Understanding dollarization: a Keynesian/Kaleckian perspective

Marco Missaglia, Universita' di Pavia, Italy (marco.missaglia@unipv.it)*

October 2019

Preliminary draft. For comments only

1. Introduction

Much has been written on dollarization and most contributions target, so to speak, the practitioners, addressing issues like the optimal level of official international reserves the central bank of a dollarized economy should try to hold. Still, in most cases these contributions are not related to a clear and transparent model of the economy at hand and, above all, they do not deal with the growth implications of dollarization. Dollarization is usually thought of as a short-run constraint on the possibility to implement standard stabilization macro policies, but it is not generally considered a constraint on longer-run growth. On the contrary, it is generally believed that dollarization, by stabilizing inflation at low levels, helps the economy return to a sustained growth path.

On top of this, it is not always realized that dollarization as such does not change the endogenous nature of money. *Money is endogenous everywhere*, regardless of whether a country has its own currency or not. The banking system in a dollarized economy, as it is the case in any other economy, may and does create deposits *ex-nihilo* each time making loans to the economy is perceived as profitable. After having extended the loan, banks look for funds and, for the private banking system *as a whole* operating in an economy with its own currency, the usual source of funding is the central bank[1]. The "central bank" of a dollarized economy is the rest of the world,

* Many thanks to Elena Molho and Giorgio Rampa for useful suggestions. Errors are mine.

[1] For a single bank, the other sources of funds are deposits and the interbank market.

since in such a case reserves are to be borrowed from abroad. As we will see, this is a crucial difference, but money is endogenous in both cases.

The aim of this paper is to fill these two gaps and build a theoretical growth model for a dollarized economy in a framework of endogenous money. We will show that, *ceteris paribus*, the steady-state medium-term growth rate of a dollarized economy is lower than that of a country with its own currency. We will also show that a dollarized economy is more likely to be unstable than an economy with its own currency, in the specific sense that, everything else being equal, it is more likely for a dollarized economy to fall into a debt trap.

What the experience of dollarized economies shows is that in times of crisis people preference for cash (the legal tender) as a store of value drastically increases (see Acosta and Guijarro, 2018, p. 236-37, for an illustration of the Ecuadorian case). Scared people just go to ATM or the bank counter and withdraw, and then hold money (cash) under the pillow or in a safe at home. The legal tender is (perceived to be) like gold – a safe asset to be held in times of troubles. Referring to "people" or "households" is not very satisfactory, and one should incorporate a class dimension in a serious discourse on dollarization. The way rich people and the middle class react to uncertainty is not the same. The latter increases its preference for the legal tender, whereas the former strengthen their preference for holding foreign assets ("capital flights"). This is an extremely interesting and important aspect of dollarization. However, this is beyond the scope of the paper, where we intend to present a first theoretical reflection on this topic and concentrate on the growth implications of "people" having a more or less pronounced preference for cash as a store of value. For this reason, and to keep the model analytically tractable, income distribution will be taken as given. Keeping in mind that in the post-Keynesian tradition income distribution and inflation are strongly correlated – the latter being the outcome of the conflict over the former[2] - inflation will be treated exogenously as well. Taking the (given) inflation rate to be zero is not only (and not mainly) justified by some need of analytical tractability. What we really want is to compare two economies with the very same essential macro features (same inflation and real exchange rate, same income distribution, etc.) and this way understand the

[2] Weintraub (1958) and Rowthorn (1977) are the pioneers of this "conflicting claim" view on inflation.

“pure” implications of having an own currency or not on the growth path of the system. In other words, imagine that dollarization has properly done its job, i.e. stabilized inflation in the dollarized country at the same (or comparable), low level as that prevailing in other economies (taking this level to be zero is just a matter of simplicity): once this is done, what are the consequences of *remaining* dollarized in terms of medium-term growth and short-run fluctuations prompted by some possible shock?

2. Preference for cash

In a world of endogenous money (the world as it is), being “dollarized” only constitutes a limitation to the extent that people are willing to hold some cash as a store of value. Indeed, should people be happy with their bank deposits, the limitation of sovereignty associated to dollarization would be a purely formal one: you cannot print a piece paper people are just not willing to hold. In a cashless (better: in a framework where cash never constitutes a store of value), endogenous money world, being dollarized or not would not make any relevant difference.

Cash (as a store of value), however, does exist. Especially in time of troubles, people go to the ATM and withdraw cash to be held in some safe at home. This is of course a perfectly rational behavior. When you are scared and think your bank and/or the bank system is going to collapse or simply suffer, you want cash. You want the legal tender under your pillow. All the more so when the legal tender is the US dollar, a powerful currency trusted around the world. It is true that faced with the fear of some financial troubles, people increase their preference for cash everywhere, both in dollarized and non-dollarized economies. However, there is a huge difference between the two cases. In a non-dollarized economy, people know it will be always possible for monetary authorities to satisfy the increased demand for cash by just printing more of it. In a dollarized economy, people know this is not possible and then the extra-cash will have to be borrowed from abroad. Will the country be able to get these loans? At which conditions? Will the economy and households suffer from being more indebted toward the rest of the world? These are the reasons why, instead of “waiting too long” (waiting the concrete occurrence of some unhappy event) to withdraw cash, people in a dollarized economy, compared to those living in non-dollarized countries, are likely to keep "on average" a higher fraction of their liquid wealth in the form of cash. Let us call α, with 0 < α < 1, the fraction of their liquid wealth households want to keep in cash. What we are saying is that on average α is higher in dollarized than in non-dollarized economies. What are the implications of having α at different, exogenous levels? This is what we are going to discuss in the basic model presented in sections 3 and 4. After all, the very fact of wishing to hold cash (which has to be borrowed from abroad) rather than deposits (which have not) might well increase the stock of foreign debt and worsen the current account, in a potentially destabilizing spiral we want to study and understand carefully.

3. The basic model in the short-run

Loosely speaking, the model we are going to build is Keynesian/Kaleckian. Keynesian, because the level of activity fundamentally depends on entrepreneurs expectations; Kaleckian, because at the macro level entrepreneurs end up earning what they decide to spend in the first place.

Short run

A minimalist structure to think about dollarization is illustrated in the Stock Matrix (SM) and Flow Matrix (FM) below. In order to focus on dollarization, there are no securities and households may only hold their financial wealth either in cash (the legal tender, H) or in a bank deposit, D. Without losing any generality, we assume the interest rate on deposits is zero.

SM (dollars)					
	Households	Firms	Banks	RoW	TOTAL
Deposits	D		-D		0
Cash	H			-H	0
Loans		-L	L		0
Foreign loans			$-L_f$	L_f	0
Capital		pK			pK
TOTAL	V	0	0	-NFA	V = pK + NFA

In this open economy, households' nominal wealth (V) is the sum of the value of the capital stock (pK) and the net foreign asset position (NFA) of the country. The latter, in our framework, is the difference between cash holdings (H, a liability for the Rest of the World in our dollarized framework) and net foreign debt (L_f, which in principle may be positive or negative; together with deposits, D, foreign debt is the source of banks' loans to firms, L). So:

$$V = PK + H - L_f$$

Using the definitions v = V/PK (normalized real wealth or wealth-to-capital ratio), h = H/PK (normalized real cash in the hand of households) and $l_f = L_f/PK$ (external debt-to-capital ratio), the above identity may be written as

$$v = 1 + h - l_f \qquad (1)$$

In each moment in time (in the "short-run"), the stock of existing wealth is what it is. It is given (by the past history of the economy). Obviously, it evolves over time (in the "medium-run"). The fact that V (and then v) is given in the short-run has an important implication. The (short-run) variations of h and l_f must be the same. The economic rationale is very simple. If people go to the ATM or the bank counter and withdraw (h goes up), the only way for the banking system of a dollarized economy to satisfy this increased demand for cash is to borrow it from abroad (l_f increases by the same amount). This certainly produces real effects because the interest bill to be paid to the foreigners rises and GNI correspondingly falls, but in the short-run the economy is becoming neither richer nor poorer as a consequence of the decision of people to hold their wealth under the pillow rather than in a bank account. The same concept may be expressed in a different way, which will reveal convenient for a deeper grasping of the dynamic analysis we will undertake at a later stage. We know α represents the fraction of their (liquid) wealth people wish to hold in cash, i.e. H = αV. Normalized real cash in the hand of households, h, may then be expressed as h = αv. Hence, it is possible to rewrite (1) as

$$v = \frac{1-l_f}{1-\alpha}$$

or, re-arranging

$$l_f = 1 - v(1-\alpha) \qquad \text{(1bis)}$$

This formulation will reveal useful to develop our dynamic analysis and makes it clear that in the short-run, for a given level of v, any increase in people preference for cash translates into a higher external debt-to capital ratio. To put the same thing differently: for a given level of v, a country becomes a debtor when α > (v – 1)/v.

FM (dollars)								
	Hous	Firms		Banks		RoW		TOT
		Curr	Cap	Curr	Cap	Curr	Cap	
CONS	-pC	pC						0
INV		pI	-pI					0
EXP		pX				-pX		0
IMP		-M				M		0
[memo]	Nominal GDP = pC+PI+pX –M = W + Pf + Pb + i*Lf							
Int on L		-iL		iL				0
Int on Lf				-i*Lf		i*Lf		0
Wages	W	-W						0
Profits F	Pf	-Pf						0
Profits B	Pb			-Pb				0
	Flows-of-funds							
VarCash	$-\dot{H}$						$\dot{H}$	0
VarDep	$-\dot{D}$				$\dot{D}$			0
VarL			$\dot{L}$		$-\dot{L}$			0
VarLf					$\dot{L}_f$		$-\dot{L}_f$	0
TOT	0	0	0	0	0	Sf = - NCA	$N\dot{F}A$	0

Call Y_d real GDP (expressed in units of the domestic commodity). The fundamental identity of national accounts for our open economy with no government is

$$Y_d = C + I + (X - qM) = C + I + TB$$

M represents imports expressed in units of foreign commodity; this is why they are converted into units of domestic commodity by applying the real exchange rate q, i.e. the price of the foreign commodity expressed in units of the domestic commodity[3]. In general, q = eP*/P. Here, however, e = 1 (the economy is dollarized) and we are assuming P* = 1 (as is clear from table

[3] This "fundamental identity" is far from being obvious. The way it is written, it implicitly assumes that "our" exports' price is P (as can be seen from table FM), which is essentially determined by internal factors. However, as stressed by Taylor (1983, p. 128), "a pure primary product exporter would have its export price determined from abroad", and therefore our model "can best be interpreted as referring to a semi-industrialized country".

FM), so q = 1/P. The trade balance expressed in units of domestic commodity is TB = X – qM. Imports are to be thought of as domestic firms' purchases of the foreign commodity. This commodity, as it is the case for the domestically produced commodity as well, may be used for both consumption and investment purposes.

L_f is the stock of net foreign debt (or credit, when $L_f < 0$) expressed in dollars or, given that P* = 1, in units of foreign commodity. Real GNI (Y_n, expressed in units of domestic commodity) can then be defined as:

$$Y_n = Y_D - i^* q L_f = C + I + \left[X - q\left(M + i^* L_f\right)\right] = C + I + CA,$$

where CA is the real current account expressed in units of domestic commodity (NCA in table FM is the nominal current account). In general, domestic banks borrow from (or lend to) the RoW at the rate i*, whereas at home they lend money at the rate i. How are i* and i determined? As to i*. one could assume the interest rate paid to foreigners to increase with the external debt-to-capital ratio: at the end of the day, foreigners know they are lending dollars to a country unable to print them and it would make sense to think they want to get a higher interest rate (risk premium) when the external debt-to-capital ratio goes up. However realistic, this assumption would not add that much to the argument we want to develop and therefore we will postulate that the economy at hand borrows from the RoW at the *exogenous* world interest rate i*, without adding any risk premium to the picture. To understand this choice (which greatly simplifies the algebra), think this way: what happens when α goes up (when people go the ATM and withdraw)? As we already saw, the external debt-to-capital ratio increases and then, for any *given* i*, GNI will fall. As a consequence, aggregate consumption and then (in a demand-led model) GDP will also decrease. With a falling activity level and for given distributive shares, the macro profit rate will go down as well. This, in turn, will depress accumulation and the economy could enter a recessionary phase. All this – let us insist on this point – is what is likely to happen for a *given* i*. Well, putting a risk premium into the picture and allowing the interest rate paid to foreign creditors to increase with the external debt-to-capital ratio would only *strengthen* this mechanism, without changing its nature. Instead of falling by, say, 1%, GNI would fall by 1.2%, since on top of having to pay interests on a mounting stock of foreign debt, this would have to be done at an increasing interest rate. It follows that, for the sake of *theoretical* modelling, nothing is lost by abstracting from this magnifying effect. Moving to the domestic rate, i, it would certainly be reasonable to think this is determined as a markup on the world interest rate (domestic banks are indeed monopolists in the domestic market[4]), $i = i^* + m$. Once again, however, would having $m > 0$ add something relevant to our argument? Observe, first, that having $m > 0$ is not required to guarantee some positive profit to the banking system. Indeed, using $L = pK$ (matrix SM), $L_f = pK + H - V$ (again matrix SM) and our assumption $H = \alpha V$, one can immediately see that banks' profits may be expressed as $iL - i^*L_f = pK(i - i^*) + i^*V(1 - \alpha)$: provided that $\alpha < 1$, you do not need $i > i^*$ to have positive banks' profits. Second, when people preference for cash increases (α goes up; a similar argument may be applied to the case of a falling α), banks' profits will suffer both directly and indirectly. Directly, because for any given stock of loans to the economy banks will have to borrow more from abroad; indirectly, because the reduction of the macro profit rate (explained above) will depress banks' lending activity. Well, it is pretty unlikely that banks react to this recessionary scenario by increasing m and then i, which would further slowdown their lending activity. It makes more sense, therefore, to think that the markup rate applied by banks depends on other considerations we are not taking into account in the present framework (the degree of competition in the banking sector, for instance). Consequently, we will take m to be exogenous and, for the sake of the argument, nothing is lost by postulating $m = 0$ ($i = i^*$).

