Ecological Transition in Natural-Resource Exporter Countries: a Structural Stock and Flow Consistent Model

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Résumé

La transition écologique se caractérise par l'émergence de certaines industries et le déclin d'autres. La dynamique globale conduira à des contraintes inter-industrielles avec des boucles de rétroaction entre dynamique réelle et dynamique financière. Cela est particulièrement vrai pour les économies en développement confrontées à de fortes contraintes de balance des paiements, tant du côté des comptes courants que des comptes financiers.

Nous développons un modèle SFC visant à comprendre les conséquences et la dynamique de la transition bas carbone dans les économies en développement. Les modèles SFC traditionnels sont utiles pour comprendre les contraintes financières car ils modélisent explicitement la structure financière des entreprises et des pays. Néanmoins, à quelques exceptions notables près, ces modèles ne tiennent pas compte des différences structurelles dans les secteurs productifs.

Nous proposons un modèle prototype qui désagrége le côté productif en trois secteurs : les exportations de ressources naturelles, les biens non échangeables et les autres biens et services. Le modèle est structurel car il considère différentes dynamiques sectorielles en termes de concurrence sur le marché et de comportement d'investissement. Ceci est important car ces dynamiques déterminent comment les politiques de transition écologique affecteront différemment les industries.

L'apport du modèle est double. Premièrement, il fournit des preuves que les mesures qui réduisent les contraintes financières sectorielles sont plus adéquates que le prix du carbone. Deuxièmement, il fournit un cadre facilement adaptable selon les particularités des pays pour analyser les politiques dans différents contextes.

Mots-clés: Transition bas carbone, Exposition externe, Exposition fiscale, Exposition socio-économique, Vulnérabilités macroéconomiques, Risque pays, Tableau d'entrées-sorties multirégional.

Abstract

The Ecological Transition is characterised by the emergence of some industries and the decline of others. The overall dynamics will leads to inter-industrial constraints with feedback loops between real and financial dynamics. This is particularly true for developing economies facing strong balance of payment constraints, both in the current and financial accounts sides.

We develop a SFC model aiming to understand the consequences and dynamics of the low-carbon transition in developing economies. Traditional SFCs models are useful to understand financial constraints because they model explicitly the financial structure of firms and countries. Nevertheless, with some notable exceptions, these models do not account for structural differences in productive sectors.

We propose a prototype model that disaggregates the productive side into three sectors: Natural-resource exports, Non-tradable and Other Goods and services. The model is structural because it considers different sectoral dynamics in terms of market competition and investment behaviour. This is important because these dynamics determine how ecological transition policies will affect industries differently.

The model's contribution is twofold. Firstly, it provides evidences that measures that reduce sectoral financial constraints are more adequate than carbon-price. Secondly, it provides a framework easily adaptable according to countries' idiosyncrasies to analyse policies in different contexts.

Keywords: Ecological transition, Natural-resources, Balance-of-Payment constraints, Fiscal constraints, Stock and Flow Consistent Models, Developing countries.

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1. Introduction

The Paris Agreement set the stabilization of climate change below 2°C as a global objective, which will demand deliberate policies to reach net-zero carbon emissions (UNFCCC, 2015). The consequences of these policies are very diverse and depend on the productive and financial structure of the impacted industries. High-emission industries may face chronic overcapacity, bringing an imminent risk of default due to assets' unanticipated and premature write-downs (Caldecott, 2018). On the other hand, investments in green industries may not compensate for these losses, as investments are risky and very capital-intensive (Mazzucato and Semieniuk, 2018).

The Ecological Transition can be seen as a process of emergence of some (sunrise) industries and the decline of others (sunset industries). The overall dynamics will lead to inter-industrial constraints with the emergence of idle or over-utilised capital and employment tensions but also feedback loops between real and financial dynamics (Semieniuk et al., 2021). This is particularly true for developing economies facing strong balance of payment constraints, both in the current and financial accounts sides and difficulties to rely on domestic private banking only (Ameli et al., 2021).

We develop a Structural Stock-Flow Consistent (SFC) model aiming to understand the consequences and dynamics of the low-carbon transition in developing economies. Traditional SFCs models are useful to understand financial constraints because they explicitly model the financial structure of firms and countries (Godley and Lavoie, 2007). In the case of financialized open economies, these models bring special insights as they are truly monetary. These models distinguish resource constraints (i.e current account) from financial constraints (i.e lending for investment/consumption) (Borio and Disyatat, 2015) by assuming that current account and capital account are the outcome of decisions taken by different agents (Yilmaz and Godin, 2020). Nevertheless, with some notable exceptions (Jackson and Jackson, 2021; Dunza et al., 2021), SFC models rarely account for multiple productive sectors and industries, even if they usually account for different institutional sectors.

The prototype model developed here seeks to understand the fundamental dynamics of the low-carbon transition in developing economies. It desegregates the productive side into three structurally different sectors: Natural-resource exports, Non-tradable and other Goods and services. The model is structural not only because it considers multiple productive sectors, but because it considers different sectoral dynamics in terms of market competition and investment behaviour. This is important because the dynamics of prices and other behaviour decisions determine how ecological transition policies will affect these industries differently (Savona and Ciarli, 2019), implying different dynamics according to countries' productive and trade structures (IMF, 2020; Peszko et al., 2020).

We calibrate the model for a theoretical Natural-resource export country, and run different tests to analyse how different green measures are more effective in specific contexts. The contribution of the prototype model is twofold. Firstly, it provides evidence that carbon-price may not be effective in economies that rely excessively on carbon-intensive industries. Complementary measures need to be adopted to reduce its contractionary impacts on low-carbon intensive industries. Secondly, it provides a framework easily adaptable according to countries' idiosyncrasies to analyse policies in different contexts.

The paper is divided into four sections besides this introduction and the concluding remarks. The next section describes the sectoral structure of the model. The following section is dedicated to the determination of the balanced growth path and the model calibration. Section 4 presents the simulations with the aim of showing the applicability of the model. Finally, the concluding remarks discusses the advances and contribution of this approach.

2. Sectoral Structure

Structural change plays a very important role in developing economies, and it is strongly determined by world dynamics, especially in financialized countries. Frankel (2010) presents a systematization of the rounds of capital inflows to developing countries showing at least three large waves from 1970 to 2009. The impact of these waves in developing countries is well documented in the literature, with booming periods being characterised by currency appreciation, falling trade and current account deficits, increases in consumption and investment (Reinhart and Reinhart, 2008), as well as by credit booms (Magud et al., 2014; Baskaya et al., 2016) and asset-price booms (Caballero, 2016). Frankel (2010) and de La Torre et al. (2015) shows that these booms are followed by structural changes in these economies, with a shift from growth in tradables to higher growth in non-tradables. In the same vein, Lartey (2008) and Benigno and Fornaroy (2014) argue that access to foreign capitals may give rise to a Dutch disease-like phenomena, defining it as a "financial" resource curse. Benigno et al. (2015) found that periods of net capital inflows bonanza are associated with a relative squeeze of manufacturing, leading to premature deindustrialization, such as discussed by Rodrik (2016) and Palma (2014).

The dynamics of this structural transformation process needs to account for at least three structurally different sectors. Firstly, it is necessary to divide the economy into traded and non-traded sectors, as the impacts of the world dynamics affect them from different channels. Even though both sectors benefit from booms due to lower cost to access credit, inputs and capital goods, and higher domestic demand, the impact on traded goods is ambiguous. Because these industries compete with imports for the domestic market and with other economies to export, they may not profit from booms as industries that produce non-tradables. Moreover, it is necessary to account for a specific sector within tradables: the sector that produces natural-resource based goods. During the cycles these industries are particularly impacted because they export predominantly commodities with very volatile prices. Different from other tradables, these industries are positively over-impacted in booms due to the increase in prices.

Based on Yilmaz and Godin (2020), we develop a continuous-time[1] SFC model for an open developing economy. However, this model does not account for sectoral differences as it considers only one productive sector. Following Skott (2021), we divide into three productive sectors: non-tradables, natural-resource exporters and other tradables. These productive

[1] For a long discussion on the advantages of using a continuous-time rather than a discrete-time model see Gandolfo (2012) and Yilmaz and Godin (2020).

sectors hire labour and produce goods and services either for the domestic market or for exports. The main structural characteristics productive sectors are the following:

- Natural-resource exporters (r): produces a homogenous good for export market only; price-taker (commodity); investment is profit-driven; operates at full-capacity
- Non-traded goods and services sector (n): produces heterogeneous goods only for the domestic market; price-maker (imperfect competition); investment is driven by demand, but it depends on profitability; operates bellow full-capacity
- Other manufactured goods and traded services (m): produces heterogeneous goods and services for export and domestic market markets; price-maker (imperfect competition); investment is driven by demand, but it depends on profitability; operates bellow full-capacity

Besides the productive sectors we also consider institutional sectors. These sectors do not hire labor nor produce. Instead, they are responsible for generating final demand and organizing the financial transactions:

- Households (H): consumes all goods and services but commodities; income comes from wages, profits, interest on deposits and social transfers; pay income taxes, social contributions, interest on lending; invest on firms and banks
- Government (G): taxes production and income, consumes only non-traded goods and services, pays unemployed benefits, interest on bonds; absorb Central Bank profits and receive dividends from firms and banks
- Banks (B): lend money for firms, consumers and government, borrow from Central Bank according to their financial needs
- Central Bank (C): accommodates banks' money demand and determines the policy rate according to a Taylor Rule
- Rest of the World (W): besides imports and exports, also finance firms by FDI and government debt

2.1. Productive Sectors

2.1.1. Production and investment

Even though capital and labour may be substitutes in the production process, recent empirical evidence shows a low level of substitution between them even at the macroeconomic level. Knoblach et al. (2020), for example, presents evidence that the elasticity of substitution between labour and capital is about 0.3. In the sectoral level one can expect an even lower substitution. Therefore, we assume that production process In all sectors production is determined by a Leontief function where capital is partially employed, $Y_j^P = \min(a_j N_j, u_j K_j / b_j)$.

In most developing economies, even when unemployment rate is low, labour shortage is not an important constraint to growth. Because these economies are dual with large

Table 1: Transactions Flows Matrix (TFM)

