Debt hierarchy: autonomous demand composition, growth and indebtedness in a Supermultiplier model

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Abstract

Supermultiplier models contend that non-capacity creating autonomous expenditures drive economic growth in the long run. In contrast, capacity-creating expenditures are mainly responsive to the principle of capital stock adjustment associated with flexible accelerator models of induced investment. The financial implications of this theoretical closure to the economic system arguably need further development. This paper develops a closed economy stock-flow consistent (SFC) Supermultiplier model featuring households', firms' and government's indebtedness and two sources of autonomous demand (government and household consumption); and explores analytically: (i) the stability conditions of the system; (ii) the impact of growth in the fully adjusted position, including considerations about the traverse to such a state; and (iii) the effects of a change in the composition of autonomous expenditures over debt-to-output ratios. The main results suggest that: (i) the framework we develop is compatible with empirical evidence regarding the relation between private and public debt-to-GDP ratios; (ii) the Supermultiplier model is capable of generating unstable trajectories of indebtedness that are typically associated with Minsky-Godley views of unsustainable processes, although through different economic mechanisms; and (iii) the composition of total autonomous demand matters for the distribution of debt in the economy, in a way that public debt plays a stabilizing role over household debt.

Keywords: growth; aggregate demand; Supermultiplier models; stock-flow consistent; public debt; private debt.
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1 Introduction

Increasing government deficit and debt to GDP ratios have been in the spotlight since the COVID-19 outbreak and subsequent pandemics urged governments to adopt spending and revenue fiscal measures to deal with the health and economic impacts of the crisis on the population (International Monetary Fund, 2020, 2021). One more time, the role of public spending, its limits, and implications have come to the forefront of the economic debate. At the same time, mounting accumulated private debt due to the drastic reduction in aggregate demand and significant shifts in its sectoral composition will likely pose pressures for the deleveraging of the private sector in the aftermath of the pandemics.

This context makes even more patent the need to address the relation of public debt and spending and GDP growth and the interactions of public and private debt in a (lack of) growth scenario. With that in mind, we build a stock-flow consistent (SFC) Supermultiplier model with two debt-financed autonomous expenditures – government and household autonomous consumption – to explore some consequences of changes in the growth rate and composition of autonomous expenditures on the institutional sectors' debt ratios and on aggregate demand.

The Supermultiplier approach represents growth as a demand-led process, where non-capacity creating autonomous expenditures lead growth in the long run (Serrano, 1995; Bortis, 1997). In turn, capacity-creating expenditures are mainly responsive to the principle of capital stock adjustment associated with some version of a flexible accelerator investment function (Freitas & Serrano, 2015; Allain, 2015; Lavoie, 2016; Fazzari et al., 2020). We rely on this approach because it reproduces straightforwardly important regularities found in empirical studies from different theoretical strands. Namely: (i) a positive relationship between autonomous demand and GDP; (ii) a positive relationship between GDP and the investment to GDP ratio (Girardi & Pariboni, 2016; Stirati & Meloni, 2018; Wen, 2007); and (iii) the unmatched strength of the income-accelerator effect on aggregate investment (Chirinko, 1993; Bivens, 2017; Girardi et al., 2020).

To deal with the interactions of private and public debt, the model we construct assumes a hierarchical relation, with national government debt better positioned than private debt (both issued in the domestic currency). Accordingly, debt-financed government expenditures are not subject to the same constraints of debt-financed private expenditures. The reasoning for this hierarchy builds on Lerner's idea of monetary sovereignty and theory of functional finance (Lerner, 1943, 1947) but also on other authors' contributions, such as Minsky's view on the greater liquidity of government assets in comparison to private ones (Minsky, 1986). In a way, this assumption is hardly controversial since rarely the risk premium on national government debt issued in its own currency surpasses that of the *collective* of private agents'.

Our main contribution is twofold: (i) our model contributes to the Supermultiplier literature (Freitas & Serrano, 2015; Lavoie, 2016; Allain, 2015; Nah & Lavoie, 2019; Fazzari et al., 2020), as it addresses the typical absence of the financial counterparts of autonomous expenditures often highlighted as a caveat of these growth models (Brochier & Macedo e Silva, 2019; Hein & Woodgate, 2021). It also contributes more generally to demand-led growth models as it provides an in-depth discussion of the relation between public and private debt; (ii) it gives substance to relationships that are vastly explored empirically but remain under-explored theoretically.

First, canonical versions of the Supermultiplier model assume private consumption (Freitas & Serrano, 2015; Lavoie, 2016), government consumption (Allain, 2015), or exports (Nah & Lavoie, 2017) as the autonomous demand component leading growth. However, since these models are designed to be simple, they do not deal explicitly with the financial implications of such expenditures. In that regard, there have been important contributions highlighting the implications of government debt accumulation in Supermultiplier models where autonomous government expenditures lead growth (Dutt, 2019, 2020; Hein, 2018; Hein & Woodgate, 2021; Fiebiger, 2021; Cassetti, 2020). Some studies also explore the implications of private debt accumulation in the model, with the inclusion of credit-financed private consumption as the autonomous growth driver (Pariboni, 2016; Fagundes, 2017; Mandarino et al., 2020; Brochier & Freitas, 2021).

Freitas and Christianes (2020) take a step further by addressing the implications of government debt dynamics in a Supermultiplier model with two autonomous demand components, government and household consumption. This version of the Supermultiplier, in contrast to Cassetti (2020), Hein and Woodgate (2021), and Fiebiger (2021), shows that self-defeating austerity and the paradox of debt are possible, although not necessary, long run outcomes of Supermultiplier models when the model deals with more than one source of autonomous expenditures. That said, Freitas and Christianes (2020) do not take into account the financing of household autonomous consumption; consequently, their model rules out the demand effects of the process of household debt accumulation and the features related to public and private debt *interactions* in a growth setting.

Since we are interested precisely in analyzing the interactions of the institutional sectors' indebtedness and growth, we assume there are two sources of debt-financed autonomous expenditures, government and household consumption, and also that firms fill the gap of their investment financial needs with loans.[1]

Secondly, our model also explores issues that relate to at least three topics subject to rich scrutiny in empirical studies on debt to income ratios: (a) the linkages of government debt and GDP growth; (b) the relation of private debt, economic activity, and the likelihood of crises; and (c) the public-private debt nexus.

(a) GDP growth and government debt to GDP ratio unconditional correlation appear to be negative (Panizza & Presbitero, 2014). Indeed, it is possible to find empirical studies backing causality running both ways. More specifically, the seminal work of Reinhart and Rogoff (2010) found a detrimental 90% threshold of government debt to GDP on growth. Herndon et al. (2014) found spreadsheet flaws invalidating Reinhard and Rogoff's results correlation-wise. Econometrically, the discussion around these works has originated a vast body of empirical studies exploring the relation of government debt to GDP ratio and GDP growth. However, many of these studies do not address appropriately critical econometric issues, such as endogeneity, heterogeneity of countries,

[1]For Supermultiplier models that also deal with firms' indebtedness, see Brochier and Macedo e Silva (2019), Brochier and Freitas (2021), Mandarino et al. (2020), Cassetti (2020), and Fiebiger (2021).

and identification of causality (Amann & Middleditch, 2020; Jacobs et al., 2020). Recent studies that handle those issues tend to find causality running from GDP growth to government debt to GDP ratio (Jacobs et al., 2020; Guerini et al., 2020), or at least fail to find a robust relation running from government debt to GDP ratio to GDP growth (Panizza & Presbitero, 2014). Guerini et al. (2020), for instance, find that public debt shocks have positive and persistent effects on the level of economic activity.

(b) The effects of expanding private debt on GDP are positive but short-lived, if not detrimental to economic activity in the longer run (Guerini et al., 2020; Mian et al., 2017). Debt accumulation, particularly in the household sector, is related to an increased likelihood of financial crises (Schularick & Taylor, 2012; Jordà et al., 2016; Mian et al., 2017; Mian & Sufi, 2018).

(c) At last, available evidence shows that public and private debt tend to present a significant negative correlation. Accordingly, deleveraging attempts by the private sector are usually met by increasing government indebtedness. Conversely, credit booms are typically compatible with falling government debt to GDP ratios (Mbaye et al., 2018). Reinhart and Rogoff (2010) found this relation for the aftermath of financial crises.

To address these issues, we extend Freitas and Serrano (2015) by including in the model households', firms', and government's indebtedness. More specifically, we combine previous contributions to the Supermultiplier model that address household debt (Pariboni, 2016; Fagundes, 2017; Brochier & Freitas, 2021; Mandarino et al., 2020), government debt (Freitas & Christianes, 2020) and firms' debt (Brochier & Freitas, 2021; Brochier & Macedo e Silva, 2019; Mandarino et al., 2020). By integrating two sources of debt-financed autonomous expenditures, the resulting SFC model sheds light on novel aspects regarding the interrelations between private and public debt. We solve the SFC Supermultiplier model analytically and examine the stability conditions, focusing on the steady-state stock-flow norms.

Based on this procedure, we reach three main results: (i) a more complex Supermultiplier model, which deals with more than one source of autonomous expenditures, can deal with financial developments and their implications for an economic system. For one thing, in this two-autonomous demand setting, the paradox of debt is no longer a necessary result neither for the government nor for the household sector, as typical in Supermultiplier models; (ii) the inclusion of the financial counterpart of these autonomous expenditures, in this case, debt-financed government and household consumption, changes the overall stability conditions of the Supermultiplier model, as in Hein and Woodgate (2021). More specifically, the financial ratios will add a lower bound for a stable growth path, with government debt stability posing the most binding constraint; and (iii) the model also provides a theoretical explanation for the empirical findings on the relations of private and public debt and GDP growth. The model's results shed light on interactions of fiscal policy and private sector's willingness to deleverage. For instance, when a credit boom comes to an end and households deleverage while the government increases spending to countervail the demand leakages, households debt to GDP ratio falls, and government debt to GDP rises. Besides, when there is a decrease in the trend of output (determined in the model by the growth rate of autonomous expenditures), both household and government debt ratios increase.

