Assessing the Speed of the Green Transition: Directed Technical Change Towards Decarbonization

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Abstract

This paper employs a multi-sectoral growth model of linear production with changing technology to study the technical substitution of carbon sectors by green sectors, that is, the Green Transition. The framing of this transition is based on a dynamic cross-dual adjustment between prices and quantities in the form of a law of excess demand and a law of excess profitability, which produces a complex pattern of oscillations around their equilibrium values. The linear adjustment coefficients of the model are empirically estimated for six countries using a mixed-effects varying-slopes model on EU KLEMS data. The speed of green substitution that allows decarbonization to meet the targets of the UN Intergovernmental Panel on Climate Change is evaluated analytically and computationally with respect to four varying parameters (relative cost efficiency, carbon tax and green subsidy rates, and initial investment ratio), with highly significant results: carbon taxes have the largest impact on the speed of decarbonization, followed by green subsidies; relative cost efficiency has a negligible impact on speed within realistic time frames. Directed technical change is enforced by a revenue-neutral, pro-active fiscal policy of a tax-subsidy form, which has the effect to greatly accelerate the phase-out of the carbon sector and the phase-in of green energy. Without fiscal policy, this substitution process will be too slow to reach the IPCC targets on time. **JEL Codes** C63, O25, O41, Q55

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1. Introduction

The climate crisis is one of the defining issues of our time. According to a recent landmark report by the UN Intergovernmental Panel on Climate Change (IPCC), 16 years are left for a rapid structural change of the economy towards decarbonization in order to avoid climate catastrophe by keeping world temperatures below the 1.5-Celsius-degree target. Consensus among economists views climate change as a negative externality that can be corrected by a Pigouvian tax on carbon internalized by polluting firms, which may be further used to subsidize and direct structural technical change towards green technology. Recently, most economic analysis on climate change policies operates in the tradition of computational general equilibrium and new growth theory, which endogenizes innovation (Goulder & Schneider, 1999; Nordhaus & Boyer, 2000; Nordhaus, 2002; Jaffe et al., 2002; Goulder & Parry, 2008; Gillingham et al., 2009; Acemoglu et al., 2012; Gans, 2012; Golosov et al., 2014; Acemoglu et al., 2016). These contributions envision optimal taxation based on the fundamental neoclassical trade-off between consumption today and consumption in the future, leading to the design of damage functions that relate climate change with losses in economic output (Keen, 2020). More recent studies study the crucial role of public finance and the financial market, for example in the form of green bonds, in promoting the green transition (Heine et al., 2019; Deleidi et al., 2020; Semmler et al., 2021).

In particular, Acemoglu et al. (Acemoglu et al., 2012, 2016) posit two pressing questions: (1) how much time the structural transition from carbon to renewable energy may take, and (2) to what extent fiscal policy of the tax-subsidy kind can accelerate such a process under directed technical change. Our contribution addresses these two questions using an extension of a dynamical model of multi-sector growth that incorporates technological dynamics in the form of process innovation and extinction (Flaschel & Semmler, 1987, 1992). In contrast to the contributions based on new growth theory, the multi-sectoral character of the model allows to explicitly explore the effect of input-output linkages in production in the context of directed technical change. Within the broader literature on competitive dynamical adjustments in prices and quantities (Jorgenson, 1960; Hahn, 1970; Morishima, 1981; Mas-Colell, 1986; Duménil & Lévy, 1987; Flaschel & Semmler, 1987; Fisher, 1989a; Flaschel, 1990; Duménil & Lévy, 1993), the linear production system theoretically relies on the laws of excess demand and excess profitability, which imply cross-dual linear adjustments in prices with respect to imbalances between supply and demand and in quantities with respect to deviations from normal profitability.

Using a mixed-effects model with varying slopes on EU KLEMS data, the model is calibrated by estimating empirically the linear adjustment coefficients for six developed economies (Germany, France, Japan, Italy, Netherlands, and the US). Input-output WIOD data is employed to set the technology structure. The empirical adjustment coefficients and initial conditions, extracted from EU KLEMS, are then used to compute simulations of the dynamic substitution of two carbon-based sectors of economic production by green, carbon-free synthetic equivalents: in particular, "Electricity, gas, steam and air conditioning supply" (D) and "Manufacture of coke and refined petroleum products" (C19). The theoretical model can be thus conceived as a machine learning algorithm that uses the WIOD and EU KLEMS datasets as training data.

Finally, the speed of decarbonization is evaluated analytically and computationally with respect to a variety of production and fiscal-policy parameters: the relative efficiency in production of the green sector with respect to the carbon sector (i.e. capital intensity, which can also be understood as a nominal carbon-pricing tax), the (real) carbon tax rate, the green subsidy rate, and the initial output ratio or investment. For a synthetic dataset of 14,250 simulations, the dependence of decarbonization speed on all these parameters is highly significant, with varying degrees of intensity: technical efficiency has the smallest impact and a carbon tax on profits has the largest impact. In absence of fiscal policy, efficiency in production, is, as expected, the critical parameter that regulates the speed of the green transition, by imposing sustained profit and growth rate differentials between the carbon and the green sector. However, lower costs in production alone induce too slow of an adjustment speed in the phase-in of the green energy sector to fall within IPCC targets or any reasonable time horizon, even when starting many magnitudes above its current levels of output relative to the carbon sector.

The findings of the paper show to what extent fiscal policy that taxes carbon output to subsidize green output is indispensable to expand the profit and growth rate differentials enough to meet the IPCC targets for decarbonization in time. Without fiscal policy, it is not possible for any economy to reach the IPCC targets on time. Furthermore, tax-subsidy policy shows its greatest effect at the earliest stages of the green transition, suggesting that public investment is necessary to kickstart and mobilize private funds into moving on green technologies over carbon ones (Acemoglu et al., 2016; Heine et al., 2019; Deleidi et al., 2020; Semmler et al., 2021). The paper finally suggests appropriate fiscal policy mixes to accelerate the green transition within IPCC targets.

2. A multi-sector growth model of the green transition

2.1. Dual and Cross-dual Adjustment Models

Starting in the 1960s and 1970s, models of dual and cross-dual adjustment in prices and quantities have a long history in economics (Jorgenson, 1960; Hahn, 1970; Burmeister et al., 1973; Morishima, 1981; Mas-Colell, 1986; Duménil & Lévy, 1987; Flaschel & Semmler, 1987; Fisher, 1989a; Flaschel, 1990; Flaschel & Semmler, 1992; Duménil & Lévy, 1993). In such models, dynamic stability is studied by considering specific adjustment processes in the form of stylized facts as laws. Inspired by the short-run Walrasian process of price groping or tâtonnement, within the neoclassical analysis of temporary general equilibrium the so-called "law of demand and supply" became the most popular form of adjustment process, where excess demand triggers a change in prices.

However, Hahn noted that the study of Walrasian groping has not been very fruitful (Hahn, 1970). Subsequent investigations within the context of neoclassical growth models with heterogeneous capital goods have generally revealed the possibility of a saddlepoint behavior of their dynamics, where asymptotic stability to equilibrium is not guaranteed (Burmeister et al., 1973). In the context of input-output analysis, Jorgenson contributed his famous dual (in)stability theorem, where if the output system is stable, the price system is unstable, and vice versa (Jorgenson, 1960). As Morishima noted, Jorgenson's adjustment processes were of the dual form only, with uncoupled dynamic adjustment in prices and quantities (Morishima, 1981). Further, prices and quantity adjustments could be made stable by removing two implicit assumptions in Jorgenson's model, namely the full utilization of capital and perfect-foresight price expectations (Fukuda, 1975).

Morishima showed that equilibrium could be asymptotically stable following a cross-dual formulation, where the Walrasian law of excess demand was to be supplemented by a rule describing how quantities are adjusted, in particular in the form of a "law of excess profitability", and the analysis is restricted to the goods market and long-run equilibrium positions, i.e. without considering short-run temporary equilibria (Morishima, 1981). Contributions in a more classical perspective, where stability is understood not in asymptotic terms but as a self-restricted, gravitational movement of quantities, prices, and profitability differentials around their equilibrium values, feature the work of Steedman, Nikaido or Duménil and Levy (Steedman, 1984; Nikaido, 1985; Duménil & Lévy, 1987, 1993). In addition, more recent contributions in neoclassical theory have also formulated similar laws of profitability, where an excess of prices over costs triggers supply responses of firms (Mas-Colell, 1986).

In the Flaschel-Semmler model (Flaschel & Semmler, 1987), a deviation of quantities from equilibrium will trigger a response in prices (law of excess demand), while a deviation of unit profits from equilibrium will induce a response in quantities (law of excess profitability). The dynamic process of the free mobility of profit-seeking capital among sectors of production induces fluctuations in outputs: if an industry earns higher-than-average profits, capital will move there raising output. Since market prices react positively to excess demand and negatively to excess supply by the law of demand, the increase in supply caused by capital inflows will eventually drive prices down, reducing industry profitability and eventually forcing capital to flow out to other sectors of production with higher returns on capital, and in the process reducing industry output. Market prices and relative quantities gravitate around their equilibrium values, which are ultimately determined by the technological structure in line with the von Neumann-Sraffa input-output model (Von Neumann, 1945; Sraffa, 1960). This is the basic structure of market dynamics in the theoretical model.

2.2. Constant-Technology Dynamics

The dynamical model of multi-sector growth with circulating capital and constant technology describes price and quantity oscillations over time around the equilibrium values of N prices p (a row vector) and N quantities x (a column vector) of the Sraffa-von Neumann system (Von Neumann, 1945; Sraffa, 1960). These price and quantity oscillations are of a Lotka-Volterra form, following a dynamical cross-dual adjustment in a linear model of production with matrix A as inputs and matrix B as outputs[1], which take real positive values. By the Perron-Frobenius theorem, unique positive equilibrium values p^* and x^* solve for the positive gross rate of return or expansion rate $R > 1$, in matricial notation:

$$Bx^* = RAx^* \quad \textit{supply equals demand} \tag{1}$$

$$p^*B = Rp^*A \quad \textit{profit rates are uniform across sectors} \tag{2}$$

where p^*B is the equilibrium unit revenue and Rp^*A is the equilibrium unit cost in relation to the expansion rate R. In scalar notation,

$$\sum_j b_{ij}x_j^* = R\sum_j a_{ij}x_j^* \quad i = 1, ..., N \tag{3}$$

$$\sum_i p_i^* b_{ij} = R\sum_i p_i^* a_{ij} \quad j = 1, ..., N \tag{4}$$

[1]These matrices are "augmented" in the sense that they also incorporate labor supply and its price, the wage rate. This contribution emphasizes issues of multi-sector growth and technical change over distribution.

