A Baseline Supermultiplier Model for the Analysis of Fiscal Policy and Government Debt

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Abstract

The article presents a basic Sraffian Supermultiplier model for the analysis of fiscal policy and government debt. We first discuss the assumptions and the equilibrium and stability properties of the model. Next, we investigate the effects on the main endogenous variables of the model (including the primary government deficit and debt ratios) of changes in the rate of growth and composition of autonomous demand, in the tax rates on profits and wages, and in the rate of interest. The analysis of the impacts of changes in the interest rate is conducted according to two possible closures for the Classical/Sraffian theory of income distribution. In the first closure the changes in the rate of interest do not affect income distribution between wages and profits, which implies that its influence over the endogenous variables operates only through the financial component of total government deficit. The second closure is the monetary theory of distribution according to which there is an inverse relationship between the rate of interest and the wage share. In this case, besides the pure financial effect, we show that the change in the rate of interest affects the economy through the equilibrium value of the supermultiplier and the tax burden. The analysis is carried out both for the case in which the interest rate is lower than the trend rate of growth of output as well as for the opposite case.

Keywords: Demand led growth; Fiscal policy; Government debt.

JELCode: E11; E12; O41

1. Introduction

This article presents an extension of Freitas & Serrano (2015) which provides a baseline Supermultiplier model for the analysis of fiscal policy and government debt. It has some distinctive features in relation to the available literature on Supermultiplier models with a government sector. In fact, our model deals with a more general economic setting than Allain (2015), since the latter restricts his analysis to the case in which there is zero primary government deficit/surplus and no government debt, while our model allows the discussion of non-zero primary deficit/surplus situations and the dynamics of government indebtedness. On the other hand, we analyze the fully adjusted/long run equilibrium of the Supermultiplier model with government, whereas Hein (2018) restricts his investigation to the cases of the short and medium run equilibria. Moreover, it also differs from Dutt (2015 and 2019) and Cassetti (2017) Supermultiplier models since we suppose the existence two sources of autonomous demand, capitalist autonomous consumption and government consumption. We investigate the effects of changes in the composition of autonomous non-capacity creating expenditures and, as we will see, these composition changes is important because they allow us to show that, among other things, the debt paradox and the self-defeating austerity scenarios are possibilities but not necessary outcomes of supermultiplier models with a government sector as would be the case if government expenditures were the only source of autonomous demand in the model. Finally, in contrast to the other contributions just mentioned, we will address both the case in which the economy has rate of interest lower than the trend rate of growth of output and the converse case. Although in the latter case the equilibrium debt to output ratio is unstable, we believe that it should be subject to analysis since this is a relevant situation in reality.

As a first approximation to the investigation of fiscal policy and government indebtedness based on the specific version of the Supemultiplier model here developed, we decided to simplify our analysis in various directions as we shall see shortly. In particular, we will focus our attention on the equilibrium properties of the fully adjusted equilibrium of the model. This focus limits our ability to evaluate the economic effects and relative merits of different stylized versions of fiscal policy rules (and conventions) adopted by governments in practice, and explains why we will avoid this kind discussion below. Clearly, the latter requires a full dynamic (disequilibrium) analysis based on the model. However, we believe that the kind of static (or equilibrium) analysis that will be undertaken here is a required step before a full dynamic (disequilibrium) analysis is adopted. Since limitations of space prevent us from adopting both approaches here, we will postpone the more dynamic (disequilibrium) analysis based on the model to future works.

The structure of the paper is the following. In the second section, we present our version of the Supermultiplier model with a government sector covering the discussion of its basic hypothesis and relations, of the assumptions and relations concerning government deficit and indebtedness, and of the features of the fully adjusted steady-state position and its stability properties. In the third section, we analyze the steady-state (or equilibrium) effects of changes in some exogenous variables of the model over its main endogenous variables, including the primary deficit and debt ratios. This analysis will address both the case in which the economy is characterized by a rate of interest greater than the trend rate of growth of output and the converse situation. We conclude by summarizing our main results and indicating the directions for future research along the lines of the model

2. A Baseline Supermultiplier model with a Government Sector

2.1. Basic Assumptions and Relations

To maintain our analysis as simple as possible in the first approximation to the issue under investigation we adopt some simplifying assumptions. First, the model deals with a closed capitalist economy with a government sector. Secondly, we restrict government activities in the model to real government consumption, taxation over wages and profits, and the issuance of public debt.[1,2] Further, we suppose that the only method of production in use requires a fixed combination of a homogeneous labor input with homogeneous fixed capital to produce a single product. Natural resources are supposed to be abundant, constant returns to scale prevails, there is no technological progress and economic growth is not constrained by labor scarcity. Finally, the model is specified in real terms in the sense that all relevant variables of the model are real magnitudes.

Since natural resources and labor are not scarce in our model, the only remaining potential supply constraint is the productive capacity available in the economy (i.e., a potential capital constraint). It follows that the level of potential output depends on the level of the capital stock K and on the technical capital to capacity output ratio v as:

$$Y_{Kt} = \frac{1}{v} K_t \tag{1}$$

where Y_K is the level of capacity output. Since v is given, then the rate of growth of capacity output is equal to the rate of capital accumulation, g_{Kt}, given by:

[1] Thus, in a first approximation, we are ignoring the existence of various government activities such as investment, social transfers, taxation over interest income and indirect taxation. The incorporation of the issues in the Supermultiplier model will be the subject matter of future research.

[2] In the same spirit of simplifying our investigation, we are also assuming that there are no public owned enterprises in our framework of analysis. Thus, in our simplified setting, there is no difference between the Government sector and the Public sector.

$$g_{Kt} = \left(\frac{I_t/Y_t}{v}\right) u_t - \delta \tag{2}$$

where $u_t = Y_t/Y_{Kt}$ (with $0 \leq u_t \leq 1$) is the actual degree of capacity utilization defined as the ratio of the current level of output (Y_t) to the current level of capacity output; I_t/Y_t is the investment share on aggregate output defined as the ratio of gross investment (I_t) to the level of output; and δ (> 0) is the exogenously given depreciation (or replacement) rate of the capital stock. According to equation (2), there is a necessary formal relationship between the rate of capital accumulation, the actual degree of capacity utilization and the investment share of output. Given the capital to capacity output ratio, the change in the actual degree of capacity utilization depends on the difference between the rate of growth of output and the rate of capital accumulation:

$$\dot{u} = u_t(g_t - g_{Kt}) \tag{3}$$

where g_t is the rate of growth of output.

As regards the demand side of the model, the components of aggregate demand are private consumption, private investment and government consumption. First, let us discuss the determination of private investment expenditures in the model. Indeed, in order to further simplify our analysis, we assume that there is no residential investment. Thus, the only type of investment (i.e., capacity creating expenditure) being considered here is the one realized by capitalist firms.[3] We suppose that capitalist firms' investment follows the capital stock adjustment principle. According to this principle, inter-capitalist competition influences the process of investment leading to the tendency towards the adjustment of productive capacity to meet demand at a price that covers production expenses and allows, at least, the obtainment of a minimum required profitability.[4]

According to the version of the investment function based on the capital stock adjustment principle used in this work,[5] the determination of investment by capitalist firms is done as follows:

$$I_t = h_t Y_t \tag{4}$$

where h_t (with $0 \leq h_t < 1$) is the marginal propensity to invest. Moreover, the marginal propensity to invest changes endogenously in response to deviations of the actual degree of capacity utilization from its normal level as follows:

$$\dot{h} = h_t \gamma (u_t - u_n) \tag{5}$$

where γ is a parameter that measures the reaction of the growth rate of the marginal propensity to invest to the deviation of the actual degree of capacity utilization u_t from its normal or planned level

[3] Remember that we are supposing that there are no public owned enterprises in our model. Hence, we are also ignoring investment expenditure made by these enterprises.
[4] See Matthews (1959) for a very clear account of the basic idea behind the capital stock adjustment principle.
[5] Here we follow the version discussed in more details in Freitas & Serrano (2015).

u_n. We assume that the normal degree of capacity utilization has a positive but smaller than one value (i.e., $0 < u_n < 1$).[6] This is explained by the hypothesis that, under the pressure of competition, firms try to maintain margins of planned spare capacity to avoid the risk of losing market shares for not being able to supply the demand for their products at its peak levels. On the other hand, we suppose that the parameter γ has a small positive value (i.e., $\gamma > 0$). In our view, there are two main reasons for this specification of the reaction parameter. The first is that firms want normal utilization to prevail on average over the lifetime of the productive equipment and not in every single period of use. The second one is that firms also know that demand does change and some of the changes are temporary, while others are not.

From the combination of equations (4) and (5),[7] it follows that the growth rate of aggregate investment will be higher than the rate of growth of output whenever the actual degree of capacity utilization is above its normal level and *vice-versa*. Indeed, the pressure exerted by competition would ensure that firms as a whole would be compelled to invest in order to ensure that they can meet future peaks of demand when the degree of capacity utilization is above the normal one and the margins of spare capacity are getting too low. Conversely, firms would not want to keep accumulating costly unneeded spare capacity when the actual degree of capacity utilization remains below the profitable normal or planned level.