	NR exporters curr	NR exporters cap	Non-traded G&S curr	Non-traded G&S cap	Other G&S curr	Other G&S cap	HH	Gov	Commercial Banks curr	Commercial Banks cap	Central Bank	Rest of the World	$\sum$
NR export. (r)	$+X_r$											$-X_r$	0
Non-traded (n)	$-IC_r^n$		$+Y_n^P - IC_n^n$	$-\dot{V}_n$	$-IC_m^n$		$-C_n$	$-G$					0
Other G&S (m)	$-IC_r^m$	$-I_r^D$	$-IC_n^m$	$-I_n^D$	$+Y_m^P - IC_m^m$	$-I_g^D - \dot{V}_g$	$-C_g^D$					$-X_m$	0
Imports	$-IC_r^{m,IM}$	$-I_r^{IM}$	$-IC_n^{m,IM}$	$-I_r^{IM}$	$-IC_g^{m,IM}$	$-I_r^{IM}$	$-C_m^{IM}$	$+T_m^{IM}$				$+IM_m$	0
	$[GVA_r]$		$[GVA_n]$		$[GVA_m]$								$[GVA]$
		$[I_r]$		$[I_n + \dot{V}_n]$		$[I_m + \dot{V}_n]$							$[GKF]$
Wages	$-W_r$		$-W_n$		$-W_m$		$+W$						0
Taxes (output)	$-t_r^Y X_r$		$-t_n^Y Y_n^D$		$-t_m^Y Y_m^D$			$+T_Y$					0
Contributions							$-SC$	$+SC$					0
Social transf.							$+ST$	$-ST$					0
Inv. Accumul.			$-\dot{V}_n$	$+\dot{V}_n$	$-\dot{V}_m$	$+\dot{V}_m$							0
Interests, L	$-i^L L_r$		$-i^L L_n$		$-i^L L_m$		$-i^L L_H$		$+i^L L$				0
Interests, D							$+i^D D_H$		$-i^D D_H$				0
Interests, B^B								$-i^B B^B$	$+i^B B^B$				0
Interests, B^F								$-i^B B^F$				$+i^B B^F$	0
Interests, L^{FX}	$-i^{FX} L_r^{FX}$		$-i^{FX} L_n^{FX}$		$-i^{FX} L_m^{FX}$							$+i^{FX} L^{FX}$	0
Interests, B^{FX}								$-i^{FX} B^{FX}$				$+i^{FX} B^{FX}$	0
Interests, R_B									$+i^P R_B$		$-i^P R_B$		0
Interests, A									$-i^P A$		$+i^P A$		0
	$[NF_r]$		$[NF_n]$		$[NF_g]$				$[NF_B]$		$[NF_{CB}]$		$[NF]$
Dividends (H)	$-Div_r^H$		$-Div_n^H$		$-Div_m^H$		$+Div^H$		$-Div_B^H$				0
Dividends (G)	$-Div_r^G$		$-Div_n^G$		$-Div_m^G$			$+Div^G$	$-Div_B^G$				0
Dividends (F)	$-Div_r^F$		$-Div_n^F$		$-Div_m^F$				$-Div_B^F$			$+Div^F$	0
Dividends (CB)								$+NF_{CB}$			$-NF_{CB}$		0
Taxes (income)							$-T_H$	$+T_H$					0
Ret. Earnings	$-RE_r$	$+RE_r$	$-RE_n$	$+RE_n$	$-RE_m$	$+RE_m$			$-RE_B$	$+RE_B$			0
[Financial Needs]		$[TFN_r]$		$[TFN_n]$		$[TFN_m]$	$[NLP_H]$					$[CA = KA]$	
Domestic Inv.		$+DDI_r$		$+DDI_n$		$+DDI_m$	$-DDI$			$+DDI_B$			0
Public Inv.		$+PDI_r$		$+PDI_n$		$+PDI_m$		$-PDI$		$+PDI_B$			0
Foreign Inv.		$+FDI_r$		$+FDI_n$		$+FDI_m$				$+FDI_B$		$-FDI$	0
								$[GFN]$		$[OF_B]$			
Deposits							$-\dot{D}_H$			$+\dot{D}_H$			0
Lending, no FX		$+\dot{L}_r$		$+\dot{L}_n$		$+\dot{L}_m$				$-\dot{L}_i$			0
Lending, FX		$+\dot{L}_r^{FX}$		$+\dot{L}_n^{FX}$		$+\dot{L}_m^{FX}$						$-\dot{L}^{FX}$	0
Bonds, B^B								$+\dot{B}^B$		$-\dot{B}^B$			0
Bonds, B^F								$+\dot{B}^F$				$-\dot{B}^F$	0
Bonds, B^{FX}								$+\dot{B}^{FX}$				$-\dot{B}^{FX}$	0
Operating Acc.								$-\dot{OA}$			$+\dot{OA}$		0
Bank Reserves										$-\dot{R}_B$	$+\dot{R}_B$		0
Int'l Res		$-\dot{R}_r^{FX}$		$-\dot{R}_n^{FX}$		$-\dot{R}_m^{FX}$				$-\dot{R}_B^{FX}$	$-\dot{R}_{CB}^{FX}$	$+\dot{R}^{FX}$	0
Advances										$+\dot{A}$	$-\dot{A}$		0
$\sum$	0	0	0	0	0	0	0	0	0	0	0	0	0

Notes:

$T_H = t_H[(1-s)wN + Div^H + i^D D_H]$, taxes (income)

$T_Y = t_r^Y X_r + t_n^Y Y_n^D + +t_n^Y Y_m^D$, taxes (output)

Operations involving purchasing of goods and services and production are expressed in real terms, but it is necessary to consider their respective prices. They are: IC, Y, C, G and I. V, X and M are already expressed in nominal terms.

amount of hidden unemployment (Skott, 2021), the unemployment rate is underestimated, and these economies can grow for long periods without facing labour constraints. Therefore, production is determined by actual capital (K), the capital-output ratio (b) and the capacity utilization rate (u), and hence labour employed (N) is determined by production and labour productivity (a):

$$N_j = \frac{Y_j^P}{a_j k} \tag{1}$$

where j stands for all productive sectors ($j = r, n, m$).

Besides labour and capital, intermediate inputs are also used for production. For simplicity, we assume that natural-resources are not used as inputs, and hence domestic intermediate consumption is divided into non-traded goods and services (IC^n) domestically produced manufacturing goods and services (IC^m), and imported manufacturing goods and services ($IC^{m,IM}$):

$$IC_j^n = c_j^n Y_j^P \tag{2}$$

$$IC_j^m = c_j^m Y_j^P \tag{3}$$

$$IC_j^{m,IM} = c_j^{m,IM} Y_j^P \tag{4}$$

where c_j^n is the technical coefficient of the sector j for products of the the sector n, c_j^n is the technical coefficient of the sector j for products of the the sector m and the superscript IM indicates that it is imported.

Capital (K) accumulates according to investments (I) and the depreciation rate (δ). Investment increases capital, whilst it depreciates as a proportion of the current stock, as follow:

$$\dot{K}_j = I_j - \delta_j K_j \tag{5}$$

In the case of natural resources, we assume for simplification that all production is exported, there is no change in inventories in this sector. Therefore, as it does not play any role in the dynamics. In the case of the other two sectors, actual inventories evolves according to actual demand (Y^D) and production:

$$\dot{V}_{j'} = Y_{j'}^P - Y_{j'}^D \tag{6}$$

where j' stands for all productive sectors but natural-resources ($j' = n, m$)

Sectoral investment is determined by the expected gross profitability (r^e) and the average cost of third-part capital, which is given by the average of the interest rate of new contracts ($i^{L,a}$) and the leverage ratio (l). The higher is the expected profitability in relation to the cost of third-part capital, the higher will be the investment in new capital:

$$I_j = \text{max}[0, K_j(\kappa_0 + \kappa_1(r_j^e - l_r i_j^{L,e}) + \delta_j)] \tag{7}$$

where κ_0 is the autonomous investment, κ_1 is the sensitivity of the investment rate to net expected profitability (expected profitability discounted by interest payments).

Even though firms decide how much they will invest based on expected net profitability, if capacity utilization is too high (beyond the maximum tolerated by firms, which is given by the maximum engineering capacity), depreciation will increase faster, and hence firms will have to invest more to compensate for that.[2] Therefore, the investment function is not continuous, as it will increase faster if capacity utilization is beyond a tolerable capacity ($\bar{u}$):

$$\delta_j = \max\{\delta_0, \delta_0 \exp[\gamma_\delta(u_j - \bar{u})]\} \tag{8}$$

The leverage ratio and the average interest rate of new contracts is given by

$$l_j = \frac{L_j + L_j^{FX} e}{K_j p^K} \tag{9}$$

and

$$i_j^{L,e} = \sigma_j^{FX}(i^{FX} + \mu^{FX})e^e + (1 - \sigma_j^{FX})i^L \tag{10}$$

where L is the total lending in domestic currency, L^{FX} is the total lending in foreign currency, i^L and i^{FX} are the domestic lending interest rate and the world interest rate (in foreign currency), μ^{FX} is the mark-up over foreign foreign lending and $i^{L,e}$ is the expected interest rate.

Expected gross profits depends, on the revenue side, of expected sales, expected prices and *ad valorem* taxation on sales (t^Y). Following Godley and Lavoie (2007) and Yilmaz and Godin (2020), we assume a first-in-first-out accounting procedure, and hence costs are calculated based on the historical unit costs (HUC).

Producers of commodities, however, know that all production will be exported, and hence the uncertain variables are expected prices ($p^{W,e}$) and expected nominal exchange rate (e^e). Therefore, expected profitability is given by:

$$r_r^e = \frac{Y_r^P[(1 - t_r^Y)p_r^{W,e}e^e - HUC_r]}{K_r p^K} \tag{11}$$

where p^K is the current price of capital.

Producers of non-traded and other goods and services, on the other hand, are price-makers, and hence they will receive the price they charge. However, differently than natural resource exporters, they not necessarily sell all their production, and hence expected profitability depends on expected sales. Firms do not want to invest only to produce for the short-term fluctuations, and hence they will decide their investment based on the middle and long-term expected sales. The expected profitability will account for future expected sales, as following:

$$r_{j'}^e = \frac{Y_{j'}^{e,f}[(1 - t_{j'}^Y)p_{j'} - HUC_{j'}]}{p^K K_{j'}} \tag{12}$$

For all productive sectors, historical unit costs are given by actual unit costs (UC), the

[2] Setterfield (2019) discuss the boundaries of tolerable intervals where capacity utilization is acceptable, whilst Lavoie (2014), p. 148, argue that there is the need for having an idle capacity.

adjustment speed (β_{UC}) and the historical inflation of the unit costs:

$$H\dot{U}C_j = \beta_{UC}(UC_j - HUC_j) + HUC\pi^c \tag{13}$$

Actual unit costs depends on labour costs and input costs as a proportion of production. Unit labour costs are given by wages (w) and labour productivity and unit input costs are given by input prices and the technical coefficients

$$UC_j = \frac{w}{a_j} + c_j^n p_n + c_j^m p_m + c_j^{m,IM} p_j^{IM} \tag{14}$$

2.1.2. Foreign trade and actual sales

Natural-resource exporters produce only for the external market and sell all their production at a given price, as they produce commodities. Export revenue (X) is given by production (Y^P), world price (p^W) and the nominal exchange rate (e):

$$X_r = Y_r^P p_r^W e \tag{15}$$

where the subscipt r refers to operations of natural-resource exporters.

Non-traded goods and services produce only for the domestic market and there is no imports of these goods. Actual sales in this sector is given by the summation of intermediate consumption of all three sectors and final demand. Because this sector does not export and do not produce capital goods, only government and household consumption contributes for the final demand:

$$Y_n^D = IC_r^n + IC_n^n + IC_m^n + C_n + G_n \tag{16}$$

Other traded goods and services produce for the domestic market and for exports, besides competing with imports. Different from natural resources, they are price makers, and hence price competitiveness and demand matter for determining the volume of exports and imports. Even though developing countries tend to produce less sophisticated goods than developed economies, non-price competitiveness is an important determinant of their capacity to export,(Fagerberg, 1988; Benkovskis and Wörz, 2016). Export revenues can thus be written as a function of price and non-price competitiveness, as follow:

$$X_m = \zeta_X \left(Y^W\right)^{\varepsilon} \left(\frac{p_m}{p_m^W e}\right)^{\eta_X} p_m \tag{17}$$

where ε is the income elasticity of demand for exports, which measures non-price competitiveness, η_X is the price elasticity of demand for exports, which measures the price competitiveness, and Y^W is the world GDP measured in constant prices.

World GDP evolves according to world productivity growth and population growth:

$$\dot{Y^W} = Y^W(\alpha_a + \alpha_{Pop}) \tag{18}$$

Import penetration (σ_m^{IM}) is the share of imports in total demand excluding exports, which includes domestic absorption and intermediate consumption. Import penetration depends on relative prices and on the price elasticity of demand for imports (η_{IM}):

$$\sigma_m^{IM} = \zeta_{IM} \left(\frac{p_m^{IM}}{p_m} \right)^{\eta_{IM}} \tag{19}$$

where the price of imported goods in national currency is given by its world price, the exchange rate and the import tax (t_m^{IM}):

$$p_m^{IM} = (1 + t_m^{IM}) p_m^W e \tag{20}$$

Total imports is, therefore, the summation of import share of domestic absorption and imported intermediate consumption:

$$IM_m = (\sigma_m^{IM} Y_g^A + IC_r^{m,IM} + IC_n^{m,IM} + IC_g^{m,IM}) p_m^W e \tag{21}$$

where absorption includes demand from household consumption (C_m) and the summation of capital investment of productive sectors:

$$Y_m^A = C_m + I_r + I_n + I_m \tag{22}$$

Domestic intermediate consumption is given by the domestic technical coefficients, which depends on the import penetration and the technical coefficient:

$$c_j^m = (1 - \sigma_m^{IM}) c_j^{m,T} \tag{23}$$

and

$$c_j^{m,IM} = \sigma_m^{IM} c_j^{m,T} \tag{24}$$

where T stands for total.

