain[6]:

$$Y_t = \frac{1}{(s-h)}\left[(g_l + a)L_t^w + bL_t^f + dL_t^m\right] \tag{27}$$

Where we have defined: $s = 1 - c$; $c = \Big[(\alpha_1^r + \eta_1)\mu\pi + \frac{(1-\pi)}{(\alpha+1)}(1 + (\alpha_1^m + \eta_0)\alpha + r(\alpha_1^r + \eta_1 - 1))\Big]$; $a = (\alpha_1^r + \eta_1 - 1)r_w$; $b = (\alpha_1^r + \eta_1)r_f$ and; $d = (\alpha_1^r + \eta_1 - \alpha_1^m)r_m$. $s = 1 - c$.

Finally, if we define the capital to output ratio as $v = \frac{K_t}{Y_t}$, the low-income household loans to capital ratio as $l_w = \frac{L^w}{K}$, the middle-class loans to capital ratio as $l_m = \frac{L^m}{K}$, the loan of firms to capital ratio as $l_f = \frac{L^f}{K}$, and can obtain an short-run solution of

[6]See mathematical Appendix for further derivations

the rate of capacity utilization ($u_t$), from equation 27:

$$u_t = \frac{v[(g_l + a)l_w + bl_f + dl_m]}{s - h} \tag{28}$$

## 3.2 Long run equilibrium

In the analysis of the long-run equilibrium we must factor the economy's growth. Recalling equation 9 we can state that:

$$g_z^* = g_l^* = \varphi_0 - \varphi_1' \frac{r_w l_w^*}{u_n} \tag{29}$$

Where $\varphi_1' = \frac{\varphi_1(\alpha+1)v}{(1-\pi)}$

$$h^* = \frac{g_k^* v}{u_n} = \frac{g_l^* v}{u_n} \tag{30}$$

The actual rate of capacity utilization equals its normal level. Thus:

$$u^* = u_n = \frac{v[(g_l^* + a)l_w^* + bl_f^* + dl_m^*]}{s - h^*} \tag{31}$$

Which can be written as:

$$l_w^* = \frac{\frac{u_n}{v}(s - h^*) - bl_f^* - dl_m^*}{g_l^* + a} \tag{32}$$

For a final description of our steady state, we are only missing an equation that explains the behavior of $l_f^*$. However, in steady-state we must have constant ratios for all variables (Godley and Lavoie (2006)). Thus:

$$\frac{\dot{L}^m}{L^m} = \frac{\dot{L^f}}{L^f} = \frac{\dot{L^w}}{L^w} = g_l^* \tag{33}$$

Therefore, from equation 20 and 12, we will have that[7]:

$$l_f^* = \frac{(1-\epsilon)(h^* - (1-\mu)\pi)u_n}{(g_l^* - r_f)v} \tag{34}$$

[7]See the details of the derivation in the mathematical appendix

and

$$l_m^* = \frac{\eta_0 \alpha (1-\pi) u_n}{(\alpha+1) g_l^* v} \tag{35}$$

We now have a full description of the model's short-run equilibrium, and of its steady-state solution. In what follows, we will derive the model's steady state dynamics, and define the values of the parameters we will test when we run the model. We will also measure the effects of an increase in interest rates on the steady-state values of certain variables. To that effect, we are interested in $\frac{C^w}{C}$ to measure consumption inequality, $\frac{Y^w}{Y}$ to measure income inequality, and $g$ to measure economic activity. This analysis will allow us to understand the effect of a contractionary monetary policy, as the monetary authority increases interest rates.

## 3.3 Steady state dynamics

As usual in these types of models we start with the dynamic equation that explains the behavior of the marginal propensity to invest (equation 23):

$$\dot{h} = \gamma h (u_t - u_n) \tag{36}$$

Where $u_t$ is defined as in equation 28 above. As a result, our first steady state equation is defined as a function of four different variables: $l_m$, $l_f$, $l_w$ and $g_l$. Therefore, to have a complete dynamical system we will need to find a dynamic equation for each one of these variables. We start with $\dot{l_f}$, for which we can obtain the following equation:

$$\dot{l}_f = \frac{\dot{L}^f}{K_t} - \frac{\dot{K}}{K_t}\frac{L^f}{K_t} \tag{37}$$

Which results in:

$$\dot{l}_f = [(1-\epsilon)(h-(1-\mu)\pi) - h l_f]\frac{u_t}{v} + r_f l_f \tag{38}$$

We find another equation to explain the dynamical behavior of $l_w$:

$$\dot{l}_w = \frac{\dot{L}^w}{K_t} - \frac{\dot{K}}{K_t}\frac{L^w}{K_t} \tag{39}$$

Which results in:

$$\dot{l}_w = (g_l - h\frac{u_t}{v})l_w \tag{40}$$

The same procedure leads to the following equation for $\dot{l}_m$:

$$\dot{l}_m = \frac{\dot{L}^m}{K_t} - \frac{\dot{K}}{K_t}\frac{L^m}{K_t} \tag{41}$$

Which results in:

$$\dot{l}_m = \left(\frac{\eta_0\alpha(1-\pi)}{(\alpha+1)} - hl_m\right)\frac{u_t}{v} \tag{42}$$

We now construct an equation that describes the dynamic behavior of $\dot{g}_l$. To that effect, we recall the definition of $g_l = \frac{\dot{L}^w}{L^w} = \frac{Z}{L^w}$, in which case we infer that:[8]

$$\dot{g}_l = (g_z - g_l)g_l \tag{43}$$

It follows that our dynamical system is given by:

$$\begin{aligned}
\dot{h} &= \gamma h\left(\frac{v[(g_l+a)l_w+bl_f+dl_m]}{s-h} - u_n\right) \\
\dot{l}_w &= \left(g_l - \frac{h[(g_l+a)l_w+bl_f+dl_m]}{s-h}\right)l_w \\
\dot{l}_f &= [(1-\epsilon)(h-(1-\mu)\pi) - hl_f]\frac{[(g_l+a)l_w+bl_f+dl_m]}{(s-h)} + r_f l^f \\
\dot{l}_m &= \left(\frac{\eta_0\alpha(1-\pi)}{(\alpha+1)} - hl_m\right)\frac{[(g_l+a)l_w+bl_f+dl_m]}{(s-h)} \\
\dot{g}_l &= (g_z - g_l)g_l
\end{aligned} \tag{44}$$

Since this is a complex dynamical system, further exploring its steady state stability in its analytical format produces results that are quite difficult to interpret in an economically meaningful way. Furthermore, since Avritzer and Brochier (2022) have already explored the steady-state stability of a simpler version of this model in its

[8] The details of all the dynamic equations above can be found in the mathematical appendix.

analytical format, we thought it would be more useful at this point to turn ourselves to a numerical illustration of our model. As can be seen further in the paper this numerical illustration will actually provide us with interesting interpretations for our results. However, before we do that, we must first turn our attention to some important ratios that will help us understand the effect of monetary contraction on inequality in our model.