We need a theory for C, one for I and one for the trade balance. In this version of the model, we do not consider the effects of income distribution on aggregate consumption (a core principle of post-Keynesian economics) and, as made explicit in tables SM and FM, we rather refer to a generic "households" earning national income (the sum of wages and non-financial firms' and banks' profits) and accumulating national wealth. This way, we may write a generic aggregate consumption function (often employed in the so-called SFC models) as

$$C = c_1 Y_n + c_2 \mathrm{qV}$$

[4] In this basic model, domestic banks' loans are indeed the only way for domestic firms to finance accumulation (see tables SM and FM).

Using the definitions of real GDP and (1), we get

$$C = c_1\left[Y_d - i^* q L_f\right] + c_2 qV$$

In a growth model, it is convenient to normalize variables dividing by the capital stock. Using the definitions c = C/K (normalized real consumption), u = Y_d/K (output-capital ratio, used as a proxy for the degree of capacity utilization) and l_f = qL_f/K (the external debt-to-capital ratio), we may write

$$c = c_1\left[u - i^* l_f\right] + c_2 v, \tag{2}$$

Let us move to investment. In fairly general terms and without entering here the infinite debate over the appropriate form of the investment function, one might postulate (following Joan Robinson (1962)) that non-financial firms' investments depend on their expected profit rate (r^e). Using a simple linear formulation and defining g = I/K (the accumulation rate), we have

$$g = g_0 + g_1 r^e \tag{3}$$

At any point in time, the expected profit rate is what it is and therefore is taken as given in the short-run. The actual (current) macro profit rate is the ratio between the profit bill and the value of installed capital

$$r = \frac{pC + pI + pX - M - W - iL}{PK} = \frac{pC + pI + pX - M - W - i^* L}{PK},$$

(where we used i = i*). Calling w the nominal wage, ω = w/P the real wage, N the employment level, a = N/Y_d the inverse of labor productivity (i.e. the labor-to-GDP ratio) and ψ = ωa the wage share in total GDP, one can easily calculate the profit rate as (do not forget that in our framework L = PK, look at table SM)

$$r = [(1 - \psi)u - i^*] \tag{4}$$

The interpretation is somewhat obvious: since ψ is the share of GDP going to workers and the interest rate i* measures the rent going to domestic bankers, (4) says that when more is to be left to either workers or *rentiers*, industrialists[5] get less.

Finally, a theory for the trade balance. Calling b = TB/K the normalized real trade balance, the simplest possible one is

$$b = b_0 - b_2 u \qquad (5)$$

The above relation, where $b_2 > 0$, says that the trade balance worsens when GDP goes up[6]. What about the effect of a real depreciation? Here, we are treating the real exchange rate as given, and it is possible to think that its effect is somewhat hidden in the parameter b_0, a shift parameter that may also represent any kind of external shock (variations in the world income, in the world price of some key commodities, etc.). We do not know whether a real depreciation makes b_0 bigger or smaller – it depends on whether the Marshall-Lerner condition holds or not.

Moving from the trade balance to the current account is easy. Call ca = CA/K the normalized real current account:

$$ca = b - i^* l_f \qquad (6)$$

The equilibrium in the commodity market requires

$$u = c + g + b \qquad (7)$$

The system (1bis)-(7) is a complete and tremendously simple short-run model for the determination of l_f, c, g, b, ca, r, and u. The structure of causation reveals the Keynesian/Kaleckian nature of this short-run scheme: (3) determines g in accordance with entrepreneurs' expectations (the Keynesian side) and (1 bis) makes it clear that preference for cash determines the external debt-to-capital ratio l_f; then, the sub-system (2)-(5)-(7) gives c, u and b; finally, (6)

---

[5] We prefer to use the word "industrialists" (or managers) rather than "capitalists" because in our model there are no capitalists *strictu sensu*. Investments are fully financed by making recourse to bank loans and non-financial firms distribute profits to managers' households. There are no shares, and no shareholders.

[6] For any given level of i*, q and L_f, an increase in GDP implies a higher GNI as well. Households' consumption demand goes up and as a result imports of consumption items increase. Moreover, *ceteris paribus* a higher u increases the current profit rate and then improves profit expectations, thereby stimulating firms' investment demand. This, in turn, will push imports of investment goods up.

fixes ca and (4) tells entrepreneurs their actual profit rate. Entrepreneurs' investment expenditures are at the beginning of this causal chain, their profits at the end: at the end of each period, entrepreneurs' get what they spend at the beginning (the Kaleckian side). In this simple model, as in the real world, entrepreneurs are the alpha and the omega of the economy.

The short-run solution of the system (indicated by the subscript "s") is:

$$u_s = \frac{g_0 + b_0 + g_1 r^e + v[c_1 i^*(1-\alpha) + c_2] - c_1 i^*}{1 - c_1 + b_2}$$

$$r_s = (1 - \psi) u_s - i^*$$

$$g_s = g_0 + g_1 r^e$$

$$b_s = b_0 - b_2 u_s$$

$$c_s = c_1 u_s + v[c_1 i^*(1-\alpha) + c_2] - c_1 i^*$$

$$ca_s = b_s - i^*[1 - v(1-\alpha)]$$

The (normalized) excess demand for commodities is ed = (c + g + b – u) and short-run stability requires

$$\frac{\partial ed}{\partial u} = c_1 - b_2 - 1 < 0,$$

which is the same condition for u to be positive in equilibrium. We will assume this standard Keynesian stability (positivity) condition holds.

The short-run impact of a higher α is clearly negative. On top of reducing bankers' profits (as we already saw), it lowers the level of economic activity and the industrialists' profit rate. A higher α has a negative effect on GNI and then consumption spending - this is the reason why in a demand-led model it lowers the level of activity. One could be tempted to claim that the impact on the current account is ambiguous, since the interest bill to be paid to the foreigners goes up but the trade balance improves with the contraction of the economy following a more pronounced preference for cash. It is easy to see, however, that the former effect is stronger than the latter. The relevant derivative is:

$$\frac{\partial ca}{\partial \alpha} = -i^* v \left\{ \frac{(1-c_1)(1+b_2)}{(1-c_1+b_2)} \right\}$$

If the propensity to consume out of income is less than 100 percent we may safely conclude that $\partial ca/\partial \alpha < 0$: a stronger preference for cash worsens the current account[7].

In light of the subsequent dynamic analysis, it might be useful to see how the system reacts to higher i* and v. As to the world interest rate i*, it is not surprising to see that the answer depends on whether the economy at hand is a net debtor or a net creditor in the international financial markets:

$$\frac{\partial u}{\partial i^*} = -c_1 \frac{1-v(1-\alpha)}{1-c_1+b_2} = -c_1 \frac{l_f}{1-c_1+b_2}$$

Obviously, the activity level will be stimulated in a creditor country ($l_f < 0$) and depressed in a debtor one ($l_f > 0$). As to the industrialists' profit rate, it falls unambiguously in a debtor economy, whereas in a creditor economy even industrialists (on top of bankers) could benefit from a higher interest rate, provided that

$$-\frac{(1-\psi)c_1 l_f}{1-c_1+b_2} > 1 \, .$$

In words: if the wage share in total GDP is sufficiently low and/or the credits of the country at hand toward the rest of the worlds are sufficiently important, the stimulus to economic activity prompted by a higher interest rate more than compensates the heavier interest bill industrialists must pay to bankers. That said, one should not forget that the case of a dollarized economy with positive claims toward the rest of the world is quite unlikely and, empirically, not that relevant. This is due, *inter alia*, to the fact we stressed several times and constitutes the starting point of our reflection: contrary to what happens in a country with its own currency, when people in a dollarized economy decide to keep a higher fraction of their liquid wealth in the form of cash, this immediately produces a rise of the country's external debt.

[7] The model is built in such a way that we cannot say that much on the impact of a real depreciation. On the one hand, a higher q increases l_f, and as we just saw this has a recessionary impact. On the other, it might increase b_0 (provided that the Marshall-Lerner condition holds), which prompts an expansionary impact. A priori, the net effect is unclear and the risk of a contractionary devaluation is always there, as first emphasized by Taylor and Krugman (1979) in their seminal paper on this topic.

It is important to establish how the current account reacts in our model economy to a higher wealth-to-capital ratio, v. Again, there are two forces moving the current account in opposite directions. On the one hand, richer people consume more and the trade balance worsens with a more buoyant economic activity. On the other, for any given α, l_f goes down: a debtor country will have to pay less interest to the foreigners (a creditor country will receive more interests from the foreigners) and this improves the current account. The net effect is measured by

$$\frac{\partial ca}{\partial v} = \frac{i^*(1-\alpha)(1-c_1)(1+b_2)-b_2c_2}{(1-c_1+b_2)}$$

meaning that the current account will improve if and only if

$$i^* > \frac{b_2c_2}{(1-\alpha)(1+b_2)(1-c_1)} \tag{8}$$

It might be noted that, regardless of whether a country is a debtor or a creditor in the international financial markets, a higher interest rate helps the current account to improve with higher wealth, whereas a stronger preference for cash (higher α) may prevent the current account from improving under the same circumstances. These are important elements for a deep understanding of the dynamics of the system we are going to analyze.

4. The dynamics of the model

In the model (1bis)-(7) there are two state variables, v and r^e.

Studying the evolution over time of the wealth-to-capital ratio, v, clearly requires a full understanding of the dynamics of the external debt-to-capital ratio, l_f, since foreign debt is a (negative) component of wealth. Formally, (1bis) implies

$$\dot{v} = -\frac{1}{1-\alpha}\dot{l}_f \tag{9}$$

Of course, the stationarity of the external debt-to-capital ratio implies that of the wealth-to-capital ratio (and vice versa). Note, also, that in an economy where people hold cash as a store of value (α > 0), the relation between the variations of the external debt-to-capital ratio and those of the wealth-to-capital ratio is not one-to-one, as it would be in case people decide not to hold cash as a store of value (α = 0). From the definition $l_f = qL_f/K$ and remembering that q = 1/P is taken as given ($\hat{q} = 0$) we get

$$\hat{l}_f = \hat{L}_f - g \tag{10}$$

Using national accounts (the flows-of-funds in table FM are enough), one gets

$$\dot{L}_f = \dot{H} - N\dot{FA} = \dot{H} - NCA \tag{11}$$

However simple, the notion incorporated in equation (11) is key. Even if the net foreign asset position does not change (the nominal current account is zero), in a dollarized economy foreign debt increases to the extent that people want to hold more cash as a store of value.

Dividing by the stock of foreign debt and normalizing by the capital stock, (11) becomes

$$\hat{L}_f = \frac{\frac{\dot{H}}{PK} - ca}{l_f} \tag{12}$$

Let us insert (12) into (10):

$$\dot{l}_f = \frac{\dot{H}}{PK} - ca - gl_f \tag{13}$$

Now, using our assumption that people keep a fraction α of their wealth in the form of cash (H = αV), (13) may be written as

$$\dot{l}_f = \alpha\frac{\dot{V}}{PK} - ca - gl_f \tag{14}$$

By the definition of v we get

$$\frac{\dot{V}}{PK} = \dot{v} + vg$$

Hence,

$$\dot{l}_f = \alpha(\dot{v} + vg) - ca - gl_f \quad (15).$$

This is not yet the end of the story. Using (1bis), (15) may be written as

$$\dot{l}_f = \alpha\left(\dot{v} + \frac{1-l_f}{1-\alpha}g\right) - ca - gl_f$$

Rearranging:

$$\dot{l}_f = \alpha\dot{v} - ca + g\frac{(\alpha - l_f)}{(1-\alpha)} \tag{16}$$

Before dropping (16) into (9) to have a complete differential equation for the dynamics of the wealth-to-capital ratio, let us have a quick look to the meaning of (16) itself. To begin with, think to what happens with α = 0, i.e. in a world where people do not want to hold cash as a store of value. Equation (16) would get a much more familiar face:

$$\dot{l}_f = -\left(ca + gl_f\right),$$

meaning that the external debt-to-capital ratio diminishes with positive growth (the denominator increases) and a positive current account (the numerator goes down because less has to be borrowed). What (16) shows is that in a dollarized economy where people want to keep some liquid wealth in the form of cash, things are not that simple. There are essentially two important messages. First, the growth of (normalized) wealth as such worsens the external debt-to-capital ratio to the extent that people want to hold cash as a store of value. This happens simply because cash has to be borrowed from abroad. Second, the positive effect of the growth of capital (and output) on the external debt-to-capital ratio is mitigated, once again, by the willingness of people to hold cash. Even more than that: if this willingness is very pronounced and/or the external debt-to-capital ratio is initially low ($\alpha > l_f$), real growth would go hand in hand with an increasing external debt-to-capital ratio (sooner or later, that growth would then become unsustainable).