Let us now turn our attention to the determinants of private consumption. The latter comprises consumption of workers and capitalists. The former is equal to the wage bill after tax deductions and, therefore, it is an induced expenditure, since production decisions by capitalist firms fully determine the wage bill of the private sector of the economy. On the other hand, we suppose that capitalist consumption Z_{Kt} (> 0) is an autonomous non-capacity creating expenditure financed by credit and, therefore, unrelated to the current level of output resulting from firms' production decisions (this implies that the marginal propensity to consume out of capitalists disposable income is zero).

From the hypothesis above, the level of private consumption is given by:

$$C_t = Z_{Kt} + (1 - t_w)\omega Y_t \tag{4}$$

where ω ($0 < \omega < 1$) is the wage share of total income and t_w ($0 < t_w < 1$) is the tax rate on wages. Observe that the marginal propensity to consume out of total income (i.e., $0 < (1 - t_w)\omega < 1$)

[6] Following Ciccone (1986; 1987), we interpret the normal or planned degree of capacity utilization as determined, among other things, by the historically 'normal' ratio of peak to average demand. This latter ratio is assumed to be unaffected by current oscillations of demand since it is presumably based on the observation of the actual cyclical and seasonal patterns of the market over a long period of time.

[7] This combination gives us an equation for the determination of the rate of growth of capitalist investment according to which $g_{It} = g_t + \gamma(u_t - u_n)$.

depends on the tax rate on wages and on the wage share. The tax rate over wages is an exogenous variable determined by the government and is one of its fiscal policy instruments.

On the other hand, (primary) income distribution is determined along Classical (Sraffian) lines. More specifically, the normal rate of profit (denoted ρ, where $0 < \rho < R = u_n/v$ and R is the maximum rate of profit) and the wage share are negatively related as follows:

$$\omega = 1 - \left(\frac{v}{u_n}\right)\rho$$

We also postulate the existence of a relationship between the normal rate of profit, the (real) interest rate (r) and the rate of profit of enterprise ($\eta \geq 0$) captured by the following expression:

$$\rho = r + \eta$$

Further, we assume that the monetary authority exogenously determines the real rate of interest.[8] Given that, there exist two polar closures for this relationship. According to the monetary theory of distribution (c.f., Pivetti, 1991 and Serrano, 1993) the rate of profit of enterprise is determined by various elements of risk specific to each alternative productive employment of capital. It would be characterized for not being negatively related to the rate of interest and for being a relatively persistent magnitude, which only changes sporadically. It follows that, according to this view, the rate of interest rate and the (normal) rate of profit of enterprise "[…] should be regarded as the *autonomous* determinants of the rate of profit" (Pivetti, 1991, p. 25). By its turn, the alternative closure suggests that rate of profit is influenced by the wage bargain process and the technical conditions of production in the economy, which lead to the endogenous determination of the rate of profit of enterprise in the equation above.[9] Thus, by substitution we obtain:

$$\omega = 1 - \left(\frac{v}{u_n}\right)(r + \eta) \tag{6}$$

Hence, the monetary theory of distribution implies that, given the technical conditions of production, the normal rate of profit determines the wage share and, therefore, there exists an inverse relationship between the wage share and the rate of interest. On the other hand, according to the alternative closure indicated above there is no direct relationship between these two variables. In fact, given the technical conditions of production, the wage share determines the normal rate of profit. Thus, a change in the rate of interest provokes a change of the same magnitude and in the opposite direction in the rate of profit of enterprise, leaving the normal rate of profit unchanged.[10] It must be remarked, however, that

[8] In fact, it should be noticed, monetary authorities directly regulate nominal interest rates and only indirectly the real interest rate. To simplify our analysis we will assume that monetary authorities control the real interest rate without discussing the details of the relationship between real and nominal interest rates. Moreover, the present article will not discuss the operation of monetary policy both in terms of its transmission mechanism to aggregate demand variables and in terms of its reaction to indicators of economic performance. In this sense, the model is purposefully incomplete in order to focus our attention on the fiscal impact of real interest rates.

[9] Pivetti (1991, Part II, chapter 7) attributes this position to Marx.

[10] In this case, the rate of profit of enterprise (η) would be the endogenous variable in equation (6).

in both cases the wage share does not depend directly on the endogenous variables of the model, in particular on the rate of growth of output. Therefore, since the tax rate on wages and the wage share are exogenous variables, the marginal propensity to consume out of total income is also an exogenous variable in the model under investigation.

In equilibrium between real aggregate output and demand we have:

$$Y_t = C_t + I_t + Z_{Gt} \tag{7}$$

where Z_{Gt} is positive level of government consumption, which is also an autonomous non-capacity creating demand component. Substituting (4) and (5) into equation (7) we obtain:

$$Y_t = Z_{Kt} + Z_{Gt} + (1 - t_w)\omega Y_t + h_t Y_t \tag{8}$$

Further, the level of total autonomous non-capacity creating expenditure (Z_t) is given by:

$$Z_t = Z_{Kt} + Z_{Gt} \tag{9}$$

Now let us represent government and capitalist autonomous expenditures as shares of total autonomous demand as follows:

$$Z_{Gt} = \sigma Z_t \tag{10}$$

$$Z_{Kt} = (1 - \sigma) Z_t \tag{11}$$

where σ is the parameter that regulates the composition of Z_t between capitalist and government consumption (with $0 < \sigma < 1$). We suppose that Z_t increases through time according to an exogenous and positive rate of growth g_z. Thus we can emulate, from a modeling point of view, the different patterns of changes associated with differences in the rates of growth of capitalist and government consumption expenditures. With this assumption, we aim to avoid the possibility that the total autonomous demand share of one of these expenditures tends towards one hundred percent, while its complement tends to zero. Therefore, we are assuming that, although they can be different, there is a tendency for the rates of growth of both autonomous consumption expenditures being equal on average over time.

Now substituting (9) into (8) and solving the resulting expression for the level of real output we determine the level of real output as follows:

$$Y_t = \left[\frac{1}{1 - \left((1 - t_w)\omega + h_t\right)}\right] Z_t \tag{12}$$

where the term within the bracket is the supermultiplier incorporating the inducement effects over consumption and investment regulated, respectively, by the marginal propensity to consume $(1-t_w)\omega$ and by the marginal propensity to invest h_t. Observe that in order to have a positive solution for the level of output the value of the marginal propensity to spend must be lower than one (i.e., $(1-t_w)\omega+h_t<1$).

Moreover, from equations (5) and (8) (or (12)) we obtain an expression for the growth rate of output/demand given by:

$$g_t = g_Z + \frac{h_t\gamma(u_t-u_n)}{1-\left((1-t_w)\omega+h_t\right)} \quad (13)$$

The rate of growth of output/demand depends on the rate of growth of autonomous (non-capacity creating) expenditures and the growth of the supermultiplier outside the fully adjusted equilibrium path of the model (i.e., the second term on the RHS of the equation).

2.2. Government Deficit and Debt Dynamics

Supposing that there is no monetary financing of government deficit, the following differential equation gives us the relation between total government deficit and the change in its net debt position, $\dot{B}$:[11]

$$\dot{B} = Z_{Gt} - T_{wt} - T_{kt} + rB_t \quad (14)$$

where T_{wt} is the total amount of tax over wages and T_{kt} is the total amount of tax over profits. The term $Z_{Gt}-T_{wt}-T_{kt}$ in the RHS of equation (14) is the primary government deficit and the last term (i.e., rB_t) is the financial component of total government deficit.[12] As we saw above T_{wt} is proportional to total wage income (i.e, $T_{wt}=t_w\omega Y_t$). Let us suppose here that T_{kt} is also proportional to total profits (i.e, $T_{kt}=t_k(1-\omega)Y_t$, with $0<t_k<1$).

In a growing economy it is convenient to work with a version of equation (14) in which we represent the dynamics of the government debt to output ratio (hereafter referred as debt ratio) as follows:[13]

[11] Note that we are ignoring the possibility of monetary financing of the deficit, which is done in order to simplify our analysis of the behavior of government debt.

[12] Indeed, the presence of the last term on the RHS of equation (14), implies that, departing from a situation in which the government has a net debtor position, debt would increase even with a zero primary deficit. In this latter situation, government debt would increase to finance the interest payments due to the service of the existing debt and its rate of growth would be determined by the rate of interest.

[13] Let us define the government debt ratio as $b_t=B_t/Y_t$. It follows that $\dot{B}=\dot{b}Y_t+b_t\dot{Y}$. Thus, dividing both sides of the (14) by the level of output and solving for the change in the government debt ratio, we obtain an equation relating the change of the debt ratio to the ratio of total government deficit to output.

$$\dot{b} = \sigma\frac{Z_t}{Y_t} - t + (r - g_t)b_t \quad (15)$$

where $t = t_w\omega + t_k(1-\omega)$ is the average tax rate or the tax burden, $Z_{Gt}/Y_t = \sigma(Z_t/Y_t)$ is the government consumption-output ratio, $\sigma(Z_t/Y_t) - t$ is the primary government deficit to output ratio (hereafter referred as primary deficit ratio) and $(r - g_t)b_t$ is the financial component of the total deficit to output ratio (hereafter referred as total deficit ratio). From equation (12) we can infer that $Z_t/Y_t = 1 - \left((1-t_w)\omega + h_t\right)$. On the other hand, we may express the average tax rate as follows:

$$t = t_w + \tau(1-\omega) \quad (16).$$

where $\tau = t_k - t_w$ is a variable that we will use to capture changes in the tax rate on profits in relation to the tax rate on wages, which implies changes in the after-tax distribution between wages and profits promoted by government tax policy. Thus, we obtain the final expression for the equality between changes in the debt ratio (in the LHS of the equation) and total deficit ratio (in the RHS of the equation):

$$\dot{b} = \sigma\left(1 - \left(1 - t + \tau(1-\omega)\right)\omega - h_t\right) - t + (r - g_t)b_t \quad (15').$$

We may now analyze the fully adjusted steady-state of the Supermultiplier model with a government sector.