## 3.4 Consumption and Income Inequality

To understand how monetary policy affects income distribution we propose the observation of five very important ratios in our model, namely, (i) the low-income workers consumption to middle-class consumption ratio $\frac{C_t^w}{C_t^m}$; (ii) the low-income workers consumption to rentiers consumption ratio $\frac{C_t^w}{C_t^r}$; (iii) and the workers income to total income ratio $\frac{Y_t^w}{Y_t}$, (iv) the middle-class to the rentier class consumption ratio $\frac{C_t^m}{C_t^r}$, and (v) the low-income workers to the middle-income workers income ratio, $\frac{Y_t^w}{Y_t^m}$. In table 3 we present the defining equations for these ratios, as well as their steady-state dynamics. The details of each of these derivations can be found in the mathematical appendix.

| Ratio | Equation | Dynamic Equation |
| --- | --- | --- |
| | Workers to middle class consumption ratio | |
| $\frac{C_t^w}{C_t^m}$ | $\frac{(1-r)(1-\pi)u_t+(g_l-r_w)l_w v(\alpha+1)}{\alpha_1^m\left(\alpha(1-\pi)u_n-r_m l_m v(\alpha+1)\right)}$ | $\frac{(1-r)(1-\pi)\dot{u}+\dot{g}_l(\alpha+1)v+(g_l-r_m)\dot{l}_w v(\alpha+1)}{\alpha_1^m(\alpha(1-\pi)u_t-r_m l_m v(\alpha+1))} - \frac{(1-r)(1-\pi)u_t+(g_l-r_w)l_w v(\alpha+1)}{\alpha_1^m(\alpha(1-\pi)u_t-r_m l_m v(\alpha+1))} * \frac{\alpha_1^m(\alpha(1-\pi)\dot{u}_t-r_m\dot{l}_m v(\alpha+1))}{\alpha_1^m(\alpha(1-\pi)u_t-r_m l_m v(\alpha+1))}$ |
| | Workers to rentier consumption ratio | |
| $\frac{C_t^w}{C_t^r}$ | $\frac{(1-r)(1-\pi)+(g_l-r_w)l_w\frac{v}{u_t}(\alpha+1)}{\alpha_1^r((\alpha+1)(\pi\mu+(r_w l_w+r_m l_m+r_f l_f)\frac{v}{u_t})+r(1-\pi))}$ | $\frac{(\alpha+1)v\left(\dot{g}_l l_w+(g_l-i_l)\dot{l}_w\right)+(1-r)(1-\pi)\dot{u}_t}{\alpha_1^r((\alpha+1)(\pi\mu u+(r_w l_w+r_m l_m+r_f l_f)v)+r(1-\pi)u)} - \frac{(1-r)(1-\pi)u_t+(g_l-r_w)l_w v(\alpha+1)}{\alpha_1^r((\alpha+1)(\pi\mu u+(r_w l_w+r_m l_m+r_f l_f)v)+r(1-\pi)u)} * \frac{\alpha_1^r\left((\alpha+1)(\pi\mu\dot{u}_t+(r_w\dot{l}_w+r_m\dot{l}_m+r_f\dot{l}_f)v)+r(1-\pi)\dot{u}_t\right)}{\alpha_1^r((\alpha+1)(\pi\mu u+(r_w l_w+r_m l_m+r_f l_f)v)+r(1-\pi)u)}$ |
| | Middle class to rentier consumption ratio | |
| $\frac{C_t^m}{C_t^r}$ | $\frac{\alpha_1^m(\alpha(1-\pi)u_t-r_m l_m v(\alpha+1))}{\alpha_1^r((\alpha+1)(\pi\mu u_t+(r_w l_w+r_m l_m+r_f l_f)v)+r(1-\pi)u_t)}$ | $\frac{\alpha_1^m\left(\alpha(1-\pi)\dot{u}_t-r_m\dot{l}_m v(\alpha+1)\right)}{\alpha_1^r((\alpha+1)(\pi\mu u+(r_w l_w+r_m l_m+r_f l_f)v)+r(1-\pi)u)} - \frac{\alpha_1^m\left(\alpha(1-\pi)u_t-r_m l_m v(\alpha+1)\right)}{\alpha_1^r((\alpha+1)(\pi\mu u+(r_w l_w+r_m l_m+r_f l_f)v)+r(1-\pi)u)} * \frac{\alpha_1^r\left((\alpha+1)(\pi\mu\dot{u}_t+(r_w\dot{l}_w+r_m\dot{l}_m+r_f\dot{l}_f)v)+r(1-\pi)\dot{u}_t\right)}{\alpha_1^r((\alpha+1)(\pi\mu u+(r_w l_w+r_m l_m+r_f l_f)v)+r(1-\pi)u)}$ |
| | Workers income to total income ratio | |
| $\frac{Y_t^w}{Y_t}$ | $\frac{(1-\pi)}{(\alpha+1)}(1-r) - r_w l_w^* \frac{v}{u_n}$ | $-r_w\dot{l}_w\frac{v}{u_t} + r_w l_w\frac{v\dot{u}_t}{u_t^2}$ |
| | Workers to middle class income ratio | |
| $\frac{Y_t^w}{Y_t^m}$ | $\frac{(1-r)(1-\pi)u_n-r_w l_w v(\alpha+1)}{\alpha_1^m\left(\alpha(1-\pi)u_n-r_m l_m v(\alpha+1)\right)}$ | $\frac{(1-r)(1-\pi)\dot{u}-r_m\dot{l}_w v(\alpha+1)}{\alpha_1^m(\alpha(1-\pi)u_t-r_m l_m v(\alpha+1))} - \frac{(1-r)(1-\pi)u_t-r_w l_w v(\alpha+1)}{\alpha_1^m(\alpha(1-\pi)u_t-r_m l_m v(\alpha+1))} \frac{\alpha_1^m(\alpha(1-\pi)\dot{u}_t-r_m\dot{l}_m v(\alpha+1))}{\alpha_1^m(\alpha(1-\pi)u_t-r_m l_m v(\alpha+1))}$ |