Import penetration also determines the price of capital, since it is the weighted price of domestic and imported goods:

$$p^K = \sigma_m^{IM} p_m^{IM} + (1 - \sigma_m^{IM}) p_m \tag{25}$$

Actual sales of domestic producers in other traded goods and services sector (Y^D), is, therefore, given by final demand absorbed by domestic producers and demand for domestic inputs:

$$Y_m^D = \frac{X_m}{p_m} + (1 - \sigma_m^{IM}) Y_m^A + IC_r^m + IC_n^m + IC_m^m \tag{26}$$

2.1.3. Demand, expectations and pricing

In the case of natural resources, production is determined by the productive capacity, which is given by the stock of capital and the maximum capacity utilization that avoids over-depreciation of capital. Therefore, total production is given by:

$$Y_r^P = \frac{K_r \bar{u}}{b_r} \tag{27}$$

In the other two sectors, however, there is no need of full-employment, and hence production is not necessarily determined by actual capital. Firms will produce (constrained by the stock of capital) according to expected sales (Y^e) and current inventories (V). However, because firms can face fluctuations on demand, we assume that firms invest in inventories to guarantee their sales (Charpe et al., 2011). Therefore, firms produce given by their expected sales, the desired rates of inventories and the actual inventories (v^d):

$$Y_{j'}^P = \text{min}[Y_{j'}^e + I_{j'}^V, \frac{K_{j'}}{b_{j'}}] \tag{28}$$

where

$$I_{j'}^V = (Y_{j'}^e + \dot{Y}_{j'}^e) v_{j'}^d - V_{j'} \tag{29}$$

is the investment in inventories.

Expected sales follows a backward looking process where firms adjust their expectation according to actual demand. However, knowing that the economy is growing, they also account for historical growth rate of sales, which has a long-term factor, given by the historical growth rate of capital (g^K) and a medium-term factor, given by the historical growth rate of capacity utilization (g^u):

$$\dot{Y}_{j'}^e = \beta_e (Y_{j'}^D - Y_{j'}^e) + Y_{j'}^e (g_{j'}^K + g_{j'}^u) \tag{30}$$

where

$$\dot{g}_{j'}^K = \beta_g \left(\frac{\dot{K}_{j'}}{K_{j'}} - g_{j'}^K \right) \tag{31}$$

and

$$\dot{g}_{j'}^u = \beta_g \left(\frac{u_{j'}}{u_{j'}^h} - 1 \right) \tag{32}$$

Actual and historical capacity utilization rates are given respectively by:

$$u_{j'} = \frac{b_{j'} Y_{j'}^P}{K_{j'}} \tag{33}$$

and

$$\dot{u}_{j'}^h = \beta_h \left(u_{j'} - u_{j'}^h \right) \tag{34}$$

Expected gross profitability depends on prices and expected sales. Firms fix prices according to a mark-up (μ_n) over historical unit costs:

$$p_{j'} = (1 + \mu_{j'}) HUC_{j'} \tag{35}$$

However, the mark-up adjusts to reduce the distance between current and desired inventories. The higher is the current inventories compared to desired inventories, the lower is the mark-up, as firms decide to reduce prices to sell more:[3]

$$\mu_{j'} = \mu_0 + \mu_1 \left(\frac{Y_{j'}^{e} v_{j'}^{d}}{V_{j'}} - 1 \right) \tag{36}$$

2.1.4. Productivity dynamics and non-price competitiveness

Because capital is measured in term of their efficiency, capital productivity, which is the inverse of capital-output ratio, is considered as constant for all sectors, as well as technical coefficients. Technological change, therefore, only impact on labour productivity, which depends on three factors: the adoption of new capital, which embodies new technologies, and the learning process, which makes the new capital more productive.

In the NR exporter sector, only the first factor take place. Because this sector does not produce capital, productivity grows according to the rate of investment and the exogenous marginal productivity growth of capital, as follow:

$$\dot{a_r} = a_r \frac{I_r}{K_r} \alpha_a^{mg} \tag{37}$$

where α_a^{mg} is the exogenous marginal productivity growth rate.[4]

In the non-traded goods and services sector, one can assume that productivity is given by the real wage growth. Since this sector produces predominantly non-tangible goods, aggregate productivity is not measurable(Maroto and Rubalcaba, 2008; Maroto-Sánchez, 2012). Thereby, productivity dynamics in this sector can be defined as:

$$\dot{a_n} = a_n \left(\frac{\dot{w}}{w} - \frac{\dot{p}}{p} \right) \tag{38}$$

Finally, the most complex sector to be analysed is the traded goods sector. We will assume that maximum productivity ($\overline{a}$) grow similarly to the NR exporter sector:

$$\dot{\overline{a_m}} = \overline{a_m} \frac{I_m}{K_m} \alpha_a^{mg} \tag{39}$$

However, effective productivity growth also depends on the learning process and on the distance to the technological leaders. Following a typical learning-by-doing process applied domestically and abroad, productivity will grow faster than the rest of the world if the sector under consideration is historically growing faster than world growth, and it will grow slowly if the economy is growing slowly:

[3] As discussed by Yilmaz and Godin (2020), even though there is the possibility of counter-cyclical mark-ups due to collusion by good producers, we assume that mark-ups work as equilibrators, and hence they are pro-cyclical.

[4] This equation is obtained considering that actual productivity is the weighted average of the productivity of new capital (given by I) and the productivity of old capital (given by $K - I$). Actual productivity growth is, thus, given by: $\hat{a_j} = \frac{I_j}{Y_j^P v_j} a_j^{\hat{m}g}$, where the hat indicates growth rates and j stands for the sector under consideration.

$$\dot{a_m} = a_m \left[\frac{\dot{\overline{a_m}}}{\overline{a_m}} + \gamma_a \left(g_m - \alpha_a - \alpha_{Pop} \right) \right] \tag{40}$$

where γ_a measures the dynamic increasing returns to scale, and the sectoral growth rate is given by the growth rate in the medium term

$$g_m = \frac{\dot{Y_m^{P,h}}}{Y_m^{P,h}} \tag{41}$$

and

$$\dot{Y_m^{P,h}} = \beta_h (Y_m^P - Y_m^{P,h}) \tag{42}$$

Because productivity in the non-traded goods and services sector is given by the productivity growth of the economy in the long run, overall productivity growth is, therefore, given by the weighted average of the productivity of the other sectors:

$$\dot{a} = a \left(\frac{Y_r^P}{Y_r^P + Y_m^P} \frac{\dot{a_r}}{a_r} + \frac{Y_m^P}{Y_r^P + Y_m^P} \frac{\dot{a_m}}{a_m} \right) \tag{43}$$

In the context of homogeneous goods, labour productivity determines only the volume of goods produced by unit of labour employed. This is the case for NR exporters, where productivity, in the end, determines only profitability.

Nevertheless, for heterogeneous goods, productivity is also related to other factors, such as capacity to deliver, product quality and variety. The income elasticity of demand for exports measures these factors, which are related to the non-price competitiveness. By assuming that export elasticity (ε) changes according to productivity growth differential, one have that:

$$\dot{\varepsilon} = \varepsilon \gamma_\varepsilon \left(\frac{\dot{a_m}}{a_m} - \alpha_a \right) \tag{44}$$

2.1.5. Firms financing

Firms borrow to produce and invest. For simplicity, however, we will abstract from lending for production (working capital), and we will focus only on long-term lending. Total Financial Needs (TFN) of firms is given by the investment multiplied by the price of capital (p^K) discounted by non-distributed profits, which is given by difference between net profits (NF) and dividends.

$$TFN_j = p^K I_j - (1 - \sigma_D) NF_j \tag{45}$$

where σ_D is the share of profits distributed as dividends.

Net profits are calculated as total sales discounted by all costs (taxation, wages, input costs and interest payments):

$$NF_j = Y_j^D (1 - t_j^Y) - Y_j^P UC_j - i^L L_j - (i^{FX} + \mu^{FX}) L_j^{FX} e \tag{46}$$

Dividends are distributed according to the share of investors in total equity (EQ) and it is proportional to net profits, as follow:

$$Div_j^i = \sigma_D NF_j \frac{EQ_j^i}{\sum_i EQ_j^i} \tag{47}$$

where i stands for the different investors (H, G and F stand for households, government and foreign, respectively).

Firms will first try to finance their financial needs by the equity market (it will be discussed for each institutional sector). Then, they will try to do it by foreign lending and the remaining financial needs are closed by domestic lending. Access to foreign lending depends on the export share on production, since the higher access to international markets in goods and services increases the opportunities for accessing international financial markets.

However, before deciding how much they will borrow in domestic and foreign currency, firms also save a share of the revenue of their exports as international reserves (R^{FX}) in order to have liquidity to import inputs and pay interests. The amount they want to save varies accordingly to their exports, as follow:

$$\dot{R_j^{FX}} = \sigma^R \frac{X_j}{e} \tag{48}$$

where X_j is exports measured in foreign currency.

The share of domestic lending is a proportion of total financial needs not covered by equity. This proportion (σ^{FX}) is upper bounded by the supply of lending, which is given by the share of exports in production (σ^X), and it will depends also on the demand for foreign lending:

$$\sigma_j^{FX} = \text{min}(\sigma_j^X, \sigma_j^{FX,D}) \tag{49}$$

where

$$\sigma_j^X = \frac{X_j}{Y_r p_j e} \tag{50}$$

Assuming that firms will use foreign lending to avoid a mismatch between revenues and costs in foreign and domestic currency, but they have a zero lower bound, and they will accept an additional foreign lending if these rates diverge:

$$\sigma_j^{FX,D} = \text{max}\left[0, \sigma_j^X - \sigma_c^{IM} c_j^m\right] \tag{51}$$

where σ_c^{IM} is the share of traded inputs that firms believe to be affected by the exchange rate path-through.

Therefore, lending in foreign and domestic currencies evolves as following:

$$\dot{L_j^{FX}} = \sigma_j^{FX} \left[\frac{TFN_j - (DDI_j + PDI_j + FDI_j)}{e} + \dot{R_j^{FX}}\right] \tag{52}$$

and

$$\dot{L_j} = (1 - \sigma_j^L)[TFN_j - (DDI_j + PDI_j + FDI_j) + \dot{R_j^{FX}} e] \tag{53}$$

where DDI, PDI and FDI are respectively household domestic direct investment, public direct investment and foreign direct investment.

2.2. Institutional sectors

2.2.1. Households

Households consume both non-traded and other traded goods based on their disposable income, their wealth and the access to new loans. Households disposable income (YD_H) includes wages, dividends (Div), interest on their deposits (i^D) and social transfers from the government (ST). However, they have to pay income taxes (t_H), social contributions (sc) and interest on their loans (i^L).

$$YD_H = (1 - t^H)[(1 - sc)wN + \sum_{j,B} Div_j^H + i^D D_H] + ST - i^L L_H \tag{54}$$

where dividends includes those received from productive sectors (j) and banks (B), and

$$N = \sum N_j \tag{55}$$

Households will first decide how much they will consume and then distribute between the two sectors. Furthermore, consumption takes time to adjust to income and wealth, and hence target consumption is given by

$$C^T = \gamma_0 Pop + \gamma_1(YD_H + \dot{L_H}) + \gamma_2 D_H \tag{56}$$

where

$$\dot{Pop} = \alpha_{Pop} Pop \tag{57}$$

and actual consumption evolves as following:

$$\dot{C} = \beta_C(C^T - C) \tag{58}$$

New lending is available according to income obtained from wages, and hence one can considered that lending grow at the same rate a the wage bill:

$$\dot{L_H} = L_H \left(\frac{\dot{w}}{w} + \frac{\dot{N}}{N} \right) \tag{59}$$

The difference between disposable income and consumption (excluding consumption financed by new lending) gives household available funds for investing. Based on the share of firms investment in total investment, households will distribute the composition of their investment in productive sectors as follow:

$$DDI_j = \sigma_j^H(YD_H - C + \dot{L_H}) \tag{60}$$

Moreover, households also invest in banks and save the remaining funds available as deposits. Different from investment in productive activities, investment in banks is guided by a portfolio decision based on the difference between expected banks profitability and the deposit rate:

$$DDI_B = \left(1 - \frac{1}{\exp[\gamma_{BI}(r_B - i^B)]}\right)(1-\sigma_H)(YD_H - C + \dot{L_H}) \quad (61)$$

The remaining funds are saved as deposits:

$$\dot{D_H} = YD_H - C + \dot{L_H} - \sum DDI_{j,B} \quad (62)$$

Household equity evolves due to new investments, as discussed before, but also due to non-distributed profits. Thereby, it is given by:

$$\dot{EQ}^H_{j,B} = DDI_{j,B} + (1-\sigma_D)NF_{j,B}\frac{EQ^H_{j,B}}{EQ_{j,B}} \quad (63)$$

The share of spending in non-traded goods (σ^C_n) consider relative prices... (how to make it in an easy way?).