The expansion rate R is the inverse of the unique largest positive real eigenvalue of the input-output matrix A/B, equilibrium prices p^* are its associated positive row eigenvector, and equilibrium output x^* its the associated positive column eigenvector. The second largest eigenvalue of the input-output matrix A/B determines the speed of convergence to equilibrium (Bródy, 1997). R thus can be considered the "maximum expansion rate" (Shaikh, 2016), which is associated to the aggregate profit rate when wages are zero and there is no capitalist consumption, that is, all capitalist profit is re-invested:

$$1 + r = R = \frac{pBx}{pAx} \tag{5}$$

When the system is in maximum expanded reproduction, the demand for final goods will be proportional to the total output vector (Shaikh, 2016). The empirical statistical analysis of the input-output matrix for the US between 1967 and 2007 shows a remarkably persistent exponential distribution for the distribution of its (moduli) eigenvalues, which cluster around zero in the complex plane (Torres-González & Yang, 2019). From a dynamical-systems perspective, this suggests a very complex pattern of oscillations operating at different speeds of adjustment, which the linear production system attempts to capture by appealing to two abstract laws: the law of excess demand and the law of excess profitability.

Cross-duality in its simplest form gives rise to stability, but not to asymptotic stability for the equilibrium assumed, but rather ceaseless over- and undershooting of prices, quantities, and profit rates around their natural values as centers of gravity (Flaschel & Semmler, 1987; Shaikh, 2016), see figure 1.

Following the law of excess demand, market prices p will decline (rise) if supply Bx is greater (smaller) than demand:

$$\left(\frac{\dot{p}}{p}\right)^T = -\delta_p[B - RA]x = \delta_p[\underbrace{RAx}_{\text{demand}} - \underbrace{Bx}_{\text{supply}}] \tag{6}$$

Following the law of excess profitability, quantity x_i will rise (decline) if unit revenues pB are greater (smaller) than unit costs times R, RpA, since capital will flow out of the sectors with below-normal profitability into the sectors with above-normal profitability:

$$\frac{\dot{x}}{x} = +\delta_x[B - RA]^T p^T = \delta_x[\underbrace{B^T p^T}_{\text{revenue}} - \underbrace{RA^T p^T}_{cost}] \tag{7}$$

where $\frac{\dot{x}}{x}$ is the column vector of the growth rates in relative quantities, $\frac{\dot{p}}{p}$ is the row vector of the growth rates in relative prices, and δ_p and δ_x are diagonal matrices with N positive adjustment coefficients (so they can also be understood as vectors).

In discrete-time, scalar form,

$$\frac{p^i_{t+1}}{p^i_t} - 1 = -\delta^i_p \sum_j (b_{ij} - Ra_{ij}) x^j_t \quad i = 1, ..., N \tag{8}$$

$$\frac{x^i_{t+1}}{x^i_t} - 1 = +\delta^i_x \sum_j p^j_t (b_{ji} - Ra_{ji}) \quad i = 1, ..., N \tag{9}$$

In order to retrieve dynamic convergence to equilibrium, an adjustment is added, with parameter $\gamma > 0$, where capitalists also take account of the sign of change of extra profits (or losses) when moving their capitals between sectors, so the law of excess profitability is modified (Flaschel & Semmler, 1987):

$$\frac{\dot{x}}{x} = +\delta_x[B - RA]^T (p^T + \gamma \dot{p}^T) \tag{10}$$

This modified cross-dual adjustment process is proven to be globally asymptotically stable as a special case of what Hahn and Fisher (Hahn, 1982; Fisher, 1989b) call quasi-global stability (Flaschel & Semmler, 1987, p.26).

2.3. Technical change with process innovation and extinction

In a subsequent contribution (Flaschel & Semmler, 1992), a generalization is proposed following the classical competitive process of dynamical adjustment of the model of technical change presented in Silverberg (Silverberg, 1984), which is based on the Goodwin model of class struggle and capital accumulation (Goodwin, 1982). The Goodwin model assumes neutral, exponential, disembodied technical progress, under fixed coefficients production, with fluctuating unemployment regulating changes in the level of real wages. Instead of disembodied technical progress, the contribution by Silverberg presents an economy with a fixed production process and then proceeds by examining the stability of the resulting equilibrium state when a second production process embodied in a new capital good with different technical coefficients is introduced.

The linear production system with technical change allows the input-output A, B to be $K \times N$ rectangular and evolve over time, with $i = 1, ..., K$ commodities (corresponding to rows) and $j = 1, ..., N$ processes (corresponding columns). Hence, the jth column of A corresponding to the jth process indicates the input requirements of commodities $i = 1, ..., N$. Now the quantity vector is of dimension K for each process (as adjustment parameter vector δ_x), while the price vector has dimension N for each commodity (as adjustment parameter vector δ_p). The output matrix B has the same dimensions than the input matrix A. If the system is single-product, the $K \times N$ output matrix is composed exclusively by 0 and 1s, that is, b_{ij} is 1 if the jth process produces commodity i and 0 if not. For joint-product systems, b_{ij} can take any real number between 0 and 1.

In particular, the model considers material (or wage) saving innovations (i.e. capital- or labor-saving), substitution effects where a more efficient process competes to take over a less efficient process, and innovation in a joint-product system. If a new process is introduced, a square $K \times K$ A matrix at time t is replaced at time $t+1$ by a rectangular $K \times N$ A matrix where now $N = K+1$. A newer, more efficient process j' emerges to compete with an older, less efficient process j: formally, $a_{ij'} \leq a_{ij} \quad \forall i = 1, ..., K$ commodities. Three scenarios can be examined separately as in the original contribution (material/wage-saving innovations, substitution effects, and joint-product innovation), but the three cases can also be simulated altogether to model innovation in more general terms.

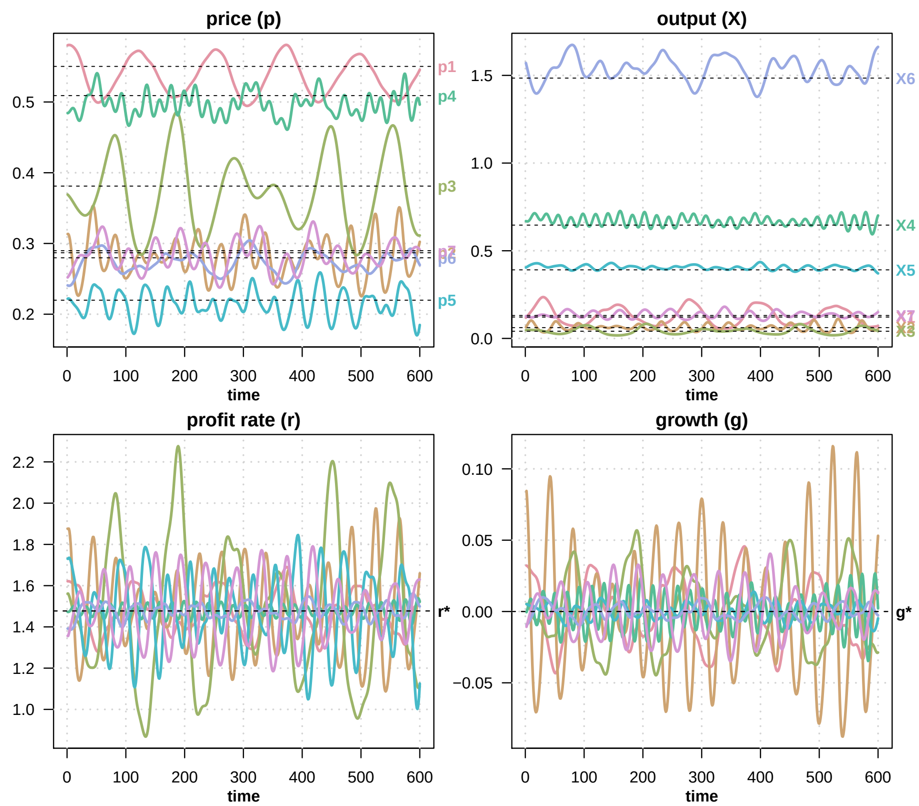

Figure 1: **Flaschel-Semmler Constant-Technology Dynamics for the 2003 US Direct Requirements Matrix** Dashed horizontal lines indicate equilibrium values for the profit rate r^*, prices p^*, quantities x^*, and aggregate growth $g^* = 0$. [color, 2-column]

	1	2	3	4	5	6	7
1 Agriculture	.2008	.0000	.0011	.0338	.0001	.0018	.0009
2 Mining	.0010	.0658	.0035	.0219	.0151	.0001	.0026
3 Construction	.0034	.0002	.0012	.0021	.0035	.0071	.0214
4 Manufacturing	.1247	.0684	.1801	.2319	.0339	.0414	.0726
5 TTU	.0855	.0529	.0914	.0952	.0645	.0315	.0528
6 Services	.0897	.1668	.1332	.1255	.1647	.2712	.1873
7 Other	.0093	.0129	.0095	.0197	.0190	.0184	.0228

[a] TTU: Trade, Transportation, and Utilities

Table 1: **2003 US Direct Requirements Matrix**

3. Model Calibration

3.1. Simple Example with Synthetic Data

As a first example, constant-technology dynamics are simulated for the 2003 US direct requirements matrix, disaggregated into 7 industries (table 1) (Miller & Blair, 2009, p.29) in figure 1. The equilibrium values for prices, quantities, and the expansion rate are:

$$p^* = (0.550, 0.287, 0.381, 0.509, 0.220, 0.280, 0.290) \quad (11)$$

$$x^* = (0.0727, 0.0367, 0.0244, 0.3854, 0.2331, 0.8853, 0.0780) \quad (12)$$

$$R = 2.477343 \quad (13)$$

The resulting simulations produce a complex pattern of deterministic coupled cyclic oscillations in the price and quantity vector around its equilibrium values. Oscillations vary in amplitude and frequency depending on the adjustment parameters chosen, in this case:

$$\delta_p = (0.1, 1, 0.5, 0.5, 1, 0.1, 0.5) \quad (14)$$

$$\delta_x = (1, 2, 0.5, 2, 0.25, 1, 1) \quad (15)$$

$$\gamma = 1 \tag{16}$$

The N adjustment parameters for prices $\delta_{p,i}$ [equation 8] can be estimated from the simulation data over time interval Δt with an ordinary linear regression without intercept for each industry i:

$$y_{i,t} = \alpha_i x_{i,t} + \epsilon_{i,t} \quad i = 1, ..., N; t \in \Delta t \tag{17}$$

where the dependent variable is the growth rate of prices, the independent variable is excess demand, and the linear slopes α_i correspond to the adjustment parameters $\delta_{p,i}$ for each industry i. Likewise, the N adjustment parameters for quantities $\delta_{x,i}$ [equation 9] can be estimated with a linear regression:

$$y_{i,t} = \alpha_{1i} x_{i,t} + \alpha_{2i} x_{2,i,t} + \epsilon_{i,t} \quad i = 1, ..., N \tag{18}$$

where the dependent variable is the growth rate of quantities, the independent variable is the excess unit profit, and the linear slopes α_{1i} are the adjustment parameters $\delta_{x,i}$, while $\alpha_{2i} = \gamma\delta_{x,i}$.