Now drop (16) into (9). Using (1bis), one gets the following very simple expression for the dynamics of the wealth-to-capital ratio

$$\dot{v} = ca + g(1 - v) \tag{17}$$

Let us then turn to the evolution of profit expectations. We will simply assume they are revised upward (downward) each time the actual profit rate is higher (lower) than expected ($\varphi > 0$):

$$\dot{r}^e = \varphi[r - r^e] \tag{18}$$

Expressions (17) and (18) constitute the system of two differential equations we are going to use to study the dynamics of our model economy. Studying (18) is very easy, whereas the behavior of (17) is much more complicated. Let us start simple. Use the short-run solution of the model for the macro profit rate and impose r = r^e. This way, you immediately get an explicit function for the isocline (demarcation line) associated to (18), i.e. the set of infinite pairs (r^e, v) such that the expected profit rate is constant over time ($\dot{r}^e = 0$) and coincides with the actual profit rate:

$$r^e = \frac{(1-\psi)(g_0+b_0)-(1+b_2-\psi c_1)i^* + (1-\psi)[c_1 i^*(1-\alpha)+c_2]v}{(1-c_1+b_2)-(1-\psi)g_1} \quad (19)$$

First of all, a comment on the denominator. In this model – as it is the case in most Keynesian models – there is a source of potential instability (we would call it "internal instability") coming from the traditional accelerator effect. In our model economy, higher expected profits stimulate investments, then output and sales, then actual profits. Higher actual profits, in turn, translate into higher expected profits, and this stimulates investments again, and so on and so forth. Clearly, this is a potentially explosive dynamics. The importance of this cumulative effect depends on the slope parameter of the investment function (g_1) and on the profit share (1 – ψ) – in words: on how strongly investments respond to a wave of optimism and on how much of the extra-income generated by investments ends up into the hands of industrialists (those who decide investments). For the model to be dynamically stable, these two parameters cannot be too high. Imposing the condition

$$(1-\psi)g_1 < (1-c_1+b_2) \quad (20),$$

i.e. the positivity of the denominator of (19), serves exactly the purpose of avoiding this kind of internal instability[8]. For future reference, observe that, provided that (20) holds, the intercept of (19) is positive if

$$i^* < \frac{(1-\psi)(g_0+b_0)}{(1+b_2-\psi c_1)} = i_1 > 0 \quad (21)$$

Once this is clear, the implications of (19) are straightforward: (a) the demarcation line $\dot{r}^e = 0$ is a straight line with positive slope in the (v, r^e) space, and the economics is simple: an increase in

[8] The careful reader might have noticed that (20) is a bit more restrictive than the short-run stability condition.

v, which produces a higher actual and then expected profit rate, must be compensated by a higher r^e that, as such, slows the rate of variation of the expected profit rate; (b) the stronger the preference for cash (the higher α), the lower the slope of the isocline, because the expansionary impact of a higher v is now reduced; (c) as expected, the intercept of the isocline (which does not depend on α) increases with the shift parameters g_0 an b_0 and decreases with the interest rate. It might be noted that the slope of the isocline increases with the interest rate: indeed, the higher the interest rate, the stronger the expansionary impact of a given increase in v (a given reduction of foreign debt or a given rise in credits towards the rest of the world). In figure 1, the isocline $\dot{r}^e = 0$ is drawn under assumption (20). The fact that

$$\frac{\partial \dot{r}^e}{\partial v} = \frac{\varphi(1-\psi)[c_1 i^*(1-\alpha)+c_2]}{1-c_1+b_2} > 0$$

shows that the adjustment of the expected profit rate is stable: whenever the economy lies below (above) the isocline $\dot{r}^e = 0$, the expected profit rate goes up (down), as indicated by the small vertical arrows:

[FIGURE 1 TO BE PUT HERE]

As to (17) – by far the most complicated part of the story – use the short-run solutions of the model for ca and g and impose stationarity ($\dot{v} = 0$). After tedious calculations, you get an explicit function for the demarcation line associated to (17), i.e. the set of infinite pairs (v, r^e) such that the wealth-to-capital ratio is constant over time:

$$\{i^*(1-\alpha)(1-c_1)(1+b_2) - b_2 c_2 - g_0(1-c_1+b_2)\}v + [(1-c_1)g_1]r^e - [(1-c_1+b_2)g_1]vr^e = (1-c_1)[i^*(1+b_2) - b_0 - g_0] \quad (22)$$

Equation (22) defines a rectangular hyperbola in the (v, r^e) space. Studying its behavior needs a good deal of patience, and the algebraic details are then relegated to the Appendix[9]. There, we proof that, as expected, the crucial determinants of the exact position and slope of the isocline $\dot{v} = 0$ (of our rectangular hyperbola) are the world interest rate, i*, and the preference for cash

[9] The reader, for a better understanding of what follows, is warmly suggested to go through the Appendix.

parameter, α. The model is fairly rich and general: there are eight possible sub-cases – illustrated in figures from 2 to 9 - each of them belonging to one of the following two larger cases:

1) A "weak" preference for cash, where

$$\alpha < \alpha_T = \frac{b_0(1-c_1)-b_2(g_0+c_2)}{b_0(1-c_1)+g_0(1-c_1)}$$

Observe that $\alpha_T < 1$, as it must be. In principle, α_T (whose definition has been derived in the Appendix) could also be negative, but under these circumstances we would immediately fall into the other, following case;

2) A "strong" preference for cash, with

$$\alpha > \alpha_T.$$

[FIGURES FROM 2 TO 9 TO BE PUT HERE]

Figures from 2 to 5 illustrate the inter-temporal behavior of an economy characterized by a weak preference for cash, whereas figures from 6 to 9 do the same for an economy with a strong preference for cash. The small horizontal arrows in the different diagrams are intended to show that, as properly proved in the Appendix, the only branch of the rectangular hyperbola attracting the economy when it stays outside the isocline $\dot{v} = 0$ is that lying above its horizontal asymptote.

The different subcases are associated to different levels of the world interest rate, i^*. Moving from figure 2 to figure 5 (weak preference for cash), the world interest rate goes down, from a "very high" to a "very low" level (in the Appendix these vague expressions are given a rigorous meaning and quantitative definition, by establishing appropriate thresholds; that said, however, these "vague expressions" are here more than enough to understand the relevant economics we are going to explain). The same happens when moving from figure 6 to figure 9 (strong preference for cash) – the interest rate declines from a very high to a very low level[10].

An intertemporal equilibrium (steady-state) for this economy is a point of intersection between the isoclines $\dot{r}^e = 0$ and $\dot{v} = 0$. What we are really interested in – the essential purpose of this

[10] In moving from Figure 2 to Figure 5 (from 6 to 9), nothing else changes. With the exception of the interest rate, the parameters of the economy (its structural features) remain unchanged.

paper – is to see what happens to this steady-state when α varies, knowing that in a dollarized economy α is typically higher. Before doing so, however, let us try to understand whether such a steady state exists or not and discuss some of its features.

Start with the case of a weak preference for cash. A simple visual inspection of our diagrams should persuade the reader that whereas in the sub-cases illustrated in figures 3, 4 and 5, a locally stable steady-state certainly exists (simply because the slope of $\dot{r}^e = 0$ is always positive and finite and then, sooner or later, this isocline must intersect with the stable branch of $\dot{v} = 0$), this is not necessarily true when the interest rate is very high (figure 2). In this sub-case, a stable steady state might not exist. In figure 2a, we represent exactly this possibility of non-existence: the two isoclines do not intersect. A couple of observations. First, in the case we are dealing with, the intercept on the vertical axis of the isocline $\dot{r}^e = 0$ is certainly negative[11]. Second, and more importantly, an even higher world interest rate (compared to that implicitly assumed in figure 2a) would further lower the vertical intercept of $\dot{r}^e = 0$ and increases its slope. This way, in principle, a stable steady-state could be reached, as represented in figure 2b (starting from figure 2a, imagine to shift the $\dot{r}^e = 0$ isocline downward and make it steeper and steeper; it is true that in the meanwhile the stable branch of the rectangular hyperbola moves upward[12], but sooner or later the two isoclines must intersect, and this is exactly what happens in figure 2b). The (locally) stable steady state is S, whereas U is unstable (a saddle point, to be more precise). Which kind of economy is, however, the one represented in figure 2b? Do not forget that S is a *locally* stable steady state, which means that the economy will actually reach that point only starting from a "sufficiently small" neighborhood. So, the economy we are talking about is one which is indebted toward the rest of the world[13] and that, despite having to pay a very high interest rate to foreign creditors, is able to reach a steady-state characterized by a relatively high profit rate. This may only happen (look at (4)) when income distribution is tremendously skewed in favor of industrialists and against workers: an otherwise financially unsustainable scenario becomes

[11] Just use (21) and the result (derived in the Appendix) according to which $i_2 > i_1$.

[12] D increases with the interest rate and B remains unchanged.

[13] Observe that, in S, $v < v_D < 1 < 1/(1 - \alpha)$.

sustainable in that workers accept to remain silent[14]. If this is not the case and such an awful steady-state is not achieved because of some kind of real wage resistance, we are back to figure 2a, to the case of an economy that, once reached the isocline $\dot{r}^e = 0$ (an attractor), goes towards minus infinity: even if the preference for cash is weak, the interest rate is so high that sooner or later the process of accumulation ceases and foreign debt becomes unsustainable. In sum, the case of a very high interest rate represented in figures 2a and 2b gives people very few options. Either they accept the *Scylla* of a socially awful equilibrium or they go for the *Charybdis* of a financial disaster (from the perspective of workers the difference, if any, is probably irrelevant).

Luckily enough, things are different when the world interest rate lowers. Take figure 3. Though higher, the intercept on the vertical axis of the isocline $\dot{r}^e = 0$ is again negative (same argument as in footnote 11). There are two steady states, but only S is (locally) stable (the other is, again, a saddle point). The difference with the scenario depicted in figure 2 is not only that in this case a locally stable steady state certainly exists. There are at least two other important issues to be mentioned. First, even in this "improved" scenario, it is still possible for the economy to go towards minus infinity (foreign debt becomes unsustainable and stops accumulation). This is for instance the case of an economy starting from very low (however positive) levels of both the expected profit rate and the wealth-to-capital ratio, in a point like P somewhat close to the origin (and below the unstable branch of the hyperbola). There is little doubt, however, that this scenario is less likely than it was in the previous case, characterized by a higher interest rate (figure 2): even when the starting levels of r^e and v are relatively low - in a point like Q, for instance (lying this time above the unstable branch of the hyperbola), a point that would push the economy towards minus infinity in the case of figure 2 - the economy might well end up in the steady-state S, especially when profit expectations do not adjust too rapidly. Second, in the steady state S the economy might result to be net creditor towards the rest of the world. Of

[14] In a model where income distribution affects aggregate demand and output and growth happen to be "wage-led", not even workers' silence would be enough.

course, this is not to be taken for granted[15], but as we saw this possibility was just to be ruled out in the case illustrated by figure 2.

Things are even better in figure 4, with a still lower interest rate. In this case we cannot say whether the intercept on the vertical axis of the isocline $\dot{r}^e = 0$ is positive or negative[16], but this is not important to know. In any case, indeed, the economy is very likely to reach a stable steady-state like S, with positive v and r. Observe, in particular, that even in case the economy starts from a point like P – a point that would have pushed the system towards minus infinity in the case illustrated by figure 3 – it will end up in the steady-state S.