Table 3: Income and Consumption Distribution Ratios

In this scenario, we know that the rate of capacity utilization varies with respect to time as follows:

$$\frac{du}{dt} = \frac{v(g_l + a)\dot{l}_w + v\dot{g}_l l_w + bv\dot{l}_f + dv\dot{l}_m}{(s-h)} + \frac{v[(g_l + a)l_w + bl_f + dl_m]\dot{h}}{(s-h)^2} \tag{45}$$

As emphasized in Meyer and Sullivan (2017) among others, beyond measures of disparities in income distribution, it is also important to look into consumption inequality, as an indicator of households' well-being. With that in mind, we have decided to look at the effect of monetary policy on the three consumption ratios described above. In the following section, we then present a numerical illustration of the effects of monetary policy on these consumption ratios.

# 4 Numerical illustration of our model

Based on the analysis produced thus far, we propose here a numerical illustration of the model using parameters widely shared in the literature. A detailed description of the parameters, their values, as well as the source can be found in table 4.

| Parameter | Meaning | Value (Initial) | Sources |
|---|---|---|---|
| $\alpha_1$ | Middle class to low income class hours of work ratio | 0.25 | Setterfield and Kim (2020) [a] |
| $\alpha_2$ | Middle class to low income class wage ratio | 2.27 | Setterfield and Kim (2020) [a] |
| $r_{CB}$ | Federal funds rate | 0.01 | Author's suggestion [b] |
| $s_0$ | Baseline spread on federal funds rate | 0.2 | Author's suggestion [c] |
| $s_1$ | Spread on mortgage rates | 0.05 | Author's suggestion [c] |
| $u_n$ | Normal rate of capacity utilization | 0.8 | Haluska et al. (2021) |
| $v$ | Capital output ratio | 2.7 | Avritzer and Brochier (2022) |
| $\varphi_0$ | Autonomous component of consumption | 0.09 | Avritzer and Brochier (2022) |
| $\varphi_1$ | Low income households sensitivity to debt burden | 0.06 | Avritzer and Brochier (2022) |
| $\pi$ | Profit share | 0.4 | Fed's Database |
| $\alpha_1^m$ | Middle class marginal propensity to consume | 0.75 | Szymborska (2022) |
| $\alpha_1^r$ | Rentier class marginal propensity to consume | 0.2 | Szymborska (2022) and Setterfield and Kim (2020) |
| $r$ | Share of rent payment on low income wages | 0.2 | Author's suggestion[d] |
| $\eta_0$ | Middle class mortgage as a share of their wage | 0.1 | Author's suggestion[d] |
| $\eta_1$ | Rentier class spending with house as a share of their earnings | 0.05 | Author's suggestion[d] |
| $\epsilon$ | Share of firm financing through new equity issue | 0.5 | Skott and Ryoo (2008) |
| $\mu$ | Share of firm profits that are distributed | 0.3 | Skott and Ryoo (2008) |
| $\gamma$ | Speed of adjustment of $u$ to its normal level $u_n$ | 0.02 | Avritzer and Brochier (2022) |

[a] These parameters are furthermore consistent with Szymborska (2022) as it results in a low income wage bill share of 0.36;
[b] Based on Fed's Database. This is just an initial value which will be further increased to 0.02;
[c] Based on Szymborska (2022) and Teixeira and Petrini (2021). These are just initial values which will be further increased;
[d] Values were calculated to better fit the numerical simulation;

Table 4: Parameter values

Initially, we consider the steady state of the model without the middle-class mortgage (i.e.: $\eta_0 = 0$). The results of this first simulation are described in table 5 and illustrated in figure 5. It is interesting to observe that even though we have already included firm loans in this model, workers' debt-to-capital ratio is quite high at 0.75.

The inclusion of loans in yet another sector of the economy, middle-class mortgage, will make our model more realistic and decrease the steady state value of $l_w$. However, the steady-state solution will still require a high accumulation of workers' debt ($g_l$), as this is the only driver of economic growth. Subsequently, we introduced a series of modifications in the parameters to analyze how the variables of interest change.

| Variable | Steady state results (Initial) |
|---|---|
| $h^*$ | $0.2876$ |
| $g_l^*$ | $0.0852$ |
| $l_w^*$ | $0.7543$ |
| $l_f^*$ | $0.0149$ |

Table 5: Steady state results

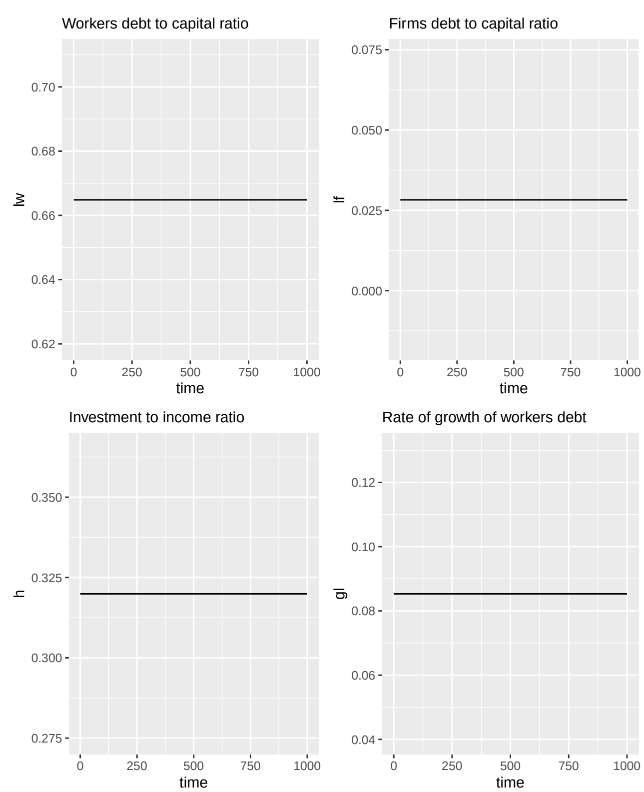


Figure 5: Steady state stability of the model

In figure 6, we include middle-class mortgages to the model. First, low-income workers' debt-to-capital ratio plunges from 0.75 to 0.66, while the rate of growth of workers' debt rises quickly from 0.0852 to 0.0858. Finally, the firms' debt-to-capital ratio grows from 0.015 to 0.018. In other words, with the presence of middle-class mortgages, our model requires higher economic growth, which must come from a higher rate of growth of debt for low-income households. However, low-income workers' debt-to-capital ratio has decreased as a result of adding another loan possibility to our model.

