Wage inequality and induced innovation in a classical-Marxian growth model

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Abstract The present paper works out a classical-Marxian growth model with heterogeneous labour force and endogenous technical change. It draws on the Kaleckian mark-up pricing to link wage inequality to the relative unit labour cost; on growth cycle models *à la* Goodwin to formalize the dynamic interaction between labour market and income shares; on the induced innovation literature to link the bias of technical change to the firm's microeconomic choices. We argue that both wage inequality and the direction of technical change are determined by the institutional variables affecting capitalists' bargaining strength and workers' differential bargaining power. A decline in low-skilled workers' bargaining strength or a rise in capitalists' bargaining power lead to both an increase in wage inequality and a bias of technical change favouring high-skilled over low-skilled labour productivity growth. As opposed to the Goodwin model with induced technical change, labour market institutions may thus affect steady-state income distribution, capital accumulation and labour productivity growth. Finally, if the steady-state value of wage inequality exceeds a critical value, an exogenous increase in the mark-up or in the high-skilled workers' bargaining power allow both capitalists and high-skilled workers to increase their income shares at the expense of the low-skilled workers.

Keywords Wage inequality, growth, distribution, endogenous technical change

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1. INTRODUCTION

Over the past decades, the US and the major European economies experienced a sharp rise in personal income inequality. This was the result of a decline in the labour share in income, an increase in the income share accruing to the top 1% of income recipients, and an increase in personal income inequality within the bottom 99% of the income spectrum. The rise in inequality at the very top of income distribution reflects the growth of executive compensation, the expansion of the financial sector, and the income redistribution from wages to profits. Outside the top 1%, the increase in personal income inequality is a result of rising wage dispersion between high-wage and low-wage earners (Piketty and Saez, 2003; Piketty, 2014; Mishel and Bivens, 2021).

The neoclassical approach to skill-biased technical change and human capital explains the trend of intra-working-class income distribution through the lenses of relative factor scarcity. According to this interpretation, the distribution of wages is shaped by the interaction between relative demand and relative supply of skills. Since, in the absence of exogenous shifts in the relative demand, the large increase in the relative supply of high-skilled labour that occurred over the last decades would have reduced the skill premium, neoclassical authors deduce that technical change must have been skill-biased.

The standard explanation for skill-biased technical change invokes the concept of "capital-skill complementarity". As capital is supposed to be more complementary to high-skilled labour than to low-skilled labour, the decline in the price of capital goods due to innovations in information and communication technologies would have caused firms to adopt more capital-intensive technologies and to substitute away from low-skilled labour. The implication is that an increase in the capital stock would lead to a constant rightward shift in the relative demand for high-skilled labour. Wage inequality between high-skilled and low-skilled workers would then rise, unless the increase in the supply of human capital keeps up with the pace of skill-biased technical change (Tinbergen, 1975; Katz and Autor, 1999; Krusell, *et al.*, 2000; Card and Lemieux, 2001; Goldin and Katz, 2008). Therefore, in a perfectly-competitive framework, the only role for policy is to make the relative endowment of high-skilled labour be "less scarce", namely to implement educational policies aiming to upgrade workers' skills (Goldin and Katz, 2008; Acemoglu and Autor, 2012). Besides reducing wage inequality, investment in human capital is supposed to be a central determinant of economic growth (Uzawa, 1965; Lucas, 1988; Acemoglu, 2009; Aghion and Howitt, 2010). In a non-perfectively competitive framework, other institutional factors like the decline in unionization and the decentralization of collective bargaining are argued to interact with skill-biased technical change in determining labour market outcomes. However, the role of labour market institution is explicitly neglected as a direct cause of changes in wage inequality. By altering the effectiveness of union activity or the internal organization of firms, labour market institutions may at most amplify the direct effect of skill-biased technical change on income distribution (Lindbeck and Snower, 1996; Acemoglu, *et al.*, 2001; Acemoglu, 2002b; Hornstein, *et al.*, 2005; Ortigueira, 2013).[1]

From a classical-Marxian standpoint, the conventional debate on wage inequality is largely unsatisfactory, because it ignores the role of the conflict over income distribution among social classes in determining the direction of technical change. In this view, induced technical change is

[1] Even changes in wage dispersion among workers with the same educational level are ultimately ascribed to a purely technological process, with no room for labour market institutions, as residual inequality is supposed to reflect returns to unobserved individual abilities (Nelson and Phelps, 1966; Card and Lemieux, 1996; Acemoglu, 2002b; Violante, 2002; Lemieux, 2006). Some neoclassical authors acknowledge that institutional factors like unionization, the degree of centralization of wage bargaining, and employment protection legislation affect labour market outcomes (see, for instance, Koeniger, *et al.*, 2007; Checchi and García-Peñalosa, 2008), but they don't investigate the role of labour market institutions jointly with skill-biased technical change.

regarded as a weapon of capitalists in the class conflict for breaking the bargaining power of the working class. An increase in unit labour cost stimulates labour-saving innovations, since replacing workers with machines allow capitalists to regenerate the reserve army of labour and restore profitability. Thus, labour productivity growth is an increasing function of the labour share, namely the counterpart of unit labour cost at a macro level (Marx, 1976; Foley and Michl, 1999; Brugger and Gehrke, 2018; Tavani and Zamparelli, 2018).

The present paper extends the classical-Marxian approach to induced innovation to the case of a heterogenous labour force, made up of high-skilled and low-skilled workers. It works out a classical growth model of a high-skilled-labour-constrained economy, based on a Kaleckian mark-up pricing, on a Goodwin-type interaction between labour market dynamics and income distribution, and on the induced innovation literature as a way to formalize endogenous cost-driven technical change. Since the direction of technical change is determined by the shares of high-skilled labour and low-skilled labour in total costs, high-skilled labour productivity growth turns out to be an increasing function of the high-skilled labour share in income, and low-skilled labour productivity growth becomes an increasing function of the low-skilled labour share.

In contrast to the conventional wisdom, we show that both wage inequality and the direction of technical change are jointly determined by the institutional and political factors affecting the conflict over income distribution between capitalists and (heterogeneous) workers. A fall in the low-skilled workers' bargaining strength or a shift in the balance of power from the working class to capitalists are conducive to both an increase in wage inequality and an induced bias of technical change favouring high-skilled over low-skilled labour productivity growth. Thus, labour market institutions are found to play a relevant role in both the search for new techniques and the shape of long-run income distribution. The causality direction among technology, institutions and wage inequality predicted by the neoclassical authors is reversed. In a standard neoclassical framework, the distribution of wages is shaped by technological factors, and labour market institutions only act as a mediating factor between skill-biased technical change and wage inequality. Conversely, in the proposed framework, institutional factors related to labour market regulation affect wage inequality both directly, by altering the relative bargaining positions of high-skilled and low-skilled workers in the labour market, and indirectly, by inducing different rates of high-skilled- and low-skilled-labour-augmenting technologies. As the different growth rates of high-skilled and low-skilled labour productivity are totally passed through to real wages at the steady state, it is technical change that acts as a mediating factor between labour market institutions and income distribution. Thus, changes in the institutional framework governing the distributive conflict are the primary cause of changes in wage inequality.

We show that, in the presence of a high level of wage inequality, an increase in the bargaining power of the high-skilled workers, as compared to the low-skilled workers, or an increase in the capitalists' bargaining strength allow both capitalists and high-skilled workers to raise their income shares at the expense of the low-skilled workers. Moreover, in contrast to both the conventional wisdom and the Goodwin model with induced technical change and homogenous labour force, no necessary trade-offs arise between labour market regulation and employment in the long run. An increase in the bargaining power of a fraction of the working class needs not imply employment losses, particularly in the presence of a high level of wage inequality.

The remainder of this paper is organized as follows. Section 2 provides an extensive discussion of the related literature and the main contributions of this paper. Section 3 proposes a theoretical model and derives the basic equations for the analysis. Section 4 discusses the characteristics of the dynamical system and the steady state. Section 5 derives the necessary and sufficient conditions for the local stability of the equilibrium. Section 6 details the main results of comparative statics analysis. Section 7 then concludes.

2. RELATED LITERATURE

The present paper relates to different strands of literature.

First, the goods market is formalized along classical-Marxian lines. A distinctive feature of this approach is the close connection between capital accumulation and the conflict over income distribution among social classes. Capitalists own the capital stock of the economy and are supposed to have a larger propensity to save than workers, as a large fraction of profits is retained for investment purposes. Class-based saving behaviour implies that changes in functional income distribution affect capital accumulation. In its simplest formulation, i.e. the case of an exogenous real wage rate and constant technical coefficients of production, a classical-Marxian growth model consists of a system of four equations in four variables: (i) an inverse relationship between profit rate and real wage rate, for given labour productivity and output-capital ratio; (ii) an inverse relationship between capital accumulation and consumption per employed worker; (iii) a positive relationship between capital accumulation and profit rate, for a given propensity to save; (iv) a distributional closure stating that the real wage rate is set at the (socially and historically determined) subsistence level. The profit rate, that is, what is left after workers are paid their subsistence real wage, determines the equilibrium capital accumulation and hence the equilibrium consumption per worker. Thus, the validity of Say's law in its classical version is assumed: all savings are invested to increase the capital stock of the economy, so that no problems of lack of effective demand arise in the long run.[2] However, the assumption of a fixed-coefficients production function implies that even in a one-sector economy there is no spontaneous tendency towards full employment of labour (Kurz and Salvadori, 1995; 2003; Blecker and Setterfield, 2019).[3]

In this paper, I modify the basic classical-Marxian model in order to include endogenous technical change, a Kaleckian mark-up pricing, and heterogeneity in skill levels across workers. The model economy includes three distinct social classes with different saving behaviour: capitalists, high-skilled workers and low-skilled workers. The high-skilled labour real wage rate is supposed to be greater than the low-skilled labour real wage rate, since the acquisition of skills allow high-skilled workers to have both higher productivity and stronger bargaining power than low-skilled workers. Only the growth rate of the low-skilled workers' nominal wage is fully exogenous, whereas the growth rate of the high-skilled workers' nominal wage is made to depend on the high-skilled employment rate. The assumption that firms set the price by charging a mark-up over unit labour cost implies that the income shares of the three classes in the economy are anchored to the mark-up and to the relative unit labour cost. Thus, the Kaleckian mark-up pricing provides a link between the micro level of the firm's price-setting decisions and the macro level of income distribution. A rise in the mark-up affects functional income distribution, since it implies income redistribution from wages to profits, whereas an increase in the relative unit labour cost only worsens the intra-working-class

[2] However, the relation between the role of effective demand and the classical-Marxian approach to economic growth is more problematic than it appears in the simplified theoretical framework presented here. It has been argued that if one allows aggregate demand to affect the short-run equilibrium of an economy, it is only in special cases that the long-run equilibrium can be taken as totally independent of effective demand in classical growth models (Dutt, 2011).

[3] Post-Keynesian growth models reverse the causality direction between profit rate and capital accumulation predicted by the classical authors. With an independent investment function, the profit rate is determined by capital accumulation, as well as by the exogenous propensity to save out of profits. In the Kaleckian approach, that was adopted to formalize the short-run equilibrium of the economy in Chapter 1, the assumption of an endogenous rate of capacity utilization allows the profit rate and the profit share to move in opposite directions. Thus, Kaleckian models can generate different demand and growth regimes, whereas classical-Marxian models only allow for a "profit-led" growth regime. This implies that the classical-Marxian approach and the post-Keynesian theories of distribution and growth can be considered as different model closures of the same class-based framework for analyzing the issue of economic growth. For a survey of the heterodox models in this sense, see Hein (2017) or Dutt (2018).

distribution of wages. The endogenization of technical change gives back a steady-state growth path characterized by high-skilled- and low-skilled-labour-augmenting technical change, along with a constant output-capital ratio.

The classical-Marxian tradition has investigated the implications of two alternative distributional closures: (i) a closure with exogenous income distribution, in which an infinitely elastic labour supply always accommodates labour demand at a constant real wage rate or a constant wage share; (ii) a closure with endogenous income distribution, in which the distributive variable adjusts so as to maintain a constant employment rate in the long run. Closure (i) is considered a realistic assumption for both a dual economy in the sense of Lewis (1954), in which a rising industrial sector can always draw workers from the substantially unlimited reserve of labour of the rural sector, and a mature economy with a loose immigration policy, in which the traditional sector has depleted its pool of labour but foreign labour inflows preserve the economy from labour scarcity. Closure (ii) is typically adopted for a mature industrialized economy in which foreign labour inflows are not available to accommodate labour demand, and capital accumulation responds to signals from both goods and labour markets. This closure corresponds to the case of a labour-constrained economy, where the supply side imposes a binding constraint to the demand side of the labour market, and economic growth is constrained by the growth rate of effective labour supply (Skott, 2010; Foley and Michl, 1999; Tavani and Zamparelli, 2016; 2018).[4]

In this paper, I adopt a slightly modified version of both closures (i) and (ii). The growth rate of the low-skilled workers' nominal wage is supposed to be exogenously determined by the institutional and political factors that affect the low-skilled workers' bargaining strength. Thus, low-skilled labour supply is perfectly elastic and always accommodates low-skilled labour demand, irrespective of the real wage rate or the income share that low-skilled workers are able to attain in the distributive conflict. Conversely, the mechanism of wage formation of the high-skilled workers is assumed to be described by a nominal Phillips curve, that relates the growth rate of the high-skilled workers' nominal wage to the high-skilled employment rate. A constant high-skilled employment rate in the long run implies that the economy is high-skilled-labour constrained: capital accumulation is constrained by the growth rate of the high-skilled effective labour supply. The rationale for this assumption is that the acquisition of high skills involves some costly activity, either for the individual or for the government, that does not make high-skilled labour supply be immediately available to accommodate high-skilled labour demand. Thus, the growth rate of high-skilled labour supply imposes a binding constraint to the demand side of the labour market. Conversely, individuals can always acquire low skills costlessly, so that not even an advanced economy faces a supply-side constraint in the low segment of the labour market. However, in contrast to closure (ii) with constant technical coefficients of production, and like all classical growth models with induced technical change, it is the output-capital ratio, rather than the distributive variable, that adjusts so as to keep the employment rate constant in the long run.

Second, this paper relates to the induced innovation literature. The core idea of the induced innovation theory is that technical change is cost-driven, that is, the direction of technical change is determined by the relative size of the labour and capital shares in total costs. An increase in unit labour cost is then supposed to foster labour productivity growth.