Besides this introduction, the paper is structured as follows: section 2 presents the accounting framework and the behavioral equations of the model as well as the short run goods' market equilibrium. Section 3 deals with the system local stability conditions, presents the short run dynamic equations of the institutional sectors' financial ratios, and analyses the effects of the autonomous demand growth rate and composition on the long run financial ratios. Section 4 discusses the economic meaning of the effects of the autonomous demand growth rate and composition on the average growth rate and on the debt ratios through the analysis of different scenarios. At last, section 5 sums up the main findings of the paper.

2 A model with two debt-financed autonomous expenditures

The model assumes a hierarchy between government and private debt (both issued in the domestic currency), with government debt better positioned than private debt. Accordingly, debt-financed government expenditures are not subject to the same constraints as debt-financed private expenditures.[2] The reasoning is as follows.

Credit-financed private expenditures, be that of households or firms, are oftentimes subject to credit rationing or subtle tightening of credit conditions imposed by banks and financial institutions. A higher indebtedness in these sectors may eventually contribute to mounting financial fragility (Minsky, 1982) and precipitate a reversal in these aggregate spending components. Besides, in the case of households, there is a further restriction to credit-financed expenditures imposed by the fact that they have to provide for their subsistence. Consequently, a feasibility condition is that debt servicing cannot surpass a certain share of their average income (Setterfield & Kim, 2020).

As for a (national) government that has monetary sovereignty, issuing debt denominated in the currency it creates, the same restrictions do not apply (Lerner, 1943). The reason is that the government pays for its cash commitments using its currency (Lerner, 1947), making it implausible to assume a risk of default commensurable with the private sector's. Open economies, especially peripheral ones, do face constraints that are more binding than a closed economy (see Prates, 2020). Macroeconomically, however, external constraints would hurt the private sector's degrees of freedom at least as much as it would for the government. Accordingly, even in open economies, the assumption that government expenditure is less constrained than private's one should hold.

The model is framed in continuous time[3] and deals with a pure credit closed capitalist economy featuring four institutional sectors: households, non-financial firms, banks, and government. We also implicitly consider that the central bank determines the interest rate, which is homogeneous across different assets for simplicity. The households sector comprises workers and capitalists, which we model as a unified institutional sector.

[2] A similar view is shared by Cassetti (2020).

[3] The convention of models in continuous is to express variables changing in time as $x(t)$. We write x_t to make it easier to read while still allowing for the distinction of parameters and time-changing variables.

2.1 Accounting framework and behavioral equations

Tables 1 and 2 present the basic accounting framework of the model, displaying respectively the balance sheet of the four institutional sectors the transactions between these sectors, including current, capital and flow of funds accounts. In what follows, we present the behavioral assumptions.

Table 1: Balance sheet of the economy.

	Households	**Firms**	**Banks**	**Govt.**	$\sum$
Bills			$+B$	$-B$	0
Deposits	$+D$		$-D$		0
Loans	$-L_h$	$-L_f$	$+L$		0
Physical capital		$+K$			$+K$
Net worth	$+V_h$	$+V_f$	0	$-B$	$+K$

Table 2: Current accounts, capital account and flows of funds.

	Households	**Firms**		**Banks**	**Government**	$\sum$
		Current	Capital			
Wages	$+W$	$-W$				0
Consumption	$-C_h$	$+C+G$			$-G$	0
Investment		$+I$	$-I$			0
Taxes	$-T_h$	$-T_f$			$+T$	0
Interest (loans)	$-rL_h$	$-rL_f$		$+rL$		0
Interest (bills)				$+rB$	$-rB$	0
Profits (firms)	$+P_f^d$	$-P_f$	$+P_f^u$			0
Profits (banks)	$+P_b$			$-P_b$		0
Subtotal	$+SAV_h$	0	$+SAV_f$	0	$+SAV_b$	0
Change bills				$-\dot{B}$	$+\dot{B}$	0
Change deposits	$-\dot{D}$			$+\dot{D}$		0
Change loans	$+\dot{L}_h$		$+\dot{L}_f$	$-\dot{L}$		0
Column total	0	0	0	0	0	0

The autonomous demand injections (Z_t) grow, for simplicity, at an exogenously given rate (g_z) in the steady-state:

$$Z_t = Z_0 e^{g_z t} \tag{1}$$

As in Freitas and Christianes (2020), Z_t splits into two sub-components, namely households autonomous consumption (Z_{h_t}) and government autonomous consumption (Z_{g_t}):

$$Z_t = Z_{h_t} + Z_{g_t} \tag{2}$$

Equation (2) allows us to have two sources of autonomous demand while impeding the share of either of these expenditures to total autonomous demand to tend endogenously towards zero or one in the long run.

Government autonomous consumption is defined as a fraction (σ) of total autonomous demand injections:

$$Z_{g_t} = \sigma Z_t \tag{3}$$

Notice that σ also determines the composition of autonomous demand injections between government and household consumption (with $0 < \sigma < 1$). With σ exogenously given, we are implicitly assuming the same average autonomous demand growth rate in the long run (Freitas & Christianes, 2020). Despite that, this modeling strategy allows us to compare different growth patterns of autonomous demand as the growth rate of each component can be temporarily distinct.

Substituting equation (3) into (2), household autonomous consumption can also be expressed as a fraction of total autonomous demand injections:

$$Z_{h_t} = (1 - \sigma) Z_t \tag{4}$$

Households income (Y_h) is the sum of wages and financial income accruing from distributed profits of firms (P_f^d) and banks (P_b):

$$Y_{h_t} = W_t + P_{f_t}^d + P_{b_t} \tag{5}$$

Households pay taxes (T_h) on both wages (W) and dividends:

$$T_{h_t} = t_h W_t + t_p (P_{f_t}^d + P_{b_t}) \tag{6}$$

where $0 \leq t_h < 1$ and $0 \leq t_p < 1$ are respectively the marginal tax rate on wages and property income. For simplicity, we assume that interest expenses (rL_h) are non-deductible of the tax basis.

Income distribution is given exogenously, with a fixed wage share of pre-tax income ($0 < \omega < 1$).[4] The total wage income is then given by:

$$W_t = \omega Y_t \tag{7}$$

where Y is the total output.

Following Fagundes (2017) and Pariboni (2016), we assume that households take on new loans to finance autonomous consumption and that they repay part of their accumulated debt (L_{h_t}) at a given amortization rate (λ, with $0 \leq \lambda \leq 1$). Households' debt dynamics is then given by:

$$\dot{L}_{h_t} = Z_{h_t} - \lambda L_{h_t} \tag{8}$$

[4]This is the case in both neo-Kaleckian and Sraffian income distribution theories. They generally assume no regular or systematic relationship through which the growth of output affects income distribution.

Households' new loans correspond to Z_h, implying that the household sector takes new loans to consume. In its turn, λ reflects an exogenous duration of debt. In case we have $\lambda = 0$, loans are fully rolled over.

Besides the autonomous consumption, households consume a fraction of their after-tax disposable wage income:

$$C_t = Z_{h_t} + c_h \left[(1 - t_h)\omega Y_t - (r + \lambda)L_{h_t}\right] \tag{9}$$

where $0 \ll c_h \leq 1$ is the marginal propensity to consume and C is the aggregate household consumption. From equation (9), one can notice that debt servicing (interest payments on loans and principal repayments) reduces households' disposable wage income for consumption.[5]

Since we assume away capital gains, households net wealth variation depends only on their financial balance (SAV_h):

$$\dot{V}_{h_t} = SAV_{h_t} \tag{10}$$

The financial balance of households can be readily retrieved from table 2:

$$SAV_{h_t} = W_t + P^d_{f_t} + P_{b_t} - T_{h_t} - T_{p_t} - C_{h_t} - rL_{h_t} \tag{11}$$

As for non-financial *firms*, their investment behavior is based on the flexible accelerator principle, as usual in the Supermultiplier approach. Aggregate investment of firms is induced by income (12) and the marginal propensity to invest (h, with $0 \leq h < 1$) endogenously reacts to the discrepancies between the actual (u) and the normal capacity utilization rate ($\bar{u}$, with $0 < \bar{u} < 1$) according to the sensitivity parameter γ (with $\gamma > 0$) (13) (Freitas & Serrano, 2015). For avoiding instability in firms' adjustment of capacity to (expected) demand, γ must assume sufficiently low values (Freitas & Serrano, 2015).[6,7]

$$I_t = h_t Y_t \tag{12}$$

$$\dot{h}_t = h_t \gamma (u_t - \bar{u}) \tag{13}$$

Assuming that the production is not curbed by labor or natural resources limitations, the only constraint to output is the stock of physical capital. Accordingly, the potential or full capacity output of the economy is given by:

$$Y_{K_t} = \frac{1}{v} K_t \tag{14}$$

where Y_K is the full potential output, v is a given capital-output ratio, and K is the capital stock.

[5]There is an extensive neo-Kaleckian literature that develops mainly after the 2007-08 financial crisis, that deals with household credit-financed consumption and indebtedness. See, among others, Dutt (2006), Hein (2012), Setterfield et al. (2016), and Setterfield and Kim (2020).