As to figure 5 – the lowest interest rate with a weak preference for cash – we can observe that, once again, we are not sure that the intercept on the vertical axis of the isocline $\dot{r}^e = 0$ becomes positive[17]. In case it is positive (figure 5a: preference for cash is extremely low, see footnote 17), the economy reaches a stable steady state like S almost from everywhere. In case it is negative (figure 5b: preference for cash is not as low as it is in figure 5a), the economy could in principle end up in a stable steady state like T, with a *negative* profit rate. Again, which kind of economy we are talking about? How is it possible to have a negative profit rate with such a low interest rate? Even if this is not to be taken for granted, the economy we are dealing with is likely to be a net creditor in international financial markets[18] (or, at worst, a “weak” debtor). In this case, the profit rate might well lower with falling interest rates, especially when the wage share is high (as it can be easily seen from the short-run solution of the model). Needless to say, the case illustrated by figure 5b is extremely unlikely to materialize. It is more a theoretical *curiosum* than

---

[15] Observe that, in S, $v > v_D$, but we do not whether $v > 1/(1 - \alpha)$. This depends (and obviously so) on several other parameters of the economy.

[16] The argument goes as follows. To be sure that the intercept continues to be negative, we should be able to say that $i^* > i_1$, but we only know that $i^* > i_3$. This would be enough when $i_3 > i_1$ but, as proved in the Appendix, for this to be true it must be $\alpha > \alpha_H$, with $\alpha_H < \alpha_T$. Well, we just know that, in the case illustrated by Figure 4, $\alpha < \alpha_T$ and therefore we cannot conclude.

[17] The argument is essentially the same we developed in footnote 16. To be sure that the intercept becomes positive, we need $\alpha < \alpha_H$, but we only know that $\alpha < \alpha_T$, and $\alpha_H < \alpha_T$.

[18] Observe, indeed, that in the steady-state T we have $v > v_N > v_D$. In words, the wealth-to-capital ratio is very high, as it is the case for a creditor economy. That said, we cannot take it for granted that the economy at hand is a net creditor, since $v > v_N$ certainly implies $v > 1/(1 - \alpha)$ only when $\alpha > \alpha_T$ (i.e. in the case of a strong preference for cash), as the reader can easily prove by using the definition of v_N given in the Appendix.

a concrete possibility, especially because it is very hard to think of a steady state with negative profit rates.

Let us move now to the case of a strong preference for cash (figures from 6 to 9). Figure 6 looks very much like figure 2 (and, again, the intercept on the vertical axis of the isocline $\dot{r}^e = 0$ is negative), with its large corridor of instability: the world (and then domestic) interest rate is extremely high and, once again, the economy oscillates between the *Scylla* of a socially awful equilibrium and the *Charybdis* of a financial disaster. The only and relevant difference is that with a strong preference for cash this alternative is even more dramatic. This can be understood from both an economic and a formal perspective. Economically, the point is obvious: here people go more often to the ATM and, as we already know, this is bad for the external debt of the country. From a formal perspective, take figure 2 and study what happens when α goes up. Using the definitions given in the Appendix, it is easy to see that: a) the horizontal asymptote of the rectangular hyperbola lowers (since A goes down); b) D, B and v_D do not change; c) v_N increases (moves to the right), since A is lower than before. On top of this, the isocline $\dot{r}^e = 0$ becomes flatter, whereas its intercept does not change. As a result, the branch of the rectangular hyperbola lying in the northeast section of the diagram rotates to the left around the same intercept (D/B) as before and becomes steeper. The other branch, lying in the south-west part of the diagram, moves to the right and becomes flatter. The implication is that even an economy that was staying in its awful, *Scylla* steady state (point S in figure 2b) now easily enters the corridor of instability and runs towards *Charybdis*. The only way to avoid this destiny is to have an even higher interest rate and, possibly, an even worse income distribution.

Move to figure 7. The interest rate is going down, the same way it was doing in moving from figure 2 to figure 3. This time, however, the perspectives of the economy remain bleak – the *Scylla-Charybdis* nightmare does not disappear. It is true that the north-west branch of the hyperbola lowers a bit (D/B is less than before and the horizontal asymptote becomes negative) and this, for any *given* position of the isocline $\dot{r}^e = 0$, makes it more likely for the economy to reach a (*Scylla*) steady-state. It is also true, however, that the isocline $\dot{r}^e = 0$, while moving upward, becomes flatter. It is then perfectly possible for this economy not to achieve any steady state and precipitate towards *Charybdis*. This is exactly the case represented in figure 7. In terms of geometric intuition, one can observe that in the movement from figure 2 to figure 3 (the interest rate lowers and there is a weak preference for cash) the north-west/south-east hyperbola transforms into a south-west/north-east one. This way, it becomes much easier for the economy to have and reach a steady state, since the slope of the isocline $\dot{r}^e = 0$ is positive. On the contrary, in the movement from figure 6 to figure 7 (the interest rate lowers, but there is a strong preference for cash), the hyperbola remains of the north-west/south-east typology and the economy is less likely to benefit from the reduction of the interest rate. It is more likely to remain exposed to the risk of unsustainability. This is very easy to understand from an economic point of view. In a dollarized economy, the dynamics of foreign debt is (or becomes) sustainable when the interest rate and/or the preference for cash as a store of value are sufficiently low (or lower sufficiently). This is the reason why, in a framework with a relatively strong preference for cash, the very same reduction of the interest rate cannot be as beneficial as it would be when people have a weak preference for cash.

The interest rate must reduce "a lot" – figures 8 and 9 – for the economy to be more likely to have and reach a stable steady state instead of falling into a debt trap. That said, however, it is interesting to compare the situations illustrated by figures 4 and 8. The interest rate is the same and in both cases the economy reaches a stable steady-state. With a strong preference for cash, however (figure 8), the steady-state level of v is less than v_D, meaning that the economy at hand is certainly a net debtor toward the rest of the world, whereas, as we saw, the economy illustrated in figure 4 (weak preference for cash) might well be a net creditor. It is only when the interest rate is extremely low (figure 9) that this economy with a strong preference for cash *could* become a net creditor[19].

The above analysis proved that, *ceteris paribus*, it is more likely for an economy with a strong preference for cash (a dollarized economy) to fall into a debt trap. It was also shown that in order to avoid such an unpleasant destiny, an economy with a strong preference for cash should accept

[19] Remaining in the realm of geometric intuition, the reader might note that figures 2 and 6 on the one hand, and 4 and 5 and 9 on the other, are pretty the same. This is clearly not true for figures 3-7 and 4-8. In words: should the interest rate be *very* high (figures 2-6) or *very* low (figures 5-9), having a strong or weak preference for cash does not change significantly the likely dynamics of the economy. Out of these extreme cases, however, preference for cash becomes a key parameter.

a probably unacceptable equilibrium with a very bad income distribution. We are now left with a last point to be understood. Assume that none of these bad scenarios materializes. There is a stable steady state with a somewhat acceptable income distribution. Well, what about the steady-state growth rate compared to an economy where, all the rest being the same, preference for cash is weak?

The most rigorous way to answer this question would be to rewrite the system (17)-(18) or, equivalently, that formed by equations (19) and (22) in differential terms and calculate the partial derivative $dr/d\alpha$. This route would add further technicalities to an already heavy body of algebra but, luckily enough, a much simpler graphical analysis is more than enough. Take for instance figure 4. Imagine the economy is in its steady-state S. At a point, for some reason and all the rest remaining the same, preference for cash goes up. There are two possibilities: α increases but it remains true that $\alpha < \alpha_T$ or, second possibility, α increases more significantly and the system moves to a regime where $\alpha > \alpha_T$ (and then the appropriate diagram to describe the economy becomes figure 8). In the first case – look at figure 10 for an illustration – the isocline $\dot{r}^e = 0$ becomes flatter and the stable branch of the isocline $\dot{v} = 0$ shifts downward (v_D does not change, whereas A/C lowers and this shifts the horizontal asymptote down). Therefore, we do not know a priori whether the steady-state level of the wealth-to-capital ratio goes up or down, but it is certainly true that $dr/d\alpha < 0$. There are no ambiguities at all: a stronger preference for cash reduces the steady-state profit rate and then, just look at (3), the steady-state growth rate of the economy.

[FIGURE 10 TO BE PUT HERE]

In the other case – α increases more significantly and the economy moves from figure 4 to figure 8, the analysis is even easier. Just observe that in the steady state described by figure 4, $v > v_D$, whereas in the steady-state of figure 8 we have $v < v_D$. Since v_D does not depend on α, we must conclude that the steady-state level of the wealth-to-capital ratio is lower in the case described by figure 8. With a lower v and a higher α, the steady state profit rate must be lower as well (just look at (19)). We must conclude, once again, that a stronger preference for cash decelerates the steady-state growth of the system.

Things would not change should we assume that the economy initially lies in a steady state like S in figure 5a. Again, what would happen should preference for cash increase, all the rest remaining unchanged? Once more, the rise of α could be such that the regime does not change (α remains lower than α_T) or so significant that it does and we get to $\alpha > \alpha_T$. Take the first case: as illustrated in figure 11, the mechanics is the same as before. The isocline $\dot{r}^e = 0$ becomes flatter and the stable branch of the isocline $\dot{v} = 0$ shifts downward (v_D does not change, whereas A/C lowers to A'/C). Once again, the effect on the wealth-to-capital ratio is ambiguous, but we certainly have dr/dα < 0 and the steady-state growth rate diminishes. In the second case (we get to $\alpha > \alpha_T$), things are even simpler: the steady state S' of figure 11 is nothing but the steady state S' of figure 9.

[FIGURE 11 TO BE PUT HERE]

5. Conclusions and extensions

In a world of endogenous money, where commercial banks may and do create money by making loans and make loans each time this is perceived as profitable, what are the consequences of being "dollarized", i.e. not having the right to print an own currency? Of course, should people not be interested in holding cash as a store of value and be content with their bank deposits, nothing relevant would change. You do not have the right to print something people do not want to hold.

However, especially in dollarized economies, people preference for cash (i.e. the desire of people to hold cash *as a store of value*), is inevitably higher than in countries with their own currencies. In the paper we have shown that it is more likely for an economy with a strong preference for cash (a dollarized economy) to fall into a debt trap. It was also shown that in order to avoid such an unpleasant destiny, an economy with a strong preference for cash should accept a probably unacceptable equilibrium with a very bad income distribution. It was also shown that when a dollarized economy reaches a stable steady state, this is characterized by a lower growth rate compared to that of an economy with its own currency.

These are important results. Yet, the analysis proposed in the paper could be extended by loking at what happens *during* a crisis (and not only, so to speak) in response of the fear of a crisis. During a crisis people preference for cash as a store of value drastically increases. This means that α could be treated as endogenous, varying counter-cyclically. This would probably magnify the short-run impact of exogenous shocks. If, say, the world price of some important export item falls and people react by increasing their preference for cash, this is likely to amplify the recessionary impact of the shock. This is a possible extension of this paper, where in any case we decided to concentrate on the medium-run, growth impact of having, on average, a higher preference for cash.