Consumption in real terms is thus given by:

$$C_n = \sigma^C_n \frac{C}{p_n} \quad (64)$$

and

$$C_m = \frac{C - C_n p_n}{p^K} \quad (65)$$

where

$$\sigma^C_n = \frac{\gamma^n_0}{\exp(\gamma^n_1 * (pn/pK - 1)} \quad (66)$$

Wages are not determined internally to each sector, but for the economy as a whole. The higher is the unemployment rate, the lower is wage bargain power, and hence real wages can grow at a different rate of productivity growth. Moreover, nominal wages grow according to expected inflation. Thereby, we have that:

$$\dot{w} = w\left[\frac{\dot{a}}{a} + \frac{\dot{p^{C,e}}}{p^{C,e}} + \beta_w\left(\frac{N}{Pop} - \gamma_N\right)\right] \quad (67)$$

where γ_N is the participation rate in which bargaining power is capable of guaranteeing that all expected consumers inflation and productivity growth is transferred to wages, p^C is the average consumer prices, and $p^{c,e}$ is the expected consumer prices:

$$p^C = \sigma^C_n p_n + (1-\sigma^C_n)p^K \quad (68)$$

and

$$\dot{p^{C,e}} = p^{C,e}\beta_{pC}(p^C - p^{C,e}) + p^{C,e}\lambda_p \quad (69)$$

2.2.2. Government

Assuming that government has a strict fiscal rule for its consumption, where it changes according to expected inflation, population growth and productivity growth, we have that:

$$\dot{G} = G\left(\frac{\dot{p^{C,e}}}{p^{C,e}} + \alpha_{Pop} + \frac{\dot{a}}{a}\right) \tag{70}$$

Government consumes only non-traded goods and services, which includes all governmental activities (public health, public education and public administration), thereby

$$G_n = G/p_n \tag{71}$$

Government also pays a basic revenue for unemployed people (social transfers), and the value grows with consumers inflation and productivity:

$$ST = st(Pop - N) \tag{72}$$

where

$$\dot{st} = st\left(\frac{\dot{a}}{a} + \frac{\dot{p^{C,e}}}{p^{C,e}}\right) \tag{73}$$

As a source of revenue government taxes household income and firm's sales, imports and social contributions:

$$T_G = t^H[(1-sc)wN + Div^H + i^D D_H] + t_r^Y X_r + t_n^Y Y_n^D p_n + t_m^Y Y_m^D p_m + t_m^{IM} M_m + scNw \tag{74}$$

Government primary deficit evolves according to taxation, social contributions, government consumption and social transfers, as presented before. Nevertheless, government also invest directly in productive activities and banks (PDI). Public Direct Investment is a proportion of government expenses, whilst its distribution follows the current distribution of government equity:

$$PDI_{j,B} = \sigma_{j,B}^P G \tag{75}$$

Government equity evolves due to new investments, as discussed before, and due to non-distributed profits:

$$\dot{EQ}_{j,B}^G = PDI_{j,B} + (1-\sigma_D)NF_{j,B}\frac{EQ_{j,B}^G}{EQ_{j,B}} \tag{76}$$

Besides the primary deficit, government also needs to finance its spending with interest payments of bonds. Central bank profit and dividends received from firms, on the other hand, reduces Government Financial Needs, as follow:

$$GFN = G + ST + PDI + i^B(B^B + B^F) + i^{FX}B^{FX}e - T_G - NF_{CB} - Div^G \tag{77}$$

where B^B is government bonds with banks, B^F is government bonds with foreigners in domestic currency and B_G^{FX} in foreign currency,

$$Div^G = Div_r^G + Div_n^G + Div_m^G + Div_B^G \tag{78}$$

and

$$PDI = PDI_r + PDI_n + PDI_m + PDI_B \tag{79}$$

For financing its deficit, government issue bonds. Firstly, the government decides how much bonds are issued in foreign currency. Because these bonds are almost free of risk, the foreign financial markets will absorb these bonds, but they can be very costly for the government, and hence, government avoids issuing these bonds, only using it when differential of interest rates including expected devaluation is too high. Thereby, if the interest rate is the same, the share of bonds issued in foreign currency is zero. The higher is the interest rate differential, the higher is the share of bonds issued in foreign currency. Bonds issued by the government (and absorbed by the market) in foreign currency and absorbed by the market is given by:

$$\dot{B^{FX}} = \left(1 - \frac{1}{\exp(\gamma_B i_B^{dif})}\right) GFN \tag{80}$$

where

$$i_B^{dif} = \frac{1 + i^B}{(1 + i^{FX})\frac{e^e + \dot{e^e}}{e}} - 1 \tag{81}$$

Total supply of bonds in domestic currency (B^S) will be given by the GFN discounted by bonds issued in foreign currency added by the difference between the target and the actual Operating Account (OA). The Operating Account is necessary to guarantee that government will be able to pay its expenses, and it will vary according to the difference between the supply and the demand of bonds:

$$B^S = GFN - \dot{B^{FX}} e + (\lambda_O - OA) \tag{82}$$

where λ_O is the target operating account that government want to keep in order to guarantee liquidity, which is a proportion of government spending

$$\lambda_O = \bar{\lambda_O} \tag{83}$$

The demand of bonds by banks is a share of the supply of bonds in national currency discounted by bonds bought by foreigners (discussed latter). Therefore, bonds issued by the government and absorbed by banks is given by:

$$\dot{B^B} = \min\left[B^S - \dot{B^F}, \frac{S^B - \dot{B^F}}{\exp[\gamma_B\,(i^B - i^{B,d})]}\right] \tag{84}$$

where $i^{B,d}$ is the desired interest rate by which the market accept to absorb all supply of bonds.

The desired interest rate is given by the policy rate plus a risk of default premium, which depends on the gross debt of the government (DG) to GDP ratio.

$$i^{B,d} = i^P + \gamma_{Bd}\frac{DG}{GDP} \tag{85}$$

where

$$DG = B^B + B^F + B^{FX}e \tag{86}$$

and

$$GDP = C + I + G + X_r + X_m - M_m \tag{87}$$

where

$$I = (I_r + I_n + I_m)p^K \tag{88}$$

Government operating account and the bonds interest rate evolves as following:

$$\dot{OA} = \dot{B^B} + \dot{B^F} + \dot{B^{FX}}e - GFN \tag{89}$$

Finally, government adjusts the actual interest rate according to the supply and demand Of bonds to avoid letting the government operating account (OA) to become too low, bounded by the desired interest rate:

$$\dot{i^B} = \max\left[\gamma_{iB}\left(\frac{\lambda_O - OA}{GDP}\right), i^{B,d} - i^B\right] \tag{90}$$

2.2.3. Commercial Banks

Banks finance firms and household financial needs by lending in ex-post interest rates, which are given by the marginal capitation cost and a bank mark-up. As we will see, the marginal capitation cost is given by the policy rate, as this rate determines how banks close their financial needs. The mark-up is constant (banks spread), and hence we have that:

$$i^L = \mu_B + i^P \tag{91}$$

Banks are obligated to keep compulsory deposits with the central bank according to the required reserves ratio (σ_{rr}) and their total deposits:

$$R_B = \sigma_{rr}D_H \tag{92}$$

If deposits and own funds are not sufficient to cover their lending and reserves, they need advances from Central Bank. On the other hand, if there is excess of liquidity, they borrow it to the Central Bank, which pays the policy rate as interest rate

$$A_{CB} = L_H + L_r + L_n + L_m + B^B + R_B - D_H - OF_B + R_B^{FX}e \tag{93}$$

(are you sure about the FX reserves?)

Such as non-financial firms, banks distribute profits according to the share of equity, as following:

$$Div_B^i = \sigma_D NF_B \frac{EQ_B^i}{EQ_B} \tag{94}$$

where

$$EQ_B = EQ_B^H + EQ_B^G + EQ_B^F \tag{95}$$

For simplification, we assume that deposits interest rate is equal to the policy rate,

$$i^D = i^P \tag{96}$$

and, hence, banks profits and its profitability can be written as:

$$NF_B = (i^L - i^P)(L_H + L_r + L_n + L_m) + (i^B - i^P)B^B + i^P OF \tag{97}$$

and

$$r_B = \frac{NF_B}{OF_B} \tag{98}$$

The summation of distributed profits is given by

$$Div_B = \sum Div_B^i \tag{99}$$

Banks own funding (OF) evolves according to new investments and retained profits

$$\dot{OF_B} = DDI_B + PDI_B + FDI_B + (1 - \sigma_D)NF_B \tag{100}$$

2.2.4. Central Bank

Central bank is responsible for the monetary policy, besides guaranteeing liquidity through advances to commercial banks. Central bank profit is given by the difference between revenue from these advances and the interest of compulsory deposits:

$$NF_{CB} = i^P A_{CB} - i^P R_B \tag{101}$$

Policy rate follows a simplified Taylor rule, where the distance between expected inflation and the inflation target is used as reference:

$$i^P = \max\left[0, \iota_0 + \iota_1 \left(\frac{\dot{p^{C,e}}}{p^{C,e}} - \lambda_p\right)\right] \tag{102}$$

where λ_p is the inflation target. (distance between current and the capital utilization rate the CB think is adequate can also be used to have the complete Taylor rule)

Central bank also do open market operations with foreign reserves (R_{CB}^{FX}) to reduce the volatility of the nominal exchange rate. If Central Bank want to keep the nominal exchange rate fixed, it absorbs all excess of foreign currency supply (FX^S) in relation to demand (FX^D), increasing its reserves. If it want to let it completely floating, it only keeps a constant share of country's imports as reserves with the aim of guaranteeing liquidity. Thereby, we have that:

$$\dot{R_{CB}^{FX}} = \sigma_0^{FX}(FX^S - FX^D) + \sigma_1^{FX}\frac{\dot{IMm}}{e} \quad (103)$$

where σ_0^{FX} varies from zero to one according to Central Bank intention to keep e fixed or floating, and, if it is zero (the country is in a floating exchange rate regime), σ_1^{FX} is the share of imports that central banks want to keep as reserves.

2.2.5. Rest of the World

Capital flows are attracted to finance either firms of productive sectors or government debt. Firms can be financed either by portfolio or foreign direct investments, what we call here equity. In the case of capital flows that are attracted to finance government debt, they will reduce the dependence of banks, as we seen before.