2.3. Fully Adjusted Steady-State

The behavior of the model can be described by the following dynamical system with three differential equations in three variables, h, u and b.

$$\dot{h} = h_t\gamma(u_t - u_n) \quad (5)$$

$$\dot{u} = u_t\left(g_Z + \frac{h_t\gamma(u_t - u_n)}{1 - \left(\left(1 - t + \tau(1-\omega)\right)\omega + h_t\right)} - \frac{h_t}{v}u_t + \delta\right) \quad (3)$$

$$\dot{b} = \sigma\left(1 - \left(\left(1 - t + \tau(1-\omega)\right)\omega + h_t\right)\right) - t + \left(r - g_Z - \frac{h_t\gamma(u_t - u_n)}{1 - \left(\left(1 - t + \tau(1-\omega)\right)\omega + h_t\right)}\right)b \quad (15')$$

It should be noticed, that, in the above system, the first two equations are independent of the third one. Thus, the first two equations can be separately analyzed as an independent subsystem. In fact, such a subsystem has been investigated in Freitas and Serrano (2015). From the latter work, we know that the fully adjusted equilibrium of the model gives us:

$$u^* = u_n \tag{16}$$

$$h^* = \frac{v}{u_n}(g_Z + \delta) \tag{17}$$

$$g^* = g_I^* = g_K^* = g_Z \tag{18}$$

$$\left(\frac{Z_t}{Y_t}\right)^* = 1 - \left(\big(1 - t + \tau(1-\omega)\big)\omega + \frac{v}{u_n}(g_Z + \delta)\right) \tag{19}$$
$$= 1 - \left(\big(1 - t + \tau(1-\omega)\big)\omega + h^*\right)$$

We may now use the equilibrium values above in equation (15’) in order to analyze the determinants of the equilibrium value of the debt ratio as follows

$$\dot{b} = \sigma\left[1 - \left(\big(1 - t + \tau(1-\omega)\big)\omega + \frac{v}{u_n}(g_Z + \delta)\right)\right] - t + (r - g_Z)b_t \tag{15''}$$

Thus the steady-state solution for b_t is given by:

$$b^* = \frac{\sigma\left[1 - \left(\big(1 - t + \tau(1-\omega)\big)\omega + \frac{v}{u_n}(g_Z + \delta)\right)\right] - t}{g_Z - r} = \frac{\sigma\left(\frac{Z_t}{Y_t}\right)^* - t}{g_Z - r} \tag{20}$$

where $PDR^* = \sigma(Z_t/Y_t)^* - t$ is the value of primary deficit ratio in equilibrium.

Note that according to the signs of the primary deficit ratio in equilibrium and $g_Z - r$ we may have steady-state values for which the government has a net debtor ($b^* > 0$) or a net creditor ($b^* < 0$) equilibrium position. Indeed, we have that:

$b^* > 0$ if $\sigma(Z_t/Y_t)^* - t > 0$ and $g_Z - r > 0$ or $\sigma(Z_t/Y_t)^* - t < 0$ and $g_Z - r < 0$

and

$b^* < 0$ if $\sigma(Z_t/Y_t)^* - t > 0$ and $g_Z - r < 0$ or $\sigma(Z_t/Y_t)^* - t < 0$ and $g_Z - r > 0$

We will focus our attention in the cases in which the government has a net debtor position in the steady-state of the model.

2.4. The Stability of the Fully Adjusted Equilibrium

As we pointed out above, the first two equations of the dynamical system form a subsystem that can be analyzed separately from the equation capturing the dynamics of the debt ratio (i.e., we have a decomposable system with an uncoupled subsystem). As the dynamic stability of this subsystem has been investigated in Freitas and Serrano (2015), the same local dynamic stability condition discussed there applies here. That is, the disequilibrium propensity to spend must be smaller than one in the steady-state:

$$(1-t_w)\omega+\frac{v}{u_n}(g_Z+\delta)+\gamma v<1 \tag{21}$$

Moreover, observe that the introduction of the government sector in the model increases the likelihood of its equilibrium being stable. The marginal propensity to consume is smaller than in the case of an economy without government sector since the after tax wage share is smaller than the wage share (i.e., $(1-t_w)\omega<\omega$) due to the leakage associated with the taxation of wages.

On the other hand, for the local dynamic stability of the debt ratio to obtain, the derivative $\mathrm{d}\dot{b}/\mathrm{d}b=r-g_Z$ must have a negative value when evaluated at the equilibrium point (i.e., the value of the derivative calculated using equation (15'')). From this condition we obtain a version of the *Domar stability (or sustainability) condition*:

$$g_Z>r \tag{22}$$

The violation of this condition (i.e. $g_Z<r$) implies that the financial component of the total deficit ratio increases when the debt ratio rises. It follows that the equilibrium debt ratio is unstable in this case. For debt ratios initially above the equilibrium one, the debt ratio rises indefinitely, while for debt ratios initially below the equilibrium the opposite occurs. It follows that there is a marked difference in the behavior of the debt ratio depending on whether the trend rate of growth of output has a value above (stable equilibrium) or below (unstable equilibrium) of the value of the rate of interest.

3. Steady-state effects of changes in some exogenous variables of the model

Now we can analyze the impact of changes in some selected exogenous variables of the model on the steady-state values of its main endogenous variables. In particular, we will discuss the economic impact of the use of different types of fiscal policy instruments and also the effects on the equilibrium values of the primary deficit (surplus) and debt ratios. As remarked above, the behavior of debt ratio is qualitatively distinct according to whether the trend rate of growth of output is higher or lower than the rate of interest. For this reason, a separate analysis of these two cases seems to be justified. Thus, in what follows, we will deal first with the case in which the rate of interest has a value below the steady-state rate of growth of output and, after that, we will discuss the converse case.

3.1. The $g^* > r$ case

In this case, the Domar stability condition is met and we have a stable process of debt accumulation that converges towards a positive debt equilibrium ratio. As we saw above, in these circumstances an equilibrium characterized by a net debtor position is only possible if the government tends to run primary deficits. Thus, throughout this subsection we will suppose that the before and after change steady-states of the model feature positive primary deficit and debt ratios.

Let us start our discussion with the analysis of the effects caused by changes in the pattern of growth of the autonomous demand. In our model, these changes are captured by modifications in the rate of growth and the composition of autonomous demand. First, let us deal with a hypothetical change in the rate of growth of autonomous expenditures in which the composition of autonomous demand between capitalist and government consumption is maintained constant (i.e., a change in g_z with σ constant). Indeed, according to our model, an increase (a decrease) in the rate of growth of autonomous demand causes an increase (a decrease) of the same magnitude in the steady-state rates of growth of output, investment and capital stock (from equation (18)). By its turn, the higher (lower) rate of growth of output leads to a higher equilibrium investment-output ratio (from equation (17)) and, therefore, to a higher (lower) equilibrium value of the supermultiplier and a lower (higher) equilibrium ratio of autonomous demand to output (from equation (19)). As a consequence, given the composition of autonomous demand σ, the rise (fall) in the rate of growth of autonomous demand causes a decrease (an increase) in the share of autonomous government consumption on output in equilibrium (i.e., $(Z_{Gt}/Y_t)^* = \sigma(Z_t/Y_t)^*$) and, given the tax burden t, a decrease (an increase) in the equilibrium value of the primary deficit ratio (i.e., $PDR^* = \sigma(Z_t/Y_t)^* - t$).[14] Therefore, departing from a situation in which the government has a positive primary deficit ratio, we conclude, based on equation (20), that an increase (a decrease) in the growth rate of autonomous demand provokes a fall (rise) in the equilibrium debt ratio for two reasons. First, as we just saw, because a rise (fall) of the growth rate of autonomous demand promotes a reduction (an increase) in the primary deficit ratio and, secondly,

[14] In fact, from the expression representing the primary deficit ratio we have $\partial PDR^*/\partial g_Z = -\sigma(v/u_n) < 0$.

because it decreases (increases) the contribution of the financial component of the total deficit ratio to the rate of growth of the debt ratio.[15,16]

The second reason for the change in the equilibrium debt ratio raised above is discussed in the economic literature since, at least, the classical contribution to the issue by Domar (1944). The main difference in relation to the literature on the issue under discussion is that in the supermultiplier model, like in the Kaleckian models, economic growth is conceived as a demand led process in which the long run growth of aggregate demand can be influenced by the expansion of government expenditures and, therefore, by fiscal policy.[17] However, the same cannot be said about the first reason appointed for the existence of an inverse relationship between the trend rate of growth of output and the debt ratio mentioned above. The negative relationship between the rate of growth of autonomous demand and the ratio of autonomous demand to output is an important distinctive feature of Supermultiplier growth models (c.f., Freitas & Serrano, 2015 and Serrano & Freitas, 2017). Given the composition of autonomous demand, such a feature implies the existence of an inverse relationship between autonomous demand growth and the primary deficit ratio. The latter results follow directly from the positive relationship between the trend rate of growth of output and the investment-output ratio, a feature of the supermultiplier models not shared by Kaleckian growth models (Serrano & Freitas, 2017).