Neoclassical authors have interpreted the concept of induced innovation as technical change being driven by relative factor endowments (Brugger and Gehrke, 2017). This interpretation dates back to Hicks's (1932) claim that a change in relative input prices stimulates innovations that use more of the factor that has become relatively more scarce. An increase in the capital stock of the economy, by raising the wage-interest ratio, would then induce a labour-saving direction of technical change. The

[4] This does not imply that the economy will achieve full employment of labour, but only that the economy will grow at the full-employment growth rate, i.e. the growth rate compatible with a constant employment rate in the long run.

concept of induced technical change has been formalized by Kennedy (1964) and Samuelson (1965) by means of a decreasing and concave "innovation possibility frontier", that represents the set of feasible combinations of factor-augmenting technologies. A profit-maximizing firm will choose the direction of technical change so as to maximize the rate of unit cost reduction given the constraint of the innovation possibility frontier. Thus, labour productivity growth turns out to be an increasing function of the wage share. However, the induced innovation theory has been proved inconsistent with the neoclassical approach to factor-pricing in a perfectly competitive framework.[5] If all factors are paid to their marginal productivities, a change in relative factor prices will not induce a particular direction of technical change (Salter, 1960).

Within the classical-Marxian tradition, induced technical change is regarded as an instrument in the hands of capitalists in the class conflict. By replacing workers with machines, capitalists actively search for innovations that allow them to reduce the bargaining power of the working class or a fraction of it (Brugger and Gehrke, 2018). As capital and labour are not treated as symmetric productive factors, the induced innovation theory is not affected by the conceptual criticisms raised by neoclassical authors. Moreover, a microfoundation of technical change based on the innovation possibility frontier has been widely adopted by the classical-Marxian literature, as the maximization of the rate of unit cost reduction is equivalent to the maximization of the rate of change in the profit rate, and hence is consistent with the Okishio (1961) rule for viable innovations of the classical analysis of the choice of techniques (Shah and Desai, 1981; van der Ploeg, 1987; Foley, 2003; Julius, 2005; Rada, 2012; Tavani, 2012; 2013; Zamparelli, 2015).[6] If the induced innovation hypothesis is integrated into a balanced growth model, income distribution is determined only by the shape of the innovation possibility frontier, and income shares adjust in order to ensure a Harrod-neutral direction of technical change in the long run (Foley and Michl, 1999; Tavani and Zamparelli, 2018).

In this paper, I modify the standard innovation possibility frontier in order to allow firms to choose among high-skilled-labour-, low-skilled-labour- and capital-augmenting technologies. The solution of the firm's maximization problem implies that a fall in the mark-up positively affects both high-skilled- and low-skilled-labour-saving innovations, while an increase in wage inequality favours high-skilled- over low-skilled-labour-saving technologies. In contrast to the basic classical growth model with induced innovation, the long-run value of the distributive variable is not determined only by the dynamic equation of the output-capital ratio, and both the distributive variable and the (high-skilled) employment rate adjust in order to stabilize the output-capital ratio in the long run. Moreover, the steady-state growth path is characterized by both high-skilled- and low-skilled-labour-augmenting technical change.

Third, this paper relates to the Goodwin (1967) model of growth cycle. As is well known, this model provides a formalization of Marx's account of the class conflict over income shares, based on the Lotka-Volterra equations for predator-prey population dynamics. It consists of two dynamic equations for the labour share and the employment rate, which describe closed orbits around the

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5 The more recent literature on directed technical change can be considered as a neoclassical attempt to overcome these criticisms, by combining an endogenous direction of technical change with production of capital goods under monopolistic competition (Acemoglu, 2003; 2015). According to this literature, the direction of technical change responds to the profitability incentives of capital goods producers. As intermediate and final goods producers use both capital and (high-skilled and low-skilled) labour inputs, the decision of a profit-maximizing firm producing capital goods will be affected by the relative price and the relative endowment of high-skilled labour in the economy. The implication is that the development of high-skilled labour complementary technologies is induced by the rising supply of high-skilled labour itself. Thus, when the directed technical change approach is applied to skill-biased technical change, neoclassical authors conclude that, in contrast to the induced innovation hypothesis, technical change will be biased towards the relatively more abundant factor (Acemoglu, 2002a; 2002b).

6 Duménil and Lévy (1995; 2003) frame the firm's choice of techniques in a stochastic set-up. Other authors simply postulate a positive dependence of labour productivity growth on the wage share at a macro level, and the output-capital ratio is assumed to be constant even along the transition path. See, for instance, Dutt (2013).

equilibrium values of the two variables. The non-trivial equilibrium solution corresponds to the long-run values of income distribution and employment in a classical-Marxian model of a labour-constrained economy with exogenous labour productivity growth and exogenous labour supply growth. Thus, the Goodwin model can be interpreted as a description of the short-run cyclical dynamics of the wage share and the employment rate around a long-run trend which is mainly the product of structural and institutional changes (Veneziani and Mohun, 2006; Mohun and Veneziani, 2008; Fiorio, *et al.*, 2013).

The original model has been extended in many directions. Most notably, some authors have explored the dynamic and steady-state implications for the growth cycle of the introduction of the induced innovation hypothesis. As emphasized by Shah and Desai (1981) and van der Ploeg (1987), induced technical change gives capitalists an additional weapon in the class conflict, other than reducing investment, to regenerate the reserve army of labour and restore profitability, thus making the equilibrium be locally stable. As a result, the labour share and the employment rate converge towards the steady state with oscillations of decreasing amplitude (Foley, 2003). The integration of the induced innovation hypothesis into the Goodwin model implies that the steady-state employment rate adjusts to the level consistent with the labour share determined by the shape of the innovation possibility frontier at the intercept. Therefore, as Tavani and Zamparelli (2015) and Zamparelli (2015) point out, by introducing an explicit policy variable into the model, it can be shown that an increase in workers' bargaining strength only reduces the employment rate, while leaving income distribution, capital accumulation and labour productivity growth unaffected.

Some recent contributions have explored the channels through which labour market institutions may affect long-run income distribution, employment, capital accumulation, and labour productivity growth in the Goodwin model of growth cycle with induced technical change. Julius (2005) finds that if labour market institutions allow for a partial pass-through of labour productivity growth to the real wage, and the wage-setting process is internalized by the firm in its choice of techniques, the workers' bargaining power has a positive effect on the long-run wage share. Tavani (2012; 2013) find that if the wage-bargaining process takes the form of a strategic interaction *à la* Nash, labour market institutions have a positive effect on labour productivity growth, but no effect on steady-state income distribution and a negative effect on long-run employment. Tavani and Zamparelli (2015) show that if firms face a trade-off between investing in capital accumulation and investing in R&D expenditure for labour-saving innovations, and the position of the innovation possibility frontier is endogenous to the amount of R&D investment, an increase in workers' bargaining strength leads to an increase in the long-run wage share at the expense of employment and capital accumulation.

In this paper, I address the issue of the steady-state effects of labour market institutions in an economy with a heterogenous labour force and a Goodwin-like dynamic interaction between labour market and income shares. The model includes two distributive variables: the mark-up, representing functional income distribution, and the relative unit labour cost, which is equivalent to the ratio of the high-skilled labour share to the low-skilled labour share. As the economy is high-skilled-labour constrained, the antagonistic relationship between capital and labour over income distribution is formalized by assuming a negative response of profitability to the high-skilled employment rate and a nominal Phillips curve for the wage formation of the high-skilled workers. The negative response of the mark-up to the high-skilled employment rate is internalized by the firm in its choice of techniques. The dynamics of wage inequality is affected by the growth rates of nominal wages of high-skilled and low-skilled workers. Thus, the dynamic equation for the wage share of the standard Goodwin model is replaced by a dynamic equation for wage inequality, whereas a dynamic equation for employment is defined only for the high-skilled workers. As in Nishi (2020), the gap between nominal wage growth and labour productivity growth is reflected in the inflation rate. I find that, in contrast to the standard Goodwin model with induced innovation, labour market institutions affect steady-state income distribution, capital accumulation and labour productivity growth. Moreover, an increase in the bargaining strength of a fraction of the working class needs not imply employment losses, particularly in the presence of a high level of wage inequality.

Finally, this paper relates to the more recent non-neoclassical research agenda on the interaction between wage inequality, personal income distribution and economic growth. The basic classical-Marxian and post-Keynesian models of distribution and growth have been extended in three directions. First, some authors have developed two-class models in which workers are allowed to own a share of the capital stock of the economy (Dutt, 2017; Palley, 2017a; 2017b), the propensity to save out of wages is affected by wage inequality (Carvalho and Rezai, 2015; Prante, 2018; Hein and Prante, 2020), or workers' saving behaviour is determined by relative consumption concerns (Kapeller and Schütz, 2014; 2015). These contributions show that even a profit-led economy may be equality-led, as the reduction in wage inequality or an increase in the workers' share of capital stock have an expansionary effect on output growth, and that the demand regime of an economy is endogenous to the level of wage or wealth inequality. Second, some authors have proposed three-class models that incorporate a middle class, in the form of managers or supervisory workers, that is located in between capitalists and ordinary workers (Lavoie, 2009; Tavani and Vasudevan, 2014; Palley, 2015a; 2015b). Building on the Marxian distinction between productive and unproductive labour,[7] managers are assumed to be rewarded according to their capability to extract surplus from ordinary workers. Thus, these models describe the additional dimensions of the conflict over income distribution in advanced economies, while maintaining the wage-profit divide as the main distinctive feature of the capitalist class structure. Third, some contributions, that are the closest references of this paper, have explicitly integrated the concept of "human capital" into Kaleckian and classical-Marxian growth models, often splitting the working class into high-skilled and low-skilled workers. Carvalho, *et al.* (2019), and Lima, *et al.* (2019), show that if growth is demand-led, the economy typically operates with excess "knowledge capacity". Then, the presence of overeducation in the labour market is the result of a lack of effective demand, and it cannot be ascribed to an occupational mismatch between demand and supply of skills. Dutt (2010), and Dutt and Veneziani (2011; 2019; 2020) find that, in contrast to the conventional wisdom, educational policies may have an expansionary effect on output growth only by altering income distribution among social classes, rather than by spurring technical change. Moreover, the effect on wage inequality is dependent on the qualitative properties of the education system: a regressive education may weaken intra-working-class solidarity and socialize workers into legitimate income inequality.

To the best of my knowledge, none of the recent contributions in classical-Marxian and post-Keynesian traditions expressly addresses the issue of the joint determination of wage inequality and direction of technical change. In Dutt (2010), and Dutt and Veneziani (2011; 2019; 2020), the productivity gap between high-skilled and low-skilled labour is exogenous, hence skill-biased technical change is ruled out by assumption. In Lima, *et al.* (2019), labour productivity growth is endogenous and linked to the government expenditure on education, but human capital is uniformly distributed across workers. Neto and Ribeiro (2019) develop a Kaleckian model of the process of technological catching-up in developing economies with skill-biased technical change, but limit themselves to assume that technical change raises productivity gaps. Other authors do not include technical change at all. The main contribution of this paper is to fill this gap in the literature, and to link the recent research agenda on wage inequality and personal income distribution to the induced innovation literature. Thus, while the line of research on human capital investigates the determinants and the macroeconomic effects of the supply of skills, this paper addresses the issue from the demand side, namely how demand for skills is affected by technical change and labour market institutions.

[7] On the notion of "unproductive labour" in the Marxian literature and for an empirical application to the US economy, see Mohun (2014) and Duménil and Lévy (2015).

3. THE STRUCTURE OF THE MODEL

3.1. PRODUCTION, INCOME DISTRIBUTION, AND EMPLOYMENT

Consider a closed economy with no government, in which only one good is produced with three inputs, low-skilled labour, high-skilled labour and a non-depreciating capital. There are three social classes: capitalists, who own the economy's capital stock and receive profits; high-skilled workers, that inelastically supply one unit of high-skilled labour in each period and receive a high-skilled wage; low-skilled workers, that inelastically supply one unit of low-skilled labour in each period and receive a low-skilled wage. The relation between inputs and the homogenous output is represented by a Leontief production function:

$$Y = \min\{a_L L, a_H H, a_K K\} \tag{1}$$

where Y denotes actual output in real terms; L, low-skilled labour employed in production; H, high-skilled labour; K, capital; $a_L = Y/L$, low-skilled labour productivity; $a_H = Y/H$, high-skilled labour productivity, with $a_H > a_L$; and $a_K = Y/K$, the output-capital ratio. The assumption of a fixed-coefficients production function implies that demands for low-skilled labour, high-skilled labour and capital are inelastic to input prices, and one or more inputs may not be fully employed.

Denoting the low-skilled nominal wage by w_L, the high-skilled nominal wage by w_H, the profit rate on capital stock by $r = \pi a_K$, where π is the profit share, the price by p, national income in real terms is given by:

$$Y = \frac{w_L}{p} L + \frac{w_H}{p} H + rK \tag{2}$$

We assume that $w_H > w_L$, which is consistent with the evidence that the higher labour productivity of high-skilled workers translates into a higher nominal wage.

National income accrues to the three social classes in the economy. We assume that capitalists save all their income, high-skilled workers have propensity to save $s \in (0,1)$, low-skilled workers devote all their income to consumption. In line with the classical tradition, capitalists have a higher propensity to save than workers, as the functional nature of profits implies that a large fraction of profits is retained for investment purposes. High-skilled workers have a higher propensity to save than low-skilled workers, consistently with the absolute income hypothesis that the propensity to save of high income individuals exceeds the propensity to save of low income earners (Keynes, 1936).

Firms set the price by charging a fixed mark-up (μ) over unit labour cost:

$$p = \left(\frac{w_H}{a_H} + \frac{w_L}{a_L}\right)(1 + \mu) = \frac{w_L}{a_L}(1 + z)(1 + \mu) \tag{3}$$

where the relative unit labour cost z is defined by:

$$z \equiv \frac{w_H a_L}{w_L a_H} \tag{4}$$

The mark-up pricing allows anchoring the macro level of income distribution to the micro level of the firm's price-setting decisions. From equations (2) and (3), we can define income distribution as follows:

$$\omega_H = \frac{w_H}{pa_H} = \frac{z}{(1+\mu)(1+z)} \tag{5}$$

$$\omega_L = \frac{w_L}{pa_L} = \frac{1}{(1+\mu)(1+z)} \tag{6}$$

$$\pi = 1 - \frac{w_L}{pa_L} - \frac{w_H}{pa_H} = \frac{\mu}{1+\mu} \tag{7}$$

where ω_H, ω_L and π are the income shares of high-skilled workers, low-skilled workers and capitalists, respectively.