The flow of new foreign equity investments (direct and indirect) is given by a share of world financial flows according to the profitability and the actual share of equity in total equity:

$$FDI_{j,B} = \phi_{j,B}(r_{j,B}^e - rsk - i^{FX})WFF \quad (104)$$

and rsk incorporates both the expected undervaluation of the domestic currency and a risk premium:

$$rsk = \min\left[i^B - i^{FX}, \frac{\dot{e^e} + e^e}{e}\left(1 + \gamma_{rsk}\frac{GDP}{R_{CB}^{FX}e}\right) - 1\right] \quad (105)$$

Besides equity investments, foreign capital flows also finance the government by buying bonds both in domestic and foreign currency. Bonds in foreign currency were discussed before, since it is a government decision (it is a low risk investment fore foreign investors, but a risk debt for the government). In the case of bonds in domestic currency, what determines the flow is the difference between the interest rate payed by the government and the world interest rate added by the external risk premium (limited by the supply of bonds):

$$\dot{B^F} = \min\{B^S, [\phi_0^F + \phi_1^F(i^B - rsk - i^{FX})]WFF\} \quad (106)$$

The world financial depends on world growth rate in nominal terms, which is given by population and productivity growth, and international inflation (α_p):

$$WFF = \phi^W(Y^W p_m^W)(\alpha_a + \alpha_{Pop} + \alpha_p)e \quad (107)$$

Change in equity is, therefore, the summation of new investments and retained profits:

$$\dot{EQ_{j,B}^F} = FDI_{j,B} + (1 - \sigma_D)NF_{j,B}\frac{EQ_{j,B}^F}{EQ_{j,B}} \quad (108)$$

Nominal exchange rate is determined by the adjustment of supply and demand for foreign currency, as follow:

$$\dot{e} = e\beta_{eN}\frac{FX^D + \dot{R_{CB}^{FX}} - FX^S}{FX^S} \tag{109}$$

where

$$FX^D = IM_m + (L_r^{FX} + L_m^{FX})(i^{FX} + \mu^{FX})e + \dot{R_r^{FX}} + \dot{R_m^{FX}} + Div^F + i^B B^F + i^{FX} B^{FX} e \tag{110}$$

and

$$FX^S = X_r + X_g + (\dot{L_r^{FX}} + \dot{L_m^{FX}})e + FDI + \dot{B^F} + \dot{B^{FX}} e \tag{111}$$

where

$$Div^F = Div_r^F + Div_n^F + Div_m^F + Div_B^F \tag{112}$$

and

$$FDI = FDI_r + FDI_n + FDI_m + FDI_B \tag{113}$$

Expected exchange rate depreciation and expected commodities prices follow a typical backward-looking expectation structure:

$$\dot{e^e} = \beta_{ee}\left(e - e^e\right) \tag{114}$$

and

$$\dot{p_r^{W,e}} = \beta_{pr}\left(p_r^W - p_r^{W,e}\right) + p_r^{W,e}\alpha_p \tag{115}$$

The remaining foreign currency will be allocated by banks as reserves:

$$\dot{R_B^{FX}} = \frac{FX^S - FX^D}{e} - \dot{R_{CB}^{FX}} \tag{116}$$

3. Balanced growth path

3.1. Investment function

The assumption of a linear investment function implies that only for a linear combination of the parameters, the model will be stable in the long run. Therefore, one need to determine these parameters, otherwise there will be either over-investment or under-investment leading the economy to explosive growth or collapse.

3.1.1. Investment sensitivity

By assuming that there is no ever-increasing or ever-decreasing unemployment rate, and that population growth is exogenous, one have that

$$g = \alpha_a + \alpha_P$$

where g is the growth rate of the economy in the long run, and α_a and α_P are the growth rates of labour productivity and population, respectively.

If one assume that productivity growth depends on capital accumulation, as capital embodies world technological advances, and it also depends on the difference of the domestic and international growth rates, as a faster growing country will be able to promote technological changes faster than the world, one have that

$$\alpha_a = \frac{I}{K}\alpha_a^{mg} + \gamma_g(g - g^*)$$

where I is new investment, K is the current stock of capital, α_a^{mg} is the difference between the productivity of new capital compared to the current stock of capital, which gives the marginal growth of productivity, γ_g is the impact of the growth differential on productivity, and g^* is the world growth rate.

Based on this, we have that

$$g = \frac{I\alpha_a^{mg} + (\alpha_P - \gamma_g g^*)K}{(1-\gamma_g)K}$$

From the accumulation perspective, if one assume a constant capital-output ratio, growth in the long run is given by the growth of rate capital, and hence

$$g = \frac{\dot{K}}{K} = \frac{I - \delta K}{K}$$

In the long run, these two growth rates have to be the same, otherwise the economy is not stable. Therefore, we have that investment, from a supply-side perspective is given by

$$I = K\frac{\delta(1-\gamma_g) + \alpha_P - \gamma_g g^*}{1 - \gamma_g - \alpha_a^{mg}}$$

Nevertheless, the demand for investment is assumed to be linearly dependent of expected profit rate net of interest ($r^{e'}$):

$$I = K(\kappa_0 + \kappa_1 r^{e'} + \delta)$$

For stability we need that investment from a supply-side perspective is equal to the demand for investment. Because we are assuming a constant capital-output ratio, $K = \frac{bY}{u}$, we have the first stability equation for the system:

$$\kappa_1 = \frac{\alpha_P + \delta\alpha_a^{mg} - \gamma_g g^* - \kappa_0(1 - \gamma_g - \alpha_a^{mg})}{(1 - \gamma_g - \alpha_a^{mg})r^{e'}} \quad (117)$$

where $r^{e'} = r^e - li^{L,a}$

3.1.2. Profitability and unit costs

Net profitability ($r^{e'}$) depends on expected prices, historical unit costs and expected sales, besides the interest payments.

In the case of price takers, all production is sold, but they are not aware of the price received by their sales, as they sell their production at the international prices, which are exogenous.

For these sectors, as $K = \frac{Yb}{u}$, expected net profitability is given by

$$r_r^{e'} = [p_r^{W,e} e(1-t) - HUC]\frac{u_r}{p^K b_r}$$

where $e = e^e$ in the balanced growth path.

If one assume $p^K = 1$ as the numerator, expected net profitability for these sectors can be written as:

$$r_r^{e'} = \Pi_r^e \frac{u_r}{b_r} - l_r i_r^{L,a} \tag{118}$$

where Π_r^e is the expected profit margin, given by:

$$\Pi_r^e = (1-t)p_r^W e - \frac{w}{a_i} + \sum c_r^j p_j$$

given that in the long run, $p_r^{W,e} = p_r^e$ and $HUC = UC$.

In the case of price makers' sectors, prices are fixed over historical unit costs, and hence the expectation components of profitability are expected sales and historical unit costs.

Production is given by expected sales and the desired inventories discounted by current inventories. Because in a balanced growth path expected evolves as

$$\dot{Y_i^e} = Y_i^e g$$

and because in the long run expected sales and demand are equal, we have that

$$\dot{V_i} = Y^e v_i^d g$$

which implies that production is given by

$$Y_i^P = Y_i^e(1 + v_i^d g)$$

Therefore, expected profitability is given by

$$r_i^{e'} = \frac{Y_i^P\left[(1-t_i^Y)p_i - HUC_i\right]}{K_i(1+v_i^d g)} - l_i i_i^{L,a}$$

In these sectors, however, prices are fixed according to the mark-up rule. Therefore, net expected profitability can be written as

$$r_i^{e'} = \left[(1-t_i^Y)(1+\mu_i) - 1\right]\left(\frac{w}{a_i} + \sum c_i^j p_j\right)\frac{u_i}{b_i(1+v_i^d g)} - l_i i_i^{L,a}$$

The mark-up is composed by two factors: one autonomous, μ_0, which depends on an exogenous price-elasticity of demand for the product, and is sector-specific, and another tghat varies according to the difference between desired and the current inventories. Given that in the long run desired inventories are equal to current inventories, we have that the autonomous component of the mark-up is given by

$$\mu_{0i} = \left[\frac{(r_i^{e'} + l_i i_i^{L,a})(1 + v_i^d g) b_i}{\left(\frac{w}{a_i} + \sum c_i^j p_j\right) u_i} + 1 \right] \frac{1}{1 - t_i^Y} - 1$$

3.1.3. Balanced growth path

Because sectors have different leverage ratios, pay different interest rates, present different levels of capacity utilization and diverge in many other aspects, if one assumes that the investment parameters are the same, there is nothing that guarantees that the economy will be in a balanced path. For this to happen, one parameter has to adjust in order to make sure that all sectors will grow at the same rate.

A possibility is to have different values for the parameters of the investment function, but there is no reason to believe that investment is more or less attractive in different sectors given their expected net profitability. If one assume that κ_0 and κ_1 is determined in sector r, there will be only balanced growth path $(g_r = g_i)$ if net expected profitability is the same. It implies there is the need for one variable to adjust sectoral net profitability in order to keep the balanced growth path.

In a market economy, it is expected that mark-up varies according to sector to guarantee the same profitability. If profitability is higher in one sector, firms will invest more in this sector, which will lead to increasing competition and, consequently, reduction in margins. Therefore, we will assume that mark-up varies across sectors to guarantee the same profitability after interest payments.

Assuming that margin in sector r is given as it is a price taker, if $r_r^{e'} = r_i^{e'}$, the long-term component of the mark-up of every sector i will adjust to guarantee that all sectors present the same net profitability.

Based on equations (2) and (3), one can write it as a system of equations in terms of sectoral mark-ups and the net profitability. If one assume that $p_m^{IM} e = p_m$, which is necessary for the balanced growth path, it will imply that $p_m = 1$. This system of three equations has only one solution, as price-makers' mark-ups and the relative the profit margin of sector r are the only variables.

3.1.4. Productivity dynamics

If one assumes that world productivity evolves as domestic productivity and that capital-output ratio is also constant, we have that

$$\alpha_a^{mg} = \frac{g^* - \alpha_P^*}{g^* + \delta} \tag{119}$$

Because $\frac{I}{K} = g + \delta$, domestic productivity growth can be written as

$$\alpha_a = \alpha_a^{mg}\delta - \gamma_g g^* + (\gamma_g + \alpha_a^{mg})g$$

where $(\gamma_g + \alpha_a^{mg})$ is the induced technological change.

From the empirical literature on Verdoorn's law, we have that it is about 0.5, and hence we have that

$$\gamma_g = 0.5 - \frac{g^* - \alpha_P^*}{g^* + \delta} \tag{120}$$

3.2. Debt Sustainability

In a balanced growth path, all variables have to grow at the same rate. The growth rate of nominal variables (g_N), such as lending, deposits and consumption, has to be equal to the summation of real growth and inflation, therefore,

$$g_N = \alpha_{Pop} + \alpha_a + \alpha_p$$

3.2.1. Households

In the case of households, the variable that closes their current balance is deposits (D_H). Household deposits evolves as

$$\dot{D} = YD_H - C + \dot{L_H} - DDI$$

Domestic direct investment in productive sectors depends on household income after consumption and new lending

$$DDI' = \sum \sigma_i^H (YD_H - C + \dot{L_H})$$

where $i = r, n, m$.

Domestic direct investment in banks depends on the same variables discounted by the domestic direct investment in productive sectors

$$DDI_B = \left(1 - \frac{1}{\text{exp}[\gamma_{BI}(r_B - i^B)]}\right)\left(1 - \sum \sigma_i^H\right)(YD_H - C + \dot{L_H})$$

Thereby, we have that household deposits (given the difference between bank profitability and bonds interest rate), increases according only to

$$\dot{D_H} = \Omega(YD_H - C + \dot{L_H})$$

where $\Omega = \frac{1 - \sum \sigma_i^H}{\text{exp}[\gamma_{BI}(r_B^e - i^D)]}$.

Defining θ_D as the ratio of total households deposits to consumption, and knowing that disposable income is given by

$$YD_H = \Lambda p^C Pop + (1 - t^H)D_H i^D + (1 - t^H)Div^H$$

where $\Lambda = [(1 - t^H)(1 - sc) - i^L \sigma_L]w\sigma_N + st(1 - \sigma_N)$, $\sigma_N = N/Pop$ and $\sigma_L = L_H/(wN)$.

The level of consumption that makes deposits to grow at the same nominal level of the GDP, $\dot{D_H} = g_N D$, is given by

$$C = \frac{\Lambda p^C Pop + (1 - t^H) Div^H + \dot{L_H}}{1 - \theta_D [i^D (1 - t^H) - \frac{g_N}{\Omega}]} \tag{121}$$

Once the level of consumption that stabilizes the debt is determined (dividends will be discussed later as it is related to firms' debt sustainability), one can determine deposits and, consequently, disposable income.