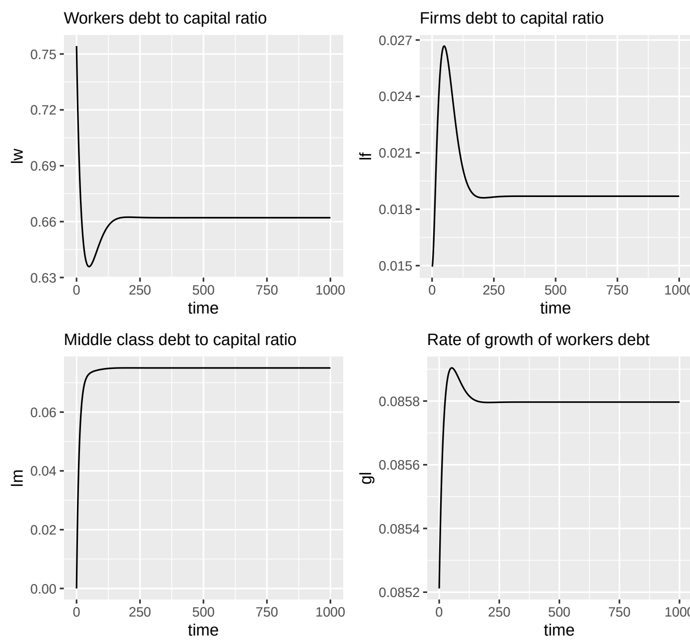


Figure 6: Changes in the model after the inclusion of middle-class mortgages

Aiming to reproduce realistic conditions, we now insert in the model a modest increase in $r_{CB}$, the federal funds rate, from 0.01 to 0.0125, as presented in figure 7. Workers' debt to capital ratio, $l^w$, jumps from 0.64 to almost 0.70, and so does the consumption ratio of workers to rentiers. At the same time, both the workers' total income ratio and the rate of growth of workers' debt, $g_l$, fall, from 0.238 to 0.236 and from 0.0855 to 0.0842, respectively. The slight increase in the federal funds rate led workers to accumulate less debt, as they react to a higher debt burden.

However, as the growth engine of this economy is the debt of the working class, a reduction in this group's consumption provokes a fall in output. As a result, the rentier class will also consume less. Thus, proportionally, rentiers reduce more their consumption than workers. This is also represented in the increase in $l_w$. Even though low-income workers have decreased their accumulation of debt, the effect that this has on output growth compensates for it and the overall effect is an increase in workers' debt-to-capital ratio.

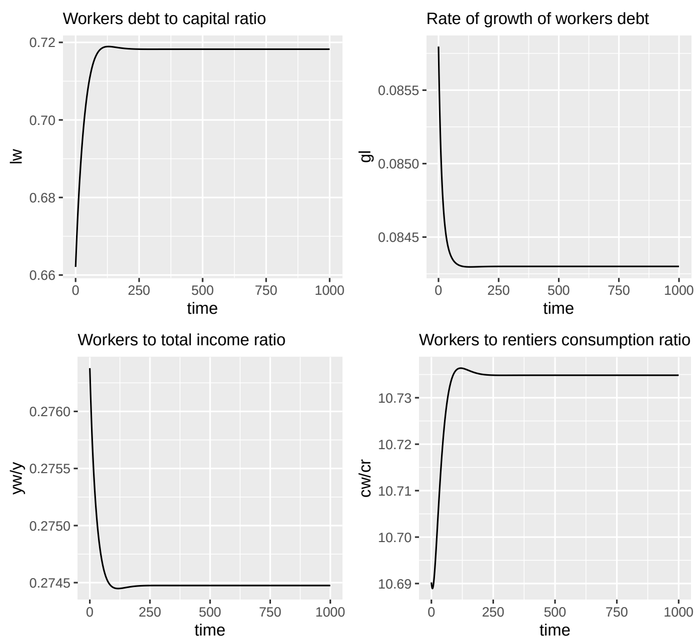


Figure 7: Changes in the model after a small increase in $r_{CB}$, from $1\%$ to $1.25\%$

Subsequently, we simulated a larger increase in the federal funds rate, from 0.01 to 0.02. Figure 8 contains the results of the changes in the variables of interest. In contrast with the previous simulation, all the ratios remain the same, except for the workers-to-rentiers consumption ratio. With the larger increase in $r_{CB}$, that ratio falls sharply, later stabilizing at a still lower rate. The sharper increase in the federal funds rate led workers to consume much less than the rentier class, even though the latter also reduce their consumption, as outlined above.

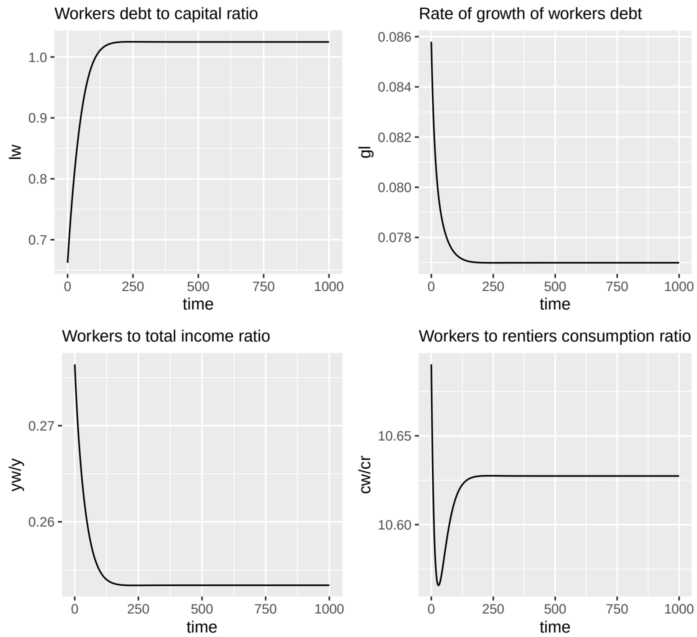


Figure 8: Changes in the model after a large increase in $r_{CB}$, from $1\%$ to $2\%$

We then proceeded to analyze the variables and ratios of interest in a context in which the funds rate rose, leading to an increase in $s_0$, the baseline spread on the federal funds rate. To address the notions that are pertinent to our focus here, we see that the rate of growth of workers' debt drops, a trend that persists in all the other simulations, except for the first one, which introduced the middle-class mortgage. We see in this fact, the cautious reaction workers have when interest rates rise.