The mark-up μ and the relative unit labour cost z are the key distributive variables of the model. From equation (4), or equations (5) and (6), we have $z = \omega_H/\omega_L$. Thus, an increase in μ only affects functional income distribution, as it implies an increase in the profit share, while leaving wage inequality unaltered. An increase in z only affects the distribution of wages, as it implies an increase in the high-skilled workers' wage share at the expense of the low-skilled workers, while leaving functional distribution unchanged.

Along classical-Marxian lines, we assume that savings are identically equal to investment. Thus, the growth rate of the capital stock is identically equal to the ratio of savings to the capital stock:

$$g \equiv \frac{I}{K} \equiv \frac{S}{K} = \frac{1}{1+\mu}\left(\mu + \frac{sz}{1+z}\right)a_K \tag{8}$$

The actual output is then assumed to be at its potential level, determined by the full utilization of the productive capacity of the economy. However, the assumption of a fixed-coefficients production function (equation (1)) still allows for unemployment of high-skilled and low-skilled labour.

At each point in time, high-skilled labour supply is $N = N_0 e^{nt}$, where N_0 denotes the initial value of high-skilled labour supply and $n > 0$ denotes the exogenous growth rate of N. Low-skilled labour supply is supposed to be infinitely elastic. The rationale for these assumptions is that the acquisition of a high level of skills requires some costs, either for the individual or for the government, so that the growth rate of high-skilled labour supply imposes a constraint to the growth rate of high-skilled labour demand. Conversely, an individual can always acquire the minimum level of skills required by the labour market with no costs, so that an economy with a pool of unemployed workers does not face a supply-side constraint in the low segment of the labour market.

From equation (1), the high-skilled employment rate e is given by:

$$e = \frac{a_K K}{a_H N} \tag{9}$$

The growth rate of the high-skilled workers' nominal wage is assumed to be an increasing function of the high-skilled employment rate e and an exogenous variable α:[8]

$$\hat{w}_H = h(e, \alpha), \quad h'_e > 0, \; h'_\alpha > 0 \tag{10}$$

Equation (10) formalizes the Marxian profit-squeeze mechanism with a nominal Phillips curve, as in Desai (1973), limited to the high-skilled workers. We interpret the exogenous variable α in a broad sense as a parameter that captures all institutional factors favouring the high-skilled workers' bargaining power.

The growth rate of the low-skilled workers' nominal wage is equal to an exogenous variable β:

[8] For any variable x, $\dot{x} = dx/dt$ and $\hat{x} = \dot{x}/x$.

$$\hat{w}_L = \beta, \qquad \beta > 0 \tag{11}$$

The exogenous variable β represents all institutional factors that positively affects the low-skilled workers' bargaining strength in the conflict over income distribution.

The mark-up set by the firm is supposed to be a decreasing function of the high-skilled employment rate e and an increasing function of an exogenous variable γ:

$$\mu = \mu(e, \gamma), \qquad \mu'_e < 0, \mu'_\gamma > 0 \tag{12}$$

Equation (12) postulates that an increase in labour market tightness, which is measured by the employment rate of the high-skilled workers, has a negative effect on profitability. The exogenous variable γ is an institutional parameter representing the capitalists' bargaining power *vis-à-vis* the working class as a whole.

3.2. *Direction of technical change*

We generalize the induced innovation hypothesis (Kennedy, 1964; Samuelson, 1965; Foley, 2003) in order to consider a heterogeneous labour force, and consequently technical change directed towards high-skilled or low-skilled labour. As firms face a trade-off in their choice of techniques, the set of feasible combination of factor-augmenting technologies can be summarized by a continuous, decreasing and concave innovation possibility frontier in a three-dimensional space:

$$\hat{a}_K = \phi(\hat{a}_H, \hat{a}_L, \tau), \qquad \phi'_{\hat{a}_H} < 0,\ \phi'_{\hat{a}_L} < 0,\ \phi''_{\hat{a}_H\hat{a}_H} < 0,\ \phi''_{\hat{a}_L\hat{a}_L} < 0,\ \phi''_{\hat{a}_H\hat{a}_L} = 0,\ \phi'_\tau = 1 \tag{13}$$

where the assumption of a null cross derivative is made for the sake of simplicity and without loss of generality.

Firms maximize the rate of unit cost reduction, or equivalently the rate of change in the profit rate (Julius, 2005), given the constraint of the innovation possibility frontier, taking into account the negative response of profitability to labour market tightness:

$$\max_{\hat{a}_K, \hat{a}_H, \hat{a}_L} \frac{\mu(e,\gamma)}{1+\mu(e,\gamma)}\hat{a}_K + \frac{z}{[1+\mu(e,\gamma)](1+z)}\hat{a}_H + \frac{1}{[1+\mu(e,\gamma)](1+z)}\hat{a}_L \tag{14}$$
$$s.t.\ \hat{a}_K = \phi(\hat{a}_H, \hat{a}_L, \tau)$$

The solution to this problem yields the two first-order conditions:

$$\phi'_{\hat{a}_H}(\hat{a}_H, \hat{a}_L) = -\frac{z}{\mu(e,\gamma)(1+z)} \tag{15}$$

$$\phi'_{\hat{a}_L}(\hat{a}_H, \hat{a}_L) = -\frac{1}{\mu(e,\gamma)(1+z)} \tag{16}$$

The optimal direction of technical change (equations (15) and (16)) identifies the growth rates of high-skilled labour productivity growth $\hat{a}_H$ and low-skilled labour productivity growth $\hat{a}_L$ as implicit functions of wage inequality z and high-skilled employment rate e:

$$\hat{a}_H = f^H[z, \mu(e,\gamma)], \qquad f_z^{H\prime} > 0,\ f_\mu^{H\prime} < 0 \tag{17}$$

$$\hat{a}_L = f^L[z, \mu(e,\gamma)], \qquad f_z^{L\prime} < 0,\ f_\mu^{L\prime} < 0 \tag{18}$$

From equations (17) and (18), we have that an increase in wage inequality favours the adoption of high-skilled- over low-skilled-labour saving innovations, whereas an increase in the high-skilled employment rate stimulates the adoption of both high-skilled- and low-skilled-labour-saving techniques.[9] The reason is that an increase in z implies a higher high-skilled labour share and a lower low-skilled labour share in total costs, hence a stronger incentive for firms to direct technical change towards the high-skilled labour, at the expense of the low-skilled labour; an increase in e reduces the capital share in total costs, thus inducing technological improvements of both high-skilled and low-skilled labour productivity growth.

4. Dynamical system and steady state

The dynamic behaviour of the system can be represented as a three-dimensional system of differential equations in the output-capital ratio a_K, the relative unit labour cost, i.e. wage inequality z, and the high-skilled employment rate e. From equations (4), (9), and (13), we have:

$$\frac{\dot{a}_K}{a_K} = \phi\left(\frac{\dot{a}_H}{a_H}, \frac{\dot{a}_L}{a_L}, \tau\right) \tag{19}$$

$$\frac{\dot{z}}{z} = \frac{\dot{w}_H}{w_H} + \frac{\dot{a}_L}{a_L} - \frac{\dot{w}_L}{w_L} - \frac{\dot{a}_H}{a_H} \tag{20}$$

$$\frac{\dot{e}}{e} = \frac{\dot{a}_K}{a_K} + g - \frac{\dot{a}_H}{a_H} - n \tag{21}$$

From equations (5), (6), and (20), it is immediate to check that, as in Nishi (2020), the gap between nominal wage growth and labour productivity growth is reflected in the inflation rate. Thus, at the steady state, real wage growth is equal to labour productivity growth. As the mark-up is dependent on the high-skilled employment rate (equation (12)), we have that $\dot{z}/z = \dot{e}/e = 0$ implies $\dot{\omega}_H/\omega_H = \dot{\omega}_L/\omega_L = \dot{\pi}/\pi = 0$ and $\dot{p}/p = \dot{w}_H/w_H - \dot{a}_H/a_H = \dot{w}_L/w_L - \dot{a}_L/a_L$.[10] Therefore, the inflation rate adjusts so as to stabilize the high-skilled and low-skilled labour shares and make them consistent with the mark-up (i.e. the profit share) set by the firm, for a given employment rate.
Substituting from equations (8) (along with equation (12)), (10), (11), (17), and (18), into equations (19), (20), and (21), we obtain the equations of motion for output-capital ratio, wage inequality, and high-skilled employment rate:

$$\frac{\dot{a}_K}{a_K} = \phi\{f^H[z, \mu(e,\gamma)], f^L[z, \mu(e,\gamma)], \tau\} \tag{22}$$

$$\frac{\dot{z}}{z} = h(e,\alpha) - \beta - f^H[z, \mu(e,\gamma)] + f^L[z, \mu(e,\gamma)] \tag{23}$$

$$\frac{\dot{e}}{e} = \frac{\dot{a}_K}{a_K} + \frac{1}{1+\mu(e,\gamma)}\left[\mu(e,\gamma) + \frac{sz}{1+z}\right] a_K - f^H[z, \mu(e,\gamma)] - n \tag{24}$$

The resulting dynamic interaction among the three variables is quite different from the Goodwin model with induced technical change and homogenous labour force. A dynamic equation for

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[9] See Appendix A for the calculation of the expressions for $f_z^{H\prime}$, $f_\mu^{H\prime}$, $f_z^{L\prime}$, and $f_\mu^{L\prime}$.

[10] For a formal proof, see Appendix B.

employment is defined only for high-skilled labour, as low-skilled labour is available in unlimited supply (equation (24)). Moreover, the dynamic equation for the wage share is replaced by a dynamic equation for wage inequality (equation (23)). Thus, the dynamic behaviour of the distributive variable captures the dynamics of intra-working-class income distribution, rather than the conflict over income distribution between workers and capitalists. The profit-squeeze effect of labour market tightness affects the dynamic behaviour of all variables, through the negative response of the mark-up to the high-skilled employment rate.

Equation (24) implies that, at the steady state, the economy will grow at the rate that ensures a constant high-skilled employment rate in the long run. Therefore, the economy is high-skilled-labour-constrained, namely economic growth is constrained by the growth rate of the high-skilled effective labour supply.

In the equilibrium, we have $\dot{a}_K = \dot{z} = \dot{e} = 0$. Thus, the steady-state values of output-capital ratio $a_K^*(\alpha, \beta, \gamma, \tau, s, n)$, wage inequality $z^*(\alpha, \beta, \gamma, \tau, s, n)$, and high-skilled employment rate $e^*(\alpha, \beta, \gamma, \tau, s, n)$ solve the following three equations:

$$\phi\{f^H[z, \mu(e, \gamma)], f^L[z, \mu(e, \gamma)], \tau\} = 0 \tag{25}$$

$$h(e, \alpha) - \beta - f^H[z, \mu(e, \gamma)] + f^L[z, \mu(e, \gamma)] = 0 \tag{26}$$

$$\frac{1}{1 + \mu(e, \gamma)}\left[\mu(e, \gamma) + \frac{sz}{1 + z}\right] a_K - f^H[z, \mu(e, \gamma)] - n = 0 \tag{27}$$

The dynamic equations of a_K and z evaluated at the steady state (equations (25) and (26)) give the conditions on z and e that keep output-capital ratio and wage inequality constant. As the dynamic equation of e only determines the equilibrium value of a_K (equation (27)), we can investigate the steady-state properties of the system by focusing only on the equations (25) and (26). We call the two isoclines AK and Z, respectively. The AK isocline represents the values of z and e that are consistent with the firm's technical choices and a constant output-capital ratio. The Z isocline gives the conditions on z and e that are consistent with the equilibrium in the labour market.

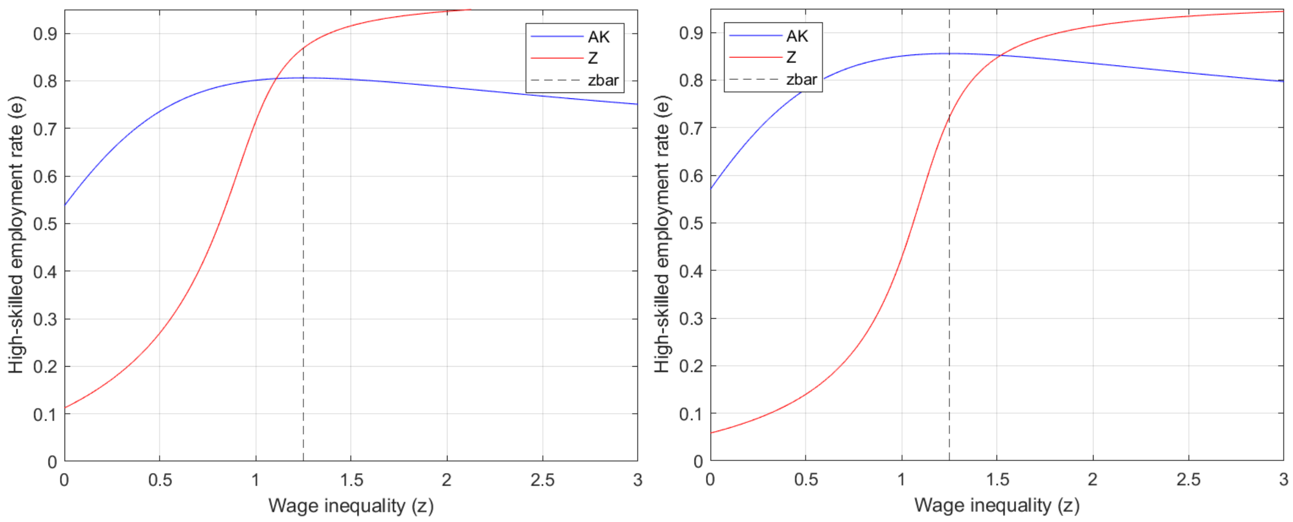
Let us define $\rho \equiv \phi''_{\hat{a}_L\hat{a}_L}/\phi''_{\hat{a}_H\hat{a}_H}$, $\bar{z} \equiv 1/\rho$, and $\Gamma_\mu \equiv (1 - \rho z) f_\mu^{L'}$. $\Gamma_\mu > 0$ if, and only if, $z > \bar{z}$.

In the (z, e) plane, the AK isocline is depicted by an inverted U-shaped curve with a maximum point in $z = \bar{z}$ (Fig. 1).[11] An increase in the high-skilled employment rate e always exerts downward pressure on the output-capital ratio, as a reduction in the capital share in income tends to stimulate both high-skilled- and low-skilled-labour-saving innovations at the expense of capital-saving techniques. Conversely, an increase in wage inequality z gives rise to two counteracting effects on the output-capital ratio: on the one hand, it induces a direction of technical change towards high-skilled labour, which puts downward pressure on the output-capital ratio; on the other hand, it induces less low-skilled-labour-saving innovations, thus exerting upward pressure on the output-capital ratio. As the first effect is non-linear and (in absolute value) increasing in z (equation (A1)), there is a critical value $\bar{z}$ such that if $z > \bar{z}$ the first effect will dominate over the second one. Thus, if $z > \bar{z}$, an increase in wage inequality has to be counteracted by a decrease in the employment rate in order to keep the economy on a steady-state growth path, whereas if $z < \bar{z}$ a constant output-capital ratio requires wage inequality and employment rate to go in the same direction.