We will now analyze the effects of modifications in the pattern of growth of autonomous demand associated with a 'pure' change in its composition between government and capitalist consumption (i.e., a change in σ given the rate of growth of autonomous demand, g_Z). According to our model, such a change does not have a direct permanent effect on the rate of growth of output/demand, the investment-output ratio, the supermultiplier and the ratio of autonomous demand

[15] From equation (20) we obtain the derivative $\partial b^*/\partial g_Z = -[\sigma(v/u_n)/(g_Z - r)] - [(\sigma(Z_t/Y_t)^* - t)/(g_Z - r)^2]$. Since we are supposing that the primary deficit ratio is positive and that $g_Z > r$ then the derivative must have a negative sign. The first reason for the change in b^* mentioned in the text is captured by the first term on the RHS of the derivative (i.e., $(\partial b^*/\partial PDR^*)(\partial PDR^*/\partial g_Z) = -\sigma(v/u_n)/(g_Z - r)$) and the second one by the second term on the RHS (i.e, $(\partial b^*/\partial(g_Z - r))(\partial(g_Z - r)/\partial g_Z) = -(\sigma(Z_t/Y_t)^* - t)/(g_Z - r)^2$).

[16] Dividing both sides of (15) by the debt ratio we obtain an expression for the rate of growth of the debt ratio given by $g_{bt} = \dot{b}/b_t = (\sigma(Z_t/Y_t) - t)/b_t + r - g_Z$, where the term $r - g_Z$ is the contribution of the financial component of the total deficit ratio to the rate of growth of the debt ratio and $(\sigma(Z_t/Y_t) - t)/b_t + r$ is the rate of growth of government debt (i.e., of $g_{Bt} = \dot{B}/B_t$).

[17] However there is a difference in the way according to which government expenditures may affect the trend rate of growth of output in the Supermultiplier and traditional Kaleckian models. In the case of the traditional Kaleckian models with a government sector (c.f., You & Dutt, 1996; Palley, 2013; and Blecker, 2002), government expenditure is essentially an endogenous variable whose proportion to the capital stock of the economy can change according to the fiscal policy orientation (expansionary or contractionary). Such changes affect the equilibrium rate of growth of output by causing a permanent modification in the equilibrium level of the degree of capacity utilization, which can be different from its normal level. Thus, for instance, an increase in the ratio of government demand to the stock of capital leads to a rise in the equilibrium level of the degree of capacity utilization and, therefore, to an increase in the equilibrium rate of growth of private investment according to the usual specification for the investment function of traditional Kaleckian models. In fact, if capitalist investment were completely insensitive to the degree of capacity utilization changes in the ratio of government expenditure to capital stock would not affect the equilibrium rate of growth of the economy. These models are truly investment-led growth models. In the case of the Supermultiplier models government expenditure is an autonomous demand component whose rate of growth exert a direct influence on the trend rate of growth of output without any effect on the equilibrium value of the degree of capacity utilization, but with as lasting effect over the investment to output ratio.

to output. Hence, besides the lack of growth effects, it does not have also a level effect over equilibrium output, capacity output and capital stock. In fact, a change in the composition of autonomous demand expenditures only exerts its influence on the equilibrium values of the primary deficit and debt ratios, our main fiscal endogenous variables. A change in σ affects the primary deficit ratio in the same direction since both the autonomous demand-output ratio and the tax burden are unaffected by such a change.[18] It follows that an increase (a decrease) in the share of government consumption on autonomous demand σ raises (reduces) the equilibrium value of the debt ratio provided the trend rate of growth of output exceeds the rate of interest.[19]

The general case involves changes in both the rate of growth of autonomous demand and its composition (i.e., changes in g_Z and σ). As our previous analysis shown, the impacts of these two sources of changes over the primary deficit and debt ratios go in different directions. Thus, changes in the pattern of growth of autonomous demand that involve modifications of g_Z and σ in different directions have unambiguous impacts on the equilibrium primary deficit and debt ratios. In contrast, while an increase (a decrease) of the rate of growth of autonomous demand brings about a decrease (an increase) in the primary deficit and debt ratios, a rise (fall) in the government share of autonomous demand leads to an increase (a reduction) in both ratios. Hence, the impacts of the alternative sources of change go in different directions and, therefore, the final impact over the primary deficit and debt ratios depends on the relative strength of the effects of the changes in rate of growth and autonomous demand composition.[20]

There are two implications of the previous results that we would like to address here. First, there is the issue of (allegedly) paradoxical results concerning the government indebtedness as measured by the debt ratio. In Supermultiplier models with a government sector suggested by Dutt (2015 and 2019) and Cassetti (2017), government spending is the only source of autonomous demand in the model (in our notation we would have $\sigma = 1$ and $d\sigma = 0$).[21] In these circumstances, an increase (a decrease) in the rate of growth of government spending will necessarily provoke a fall (rise) in the

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[18] From the expression representing the primary deficit ratio we obtain the partial derivative $\partial PDR^*/\partial\sigma = (Z_t/Y_t)^* = 1 - [(1 - t + \tau(1-\omega))\omega + h^*]$. Then, if the stability condition for the supermultiplier model holds, the propensity to spend in equilibrium must have a positive value lower than one (i.e., $(1 - t + \tau(1-\omega))\omega + h^* < 1$) and, therefore, $\partial PDR^*/\partial\sigma > 0$.

[19] From equation (20) we obtain the partial derivative $\partial b^*/\partial\sigma = (Z_t/Y_t)^*/(g_Z - r)$, which has a strictly positive value provided that $g_Z > r$.

[20] The effect on the primary deficit equilibrium ratio resulting from combined changes in g_Z *and* σ can be expressed as $dPDR^*|_{dg_z,d\sigma\neq 0} = [\partial PDR^*/\partial g_Z]dg_Z + [\partial PDR^*/\partial\sigma]d\sigma = -[\sigma(v/u_n)]dg_Z + [(Z_t/Y_t)^*]d\sigma$. By its turn, the effect on the equilibrium debt ratio resulting from the same sources of change is $db^*|_{dg_z,d\sigma\neq 0} = [\partial b^*/\partial g_Z]dg_Z + [\partial b^*/\partial\sigma]d\sigma = -[\sigma(v/u_n)/(g_Z - r) + (\sigma(Z_t/Y_t)^* - t)/(g_Z - r)^2]dg_Z + [(Z_t/Y_t)^*/(g_Z - r)]d\sigma$. Thus, since, as we argued, $\partial PDR^*/\partial g_Z, \partial b^*/\partial g_Z < 0$ and $\partial PDR^*/\partial\sigma, \partial b^*/\partial\sigma > 0$, then $dPDR^*|_{dg_z,d\sigma\neq 0}, db^*|_{dg_z,d\sigma\neq 0} \gtreqless 0$ according as $dg_Z \lesseqgtr 0$ and $d\sigma \gtreqless 0$. Finally, for $dg_Z, d\sigma \gtreqless 0$ we have $dPDR^*|_{dg_z,d\sigma\neq 0} \gtreqless 0$ according as $|[\partial PDR^*/\partial g_Z]dg_Z| \lesseqgtr |[\partial PDR^*/\partial\sigma]d\sigma|$ and $db^*|_{dg_z,d\sigma\neq 0} \gtreqless 0$ according as $|[\partial b^*/\partial g_Z]dg_Z| \lesseqgtr |[\partial b^*/\partial\sigma]d\sigma|$.

[21] In these models, consumption by capitalists is an endogenous expenditure, supposed to be a positive function of their disposable income (i.e., profits after tax plus interest payments by government).

debt ratio. Hence, the paradox of debt is a necessary corollary of the model. In our model, the government conducting fiscal policy by using government expenditures as an instrument may affect the trend rate of growth of output. However, as argued above, when the trend rate of growth of output changes it may lead to a modification of the debt ratio in the same or in the opposite direction depending on what happens to the composition of total autonomous demand between government and capitalist consumption. In this sense, we may say that the debt paradox is a possibility but not a necessary outcome of our supermultiplier model with a government sector.

The second implication refers to a situation in which the government wants to control its expenses to reduce the primary deficit and debt ratios through an austere fiscal policy.[22] In the case of a supermultiplier model in which government spending is the only source of autonomous demand, an attempt to reduce both ratios by diminishing the rate of growth of government expenditures would be necessarily frustrated. Indeed, self-defeating austerity would be a necessary outcome in a model with such specification. This is not the case, however, in our version of the supermultiplier model with a government sector. As we argued, a decrease in the pace of expansion of government consumption that leads to a reduction in the trend rate of growth of autonomous demand may have a negative effect on the primary deficit and debt ratios if there is a sufficiently large fall in the government consumption share of total autonomous demand (i.e., a fall in σ).[23] Hence, the self-defeating austerity scenario is not a necessary corollary of our model.