Also in line with the other simulations is the workers' debt-to-capital ratio: it naturally rises in tandem with an increase in the $r_{CB}$. The $\frac{C^w}{C^m}$ ratio presents a behavior similar to the one we observed in figure 7, in the panel that related the consumption of workers to that of rentiers. In figure 9, however, the relationship is between the consumption of workers and the consumption of the middle class. The same trend, here, follows the fact that the middle class can accommodate a rise in its mortgage interest better than workers can handle an increase in credit card rates. The resulting graph shows a rise in the consumption ratios between low-income workers and the middle class.

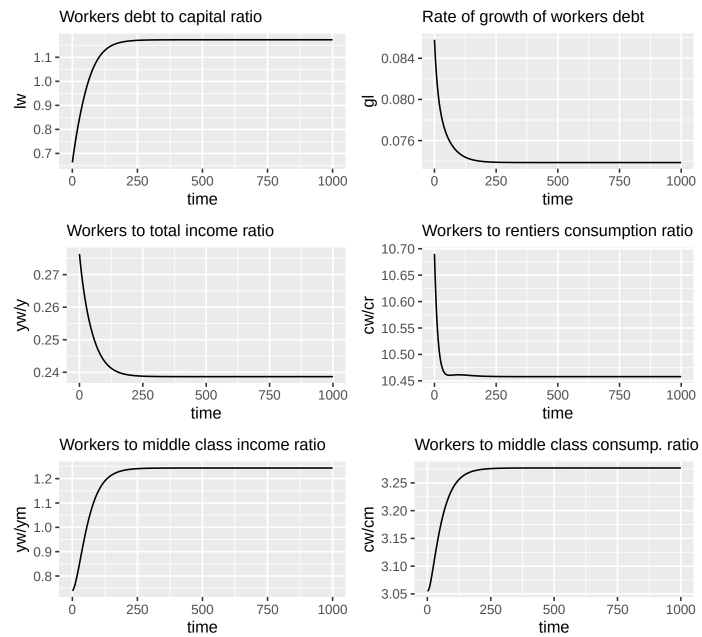


Figure 9: Changes in the model after $s_0$ increases as a result of an increase in $r_{CB}$

Finally, figure 10 shows a comparison of variables and ratios that relate to the middle class and the rentiers. The reaction to interest rate rise is more in line with predicted results, without the nuances we observed in previous simulations. An increase in the interest rate leads to an increase in the low-income class debt-to-capital ratio, a fall in the rate of growth of the low-income workers' debt, a fall in the low-income class to rentiers' consumption ratio, and a decrease in the middle-class to rentiers' consumption ratio.

We see in these experiments a manifestation of different effects of the interest rate variation. In instances when all the socioeconomic groups react similarly, the specificities of their debt and income positions provoke effects that would not have been intuited immediately. We propose that the different behavior results from the conditions we first highlighted at the beginning of the paper, and these conditions become more pronounced as we compare them to the data introduced earlier.

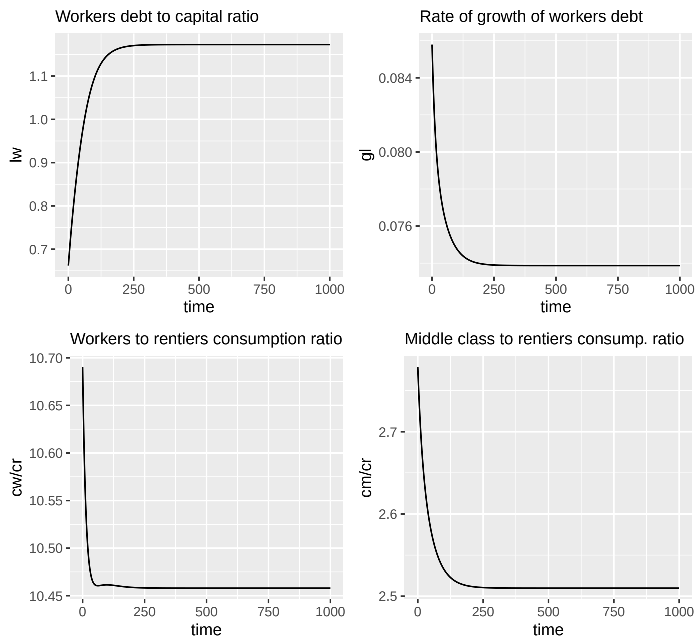


Figure 10: The middle class versus the rentier class

# 5 Conclusion

Research on household debt dynamics has been increasingly concerned with the detailed analysis of the effects of monetary policy. Instead of simply describing the large-scale mechanics of such policy, researchers seek to understand the minutiae of how policies affect various segments of the population. This paper has offered an angle for the analysis, assuming the principle that different households will be affected by and react differently to movements in the interest rate set by the monetary authority. We have shown through preliminary experiments that the effect of interest rate hikes is not uniform across the different socioeconomic classes. Debt and income composition play a critical role in this process.

To account for the configuration of inequality in contemporary American society, we proposed and ran an SFC-PK model with three household segments: low-income, middle-income, and rentier classes. Furthermore, we have assumed a type of supermultiplier growth model where credit-financed consumption of low-income households is the driver of growth as suggested in Avritzer and Brochier (2022). We showed that a rise in interest rates can be absorbed more easily by the rentier class, as they are the ones least affected by an increase in the burden of debt servicing. The same interest rate hike, however, has a more damaging impact on the low-income and middle classes.

The numerical simulations presented in the third section of this paper show that while interest rates increase four main dynamics are observed in our model. First of all, due to the increase in debt burden, low-income households reduce their credit-financed consumption and, therefore, the rate of growth of their debt. Secondly, as a result of this last point, the rate of growth of the economy falls resulting in a higher debt-to-capital ratio for this same low-income class. These two results held across all simulations for increased interest rates: i) small increase of 25 bps; ii) large increase of 100 bps; and iii) again a large interest hike, but now followed by an increase in the baseline spread charged on low-income workers by commercial banks.