The Z isocline is upward (downward) sloping if $h'_e + \Gamma_\mu \mu'_e > (<)\ 0$. The effect of an increase in the employment rate e in the labour market is ambiguously signed: on the one hand, it strengthens the high-skilled workers' bargaining power, thus raising the growth rate of the high-skilled workers' nominal wage for given labour productivity growth; on the other hand, by reducing the capital share

[11] For the computation of the slopes of the AK and Z isoclines, see Appendix C.

Fig. 1. *The AK and Z isoclines in the baseline scenario*



a) Case I: $z^* < \bar{z}$ *b) Case II:* $z^* > \bar{z}$

Note: In Case I, the long-run equilibrium values are $z^* = 1.1079$, $e^* = 0.8048$, $g^* = 0.0429$, $\hat{a}_H^* = 0.0329$, $\hat{a}_L^* = 0.0167$, $\hat{w}_H^* = 0.0562$, $\hat{w}_L^* = 0.04$, $\mu^* = 0.4287$, $\pi^* = 0.3$, $\omega_H^* = 0.3679$, $\omega_L^* = 0.3321$, $a_K^* = 0.1148$, and $\hat{p}^* = 0.0233$. In Case II, the long-run equilibrium values are $z^* = 1.5178$, $e^* = 0.8519$, $g^* = 0.0581$, $\hat{a}_H^* = 0.0531$, $\hat{a}_L^* = 0.0255$, $\hat{w}_H^* = 0.0676$, $\hat{w}_L^* = 0.04$, $\mu^* = 0.54$, $\pi^* = 0.3506$, $\omega_H^* = 0.3915$, $\omega_L^* = 0.2579$, $a_K^* = 0.1355$, and $\hat{p}^* = 0.0145$.

in income, it stimulates both high-skilled- and low-skilled-labour-saving innovations. While the direct effect of employment on wage inequality in the labour market is always positive, the indirect effect, resulting from the induced bias of technical change, is ambiguously signed. As the response of high-skilled labour productivity growth to the mark-up is non-linear and (in absolute value) increasing in z (equation (A5)), the indirect effect is positive if, and only if, $z < \bar{z}$. Accordingly, if $z < \bar{z}$, the Z isocline is unambiguously upward sloping, whereas if $z > \bar{z}$ the Z isocline is upward sloping if, and only if, the direct effect of employment on wage inequality offset the indirect one. Therefore, the equilibrium in the labour market requires wage inequality and employment to go in the same direction if an increase in the high-skilled employment rate results in an overall improvement of the bargaining position of the high-skilled workers, relative to the low-skilled workers, despite the negative impact of the induced high-skilled-labour-saving innovations on wage inequality. In the numerical simulations, the parameter values are such that the Z isocline is always upward sloping.[12] However, the results of comparative statics analysis are independent of the slope of the Z isocline.

In what follows, we assume that non-trivial equilibrium values (a_K^*, z^*, e^*) exist and are economically meaningful.

[12] For the details of the numerical simulations, see Appendix D.

5. LOCAL STABILITY ANALYSIS

Let us define $\theta_z \equiv (1-\rho z)f_z^{L'}$, $\theta_\mu \equiv (1+\rho z^2)f_\mu^{L'} < 0$, $\Gamma_z \equiv (1+\rho)f_z^{L'} < 0$. Remind that $\Gamma_\mu \equiv (1-\rho z)f_\mu^{L'}$. $\theta_z > 0$ and $\Gamma_\mu > 0$ if, and only if, $z > \bar{z}$.
We investigate the local stability of the equilibrium linearizing the system of differential equations (22), (23), and (24) around the equilibrium values (a_K^*, z^*, e^*):

$$\begin{bmatrix} \dot{a}_K \\ \dot{z} \\ \dot{e} \end{bmatrix} = \begin{bmatrix} 0 & J_{12} & J_{13} \\ 0 & J_{22} & J_{23} \\ J_{31} & J_{32} & J_{33} \end{bmatrix} \begin{bmatrix} a_K - a_K^* \\ z - z^* \\ e - e^* \end{bmatrix} \tag{28}$$

where the elements of the Jacobian matrix $\boldsymbol{J}$ evaluated at the steady-state values $a_K^*(\alpha,\beta,\gamma,\tau,s,n)$, $z^*(\alpha,\beta,\gamma,\tau,s,n)$, and $e^*(\alpha,\beta,\gamma,\tau,s,n)$ are given by:

$$J_{12} \equiv \left.\frac{\partial \dot{a}_K}{\partial z}\right|_{a_K=a_K^*, z=z^*, e=e^*} = \theta_z^* \phi'_{\hat{a}_L} a_K^* \tag{29}$$

$$J_{13} \equiv \left.\frac{\partial \dot{a}_K}{\partial e}\right|_{a_K=a_K^*, z=z^*, e=e^*} = \theta_\mu^* \phi'_{\hat{a}_L} \mu'_e a_K^* < 0 \tag{30}$$

$$J_{22} \equiv \left.\frac{\partial \dot{z}}{\partial z}\right|_{a_K=a_K^*, z=z^*, e=e^*} = \Gamma_z^* z^* < 0 \tag{31}$$

$$J_{23} \equiv \left.\frac{\partial \dot{z}}{\partial e}\right|_{a_K=a_K^*, z=z^*, e=e^*} = (h'_e + \Gamma_\mu^* \mu'_e) z^* \tag{32}$$

$$J_{31} \equiv \left.\frac{\partial \dot{e}}{\partial a_K}\right|_{a_K=a_K^*, z=z^*, e=e^*} = g'_{a_K} e^* > 0 \tag{33}$$

$$J_{32} \equiv \left.\frac{\partial \dot{e}}{\partial z}\right|_{a_K=a_K^*, z=z^*, e=e^*} = (\theta_z^* \phi'_{\hat{a}_L} + g'_z - f_z^{H\prime}) e^* \tag{34}$$

$$J_{33} \equiv \left.\frac{\partial \dot{e}}{\partial e}\right|_{a_K=a_K^*, z=z^*, e=e^*} = \left(\theta_\mu^* \phi'_{\hat{a}_L} + g'_\mu - f_\mu^{H'}\right) \mu'_e e^* < 0 \tag{35}$$

Only partial derivatives (30), (31), (33), and (35) are unambiguously signed, whereas the signs of (29), (32) and (34) are crucially dependent on the level of wage inequality, on the effect of the high-skilled employment rate on the growth rate of the high-skilled nominal wage and the rates of high-skilled- and low-skilled-labour-saving innovations, and on the effect of wage inequality on capital accumulation and the rate of high-skilled-labour-saving techniques.
Equation (29) shows that an increase in wage inequality has a stabilizing effect on the dynamics of the output-capital ratio if and only if $z > z^*$. Indeed, an increase in z has two opposite effects on the rate of change of the output-capital ratio: on the one hand, it stimulates the development of high-skilled-labour-saving techniques, thus exerting downward pressure on the output-capital ratio; on the other hand, it reduces the adoption of low-skilled-labour-saving innovations, thus putting upward pressure on the output-capital ratio. Since the first effect is non-linear and (in absolute value) increasing in z (equation (A1)), the first effect will offset the second one if wage inequality exceeds the critical value $\bar{z}$.

Equation (32) shows that the effect of the high-skilled employment rate on the dynamics of wage inequality is mediated by its impact on the growth rate of the high-skilled workers' nominal wage and on the rates of adoption of high-skilled- and low-skilled-labour-saving innovations. An increase in e raises the growth rate of the high-skilled workers' nominal wage and, by reducing the profit share, stimulates both high-skilled- and low-skilled-labour-saving innovations. As the response of high-skilled labour productivity growth to profitability is non-linear and (in absolute value) increasing in z (equation (A5)), the overall effect of an increase in e is crucially dependent on the level of wage inequality: if $z < z^*$, an increase in e always has a destabilizing effect on the dynamics of wage inequality; if $z > z^*$, an increase in e has a stabilizing effect if and only if the stimulus to the development of high-skilled-labour-saving innovations offset the impact on the growth rates of high-skilled nominal wage and low-skilled labour productivity.

From equations (30) and (31), we have that the effect of the high-skilled employment rate on the dynamics of the output-capital ratio and the effect of wage inequality on its rate of change act as stabilization factors of the equilibrium. An increase in e lowers the capital share in total costs, putting downward pressure on the output-capital ratio. A rise in z has a negative feedback on itself, as it induces the development of high-skilled-labour-saving innovations at the expense of the low-skilled-labour-saving innovation, thus reducing wage inequality.

Equation (34) shows that an increase in wage inequality has a stabilizing effect on the dynamics of the high-skilled employment rate if $z > z^*$ and high-skilled labour productivity growth is more responsive than capital accumulation to wage inequality.

The characteristic equation of the Jacobian matrix $\boldsymbol{J}$ in (28) is given by:

$$\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 = 0 \tag{36}$$

where λ denotes a characteristic root. The coefficients of equation (36) are:

$$a_1 = -\mathrm{Tr}(\boldsymbol{J}) = -(J_{22} + J_{33}) = -\left[\Gamma_z^* z^* + \left(\theta_\mu^* \phi'_{\hat{a}_L} + g'_\mu - f_\mu^{H'}\right)\mu'_e e^*\right] \tag{37}$$

$$\begin{aligned} a_2 = \begin{vmatrix} J_{22} & J_{23} \\ J_{32} & J_{33} \end{vmatrix} + \begin{vmatrix} 0 & J_{13} \\ J_{31} & J_{33} \end{vmatrix} &= \Gamma_z^*\left(\theta_\mu^* \phi'_{\hat{a}_L} + g'_\mu - f_\mu^{H'}\right)\mu'_e e^* z^* - \theta_\mu^* \phi'_{\hat{a}_L} g'_{a_K} \mu'_e e^* a_K^* + \\ &- \theta_z^* \phi'_{\hat{a}_L}\left(h'_e + \Gamma_\mu^* \mu'_e\right)e^* z^* - \left(g'_z - f_z^{H'}\right)\left(h'_e + \Gamma_\mu^* \mu'_e\right)e^* z^* \end{aligned} \tag{38}$$

$$a_3 = -\mathrm{Det}(\boldsymbol{J}) = J_{31}(J_{13}J_{22} - J_{12}J_{23}) = -g'_{a_K}\phi'_{\hat{a}_L}\left[\theta_z^*\left(h'_e + \Gamma_\mu^* \mu'_e\right) - \theta_\mu^* \Gamma_z^* \mu'_e\right]e^* z^* a_K^* \tag{39}$$

The necessary and sufficient condition for the local stability of the dynamic system is that all characteristic roots are negative or have a negative real part,[13] which occurs when:

$$a_1 > 0, \quad a_2 > 0, \quad a_3 > 0, \quad a_1 a_2 - a_3 > 0 \tag{40}$$

Proposition 1 The equilibrium is locally stable if $\left(g'_z - f_z^{H'}\right)\left(h'_e + \Gamma_\mu^* \mu'_e\right) < 0$ and $\theta_z^*\left(h'_e + \Gamma_\mu^* \mu'_e\right) < 0$, and only if $\theta_z^*\left(h'_e + \Gamma_\mu^* \mu'_e\right) > \theta_\mu^* \Gamma_z^* \mu'_e$. Then, if $z^* < \bar{z}$, local stability requires $\theta_z^*\left(h'_e + \Gamma_\mu^* \mu'_e\right) > \theta_\mu^* \Gamma_z^* \mu'_e$, whereas $g'_z < f_z^{H'}$ is sufficient for the equilibrium to be locally stable; if $z^* > \bar{z}$, a sufficient condition for the local stability is $g'_z > f_z^{H'}$ and $h'_e + \Gamma_\mu^* \mu'_e < 0$.

Proof The condition $a_1 > 0$ is always satisfied. The condition $a_3 > 0$ is satisfied if and only if $\theta_z^*\left(h'_e + \Gamma_\mu^* \mu'_e\right) > \theta_\mu^* \Gamma_z^* \mu'_e$. After rearranging:

[13] See Gandolfo (2009).

$$a_2 = \underbrace{-\phi'_{\hat{a}_L}\left[\theta_z^*\left(h'_e + \Gamma_\mu^*\mu'_e\right) - \theta_\mu^*\Gamma_z^*\mu'_e\right]e^*z^*}_{\gtreqless 0} \underbrace{+\Gamma_z^*\left(g'_\mu - f_\mu^{H'}\right)\mu'_e e^* z^*}_{>0} \underbrace{-\theta_\mu^*\phi'_{\hat{a}_L} g'_{a_K}\mu'_e e^* a_K^*}_{>0} + \underbrace{-(g'_z - f_z^{H'})(h'_e + \Gamma_\mu^*\mu'_e)e^*z^*}_{\gtreqless 0} \tag{41}$$

If $a_3 > 0$, $\left(g'_z - f_z^{H'}\right)\left(h'_e + \Gamma_\mu^*\mu'_e\right) < 0$ is a sufficient condition for $a_2 > 0$. After some algebra, we have:

$$\begin{aligned} a_1a_2 - a_3 = {} & \underbrace{\phi'_{\hat{a}_L}\left[\Gamma_z^*z^* + \left(\theta_\mu^*\phi'_{\hat{a}_L} + g'_\mu - f_\mu^{H'}\right)\mu'_e e^*\right]}_{>0} \underbrace{\left[\theta_z^*\left(h'_e + \Gamma_\mu^*\mu'_e\right) - \theta_\mu^*\Gamma_z^*\mu'_e\right]e^*z^*}_{\gtreqless 0} + \\ & \underbrace{-\Gamma_z^*\left[\Gamma_z^*z^* + \left(\theta_\mu^*\phi'_{\hat{a}_L} + g'_\mu - f_\mu^{H'}\right)\mu'_e e^*\right]}_{<0} \underbrace{\left(g'_\mu - f_\mu^{H'}\right)\mu'_e e^* z^*}_{<0} + \\ & \underbrace{+\theta_\mu^*\phi'_{\hat{a}_L} g'_{a_K}\left(\theta_\mu^*\phi'_{\hat{a}_L} + g'_\mu - f_\mu^{H'}\right)\mu_e'^2 e^{*2} a_K^*}_{>0} + \\ & \underbrace{+g'_{a_K}\theta_z^*\phi'_{\hat{a}_L}\left(h'_e + \Gamma_\mu^*\mu'_e\right)e^*z^*a_K^*}_{\gtreqless 0} + \\ & \underbrace{+\left[\Gamma_z^*z^* + \left(\theta_\mu^*\phi'_{\hat{a}_L} + g'_\mu - f_\mu^{H'}\right)\mu'_e e^*\right]}_{<0} \underbrace{\left(g'_z - f_z^{H'}\right)\left(h'_e + \Gamma_\mu^*\mu'_e\right)e^*z^*}_{\gtreqless 0} \end{aligned} \tag{42}$$

If $a_2 > 0$ and $a_3 > 0$, $\theta_z^*\left(h'_e + \Gamma_\mu^*\mu'_e\right) < 0$ is a sufficient condition for $a_1a_2 - a_3 > 0$. We have thus proved the first part of Proposition 1.
If $z^* < \bar{z}$, then $\theta_z^* < 0$ and $h'_e + \Gamma_\mu^*\mu'_e > 0$. Therefore, $g'_z < f_z^{H'}$ is sufficient for the equilibrium to be locally stable. If $z^* > \bar{z}$, we always have $\theta_z^*\left(h'_e + \Gamma_\mu^*\mu'_e\right) > \theta_\mu^*\Gamma_z^*\mu'_e$, since $\theta_z^* > 0$ and $\theta_z^*\Gamma_\mu^* < \theta_\mu^*\Gamma_z^*$. Therefore, $g'_z > f_z^{H'}$ and $h'_e + \Gamma_\mu^*\mu'_e < 0$ are a sufficient condition for the local stability. We have thus proved the second part of Proposition 1.