Let us now discuss the effects of changes in tax rates over wages and profits. First, we will analyze the case in which the average tax rate (i.e. the tax burden in our model) changes without any modification in the distribution between the after-tax wages and profits.[24] In this case, from our analysis of the steady-state of the model, we can see that pure ($d\tau = 0$) changes in the tax burden do not have any effect on the rates of growth of output, investment and capital stock and, therefore, on the investment to output ratio. However, a pure increase (decrease) in the tax burden has a negative (positive) effect over the marginal propensity to consume out of income (i.e., $\big(1 - t + \tau(1 - \omega)\big)\omega$) and, thus, a negative (positive) effect on the equilibrium value of the supermultiplier and a positive (negative) impact on the equilibrium ratio of autonomous demand to output. Hence, although a change in the tax burden does not have a growth effect, it has a level effect. A higher (lower) tax burden is associated with a lower (higher) steady-state level of output when compared, at each instant of time, to the steady-state that we would have if there were no change in the tax burden. Finally, from the

[22] Note that our analysis here is of a positive nature and not a normative one. In this sense, our model should be able to deal with the situation described even when we disagree with it from a normative point of view.

[23] By sufficiently large reduction in σ we mean $d\sigma < 0$ such that $|[\partial PDR^*/\partial g_Z]dg_Z| < |[\partial PDR^*/\partial \sigma]d\sigma|$ and $|[\partial b^*/\partial g_Z]dg_Z| < |[\partial b^*/\partial \sigma]d\sigma|$.

[24] This is the case in which we have $dt = dt_w = dt_k$ and, therefore, $d\tau = dt_k - dt_w = 0$.

existence of this level effect it follows that the change in the tax burden affects the primary deficit and debt ratios, a (pure) increase (decrease) of the tax burden diminishes (raises) the equilibrium value of these ratios.[25]

Next, we will consider the effects of a change in tax rates that does not lead to a change in the tax burden, a kind of pure redistributive change.[26] This change promotes the redistribution between after tax wages and profits, which affects the marginal propensity to consume out of income. From our discussion of the steady-state of the model it can be verified that a change in the latter variable has neither a growth effect over output, investment and capital stock, nor an impact on the investment-output ratio. Moreover, similarly to what occurred in the case of pure tax burden change, the redistribution of the after tax income provoked by the specific changes in the tax rates here analyzed also has a level effect on steady-state output. In fact, an increase (a decrease) in the tax rate on wages in relation to the tax rate on profits leads to a fall (rise) in the marginal propensity to consume, a reduction (an increase) in the equilibrium value of the supermultiplier and an increase (decrease) in the autonomous demand to output ratio. Given the government share of autonomous demand and the tax burden, the change under investigation modifies the primary deficit and debt ratios in the same direction. That is, redistribution in favor to (against) the wage earners reduces (augments) the primary deficit and debt ratios.[27]

In the general case, changes in the taxes rates over wages and profits causes modifications in the tax burden and in the after tax distribution of income between wages and profits. As can be verified from our presentation of the steady-state properties of the model, this kind of change does not have a growth effect and does not affect the investment-output ratio. It has, however, an effect over the marginal propensity to consume[28] and on government revenue and, through them, over the equilibrium values of the level of output and primary deficit and debt ratios. An increase (a decrease) in the tax burden ($dt \neq 0$ and $dt_w \neq 0$) has a negative (positive) level effect, and reduces (raises) the primary deficit and debt ratios. On the other hand, an increase (a decrease) in the tax rate on profits in relation

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[25] From the expression for the primary deficit ratio we can deduce that $\partial PDR^*/\partial t|_{d\tau=0} = \sigma\omega - 1 < 0$ because $0 < \sigma, \omega < 1$. By its turn, from equation (20) we obtain $\partial b^*/\partial t|_{d\tau=0} = [\partial b^*/\partial PDR^*|_{d\tau=0}][\partial PDR^*/\partial t|_{d\tau=0}] = (\sigma\omega - 1)/(g_Z - r)$. Thus, since, as we just saw, $\sigma\omega - 1 < 0$ and, in the case under analysis, $g_Z > r$ then $\partial b^*/\partial t|_{d\tau=0} < 0$.

[26] That is, we have $dt_w \neq dt_k$ such that $dt = 0$. Recalling that $t = t_w + \tau(1 - \omega)$, it follows that $dt = dt_w + (1 - \omega)d\tau = 0$ and, therefore, that $d\tau|_{dt=0} = -dt_w/(1 - \omega) \neq 0$. More specifically, from the latter result it follows that $d\tau|_{dt=0} \gtreqless 0$ according as $dt_w \lesseqgtr 0$ (and, concomitantly, $dt_k \gtreqless 0$).

[27] From the expression for the primary deficit ratio we obtain $\partial PDR^*/\partial\tau|_{dt=0} = -\sigma(1 - \omega)\omega < 0$ since $\sigma > 0$ and $0 < \omega < 1$. On the other hand, from equation (20) we have the partial derivative for the debt ratio $\partial b^*/\partial\tau|_{dt=0} = [\partial b^*/\partial PDR^*][\partial PDR^*/\partial\tau|_{dt=0}] = -(\sigma(1 - \omega)\omega)/(g_Z - r) < 0$ because $\partial PDR^*/\partial\tau|_{dt=0} = -\sigma(1 - \omega)\omega < 0$ and, in the case under analysis, $g_z > r$.

[28] Note that the marginal propensity to consume depends on the tax rate over wages and not on the tax rate on profits (i.e., $c = (1 - t_w)\omega = \big(1 - t + \tau(1 - \omega)\big)\omega$). Thus, if $dt_k \neq dt_w = 0$ then $dc = 0$. In fact, from $dt_k \neq dt_w = 0$ it follows that $dt = (1 - \omega)dt_k$ and $d\tau = dt_k$ and, therefore, we obtain $dc = -\omega dt + (1 - \omega)\omega d\tau = -\omega(1 - \omega)dt_k + (1 - \omega)\omega dt_k = 0$. The marginal propensity to consume is not affected by changes in the tax rate on profits when there is no variation in the tax rate on wages. The only transmission channel functioning in this case is the one through government revenues.

to the tax rate on wages (i.e., $d\tau \neq 0$ and $dt_w \neq 0$) has a negative (positive) level effect and diminishes (augments) the primary deficit and debt ratios. Thus, government may use different combinations of changes in the tax rates over wages and profits as fiscal policy instruments in order to produce tax burden and redistribution effects over the level of output, and the primary deficit and debt ratios. Both effects may reinforce or compensate each other depending on whether they go in the same or different direction, respectively.[29]

Now we are going to deal with the effects of changes in the (real) interest rate on our main endogenous variables. This will be done in connection with our discussion about the two polar closures for the classical (Sraffian) theory of distribution briefly presented above, allowing us to deal with the isolated effect of rate of interest changes and with its indirect effects related to modifications in the wage share.

First, we will deal with the case in which the wage share explained by labor bargain and technical conditions determine the rate of profit independently of the rate of interest (i.e., the second closure discussed above). In this circumstance, a change in the rate of interest has no repercussion on the wage share and, therefore, on the primary income distribution. Thus, we can analyze the pure or direct effect of interest rate change on these variables. Indeed, from equations (16) to (19) we can verify that a change in the rate of interest has no impact on the equilibrium values of the growth rates of output, investment and capital stock, and, hence, on the investment-output ratio. It also shows that the equilibrium values of the supermultiplier or the autonomous demand to output ratio are not affected by such change. Thus, in addition to not having a growth effect, a change in the rate of interest does not have even a level effect on the steady-state output level of the simple version of the model discussed here. Moreover, as the expression for the primary deficit ratio shows (i.e., $\sigma\left[1-\left(\left(1-t+\tau(1-\omega)\right)\omega+(v/u_n)(g_Z+\delta)\right)\right]-t$), the latter ratio is not affected by interest rate changes too. Such a change has, nonetheless, an impact on the equilibrium value of the debt ratio. Indeed, an increase (a decrease) of the interest rate has a positive (negative) impact on the financial component of total deficit ratio and, therefore, has a positive (negative) effect over government indebtedness as