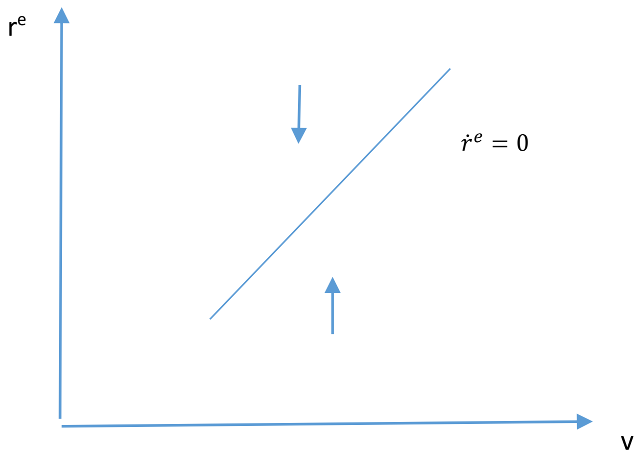

Figure 1: The demarcation line $\dot{r}^e = 0$

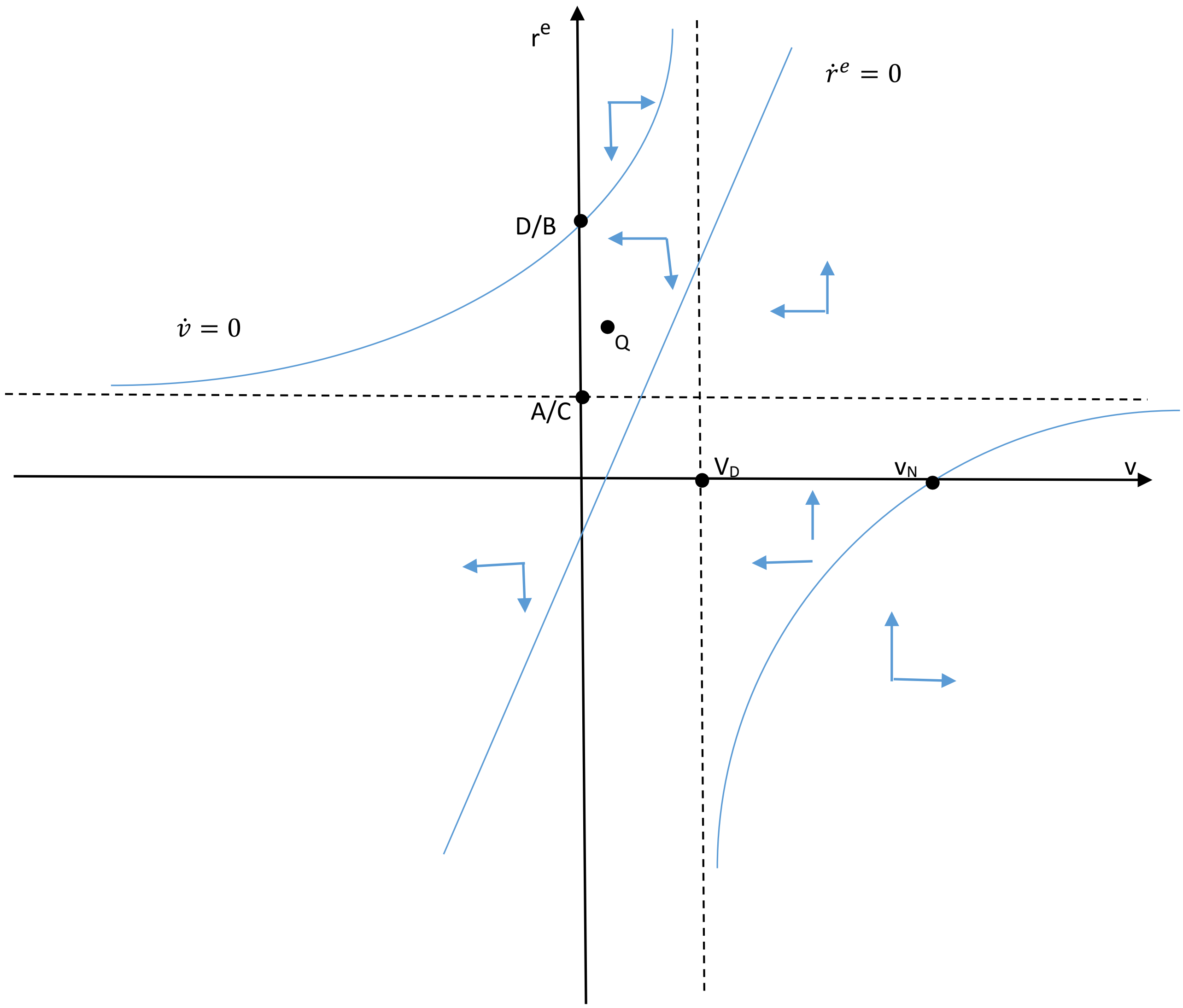

Figure 2a: The demarcation line $\dot{v} = 0$ when $\alpha < \alpha_T$ and $i^* > i_T > i_2 > i_3$. An economy with no steady-state.

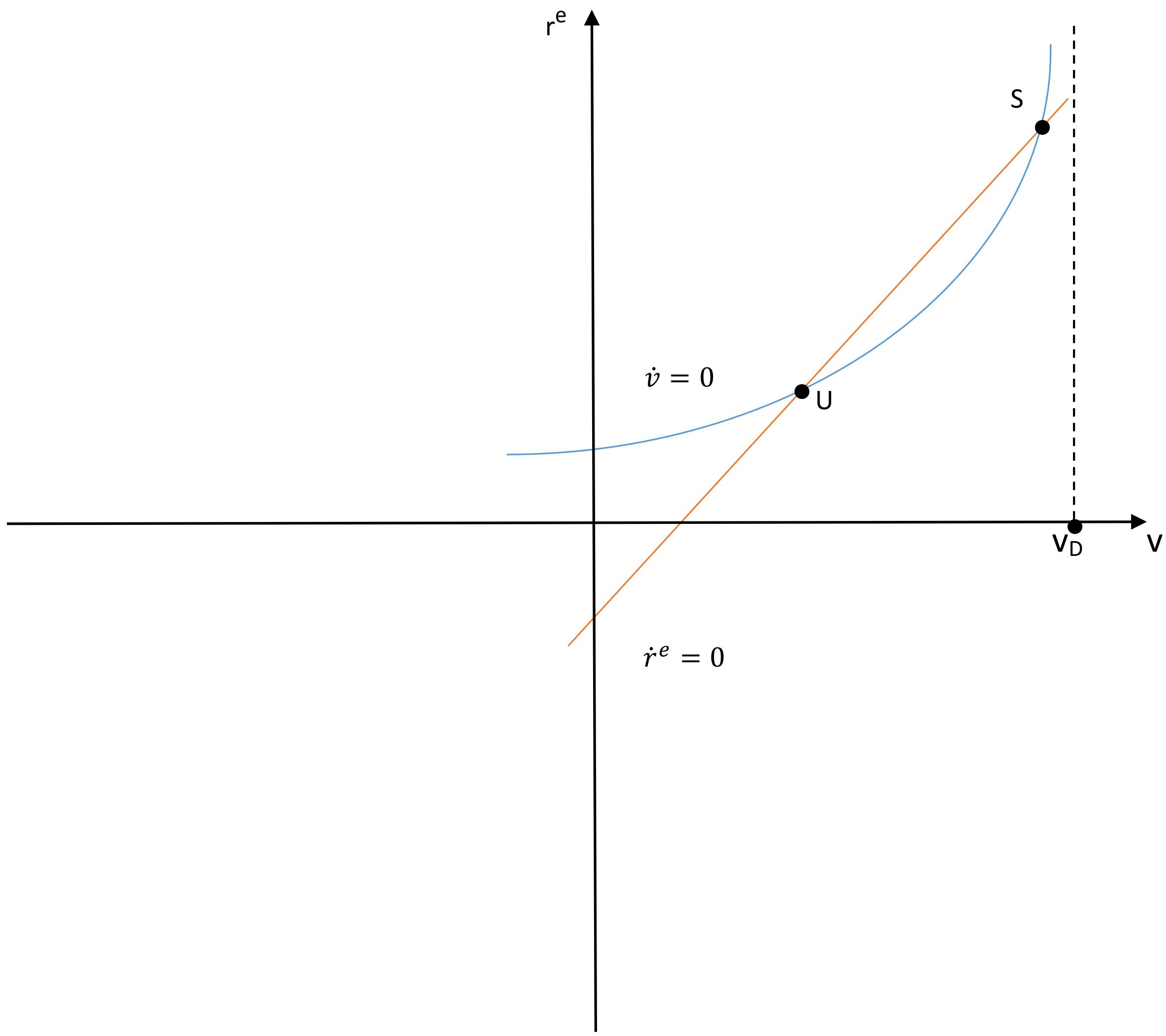

Figure 2b: The demarcation line $\dot{v} = 0$ when $\alpha < \alpha_T$ and $i^* > i_T > i_2 > i_3$. An economy with a steady-state.

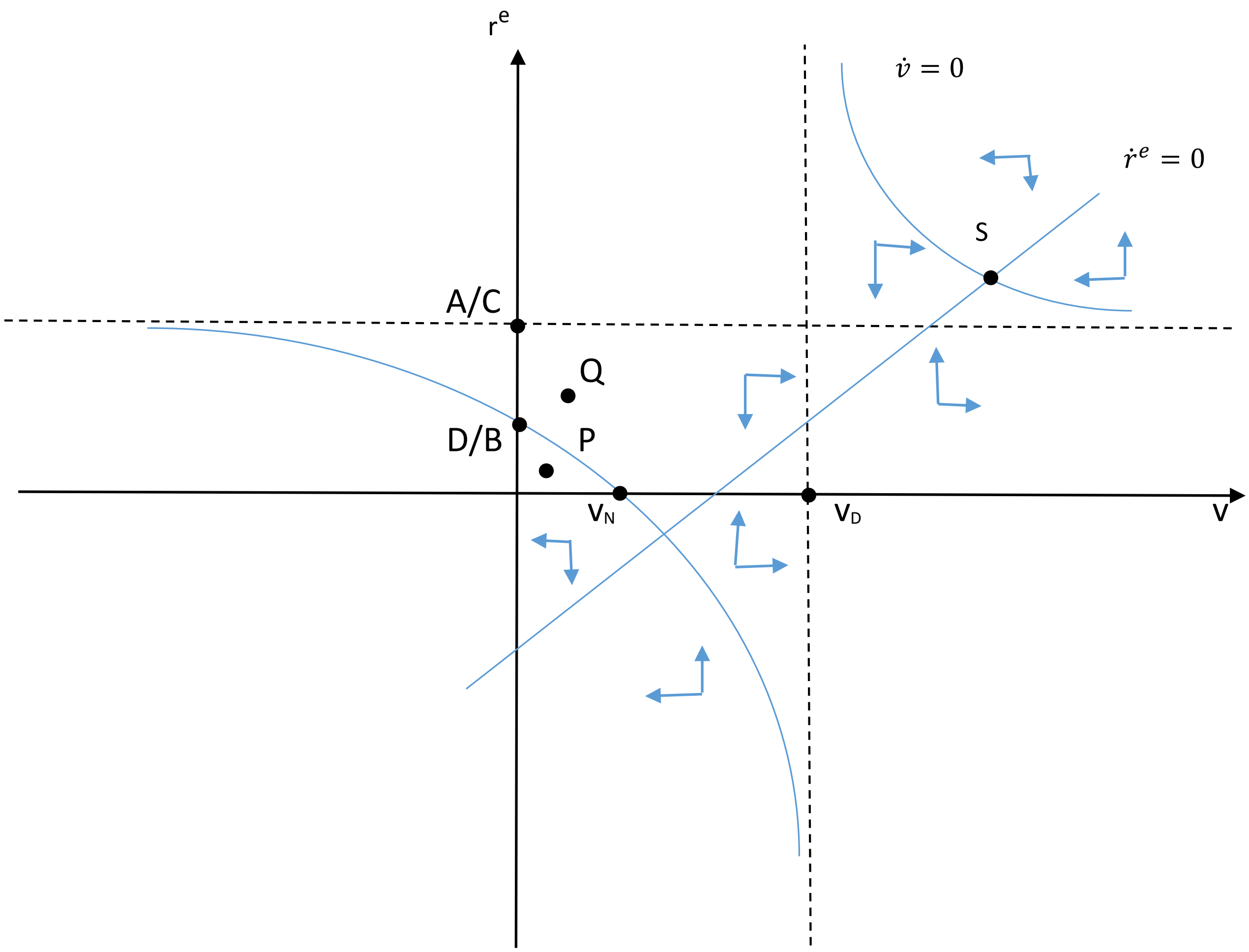

Figure 3: The demarcation line $\dot{v} = 0$ when $\alpha < \alpha_T$ and $i_T > i^* > i_2 > i_3$.

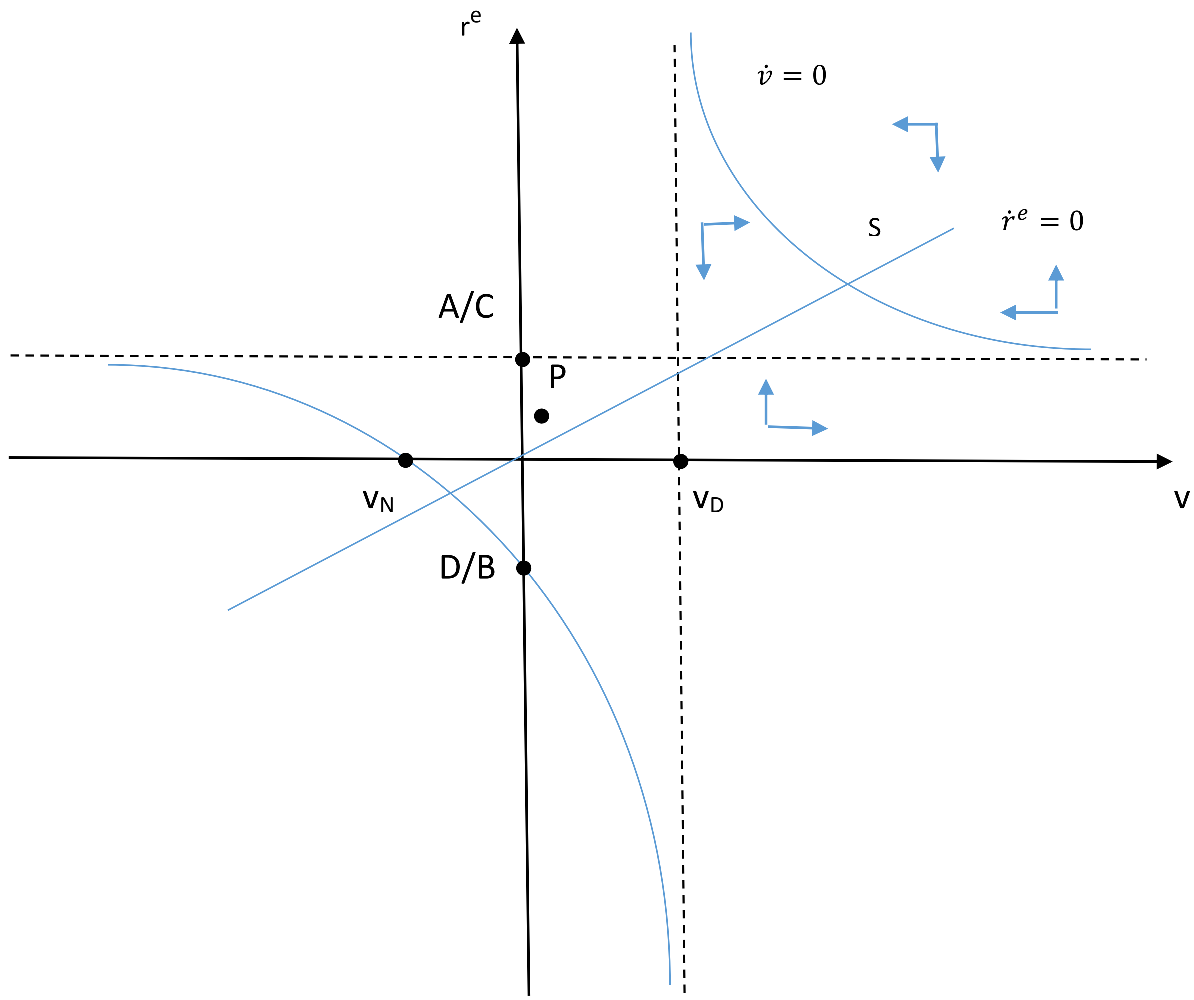

Figure 4: The demarcation line $\dot{v} = 0$ when $\alpha < \alpha_T$ and $i_T > i_2 > i^* > i_3$.