[6]Allain (2015) and Lavoie (2016) present a similar adjustment process of the actual capacity utilization towards the normal one with similar implications. However, in their case, the adjustment happens directly in the accumulation rate through the endogenous change in the expected trend rate of sales. Also similarly, in their case, the parameter that represents the speed of adjustment must be low enough not to trigger instability.

[7]Fazzari et al. (2020) use $\gamma \simeq 0.09$ (λ in their notation) as the benchmark estimation for the US economy.

Thus, the actual capacity utilization rate is defined as the ratio of output to full-capacity output $u = Y/Y_K$. From these equations, we can draw the capital accumulation rate, which depends on non-financial firms investment and on the capital depreciation rate ($0 < \delta \ll 1$):

$$g_{K_t} = \frac{h_t u_t}{v} - \delta \tag{15}$$

Firms pay taxes (T_f) on their profit net of interest payments on loans:

$$T_{f_t} = t_f(\pi Y_t - rL_{f_t}) \tag{16}$$

where $0 \leq t_f < 1$ is the marginal tax rate applicable to firms' income and $\pi = 1 - \omega$ is the profit share of income. Naturally, pre-tax profits before depreciation and amortization and after interest payments corresponds $\pi Y_t - rL_{f_t}$.

A portion $0 < s_f < 1$ of firms' profits gets retained ($P^u_{f_t}$), whereas $1 - s_f$ (dividend payout ratio) gets distributed to households:

$$P^u_{f_t} = s_f(1 - t_f)(\pi Y_t - rL_{f_t}) \tag{17}$$

$$P^d_{f_t} = (1 - s_f)(1 - t_f)(\pi Y_t - rL_{f_t}) \tag{18}$$

The budget constraint of firms gets closed with akin to Wood's (1975) and pecking order's (Myers, 1983) theories. Namely, firms take on credit from banks whenever available internal funds are not enough to pay for new investment plans. Accordingly, firms' sector debt (L_f) dynamics is given by:

$$\dot{L}_{f_t} = I_t - P^u_{f_t} \tag{19}$$

Banks accommodate both households and firms' demand for credit and buy government bills. They distribute the entire amount of profits resulting from these operations (20) to households. Households' deposits at commercial banks do not pay interest, which seems to be a fairly realistic assumption. For the sake of simplicity, we assume that the real interest rate, as prices are held constant, is set exogenously by the monetary authority, as in Ryoo and Skott (2013); and that it is the same for government bills, household and firms' loans.

$$P_{b_t} = rB_t + rL_t = rB_t + r(L_{h_t} + L_{f_t}) \tag{20}$$

Government consumption is fully autonomous, as given by equation (3) ($Z_{g_t} = G_t = \sigma Z_t$). As already defined above, taxes are levied on wages, firms' profits and property income (banks' and firms' distributed profits):

$$T_t = T_{h_t} + T_{f_t} = t_h \omega Y_t + t_f \left(\pi Y_t - rL_{f_t}\right) + t_p \left[(1 - s_f)(1 - t_f)\left(\pi Y_t - rL_{f_t}\right) + rB_t + rL_t\right] \tag{21}$$

All government debt is short-term, for simplicity. The government issuance (withdraw) of bills goes in line with its nominal balance, giving its debt (B) dynamics:

$$\dot{B}_t = \sigma Z_t - T_t + rB_t \tag{22}$$

Finally, we assume that the banking sector will hold all debt issued by the government, for the following reasons. Implicitly, we consolidate the central bank into the banking sector to tame the model's complexity. Considering this setting, if the central bank operates the monetary policy by fixing the interest rate, as they typically do,[8] it must accommodate the demand for liquid assets from the "commercial" banks. In our model, this liquid asset is the government bills. Accordingly, the value of the central bank's public bills holdings functions as a buffer to meet commercial banks' demand and government bills supply. Hence, this ensures $B^s = B^d$, as we assume.

2.2 Short run goods' market equilibrium

For analyzing the goods market equilibrium in the short run, we define three auxiliary equations that represent short-term variables, and that will also be essential to simplify the stability analysis of the system.

Writing $\dot{L}_{h_t}$ as $g_{l_{h_t}} L_{h_t}$ – with $g_{l_{h_t}}$ representing the growth rate of household debt – in the equation for changes in household debt (8) and then solving it for Z_t and L_{h_t}, we get respectively:

$$Z_t = \frac{L_{h_t}(g_{l_{h_t}} + \lambda)}{1 - \sigma} \tag{23}$$

$$L_{h_t} = \frac{Z_t(1 - \sigma)}{g_{l_{h_t}} + \lambda} \tag{24}$$

Also if we divide equation (8) by L_{h_t}, we will get an equation for household debt growth rate. By taking the total derivative of this equation, considering that $\dot{Z} = g_z Z_t$, and then substituting equation (23) into it, we get an equation for the rate of change of household debt growth rate:

$$\dot{g}_{l_{h_t}} = (g_{l_{h_t}} + \lambda)(g_z - g_{l_{h_t}}) \tag{25}$$

When the goods' market is in equilibrium, we will have that real output is equal to real aggregate demand, which is the sum of private consumption (9), government consumption (3) and non-financial firms investment (12):

$$Y_t = Z_t - c_h(r + \lambda)L_{h_t} + h_t Y_t + c_h(1 - t_h)\omega Y_t \tag{26}$$

[8]See Bindseil (2004) and Lavoie (2014, ch. 4) for a detailed discussion of how central banks implements the monetary policy.

Solving for Y_t we get the short run equilibrium real output:

$$Y_t = \left(\frac{1}{s - h_t}\right)\left[Z_t - c_h(r + \lambda)L_{h_t}\right] \tag{27}$$

Where $s = 1 - c_h(1 - t_h)\omega$ is the marginal propensity to save and the expression between parentheses on the RHS of equation (27) represents the supermultiplier, which combines the multiplier effect of induced consumption and the accelerator effect of induced investment on income.

From the equation above, we notice that for total autonomous expenditures to be positive $(Z_t - c_h(r + \lambda)L_{h_t} > 0)$, autonomous injections from household and government consumption must exceed autonomous leakages arising from the negative effect debt servicing exerts on disposable wage income and, consequently, on induced consumption.

By substituting equation (24) into (27) and normalizing it by the capital stock, we get the following equation for the short-run capacity utilization rate:

$$u_t = \frac{1}{(1 - \sigma)(s - h_t)} \upsilon \left[g_{l_{h_t}} + \lambda - c_h(r + \lambda)(1 - \sigma)\right] l_{h_{k_t}} \tag{28}$$

Where $l_{h_{k_t}} = L_{h_t}/K_t$ is household debt to capital ratio.

The existence of a goods' market stable equilibrium requires the marginal propensity to spend to be lower than unity. That is, the denominator of equation (28) must be positive. Since we are assuming $0 < \sigma < 1$, the standard condition for stability applies:

$$c_h(1 - t_h)\omega + h_t < 1 \tag{C1}$$

Assuming this stability condition holds and that household debt to capital ratio is positive ($l_{h_{k_t}} > 0$), positive values for the level of output and the capacity utilization rate also require a positive level of total autonomous expenditures. For guaranteeing total autonomous demand will assume positive figures, the following condition must be satisfied:

$$g_{l_{h_t}} > c_h(r + \lambda)(1 - \sigma) - \lambda \tag{C2}$$

Note that this condition is the same one that has to be satisfied for household debt to capital ratio to positively affect the capacity utilization rate.[9] Yet this condition should not be interpreted as saying that higher debt raises aggregate income, instead it should be read as saying that higher household autonomous consumption raises both household debt (through new loans needed to finance it) and aggregate income (Fagundes, 2017).

From equation (28), we also observe that the real interest rate has an unambiguously negative effect on the capacity utilization rate in the short run. This happens since higher interest rates translate into higher interest payments on loans that drag disposable wage income and induced

[9] $\frac{\partial u_t}{\partial l_{h_{k_t}}} = \frac{1}{(1 - \sigma)(s - h_t)} \upsilon [g_{l_{h_t}} + \lambda - c_h(r + \lambda)(1 - \sigma)]$

consumption. Due to that, one may also notice that an increase in the share of government consumption to total autonomous injections will positively affect the capacity utilization rate as long as the household debt growth rate exceeds the amortization rate with a minus sign. For a positive amortization rate, this means that a positive household debt growth rate is a sufficient condition for an increase in the share of government consumption to raise the capacity utilization rate.[10]

Plugging (24) into (26), solving for the total autonomous demand injections and dividing the result by output, we obtain the total autonomous demand injections to output ratio (z_t) in the short run:

$$z_t = \frac{Z_t}{Y_t} = \frac{(g_{l_{h_t}} + \lambda)(s - h_t)}{g_{l_{h_t}} + \lambda - c_h(r + \lambda)(1 - \sigma)} \tag{29}$$

By substituting equation (23) into (8), solving it for household debt, then dividing the result by output, we obtain household debt to output ratio in the short run:

$$l_{h_t} = \frac{L_{h_t}}{Y_t} = \frac{(1 - \sigma)(s - h_t)}{g_{l_{h_t}} + \lambda - c_h(r + \lambda)(1 - \sigma)} \tag{30}$$

From equations (29) and (30), we see directly that an increase in household debt growth rate has a negative impact both on total autonomous demand injections and on household debt ratio. This result, although seemingly counter-intuitive, is line with the transmission mechanisms of Supermultiplier models. An increase in household debt growth rate brought about by an increase in the pace of autonomous expenditures will lead to higher output growth rate and, consequently, as firms' react, to a higher investment rate. The higher propensity to invest leads to a temporarily faster pace of capital accumulation and faster output growth than the exogenous autonomous demand growth rate. Therefore, both total autonomous demand injections and household debt ratios decrease.