As a third dynamic, we also observed that while households' consumption was overall negatively affected by the increase in interest rate, the rentiers' class was the least affected by it, since its consumption is not reduced by the increase in the debt burden. This last result is stronger, the higher the increase in interest rate. Finally, our numerical illustrations also showed that the middle class is in fact the one most affected by interest hikes. The reason for it is that mortgages in our model represent a large proportion of the debt accumulated in this economy, which is very much in line with the data presented in figure 1.

Crucially, the disparities of the effects are not circumscribed by class. They extend to gender. As shown in figures 3 and 4, households headed by females face the highest debt burdens and are also either renting homes or financing homeownership through mortgages. As a result of these observations, we can argue that households headed by women are exactly the ones facing the most disadvantageous effects of interest hikes described in the simulations when we analyzed the effects of contractionary monetary policy in middle-class and low-income class consumptions.

While labor market and income aspects of gender inequality have already been extensively explored, the contribution of this paper is to look at the role of debt inequality in gender dynamics. More precisely, this paper allows us to suggest that since houses headed by females tend to have higher debt burdens, then the result of contractionary monetary policy is to increase gender inequality. This result was measured in our paper through the increase in consumption inequality that arises from different debt accumulation combined with a scenario of interest rate hikes.

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# Mathematical Appendix

## Income Distribution Derivation

$$W = W^w + W^m = (1-\pi)Y$$
$$W^w = w^w * N^w$$
$$W^m = w^m * N^m$$
$$w^m = \alpha_1 w^w; \quad \alpha_1 > 1$$
$$N^m = \alpha_2 N^w; \quad 0 < \alpha_2 < 1$$
$$\Rightarrow W^m = w^m * N^m = \alpha_1 w^w * \alpha_2 N^w = \alpha_1 \alpha_2 w^w N^w \tag{46}$$
$$\Rightarrow W^m = \alpha_1 \alpha_2 W^w = \alpha W^w; \quad \alpha = \alpha_1 \alpha_2 > 0$$
$$\alpha W^w + W^w = (1-\pi)Y$$
$$(\alpha+1)W^w = (1-\pi)Y$$
$$W^w = \frac{(1-\pi)Y}{(\alpha+1)}$$
$$W^m = \frac{\alpha(1-\pi)Y}{(\alpha+1)}$$

### Short run equilibrium derivation

$$\begin{aligned}
Y_t &= Y_t^w + \dot{L}_t^w + \alpha_1^m Y_t^m + \alpha_1^r Y_t^r + hY_t + \dot{H}_t^m + \dot{H}_t^r; \\
Y_t &= W_t^w - r_w L_t^w - R_t + \dot{L}_t^w + \alpha_1^m W_t^m - \alpha_1^m r_m L_t^m \\
&\quad + \alpha_1^r Y_t^r + hY_t + \eta_0 W_t^m + \eta_1 Y_t^r; \\
Y_t &= W_t^w - r_w L_t^w - rW_t^w + \dot{L}_t^w + (\alpha_1^m + \eta_0) W_t^m - \alpha_1^m r_m L_t^m \\
&\quad + (\alpha_1^r + \eta_1)(FD_t + FB_t + rW_t^w) + hY_t; \\
Y_t &= (1 + r(\alpha_1^r + \eta_1 - 1)) \frac{(1-\pi)Y_t}{(\alpha+1)} - r_w L_t^w + \dot{L}_t^w + (\alpha_1^m + \eta_0)\left(\frac{\alpha(1-\pi)Y_t}{(\alpha+1)}\right) \\
&\quad - \alpha_1^m r_m L_t^m + (\alpha_1^r + \eta_1)(FD_t + FB_t) + hY_t; \\
Y_t &= Y_t\left[h + \frac{(1-\pi)}{(\alpha+1)}(1 + (\alpha_1^m + \eta_0)\alpha + r(\alpha_1^r + \eta_1 - 1))\right] - r_w L_t^w + \dot{L}_t^w \\
&\quad - \alpha_1^m r_m L_t^m + (\alpha_1^r + \eta_1)(FD_t + FB_t);
\end{aligned} \tag{47}$$

Replacing $\dot{L}^w$ for equation 9, then gives us:

$$\begin{aligned}
Y_t &= Y_t\Big[h + \tfrac{(1-\pi)}{(\alpha+1)}(1 + (\alpha_1^m + \eta_0)\alpha + r(\alpha_1^r + \eta_1 - 1))\Big] - r_w L_t^w \\
&\quad + g_l L_t^w - \alpha_1^m r_m L_t^m + (\alpha_1^r + \eta_1)(\mu F_t + r_w L_t^w + r_m L_t^m + r_f L_t^f); \\
Y_t &= Y_t\Big[h + \tfrac{(1-\pi)}{(\alpha+1)}(1 + (\alpha_1^m + \eta_0)\alpha + r(\alpha_1^r + \eta_1 - 1))\Big] + (\alpha_1^r + \eta_1 - 1) r_w L_t^w \\
&\quad + g_l L_t^w - \alpha_1^m r_m L_t^m + (\alpha_1^r + \eta_1)(\mu \pi Y_t + r_m L_t^m + r_f L_t^f) \\
Y_t &= Y_t\Big[h + \tfrac{(1-\pi)}{(\alpha+1)}(1 + (\alpha_1^m + \eta_0)\alpha + r(\alpha_1^r + \eta_1 - 1)) + (\alpha_1^r + \eta_1)\mu\pi\Big] \\
&\quad + (\alpha_1^r + \eta_1 - 1) r_w L_t^w + g_l L_t^w - \alpha_1^m r_m L_t^m + (\alpha_1^r + \eta_1)(r_m L_t^m + r_f L_t^f) \\
Y_t &= Y_t\Big[h + \tfrac{(1-\pi)}{(\alpha+1)}(1 + (\alpha_1^m + \eta_0)\alpha + r(\alpha_1^r + \eta_1 - 1)) + (\alpha_1^r + \eta_1)\mu\pi\Big] \\
&\quad + (\alpha_1^r + \eta_1 - 1) r_w L_t^w + g_l L_t^w + (\alpha_1^r + \eta_1 - \alpha_1^m) r_m L_t^m + (\alpha_1^r + \eta_1) r_f L_t^f
\end{aligned} \tag{48}$$