Actual Consumption, however, is determined by population, disposable income and deposits. One of these sensitivity parameters have to adjust in order to guarantee the sustainability of consumers' debt. Here we will assume that propensity to consume deposits (γ_2) is the one that adjusts to guarantee the balanced growth path.

Consumption adjusts to target consumption according to β_C, and hence it is given by:

$$\exp\left(\frac{g_N}{\beta_C}\right) C = \gamma_0 p^C Pop + \gamma_1 YD_H + \gamma_2 D_H$$

Isolating γ_2, which is the consumption out of deposits, we have that it will be given by the actual consumption per capita discounted by the autonomous real consumption per capita and the real disposable income per capita, we have that

$$\gamma_2 = \frac{1}{\widetilde{D}} \left[\exp\left(\frac{g_N}{\beta_C}\right) \widetilde{C} - \gamma_0 - \gamma_1 (\widetilde{YD_H + \dot{L_H}})\right] \tag{122}$$

where tilde stands for per capita in real terms.

3.2.2. Firms

For every individual firm, investments have to be financed either by new debt or by equity, which includes new equity and non-distributed profits. This equality can be written as

$$I_i = DI_i + (1 - \sigma_D) NF_i + \dot{L_i^T}$$

where σ_D is the share of distributed profits, L_i^T is the total leverage in both domestic and foreign currencies and DI_i is the summation of domestic, public and foreign direct investments.

Because $I_i = K_i(g + \delta)$, and considering that firms' leverage ratio is constant, and hence lending has to grow at the nominal growth rate of the economy, $\dot{L^T} = (L + L^{FX}) g_N$, we have that

$$DI_i = K_i(g + \delta) - (1 - \sigma_D) NF_i - (L + L^{FX}) g_N$$

The distribution of the investment follows the equity distribution to maintain the balanced growth.

$$DDI_i = DI_i \frac{EQ_i^H}{EQ_i}$$

,

$$PDI_i = DI_i \frac{EQ_i^G}{EQ_i}$$

and

$$FDI_i = DI_i \frac{EQ_i^F}{EQ_i}$$

From these equations, Domestic Direct Investment is over-determined, as it is determined by the firms financial needs and by households decision to invest in firms. One can consider an adjustment coming from households decision to invest in firms (σ_i^H). Another possible adjustment is to consider that firms will distribute profits according to their financial needs to keep the leverage ratio stable (σ_D), but profit distribution should be different for every sector.

Because the demand for investment needs to be equal to the supply for every sector, we have that

$$DDI_i = \sigma_i^H (YD_H - C + \dot{L^H})$$

and, from firms' financial needs perspective, it is given by

$$DDI_i = \frac{EQ_i^H}{EQ_i} \left[g + \delta - (1 - \sigma_D) r_i^{'} - \left(l_i - \sigma_R \sigma_i^X \frac{u_i}{b_i} \right) g_N \right] K_i$$

Because these two values for the DDI in productive sectors have to be the same, if one assume that households adjust their investment in firms to guarantee firms' debt sustainability we have that

$$\sigma_i^H = \frac{EQ_i^H}{EQ_i} \frac{\left[g + \delta - (1 - \sigma_D) r_i{'} - \left(l_i - \sigma_R \sigma_i^X \frac{u_i}{b_i} \right) g_N \right]}{YD_H - C + \dot{L^H}} K_i \tag{123}$$

3.2.3. Banks

Banks's debt sustainability depends on own funding growing at the rate of nominal GDP, otherwise either they will need proportionally more Advances from Central Bank or they will have excess of liquidity. Because own funds evolves as the summation of new equity investments and retained profits, we have that:

$$DI_B = [g_N - (1 - \sigma_D) r_B] OF$$

Given that banks distributes profits as firms (σ_D), and considering that households keep their share on equity constant, we have that

$$DDI_B = [g_N - (1 - \sigma_D) r_B] EQ_B^H$$

On the other hand, DDI in banks depends on household income after consumption and direct investments in firms, which is given by

$$DDI_B = \left(1 - \frac{1}{\exp[\gamma_{IB}(r_B - i^D)]} \right) (1 - \sigma_H)(YD_H - C + \dot{L_H})$$

Thereby, one variable have to adjust in order to avoid another over-determination. Analogously to firms, we assume here that households investment in banks adjust, and once it is given by the sensitivity of profitability, we have that

$$\gamma_{IB} = \frac{\ln\left(\frac{1}{1-\Phi}\right)}{r_B - i^D} \tag{124}$$

where $\Phi = \frac{[g_N-(1-\sigma_D)r_B]EQ_B^H}{(1-\sigma_H)(YD_H-C+L^H)}$

3.2.4. Government

Government debt has to be stable as a share of nominal GDP and the Operating Account has to be equal to its target value for the government debt to be stable. It implies that Government Financial Needs (GFN), which is financed by bonds (government debt), will determine the debt sustainability.

Government Financial needs is given by the summation of expanses discounted by revenues. Re-writing the expenses either as a share of population or in terms of bonds, we have that:

$$GFN = G(1+\sigma_P) + ST + i^B DG - (i^B - i^F X)B^{FX}e - T_G - NF_{CB} - Div^G$$

where $DG = B^B + B^F + B^{FX}$ is the government debt and σ_P is the determinant of Public Direct Investment.

Since $GFN = \dot{DG}$ for guaranteeing the sustainability of government financing ($i^B = i^{B,d}$), and $\dot{DG} = DGg_N$, meaning that debt grow at the same rate of nominal GDP, we have that

$$G = \frac{(g_N - i^B)DG + (i^B - i^{FX})B^{FX}e + NF_{CB} + Div^G + T_G - ST}{1+\sigma_P} \tag{125}$$

Moreover, once $i^{B,d} = i^B$ in the balanced growth, the sensitivity of the market to the government debt, which gives the risk premium of government yields, is given by:

$$\gamma_{B^d} = \frac{i^B - i^P}{\theta_{DG}} \tag{126}$$

where $\theta_{DG} = \frac{DG}{GDP}$.prices

Finally, because public investment is determined by firms' financial needs, σ_P has to adjust to guarantee that Government Financial Needs is balanced with their investment in firms. Therefore, we have that:

$$\sigma_P = \frac{\sum_i PDI_i}{(g_N - i^B)DG + (i^B - i^{FX})B^{FX}e + NF_{CB} + Div^G + T_G - ST - \sum_i PDI_i} \tag{127}$$

3.2.5. External

Besides households, firms and government, the balanced growth also depends on the stabilization of external accounts. Here it becomes explicitly because we are assuming the same inflation domestically and abroad, which implies that exchange rate is constant, and hence supply of foreign currency is equal to its demand.

Because variables in the capital account are exogenous to world growth or given by the level of external debt, dividends are determined by sector profitability and exports are given mainly to the size of the NR export sector, we will consider here that import propensity adjusts for guaranteeing the balanced growth.

Total import is given by the import propensity multiplied by absorption and intermediate inputs. Because we are considering that prices of imported and domestic goods after taxes are the same ($p_m^{IM} = p_m$), we have that $\sigma_M^{IM} = \zeta_{IM}$, and hence

$$\sigma_M^{IM} = \frac{FX^S - [(L_r^{FX} + L_m^{FX})(i^{FX} + \mu^{FX}) + \dot{R}_r^{FX} + \dot{R}_g^{FX} + i^{FX}B^{FX}]e - (Div^F + i^B B^F)}{(Y_g^A + IC_r^{m,T} + IC_n^{m,T} + IC_g^{m,T})p_m^W e} \tag{128}$$

In the supply side of foreign exchange, exports are given according to the economic structure. Bonds in foreign and domestic currency absorbed by the market, however, have to grow ate the same rate of the nominal GDP, which implies that:

$$\phi_1^F = \frac{B^F g_N - \phi_0^F WFF}{(i^B - rsk - i^{FX})WFF} \tag{129}$$

In the special case where $rsk = i^B - i^{FX}$, which can be convenient as expected depreciation is zero in a balanced growth path, the sensitivity of the risk to the external position of the central bank (given by its reserves) is:

$$\gamma_{rsk} = rsk \frac{R_{CB}^{FX}}{GDP} \tag{130}$$

Government will issue bonds in foreign currency will depending on the difference between foreign and domestic interest rates and the Government Financial Needs. For keeping it stable, once we have that $GFN = \dot{DG}$, one need to have the sensitivity of the differential interest rates given by

$$\gamma_{Bi} = \frac{\ln(DG) - \ln(DG - B^{FX})}{i_B^{dif}} \tag{131}$$

Finally, another source of foreign exchange is FDI. Because FDI is determined for each sector separately and it is given by the net profitability discounted by the risk and the foreign exchange rate, it is necessary to determine the linear combination of the its determinants to guarantee that it grows at the same rate of the rest of the economy (in nominal terms).

Although net expected profitability in all sectors is the same (as discussed in the sections of balanced growth path), profits before interest payments diverge. Thereby, FDI sensitivity to profitability needs to change according to the sector.

Sectoral FDI was already defined in the firms' debt sustainability section (in the case of banks they will be defined latter), and it is given by $FDI_i = DI_i \frac{EQ_i^F}{EQ_i}$. Thereby, FDI sensitivity to profitability is given by:

$$\phi_i = \frac{FDI_i}{(r_i^e - rsk - i^{FX})WFF} \tag{132}$$

3.3. Sectoral Structure

Once investment is determined by the investment function, consumption is determined by income, government expenditures is determined by its debt and dividends, and the foreign exchange market is in equilibrium, one can determine the sectoral distribution according to the final demand.

From the production side, we have that employment is determined by output and labour productivity. Thereby, the employment rate is given by

$$\sigma_N = \frac{\sum N_i}{Pop}$$

Once production have to be equal to demand plus changes in inventories, and we consider productivity and population as given, either σ_N or sectoral output have to adjust to guarantee that total supply is equal to total demand. In the case of σ_N being the adjustment variable, it is also necessary that supply and demand have to be equal in every single sector.

In the case of NR exporters, production and demand are always equal as demand is given by exports, and all production is sold, therefore

$$Y_r^P = Y_r^D = \frac{X_r}{p_r^W e} \tag{133}$$

On the other hand, for the remaining sectors, there is nothing *ex-ante* that guarantee this equality. Government consumes only non-traded goods and investment is all produced by the traded goods sectors. If one assumes that exports of traded goods are given, because imports are considered as endogenous, we need to consider that consumption distribution (σ_n^C) adjust to guarantee that demand and supply will be the same for every sector.

Here, however, we will assume that consumption distribution is given, and hence sectoral output will adjust to its demand. Sectoral total demand (which includes sales and investment in inventories) is given by the summation of final demand absorbed by domestic producers and intermediate consumption. However, final demand includes investment, which depends on output (in the balanced growth path). Therefore, one can write sectoral total demand as a function of domestic absorbed final demand (excluding investment), the domestic technical coefficient matrix and the domestic absorbed investment matrix:

$$\begin{bmatrix} Y_r^{D,T} \\ Y_n^{D,T} \\ Y_m^{D,T} \end{bmatrix} = \begin{bmatrix} Y_r^P \\ C_n + G_n + \dot{V}_n \\ C_m(1-\sigma_m^{IM}) + X_m + \dot{V}_m \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ c_r^n & c_n^n & c_m^n \\ c_r^m & c_n^m & c_m^m \end{bmatrix} \begin{bmatrix} Y_r^P \\ Y_n^P \\ Y_m^P \end{bmatrix} +$$

$$+(1-\sigma_m^{IM})\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \nu_r & \nu_n & \nu_m \end{bmatrix}\begin{bmatrix} Y_r^P \\ Y_n^P \\ Y_m^P \end{bmatrix}$$

where $\nu_i = (g+\delta)\frac{b_i}{u_i}$, with $i = r, n, m$

Defining DL as the inverse dynamic Leontief matrix, $DL = (Id - IC - D)^{-1}$, where Id is the identity matrix, IC is the technical coeffient matrix, and D is the domestic absorption investment matrix, once the system is stable only if production is equal to total demand, we have that

$$\begin{bmatrix} Y_r^P \\ Y_n^P \\ Y_m^P \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ d_r^n & d_n^n & d_m^n \\ d_r^m & d_n^m & d_m^m \end{bmatrix}\begin{bmatrix} Y_r^P \\ C_n + G_n + \dot{V}_n \\ C_m(1-\sigma_m^{IM}) + X_m + \dot{V}_m \end{bmatrix}$$

where d_i^j is the induced production of sector i by domestic final demand (excluding investment) of sector j.