We can also observe that an increase (decrease) in household (government) autonomous consumption share of total autonomous demand injections will lead to a higher total autonomous demand injections ratio and a higher household debt ratio. The same effect can be noticed considering an increase in the interest rate.

At last, by substituting equation (24) into (26), taking its total derivative, dividing it by output and then substituting equations (30) (l_{h_t}), (25) ($\dot{g}_{l_{h_t}}$) and (13) ($\dot{h}_t$) into the resulting equation, we get the following equation for the growth rate of output in the short run:

$$g_{y_t} = g_{l_{h_t}} + \frac{(g_{l_{h_t}} + \lambda)(g_z - g_{l_{h_t}})}{g_{l_{h_t}} + \lambda - c_h(r + \lambda)(1 - \sigma)} + \frac{h_t \gamma (u_t - \bar{u})}{s - h_t} \tag{31}$$

Outside the steady growth path, the output growth rate is determined by the rate of expansion of total autonomous demand (first two elements on the RHS) and by the rate of change of the supermultiplier

[10] $\frac{\partial u_t}{\partial r} = -\frac{c_h v l_{h_{k_t}}}{s - h_t}$

$\frac{\partial u_t}{\partial \sigma} = g_{l_{h_t}} + \lambda$

(third element on the RHS).

3 Stability, autonomous demand, and indebtedness

In intensive form (normalized system's flow-flow, stock-flow, and stock-stock ratios), the dynamic system of the model is composed by six differential equations: the rate of change of firms' propensity to invest (32), the rate of change of household debt to capital ratio (33), the rate of change of household debt growth rate (34), the rate of change of firms' debt ratio (46), the rate of change of government debt ratio (36) and household wealth ratio (37):[11]

$$\dot{h}_t = h_t \gamma \left\{ \frac{1}{(1-\sigma)(s-h_t)} \upsilon \left[g_{l_{h_t}} + \lambda - c_h (r+\lambda)(1-\sigma) \right] l_{h_{k_t}} - \bar{u} \right\} \tag{32}$$

$$\dot{l}_{h_{k_t}} = l_{h_{k_t}} \left\{ g_{l_{h_t}} - \frac{h_t \frac{1}{(1-\sigma)(s-h_t)} \upsilon \left[g_{l_{h_t}} + \lambda - c_h (r+\lambda)(1-\sigma) \right] l_{h_{k_t}}}{\upsilon} + \delta \right\} \tag{33}$$

$$\dot{g}_{l_{h_t}} = (g_{l_{h_t}} + \lambda)(g_z - g_{l_{h_t}}) \tag{34}$$

$$\dot{l}_{f_t} = f \left[h_t, l_{h_{k_t}}, g_{l_{h_t}} \right] \tag{35}$$

$$\dot{b}_t = f \left[h_t, l_{h_{k_t}}, g_{l_{h_t}} \right] \tag{36}$$

$$\dot{v}_{h_t} = f \left[h_t, l_{h_{k_t}}, g_{l_{h_t}} \right] \tag{37}$$

Equations 46, 36 and 37 are generic functions with the endogenous factors influencing firms' sector debt, government debt, and household wealth to output ratios. We further develop equations (46) and (36) in the current section. We do not develop results for (37) in the paper because it leads to a cumbersome equation without giving further relevant insight into the context of our model.[12]

In this system:

- The rate of change of household debt growth rate is independent of the other dynamic variables of the system;
- The rate of change of household debt to capital ratio and firms' propensity to invest depend on each other and on the rate of change of household debt growth rate. They are, however, independent of government debt ratio, firms' debt ratio, and household wealth ratio;
- The other financial ratios (eqs. 46, 36 and 37) depend on firms' propensity to invest, household debt to capital ratio, and household debt growth rate.[13]

[11]For obtaining the dynamic equation for the propensity to invest in the form presented in equation (32), we substitute (28) into (13). For obtaining the dynamic equation for household debt to capital ratio (33), we normalize equation (8) by the capital stock; we then substitute equation (28) into (15) and the latter into the dynamic equation for household debt to capital ratio.

[12]Partly because we simplify away capital gains dynamics. The results are developed in the supplementary material provided with this paper.

[13]To get firms' debt ratio dynamic equation as a function of these three variables: substitute equation (28) into (31)

Hence, there are five independent sub-systems that we can analyze separately: (a) $\dot{g}_{l_h}$ dynamics is independent; (b) $\dot{h}_t$ and $\dot{l}_{h_{k_t}}$ are mutually dependent; and (c) $\dot{l}_{f_t}$, $\dot{b}_t$ and $\dot{v}_{h_t}$. The sub-systems in (c) are evidently contingent to what happens in sub-systems (a) and (b).

At the equilibrium, we must have the growth rate of all variables converging to the growth rate of autonomous expenditures ($g_{y_t} = g_z = g^*$), and the capacity utilization rate to the normal one ($u^* = \bar{u}$).[14] Taking the features of the system into account and these equilibrium conditions, we adopt the following procedure to arrive at the stability conditions around the steady state growth path presented in table 5:

1. The first sub-system analyzed is (a). We take the derivative of (34) with respect to $g_{l_{h_t}}$ and evaluate the stability condition at equilibrium:

$$\frac{\partial \dot{g}_{l_h}}{\partial g_{l_h}} = g_z - (\lambda + 2g_{l_{h_t}}) \tag{38}$$

$$\left[\frac{\partial \dot{g}_{l_h}}{\partial g_{l_h}}\right]_{g^*_{l_h}} = -(g_z + \lambda) < 0 \tag{39}$$

For the growth rate of household debt to be stable, the growth rate of autonomous demand must be greater than minus the amortization rate. Considering the amortization rate lies between zero and one, a positive growth rate of autonomous demand would be sufficient to ensure this condition holds. Therefore, the latter is easily satisfied in a realistic setting of macroeconomic parameters. Notice that the rate of change of household debt ratio does not appear in the dynamic system of equations for its stability condition around the steady state is the same of the growth rate of household debt.

2. If the growth rate of household debt is stable around the equilibrium, we can analyze the stability of the sub-system (b) – composed by the dynamics of the propensity to invest and of household debt-to-capital ratio – separately from the dynamic equations for government and firms' debt ratio and for household wealth ratio. From this sub-system, we obtain the following Jacobian matrix:

$$\mathbb{J} = \begin{bmatrix} \left[\frac{\partial \dot{h}}{\partial h}\right]_{h^*,l^*_{h_k}} & \left[\frac{\partial \dot{h}}{\partial l_{h_k}}\right]_{h^*,l^*_{h_k}} \\ \left[\frac{\partial \dot{l}_{h_k}}{\partial h}\right]_{h^*,l^*_{h_k}} & \left[\frac{\partial \dot{l}_{h_k}}{\partial l_{h_k}}\right]_{h^*,l^*_{h_k}} \end{bmatrix} = \begin{bmatrix} \frac{h^*\gamma\bar{u}}{s-h^*} & \frac{h^*\gamma v x}{(1-\sigma)(s-h^*)} \\ -\frac{(1-\sigma)\left[\frac{s\bar{u}}{v} - (g_z+\delta)\right]\bar{u}}{xv} & -(g_z+\delta) \end{bmatrix} \tag{40}$$

where $x = gz + \lambda - c_h(r+\lambda)(1-\sigma)$. The stability conditions of this 2x2 sub-system ($D[\mathbb{J}^*] > 0; T[\mathbb{J}^*] < 0$) evaluated at the equilibrium are similar to the conditions obtained in Freitas and

and the result into (44); for government debt ratio dynamic equation: substitute equations (44) and (49) into (60). In the resulting equation for the dynamics of government debt ratio substitute (31) and (28). The same goes for the household sector's wealth ratio dynamic equation.

[14]The long run conditions $g^* = g_z$ and $u^* = \bar{u}$ also imply a long run value for the propensity to invest: $h^* = \frac{v(g_z+\delta)}{\bar{u}}$. We obtain the latter by applying the equilibrium conditions to equation (15).

Serrano (2015) and Freitas and Christianes (2020):

$$D[\mathbb{J}^*] = (g_z + \delta)\gamma\bar{u} > 0 \tag{41}$$

$$T[\mathbb{J}^*] = \left[\frac{\gamma v}{s - \frac{v}{\bar{u}}(g_z + \delta)} - 1\right](g_z + \delta) < 0 \tag{42}$$

Rearranging (42), we get:

$$T[\mathbb{J}^*] = g_z < \frac{s\bar{u}}{v} - \delta - \gamma v \tag{43}$$

For economically meaningful values for the parameters, the determinant condition is always satisfied. Thus, the stability of the sub-system depends crucially on the trace condition, which can be interpreted as imposing a ceiling to the growth rate of autonomous demand that is compatible with a stable demand-led growth process (Freitas & Serrano, 2015). As in Freitas and Christianes (2020), the inclusion of a government sector that taxes workers makes the upper limit for the autonomous demand growth less stringent as it increases the marginal propensity to save.[15]

3. Assuming the stability conditions of sub-systems (a) and (b) hold, in what follows, we analyze the local stability conditions for the remaining financial ratios (c) at equilibrium. The exception is the household wealth ratio local stability condition since it is the same of government debt ratio.