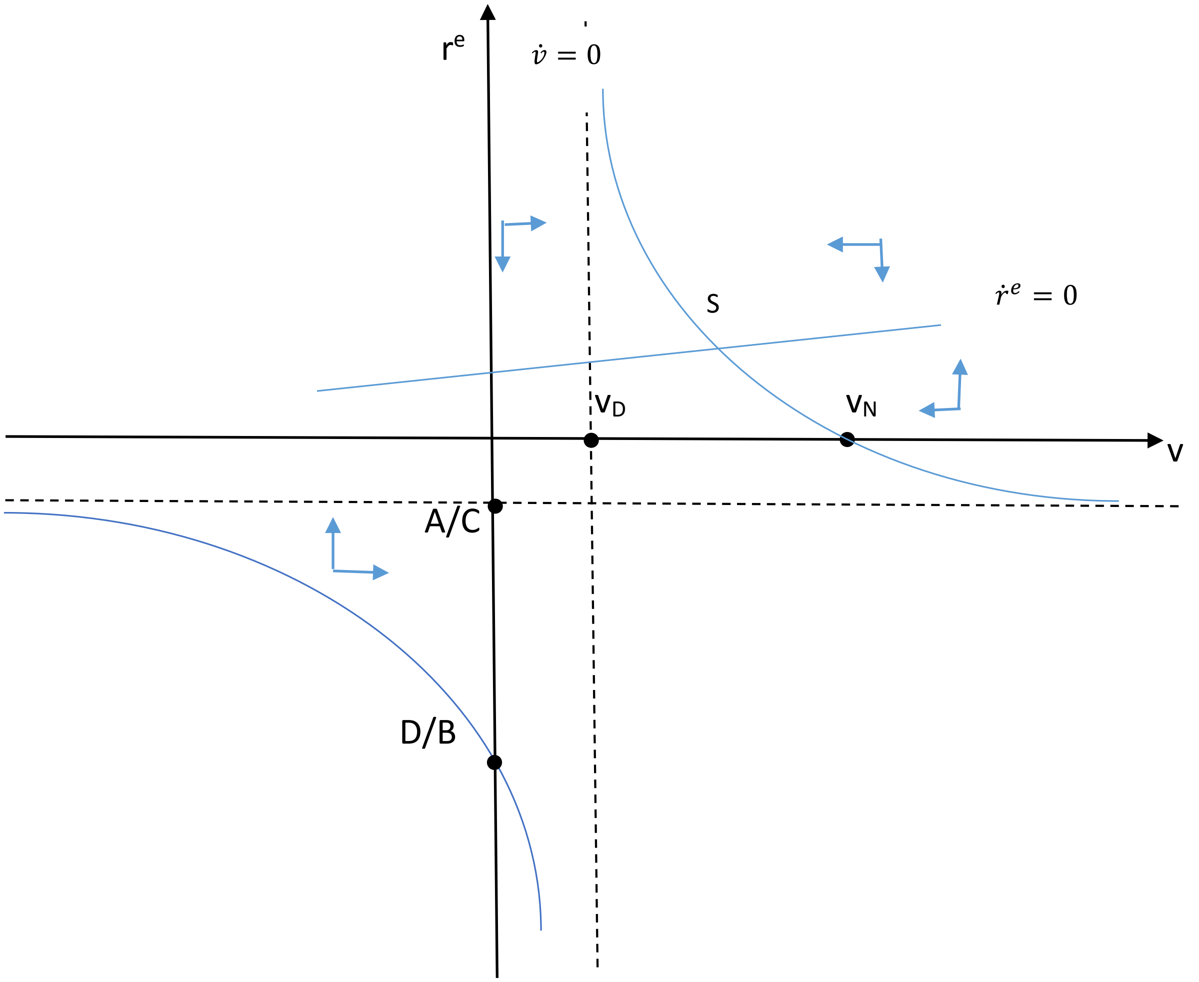

Figure 5a: The demarcation line $\dot{v} = 0$ when $\alpha < \alpha_T$ and $i_T > i_2 > i_3 > i^*$. A positive profit rate.

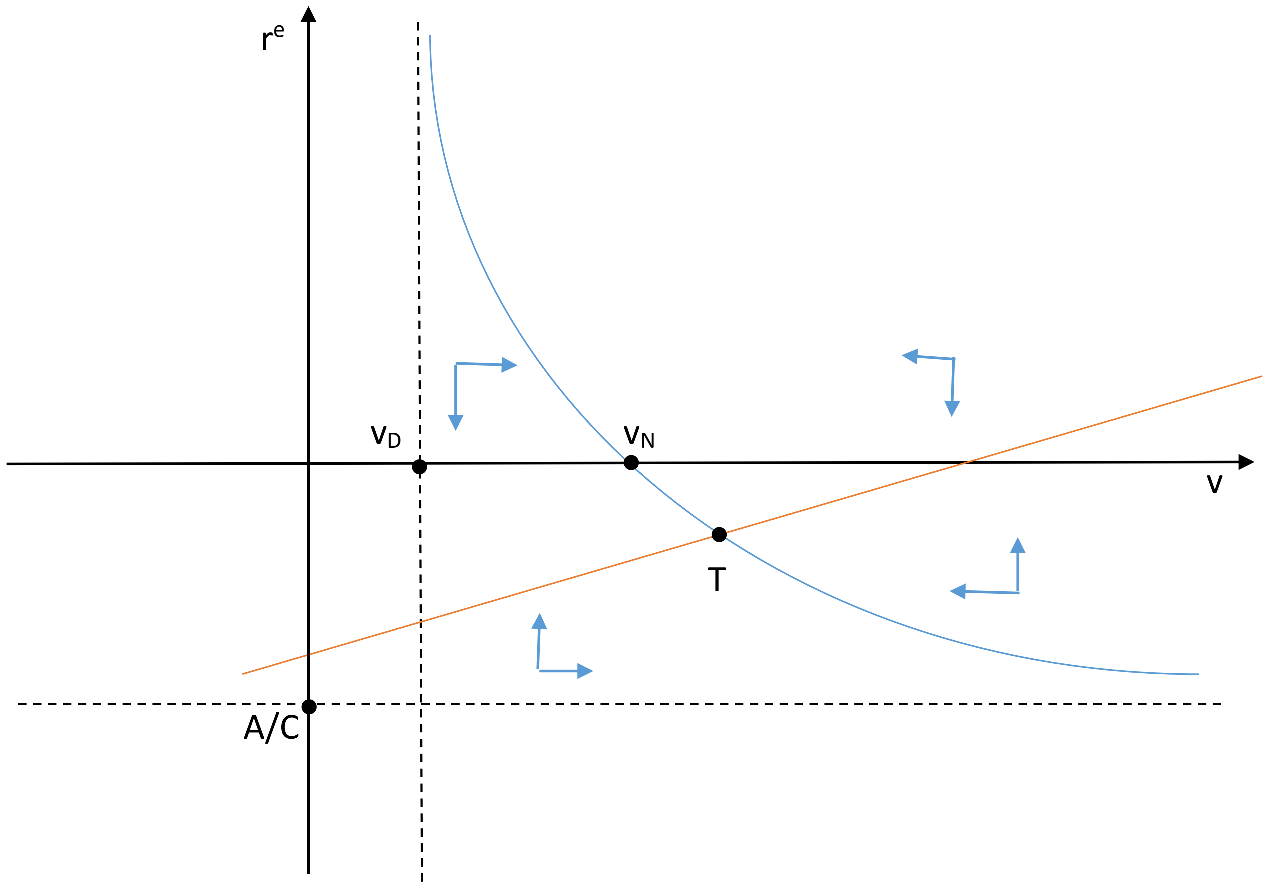

Figure 5b: the demarcation line $\dot{v} = 0$ when $\alpha < \alpha_T$ and $i_T > i_2 > i_3 > i^*$. A negative profit rate.

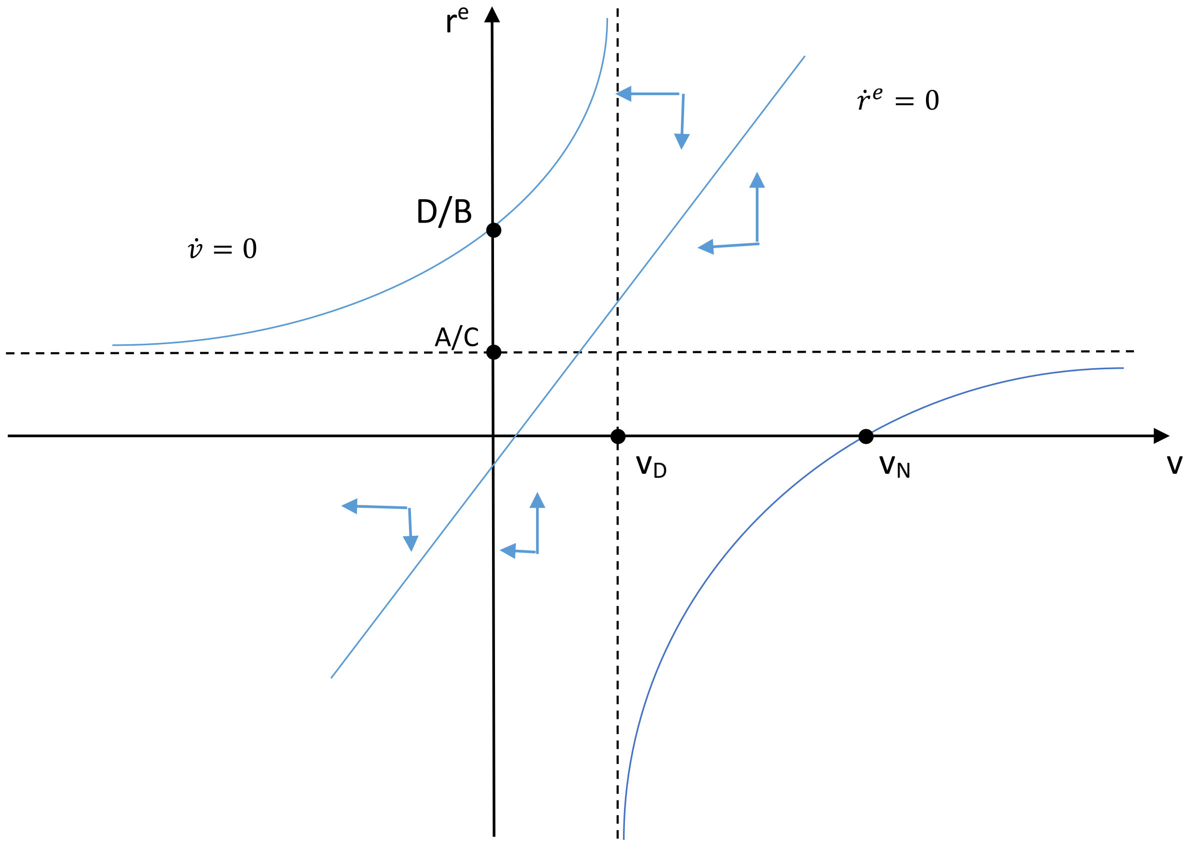

Figure 6: The demarcation line $\dot{v} = 0$ when $\alpha > \alpha_T$ and $i^* > i_3 > i_2 > i_T$. An economy with no steady-state.