Therefore, from the demand side we have that

$$Y_n^P = d_r^n Y_r^P + d_n^n(C_n + G_n + \dot{V}_n) + d_m^n[C_m(1-\sigma_m^{IM}) + X_m + \dot{V}_m]$$

and

$$Y_m^P = d_r^m Y_r^P + d_n^m(C_n + G_n + \dot{V}_n) + d_m^m[C_m(1-\sigma_m^{IM}) + X_m + \dot{V}_m]$$

Because changes in inventories is given by the difference between production and demand, we have that

$$\dot{V}_i = Y_i^P \frac{v_i^d g}{1 + v_i^d g}$$

Finally, as $X_m = \sigma_m^X Y_m^P$, and $Y_i^P = Y_i^D(1 + v_i^d g)$, sectoral demand of the other sectors are given by

$$Y_n^P = \frac{d_r^n Y_r^P + d_n^n(C_n + G_n) + d_m^n[C_m(1-\sigma_m^{IM}) + X_m + \dot{V}_m]}{1 - \frac{v_n^d g d_n^n}{1 + v_n^d g}} \quad (134)$$

and

$$Y_m^P = \frac{d_r^m Y_r^P + d_n^m(C_n + G_n + Y_n^D v_n^d g) + d_m^m C_m(1-\sigma_m^{IM})}{1 - d_m^m\left[\frac{v^d g}{1+v^d g} + \sigma_m^X(1 + v_m^d g)\right]} \quad (135)$$

Alternatively, one can define production of one sector and consider the share of consumption endogenous. In this case, we have that:

$$\sigma_n^C = \frac{d_r^n Y_r^P + d_n^n(G_n + \dot{V}_n) + d_m^n(X_m + \dot{V}_m) + d_m^n C(1-\sigma_m^{IM}) - Y_n^P}{C\left[\frac{d_n^n}{p_n} - d_m^n(1-\sigma_m^{IM})\right]} \quad (136)$$

4. Simulations

The model is applied to evaluate the impact of a decreasing trend of the world price of natural-resources. The idea behind this hypothesis is that the low-carbon transition will negatively impact the world demand and the price of some resources, such as fossil fuels, and countries that depend on these products may face macroeconomic constraints.

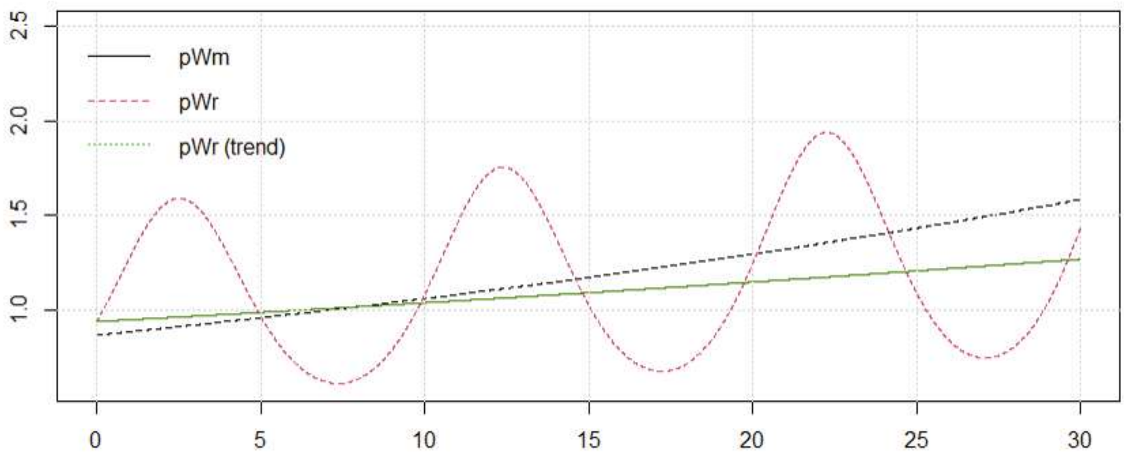
The assumption for the world price of natural resources is that it grows at a lower rate than other traded goods and services. We assume that $\hat{p_m^W} = 2\%$, while $\hat{p_r^W} = 1\%$. It implies that after 30 years, the price of the natural resources exported by the country under consideration is 34.5% lower than the price of the other traded goods and services. Even though it may be considered a not very significant price shock, it is enough to illustrate the main concerns of the analysis. Moreover, we also consider that the price of natural-resources has a cyclical component. This assumption is important both to make it clear the impacts and to force the economy that depends on these exports to have booms and boosts, amplifying both the constraints and the bonanzas.

Changes in the world price of natural resources is thus given by:

$$\dot{p_r^W} = p_r^W \left(\alpha_p + \frac{\cos \frac{2t}{3.141593}}{3.141593} - 0.01\dot{t} \right) \tag{137}$$

Figure 1 presents price dynamics. The world price of other traded goods grow at 2% per year and do not present fluctuations, whilst the price of natural resources grow on average at 1% per year (as we can see on the linear trend), but it oscillates in cycles of 10 years.

Figure 1: World prices of natural resources and other traded goods and services

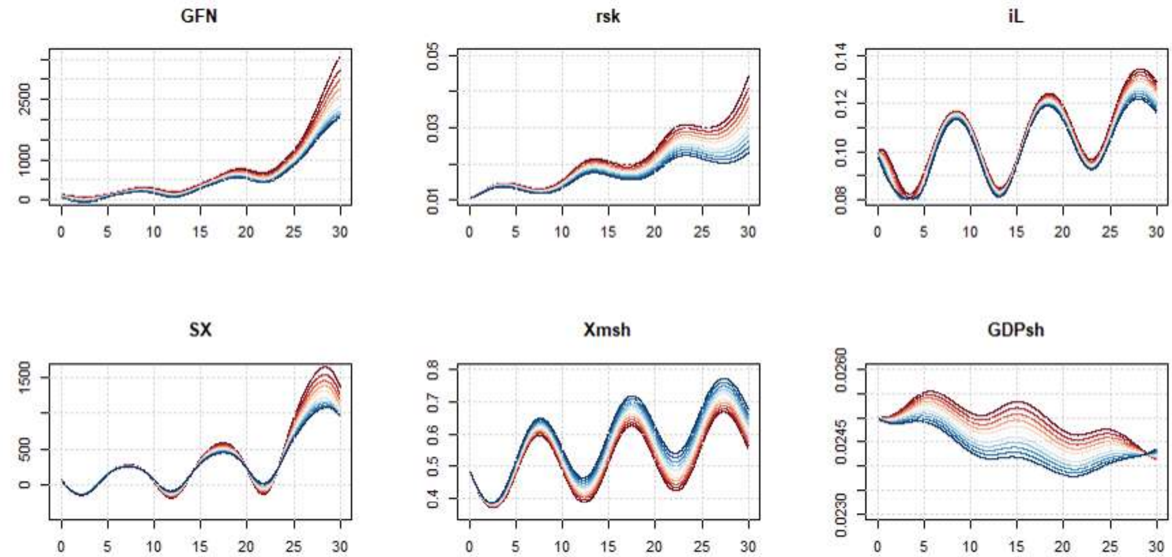


4.1. Carbon price

The drop trend of natural-resources' world price negatively impacts the economy through multiple channels. Here we will analyse two of them (the fiscal and the external channels) and investigate what happens if the government tries to mitigate this impact by taxing sales of these goods. The idea behind these simulations is that these sectors are carbon-intensive industries, and the government uses carbon price both to de-stimulate the production in these industries and to increase fiscal revenues with the aim of compensating the drop due to reduction in price.

Figure 2 presents the general dynamics of the base scenario in red and with the carbon price in blue. The dark blue line is the case with the higher carbon price (25%), which means that $t_r^Y = 0.35$, once in the base scenario, the dark red line, sales tax are the same for all sectors ($t_r^Y = t_n^Y = t_m^Y = 0.10$).

Figure 2: Fiscal and external dynamics with carbon price



The first chart in the top left presents the Government Financial Needs (GFN). In the baseline scenario, even though it decreases during the booms, it presents a growing trend, and becomes excessively high after 30 years. The drop in the price of natural resources reduces government revenues directly because the NR export sector is an important source of fiscal revenues, but also indirectly because economic growth reduces in the long run, as we can see from the chart in the bottom right (share of world GDP). This result indicates that the country may face a fiscal crisis due to the absence of alternative sources of revenue. The chart in the top middle analyses the risk of government bonds (rsk), and it is higher the higher the government debt-to-GDP ratio. Because GFN is increasing and the debt-to-GDP is also increasing, in the baseline scenario the risk also increases fast, leading to an increase in yields and, consequently, in lending interest rates (as we can see from the chart in the top right).

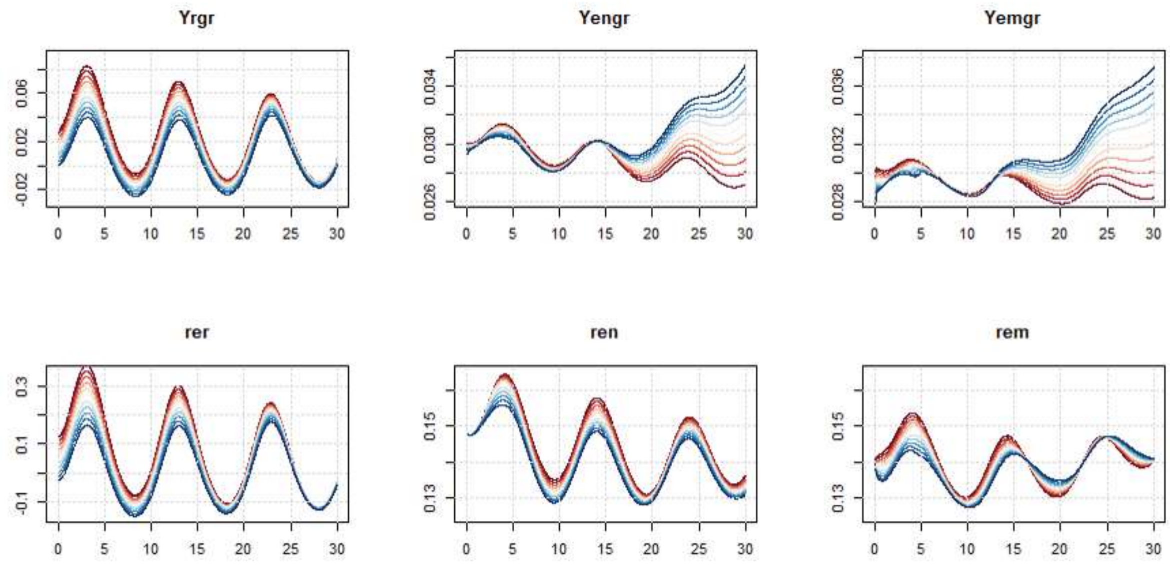
Carbon price can mitigate these negative fiscal impacts, despite its contractionary impacts in the short run (as we can see from the bottom right chart). The three charts in the top show that the country is benefiting from a fiscal point of view of an increase in tax on sales of natural resources. The government deficit (measured by the GFN) does not grow as fast as in the baseline scenario, the risk grows at a much lower pace, as well as the lending interest rate.

Even though the fiscal impact is positive, the analysis of the external dynamics shows that not necessarily the benefits of lower fiscal constraints compensate for the contractionary impacts of this policy. The chart in the bottom left presents the foreign savings (SX), which is the inverse of the current account surplus. As we can see, the carbon price reduces the volatility of the current account. The country presents a lower current account surplus in the booms, and a lower current account deficit in the boosts. The reason for that is that carbon prices de-stimulate investments in the high-carbon industry, and thus it reduces the country's dependence on these industries.