Besides the local stability conditions for the financial ratios, in the remaining subsections we also analyze the debt dynamics of each institutional sector separately and the effects of the autonomous expenditures growth rate (g_z) and their composition (σ) in the long run financial ratios. These parameters are important for the following reasons: g_z is the economy's growth driver, whereas σ is the composition of autonomous demand, through which we can represent changes in the aggregate behavior of household and government sectors. For instance, if households' autonomous spending temporarily outgrows government spending, we can show that through a fall in σ.

3.1 Non-financial firms' indebtedness and debt dynamics

We start by dealing with firms' debt ratio dynamics. We get the differential equation that shows how firms' debt ratio changes over time by substituting equations (12) (I_t) and (17) ($P^u_{f_t}$) into equation (19) ($\dot{L}_{f_t}$) and normalizing it by output:

$$\dot{l}_{f_t} = h_t - s_f(1 - t_f)\pi + [s_f(1 - t_f)r - g_{y_t}]l_{f_t} \tag{44}$$

Where $l_{f_t} = L_{f_t}/Y_t$ is firms' debt ratio.

We notice that, *cœteris paribus*, a higher propensity to invest of firms contributes to increase firms' leverage. The same goes for higher interest rates on loans that reduce the available internal

[15] Recall that $s = 1 - c_h(1 - t_h)\omega$.

funds of firms. On the other hand, the after-tax retained earnings share and the output growth rate (that increases the share of retained earnings) ease firms' indebtedness over time.[16]

Firms' debt dynamics will be stable if:

$$\frac{\partial \dot{l}_{f_t}}{\partial l_{f_t}} = s_f(1 - t_f)r - g_{y_t} < 0 \tag{45}$$

That is, firms' debt ratio will present a stable path whenever the output growth rate exceeds the after-tax interest rate times the retention rate. The first term on the RHS of (45) represents the actual interest on debt internally paid by the firms, already considering the tax-base reduction allowed by the interest payments. At equilibrium firms' debt ratio stability condition becomes:

$$\left[\frac{\partial \dot{l}_{f_t}}{\partial l_{f_t}}\right]_{l_f^*} = s_f(1 - t_f)r - g_z < 0 \tag{46}$$

Assuming the equilibrium conditions hold, we can solve equation (44) for firms' leverage ratio in the long run (l_f^*):

$$l_f^* = \frac{\frac{v}{\bar{u}}(g_z + \delta) - s_f(1 - t_f)\pi}{g_z - s_f(1 - t_f)r} \tag{47}$$

(Absent) effect of σ on firms' debt ratio. From (47), it is straightforward to notice that the composition of autonomous demand (σ) has no impact whatsoever on firms' debt ratio. For firms' leverage ratio, in the long run, only the pace of total autonomous demand (g_z) is relevant, regardless of where this demand comes from.

Effect of changes in g_z on firms' debt ratio. That said, taking the derivative of firms' debt ratio with respect to the growth rate of autonomous expenditures, we notice it will raise firms' debt ratio when the retained after-tax and interest profit rate is larger than the depreciation rate (see (A.1) and (A.2) in the Appendix for the derivation of the condition):

$$s_f(1 - t_f)\left(\frac{\pi \bar{u}}{v} - r\right) - \delta > 0 \Rightarrow \frac{\partial l_f^*}{\partial g_z} > 0 \tag{48}$$

Indeed, we can conclude that the derivative is positive because it is an economic feasibility condition. More specifically, capital accumulation requires a net (of depreciation, tax, and interest) positive retained profit rate in the long run.[17] As highlighted by Brochier and Freitas (2021), only the Minsky debt regime in firms' sector, as defined by Taylor (2004) and Lavoie (2014), makes sense economically in (the long run of) the model. The intuition behind this result is that, for a given

[16]For more on how firms' indebtedness and demand relate in a canonical Supermultiplier model see Brochier and Freitas (2021).

[17]That is, accumulation will only happen if expanding capacity is profitable.

retention rate and profit share, a higher pace of accumulation will lead to a higher investment share (accelerator effect) in comparison to the retained earnings share.

3.2 Households' indebtedness and debt dynamics

In order to analyze households' indebtedness over time, we substitute equation (4) (Z_{h_t}) into (8) ($\dot{L}_{h_t}$) and normalize it by output which leads us to the following differential equation for household debt to output ratio:

$$\dot{l}_{h_t} = (1-\sigma)z_t - \left(\lambda + g_{y_t}\right) l_{h_t} \tag{49}$$

Where $l_{h_t} = L_{h_t}/Y_t$ is households' debt ratio.

As expected, an increase in households' autonomous consumption share to total autonomous demand injections – that is, when households' autonomous expenditures temporarily grow at a faster pace than government autonomous expenditures – contributes to a faster accumulation of household debt to output. *Cœteris paribus*, higher rates of debt amortization and GDP growth slow the pace of increase of the household debt ratio.

From the derivative of equation (49) with respect to l_{h_t}, one realizes that households' debt ratio stability requires output growth to exceed minus the amortization rate (50). Therefore, for a non-negative rate of amortization, a positive growth rate of output is a sufficient condition for a stable change in household debt ratio.

$$\frac{\partial \dot{l}_{h_t}}{\partial l_{h_t}} = -(g_{y_t} + \lambda) < 0 \tag{50}$$

At the equilibrium, the stability condition for household debt ratio will be:

$$\left[\frac{\partial \dot{l}_{h_t}}{\partial l_{h_t}}\right]_{l_h^*} = -(g_z + \lambda) < 0 \tag{51}$$

Notice that this stability condition is the same of the growth rate of household debt at equilibrium (equation 39, table 5).

For arriving at the long run households' debt ratio (l_h^*): (i) we calculate the equilibrium autonomous demand injections to GDP ratio. We do that by solving equation (29) (z_t) for the steady state growth conditions to get (52); and (ii) solve (49) for households' debt ratio to obtain (53). We then substitute (52) into (53) to get (54):

$$z^* = \frac{(g_z + \lambda)\left[s - \frac{v}{\bar{u}}(g_z + \delta)\right]}{g_z + \lambda - c_h(r + \lambda)(1-\sigma)} \tag{52}$$

$$l_h^* = \frac{(1-\sigma)z^*}{\lambda + g_z} \tag{53}$$

$$l_h^* = \frac{(1-\sigma)\left[s - \frac{v}{\bar{u}}(g_z + \delta)\right]}{\lambda + g_z - c_h(1-\sigma)(r+\lambda)} \tag{54}$$

We can also obtain a long run economic feasibility condition for the household sector, assuming that it is only realistic that households have a positive consumption out of net disposable income (second term in the RHS of eq. 9). From this condition, we get an upper bound for the long run household debt ratio:

$$l_h^* < \frac{(1-t_h)\omega}{r+\lambda} \tag{55}$$

A similar condition can be found in the neo-Kaleckian literature concerned with the limits household indebtedness may impose to growth, as in Setterfield et al. (2016) and Setterfield and Kim (2020) .

Effect of changes in σ on households' debt ratio. From equation (54), we observe that when households' autonomous consumption share to total autonomous demand injections increases (decreases), as represented by a reduction (an increase) in σ, households' debt ratio will unambiguously also rise (fall). Hence, a direct relation exists between households' expenditure decisions and their debt ratio at the institutional sector level. In our model, differently from more simplified versions of the Supermultiplier model, which typically assume credit-financed consumption as the only source of autonomous expenditures (Pariboni, 2016; Fagundes, 2017; Mandarino et al., 2020; Brochier & Freitas, 2021), the paradox of debt in the household sector is not a necessary outcome. The key feature of our model that allows for this result is the assumption of more than one source of autonomous demand (as in Freitas and Christianes, 2020).

Formally, the sign of the impact that σ has on households' debt ratio depends on g_z and λ (see (A.3) and (A.4) in the Appendix for the derivation of the condition):

$$\lambda + g_z > 0 \Rightarrow \frac{\partial l_h^*}{\partial \sigma} < 0 \tag{56}$$

Accordingly, households' autonomous consumption share will positively affect its debt ratio when the growth rate of autonomous demand injections is positive, assuming a positive rate of debt amortization.

We also notice directly from equation (54) that changes in credit terms, as represented by changes in interest and amortization rates, will affect households' leverage. As expected, a larger share of principal repayment always reduces households' leverage ratio due to a faster debt repayment for an unchanged level of new loans concessions. As for the interest rate, an increase always leads to a higher debt ratio since it negatively impacts consumption out of disposable income and, therefore, temporarily reduces output leading to a higher household debt ratio.

Effect of changes in g_z on households' debt ratio. The growth rate of total autonomous demand injections negatively impacts households' debt ratio, provided the autonomous demand injections ratio is positive ($z^* > 0$). We get this result by taking the derivative of equation (53) with respect to

g_z:

$$\frac{\partial l_h^*}{\partial g_z} = \frac{-(1-\sigma)z^*}{(\lambda + g_z)^2} \tag{57}$$

To better understand what this implies, we analyze equation (54), which replaced z^* on (53). The numerator of (54) is positive as long as the Keynesian stability condition holds, provided $0 < \sigma < 1$. That means we must have a growth rate of autonomous demand injections lower than the maximum growth rate allowed by capacity ($s\bar{u}/v - \delta$). In other words, growth has to be demand-led for the equation's numerator to be positive.

In that case, for the household sector debt ratio to be positive, the denominator also has to be positive. For that to happen, the pace of autonomous demand injections must be larger than the leakages of demand arising from debt servicing minus the amortization rate. This guarantees that the level of autonomous expenditures remains on positive grounds.