The OLS regression suffices to find the parameters with p-values of 0, which is expected since it is synthetic data. Maximum-entropy linear regression obtained the same results. For the estimations using real data, the profit sign $x_{2,i,t}$ is dropped and excess unit profit is used as single regressor as in the first linear regression.

3.2. Estimation using real training data

The real dataset employed to estimate the adjustment coefficients of the model covers 36 industries in six developed economies (Germany, France, Italy, Japan, Netherlands, and the US) in the EU KLEMS database for an annual interval of 23 years between 1995 and 2017. The growth rates of prices and quantities can be directly computed from the time series of its indices, subtracting by their average so that they are relative to the average growth rate as in the theoretical model [figures 4 and 3 for Germany].

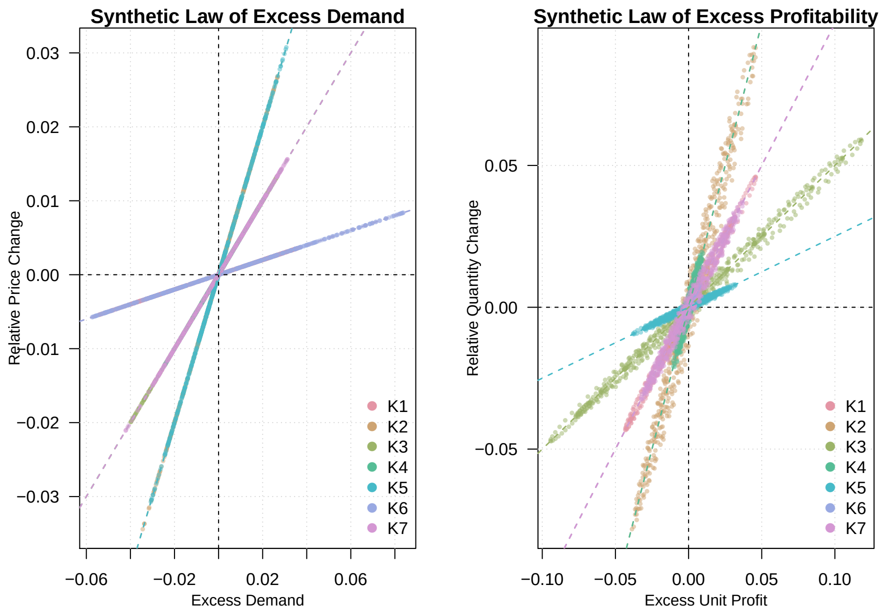

Figure 2: **Relative price and quantity changes with respect to excess demand and excess unit profit** The linear slopes correspond to the adjustment parameters δ_p and δ_x. The synthetic law of excess profitability shows a slight departure from strict linearity due to the stability adjustment γ. Dashed horizontal lines indicate equilibrium values r, x^*, p^*, and $g^* = 0$. Dashed color lines indicate the linear regressions for each sector. [color, 1.5-column]

sector	$\delta_{p,i}$	**p.val**	$\delta_{x,i}$	$\gamma\delta_{x,i}$	**p.val1**	**p.val2**
1	0.1	0	1.00	1.00	0	0
2	1.0	0	2.00	2.00	0	0
3	0.5	0	0.50	0.50	0	0
4	0.5	0	2.00	2.00	0	0
5	1.0	0	0.25	0.25	0	0
6	0.1	0	1.00	1.00	0	0
7	0.5	0	1.00	1.00	0	0

Table 2: **OLS for adjustment parameters $\delta_{p,i}$, $\delta_{x,i}$, and γ**

Figure 3: **Growth rates in industry prices, Germany** Source: EU KLEMS [color, 1.5-column]

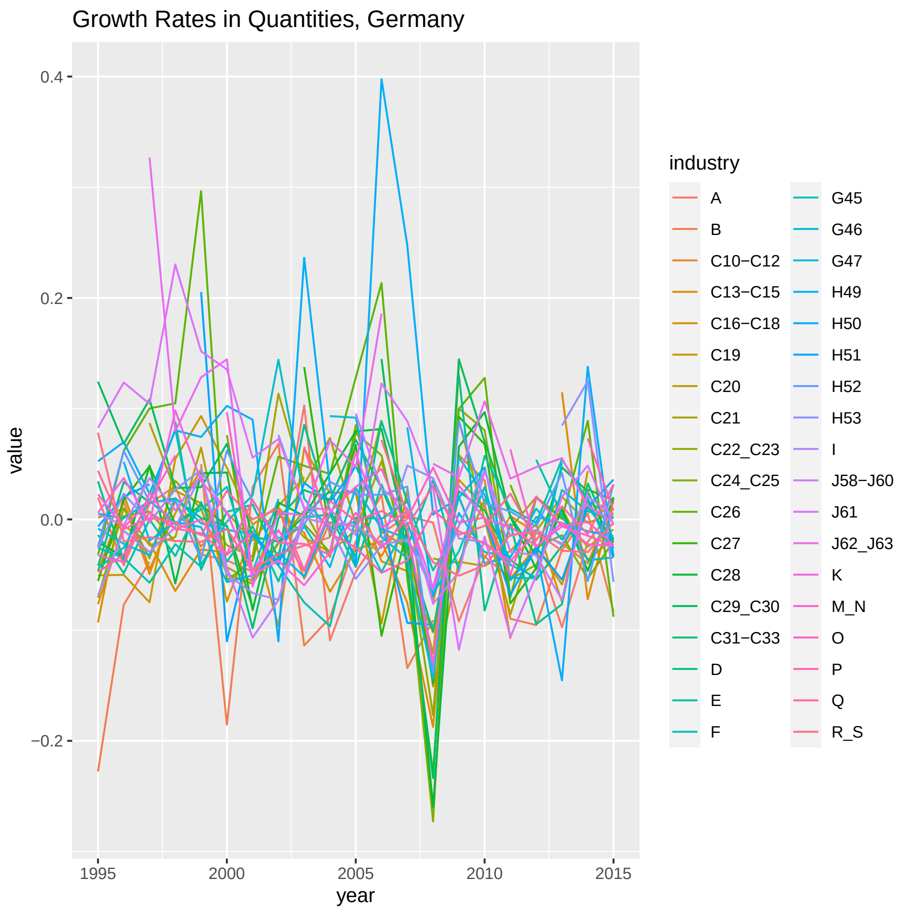

Figure 4: **Growth rates in industry quantities, Germany** Source: EU KLEMS [color, 1.5-column]

For each country and year, the general expansion factor R is computed, following its definition, as the ratio between the total monetary value of gross output over the total monetary value of the circulating capital (labor compensation plus intermediate goods). Normal profitability is just $R-1$. The expansion factor acts as the average benchmark of the model, from which to compute the imbalances between supply and demand and the deviations from normal profitability. Sector imbalances in supply and demand are obtained from the ratio of gross output (i.e. supply) to intermediates (i.e. demand) in quantity terms[2] [figure 5]. Industry deviations from normal profitability are obtained from the industry ratios of monetary value of output over the monetary value of circulating capital, which refer to sector-specific expansion factors for each industry [6]. Imbalances are computed as "unit imbalances" in relative terms to the gross output, either in quantity or monetary terms.

For real training data, a more convenient method of estimation of the linear industry adjustment coefficients employs a mixed-effects model with varying slopes and no intercept:

$$y_i = \beta_{j[i]} x_i + \epsilon_i \tag{19}$$

where the varying slopes $\beta_{j[i]}$ correspond directly to the industry adjustment coefficients δ_j in the linear production system. In this mixed-effects model, observations i are grouped by the 36 industries of each of the six countries, so that there are $36 \times 6 = 216$ groups of observations. In distribution notation,

$$y_i \sim N(\beta_{j[i]} x_i, \sigma_y^2) \beta_j \sim N(\mu_\beta, \sigma_\beta^2) \tag{20}$$

Figures 7 and 8 show the distributions of industry adjustment coefficients in prices and quantities for each country, respectively.

[2] This method of calculating excess demand may be problematic: in national income accounts, the discrepancy between demand and supply is added to one side so that the two sides balance. This discrepancy can be captured by measuring unintended inventory change (Shaikh, 2016, pp.120-128).