This lower dependence becomes clearer when we analyse the share of other goods exports in total exports, as presented in the chart in the bottom middle. Without the carbon price the share of the country in other goods exports oscillates inversely to the cycles, which is a result of relative price competitiveness losses in the booms (higher unit costs due to exchange rate appreciation and input costs). This is also true when the carbon price is included, but in this case, it presents a very positive trend, reducing the dependence of natural resources in the export basket. The blue line is always above the red line and their distance increases. The general impact of the carbon price is that it forces a short-term reduction in GDP, as can be seen from the chart in the bottom right, which presents the country's GDP as a share of world GDP. Nevertheless, by promoting a structural change towards other traded goods and services, in the long-run the impact is less negative.

The sectoral analysis makes this dynamics clearer. Figure 3 presents the growth rates and the expected profitability for the three sectors.

Figure 3: Sectoral dynamics with carbon price



As we can see from the top left chart, the carbon price directly affects the annual growth rate of natural resources, reducing it by about 3 percentage points in the first cycle. The expected profitability of this sector is negatively impacted both because of the drop in prices and because of the carbon price.

The impact on the other sectors is also negative in the short–term, as the economy loses a relevant income generated by the natural resource industry. The profitability of the non-traded goods sector is always lower in the case of having a carbon price. Moreover, even though exports of other goods and services are benefited from price-competitiveness gains, the net impact in the short run is negative due to reduction in income and in intermediate demand from both natural-resources and non-traded goods.

The results for the other sectors, however, show that the negative impact of the drop in natural resource exports is compensated for by the increase of other goods and services exports in the long run. The simulation shows that after five years the growth rate of other traded goods and services is about the same comparing the baseline and the high carbon price scenario, and after fifteen years the growth rate diverges, with an increasing growth rate for high carbon price, and decreasing in the baseline. After fifteen years, the non-traded goods and services also start growing faster in the scenario with carbon price than the scenario without it.

Based on these simulations one can affirm that introducing a carbon price is an important instrument both to reduce the fiscal constraints and to promote a structural change in the economy, leading it to less carbon intensive sectors. The simulations, however, show that the fiscal benefits and the gains of this structural change have relevant economic costs in the short and middle-term. Even though there is a reduction in the fiscal deficit and an increase in other traded goods exports, the impact on GDP is negative. In the simulations, the accumulated impact becomes positive only after almost 30 years, which means that high carbon price may not be a good policy tool in the case of countries with high external dependence on export of high-carbon intensive industries due to its contractionary impacts.

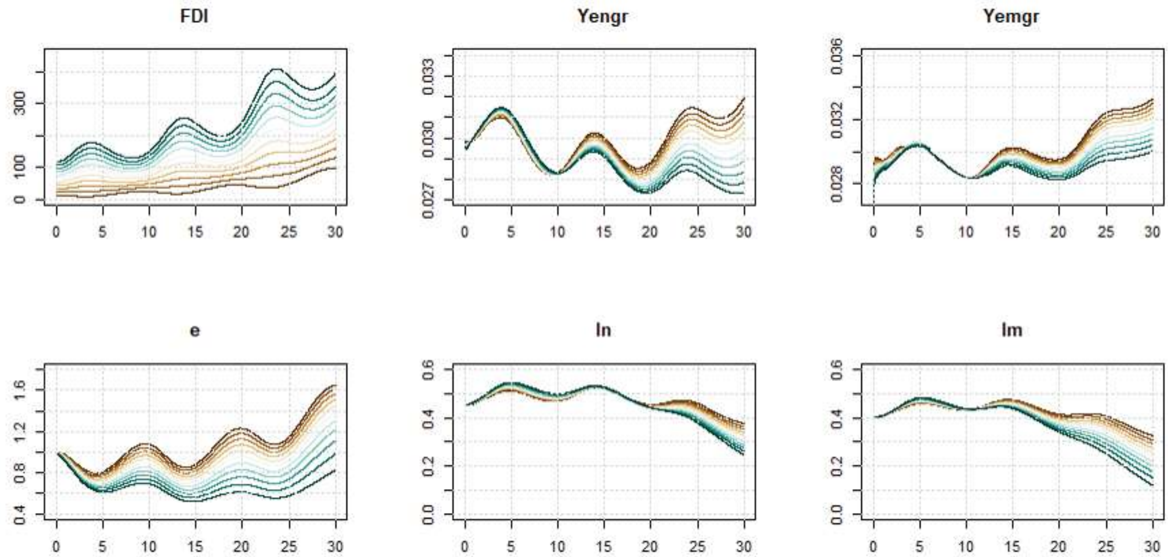
4.2. FDI attractiveness

Foreign Direct Investment (FDI) plays an important role in developing countries because lending rates are usually high. In the model developed here, FDI reduces sector financial needs, which means that firms will be able to invest more without relying on banking lending. Considering the drop in world natural resources' price, FDI becomes even more important because lending rates increase due to the increase in risk (see Figure 2). Therefore, it is important to analyse to what extent higher FDI attractiveness may promote faster growth rates.

Besides assuming the drop trend and the existence of cycles in natural resources' prices, we assume a scenario where carbon price is 12.5%, which is the average between the two extreme scenarios described before (it is shown in the lightest line in Figures 2 and 3). This is the baseline scenario, where the FDI inflows respond to profitability according to the model calibration for the balanced growth path.

We compare the baseline scenario with two other scenarios. Firstly, a scenario where all inflows of FDI are more sensitive to the profitability independently of the sector, and second, a scenario where only the FDI to other traded goods and services present higher attractiveness (measured by the sensitivity to the profitability). Figures 4 and 5 present the results for these two scenarios compared with the baseline scenario. The dark yellow line represents the baseline scenario, and the dark green represents very sensitive FDI scenarios.

Figure 4: Sectoral dynamics with high general FDI attractiveness

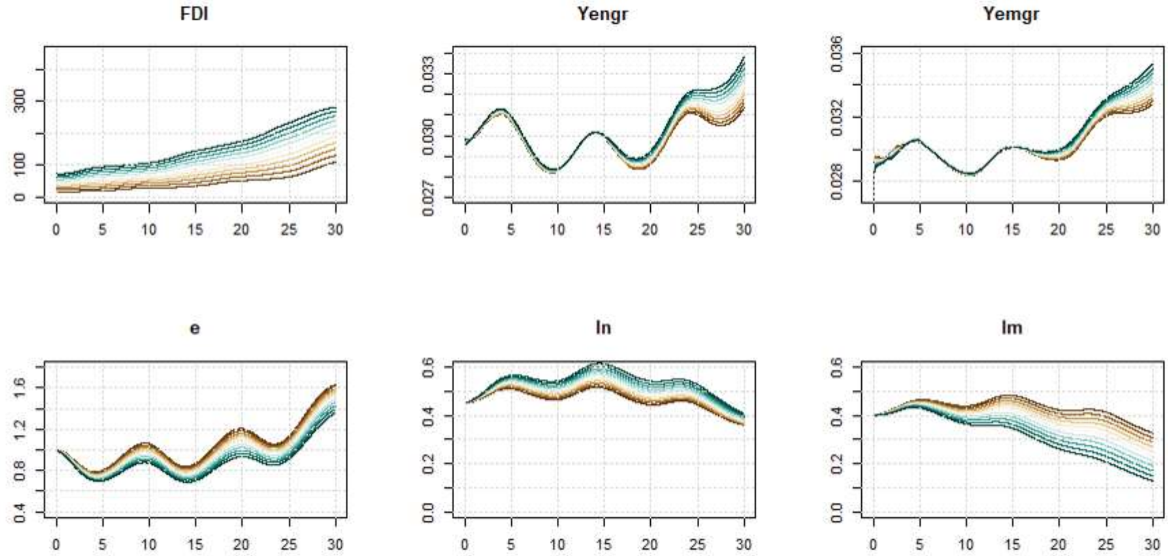


From Figure 4 we see that higher FDI attractiveness does not necessarily imply high growth rates, as we can see in the two charts in the top right, which shows the growth rate of non-traded and other traded goods and services (G&S). In the non-traded G&S sector we see a small short-term positive impact, but it is compensated for the negative impacts in the medium and long run. Even though FDI flows increase a lot among these scenarios, the impact on growth rates is not relevant in the short run because the country is not facing short-term financial constraints. In the long run, however, the impact is negative and large in both sectors (non-traded and other traded G&S).

The explanation for this result is not simple, but it is mainly led by the absence of a structural change. Higher FDI inflow reduces the constraints that lead to structural change in the baseline scenario, preventing an exchange rate devaluation, as we can see in the chart in the bottom left. In this scenario, even though financial constraints are lower as the leverage ratio is reduced due to lower financial needs, it is not enough to compensate for the negative impacts, and hence the growth rates are lower. Therefore, the benefit of having a carbon price in the long run (the stimulus to move toward the other traded G&S sector) is not effective, as the other traded G&S sector exports do not replace the natural resource sector as the driver of the economy.

This result becomes even more clear when we analyse what happens if only the FDI inflows to the other G&S sector is more sensitive, as presented in Figure 5.

Figure 5: Sectoral dynamics with high FDI attractiveness in other traded G&S



In this case, the financial constraints due to the low-carbon transition (higher lending rate) impact relatively less the other traded G&S than the other sectors, facilitating the process of structural change toward this industry. Because the volume of FDI is much lower than before, it does not prevent an exchange rate devaluation, and hence other traded G&S exports are boosted, pushing the non-traded sector and hence the economy as a whole benefit from that.

As we can see from the two charts in the bottom right, only the other traded G&S presents a decline in leverage ratio (the non-traded G&S presents an increase). Because of this, this sector grows faster in spite of the lower depreciation of the exchange rate. By producing and exporting more, the negative impacts of the low-carbon transition is reduced because the country becomes less dependent on carbon-intensive industries.

These results are important because they show how complementary policies are relevant to mitigate the negative impacts of the low-carbon transition in countries that depend on high-carbon intensive industries. These economies need to promote a structural change in the economy moving from these sunset industries to low-carbon intensive sectors, but they may face fiscal, external and financial constraints in the transition process. Therefore, carbon price policies alone are not necessarily effective as they can damage growth and constrain the transition process, demanding complementary policies that reduce these constraints for less carbon intensive industries.

5. Concluding remarks

This paper develops a prototype Structural Stock and Flow Consistent model (Structural-SFC) for natural-resource exporter countries. Because these economies present a heterogeneous productive structure, and the sectors are different in many dimensions, such as price determination, productivity and investment dynamics, a one-sector model or a multi-sectoral model where sectors are structurally the same may be misleading when analysing macroeconomic factors.

With the aim of understating the impacts of a low-carbon transition in these economies, with special regard to those that the export basket relies excessively on sunset industries, we assume that the world price of the product sold by these industries present a downward trend relative to the other products. We then analyse the effectiveness of a carbon price in the short and long run, as well as the importance of FDI attractiveness to reduce firms' financial constraints.

The main results of the analysis are that a high carbon price in these economies may be not very effective, but complementary measures to reduce financial constraints in low-carbon sectors can compensate for it. Because these economies are excessively dependent on high-emission intensive industries, and the financial and external constraints are not compensated for the lower fiscal constraints, high carbon prices alone can constrain the growth rate of these countries. Carbon price is a recessive measure, and these economies are already facing a drop on foreign revenues (to to the price drop of natural resources). Therefore, high carbon prices can be even more damaging for them in the short run – even though in the long run it may have positive impacts as the economy becomes less dependent on high-emission industries.

Nevertheless, complementary measures, such as increasing FDI attractiveness in low-carbon emission industries that produces tradable goods and services, can compensate for these negative impacts in the short and middle-term by reducing the financial constraints that emerge during the transition process. While the carbon price is effective in the long run, other policy tools can be implemented to accelerate the process of structural transition without relying on structural changes driven by balance-of-payment constraints and exchange rate depreciation.

Because the model is built upon a theoretical calibration, not representing any specific country, the simulations are only explanatory. However, sensitivity analysis may show that results can be generalised for many countries with similar structure of production. Moreover, the model is a prototype version and because it is relatively simple (although it has hundreds of equations most of them are accounting equations) it can be easily applied for specific cases.

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