The necessary condition $\theta_z^*\left(h'_e + \Gamma_\mu^*\mu'_e\right) > \theta_\mu^*\Gamma_z^*\mu'_e$, or equivalently $\theta_z^*\phi'_{\hat{a}_L}\left(h'_e + \Gamma_\mu^*\mu'_e\right) < \theta_\mu^*\Gamma_z^*\phi'_{\hat{a}_L}\mu'_e$, prevents wage inequality z and employment e from causing an explosive growth of output-capital ratio and wage inequality. Indeed, the effect of employment on the growth of the output-capital ratio (equation (30)) and the effect of wage inequality on its growth rate (equation (31)) act as stabilizing forces of the equilibrium, whereas the effect of wage inequality on the growth of the output-capital ratio (equation (29)) and the effect of employment on the growth of wage inequality (equation (32)) are not unambiguously signed. The equilibrium will be locally stable only if the effect of the stabilizing forces offset the impact of the ambiguously signed effects – a condition which is always satisfied in the presence of a high level of wage inequality (i.e. if $z^* > \bar{z}$).
The sufficient condition $\left(g'_z - f_z^{H'}\right)\left(h'_e + \Gamma_\mu^*\mu'_e\right) < 0$ and $\theta_z^*\left(h'_e + \Gamma_\mu^*\mu'_e\right) < 0$ implies that a system is locally stable in the presence of an equilibrium in the balance of power among social classes, in terms of dynamics of high-skilled employment and wage inequality (equations (31), (32), and (34)). Indeed, the system is stable if an imbalance in favour of the high-skilled workers in the dynamics of wage inequality (i.e. $h'_e + \Gamma_\mu^*\mu'_e > 0$) is counteracted by a negative effect of wage inequality on the growth rate of employment (i.e. $g'_z - f_z^{H'} < 0$) and a low level of wage inequality ($z^* < \bar{z}$), or alternatively, if an imbalance in favour of the high-skilled workers in the dynamics of employment (i.e. $g'_z - f_z^{H'} > 0$) and the level of wage inequality ($z^* > \bar{z}$) is compensated by a negative response of the growth rate of wage inequality to employment ($h'_e + \Gamma_\mu^*\mu'_e < 0$).

6. COMPARATIVE STATICS ANALYSIS

6.1. *EFFECT OF LABOUR MARKET INSTITUTIONS*

This section investigates the effects of changes in the institutional variables α, β, and γ on the steady-state values of wage inequality, high-skilled employment rate, capital accumulation, high-skilled labour productivity growth, low-skilled labour productivity growth, high-skilled nominal and real wages growth, low-skilled nominal and real wages growth, mark-up and income shares.

Let us define $\sigma \equiv \left[(1-\rho z)h_e' - \rho(1+z)^2 f_\mu^{L'} \mu_e'\right] f_z^{L'}$.[14] Since the implementation of a comparative statics analysis requires the stability of the equilibrium, we limit ourselves to the discussion of the case of $\sigma > 0$.[15]

Proposition 2 The equilibrium wage inequality, capital accumulation, high-skilled labour productivity growth, high-skilled nominal and real wages growth, and high-skilled wage share are increasing in α; the equilibrium low-skilled labour productivity growth, high-skilled real wages growth, and low-skilled wage share are decreasing in α; the equilibrium high-skilled employment rate is a positive function of α if and only if $z^* < \bar{z}$; the equilibrium values of mark-up and profit share are positive functions of α if and only if $z^* > \bar{z}$.

Proof Totally differentiating equations (25), (26), and (27) with respect to α yields:

$$\frac{dz^*}{d\alpha} = \frac{h_\alpha'(1+\rho z^2) f_\mu^{L'} \mu_e'}{\sigma} > 0 \tag{43}$$

$$\frac{de^*}{d\alpha} = -\frac{h_\alpha'(1-\rho z) f_z^{L'}}{\sigma} \tag{44}$$

Using equations (43), (44), and $g = \hat{a}_H - n$, total differentiation of equations (5) and (6), with (12), and (10), (12), (17), and (18) with respect to α yields:

$$\frac{d\mu^*}{d\alpha} = -\frac{h_\alpha'(1-\rho z) f_z^{L'} \mu_e'}{\sigma} \tag{45}$$

$$\frac{dg^*}{d\alpha} = \frac{d\hat{a}_H^*}{d\alpha} = -\frac{h_\alpha' \rho (1+z) f_z^{L'} f_\mu^{L'} \mu_e'}{\sigma} > 0 \tag{46}$$

$$\frac{d\hat{a}_L^*}{d\alpha} = \frac{h_\alpha' \rho z (1+z) f_z^{L'} f_\mu^{L'} \mu_e'}{\sigma} < 0 \tag{47}$$

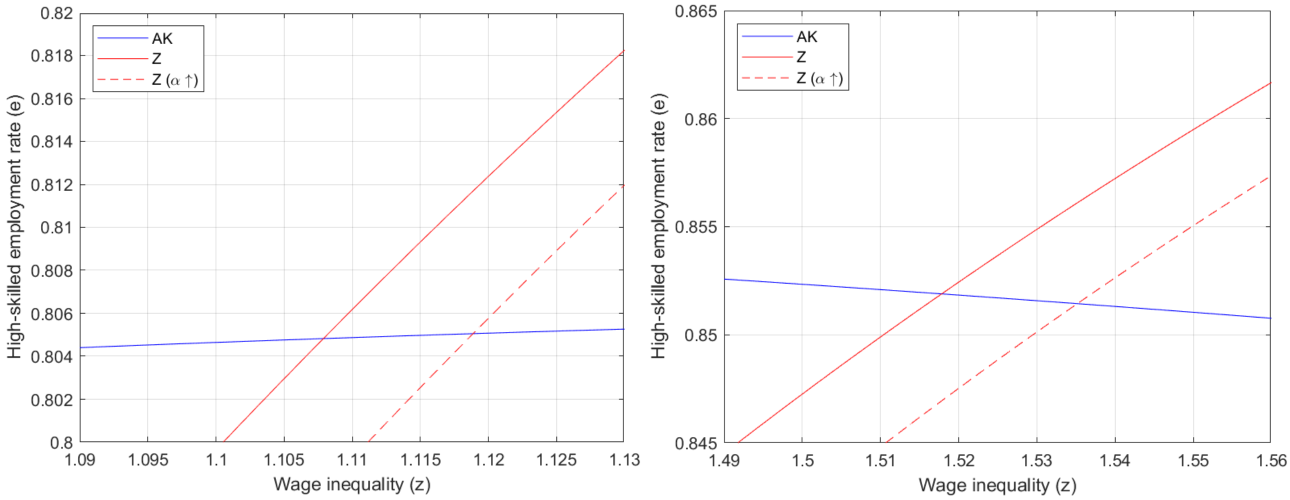
$$\frac{d\hat{w}_H^*}{d\alpha} = -\frac{h_\alpha' \rho (1+z)^2 f_z^{L'} f_\mu^{L'} \mu_e'}{\sigma} > 0 \tag{48}$$

$$\frac{d\omega_H^*}{d\alpha} = \frac{h_\alpha'[1+\rho z^2 + \mu(1+z)]\mu_e'}{\mu^2(1+\mu)^2(1+z)^3 \phi_{\hat{a}_L \hat{a}_L}'' \sigma} > 0 \tag{49}$$

[14] We omit "∗" to save notation.

[15] The coefficient a_3 of the Jacobian matrix is positive if and only if $\sigma > 0$.

Fig. 2. *The effect of an increase in the high-skilled workers' bargaining strength*



a) Case I: $z^* < \bar{z}$ *b) Case II:* $z^* > \bar{z}$

Note: In Case I, the new equilibrium values are $z^* = 1.1189$, $e^* = 0.8051$, $g^* = 0.0478$, $\hat{a}_H^* = 0.0378$, $\hat{a}_L^* = 0.0113$, $\hat{w}_H^* = 0.0665$, $\hat{w}_L^* = 0.04$, $\mu^* = 0.4285$, $\pi^* = 0.3$, $\omega_H^* = 0.3696$, $\omega_L^* = 0.3304$, $a_K^* = 0.1277$, and $\hat{p}^* = 0.0287$. In Case II, the new equilibrium values are $z^* = 1.5352$, $e^* = 0.8514$, $g^* = 0.0617$, $\hat{a}_H^* = 0.0567$, $\hat{a}_L^* = 0.0201$, $\hat{w}_H^* = 0.0766$, $\hat{w}_L^* = 0.04$, $\mu^* = 0.5403$, $\pi^* = 0.3508$, $\omega_H^* = 0.3931$, $\omega_L^* = 0.2561$, $a_K^* = 0.1436$, and $\hat{p}^* = 0.0199$.

$$\frac{d\omega_L^*}{d\alpha} = -\frac{h'_\alpha[1 + \rho z^2 + \rho\mu z(1+z)]\mu'_e}{\mu^2(1+\mu)^2(1+z)^3\phi''_{\hat{a}_L\hat{a}_L}\sigma} < 0 \tag{50}$$

An increase in the high-skilled workers' bargaining strength, as measured by the exogenous component of the high-skilled nominal wage growth (α), leads to a rightward shift in the Z isocline, while leaving the AK isocline unaffected (Fig. 2).

Consider the case of an upward sloping Z isocline. The Z isocline shifts rightward, as an exogenous increase in the high-skilled workers' bargaining power allows them to attain a higher income share, for a given employment rate. The resulting increase in wage inequality induces the adoption of high-skilled-labour-saving innovations, at the expense of low-skilled-labour-saving techniques, putting pressure on the output-capital ratio. If $z^* < \bar{z}$, the increase in wage inequality exerts upward pressure on the output-capital ratio, thus leading to an increase in the employment rate up to the level that makes the profit share consistent with the firm's technical choices and a constant output-capital ratio (Fig. 2a). Conversely, if $z^* > \bar{z}$, the increase in wage inequality put downward pressure on the output-capital ratio, that can only be stabilized by a lower long-run employment rate (Fig. 2b).

The rise in wage inequality causes the long-run skill bias of technical change to increase. Indeed, the rise in the high-skilled labour share and the fall in the low-skilled labour share in total costs translate into a higher rate of high-skilled-labour-augmenting technical change and a lower rate of low-skilled-labour-augmenting technical change at the steady state. The high-skilled employment rate negatively affects profitability, as measured by the firm's mark-up. Thus, in the presence of a low level of steady-state wage inequality (i.e. $z^* < \bar{z}$), an increase in the high-skilled workers' bargaining strength reduces the mark-up and the capitalists' share of income, whereas with a high level of wage inequality (i.e. $z^* > \bar{z}$), both capitalists and high-skilled workers increase their income shares to the detriment of the low-skilled workers. A trade-off between income distribution and employment of the high-skilled workers arises only if $z^* > \bar{z}$. Conversely, if $z^* < \bar{z}$, an increase in the bargaining power of the high-skilled workers positively affects both their income share and their employment rate.

Proposition 3 The equilibrium wage inequality, capital accumulation, high-skilled labour productivity growth, high-skilled nominal and real wages growth, and high-skilled wage share are decreasing in β; the equilibrium low-skilled labour productivity growth, low-skilled nominal and real wages growth, and low-skilled wage share are increasing in β; the equilibrium high-skilled employment rate is a positive function of β if and only if $z^* > \bar{z}$; the equilibrium values of mark-up and profit share are positive functions of β if and only if $z^* < \bar{z}$.