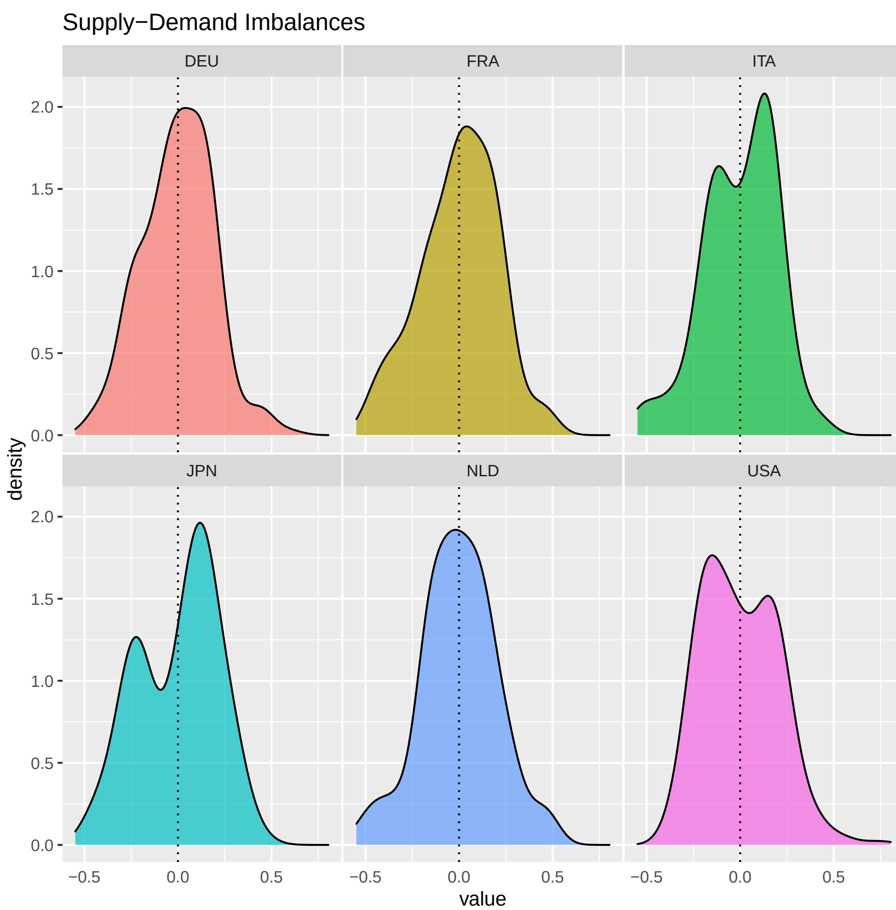

Figure 5: **Distributions of supply-demand sector imbalances for six developed economies** Source: Computed from EU KLEMS [color, 2-column]

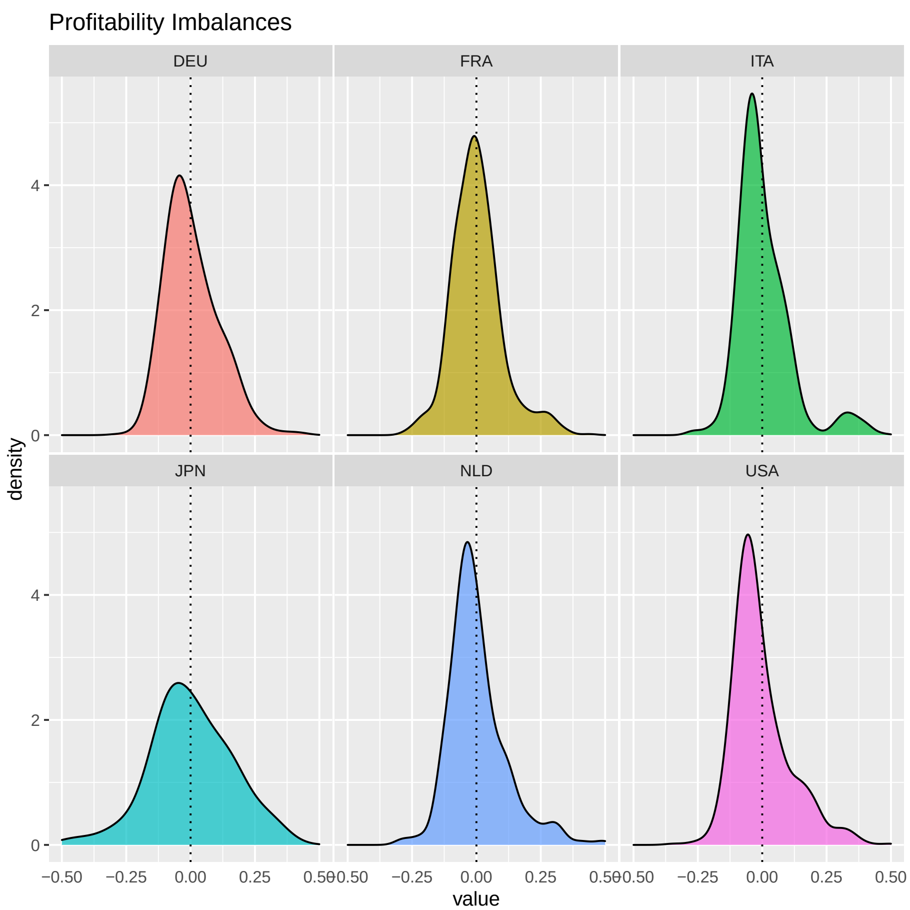

Figure 6: **Distributions of industry deviations from normal profitability for six developed economies** Source: Computed from EU KLEMS [color, 2-column]

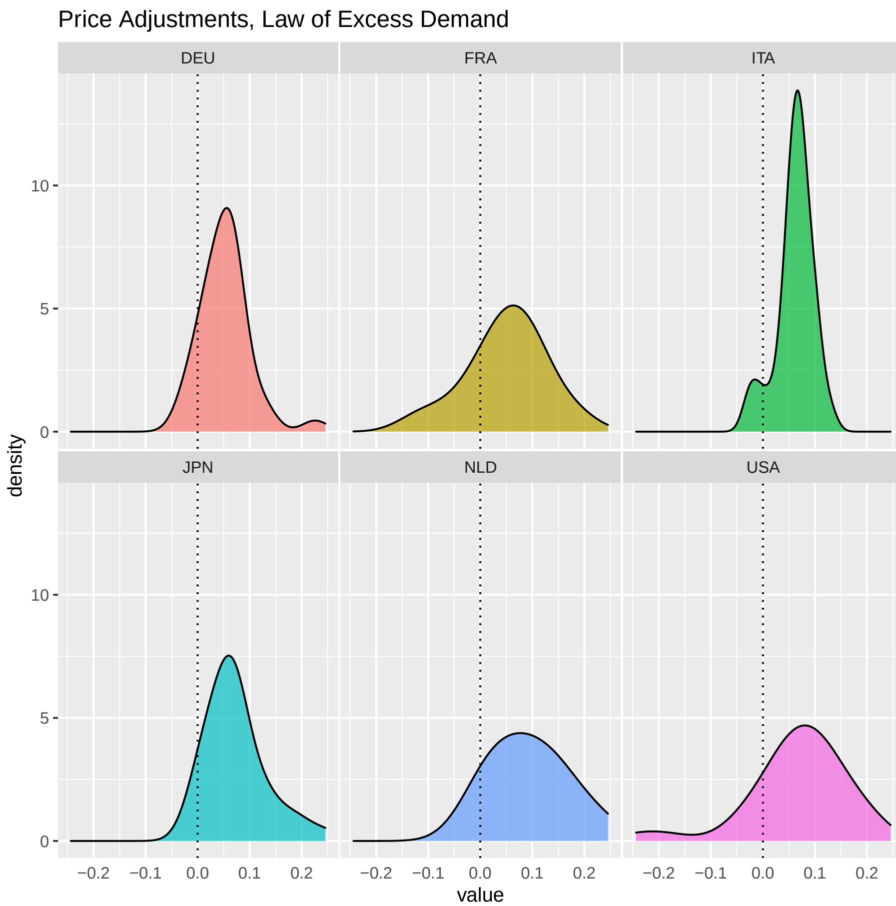

Figure 7: **Law of Excess Demand** Distributions of sector adjustment coefficients in prices for six developed economies [color, 2-column]

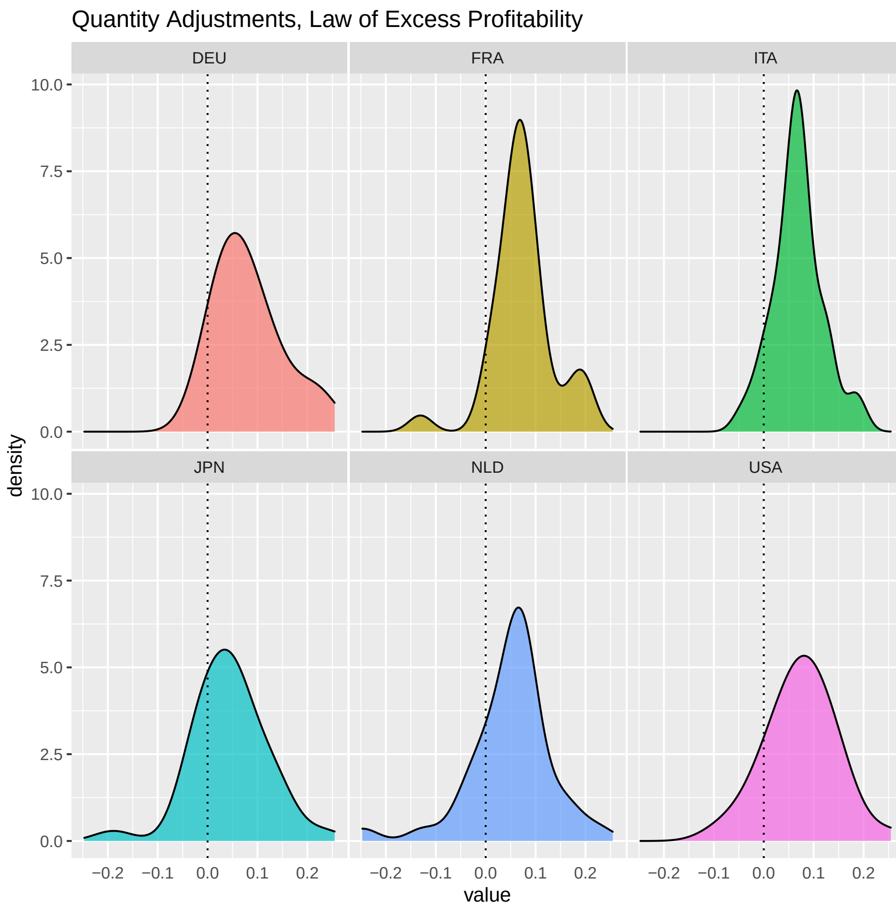

Figure 8: **Law of Excess Demand** Distributions of sector adjustment coefficients in quantities for six developed economies [color, 2-column]

4. Directed Technical Change Towards Decarbonization: The Green Transition

Once the linear adjustment coefficients are estimated for both the law of excess demand and the law of excess profitability, simulations of the dynamical process of technical substitution of specific industries with high carbon content, employing the linear production system of multi-sector growth with technological dynamics, are implemented. The input-output tables are extracted from the World Input-Output Database. The specific industries studied for substitution are "Electricity, gas, steam and air conditioning supply" (D) and "Manufacture of coke and refined petroleum products" (C19). In the simulations, they are replaced by equivalent synthetic "green" sectors with the same input-output linkages and proportional coefficients. This modeling choice allows to study the speed of substitution of the old carbon sector by the new green sector with respect to a single value, their relative technical efficiency, a parameter of great interest in the absence of fiscal policy. Employing actual input requirements of renewable energy would make the simulations considerably more complex to analyze due to the increase in parameters, while only making them marginally more realistic. In fact, the simulation results below will show that the actual technical requirements are the least relevant parameters in assessing the speed of substitution. Finally, it is important to highlight that the life-cycle carbon footprint of renewable energy is negligible compared to carbon energy, so that fully considering life-cycle greenhouse gas emissions has only modest effects on the scale and structure of power production in mitigation scenarios (Pehl et al., 2017).