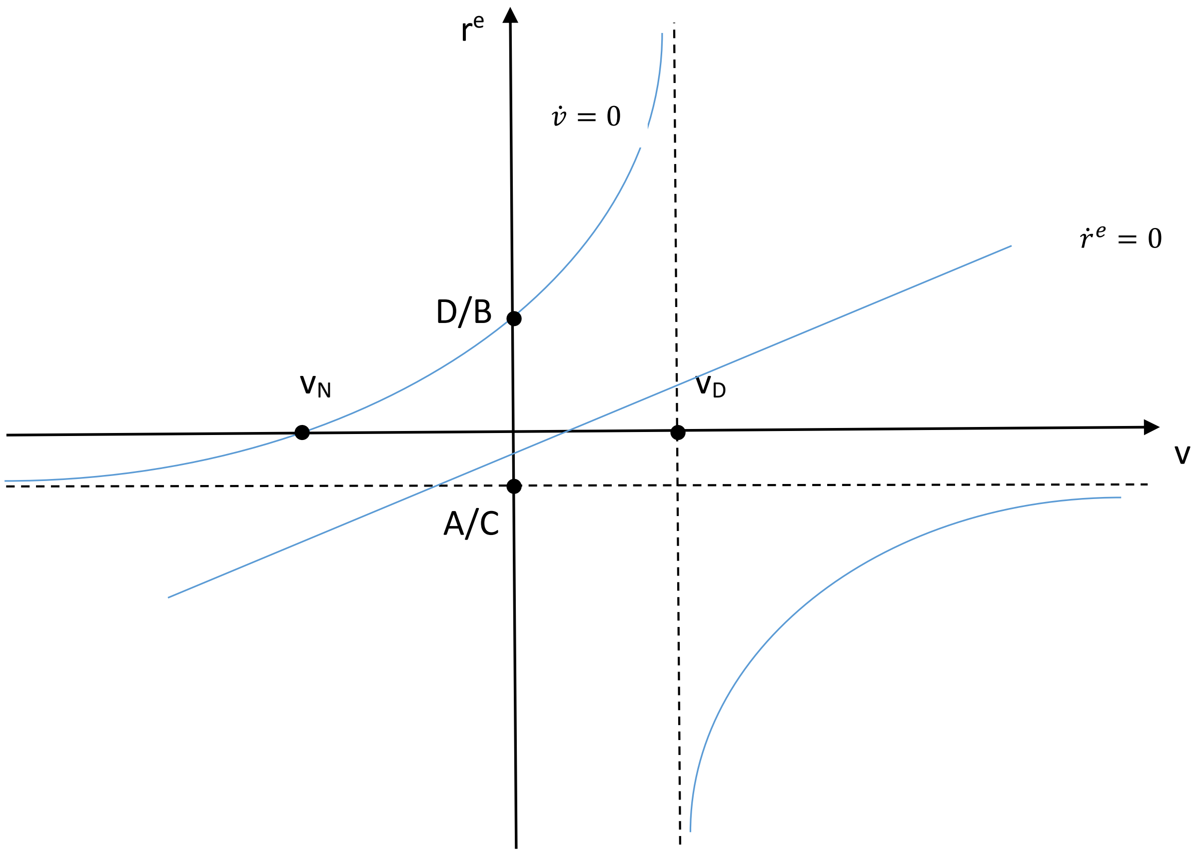

Figure 7: The demarcation line $\dot{v} = 0$ when $\alpha > \alpha_T$ and $i_T < i_2 < i^* < i_3$. Once again there is no steady-state