If we then define:

$$c = \left[(\alpha_1^r + \eta_1)\mu\pi + \frac{(1-\pi)}{(\alpha+1)}(1 + (\alpha_1^m + \eta_0)\alpha + r(\alpha_1^r + \eta_1 - 1))\right]$$

$$a = (\alpha_1^r + \eta_1 - 1) r_w = (\alpha_1^r - 1)(1 + s_0) r_{CB}$$

$$b = (\alpha_1^r + \eta_1) r_f = (\alpha_1^r + \eta_1) r_{CB}$$

$$d = (\alpha_1^r + \eta_1 - \alpha_1^m) r_m = (\alpha_1^r + \eta_1 - \alpha_1^m)(1 + s_1) r_{CB}$$

And:

$$s = 1 - c$$

Then:

$$Y_t = \frac{1}{(s-h)}\left[(g_l + a)L_t^w + bL_t^f + dL_t^m\right] \tag{49}$$

## Steady state equilibrium derivation

$$\begin{aligned}
g_l^* &= \frac{\dot{L}^f}{L^f} \\
g_l^* &= \frac{(1-\epsilon)(I-FU)}{L_t^f} + r_f \\
g_l^* - r_f &= \frac{(1-\epsilon)(hY_t-(1-\mu)F)}{L_t^f} \\
L_t^f &= \frac{(1-\epsilon)(hY_t-(1-\mu)\pi Y_t)}{g_l^*-r_f} \\
\frac{L_t^f}{K_t} &= \frac{(1-\epsilon)(h-(1-\mu)\pi)}{g_l^*-r_f}\frac{Y_t}{K_t}
\end{aligned} \tag{50}$$

$$\begin{aligned}
g_l^* &= \frac{\dot{L}^m}{L^m} \\
g_l^* L^m &= \dot{L}^m = \eta_0 W^m \\
g_l^* \frac{L^m}{K} &= \eta_0 \frac{\alpha(1-\pi)Y}{(\alpha+1)K_t} \\
\frac{L_t^m}{K_t} &= \frac{\alpha(1-\pi)u_n}{(\alpha+1)g_l^* v}
\end{aligned} \tag{51}$$

## Steady state dynamics derivation

Derivation of $\dot{l}_f$:

$$\dot{l}_f = \frac{(1-\epsilon)(I-(1-\mu)\pi Y)}{K_t} + i_l l_f - \frac{I_t}{K_t} l_f \tag{52}$$

$$\dot{l}_f = \frac{(1-\epsilon)(hY_t-(1-\mu)\pi Y_t)}{K_t} + i_l l_f - \frac{hY_t}{K_t} l_f \tag{53}$$

$$\dot{l}_f = [(1-\epsilon)(h-(1-\mu)\pi) - h l_f]\frac{Y_t}{K_t} + i_l l_f \tag{54}$$

Derivation of $\dot{l}_w$:

$$\dot{l}_w = \frac{\dot{L}^w}{L_t^w}\frac{L_t^w}{K_t} - \frac{I}{K_t} l_w \tag{55}$$

$$\dot{l}_w = g_l l_w - \frac{hY_t}{K_t} l_w \tag{56}$$

Derivation of $\dot{l}_m$:

$$\dot{l}_m = \frac{\eta_0 \alpha(1-\pi)Y_t}{(\alpha+1)K_t} - \frac{hY_t}{K_t}\frac{L^m}{K_t} \tag{57}$$

$$\dot{l}_m = \left(\frac{\eta_0 \alpha(1-\pi)}{(\alpha+1)} - h l_m\right)\frac{u_t}{v} \tag{58}$$

Derivation of $\dot{g}_l$:

$$\dot{g}_l = \frac{\dot{Z} L^w}{(L^w)^2} - \frac{Z \dot{L}^w}{(L^w)^2} \tag{59}$$

$$\dot{g}_l = \frac{\dot{Z}}{L^w} - \frac{Z}{L^w}\frac{\dot{L}^w}{L^w} \tag{60}$$

$$\dot{g}_l = \frac{\dot{Z}}{Z}\frac{Z}{L^w} - \frac{Z}{L^w}\frac{\dot{L}^w}{L^w} \tag{61}$$

Which then gives us:

$$\dot{g}_l = (g_z - g_l)\frac{Z}{L^w} = (g_z - g_l) g_l \tag{62}$$

## Consumption and Income Inequality

$$\begin{aligned}
\frac{C^w}{C^m} &= \frac{Y_t^w + \dot{L}_t^w}{\alpha_1^m Y_t^m} \\
&= \frac{W_t^w (1-r) - r_w L_t^w + \frac{\dot{L}_t^w}{L_t^w} L_t^w}{\alpha_1^m Y_t^m} \\
&= \frac{(1-r) W_t^w + (g_l - r_w) L_t^w}{\alpha_1^m (W_t^m - r_m L_t^m)} \\
&= \frac{(1-r)\frac{(1-\pi)}{(\alpha+1)} Y_t + (g_l - r_w)\frac{L_t^w}{K_t}\frac{K_t}{Y_t} Y_t}{\alpha_1^m \left(\frac{\alpha(1-\pi)}{(\alpha+1)} Y_t - r_m \frac{L_t^m}{K_t}\frac{K_t}{Y_t} Y_t\right)} \\
&= \frac{(1-r)\frac{(1-\pi)}{(\alpha+1)} + (g_l - r_w) l_w \frac{v}{u_t}}{\alpha_1^m \left(\frac{\alpha(1-\pi)}{(\alpha+1)} - r_m l_m \frac{v}{u_t}\right)} \\
\frac{C^w}{C^m} &= \frac{(1-r)(1-\pi) u_t + (g_l - r_w) l_w v (\alpha+1)}{\alpha_1^m \left(\alpha(1-\pi) u_t - r_m l_m v (\alpha+1)\right)}
\end{aligned} \tag{63}$$