In this section, the speed of decarbonization is evaluated, first analytically for the sake of clarity and then computationally, with respect to four regulating policy parameters: the relative technical efficiency in production θ, the carbon tax τ on real output, the green subsidy τ', and the initial investment ratio σ_0 (i.e. the initial ratio of green output over carbon output). First, the scenario with no policy ($\tau = \tau' = 0$) is investigated, where the relative technical efficiency in production is the crucial parameter regulating the speed of decarbonization. Then, a fiscal policy in the form of a tax-subsidy mix is introduced, where a share τ of carbon output is taxed in the form of a carbon tax and a share τ' of the carbon tax revenue is re-invested in green output in the form of a green subsidy. In this context, fiscal policy has the important effect of expanding the existing profitability and growth differentials induced by differences in production costs, that is, directing technical change towards decarbonization. Finally, the simulations allow to explore the impact of each regulating parameter on the speed of decarbonization.

4.1. Comparative Statics

For analytical convenience, input-output coefficients of the green sector g are defined as constant over time and proportional to the carbon sector c,

$$\theta = \frac{a_{ig}}{a_{ic}} \quad \forall i = 1, ..., N$$

so their unit costs are also proportional over time (labeled with superindices c and g),

$$\theta = \frac{\kappa_t^g}{\kappa_t^c} = \frac{\sum_i p_t^i a_{ig}}{\sum_i p_t^i a_{ic}} = \frac{\sum_i p_t^i \theta a_{ic}}{\sum_i p_t^i a_{ic}} \tag{21}$$

This convenient definition of the parameter θ allows to evaluate the speed of decarbonization with respect to a single value, the relative technical efficiency in production of the green sector with respect to the carbon sector, which is a parameter of interest in the absence of fiscal policy. In this situation, the relative efficiency in production θ is the main parameter regulating the speed of substitution, due to its effect on the differentials in profitability and thus growth. Parameter θ can also be understood as a nominal carbon-pricing tax that is internalized by the firms in the carbon sector. In the linear production model, the green and carbon sectors produce the same output so that they share a common price $p_t^c = p_t^g$. Hence, relative efficiency in production θ, the proportion of the capital/output ratio of the green sector with respect to the carbon sector, is the only parameter regulating their profit rate differentials,

$$1 + r_t^c = \frac{p_t^c x_t^c}{\kappa_t^c x_t^c} = \frac{p_t^c}{\kappa_t^c} \tag{22}$$

$$1 + r_t^g = \frac{p_t^c}{\theta \kappa_t^c} = \frac{1 + r_t^c}{\theta} \quad (23)$$

In short, capital will flow into and expand output faster of the green sector only if it is more cost-effective than the carbon sector. The growth rate differential between the carbon and green sector is thus dependent on parameter θ, as well as the adjustment parameters δ_x^g, δ_x^c that relate the change in quantities with deviations from the equilibrium unit profit:

$$\frac{1 + g_t^c}{1 + g_t^g} = \frac{1 + \delta_x^c (p_t^c - R\kappa_t^c)}{1 + \delta_x^g (p_t^c - R\theta\kappa_t^c)} \quad (24)$$

Lower-cost green technology ($\theta < 1$) will thus ensure a greater profit rate ($r_t^g > r_t^c$) and growth rate ($g_t^g > g_t^c$): therefore, lower costs in production alone will inevitably induce the phase-out of the carbon sector by its equivalent green one. However, there is no guarantee that, with the current differentials in production costs ($\theta \sim 0.7 - 1.1$), the speed of decarbonization will be fast enough to fall within UN IPCC time targets.

In order to keep track of the phase-in dynamics of the green sector with respect to the carbon sector, it is convenient to define the output ratio

$$\sigma_t = \frac{x_t^g}{x_t^c} \quad (25)$$

which has evolution rule

$$\sigma_{t+1} = \frac{x_{t+1}^g}{x_{t+1}^c} = \sigma_t \frac{1 + \delta_x^c (p_t^c - R\kappa_t^c)}{1 + \delta_x^g (p_t^c - R\theta\kappa_t^c)} = \sigma_t \frac{1 + g_t^g}{1 + g_t^c} = \sigma_0 \prod_{t=1}^{t+1} \frac{1 + g_t^c}{1 + g_t^g} \quad (26)$$

The Green Transition can be considered to be "successful" when the output ratio σ_t reaches and surpasses a critical value $\hat{\sigma}$ above 1, where green output is larger than carbon output. In the simulations, $\hat{\sigma} = 10$, which implies that decarbonization is achieved when green output is 10 times carbon output. The success of the Green Transition without fiscal policy not only depends on the growth rate differential regulated by the relative efficiency parameter θ, but also the initial output ratio σ_0, which corresponds to the initial investment in green technology. There is a minimum time period $t^* > 0$, that is, the duration of the

Green Transition, when the output ratio reaches its target value $\hat{\sigma}$:

$$\hat{\sigma} = \frac{x^g_{t^*}}{x^c_{t^*}} = \sigma_0 \prod_{t=1}^{t^*} \frac{1+g^c_t}{1+g^g_t} = \sigma_0 \prod_{t=1}^{t^*} \frac{1+\delta^c_x(p^c_t - R\kappa^c_t)}{1+\delta^g_x(p^c_t - R\theta\kappa^c_t)} \tag{27}$$

The welfare problem that the central planner faces consists of employing three policy variables, tax rate τ, subsidy rate τ' and initial investment σ_0, in order to expand the profitability and growth differentials between green and carbon output to bring the duration of the Green Transition t^* within the UN IPCC targets. The central planner introduces a tax $0 < \tau < 1$ on real carbon output τx_c after production, which is used to finance a subsidy of value $\tau'\tau x_c$ to the green sector (i.e. a subsidy rate of τ'). The outputs using the tax-subsidy mix (where the hat notation distinguishes the variable with and without policy) thus become:

$$\hat{x}^c_t = x^c_t - \tau x^c_t = x^c_t(1-\tau)$$

$$\hat{x}^g_t = x^g_t + \tau'\tau x^c_t = x^c_t(\sigma_t + \tau\tau')$$

Output proportion with policy τ, τ' becomes:

$$\hat{\sigma}_t = \frac{\hat{x}^g_t}{\hat{x}^c_t} = \frac{\sigma_t + \tau\tau'}{1-\tau} \tag{28}$$

The profit rates for the carbon and green sectors internalize the tax-subsidy policy:

$$1 + \hat{r}^c_t = \frac{p^c_t x^c_t(1-\tau)}{\kappa^c_t x^c_t} = (1+r^c_t)(1-\tau) \tag{29}$$

$$1 + \hat{r}^g_t = \frac{p^c_t x^c_t(\sigma_t + \tau\tau')}{\theta\kappa^c_t x^c_t \sigma_t} = (1+r^c_t)\frac{1+\frac{\tau\tau'}{\sigma_t}}{\theta} \tag{30}$$

While the negative contribution of tax τ is linear on the carbon sector, its positive effects on green profitability depend on the fraction $\frac{\tau\tau'}{\sigma_t}$, i.e. they are the largest when carbon output is much larger than green output ($\sigma_t \sim 0$), that is, at the beginning of the introduction of the policy, and they are multiplied by capital efficiency θ.

The profitability differential once the policy is introduced can be then computed in terms of capital efficiency θ, output proportion σ_t, and tax rate τ:

$$\frac{1+\hat{r}^c_t}{1+\hat{r}^g_t} = \frac{\sigma_t\theta(1-\tau)}{\sigma_t + \tau\tau'} = \frac{\theta(1-\tau)}{1+\frac{\tau\tau'}{\sigma_t}} \tag{31}$$

which shows how a tax-subsidy policy can reinforce cost-induced differentials (when $\theta < 1$) or even offset them (when $\theta > 1$). Higher policy-induced green profitability will make capital flow out of the carbon sector to the green sector faster than without policy. The growth rates with and without policy can be computed for the carbon and green sectors, so the growth rate differential with policy τ, τ' can be compared with the growth rate differential without policy:

$$\frac{1+\hat{g}_t^g}{1+\hat{g}_t^c} = \left[\frac{1+g_t^g}{1+g_t^c} + \frac{\tau\tau'}{\sigma_t}\right]\frac{1}{1-\tau} \tag{32}$$

Once again, the additive presence of the ratio $\frac{\tau\tau'}{\sigma_t}$ shows that the policy to direct technical change towards decarbonization is the most effective at the earliest stages of the phase-in (i.e. $\sigma_t \sim 0$) when the subsidy rate is nonzero. This result shows the relevance of green subsidies ($\tau' > 0$) in kickstarting and mobilizing private funds for decarbonizing the economy, in line with recent studies (Heine et al., 2019; Deleidi et al., 2020; Semmler et al., 2021). However, a tax rate τ on real output alone can already accelerate substantially the phase-out of the carbon sector without any green subsidies $\tau' = 0$, even if green capital efficiency is lower ($\theta > 1$). Numerical simulations may be more convenient to elucidate the actual differential impact of the regulating policy parameters on the speed of decarbonization.

4.2. Simulations

4.2.1. Specific Scenarios: Without policy and with policy

The simulations target two of the economic sectors with highest carbon content, "Electricity, gas, steam and air conditioning supply" (D) and "Manufacture of coke and refined petroleum products" (C19), and study their phase-out by time t^* by equivalent green sectors with relative efficiency in production θ under a carbon tax rate τ, a green subsidy rate τ' and initial output ratio σ_0.