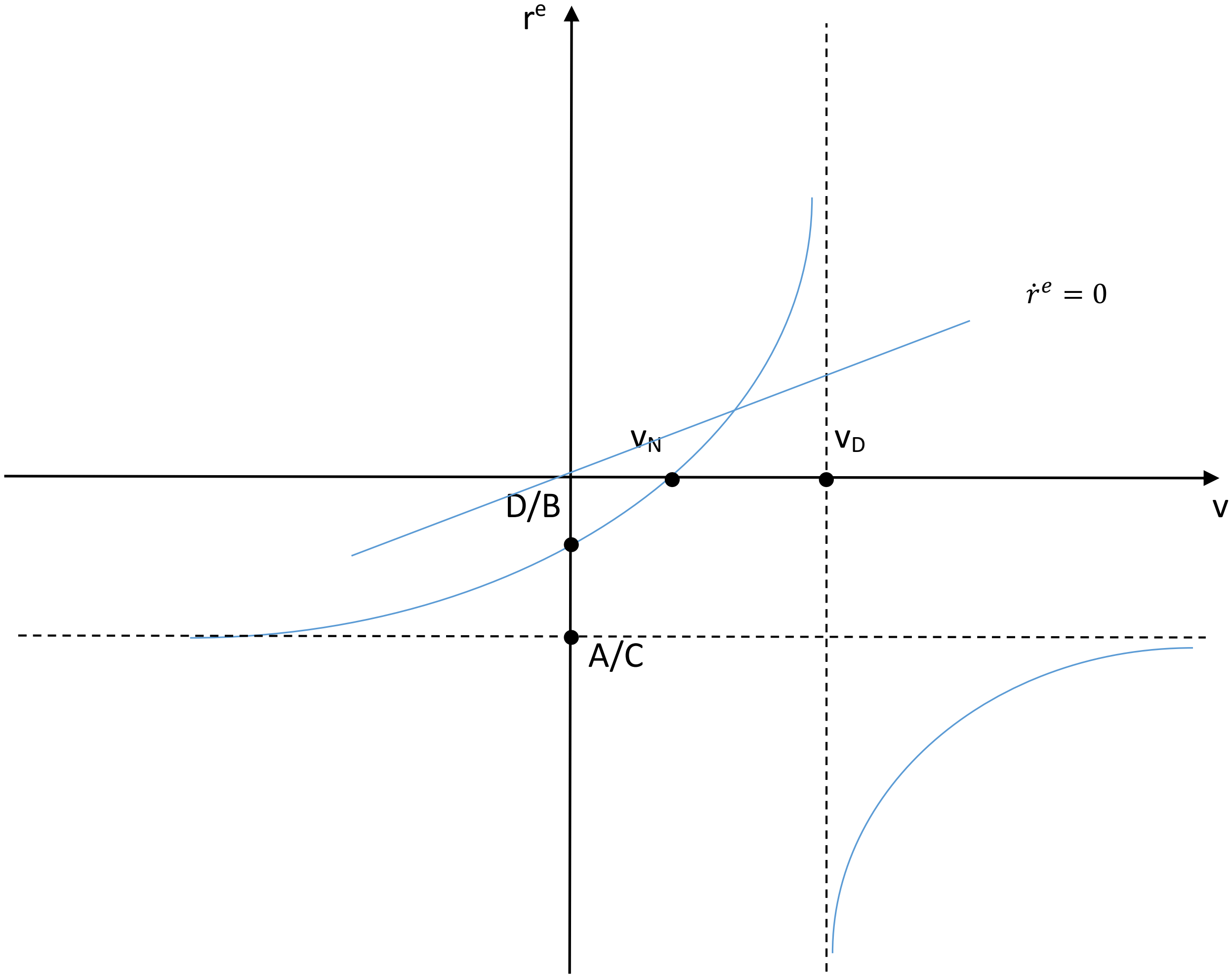

Figure 8: $\alpha > \alpha_T$, $i_3 > i_2 > i^* > i_T$

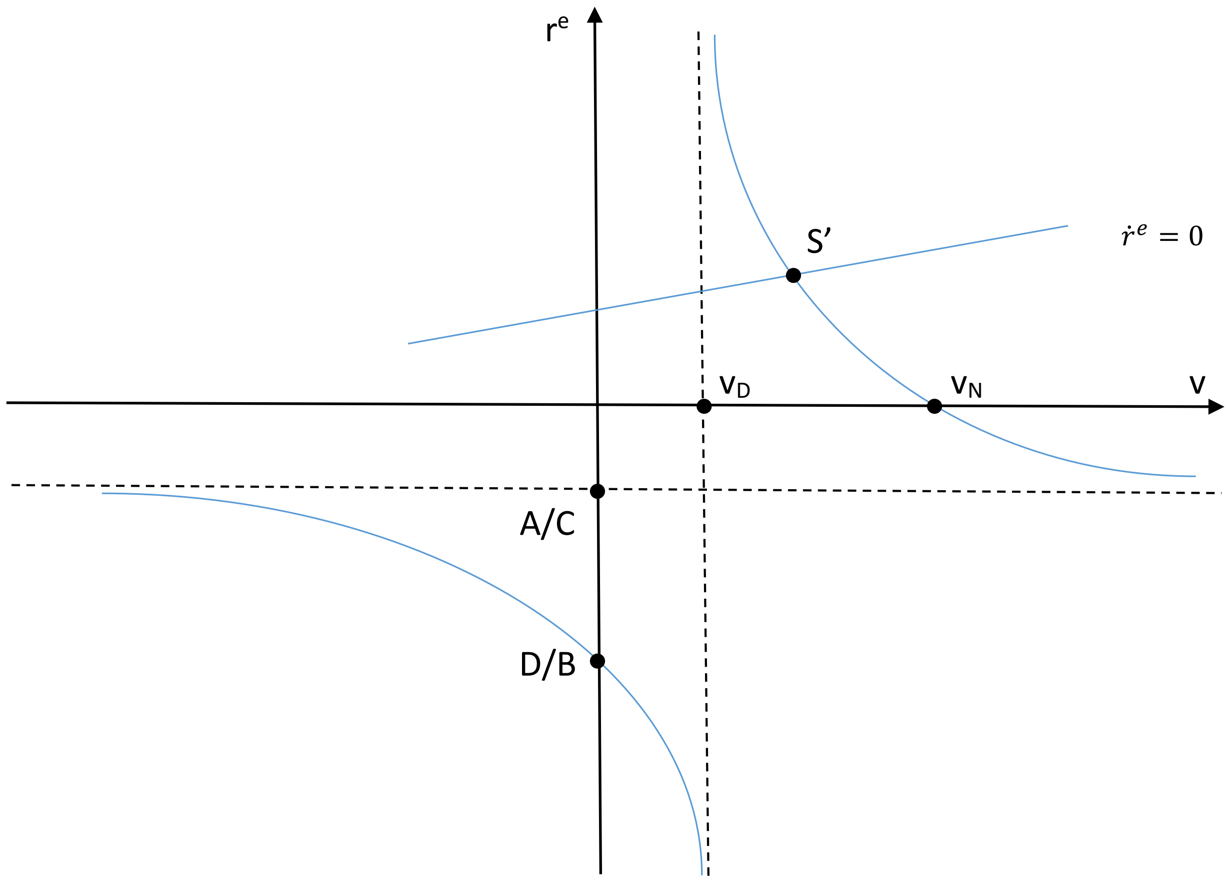

Figure 9: $\alpha > \alpha_T$, $i_3 > i_2 > i_T > i^*$

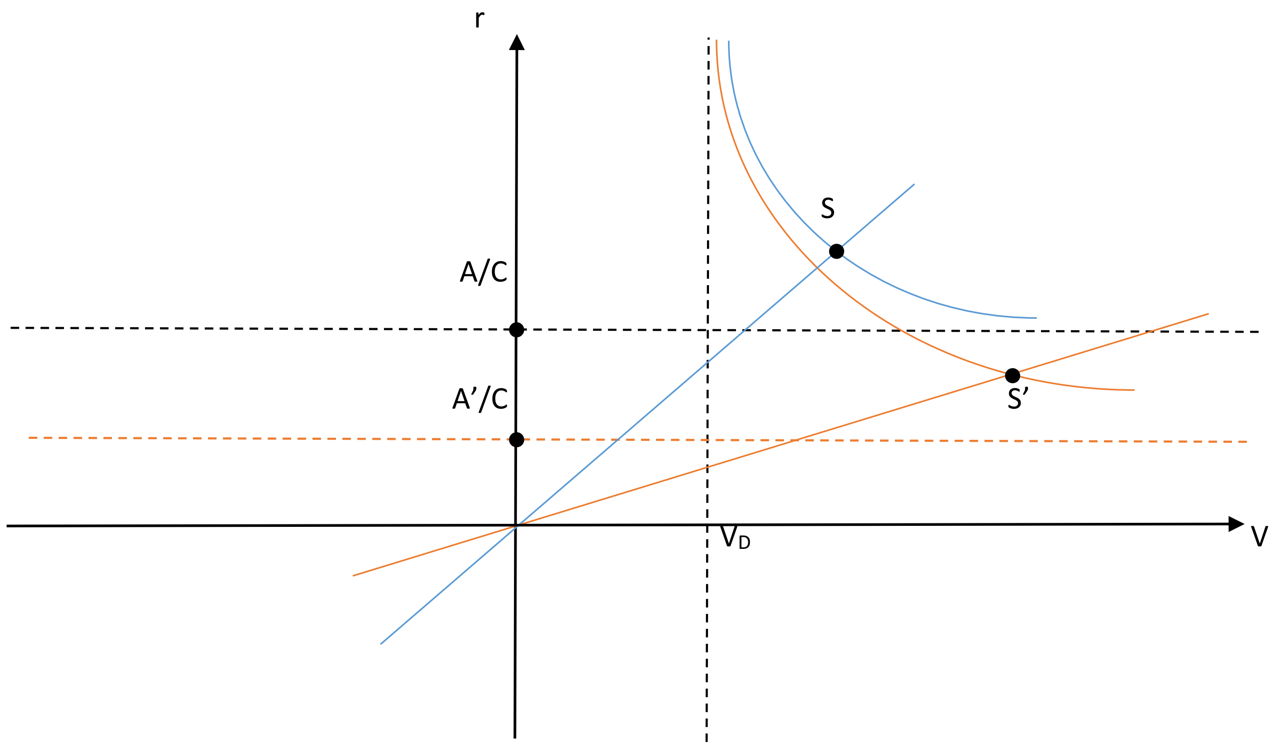

Figure 10: the impact of a higher α on profits and growth

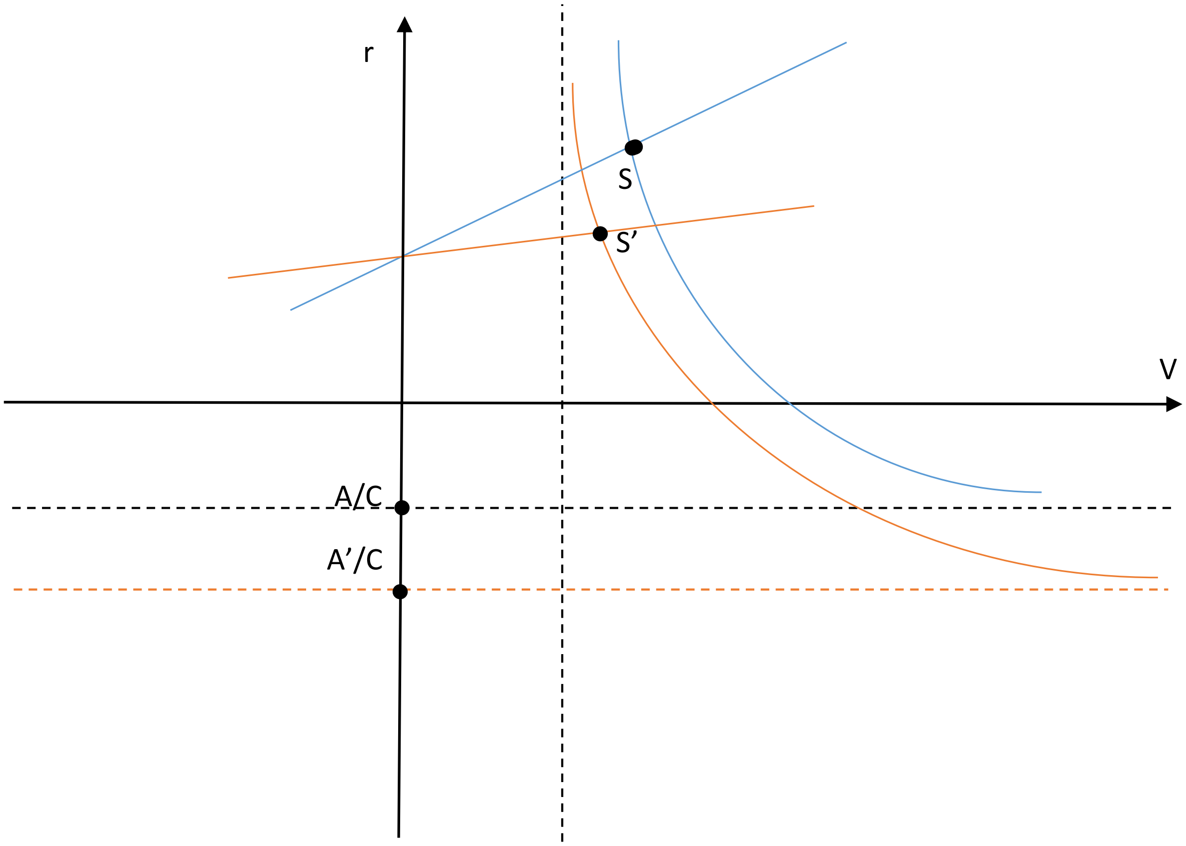

Figure 11: the impact of a higher α on profits and growth

Appendix

<u>The demarcation line $\dot{v} = 0$</u>

The equation of the demarcation line $\dot{v} = 0$ (22), i.e.

$$\{i^*(1-\alpha)(1-c_1)(1+b_2) - b_2c_2 - g_0(1-c_1+b_2)\}v + [(1-c_1)g_1]r^e - [(1-c_1+b_2)g_1]vr^e = (1-c_1)[i^*(1+b_2) - b_0 - g_0]$$

may be written as

$$Av + Br^e - Cvr^e = D,$$

having defined the parameters

$$A = \left\{i^*(1-\alpha)(1-c_1)(1+b_2) - b_2c_2 - g_0(1-c_1+b_2)\right\}$$

$$B = [(1-c_1)g_1]$$

$$C = \left[(1-c_1+b_2)g_1\right]$$

$$D = (1-c_1)\left[i^*(1+b_2) - b_0 - g_0\right].$$

Its solution is

$$r^e = \frac{D-Av}{B-Cv} \qquad \text{(A1)},$$

which is the equation of a rectangular hyperbola. We know that B > 0 and C > 0, but the signs of A and D are ambiguous. Indeed:

$$D \gtreqless 0 \quad \Leftrightarrow \quad i^* \gtreqless \frac{(b_0+g_0)}{(1+b_2)} = i_2 > 0$$

Notice that short-run stability ($1 - c_1 + b_2 > 0$) is enough to guarantee that $i_2 > i_1$, the latter being defined by (21) in the text. As to A,

$$A \gtreqless 0 \quad \Leftrightarrow \quad i^* \gtreqless \frac{b_2c_2 + g_0(1-c_1+b_2)}{(1-\alpha)(1-c_1)(1+b_2)} = i_3 > 0$$

Observe that

$$i_3 \gtreqless i_2 \quad \Leftrightarrow \quad \alpha \gtreqless \alpha_T = \frac{b_0(1-c_1)-b_2(g_0+c_2)}{b_0(1-c_1)+g_0(1-c_1)}.$$

It is easy to see that $\alpha_T < 1$ under any possible parametric configuration, but we are not sure that it is always positive. When $\alpha_T < 0$, i.e. $b_0(1-c_1) < b_2(g_0+c_2)$, it is certainly true, given that by definition $0 \leq \alpha \leq 1$, that $i_3 > i_2$.

In some cases, it might be useful to understand the relation between i_1 (defined in the text) and i_3. Well, it is easy to show that

$$i_1 > i_3 \Leftrightarrow \alpha < \alpha_H = \frac{b_0(1-c_1)(1+b_2)(1-\psi)-b_2(g_0+c_2)(1+b_2-\psi c_1)-\psi g_0(1-c_1)(1-c_1+b_2)}{b_0(1-c_1)(1+b_2)(1-\psi)+g_0(1-c_1)(1+b_2)(1-\psi)}$$

Clearly, $\alpha_H < 1$ under any possible parametric configuration. Moreover, using their definitions, one can easily calculate that $\alpha_T > \alpha_H$.

Of course, (A1) is not defined for B = Cv, i.e. for

$$v = \frac{B}{C} = \frac{(1-c_1)}{(1-c_1+b_2)} = v_D$$

The parameter v_D, with $0 < v_D < 1$, defines the vertical asymptote of our rectangular hyperbola. Note, also, that when $v < v_D$ ($v > v_D$), the denominator of (A1) is positive (negative). As to the numerator, D – Av, one should pay attention and note that

$$If\ A > 0\ (i^* > i_3)\ \Rightarrow D - Av > 0 \Leftrightarrow v < \frac{D}{A}$$

$$If\ A < 0\ (i^* < i_3)\ \Rightarrow D - Av > 0 \Leftrightarrow v > \frac{D}{A}$$

For future reference, it is convenient to define

$$v_N = \frac{D}{A} = \frac{(1-c_1)\left[i^*(1+b_2)-b_0-g_0\right]}{\left\{i^*(1-\alpha)(1-c_1)(1+b_2)-b_2c_2-g_0(1-c_1+b_2)\right\}}$$

Note, also, that the function (A1) passes through the point (v_N, 0). It is important to establish the relation between v_D and v_N. Once again it is important to pay attention and note that

$$If\ D > 0\ (i^* > i_2)\ and\ A > 0\ (i^* > i_3) \Rightarrow v_D > v_N \Leftrightarrow AB > DC \Leftrightarrow i^* < i_T = \frac{b_0(1-c_1+b_2)-b_2c_2}{(1+b_2)[b_2+\alpha(1-c_1)]}$$

$$If\ D < 0\ (i^* < i_2)\ and\ A < 0\ (i^* < i_3) \Rightarrow v_D > v_N \Leftrightarrow AB < DC \Leftrightarrow i^* > i_T = \frac{b_0(1-c_1+b_2)-b_2c_2}{(1+b_2)[b_2+\alpha(1-c_1)]}$$

$$If\ D < 0\ (i^* < i_2)\ and\ A > 0\ (i^* > i_3) \Rightarrow v_D > v_N \Leftrightarrow AB > DC\ always\ true$$

$$If\ D > 0\ (i^* > i_2)\ and\ A < 0\ (i^* < i_3) \Rightarrow v_D > v_N \Leftrightarrow AB < DC\ always\ true$$

It is now time to investigate the relation between i_T, i_2 and i_3:

$$i_T > i_2 \quad \Leftrightarrow \quad \alpha < \alpha_T$$

(note, also, that $\alpha_T > 0\ \Rightarrow i_T > 0$). As to the relation between i_T and i_3, the turning value is once again α_T:

$$i_T > i_3 \quad \Leftrightarrow \quad \alpha < \alpha_T$$

From (A1), one can see that the first and second derivative of the isocline are

$$\frac{\partial r^e}{\partial v} = \frac{DC-AB}{(B-Cv)^2}$$

and

$$\frac{\partial^2 r^e}{(\partial v)^2} = \frac{2(DC-AB)}{(B-Cv)^3}.$$

The intercept of (A1) is

$$r^e = \frac{D}{B} = \frac{i^*(1+b_2)-(b_0+g_0)}{g_1}$$

and, once again, this is positive if $i^* > i_2$.

This long technical discussion makes it clear that the crucial determinants of the exact position and slope of the isocline $\dot{v} = 0$ (of our rectangular hyperbola) are the world interest rate, i*, and the preference for cash parameter, α.

Consider first an economy with $\alpha < \alpha_T$. In this case, as we already proved, $i_T > i_2 > i_3 > 0$. There are many subcases to be analyzed, depending on where the interest rate is located in the interval $(0, \infty)$. Let us do it, and then we will move to the other case ($\alpha > \alpha_T$).

Assume $i^* > i_T$. In light of our previous discussion, this is an economy where: a) D > 0: b) A > 0; c) $v_N > v_D$; d) D/B > A/C > 0; e) the first derivative of the isocline is positive, the second is positive (negative) for $v < v_D$ ($v > v_D$). On top of this, the function (A1) is positive for $v < v_D$, negative for $v_N > v > v_D$ and again positive for $v > v_N$. Put all this together and you get exactly Figure 2.

Then move to the sub-case where $i_T > i^* > i_2 > i_3 > 0$. In this economy, a) D > 0: b) A > 0; c) $v_N < v_D$; d) D/B < A/C > 0; e) the first derivative of the isocline is negative, the second is negative (positive) for $v < v_D$ ($v > v_D$). On top of this, the function (A1) is positive for $v < v_N$, negative for $v_N < v < v_D$ and again positive for $v > v_D$. Put all this together and you get exactly Figure 3.

Next subcase is with $i_T > i_2 > i^* > i_3 > 0$. By following the same line of reasoning, the reader may easily check the isocline $\dot{v} = 0$ is now described by Figure 4. Finally, with $i_T > i_2 > i_3 > i^* > 0$, the appropriate graphical illustration of the isocline $\dot{v} = 0$ is reported in Figure 5.

Let us move now to the case $\alpha > \alpha_T$. In this case, $i_T < i_2 < i_3$. Once again, there are four subcases and, using the same argument as before, the reader may easily check that they give rise to figures from 6 to 9.

What happens when the economy lies outside the isocline $\dot{v} = 0$? Note that

$$\frac{\partial \dot{v}}{\partial r^e} = g_1[(1 - c_1) - (1 - c_1 + b_2)] \gtreqless 0 \Leftrightarrow v \lesseqgtr v_D,$$

implying that the only branch of the rectangular hyperbola that attracts the economy is that located above the horizontal asymptote (regardless of whether the latter is positive or negative), as indicated by the small arrows in our diagrams. In a sense, the reader could look at our several figures by only taking into consideration that branch of the rectangular hyperbola lying above the horizontal asymptote.

[References to be added]