$$
\begin{aligned}
\frac{C^w}{C^r} &= \frac{Y_t^w + \dot{L}_t^w}{\alpha_1^r Y_t^r} \\
&= \frac{(1-r)W_t^w}{\alpha_1^r Y_t^r} + \frac{(g_l - r_w)L_t^w}{\alpha_1^r Y_t^r} \\
&= \frac{(1-r)\frac{(1-\pi)}{(\alpha+1)}Y_t}{\alpha_1^r (FD_t + FB_t + R_t)} + \frac{(g_l - r_w)L_t^w}{\alpha_1^r (FD_t + FB_t + R_t)} \\
&= \frac{(1-r)\frac{(1-\pi)}{(\alpha+1)}Y_t + (g_l - r_w)L_t^w}{\alpha_1^r (\pi\mu Y_t + r_w L_t^w + r_m L_t^m + r_f L_t^f + r W_t^w)} \\
&= \frac{\left((1-r)\frac{(1-\pi)}{(\alpha+1)} + (g_l - r_w)\frac{L_t^w}{Y_t}\right)Y_t}{\alpha_1^r \left(\pi\mu + r_w \frac{L_t^w}{Y_t} + r_m \frac{L_t^m}{Y_t} + r_f \frac{L_t^f}{Y_t} + r\frac{(1-\pi)}{(\alpha+1)}\right)Y_t} \\
&= \frac{\left((1-r)\frac{(1-\pi)}{(\alpha+1)} + (g_l - r_w)\frac{l_w v}{u_t}\right)}{\alpha_1^r \left(\pi\mu + (r_w l_w + r_m l_m + r_f l_f)\frac{v}{u_t} + r\frac{(1-\pi)}{(\alpha+1)}\right)} \\
\frac{C^w}{C^r} &= \frac{\left((1-r)(1-\pi)u_t + (g_l - r_w)l_w v(\alpha+1)\right)}{\alpha_1^r \left(\pi\mu u_t(\alpha+1) + (r_w l_w + r_m l_m + r_f l_f)v(\alpha+1) + r(1-\pi)u_t\right)}
\end{aligned} \tag{64}
$$

$$
\begin{aligned}
\frac{C^m}{C^r} &= \frac{\alpha_1^m Y_t^m}{\alpha_1^r Y_t^r} \\
&= \frac{\alpha_1^m (W_t^m - r_m L_t^m)}{\alpha_1^r (FD_t + FB_t + R_t)} \\
&= \frac{\alpha_1^m \left(\frac{\alpha(1-\pi)}{(\alpha+1)}Y_t - r_m \frac{L_t^m}{K_t}\frac{K_t}{Y_t}Y_t\right)}{\alpha_1^r (\pi\mu Y_t + r_w L_t^w + r_m L_t^m + r_f L_t^f + r W_t^w)} \\
&= \frac{\left(\alpha_1^m \left(\frac{\alpha(1-\pi)}{(\alpha+1)} - r_m l_m \frac{v}{u_t}\right)\right)Y_t}{\alpha_1^r \left(\pi\mu + r_w \frac{L_t^w}{Y_t} + r_m \frac{L_t^m}{Y_t} + r_f \frac{L_t^f}{Y_t} + r\frac{(1-\pi)}{(\alpha+1)}\right)Y_t} \\
&= \frac{\left(\alpha_1^m \left(\frac{\alpha(1-\pi)}{(\alpha+1)} - r_m l_m \frac{v}{u_t}\right)\right)}{\alpha_1^r \left(\pi\mu + (r_w l_w + r_m l_m + r_f l_f)\frac{v}{u_t} + r\frac{(1-\pi)}{(\alpha+1)}\right)} \\
\frac{C^m}{C^r} &= \frac{\alpha_1^m \left(\alpha(1-\pi)u_t - r_m l_m v(\alpha+1)\right)}{\alpha_1^r \left(\pi\mu u_t(\alpha+1) + (r_w l_w + r_m l_m + r_f l_f)v(\alpha+1) + r(1-\pi)u_t\right)}
\end{aligned} \tag{65}
$$

$$
\begin{aligned}
\frac{Y_t^w}{Y_t} &= \frac{W_t^w - r_w L_t^w - R_t}{Y_t} \\
&= \frac{\frac{(1-\pi)}{(\alpha+1)}(1-r)Y_t - r_w L_t^w}{Y_t} \\
&= \frac{(1-\pi)}{(\alpha+1)}(1-r) - r_w \frac{L_t^w}{K_t}\frac{K_t}{Y_t} \\
\frac{Y_t^w}{Y_t} &= \frac{(1-\pi)}{(\alpha+1)}(1-r) - r_w l_w \frac{v}{u_t}
\end{aligned} \tag{66}
$$

$$\begin{aligned}
\frac{Y^w}{Y^m} &= \frac{Y_t^w}{\alpha_1^m Y_t^m} \\
&= \frac{W_t^w (1-r) - r_w L_t^w}{\alpha_1^m Y_t^m} \\
&= \frac{(1-r) W_t^w - r_w L_t^w}{\alpha_1^m (W_t^m - r_m L_t^m)} \\
&= \frac{(1-r)\frac{(1-\pi)}{(\alpha+1)} Y_t - r_w \frac{L_t^w}{K_t} \frac{K_t}{Y_t} Y_t}{\alpha_1^m \left( \frac{\alpha(1-\pi)}{(\alpha+1)} Y_t - r_m \frac{L_t^m}{K_t} \frac{K_t}{Y_t} Y_t \right)} \\
&= \frac{(1-r)\frac{(1-\pi)}{(\alpha+1)} - r_w l_w \frac{v}{u_t}}{\alpha_1^m \left( \frac{\alpha(1-\pi)}{(\alpha+1)} - r_m l_m \frac{v}{u_t} \right)} \\
\frac{Y^w}{Y^m} &= \frac{(1-r)(1-\pi) u_t - r_w l_w v (\alpha+1)}{\alpha_1^m \left( \alpha(1-\pi) u_t - r_m l_m v (\alpha+1) \right)}
\end{aligned} \tag{67}$$

$$\begin{aligned}
& W_t^w = r_w L_t^w + R_t \\
& (1-r) W_t^w = r_w L_t^w \\
& \frac{L_t^w}{(1-r) W_t^w} = \frac{1}{r_w} \\
& \frac{L_t^w}{K_t} \frac{K_t}{Y_t} \frac{Y_t}{W_t^w} = \frac{(1-r)}{r_w} \\
& l_w^{max} = \frac{(1-r)}{r_w} \frac{Y_t}{K_t} \frac{W_t^w}{Y_t} = \frac{(1-r)}{r_w} \frac{u_t}{v} \frac{(1-\pi)}{(\alpha+1)}
\end{aligned} \tag{68}$$