[29] Thus, the effect of a combined change in t and τ over the primary deficit ratio is given by $dPDR^*|_{dt,d\tau\neq 0} = [\partial PDR^*/\partial t]dt + [\partial PDR^*/\partial \tau]d\tau = [\sigma\omega - 1]dt + [-\sigma(1-\omega)\omega]d\tau$. Since, $0<\sigma<1$ and $0<\omega<1$ then $\partial PDR^*/\partial t = \sigma\omega - 1 < 0$ and $\partial PDR^*/\partial \tau = -\sigma(1-\omega)\omega < 0$. Thus, according as $dt, d\tau \gtreqless 0$ then $dPDR^*|_{dt,d\tau\neq 0} \lesseqgtr 0$. By its turn, if $dt > 0$ and $d\tau < 0$ it follows that $dPDR^*|_{dt,d\tau\neq 0} \gtreqless 0$ according as $|[\partial PDR^*/\partial t]dt| \lesseqgtr |[\partial PDR^*/\partial \tau]d\tau|$. Conversely, if $dt < 0$ and $d\tau > 0$ then $dPDR^*|_{dt,d\tau\neq 0} \gtreqless 0$ according as $(|[\partial PDR^*/\partial t]dt| \gtreqless |[\partial PDR^*/\partial \tau]d\tau|)$. On the other hand, for the debt ratio we have $db^*|_{dt,d\tau\neq 0} = [(\partial b^*/\partial PDR^*)(\partial PDR^*/\partial t)]dt + [(\partial b^*/\partial PDR^*)(\partial PDR^*/\partial \tau)]d\tau = [(\sigma\omega - 1)/(g_Z - r)]dt + [(-\sigma(1-\omega)\omega)/(g_Z - r)]d\tau$. Since, in the case under analysis, we have $g_Z > r$ then $\partial b^*/\partial t = (\sigma\omega - 1)/(g_Z - r) < 0$ and $\partial b^*/\partial \tau = (-\sigma(1-\omega)\omega)/(g_Z - r) < 0$. It follows that according as $dt, d\tau \gtreqless 0$ then $db^*|_{dt,d\tau\neq 0} \lesseqgtr 0$. Next, if $dt > 0$ and $d\tau < 0$ then $db^*|_{dt,d\tau\neq 0} \gtreqless 0$ according as $|[\partial b^*/\partial t]dt| \lesseqgtr |[\partial b^*/\partial \tau]d\tau|$. In contrast, if $dt < 0$ and $d\tau > 0$ then $db^*|_{dt,d\tau\neq 0} \gtreqless 0$ according as $|[\partial b^*/\partial t]dt| \gtreqless |[\partial b^*/\partial \tau]d\tau|$.

captured by the equilibrium debt ratio.[30] Finally, observe that a sufficiently large increase in the interest rate can lead the economy from a situation characterized by the stability of the equilibrium debt ratio towards a situation in which the rate of interest is greater than the trend rate of growth of output and, hence, the equilibrium debt ratio is unstable.[31]

Let us now discuss the effects of rate of interest changes within the context of the monetary theory of distribution (i.e., the first closure briefly discussed above). According to this theory, there exists an inverse relationship, mediated by the rate of profit, between the rate of interest and the wage share. Differently from the previous case, in this one a change in the rate of interest affects the primary distribution between wages and profits. Thus, besides the pure or direct effect of the rate of interest just analyzed, we have the repercussions of the redistribution of income, which, according to the model, affects the economy through two channels. On the one hand, a change in the wage share has an impact over the marginal propensity to consume out of total income and, on the other, it has an effect on the tax burden (i.e., on the average tax rate). In both cases, there is no growth effect in the steady-state of the model. Since, as we saw, the rate of interest does not have any direct effect on the steady-state growth rate, then we can conclude that it has no growth effect at all, at least in the specification of the Supermultiplier model used here.

Turning our attention to the indirect channels of influence previously indicated, the marginal propensity to consume is positively related to the wage share (i.e., $\partial c/\partial\omega = 1 - t_w > 0$, since $0 < t_w < 1$). Thus, an increase (a decrease) in the rate of interest (and, accordingly, in the rate of profit) leads to a fall (rise) in the wage share and a decrease (an increase) in the marginal propensity to consume out of total income. The latter, by its turn, leads to an increase (a decrease) in the equilibrium value of the supermultiplier and, therefore, a diminution (rise) in the equilibrium value of the autonomous demand to output ratio. It follows that in the case of the monetary theory of distribution the change in the interest rate has a level effect over steady-state output (and also over steady-state capacity output and capital stock). This level effect means that, everything else being constant, a rise (fall) in the interest rate brings about a steady-state level of output that is lower (higher), at each instant of time after the change, than the one that would be obtained if the interest rate were kept constant. Moreover, given the government share of autonomous demand and the level effect just mentioned, an

[30] From equation (20) we obtain the partial derivative of the equilibrium debt ratio with respect to rate of interest according to which $\partial b^*/\partial r = PDR^*/(g_z - r)^2 = [1/(g_z - r)]b^*$. Since we are supposing that economy has a positive primary deficit ratio in equilibrium, then the numerator of the latter expression is positive. Therefore, the derivative has a positive value, that is $\partial b^*/\partial r > 0$.

[31] Shortly, we will analyze this situation.

increase (a decrease) in the rate of interest raises (reduces) the share of government expenditures on output, contributing to an increase (a decrease) in the primary deficit ratio.[32]

However, the overall effect on the primary deficit ratio also depends on the other indirect channel through which a change in the interest rate affects the tax burden of the economy. Indeed, as can be easily verified, a change in primary income distribution modifies the tax burden if the tax rates on wages and profits are different. In particular, the wage share and the tax burden change in opposite directions if the tax rate on profits is greater than the tax rate on wages and in the same direction otherwise.[33] Since, according to the monetary theory of distribution, the wage share and rate of interest are inversely related, then an increase (fall) in the interest rate has a positive (negative) effect on the tax burden if the tax rate on profits is greater (lower) than the tax rate on wages. Therefore, since the primary deficit ratio and the tax burden are negatively related, an increase (reduction) in the interest rate has a negative (positive) contribution to the change in the equilibrium value of the primary deficit ratio through the tax burden channel when the tax rate on profits is greater (lower) than the tax rate on wages.[34]

The total effect of a change in the rate of interest over the equilibrium primary deficit ratio in the case of the distributive closure of the monetary theory of distribution combines the partial effects through the government expenditure share and the tax burden channels discussed above. As we saw, the partial effect through the government expenditure share of output is always positive. On the other hand, the partial effect through the tax burden may be positive or negative depending on whether the tax rate on wages is, respectively, higher or lower than the tax rate on profits. In the former case case, both the expenditure share and tax burden effects have the same sign and, hence, the overall change of the primary deficit ratio goes in the same direction of the change in the rate of interest (i.e., $\partial PDR^*/\partial r > 0$).[35] However, when the tax rate on wages is smaller than the tax rate on profits the expenditure share and tax burden effects have different signs and, therefore, the overall effect of a change in the interest rate over the primary deficit ratio depends on the relative magnitudes of the two effects. In this circumstance, for relatively high values of the tax rate on profits we have a negative effect over the equilibrium primary deficit ratio of a change in the rate of interest (i.e., $\partial PDR^*/\partial r <$

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[32] From the expression for the primary deficit ratio and the interpretation of equation (6) following the point of view of the monetary theory of distribution we obtain $\partial PDR^*/\partial r|_{dt=0} = [\partial PDR^*/\partial\omega|_{dt=0}][\partial\omega/\partial r] = [-\sigma(1-t_w)][-v/u_n] = \sigma(1-t_w)(v/u_n) > 0$ since $\sigma, 1-t_w, v, u_n > 0$.

[33] In fact, from the definition of the tax burden we have $\partial t/\partial\omega = t_w - t_k = -\tau \gtreqless 0$ as $t_w \gtreqless t_k$ or $\tau \lesseqgtr 0$.

[34] We have here $\partial PDR^*/\partial r|_{d(Z_G/Y)=0} = \left[\partial PDR^*/\partial\omega|_{d(Z_G/Y)=0}\right][\partial\omega/\partial r] = [(\partial PDR^*/\partial t)(\partial t/\partial\omega)][\partial\omega/\partial r] = [-(t_w - t_k)][-v/u_n] = (t_w - t_k)(v/u_n) = -\tau(v/u_n) \gtreqless 0$ as $t_w \gtreqless t_k$ or $\tau \lesseqgtr 0$ (besides, $v, u_n > 0$).

[35] That is, combining the two effects we have $\partial PDR^*/\partial r = \partial PDR^*/\partial r|_{dt=0} + \partial PDR^*/\partial r|_{d(Z_G/Y)=0} = \left[\partial PDR^*/\partial\omega|_{dt=0} + \partial PDR^*/\partial\omega|_{d(Z_G/Y)=0}\right][\partial\omega/\partial r] = [\sigma(1-t_w) + t_w - t_k](v/u_n)$. Therefore, if $t_w > t_k$ (or $\tau < 0$) then $\partial PDR^*/\partial r > 0$ since, by the definitions of the variables involved, $\sigma(1-t_w) > 0$ and $v/u_n > 0$.