Proof Totally differentiating equations (25), (26), and (27) with respect to β yields:

$$\frac{dz^*}{d\beta} = -\frac{(1+\rho z^2) f_\mu^{L'} \mu_e'}{\sigma} > 0 \tag{51}$$

$$\frac{de^*}{d\beta} = \frac{(1-\rho z) f_z^{L'}}{\sigma} \tag{52}$$

Using equations (51), (52), and $g = \hat{a}_H - n$, total differentiation of equations (5) and (6), with (12), and (10), (12), (17), and (18) with respect to β yields:

$$\frac{d\mu^*}{d\beta} = \frac{(1-\rho z) f_z^{L'} \mu_e'}{\sigma} \tag{53}$$

$$\frac{dg^*}{d\beta} = \frac{d\hat{a}_H^*}{d\beta} = \frac{\rho(1+z) f_z^{L'} f_\mu^{L'} \mu_e'}{\sigma} < 0 \tag{54}$$

$$\frac{d\hat{a}_L^*}{d\beta} = -\frac{\rho z(1+z) f_z^{L'} f_\mu^{L'} \mu_e'}{\sigma} > 0 \tag{55}$$

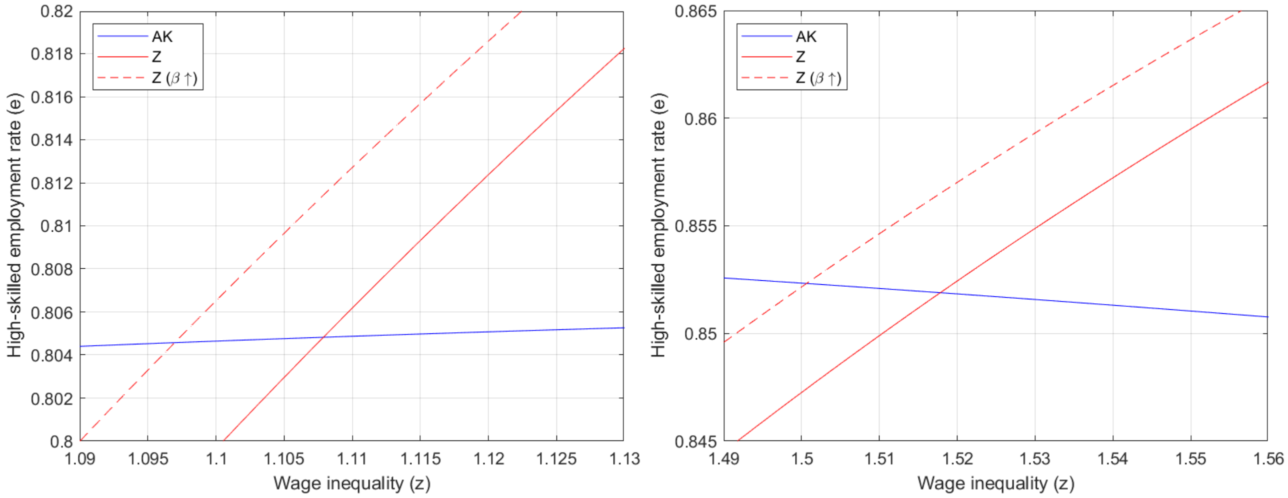
$$\frac{d\widehat{w}_H^*}{d\beta} = \frac{\rho(1+z)^2 f_z^{L'} f_\mu^{L'} \mu_e'}{\sigma} < 0 \tag{56}$$

$$\frac{d\omega_H^*}{d\beta} = -\frac{[1+\rho z^2 + \mu(1+z)]\mu_e'}{\mu^2(1+\mu)^2(1+z)^3 \phi''_{\hat{a}_L\hat{a}_L}\sigma} < 0 \tag{57}$$

$$\frac{d\omega_L^*}{d\beta} = \frac{[1+\rho z^2 + \rho\mu z(1+z)]\mu_e'}{\mu^2(1+\mu)^2(1+z)^3 \phi''_{\hat{a}_L\hat{a}_L}\sigma} > 0 \tag{58}$$

An increase in the low-skilled workers' bargaining strength, as measured by the exogenous component of the low-skilled nominal wage growth (β), leads to a leftward shift in the Z isocline, while leaving the AK isocline unaffected (Fig. 3).
The case of an increase in β is specular to the previous one. Let us consider again the case of an upward sloping Z isocline. Now, for a given high-skilled employment rate, the improved bargaining position of the low-skilled workers allow them to attain a higher income share. The resulting decrease in wage inequality induces a bias of technical change towards the low-skilled labour, at the expense of the high-skilled labour. If $z^* < \bar{z}$, the output-capital ratio must be stabilized by a decrease in the

Fig. 3. *The effect of an increase in the low-skilled workers' bargaining strength*



a) Case I: $z^* < \bar{z}$ *b) Case II:* $z^* > \bar{z}$

Note: In Case I, the new equilibrium values are $z^* = 1.097$, $e^* = 0.8046$, $g^* = 0.038$, $\hat{a}_H^* = 0.028$, $\hat{a}_L^* = 0.0221$, $\hat{w}_H^* = 0.0558$, $\hat{w}_L^* = 0.05$, $\mu^* = 0.4288$, $\pi^* = 0.3001$, $\omega_H^* = 0.3661$, $\omega_L^* = 0.3338$, $a_K^* = 0.1017$, and $\hat{p}^* = 0.0279$. In Case II, the new equilibrium values are $z^* = 1.5006$, $e^* = 0.8523$, $g^* = 0.0545$, $\hat{a}_H^* = 0.0495$, $\hat{a}_L^* = 0.031$, $\hat{w}_H^* = 0.0686$, $\hat{w}_L^* = 0.05$, $\mu^* = 0.5397$, $\pi^* = 0.3505$, $\omega_H^* = 0.3898$, $\omega_L^* = 0.2597$, $a_K^* = 0.1273$, and $\hat{p}^* = 0.019$.

high-skilled employment rate (Fig. 3a), whereas if $z^* > \bar{z}$, steady-state growth requires an increase in the high-skilled employment rate (Fig. 3b).
The reduction in wage inequality, resulting from the fall in the high-skilled labour share and the increase in the high-skilled labour share, is associated with a lower skill bias of technical change at the steady state, namely with faster low-skilled labour productivity growth and slower high-skilled labour productivity growth. As the high-skilled employment rate is inversely related to the mark-up and the profit share, if $z^* > \bar{z}$ an increase in the low-skilled workers' bargaining power leads to income redistribution from profits to wages. Thus, in the presence of a high level of wage inequality, an increase in the low-skilled workers' bargaining strength is conducive to both higher employment and a lower capital share.

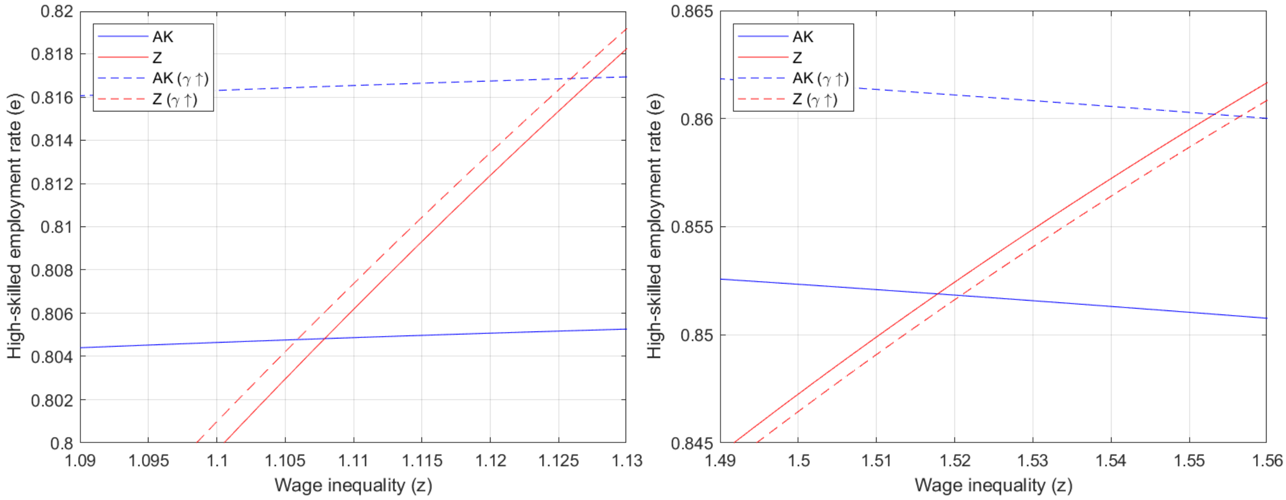
Proposition 4 The equilibrium wage inequality, high-skilled employment rate, capital accumulation, high-skilled labour productivity growth, high-skilled nominal and real wages growth, and high-skilled wage share are increasing in γ; the equilibrium low-skilled labour productivity growth, low-skilled real wages growth, and low-skilled wage share are decreasing in γ; the equilibrium values of mark-up and profit share are positive functions of γ if and only if $z^* > \bar{z}$.

Proof Totally differentiating equations (25), (26), and (27) with respect to γ yields:

$$\frac{dz^*}{d\gamma} = -\frac{h_e'(1+\rho z^2) f_\mu^{L'} \mu_\gamma'}{\sigma} > 0 \tag{59}$$

$$\frac{de^*}{d\gamma} = \frac{\rho(1+z)^2 f_z^{L'} f_\mu^{L'} \mu_\gamma'}{\sigma} > 0 \tag{60}$$

Fig. 4. *The effect of an increase in the capitalists' bargaining strength*



a) Case I: $z^* < \bar{z}$ *b) Case II: $z^* > \bar{z}$*

Note: In Case I, the new equilibrium values are $z^* = 1.1259$, $e^* = 0.8169$, $g^* = 0.0508$, $\hat{a}_H^* = 0.0408$, $\hat{a}_L^* = 0.0078$, $\widehat{w}_H^* = 0.073$, $\widehat{w}_L^* = 0.04$, $\mu^* = 0.4285$, $\pi^* = 0.3$, $\omega_H^* = 0.3708$, $\omega_L^* = 0.3293$, $a_K^* = 0.1359$, and $\hat{p}^* = 0.0322$. In Case II, the new equilibrium values are $z^* = 1.5565$, $e^* = 0.8601$, $g^* = 0.0659$, $\hat{a}_H^* = 0.0609$, $\hat{a}_L^* = 0.0135$, $\widehat{w}_H^* = 0.0874$, $\widehat{w}_L^* = 0.04$, $\mu^* = 0.5406$, $\pi^* = 0.3509$, $\omega_H^* = 0.3952$, $\omega_L^* = 0.2539$, $a_K^* = 0.1534$, and $\hat{p}^* = 0.0265$.

Using equations (59), (60), and $g = \hat{a}_H - n$, total differentiation of equations (5) and (6), with (12), and (10), (12), (17), and (18) with respect to γ yields:

$$\frac{d\mu^*}{d\gamma} = \frac{h'_e(1-\rho z)f_z^{L'}\mu'_\gamma}{\sigma} \tag{61}$$

$$\frac{dg^*}{d\gamma} = \frac{d\hat{a}_H^*}{d\gamma} = \frac{h'_e\rho(1+z)f_z^{L'}f_\mu^{L'}\mu'_\gamma}{\sigma} > 0 \tag{62}$$

$$\frac{d\hat{a}_L^*}{d\gamma} = -\frac{h'_e\rho z(1+z)f_z^{L'}f_\mu^{L'}\mu'_\gamma}{\sigma} < 0 \tag{63}$$

$$\frac{d\widehat{w}_H^*}{d\gamma} = \frac{h'_e\rho(1+z)^2 f_z^{L'}f_\mu^{L'}\mu'_\gamma}{\sigma} > 0 \tag{64}$$

$$\frac{d\omega_H^*}{d\gamma} = -\frac{h'_e[1+\rho z^2+\mu(1+z)]\mu'_\gamma}{\mu^2(1+\mu)^2(1+z)^3\phi''_{\hat{a}_L\hat{a}_L}\sigma} > 0 \tag{65}$$

$$\frac{d\omega_L^*}{d\gamma} = \frac{h'_e[1+\rho z^2+\rho\mu z(1+z)]\mu'_\gamma}{\mu^2(1+\mu)^2(1+z)^3\phi''_{\hat{a}_L\hat{a}_L}\sigma} < 0 \tag{66}$$

An increase in the capitalists' bargaining strength, as measured by the exogenous component of the mark-up (γ), leads to an upward shift in the AK isocline; if downward sloping, the Z isocline always

Tab. 1. *Results of comparative statics analysis*

	α		β		γ	
	$z^* < \bar{z}$	$z^* > \bar{z}$	$z^* < \bar{z}$	$z^* > \bar{z}$	$z^* < \bar{z}$	$z^* > \bar{z}$
z^*	$+$	$+$	$-$	$-$	$+$	$+$
e^*	$+$	$-$	$-$	$+$	$+$	$+$
$\hat{a}_H^*, g^*$	$+$	$+$	$-$	$-$	$+$	$+$
$\hat{a}_L^*$	$-$	$-$	$+$	$+$	$-$	$-$
π^*, μ^*	$-$	$+$	$+$	$-$	$-$	$+$
ω_H^*	$+$	$+$	$-$	$-$	$+$	$+$
ω_L^*	$-$	$-$	$+$	$+$	$-$	$-$
a_K^*	$+/-$	$+$	$+/-$	$-$	$+/-$	$+/-$
$\hat{p}^*$	$+$	$+$	$+/-$	$+$	$+$	$+$

Note: For the proof of the effects of a shift in parameters on a_K^* and $\hat{p}^*$, see Appendix D.

shifts upward; if upward sloping, the Z isocline rotates clockwise around the point $z = \bar{z}$ (Fig. 4).[16] As in the previous cases, we limit ourselves to discuss the case of an upward sloping Z isocline. An exogenous increase in the mark-up exerts upward pressure on the output-capital ratio, as the capital share in total costs increases. Therefore, the AK isocline shifts upward: for given wage inequality, the output-capital ratio must be stabilized by a higher employment rate. The increase in the high-skilled employment rate improves the bargaining position of the high-skilled workers in the labour market, thus leading to an increase in wage inequality. However, a rise in the capitalists' bargaining strength, by reducing the rates of adoption of high-skilled- and low-skilled-labour-saving innovations, also has a direct effect in the labour market. As the response of high-skilled labour productivity growth to the mark-up is non-linear and (in absolute value) increasing in z, the direct effect of γ depends on the level of wage inequality: for given employment, the equilibrium in the labour market requires a decrease in wage inequality if $z^* < \bar{z}$ (Fig. 4a), and an increase in wage inequality if $z^* > \bar{z}$ (Fig. 4b). However, both the analytical solutions and the numerical simulations show that the AK isocline is more responsive than the Z isocline to an increase in γ. Therefore, irrespective of the shift in the Z isocline, both wage inequality and employment increase at the steady state.

As in the case of an increase in the high-skilled workers' bargaining strength, the rise in wage inequality leads to faster high-skilled labour productivity growth and slower low-skilled labour productivity growth. In the presence of a high level of wage inequality (i.e. $z^* > \bar{z}$), the negative effect of the increase in the high-skilled employment rate on profitability is offset by the positive effect of the exogenous increase in the mark-up. Thus, an increase in the capitalists' bargaining strength also raises the capital share in income.

Tab. 1 summarizes the main results of comparative statics. As it turns out, labour market institutions affect both long-run income distribution and direction of technical change. Indeed, a fall in the low-skilled workers' bargaining strength or an increase in the capitalists' bargaining power in the distributive conflict lead to both an increase in wage inequality and an induced bias of technical change that disproportionately benefits high-skilled over low-skilled labour productivity growth. If wage inequality is high, namely if wage inequality exceeds a critical level, an increase in the bargaining power of the high-skilled workers, as compared to the low-skilled workers, or an increase

[16] For a formal proof, see Appendix C.

in the capitalists' bargaining strength also lead to income redistribution from wages to profits. Moreover, in contrast to both the conventional wisdom and the Goodwin model with homogenous labour force, no necessary trade-offs arise between labour market regulation and long-run employment. An increase in the high-skilled workers' bargaining strength reduces the high-skilled employment rate only if the current level of wage inequality is high. However, with a high level of wage inequality, an increase in the low-skilled workers' bargaining power leads to lower wage inequality, higher employment and income redistribution from profits to wages.