Combined, these two conditions show the range of values for the growth rate of autonomous demand where growth is demand-led and autonomous expenditures is surely positive:[18]

$$c_h(1-\sigma)(r+\lambda) - \lambda < g_z < \frac{s\bar{u}}{v} - \delta \tag{58}$$

The derivative of equation (54) with respect to g_z (see (A.6) and (A.7) in the Appendix for the derivation of the condition) also shows that the growth rate of autonomous demand injections will negatively impact household debt ratio, as long as the maximum rate of growth allowed by existing productive capacity is larger than the pace of autonomous demand leakages through debt servicing minus the amortization rate:

$$\frac{s\bar{u}}{v} - \delta > c_h(1+\sigma)(r+\lambda) - \lambda \Rightarrow \frac{\partial l_h^*}{\partial g_z} < 0 \tag{59}$$

Therefore, if condition (58) is warranted so is (59) and $\partial l_h^* / \partial g_z < 0$.

To sum up, an increase in the autonomous growth rate without a change in autonomosu demand composition always leads to a lower household debt ratio in the long run if autonomous demand injections share to GDP is positive and the Keynesian stability condition holds. To be sure, these two conditions mean that the inverse relationship between household debt ratio and autonomous demand growth depends on the economy being demand-led. The underlying economic mechanism is that faster household and government autonomous consumption growth will increase investment to output ratio, fastening the pace of firms' capital accumulation. As output and capital grow temporarily faster than household debt, one will observe a lower household debt ratio in the long run.

[18] A similar condition is obtained in Brochier and Freitas, 2021; Fagundes, 2017 for a Supermultiplier model with credit-financed household autonomous consumption in Hein and Woodgate, 2021 for a Supermultiplier model with debt-financed government expenditures.

3.3 Government indebtedness and debt dynamics

For analyzing government indebtedness and the long run effects of autonomous demand composition and growth on government debt ratio, we start by normalizing (22) by output. As a result, we have the following differential equation for changes in government debt ratio (60):

$$\dot{b}_t = (r - g_{y_t})b_t + \sigma z_t - t_t \tag{60}$$

Where $b_t = B_t/Y_t$ is the government debt ratio, $(r - g_{y_t})b_t$ is the financial component of changes in government debt ratio, σz_t is government consumption to output ratio and t_t is the average tax rate. The average tax rate is defined in equation (61). In order to make the equations easier to read, we grouped two sets of parameters, ψ_1 and ψ_2, in equation (61). ψ_1 (62) accounts for the average tax rate on wages and profits; ψ_2 (63) for the average tax rate on financial income related to firms' loans. The sum of the second and third terms on the RHS of (61) represents the average tax rate on financial income.

$$t_t = \psi_1 + \psi_2 r l_{f_t} + t_p r(b_t + l_{h_t}) \tag{61}$$

$$\psi_1 = t_h + \pi[(t_f - t_h) + t_p(1 - s_f)(1 - t_f)] \tag{62}$$

$$\psi_2 = s_f t_p + t_f[t_p(1 - s_f) - 1] \tag{63}$$

By substituting equation (61) into (60), the dynamic equation for the government debt to output ratio becomes:

$$\dot{b}_t = (r - g_{y_t})b_t + \sigma z_t - \psi_1 - \psi_2 r l_{f_t} - t_p r(b_t + l_{h_t}) \tag{60A}$$

We notice that household and firms' loans ratio will explicitly affect government debt dynamics since they affect property income (interests on these assets received by banks and distributed to households), which the government taxes. Therefore, property income also plays its part in the primary deficit ratio because it affects the average tax rate.

Taking the derivative of equation 60A with respect to b_t we obtain the partial stability condition of government debt dynamics (eq. 64). Similarly to Domar's (1944) stability condition, for government debt to be stable, the GDP growth rate must exceed the real interest rate net of the property income tax rate.

$$\frac{\partial \dot{b}_t}{\partial b_t} = -g_{y_t} + (1 - t_p)r < 0 \tag{64}$$

At equilibrium, the stability condition for government debt ratio is:

$$\left[\frac{\partial \dot{b}_t}{\partial b_t}\right]_{b^*} = -g_z + (1 - t_p)r < 0 \tag{65}$$

Solving (60) for government debt ratio in the long run (b^*):

$$b^* = \frac{\sigma z^* - \psi_1 - r(\psi_2 l_f^* + t_p l_h^*)}{g_z - (1 - t_p)r} \tag{66}$$

Effect of changes in σ on government debt ratio. A sufficient condition for the share of government autonomous expenditures to total autonomous expenditures to positively impact the government debt ratio, in the long run, is that the growth rate of autonomous expenditures must exceed the after-tax interest rate. This condition is warranted if the partial stability condition for the government debt ratio is satisfied (65), in which case an increase (decrease) in the government autonomous demand share will have a positive (negative) effect on government debt ratio, as in Freitas and Christianes (2020).

We arrive at condition (65) also as a condition for a positive effect of government autonomous share on government debt ratio in the long run, by analyzing the derivative of b^* with respect to σ (see (A.8) and (A.9) in the Appendix for the derivation of the condition): for the denominator of the derivative to be positive, government debt stability condition needs to hold; for the numerator to be positive as well and ensure a positive derivative, the following condition must be satisfied:

$$g_z > (c_h - t_p)r - \lambda(1 - c_h) \tag{67}$$

For an economically meaningful set of parameters, condition (67) will hold whenever government debt ratio is stable, making the latter a sufficient condition for the positive effect of σ on b^*. Since in this case: $g_z > (1 - t_p)r > (c_h - t_p)r - \lambda(1 - c_h)$.

Effect of changes in g_z on government debt ratio. From the long run government debt ratio (66), we know that the sign of $\partial b^*/\partial g_z$ is ambiguous. The effects of g_z on z^* (check eq. 52) and on the denominator of b^* are unambiguously negative. However, there are two sources of a potential positive effect: (i) firms' debt ratio increase with g_z, while ψ_2 can plausibly be either positive or negative; and (ii) households' debt ratio decrease with g_z. That said, we cannot rule out *a priori* the possibility of a positive effect of the growth rate on the government debt ratio.

Taking that into account and considering that the derivative of the long run government debt ratio with respect to the growth rate of autonomous expenditures is too complex to reach a simple analytical condition, we have run numerical simulations for a wide range of steady state settings and checked for the sign of this derivative.[19] We found that for combinations of parameters generating positive steady state values of b^* (most cases), positive derivatives are extremely rare. In combinations that yield negative values of b^*, a negative derivative also predominates by far and large. That said, we consider that $\partial b^*/\partial g_z < 0$ is the relevant result, assuming economically meaningful values for the parameters and steady state growth paths.

[19] The protocol adopted for these simulations is as follows. First, we generate a large random set of parameters within the economically relevant interval. Second, we confront the plausibility of the stock-flow norms with observed macroeconomic data (for instance, government debt to GDP should not be greater than 300% or less than -100%, which are pretty similar to real-world figures). In this step, since we are analyzing stable cases, we remove unstable cases from the data set, except those generated by the government debt ratios themselves. Third, we find the numerical derivative $\partial b^*/\partial g_z < 0$ to evaluate the frequency of negative figures. Results are available upon request.

3.4 Summary of the long run effects on debt to GDP ratios

The long run effects of the autonomous expenditures composition and growth rate on the financial stock-flow ratios of the three institutional sectors – firms, government, and households – thoroughly discussed in the previous subsections are summarized in tables 3 and 4 respectively.

Table 3: Effects of σ in financial ratios to GDP.

	Effect	Condition
Firms' debt ratio	$\frac{\partial l_f^*}{\partial \sigma} = 0$	–
Households' debt ratio	$\frac{\partial l_h^*}{\partial \sigma} < 0$	$-\lambda - g_z < 0$
Government debt ratio	$\frac{\partial b^*}{\partial \sigma} > 0$	Sufficient: $g_z - (1 - t_p)r > 0$
		Necessary: $g_z + \lambda(1 - c_h) - r(c_h - t_p) > 0$

Table 4: Effects of g_z in financial ratios to GDP.

	Effect	Condition
Firms' debt ratio	$\frac{\partial l_f^*}{\partial g_z} > 0$	$s_f(1 - t_f)\left(\frac{\pi\bar{u}}{v} - r\right) - \delta > 0$
Households' debt ratio	$\frac{\partial l_h^*}{\partial g_z} < 0$	$\left(\frac{s\bar{u}}{v} - \delta\right) - c_h(1 + \sigma)(r + \lambda) + \lambda > 0$
Government debt ratio	$\frac{\partial b^*}{\partial g_z} < 0$	$b^* > 0$

3.5 General assessment of stability

Table 5 presents the stability conditions of all sub-systems previously analyzed and gathers elements addressed throughout section 3. It merits further consideration as it provides some insights on the features of the model not explored so far.

The last two conditions of table 5 reflect two assumptions made for the analysis of both the stability conditions and the economic results: we assume the Keynesian stability condition holds and that the autonomous demand to output ratio is positive. For economically meaningful parameters,

Table 5: Synthesis of stability conditions around the steady growth path for the system and subsystems of equations in the neighborhood of the equilibrium ($g^* = g_z; u^* = \bar{u}$).