0). Otherwise, the expenditure share effect dominates the tax burden one and we obtain a positive relationship between the equilibrium primary deficit ratio and the rate of interest. Therefore, we have a negative relationship between the rate of interest and the equilibrium value of the primary deficit ratio for relatively high values of the tax rate on profits and a non-negative relationship otherwise, including the case in which the tax rate on profits has lower value than the tax rate on wages. [36]

Finally, we have to analyze the effect of a change in the rate of interest on the equilibrium debt ratio within the context of the monetary theory of distribution. As can be seen in equation (20), there exists a positive relationship between the equilibrium values of the debt and primary deficit ratios. Thus, besides the pure or direct effect of the rate of interest on the equilibrium debt ratio already analyzed in the context of the alternative closure for the income distribution, we have the indirect effects operating through the equilibrium primary deficit ratio just discussed also affecting the equilibrium value of the debt ratio. From our previous analysis of these indirect effects and the positive relationship between equilibrium values of the debt and primary deficit ratios[37] we can deduce that the effect of a change in the rate of interest over the equilibrium debt ratio through its effects over the primary deficit ratio is negative for relatively high values of the tax rate on profits and non-negative otherwise, including the case in which the tax rate on profits has lower value than the tax rate on wages. More specifically, from equation (20) we can deduce that the direct effect of a change in the rate of interest on the equilibrium debt ratio is always positive when we have a positive primary deficit equilibrium ratio.[38] Thus, it follows that whenever a change in the rate of interest has a non-negative effect on the equilibrium value of the primary deficit ratio then the equilibrium value of the debt ratio and the rate of interest are positively related. By its turn, when the rate of interest and the equilibrium value of the primary deficit ratio are negatively related the overall effect of a change in the interest rate on the equilibrium debt ratio depends on the relative magnitudes of the direct and indirect effects discussed above. In this case, for relatively high values of the tax rate on profits such as the indirect negative effects through the primary deficit ratio are greater than the positive direct effect of the modification of the rate of interest, we have a negative relationship between the rate of interest and

[36] For relatively high values of the tax rate on profits we mean a value of t_k such as $\partial PDR^*/\partial r < 0$. That is, since $\partial PDR^*/\partial r = [\sigma(1-t_w)+t_w-t_k](v/u_n)$ and $v/u_n > 0$, we must have $t_k > \sigma(1-t_w)+t_w$ or $\partial PDR^*/\partial\omega|_{d(Z_G/Y)=0} = \tau > \sigma(1-t_w) = \partial PDR^*/\partial\omega|_{dt=0}$. Therefore, the general result is that $\partial PDR^*/\partial r \gtreqless 0$ according as $t_k \lesseqgtr \sigma(1-t_w)+t_w$ or $\tau \lesseqgtr \sigma(1-t_w)$.
[37] In fact, from equation (20) we can verify that $\partial b^*/\partial PDR^* = 1/(g_z - r)$. In the case we are analyzing we have $g_z > r$ which leads to the conclusion that $\partial b^*/\partial PDR^* > 0$.
[38] See note 30 above.

equilibrium debt ratio.[39,40] Moreover, since the direct impact of the change in the rate of interest has always a positive sign, we also have the possibility of a change in the rate of interest affecting the equilibrium values of the primary deficit and debt ratio in different directions.[41]

3.2. The $r > g^*$ case

Now let us turn our attention to the case in which the rate of interest is greater than the trend rate of growth of output (i.e., $g^* = g_Z$). To begin with, we would like to note that the effects of changes in exogenous variables previously analyzed are the same for all the endogenous variables, except for the impacts on the equilibrium value of the debt ratio. In fact, it can be easily verified that the sign of the effects of changes in the exogenous variables over the equilibrium value of the debt ratio depend on the value of the expression $g_z - r$. Thus, since this expression changes its sign from positive to negative when we pass from the stable (i.e., $g_z > r$) to the unstable (i.e., $r > g_z$) case then all signs of the effects over the equilibrium debt ratio also change. Here we will focus our analysis on the investigation of these effects. Notice also that, since we are interested only in steady-states for which the government has a net debtor position (i.e, in which we have $b^* > 0$), we will suppose that, throughout this part, the government maintains a primary surplus result in equilibrium (i.e., that $PDR^* = \sigma(Z_t/Y_t)^* - t < 0$).

It follows that a change in the trend rate of growth modifies the equilibrium debt ratio in the same direction as a result of its direct and indirect contributions, the latter one being captured by the effect transmitted through the equilibrium value of the primary deficit ratio which is inversely related to the trend rate of growth of output.[42] The government share of autonomous demand (i.e., σ) and the equilibrium debt ratio are negatively related since the equilibrium primary deficit ratio is positively related to σ.[43] By its turn, pure changes in the tax burden (i.e., $dt|_{d\tau=0} \neq 0$) generate an output level

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[39] Note that from equation (20) and using our previous results we have $\partial b^*/\partial r = [\partial b^*/\partial PDR^*][\partial PDR^*/\partial r] + \partial b^*/\partial r|_{dPDR^*=0} = [1/(g_z - r)]\big[[\sigma(1 - t_w) + t_w - t_k](v/u_n)\big] + [1/(g_z - r)]b^*$. The first term in the RHS of the previous equation captures the indirect effects of the change in the rate of interest mentioned in the main text, while the second term captures the direct or pure financial effect, which is the one that is not transmitted through the primary deficit ratio. In the case under analysis (i.e., $g_z > r$ and $PDR^* > 0$), we have $\partial b^*/\partial PDR^* = 1/(g_z - r) > 0$ and $\partial b^*/\partial r|_{dPDR^*=0} = [1/(g_z - r)]b^* > 0$, but, as we saw, we may have $\partial PDR^*/\partial r \gtreqless 0$. Therefore, as argued in the main text, if $\partial PDR^*/\partial r \geq 0$ then $\partial b^*/\partial r > 0$. Otherwise, we have $\partial b^*/\partial r \gtreqless 0$ according as $t_k \lesseqgtr \sigma(1 - t_w) + t_w + b^*(u_n/v)$ or $\partial PDR^*/\partial\omega|_{d(Z_G/Y)=0} = \tau \lesseqgtr \sigma(1 - t_w) + b^*(u_n/v) = \partial PDR^*/\partial\omega|_{dt=0} + b^*(u_n/v)$.

[40] It can be shown that the likelihood of having $\partial b^*/\partial r < 0$ depends on the composition of private (capitalist) wealth. To do so let us denote $\xi^* = b^*(u_n/v) = B_t^*/K_t^*$ as the equilibrium ratio of the government debt to the capital stock, both assets owned by the capitalists according to our assumptions. Now, it is clear that the higher the value of ξ^* the greater the value of t_k or τ must be to obtain $\partial b^*/\partial r < 0$. Thus, a high government debt in relation to its capital stock diminishes the likelihood of having $\partial b^*/\partial r < 0$ and *vice-versa*.

[41] In fact, since we know that $\partial PDR^*/\partial r \gtreqless 0$ according as $t_k \lesseqgtr \sigma(1 - t_w) + t_w$ or $\tau \lesseqgtr \sigma(1 - t_w)$ and that $b^*(u_n/v) > 0$, thus it is possible to have $\partial PDR^*/\partial r < 0$ and $\partial b^*/\partial r > 0$ if $\sigma(1 - t_w) + t_w + b^*(u_n/v) > t_k > \sigma(1 - t_w) + t_w$ or $\sigma(1 - t_w) + b^*(u_n/v) > \tau > \sigma(1 - t_w)$.

[42] That is, from equation (20) we have $\partial b^*/\partial g_Z = [\partial b^*/\partial PDR^*][\partial PDR^*/\partial g_Z] + \partial b^*/\partial g_Z|_{dPDR^*=0} = -[\sigma(v/u_n)/(g_Z - r)] - [(\sigma(Z_t/Y_t)^* - t)/(g_Z - r)^2] = -[\sigma(v/u_n)/(g_Z - r)] - [1/(g_Z - r)]b^* > 0$ since we have, as before, $\partial PDR^*/\partial g_Z = -\sigma(v/u_n) < 0$ and, in the case under analysis, $r > g_z$ and $PDR^* < 0$ as stated above.

[43] Since we are supposing that the stability condition for the Supermultiplier model holds (see inequality (21)) then we must have $\partial PDR^*/\partial\sigma = (Z_t/Y_t)^* = 1 - \big[(1 - t + \tau(1 - \omega))\omega + h^*\big] > 0$. Thus, because we have here $r > g_z$, it follows that $\partial b^*/\partial\sigma = [\partial b^*/\partial PDR^*][\partial PDR^*/\partial\sigma] = (Z_t/Y_t)^*/(g_Z - r) < 0$.