The IPCC imposes many duration targets for the Green Transition (Hausfather, 2018):

- 16 years for a 66% chance of avoiding a temperature increase of 1.5 degrees Celsius,

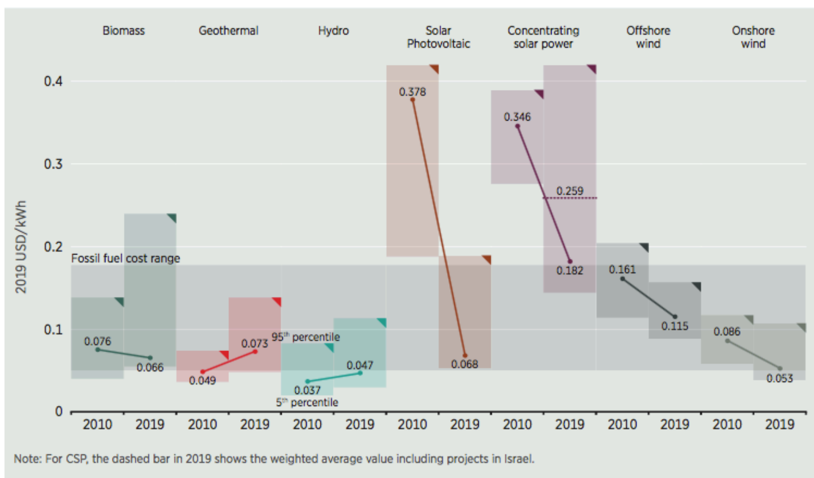

Figure 9: **Global weighted average levelised cost of electricity from utility-scale renewable power generation technologies, 2010 and 2019** Fossil fuel cost range is depicted in gray (IRENA, 2020). [color, 1.5-column]

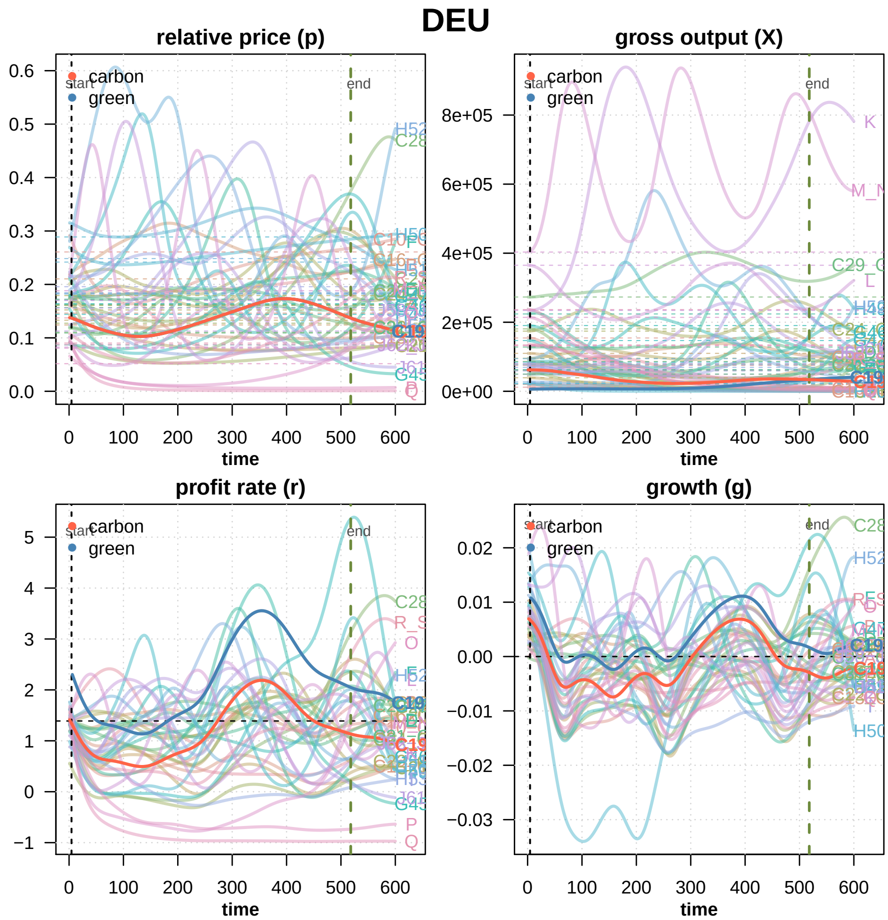

Figure 10: **Simulation of the Green Transition of sector C19 for Germany, with $\theta = 0.7$, $\sigma_0 = 0.1$, and no fiscal policy ($\tau = 0$)** Without fiscal policy, it takes more than $t^* \sim 500$ timesteps for the green sector to take over the carbon sector. [color, 2-column]

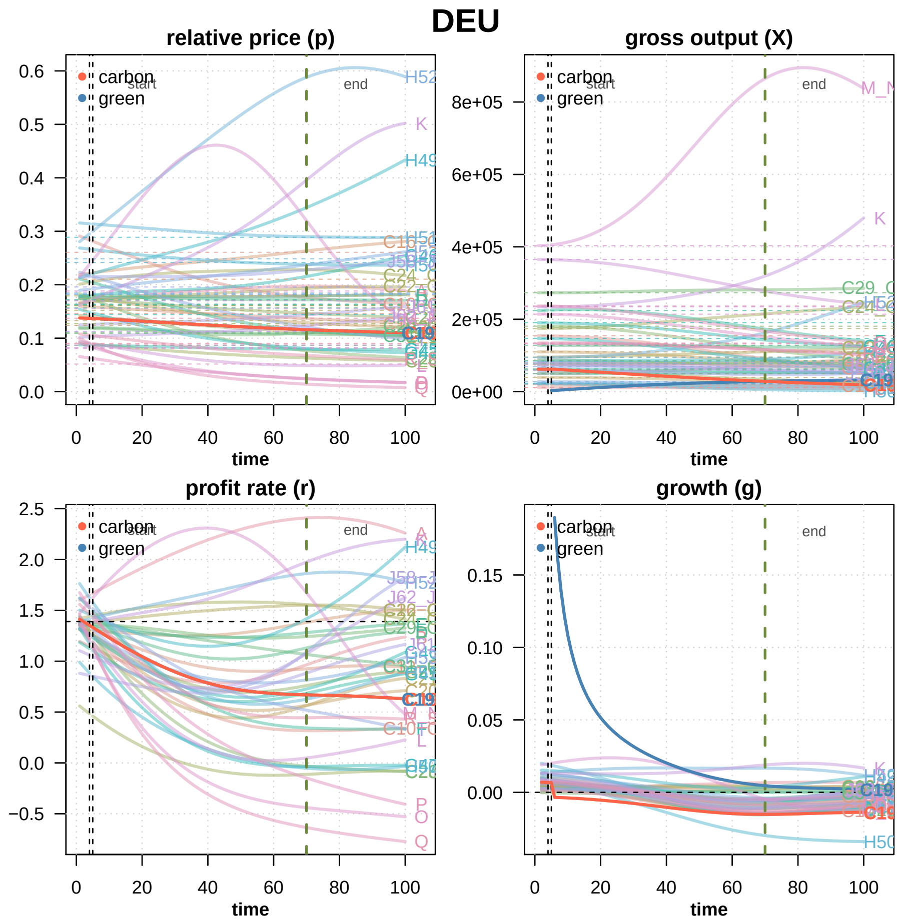

Figure 11: **Simulation of the Green Transition of sector C19 for Germany, with $\theta = 1$, $\sigma_0 = 0.055$, with fiscal policy ($\tau = 0.01$)** Despite the carbon sector being as cost efficient as the green sector, a tax rate of 1% greatly reduces the duration of the Green Transition to $t^* \sim 70$ timesteps. [color, 2-column]

- 23 years for a 50% chance of avoiding a temperature increase of 1.5 degrees Celsius,
- 51 years for a 66% chance of avoiding a temperature increase of 2 degrees Celsius, and
- 65 years for a 50% chance of avoiding a temperature increase of 2 degrees Celsius

At the current moment, green energy is increasingly outcompeting carbon energy: the cost of green energy can be considered to be between 0.7 and 1.1 times the cost of carbon energy – this the range of values chosen for θ [figure 9]. For OECD economies, the current share of final energy consumption in renewable sources over carbon sources is around 5% (Upadhyaya, 2010), which is the benchmark value that is taken for the initial output ratio σ_0 in the simulations.

Figures 10 and 11 show specific simulations of the Green Transition without and with fiscal policy for the C19 sector and the economy of Germany. Each timestep can be considered as one year, given the time dimension included in the adjustment coefficients, which were computed from yearly data. Figure 10 simulates decarbonization without fiscal policy of the most cost-efficient green sectors ($\theta = 0.7$) starting at an initial investment that is twice as the current one ($\sigma_0 = 0.1$). In spite of such advantageous situation for green technology to overtake carbon technology, decarbonization actually takes more than 500 years to occur because profitability and growth differentials are not large enough as induced by lower production costs alone. Instead, figure 11 simulates decarbonization when both technologies are equally cost-effective ($\theta = 1$), at the current initial investment ratio $\sigma_0 = 0.055$, with a minimal carbon tax rate $\tau = 0.01$ and a green subsidy rate that re-invests all revenues, $\tau' = 1$. In this scenario, even where there is no technical advantage, decarbonization only takes around 70 years. This result shows to what extent a small tax rate can greatly accelerate decarbonization. Finally, when the green sector is more cost efficient, the acceleration of decarbonization also implies a faster reduction in the relative price of the targeted sector and thus a general increase in economic efficiency.