Subsystem	Stability conditions
$\left[\frac{\partial \dot{g}_{l_h}}{\partial g_{l_h}}\right]_{g^*}$	$g_z > -\lambda$
$\mathbb{J}(\dot{h}, \dot{l}_{h_k})$	$T\,[\mathbb{J}^*] < 0 \quad \text{if:} \quad g_z < \frac{s\bar{u}}{v} - \delta - \gamma v$
	$D\,[\mathbb{J}^*] > 0 \quad \text{if:} \quad (g_z + \delta)\gamma\bar{u} > 0$
$\left[\frac{\partial \dot{l}_f}{\partial l_f}\right]_{l_f^*}$	$g_z > s_f(1 - t_f)r$
$\left[\frac{\partial \dot{b}}{\partial b}\right]_{b^*}$	$g_z > (1 - t_p)r$
$\left[\frac{\partial \dot{v}_h}{\partial v_h}\right]_{v_h^*}$	$g_z > (1 - t_p)r$
$u > 0$	$s - h > 0$
$z > 0$	$g_z > c_h(r + \lambda)(1 - \sigma) - \lambda$

the latter condition will also be satisfied whenever the government debt stability condition is satisfied because $g_z > (1 - t_p)r > c_h(r + \lambda)(1 - \sigma) - \lambda$.

The introduction of financial assets dynamics into the Supermultiplier model imposes a lower bound for the growth rate of autonomous demand needed for the overall local stability of the model's steady state.[20]

Hein and Woodgate (2021) in a more simplified framework have reached a similar conclusion. They also attribute to the introduction of the financial counterpart of autonomous expenditures the existence of an upper bound for the autonomous expenditures growth compatible with stability (Hein & Woodgate, 2021, p.7). However, this maximum growth rate for autonomous expenditures is the same that prevails in canonical versions of Supermultiplier models such as Freitas and Serrano (2015).[21] We, therefore, claim that the most significant change in terms of stability brought about by introducing financial assets and their dynamics is the lower limit it establishes for the growth rate.

This is clear from the dynamic stability conditions for the government debt, firms' debt and household wealth ratios presented in table 5, since all of these conditions individually require a

[20]Which tend to be lower than the warranted growth rate (see Haluska et al., 2021, for evidence of estimated maximum growth rate for the United States economy)

[21]That, nonetheless, does not mean that the feedback from financial assets to demand does not affect the maximum growth rate as Hein and Woodgate (2021) show by introducing consumption out of wealth into their model.

minimum growth rate for autonomous demand. This makes sense if we recall that these expenditures lead output growth in the Supermultiplier model and that, *cœteris paribus*, a higher growth rate of output and income contribute to stabilizing the debt ratios of the institutional sectors. It is also important to highlight that, for economically meaningful parameters, government debt (and household wealth) stability condition will be the most stringent: government debt stability requires a higher growth rate compared to firms' debt stability. That means that if the government debt ratio is locally stable, firms' debt ratio will also be.[22]

Since property income is taxed, the growth rate of autonomous expenditures has to exceed the after-tax (on property income) interest rate for the government debt ratio to be stable. The taxation of property income relaxes the government debt ratio stability condition, since it requires a lower growth rate of autonomous expenditures to keep the debt ratio trajectory stable in comparison to the stability condition of Supermultiplier models with debt dynamics but without taxation on property income, as in Hein and Woodgate (2021) and Freitas and Christianes (2020). Besides, it also implies that government debt can be stable for a positive primary deficit ratio even when the growth rate of autonomous demand injections is lower than the interest rate. This result is also in line with Ryoo and Skott (2013) and Godley and Lavoie (2007), who find that increasing the tax rate on property income, *cœteris paribus*, makes the public debt trajectory more stable.

4 Private debt and public debt in alternative scenarios

Given the detailed analysis of the system we performed in the previous section, we can stress the model to provide further insights into real-world economic problems. In the context of demand-led growth models, and specifically in that of Supermultiplier models, ours can explicitly represent scenarios of de-leveraging, growth reduction, and prospective fiscal policy reactions to credit market (households) behavior. The reason is that it includes more than one source of autonomous demand. Furthermore, our model can portrait straightforwardly regularities that are otherwise puzzling from supply-led models.

Focusing on the interaction of autonomous demand growth and indebtedness[23], let us consider three scenarios:

1. A change in the composition of the autonomous expenditures (σ) with a constant g_z;
2. A change in g_z with given composition of autonomous expenditures σ;
3. A reduction in g_z coupled with a change in σ.

In section 3, we dealt with the effects of autonomous demand composition on the long run financial ratios. However, changes in the composition of autonomous demand also affect the average growth rate of output and, consequently, the income *level* in the long run. To see how that happens, we

[22]Notice that the opposite is not necessarily true. Firms' debt ratio may be locally stable while government debt ratio is locally unstable.

[23]To keep the discussion within limits, we leave a detailed discussion of relation between fiscal policy from the revenue side and indebtedness as matter for future research.

substitute equation (24) into (27) and solve the latter for the level of output, assuming the equilibrium conditions hold (68):

$$Y_t^* = \left[\frac{1}{s - (\delta + g_z)\upsilon/\bar{u}}\right] \left\{\frac{Z_0 e^{g_z t}\,[g_z + \lambda - c_h(r+\lambda)(1-\sigma)]}{g_z + \lambda}\right\} \tag{68}$$

which gives the equilibrium path of income. This equation shows that the equilibrium path of output depends on the supermultiplier (first term on the RHS) and the total autonomous demand at each period. Since autonomous demand in the private sector comes from credit-financed consumption, there is an autonomous demand leakage associated with the effect debt serving has on households' disposable income and, consequently, on consumption, as discussed in sections 2 and 3. Therefore, differently from Freitas and Christianes (2020), the composition of autonomous demand between government and household consumption will affect the long run level of output since part of private autonomous demand leaks due to the impact of financial commitments on aggregate demand.

Change in the composition of the autonomous expenditures (σ) with a constant g_z. This scenario helps to comprehend the following economic situations:

(i) An episode of deleveraging[24] by the household sector through the reduction of the growth rate of new loans (Z_h), which turns out to be part of the autonomous demand injection, in a way that the government precisely compensates the households' autonomous expenditure cut (countercyclical fiscal policy). The deleveraging can be triggered by increasing banks' or households' risk aversion, thus affecting credit supply and demand. Such a configuration implies an increase in σ, as government autonomous spending temporarily outgrows households' autonomous demand injections. At the same time, due to the countercyclical nature of the fiscal policy, g_z remains constant.

And (ii) an episode of a consumer credit boom, again exactly matched by a countercyclical fiscal policy. In this case, σ decreases as households' autonomous spending temporarily outgrows the government's.

Situations (i) and (ii) are two sides of the same coin. Moreover, they are abstract but possible approximations of the United States economy in the aftermath of the 2007-8 Global Financial Crisis and in the 1990s, respectively.

In terms of output *level* (68), the key element is that σ impacts the autonomous demand leakages. More precisely, credit-fueled household consumption growth entails financial commitments that reduce disposable income, draining consumption and aggregate demand. Such an effect is unparalleled in the government autonomous expenditure's case. Because of the effect of σ in the autonomous demand leakages, in economic configuration (i), the deleveraging of the household sector diminishes the debt burden, opening room for dynamically increased consumption levels.

Conversely, in economic configuration (ii), since the reduction in government expenditures compensates the direct positive boost to private autonomous demand, what is left is the negative effect on aggregate demand associated with the increased debt burden. As consumption and

[24]See Koo (2009) and Eggertsson and Krugman (2012) for related discussions.

aggregate demand grow temporarily at a slower pace, the average growth rate of output falls. This result reasonably approximates empirical findings that private (mainly household) debt has only temporary positive effects on the level of economic activity, possibly also harming output afterwards (e.g. Jordà et al., 2016; Mian et al., 2017; Mian & Sufi, 2018; Guerini et al., 2020).

In terms of financial ratios, as discussed in section 3, we have the following unambiguous effects: (i) $\uparrow b^*$, $\downarrow l_h^*$, and $\bar{l}_f^*$; (ii) $\downarrow b^*$, $\uparrow l_h^*$, and $\bar{l}_f^*$.

Considering the normalized financial balances (see the subtotal row in table 2), these results resemble Godley's views on unsustainable processes (see, for instance, Godley, 1999),[25] although through different economic mechanisms. To illustrate, let us take the economic situation (ii). If an acceleration of household consumption is allowed by a credit-boom, the financial balance of this sector decreases, entailing increased indebtedness ($\dot{l}_h > 0$ as compared to baseline). At the same time, the nominal balance of the government improves, allowing b to fall relative to the baseline. This process cannot persist forever since it would bring about increasing household indebtedness. Although we do not model this possibly explosive scenario endogenously, we show a condition that would trigger such an unfeasible situation (55). Moreover, considering the debt hierarchy, the accumulation of household debt would increase the financial fragility of the economy, which can be related to Minsky's insights (Minsky, 1975, 1977, 1986).

The financial ratios results also display a negative relation between private and public debt arising from changes in the composition of spending. Such a result is particularly helpful to theoretically explain Mbaye et al. (2018) empirical finding that public debt typically augments in the aftermath of deleveraging episodes through the aggregate demand channel. Moreover, coupling the output level effects with the results for b^*, our model also matches the positive public debt and output level relation found in Guerini et al. (2020).

Change in g_z with given composition of autonomous expenditures σ. This scenario comprises two economic situations:

(iii) a persistent attempt of households to deleverage by cutting spending matched with austere government measures (expenditure cuts), in a way that g_z permanently reduces.

(iv) an episode of a consumer credit boom coupled with a pro-cyclical fiscal stance, in a way that g_z permanently increases instead.