6.2. Discussion: comparison with the Goodwin model with induced innovation

In this section, we provide a comparison between our model and the Goodwin model with induced technical change (Shah and Desai, 1981; van der Ploeg, 1987). As in Tavani and Zamparelli (2015), and Zamparelli (2015), we introduce an explicit policy variable related to the workers' bargaining power into the model in order to make it possible to examine the steady-state effects of labour market institutions.
In our notation, the Goodwin model with induced technical change can be represented as:

$$\frac{\dot{a}_K}{a_K} = \phi[f(\omega)] \tag{67}$$

$$\frac{\dot{\omega}}{\omega} = h(v, \delta) - f(\omega) \tag{68}$$

$$\frac{\dot{v}}{v} = \frac{\dot{a}_K}{a_K} + (1 - \omega)a_K - f(\omega) - n \tag{69}$$

where δ denotes the workers' bargaining strength in the distributive conflict; ω, the wage share; and v, the employment rate.
The steady-state equilibrium is $\omega^* = f^{-1}[\phi^{-1}(0)]$, $v^* = h^{-1}[\phi^{-1}(0), \delta]$, $a_K^* = [\phi^{-1}(0) + n]/\{1 - f^{-1}[\phi^{-1}(0)]\}$. The equilibrium capital accumulation and labour productivity growth are $g^* = \phi^{-1}(0) + n$ and $\hat{a}^* = \phi^{-1}(0)$, respectively. It is immediate to see that an increase in the workers' bargaining strength only reduces the employment rate, while leaving the long-run labour share, capital accumulation and labour productivity growth unaffected. The dynamic equation of the output-capital ratio (equation (67)) solves for the long-run value of the wage share, that does not depend on labour market institutions δ. Once the steady-state labour share is determined, the dynamic equation of the wage share (equation (68)) solves for the long-run value of the employment rate, that is dependent on labour market institutions δ. Thus, the dynamic equation of the labour share only determines the size of the reserve army of labour that makes the overall bargaining power of workers consistent with technology-determined income distribution. Capitalists react to any exogenous increase in workers' bargaining power by decreasing temporarily capital accumulation, so as to bring the labour share back to its original level. The size of the reserve army of labour then rises so as to make workers be "quiet" with their original share in income, whereas capital accumulation and labour productivity growth go back to their old steady-state values. Accordingly, labour market institutions do not affect permanently income distribution, capital accumulation and labour productivity growth.[17]
As balanced growth requires a constant output-capital ratio in the long run, steady-state labour

[17] Labour market institutions do not have persistent effects on income distribution, capital accumulation and labour productivity growth, but only reduce the long-run employment rate, even in the original model with exogenous labour productivity growth and in the Goodwin model with non-microfounded induced technical change (see Chapter 1).

productivity growth is determined by the intercept of the innovation possibility frontier. Therefore, long-run income distribution is uniquely determined by the shape of the frontier at the intercept, and the income shares change along the transitional dynamics in order to ensure a Harrod-neutral direction of technical change in the long run. As firm's technical choices are not affected by labour market institutions, an exogenous increase in the workers' bargaining power only affects income distribution along the transition path. At the new steady-state, the improved bargaining position of workers is fully neutralized by a decrease in the employment rate, that adjusts in order to make the wage share consistent with the firm's technical choices and a constant output-capital ratio.

In order to make the comparison easier, we rewrite our model as follows:

$$\frac{\dot{a}_K}{a_K} = \phi\{f^H[z, \mu(e)], f^L[z, \mu(e)]\} \tag{70}$$

$$\frac{\dot{z}}{z} = h(e, \alpha) - \beta - f^H[z, \mu(e)] + f^L[z, \mu(e)] \tag{71}$$

$$\frac{\dot{e}}{e} = \frac{\dot{a}_K}{a_K} + g[z, \mu(e), a_K] - f^H[z, \mu(e)] - n \tag{72}$$

The dynamic equation for the wage share of the Goodwin model is replaced by a dynamic equation for wage inequality, i.e. the relative unit labour costs (equation (71)), representing the dynamics of intra-working-class income distribution, rather than the conflict over income distribution between capital and labour. As the economy is high-skilled-labour constrained, a dynamic equation for employment is defined only for high-skilled labour (equation (72)), and the antagonistic relationship between capital and labour is represented by a negative response of profitability to the high-skilled employment rate – that affects the dynamic behaviour of all variables – and by a nominal Phillips curve for the wage formation of the high-skilled workers.

In contrast to the Goodwin model with induced innovation and homogenous labour force, the steady-state value of the distributive variable is not determined only by the dynamic equation of the output-capital ratio, and both the distributive variable and the employment rate adjust so as to stabilize the output-capital ratio and the distributive variable. Therefore, there are different combinations of wage inequality and employment that are consistent with the firm's optimal choice of techniques, a constant output-capital ratio, and a profile of technical change characterized by both high-skilled- and low-skilled-labour-augmenting technologies in the long run. The AK isocline is then represented by an inverted U-shaped curve, rather than a straight vertical line: due to the non-linearity of the effect of wage inequality, a constant output-capital ratio requires wage inequality and employment to go in the same direction if $z < \bar{z}$, in the opposite direction if $z > \bar{z}$.

In the Goodwin model, the steady-state value of the distributive variable is determined by technology, whereas the long-run employment rate is determined by the labour market. Thus, a trade-off arises between labour market regulation and long-run employment: any attempt by workers to increase their income share can but result in a fall in employment, since an exogenous increase in workers' bargaining strength has to be offset by an "endogenous" decrease in order to keep the overall bargaining power of workers consistent with technology-determined income distribution. Strengthening labour market regulation results in an unambiguous loss for the working class, as the decrease in the employment rate is not compensated by any gain on the distributional ground in the long run. As capital accumulation and labour productivity growth are dependent on long-run income distribution, a change in the institutional framework governing the conflict over income distribution between capitalists and workers does not have other permanent effects on the outcomes of the economy.

In our model, firm's technical choices and the equilibrium in the labour market determine both the

steady-state value of the distributive variable and long-run employment. As different combinations of wage inequality and employment are consistent with the equilibrium in the labour market and a constant output-capital ratio, our model restores a channel through which labour market institutions may affect long-run income distribution, capital accumulation and labour productivity growth.
An increase in the bargaining power of a fraction of the working class may have a positive effect on capital accumulation, high-skilled labour productivity growth and low-skilled labour productivity growth. Since, in a high-skilled-labour-constrained economy, capital accumulation is driven by high-skilled labour productivity growth, an improvement of the bargaining position of the high-skilled workers leads to faster capital accumulation, whereas an increase in the low-skilled workers' bargaining power leads to an increase in low-skilled labour productivity growth. As the direction of technical change is determined by the shares of capital, high-skilled and low-skilled labour in total costs, labour market institutions also affect long-run income distribution. An increase in the bargaining power of a fraction of the working of the working class always allows it to raise its income share. At some levels of wage inequality, the increase in the high-skilled or low-skilled labour share comes at the expense of capitalists. Therefore, as the profit share of income and long-run employment are inversely related, an improvement of the bargaining position of high-skilled or low-skilled workers needs not imply employment losses.[18] A rise in the high-skilled workers' bargaining strength reduces the employment rate only in the presence of a high level of wage inequality. However, if the current level of wage inequality is high, an increase in the low-skilled workers' bargaining power improves both the distribution of wages and functional income distribution, while increasing employment in the long run.

7. CONCLUDING REMARKS

According to the conventional wisdom on skill-biased technical change, the increase in wage inequality experienced by many advanced economies over the last decades is the result of a purely technological process, that can only be counteracted by educational policies aiming to provide workers with the skills necessary to deal with technical change. The role of the institutions governing income distribution is explicitly neglected as a primary cause of changes in income distribution. By altering the effectiveness of union activity, labour market institutions may at most amplify the direct effect of skill-biased technical change on wage inequality.
This paper proposed an alternative framework for analyzing the interaction between labour market institutions, skill-biased technical change and income distribution. We extended the basic classical-Marxian growth model in order to include a heterogenous labour force, made up of high-skilled and low-skilled workers. Furthermore, we generalized the induced innovation hypothesis in order to admit technical change directed towards high-skilled or low-skilled labour. In a classical view, induced technical change is regarded as a weapon of capitalists for breaking the bargaining power of the working class or a fraction of it. The direction of technical change is then determined by the shares of capital, high-skilled labour and low-skilled labour in total costs.
We found that, in contrast to the neoclassical literature on skill-biased technical change, both wage inequality and the direction of technical change are determined by the institutional factors affecting the conflict over income distribution among social classes. The causality direction among skill-biased

[18] It is worth noting that the argument holds for changes in the bargaining power of a *fraction* of the working class. A change in the bargaining power of capitalists *vis-à-vis* the working class as a whole (i.e. a change in γ) leads to the standard trade-off between labour market regulation and employment.

technical change, labour market institutions and wage inequality predicted by the neoclassical authors is then reversed. Institutional factors related to labour market regulation affect income inequality both directly, by altering the relative bargaining positions of the social classes in the labour market, and indirectly, by inducing different growth rates of high-skilled and low-skilled labour productivity growth, that are totally passed through to the real wages at the steady state. Thus, even if we assume that skill levels are the only source of heterogeneity across workers in the economy, and income distribution is affected by skill-biased technical change, the institutions that govern the distributive conflict are still a central determinant of both income distribution and the long-run macroeconomic outcomes. Institutional factors like the decline in unionization, the decentralization of the collective bargaining structure, the deterioration of the social protection system, and the liberalization of capital flows, that negatively affects the bargaining power of the working class, and particularly the bargaining power of the low-paid workers, lead to a rise in wage inequality and an increase in the skill bias of technical change. Furthermore, provided that the current level of wage inequality is high, an increase in the bargaining power of the high-skilled workers, as compared to the low-skilled workers, or a shift in the balance of power from the working class to capitalists alter both functional income distribution in favour of the capitalist class and the distribution of wages in favour of the high-skilled workers. Finally, we proved that, in contrast to both the conventional view and the Goodwin model with induced innovation and a homogenous labour force, no necessary trade-offs arise between labour market regulation and employment even in a supply-side framework.

Even though the proposed framework is able to address some features of the relation between wage inequality and labour market regulation, more work is needed to provide a more complete view of the interaction between functional distribution, wage inequality, technical change and stagnation in labour productivity. Indeed, as in this model the steady-state growth path is characterized by high-skilled- and low-skilled-augmenting technical change, the economy does not evolve so as to achieve steady-state average labour productivity growth. Thus, the proposed framework does not allow examining the relation between labour market institutions and stagnation in labour productivity. Furthermore, while a classical-Marxian growth model provides some useful insights into the main issue of wage inequality and technical change, it does not incorporate effective demand. A more complete understanding can come from relaxing the assumption of full capacity utilization and postulating an independent investment function. Finally, education is not formalized. The model may be extended in terms of allowing low-skilled workers to acquire skills and convert themselves into high-skilled workers. This analysis is left for future research.

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APPENDIX A

Remind that $\phi''_{\hat{a}_H\hat{a}_L} = 0$. Then, substituting equations (17) and (18) into equations (15) and (16), we find:

$$\phi'_{\hat{a}_H}\{f^H[z,\mu(e,\gamma)]\} = -\frac{z}{\mu(e,\gamma)(1+z)} \tag{A1}$$

$$\phi'_{\hat{a}_L}\{f^L[z,\mu(e,\gamma)]\} = -\frac{1}{\mu(e,\gamma)(1+z)} \tag{A2}$$

Totally differentiating equations (A1) and (A2) with respect to z and μ and rearranging, we have:

$$f_z^{H\prime} = -\frac{1}{\mu(e,\gamma)(1+z)^2}\frac{1}{\phi''_{\hat{a}_H\hat{a}_H}} > 0 \tag{A3}$$

$$f_z^{L\prime} = \frac{1}{\mu(e,\gamma)(1+z)^2}\frac{1}{\phi''_{\hat{a}_L\hat{a}_L}} < 0 \tag{A4}$$

$$f_\mu^{H\prime} = \frac{z}{[\mu(e,\gamma)]^2(1+z)}\frac{1}{\phi''_{\hat{a}_H\hat{a}_H}} < 0 \tag{A5}$$

$$f_\mu^{L\prime} = \frac{1}{[\mu(e,\gamma)]^2(1+z)}\frac{1}{\phi''_{\hat{a}_L\hat{a}_L}} < 0 \tag{A6}$$

It follows that:

$$\phi'_{\hat{a}_H} = z\phi'_{\hat{a}_L} \tag{A7}$$

$$f_z^{H\prime} = -\rho f_z^{L\prime} \tag{A8}$$

$$f_\mu^{H\prime} = \rho z f_\mu^{L\prime} \tag{A9}$$

where $\rho \equiv \phi''_{\hat{a}_L\hat{a}_L}/\phi''_{\hat{a}_H\hat{a}_H}$.