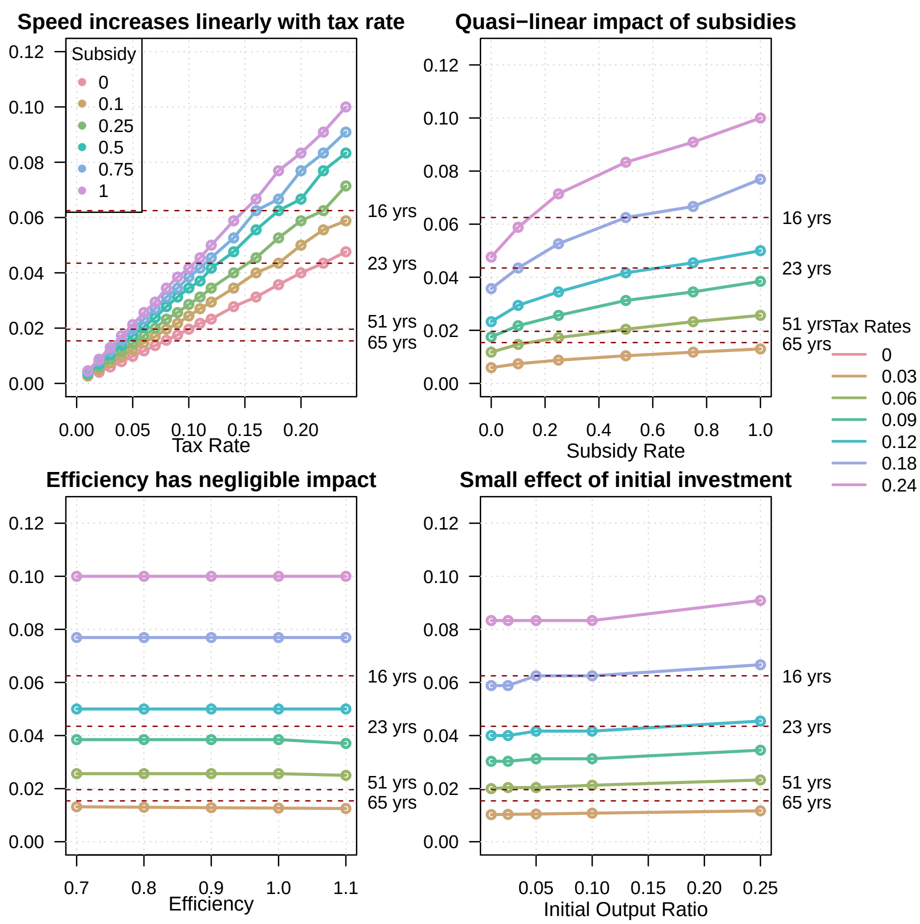

Figure 12: **Impact of Parameters on Decarbonization Speed for sector C19, Germany, for many values of the tax rate** When the tax rate is the dependent variable, the different lines correspond to different values of the subsidy rate. Horizontal dashed lines correspond to the necessary speed to meet the IPCC targets. [color, 2-column]

4.2.2. Impact of Policy Parameters on Decarbonization Speed

In this section, 14,250 simulations are computed for specific ranges of the four policy parameters at stake in order to investigate how the duration of decarbonization t^* depends on them: the range for relative efficiency θ is $(0.7, 1.1)$, the range for the carbon tax rate τ is $(0, 0.24)$ (i.e. the share of carbon output that is taxed), the range for the green subsidy rate τ' is $(0, 1)$, and the range for the initial output ratio is $(0.1, 0.25)$. For the sake of clarity, figure 12 shows the dependence of decarbonization speed $1/t^*$ (i.e. the inverse of the duration of decarbonization time) for sector C19 in Germany with respect to each parameter, for different values of the tax rate.

For instance, the top-left panel shows that, for an efficiency $\theta = 0.8$ and an initial output ratio of $\sigma_0 = 0.05$, a tax rate of $\tau = 0.22 = 22\%$ decarbonizes sector C19 within an IPCC target of 23 years for any subsidy rate, including zero green subsidies. If all carbon tax revenues are re-invested as green subsidies ($\tau' = 1$), then decarbonization within 23 years can already be achieved with half a tax rate, $\tau = 0.11$. For any subsidy rate from 0 to 100 %, a tax rate of $\tau = 0.05$ decarbonizes sector C19 within a substantially higher IPCC target time of 51 years. Many current state policy guidelines aim at decarbonizing by 2050; this would require a tax rate between 0.06 and 0.16, depending on the subsidy rate.

Further, these preliminary results show a very robust linear dependence of decarbonization speed on the tax rate and a very negligible impact for relative efficiency. The relationship between speed and subsidy rate is more complex: linear at low tax rates and logarithmic at high tax rates, showing that green subsidies are most effective when carbon taxes are the highest.

Table 3 shows the regression results of a simple OLS regression of the dependence of decarbonization speed on the four policy parameters as regressors, which is highly significant for all of them and with a very high R^2 value. All six developed economies are studied. The number of observations is lower than the number of simulations because for some values (for instance with a zero tax rate) decarbonization is not attained within the maximum time of 100 timesteps. The

Table 3: OLS regression results for decarbonization speed with respect to four policy parameters, for six developed economies

	Dependent variable:
	speed
initial.output.ratio σ_0	0.028^{***}
	(0.001)
efficiency θ	-0.002^{***}
	(0.0003)
tax τ	0.322^{***}
	(0.001)
subsidy τ'	0.023^{***}
	(0.0001)
Constant	-0.011^{***}
	(0.0003)
Observations	13,341
R^2	0.945
Adjusted R^2	0.945
Residual Std. Error	0.006 (df = 13336)
F Statistic	$57{,}437.620^{***}$ (df = 4; 13336)
Note:	*p<0.1; **p<0.05; ***p<0.01

slopes correspond to the magnitude of their impact on decarbonization speed, which confirms the preliminary results of figure 12. The impact of relative efficiency, akin to a nominal carbon pricing strategy, is negative (as expected, due to its definition) but almost negligible. The most effective fiscal strategy focuses on the carbon tax rate on real output, while the initial investment ratio and the subsidy rate have effects of similar size. It must be noted that subsidy rate and the initial output ratio have very different domains, respectively $(0, 1)$ and $(0.01, 0.25)$, and thus different effects on the speed of decarbonization.

5. Conclusions

The climate crisis is one of the defining issues of our time. Many voices are increasingly noting the existential urge to decarbonize the economy in less than 30 years, as well as the insufficiency of current economic policies to achieve those ambitious goals in time. While some influential voices indicate that carbon pricing strategies may be enough (Nordhaus, 1993), authors such as Mazzucato (Heine et al., 2019; Deleidi et al., 2020; Semmler et al., 2021; Schoder, 2021) emphasize the need of public investment to step in first in order to kickstart and mobilize massive private funds to move into green technology (that is, crowd-in instead of crowd-out by de-risking and thus facilitating private investment).

This paper examines these questions by studying the speed of substitution of a carbon-based energy sector by a renewable-energy sector under directed technical change. Fiscal policy raises taxes on carbon output and uses the revenues to subsidize green output, in the form of a tax-subsidy mix in the direction of Acemoglu (Acemoglu et al., 2012, 2016). The paper employs simulations of a dynamical model of multi-sector growth with technological dynamics as a machine-learning algorithm within the broad literature of cross-dual adjustment processes in economic dynamics. This theoretical system of linear production features a complex pattern of oscillations of prices and outputs of a Lotka-Volterra form around equilibrium values as determined by the technology structure, augmented to include the wage rate as a distributional variable (Von Neumann, 1945; Sraffa, 1960). The dynamical model of multi-sector growth relies on a linear form of cross-dual dynamic adjustment as established by two abstract laws, the law of excess demand and the law of excess profitability. The linear adjustment coefficients are empirically calibrated for six countries using a hierarchical mixed-effects linear model with varying slopes on EU KLEMS datasets.

The speed of substitution of specific carbon-based energy sectors by a green-energy sector is then evaluated analytically and computationally for the six selected countries with respect to the relative cost efficiency between the carbon and green sectors, the initial output ratio, the tax rate, and the subsidy rate. In order to find the dependence of the speed of decarbonization on these parameters, a standard OLS regression is performed on synthetic data produced by simulating the dynamic process of technical substitution for specific meaningful ranges of these parameters. The findings highlight the relevance of tax-subsidy policy mixes in regulating multi-sector growth and directing technical change in order to accommodate the needs of society, when those cannot be achieved by purely market-based solutions (that is, Pigouvian externalities). Relative cost efficiency, which can be also construed as a form of nominal carbon pricing, has a negligible impact on the speed of decarbonization within realistic time frames. By using policy to expand the existing profitability differentials and thus growth differentials of specific industries, fiscal policy has the effect to greatly accelerate the phase-out of the carbon sector, in particular at its earliest stages, in line with recent contributions (Acemoglu et al., 2012, 2016; Deleidi et al., 2020; Semmler et al., 2021).

An interesting next step in the research is to use environmentally-extended input-output tables such as EORA, which feature the carbon content of each industry. Instead of scalars addressing specific industries, vectors of subsidy-tax rates to decarbonize the whole economy can be studied. Further, there are some problems in the econometric estimation of the linear adjustment coefficients. Further, EU KLEMS and WIOD do not have data to estimate specific adjustment coefficients for the carbon and green versions of the industries studied, so at the current state of simulations they had to be assumed as identical. There may be also issues with the datasets that could be solved by relying on more precise databases for specific countries rather than international ones, where data has a higher frequency than yearly.

In the specific context of energy investments, the assumption of circulating capital is substantially stringent; the existence of fixed capital and depreciation may impact the econometric estimation and simulations. This very issue is adressed in a contribution by Flaschel and Semmler that builds on the work of Bródy (Bródy, 1974; Flaschel & Semmler, 1986). Secondly, other functional forms of adjustment could be tested, for instance where the regressions could be logistic instead of linear, retrieving a logistic kind of dynamic adjustment process,

$$y_{i,t} = \frac{1}{1 + \exp(-\beta_i x_{i,t})} \tag{33}$$

which is very interesting to explore numerically stability-wise as an extension of this linear production system. Yet, the theoretical model already works as a form of supervised machine learning using linear regressions on training data.

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References

Acemoglu, D., Aghion, P., Bursztyn, L., & Hemous, D. (2012). The environment and directed technical change. *American Economic Review*, *102*, 131–66.

Acemoglu, D., Akcigit, U., Hanley, D., & Kerr, W. (2016). Transition to clean technology. *Journal of Political Economy*, *124*, 52–104.

Bródy, A. (1974). *Proportions, prices and planning; a mathematical restatement of the labor theory of value*. Elsevier.

Bródy, A. (1997). The second eigenvalue of the Leontief matrix. *Economic Systems Research*, *9*, 253–258.

Burmeister, E., Caton, C., Dobell, A. R., & Ross, S. (1973). The "saddlepoint property" and the structure of dynamic heterogeneous capital good models. *Econometrica: Journal of the Econometric Society*, (pp. 79–95).

Deleidi, M., Mazzucato, M., & Semieniuk, G. (2020). Neither crowding in nor out: Public direct investment mobilising private investment into renewable electricity projects. *Energy Policy*, (p. 111195).

Duménil, G., & Lévy, D. (1987). The dynamics of competition: a restoration of the classical analysis. *Cambridge Journal of Economics*, *11*, 133–164.

Duménil, G., & Lévy, D. (1993). *The economics of the profit rate: Competition, crises and historical tendencies in capitalism*. Edward Elgar Publishing.