In economic situation (iii), since g_z falls, both government debt and household debt ratio to GDP increase, whereas firms' leverage ratio will diminish. The reasoning is that both government and household consumption pace slowdown, reducing the output growth rate and lowering capacity utilization in the short run. As firms understand this slowdown as a permanent decrease in the trend rate of sales, the aggregate propensity to invest falls, further feeding into a lower pace of output growth. As a result of this reaction of firms to the lower trend of autonomous demand, the output growth rate temporarily falls beyond the lower growth rate of autonomous demand, and both government and household debt ratios will be higher in the long run. As for firms, since the

[25] Godley's financial balances approach included the external sector, which we do not in this paper.

investment rate falls more than the retained profits ratio, they manage to deleverage in the long run.

This result shows an inverse relationship between government debt ratio and GDP growth, as found in many correlational empirical studies since Reinhart and Rogoff, 2010. More than that, in the model, causation runs from g_z to b^*, which is consistent with the empirical findings of Panizza and Presbitero (2014) and Jacobs et al. (2020).

Finally, in sharp contrast with economic situation (i), the attempt of households to de-leverage backfires in this case, whereas the public debt ratio still moves up.

In turn, economic situation (iv) results are opposed to those of (iii): output growth permanently increases, households and government indebtedness permanently decreases, whereas private firms' leverage rises. Accordingly, it would be a generally prosperous situation. However, this result should be taken with a grain of salt. The internal limit to locally stable growth in our model is given by the trace condition ($g_z < s\bar{u}/v - \delta - \gamma v$, see eq. 43). This condition is generally less stringent than the growth rate compatible with balance-of-payments equilibrium (Thirlwall, 2011; Bhering et al., 2019), which should then bound demand growth. Although we deal with a closed economy, this is worth highlighting to stress that the economic situation (iv) is more pertinent to economies facing sluggish economic activity, with growth below the one consistent with balance-of-payments equilibrium.

Reduction in g_z coupled with a change in σ. Besides the two previous scenarios, interesting situations can arise when both g_z and σ change.

Let us first consider households' deleveraging attempt coupled with austerity (g_z falls), but in a way that the average growth rate of government autonomous spending is higher than households' (σ increases). The increase in σ produces a higher level of output growth for a given fall in g_z. The government debt to GDP ratio unambiguously rise, since $\partial b^*/\partial\sigma > 0$ and $\partial b^*/\partial g_z < 0$. The effects on households' indebtedness are ambiguous: the fall of g_z contributes to its increment, while the rise of σ contributes to its reduction. Depending on the magnitudes of both effects, the household sector can accomplish deleveraging even with austerity. Even so, for a given set of parameters, the quantitative increase in public debt to GDP, in this case, is higher than in the economic situation (iii) because two effects in the same direction are pilling up.

A second interesting possibility reproduces the previous case, but now the average growth rate of government autonomous spending is lower than that of households', resulting in a decrease of σ. In this case, the government debt ratio can change in either direction: the fall of g_z contributes to its increment, while the fall of σ contributes to its reduction. Instead, for a given fall in g_z, households' debt ratio will further rise in comparison to situation (iii), for σ also falls. Besides that, as explored before, the output level decreases because of the larger leakages of aggregate demand associated with households' expanded debt burden. Accordingly, the hawkish attitude of the treasury, reflected in the attempt to reduce spending even further than households to prevent further debt accumulation, may end up producing a more fragile economic setting due to the accumulation of debt in the household sector.

5 Final Remarks

The model featured in this paper builds upon the idea that public and private debt ratios should not be regarded as independent phenomena. They are counterparts, and they both depend on GDP growth rates. The features of the model emphasize this interrelation since when one debt-financed autonomous expenditure (temporarily) leads, the other lags behind. In what follows, we highlight the main implications of the economic setting provided by the model:

- An increase in government autonomous expenditure share, as private (household) autonomous expenditures slowdown, may have a stabilizing effect on the private sector aggregate financial situation. This may be the case during a recession, or a period of persistently low GDP growth, when households reduce expenditures out of new borrowing, in an attempt to increase their precautionary savings and to deleverage. That does not mean that when government debt ratio goes up, household debt ratio will necessarily go down. As found by Mbaye et al. (2018) and featured in our model, an increase in government expenditures may contribute to the reduction of aggregate households debt ratio. However, if the recession is too strong, that may not be enough to bring the household debt ratio to a pre-recession level. If that is the case, both household and government debt ratios go up, with the private debt ratio partially alleviated by the counter-cyclical measures of the government. This scenario certainly looks familiar: during 2020, most governments, from developed and developing countries, heavily engaged in income transfers and credit measures aimed at the private sector (to a larger or smaller extent depending on the country) to deal with the economic crisis triggered by the COVID-19 pandemics.

- Instability can arise either from the financial or the real side of this economic system: from the real side when firms strongly react to changes in the (expected) pace of growth (Harrodian instability); and from the financial side when the growth rate of autonomous demand (both credit-financed consumption and debt-financed government expenditures) is too low in comparison to the after-tax interest rate, which represents the pace of government financial commitments from accumulated debt.

- As discussed in section 4, the framework we develop seems to be compatible with relevant empirical findings on the relation of public and private debt to GDP ratios.

- The fact that government expenditures (and debt) play a stabilizing role over household debt ratio in the model is closely related to the debt hierarchy stated in section 2. Part of the credit-financed autonomous demand of the private sector leaks as households have to pay interest on loans and repay part of their debt principal each period. The same does not (necessarily) hold for government autonomous expenditures when it is has the prerogative of issuing debt in a currency that it issues.

At this point, it should be clear that the model is ill-suited for policy recommendation due to its simplified nature, although it does provide a framework that sheds light on policy issues. It also should be clear that it is not arguing for an unbound expansion of government expenditures. Indeed, extending the model to an open economy setting is a required step to deal precisely with the limitations of debt-financed government expenditures. In an open economy scenario, even if the government takes the lead on growth, balance of payment constraints may impose a ceiling to the pace of accumulation for countries that do not issue the currency with which they pay for their international financial commitments. Extending the debt hierarchy to the relationship between domestic and foreign debt in a growth setting is the next step of our research agenda.

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A Appendix

A.1 Derivatives of long-run financial ratios

Effect of g_z on firms' debt to GDP ratio. Taking the derivative of equation (47) we get:

$$\frac{\partial l_f^*}{\partial g_z} = \frac{s_f(1-t_f)\pi - \frac{v}{\bar{u}}\left[\delta + s_f(1-t_f)r\right]}{\left[g_z - s_f(1-t_f)r\right]^2} \tag{A.1}$$

For the derivative to be positive:

$$s_f(1-t_f)\pi - \frac{v}{\bar{u}}\left[\delta + s_f(1-t_f)r)\right] > 0 \Rightarrow s_f(1-t_f)\left(\frac{\pi\bar{u}}{v} - r\right) - \delta > 0 \tag{A.2}$$

Effect of changes in σ on long run households' debt ratio. Taking the derivative of equation (54) with respect to σ we get:

$$\frac{\partial l_h^*}{\partial\sigma} = \frac{-\left[s - \frac{v}{\bar{u}}(g_z+\delta)\right]\{[\lambda + g_z - c_h(1-\sigma)(r+\lambda)] + c_h(1-\sigma)(r+\lambda)\}}{[\lambda + g_z - c_h(1-\sigma)(r+\lambda)]^2} \tag{A.3}$$

Assuming the Keynesian stability condition holds, the derivative will be negative if:

$$[\lambda + g_z - c_h(1-\sigma)(r+\lambda)] + c_h(1-\sigma)(r+\lambda) > 0 \Rightarrow \lambda + g_z > 0 \tag{A.4}$$

Effect of changes in g_z on long run households' debt ratio. Taking the derivative of (53) with respect to g_z we get:

$$\frac{\partial l_h^*}{\partial g_z} = -\frac{(1-\sigma)z^*}{(g_z+\lambda)^2} \tag{A.5}$$

Since $0 < \sigma < 1$, the derivative will be negative if $z^* > 0$.

Taking the derivative of (54) with respect to g_z we get:

$$\frac{\partial l_h^*}{\partial g_z} = -(1-\sigma)\frac{\frac{v}{v}[\lambda + g_z - c_h(1-\sigma)(r+\lambda)] + \left[s - \frac{v}{v}(g_z+\delta)\right]}{[\lambda + g_z - c_h(1-\sigma)(r+\lambda)]^2} \tag{A.6}$$

Since $0 < \sigma < 1$, the derivative will be negative if:

$$\frac{v}{v}[\lambda + g_z - c_h(1-\sigma)(r+\lambda)] + \left[s - \frac{v}{v}(g_z+\delta)\right] > 0 \Rightarrow \frac{s\bar{u}}{v} - \delta > c_h(1+\sigma)(r+\lambda) - \lambda \tag{A.7}$$

Effect of changes in σ on long run government debt ratio. Replacing equations (47), (52) and (54) into (66), rearranging, and taking the derivative with respect to σ we get:

$$\frac{\partial b^*}{\partial\sigma} = \frac{\frac{(s-h^*)(g_z+\lambda+rt_p)}{g_z+\lambda-c_h(\lambda+r)(1-\sigma)} - \frac{(s-h^*)c_h(\lambda+r)\left[(g_z+\lambda)\sigma - r(1-\sigma)t_p\right]}{[g_z+\lambda-c_h(\lambda+r)(1-\sigma)]^2}}{g_z - r(1-t_p)} \tag{A.8}$$

When the government debt is locally stable, the denominator of (A.8) is positive – see equation (64). In this case, a positive derivative requires the numerator to be also positive, which holds when:

$$g_z > (c_h - t_p)r - \lambda(1-c_h) \tag{A.9}$$