APPENDIX B

Taking logarithms of equation (5), after substituting from equation (12), and differentiating with respect to time, we find:

$$\frac{\dot{\omega}_H}{\omega_H} = \frac{\dot{z}}{z} - \frac{\mu'_e\dot{e}}{1+\mu} - \frac{\dot{z}}{1+z} = \frac{1}{1+z}\frac{\dot{z}}{z} - \frac{\mu'_e e}{1+\mu}\frac{\dot{e}}{e} = \omega_L\left[(1+\mu)\frac{\dot{z}}{z} - \mu'_e e(1+z)\frac{\dot{e}}{e}\right] \tag{B1}$$

Taking logarithms of equation (6), after substituting from equation (12), and differentiating with respect to time, we find:

$$\frac{\dot{\omega}_L}{\omega_L} = -\frac{\mu'_e\dot{e}}{1+\mu} - \frac{\dot{z}}{1+z} = -\left[\frac{\mu'_e e}{1+\mu}\frac{\dot{e}}{e} + \frac{z}{1+z}\frac{\dot{z}}{z}\right] = -\omega_L\left[z(1+\mu)\frac{\dot{z}}{z} + \mu'_e e(1+z)\frac{\dot{e}}{e}\right] \tag{B2}$$

Thus, $\dot{z}/z = \dot{e}/e = 0$ implies $\dot{\omega}_H/\omega_H = \dot{\omega}_L/\omega_L = 0$.
Taking logarithms of $\pi = 1 - \omega_H - \omega_L$ and differentiating with respect to time, we find:

$$\frac{\dot{\pi}}{\pi} = -\left(\frac{\dot{\omega}_H}{\pi} + \frac{\dot{\omega}_L}{\pi}\right) = -\left(\frac{\dot{\omega}_H}{\omega_H}\frac{\omega_H}{\pi} + \frac{\dot{\omega}_L}{\omega_L}\frac{\omega_L}{\pi}\right) = -\frac{\omega_L}{\pi}\left(z\frac{\dot{\omega}_H}{\omega_H} + \frac{\dot{\omega}_L}{\omega_L}\right) \tag{B3}$$

Thus, $\dot{\omega}_H/\omega_H = \dot{\omega}_L/\omega_L = 0$ implies $\dot{\pi}/\pi = 0$.
Taking logarithms of equation (5), after substituting from equation (12), and differentiating with respect to time, we find:

$$\frac{\dot{p}}{p}=\frac{\mu_e'\dot{e}}{1+\mu}+\frac{\dot{w}_L}{w_L}-\frac{\dot{a}_L}{a_L}+\frac{\dot{z}}{1+z}=\frac{\dot{w}_L}{w_L}-\frac{\dot{a}_L}{a_L}+\frac{\mu_e'e}{1+\mu}\frac{\dot{e}}{e}+\frac{z}{1+z}\frac{\dot{z}}{z}= \\ =\frac{\dot{w}_L}{w_L}-\frac{\dot{a}_L}{a_L}+\omega_L\left[z(1+\mu)\frac{\dot{z}}{z}+\mu_e'e(1+z)\frac{\dot{e}}{e}\right] \tag{B4}$$

Thus, $\dot{z}/z=\dot{e}/e=0$ implies $\dot{p}/p=\dot{w}_L/w_L-\dot{a}_L/a_L$.
Using equation (20), we have:

$$\frac{\dot{p}}{p}=\frac{\mu_e'e}{1+\mu}\frac{\dot{e}}{e}+\frac{z}{1+z}\frac{\dot{z}}{z}+\frac{\dot{w}_H}{w_H}-\frac{\dot{a}_H}{a_H}-\frac{\dot{z}}{z}=\frac{\dot{w}_H}{w_H}-\frac{\dot{a}_H}{a_H}+\frac{\mu_e'e}{1+\mu}\frac{\dot{e}}{e}-\frac{1}{1+z}\frac{\dot{z}}{z}= \\ =\frac{\dot{w}_H}{w_H}-\frac{\dot{a}_H}{a_H}-\omega_L\left[(1+\mu)\frac{\dot{z}}{z}-\mu_e'e(1+z)\frac{\dot{e}}{e}\right] \tag{B5}$$

Thus, $\dot{z}/z=\dot{e}/e=0$ implies $\dot{p}/p=\dot{w}_H/w_H-\dot{a}_H/a_H$.

APPENDIX C

Differentiating equations (25) and (26) with respect to z, we find:

$$\phi_{\hat{a}_H}'f_z^{H\prime}+\phi_{\hat{a}_L}'f_z^{L\prime}+\left(\phi_{\hat{a}_H}'f_\mu^{H'}+\phi_{\hat{a}_L}'f_\mu^{L'}\right)\mu_e'\left.\frac{de}{dz}\right|_{AK}=0 \tag{C1}$$

$$-\left(f_z^{H'}-f_z^{L'}\right)+\left[h_e'-\left(f_\mu^{H'}-f_\mu^{L'}\right)\mu_e'\right]\left.\frac{de}{dz}\right|_{Z}=0 \tag{C2}$$

After simplifying and rearranging, we have:

$$\left.\frac{de}{dz}\right|_{AK}=-\frac{(1-\rho z)f_z^{L'}}{(1+\rho z^2)f_\mu^{L'}\mu_e'} \tag{C3}$$

$$\left.\frac{de}{dz}\right|_{Z}=-\frac{(1+\rho z)f_z^{L'}}{h_e'+(1-\rho z)f_\mu^{L'}\mu_e'} \tag{C4}$$

Accordingly, $de/dz|_{AK}>0$ if and only if $z<\bar{z}$, whereas $de/dz|_{Z}>0$ if and only if $h_e'+\Gamma_\mu\mu_e'>0$ (remind that $\Gamma_\mu\equiv(1-\rho z)f_\mu^{L'}$).
Differentiating equations (25) and (26) with respect to γ, we find:

$$\left(\phi_{\hat{a}_H}'f_\mu^{H'}+\phi_{\hat{a}_L}'f_\mu^{L'}\right)\mu_\gamma'+\left(\phi_{\hat{a}_H}'f_\mu^{H'}+\phi_{\hat{a}_L}'f_\mu^{L'}\right)\mu_e'\left.\frac{\partial e}{\partial\gamma}\right|_{AK}=0 \tag{C5}$$

$$-\left(f_\mu^{H'}-f_\mu^{L'}\right)\mu_\gamma'+\left[h_e'-\left(f_\mu^{H'}-f_\mu^{L'}\right)\mu_e'\right]\left.\frac{\partial e}{\partial\gamma}\right|_{Z}=0 \tag{C6}$$

After simplifying and rearranging, we have:

$$\left.\frac{\partial e}{\partial \gamma}\right|_{AK} = -\frac{\mu'_\gamma}{\mu'_e} > 0 \tag{C7}$$

$$\left.\frac{\partial e}{\partial \gamma}\right|_{Z} = -\frac{(1-\rho z) f_\mu^{L'} \mu'_\gamma}{h'_e + (1-\rho z) f_\mu^{L'} \mu'_e} \tag{C8}$$

Accordingly, $\partial e/\partial\gamma|_Z > 0$ if and only if $h'_e + \Gamma_\mu \mu'_e > 0$ (remind that $\Gamma_\mu \equiv (1-\rho z) f_\mu^{L'}$).

APPENDIX D

For the numerical simulation, we specify the functional forms of equations (10), (12), and (13), as follows:

$$\phi = -\frac{1}{2\rho}\hat{a}_H^2 - \frac{1}{2}\hat{a}_L^2 - a\hat{a}_H - b\hat{a}_L + \tau, \qquad a > 0,\; b > 0,\; \rho > 0,\; \tau > 0 \tag{D1}$$

$$\frac{\dot{w}_H}{w_H} = \alpha + \frac{\lambda}{1-e}, \qquad \alpha < 0,\; \lambda > 0 \tag{D2}$$

$$\mu = \frac{\gamma}{e}, \qquad \gamma > 0 \tag{D3}$$

Equation (D1) is a quadratic function for the innovation possibility frontier with $\phi''_{\hat{a}_L\hat{a}_L} = -1$. We assume a non-linear specification for the relation between wage and employment, as in Desai, *et al.* (2006), but we express it as a nominal Phillips curve (equation (D2)). Equation (D3) is a non-linear specification for the relation between mark-up and high-skilled employment rate, such that if $e \to 1$, then $\mu \to \gamma$ and $\pi \to \gamma/(1+\gamma)$, if $e \to 0$, then $\mu \to \infty$ and $\pi \to 1$.
In this case, $\hat{a}_H$ and $\hat{a}_L$ are given by:

$$\hat{a}_H = \left[-a + \frac{ez}{\gamma(1+z)}\right]\rho \tag{D4}$$

$$\hat{a}_L = -b + \frac{e}{\gamma(1+z)} \tag{D5}$$

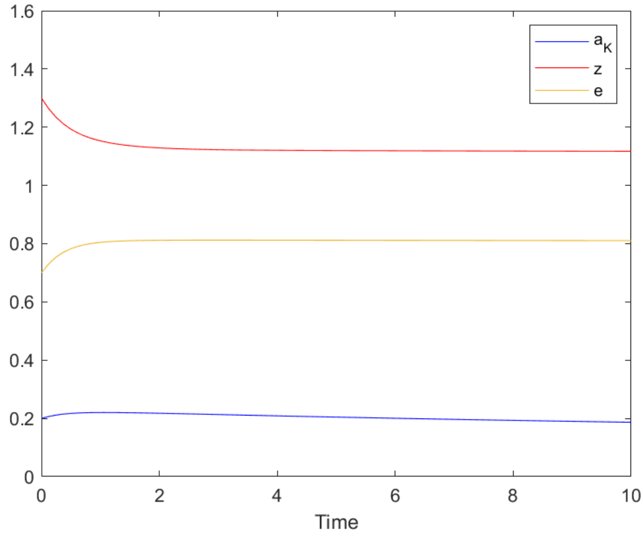
Thus, the dynamical system becomes:

$$\frac{\dot{a}_K}{a_K} = \frac{1}{2}(a^2\rho + b^2 + 2\tau) - \frac{1}{2}\left[\frac{1+\rho z^2}{\gamma^2(1+z)^2}\right]e^2 \tag{D6}$$

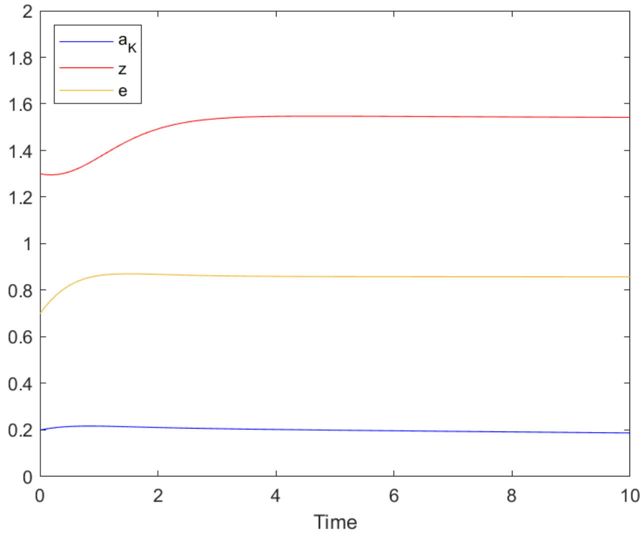
$$\frac{\dot{z}}{z} = \alpha - \beta + a\rho - b + \frac{\lambda\gamma(1+z) + (1-\rho z)(1-e)e}{\gamma(1+z)(1-e)} \tag{D7}$$

$$\frac{\dot{e}}{e} = \frac{\dot{a}_K}{a_K} + \frac{e}{\gamma+e}\left[\frac{\gamma + (\gamma + se)z}{e(1+z)}\right]a_K - \left[-a + \frac{ez}{\gamma(1+z)}\right]\rho - n \tag{D8}$$

Fig. 5. *Convergence to the steady state in the baseline scenario*



a) Case I: $z^* < \bar{z}$



b) Case II: $z^* > \bar{z}$

Note: Time series of wage inequality (z), high-skilled employment rate (e), and output-capital ratio (a_K) in the baseline scenario. Initial values: $z(0) = 1.3$, $e(0) = 0.7$, $a_K(0) = 0.2$.

Case 1: $z^* < \bar{z}$

In the baseline scenario (Fig. 1a), we set the parameters as follows:

$$\begin{array}{ccccc} \tau = 0.058 & a = 1.185 & b = 1.09 & \rho = 0.8 & s = 0.2 \\ \alpha = -0.2 & \beta = 0.04 & \gamma = 0.345 & \lambda = 0.05 & n = 0.01 \end{array} \tag{D9}$$

Fig. 2a, 3a, 4a display the long-run equilibrium values corresponding to a 1-percentage-point increase in α (i.e. $\alpha = -0.19$), a 1-percentage-point increase in β (i.e. $\beta = 0.05$), and a 0.5-percentage-points increase in γ (i.e. $\gamma = 0.35$), respectively.

Fig. 5a shows that the dynamical system is locally stable in the baseline scenario. Small changes in the parameter values do not alter the stability properties of the system.

Case 2: $z^* > \bar{z}$

In the baseline scenario (Fig. 1b), we set the parameters as follows:

$$\begin{array}{ccccc} \tau = 0.076 & a = 1.05 & b = 0.71 & \rho = 0.8 & s = 0.2 \\ \alpha = -0.27 & \beta = 0.04 & \gamma = 0.46 & \lambda = 0.05 & n = 0.005 \end{array} \tag{D10}$$

Fig. 2b, 3b, 4b display the long-run equilibrium values corresponding to a 1-percentage-point increase in α (i.e. $\alpha = -0.26$), a 1-percentage-point increase in β (i.e. $\beta = 0.05$), and a 0.5-percentage-points increase in γ (i.e. $\gamma = 0.465$), respectively.

Fig. 5b shows that the dynamical system is locally stable in the baseline scenario. Small changes in the parameter values do not alter the stability properties of the system.

APPENDIX E

Using equations (43), (44), (51), (52), (59) and (60), total differentiation of the dynamic equation of e evaluated at the equilibrium point with respect to α, β and γ yields:

$$\frac{da_K^*}{d\alpha} = \frac{h'_\alpha\left[(1-\rho z)g'_\mu - (1+\rho z^2)g'_z - \rho(1+z)f_z^{L'} f_\mu^{L'}\right]\mu'_e}{\sigma} \tag{E1}$$

$$\frac{da_K^*}{d\beta} = -\frac{\left[(1-\rho z)g'_\mu - (1+\rho z^2)g'_z - \rho(1+z)f_z^{L'} f_\mu^{L'}\right]\mu'_e}{\sigma} \tag{E2}$$

$$\frac{da_K^*}{d\gamma} = \frac{h'_e\left[(1+\rho z^2)g'_z f_\mu^{L'} - (1-\rho z)g'_\mu f_z^{L'} + \rho(1-z)f_z^{L'} f_\mu^{L'}\right]\mu'_\gamma}{\sigma} \tag{E3}$$

$z^* > \bar{z}$ implies $da_K^*/d\alpha > 0$ and $da_K^*/d\beta < 0$.
Using $\hat{p} = \widehat{w}_H - \hat{a}_H = \widehat{w}_L - \hat{a}_L$, we find:

$$\frac{d\hat{p}^*}{d\alpha} = -\frac{h'_\alpha \rho z(1+z)f_z^{L'} f_\mu^{L'} \mu'_e}{\sigma} > 0 \tag{E4}$$

$$\frac{d\hat{p}^*}{d\beta} = \frac{\left[(1-\rho z)h'_e - \rho(1+z)f_\mu^{L'} \mu'_e\right]f_z^{L'}}{\sigma} \tag{E5}$$

$$\frac{d\hat{p}^*}{d\gamma} = \frac{h'_e \rho z(1+z)f_z^{L'} f_\mu^{L'} \mu'_\gamma}{\sigma} > 0 \tag{E6}$$

$z^* > \bar{z}$ implies $d\hat{p}^*/d\beta > 0$.