Fisher, F. M. (1989a). Adjustment processes and stability. In J. Eatwell, P. Newman, & M. Milgate (Eds.), *The New Palgrave: General Equilibrium* (pp. 36–42). W. W. Norton & Company.

Fisher, F. M. (1989b). *Disequilibrium foundations of equilibrium economics*. 6. Cambridge University Press.

Flaschel, P. (1990). Cross-dual dynamics, derivative control and global stability: A neoclassical presentation of a classical theme. *Political Economy*, *6*, 73–91.

Flaschel, P., & Semmler, W. (1986). The dynamic equalization of profit rates for input-output models with fixed capital. In *Competition, Instability, and Nonlinear Cycles* (pp. 1–34). Springer.

Flaschel, P., & Semmler, W. (1987). Classical and neoclassical competitive adjustment processes. *The Manchester School*, *55*, 13–37.

Flaschel, P., & Semmler, W. (1992). Classical competitive dynamics and technical change. In J. Halevi (Ed.), *Beyond the steady state: a revival of growth theory* (pp. 198–221). Springer.

Fukuda, W. (1975). The output adjustment mechanism in a multi-sectoral economy. *Kobe University Economic Review*, *21*, 53–62.

Gans, J. S. (2012). Innovation and climate change policy. *American Economic Journal: Economic Policy*, *4*, 125–45.

Gillingham, K., Newell, R. G., & Palmer, K. (2009). Energy efficiency economics and policy. *Annu. Rev. Resour. Econ.*, *1*, 597–620.

Golosov, M., Hassler, J., Krusell, P., & Tsyvinski, A. (2014). Optimal taxes on fossil fuel in general equilibrium. *Econometrica*, *82*, 41–88.

Goodwin, R. M. (1982). A growth cycle. In *Essays in Economic Dynamics* (pp. 165–170). Springer.

Goulder, L. H., & Parry, I. W. (2008). Instrument choice in environmental policy. *Review of environmental economics and policy*, *2*, 152–174.

Goulder, L. H., & Schneider, S. H. (1999). Induced technological change and the attractiveness of co2 abatement policies. *Resource and Energy Economics*, *21*, 211–253.

Hahn, F. (1982). The Neo-Ricardians. *Cambridge Journal of Economics*, *6*, 353–374.

Hahn, F. H. (1970). Some adjustment problems. *Econometrica: Journal of the Econometric Society*, (pp. 1–17).

Hausfather, Z. (2018). Analysis: Why the IPCC 1.5 C report expanded the carbon budget. *Carbon Brief, available online at https://www.carbonbrief.org/analysis-why-the-ipcc-1-5c-report-expanded-the-carbon-budget*, .

Heine, D., Semmler, W., Mazzucato, M., Braga, J. P., Flaherty, M., Gevorkyan, A., Hayde, E., & Radpour, S. (2019). Financing low-carbon transitions through carbon pricing and green bonds. *Vierteljahrshefte zur Wirtschaftsforschung/Quarterly Journal of Economic Research*, *88*, 29–49.

IRENA (2020). *Renewable power generation costs in 2019. Report*. Technical Report. International Renewable Energy Agency, Abu Dhabi.

Jaffe, A. B., Newell, R. G., & Stavins, R. N. (2002). Environmental policy and technological change. *Environmental and Resource Economics*, *22*, 41–70.

Jorgenson, D. W. (1960). A dual stability theorem. *Econometrica*, *28*, 892.

Keen, S. (2020). The appallingly bad neoclassical economics of climate change. *Globalizations*, (pp. 1–29).

Mas-Colell, A. (1986). Notes on price and quantity tâtonnement dynamics. In *Models of economic dynamics* (pp. 49–68). Springer.

Miller, R. E., & Blair, P. D. (2009). *Input-output analysis: foundations and extensions*. Cambridge University Press.

Morishima, M. (1981). *Walras' economics: A pure theory of capital and money*. Cambridge University Press.

Nikaido, H. (1985). Dynamics of growth and capital mobility in marx's scheme of reproduction. *Zeitschrift für Nationalökonomie/Journal of Economics*, *45*, 197–218.

Nordhaus, W. D. (1993). Optimal greenhouse-gas reductions and tax policy in the "DICE" model. *The American Economic Review*, *83*, 313–317.

Nordhaus, W. D. (2002). Modeling induced innovation in climate-change policy. *Technological Change and the Environment*, *9*, 259–290.

Nordhaus, W. D., & Boyer, J. (2000). *Warming the world: Economic models of global warming*. MIT Press.

Pehl, M., Arvesen, A., Humpenöder, F., Popp, A., Hertwich, E. G., & Luderer, G. (2017). Understanding future emissions from low-carbon power systems by integration of life-cycle assessment and integrated energy modelling. *Nature Energy*, *2*, 939–945.

Schoder, C. (2021). *Regime-Dependent Environmental Tax Multipliers*. Technical Report. World Bank, Washington, DC.

Semmler, W., Braga, J. P., Lichtenberger, A., Toure, M., & Hayde, E. (2021). *Fiscal Policies for a Low-Carbon Economy*. Technical Report. The World Bank.

Shaikh, A. (2016). *Capitalism: Competition, conflict, crises*. Oxford University Press.

Silverberg, G. (1984). Embodied technical progress in a dynamic economic model: the self-organization paradigm. In *Nonlinear models of fluctuating growth* (pp. 192–208). Springer.

Sraffa, P. (1960). *Production of commodities by means of commodities: Prelude to a critique of economic theory*. Cambridge University Press. Reprint 1975.

Steedman, I. (1984). Natural prices, differential profit rates and the classical competitive process. *The Manchester School*, *52*, 123–140.

Torres-González, L. D., & Yang, J. (2019). The persistent statistical structure of the US input–output coefficient matrices: 1963–2007. *Economic Systems Research*, (pp. 1–24).

Upadhyaya, S. K. (2010). *Compilation of energy statistics for economic analysis*. Technical Report. United Nations Industrial Development Organization.

Von Neumann, J. (1945). A model of general economic equilibrium. *The Review of Economic Studies*, *13*, 1–9.

7. Appendix 1: Technical change with process innovation and extinction

In discrete time, the simulations for a $K \times N$ rectangular system proceed in the following way, where commodity 1 is produced by two competing sectors

(Flaschel & Semmler, 1992):

$$\begin{pmatrix} x_1^1 \\ x_1^2 \\ x_2 \end{pmatrix}_{t+1} = \begin{pmatrix} x_1^1 \\ x_1^2 \\ x_2 \end{pmatrix}_t + \underbrace{\begin{pmatrix} d_{x_1^1} & 0 & 0 \\ 0 & d_{x_1^2} & 0 \\ 0 & 0 & d_{x_2} \end{pmatrix}}_{\langle d_x \rangle} \underbrace{\begin{pmatrix} x_1^1 & 0 & 0 \\ 0 & x_1^2 & 0 \\ 0 & 0 & x_2 \end{pmatrix}}_{\langle x \rangle} \big[B - RA\big]^T \begin{pmatrix} p_1 \\ p_2 \end{pmatrix}_t \tag{34}$$

$$\begin{pmatrix} p_1 \\ p_2 \end{pmatrix}_{t+1} = \begin{pmatrix} p_1 \\ p_2 \end{pmatrix}_t - \underbrace{\begin{pmatrix} d_{p_1} & 0 \\ 0 & d_{p_2} \end{pmatrix}}_{\langle d_p \rangle} \underbrace{\begin{pmatrix} p_1 & 0 \\ 0 & p_2 \end{pmatrix}}_{\langle p \rangle} \big[B - RA\big] \begin{pmatrix} x_1^1 \\ x_1^2 \\ x_2 \end{pmatrix}_t \tag{35}$$

where in a single-product scenario

$$B - RA = \underbrace{\begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{B} - R \underbrace{\begin{pmatrix} a_{11}^1 & a_{11}^2 & a_{12} \\ a_{21}^1 & a_{21}^2 & a_{22} \end{pmatrix}}_{A} \tag{36}$$

As in the square $N \times N$ case with constant technology, an additional investment criterion of firms is included via the $N \times N$ term $S(p_t)$:

$$\frac{S(p_t)}{p_t} = -(B - RA)^T \delta_p (B - RA) \tag{37}$$

so that the new discrete system becomes:

$$\frac{x_{t+1}}{x_t} = 1 + \delta_x[(B - RA)^T p_t + \gamma S(p_t)] \tag{38}$$

$$\frac{p_{t+1}}{p_t} = 1 - \delta_p(B - RA)x_t \tag{39}$$

In their contribution, material-saving innovation is explored with an input matrix A that evolves over time featuring 2 commodities and 3 processes with the following coefficients:

$$A(t) = \underbrace{\begin{pmatrix} 0.4 & 0.6 \\ 0.3 & 0.5 \end{pmatrix}}_{\text{before innovation}} \rightarrow \underbrace{\begin{pmatrix} 0.4 & 0.2 & 0.6 \\ 0.3 & 0.15 & 0.5 \end{pmatrix}}_{\text{during innovation}} \rightarrow \underbrace{\begin{pmatrix} 0.2 & 0.6 \\ 0.15 & 0.5 \end{pmatrix}}_{\text{after innovation}} \tag{40}$$

Initially, there are two processes producing commodity 1 and 2 with input coefficients $(a_{11}, a_{21}) = (0.4, 0.3)$ and $(a_{21}, a_{22}) = (0.6, 0.5)$. A material-saving innovation takes place with the introduction of a newer, more efficient process producing commodity 1 $(a'_{11}, a'_{21}) = (0.2, 0.15)$ which are half of the older process. Eventually, the more efficient process drives out the older process, yielding another square matrix with smaller coefficients.

Substitution effects are computed by the following evolving matrix A:

$$A(t) = \underbrace{\begin{pmatrix} 0.4 & 0.2 & 0.6 \\ 0.3 & 0.15 & 0.5 \end{pmatrix}}_{\text{before innovation}} \rightarrow \underbrace{\begin{pmatrix} 0.15 & 0.2 & 0.6 \\ 0.1 & 0.15 & 0.5 \end{pmatrix}}_{\text{after innovation}} \tag{41}$$

Initially, the more efficient process is $(0.2, 0.15)$, which has an absolute-cost advantage over $(0.4, 0.3)$: the former process produces the same output twice more efficiently than the latter, that is, it requires half the circulating capital to produce one unit of output. The one-off innovation turns the tables by making the latter more efficient, with coefficients $(0.15, 0.1)$.