effect that leads to a change in the equilibrium value of the primary deficit ratio in the opposite direction of the change in the tax burden. Thus, we obtain a positive relationship between the tax burden and the equilibrium debt ratio in this situation.[44] On the other hand, a pure change in the tax rate on profits in relation to the tax rate on wages (i.e., $d\tau|_{dt=0} \neq 0$) causes a redistribution of after-tax income, which, by its turn, leads to an output level effect that provokes a change in the equilibrium primary deficit ratio in the opposite direction of the change in τ. It follows that the equilibrium debt ratio changes in the same direction as τ.[45] Finally, we have the effect over the equilibrium debt ratio related to the change in the rate of interest. Again, we obtain different results according to the distributive closure adopted. In the case of the distributive closure in which a change in the rate of interest does not have an effect on the wage share of income, the primary deficit ratio is not affected and we have only the direct or pure financial effect of the change in the rate of interest on the equilibrium debt ratio. However, contrary to what happens in the $g_z > r$ case analyzed above, the latter effect is always negative when the rate of interest is greater than the trend rate of growth of output (i.e., $r > g_z$) and we have a primary surplus ratio in equilibrium (i.e., $PDR^* < 0$).[46] Alternatively, according to the closure based on the monetary theory of distribution the rate of interest and the wage share are negatively related. Thus, besides the direct or pure financial negative effect of a change in the rate of interest, we also have the indirect effects resulting from the change in the wage share caused by a modification in the rate of interest. These indirect effects consist of changes in the share of government expenditure on output and the tax burden that modify the equilibrium debt ratio through their impact over the primary deficit ratio. As we saw, there is a positive relationship between the government expenditure share and the rate of interest (through the change of the supermultiplier), while the relationship between the rate of interest and the tax burden can be positive or negative depending on whether the tax rate on profits is, respectively, higher or lower than the tax rate on wages. Hence, as already discussed, for relatively and sufficiently high values of the tax rate on profits the tax burden effect dominates the government expenditure share effect and we have a negative relationship between the rate of interest and the equilibrium value of the primary deficit ratio. Otherwise, a non-negative relationship prevails between these two variables. However, differently of what we saw in the case in which $g_z > r$, when $r > g_z$ the equilibrium primary deficit ratio and debt ratios are negatively related. Therefore, since the direct or pure financial effect on the equilibrium debt ratio of

[44] In fact, we already know that $\partial PDR^*/\partial t|_{d\tau=0} = \sigma\omega - 1 < 0$ because $0 < \sigma, \omega < 1$. It follows that $\partial b^*/\partial t|_{d\tau=0} = [\partial b^*/\partial PDR^*|_{d\tau=0}][\partial PDR^*/\partial t|_{d\tau=0}] = (\sigma\omega - 1)/(g_Z - r) > 0$, since $\partial b^*/\partial PDR^*|_{d\tau=0} = 1/(g_Z - r) < 0$ when $r > g_z$.

[45] That is, since we know that $\partial PDR^*/\partial \tau|_{dt=0} = -\sigma(1-\omega)\omega < 0$ because $0 < \sigma < 1$ and $0 < \omega < 1$, then we have $\partial b^*/\partial \tau|_{dt=0} = [\partial b^*/\partial PDR^*][\partial PDR^*/\partial \tau|_{dt=0}] = -(\sigma(1-\omega)\omega)/(g_Z - r) < 0$ provided that $r > g_z$, the case under analysis.

[46] Indeed, from equation (20) we have that $\partial b^*/\partial r|_{dPDR^*=0} = PDR^*/(g_z - r)^2 = [1/(g_z - r)]b^* < 0$ since we are assuming that $PDR^* < 0$.

a change in the interest rate is always negative, the only chance of obtaining a positive relationship between the rate of interest and the equilibrium debt ratio is the prevalence of a sufficiently strong positive effect of the change in the rate of interest resulting from an equally strong negative impact of its change on the equilibrium primary deficit ratio. The latter result follows when the tax rate on profits is relatively and sufficiently high. Otherwise, we have a non-positive relation between the rate of interest and the debt ratio in equilibrium, including the case in which the tax rate on profits is lower than the one on wages.[47]

More importantly, as we argued above, when we have $r > g_z$ the Domar stability condition does not hold and, hence, the equilibrium debt ratio is unstable. In this circumstance, a change in the equilibrium value of the debt ratio gives us less information on the actual tendency of change of this variable than in the case in which the Domar condition is met. In fact, in the case under investigation the knowledge of the direction of change of the equilibrium debt ratio is not sufficient to determine the direction of change of the actual value of this variable. We also need to know the value of the debt ratio at moment of the change in the equilibrium in order to determine the behavior of the debt ratio after the change. Indeed, the debt ratio would decrease or increase after the change of its equilibrium value depending on whether the initial value of the debt ratio after such change is, respectively, below or above the new equilibrium value. It follows that an increase in the equilibrium debt ratio may cause a fall in the debt ratio and *vice-versa*.

In this sense, the inferences that we can make based on an equilibrium analysis of the impact of changes in exogenous variables over the debt ratio is much more limited in the case under analysis. That is, the kind of static (or equilibrium) analysis that we have been using so far is less satisfactory in the case under investigation. A more suitable approach would require a full dynamic (disequilibrium) analysis based on the model, combined with a discussion of the effects of some stylized versions of fiscal policy rules and conventions adopted by governments in practice. However, as we are presenting our first approximation to the subject, we decided to keep the static (equilibrium)

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[47] Note that from equation (20) we can deduce the partial derivative $\partial b^*/\partial r = [\partial b^*/\partial PDR^*][\partial PDR^*/\partial r] + \partial b^*/\partial r|_{dPDR^*=0} = [1/(g_z - r)][[\sigma(1-t_w) + t_w - t_k](v/u_n)] + [1/(g_z - r)]b^* = [1/(g_z - r)]\{[\sigma(1-t_w) + t_w - t_k](v/u_n) + b^*\}$. In the circumstances under analysis (i.e., $r > g_z$ and $PDR^* < 0$), the direct or pure financial effect $\partial b^*/\partial r|_{dPDR^*=0} = PDR^*/(g_z - r)^2 = [1/(g_z - r)]b^*$ and the derivative $\partial b^*/\partial PDR^* = [1/(g_z - r)]$ are negative. It follows that a positive relationship between the equilibrium debt ratio and the rate of interest (i.e., $\partial b^*/\partial r > 0$) requires that $\partial PDR^*/\partial r = [\sigma(1-t_w) + t_w - t_k](v/u_n) < 0$. The latter condition holds if $t_k > \sigma(1-t_w) + t_w$ or $\tau > \sigma(1-t_w)$. However, because $\partial b^*/\partial r|_{dPDR^*=0} < 0$, such condition is necessary but not sufficient, since $\partial PDR^*/\partial r$ must not only be negative but sufficiently so. The sufficient condition is that $[\sigma(1-t_w) + t_w - t_k](v/u_n) + b^* < 0$, which is met if $t_k > \sigma(1-t_w) + t_w + b^*(u_n/v)$ or $\tau > \sigma(1-t_w) + b^*(u_n/v)$. Otherwise, we obtain $\partial b^*/\partial r \leq 0$. Moreover, since $b^*(u_n/v) > 0$ then it is possible that we have, at the same time, $\partial b^*/\partial r < 0$ and $\partial PDR^*/\partial r < 0$ if $\sigma(1-t_w) + t_w + b^*(u_n/v) > t_k > \sigma(1-t_w) + t_w$ or $\sigma(1-t_w) + b^*(u_n/v) > \tau > \sigma(1-t_w)$. Finally, notice that the likelihood of having $\partial b^*/\partial r > 0$ is inversely related to the magnitude of the ratio between the government debt and the capital stock in equilibrium, $\xi^* = b^*(u_n/v) = B_t^*/K_t^*$. Everything else constant, the higher the value of ξ^* the greater is the critical value of t_k or τ above for which we obtain $\partial b^*/\partial r > 0$. Thus, the higher the weight of government debt in total capitalist wealth the more difficult it would be to obtain a positive relationship between the equilibrium debt ratio and the rate of interest in the prevailing circumstances here investigated.

approach adopted so far and postpone the use of the more complete approach just suggested to future works.[48]

Given the results for the impacts over the equilibrium debt ratio obtained above and based only on the static approach followed so far, we cannot say very much from a general point of view. However, we may say at least two things. First, we argue that those modifications in the exogenous variables that lead to a rise (a fall) of the equilibrium value of the debt ratio increase (decrease) the likelihood of a given debt ratio being on a region in which this variable does not grow indefinitely, that is on the region defined by all values of b below or equal the equilibrium value obtained after the change.[49] Secondly, for any positive value of the debt ratio strictly above the old equilibrium value at the moment of the change in the equilibrium,[50] each modification in the exogenous variables that contribute to an increase of the equilibrium value of the debt ratio lead to a tendency towards a lower average rate of growth of the debt ratio after such change than the one we would have if nothing had been altered. In particular, if the debt ratio prevailing at the moment of the change of the equilibrium happens to be also below or equal the new equilibrium value then the debt ratio has a non-increasing behavior.[51] Otherwise, with only one exception, the debt ratio continues to rise but at a slower average pace than it would increase if the change in the equilibrium had not occurred.[52] In the latter case, we

[48] Needless to say, the use of a disequilibrium approach has also benefits when applied to the case in which the equilibrium debt ratio is stable. This is particularly true when we are addressing economic policy issues in which knowing what happens in disequilibrium is essential in discussing the economic effects and relative merits of different fiscal policies. The fact that we focused our previous analysis of the stable case in the investigation of the equilibrium properties of the model explains why we avoided the latter kind of discussion above.

[49] This proposition is inspired by Pasinetti (1998) analysis of the sustainability of public debt in the Euro area. He defines a sustainable debt position as one in which the debt ratio does not increase and a sustainability area as all the combinations of primary surplus and debt ratios that generate non-increasing debt ratios for given interest rates and rates of growth of output. Here we will not discuss the concept of sustainability because we are not addressing fiscal policy issues directly in this work. For us, only the behavior of the debt ratio predicted by the model is relevant at this stage of development of our analysis. Fiscal policy issues as such will be the subject matter of future research along the lines of the model here presented.

[50] Note that, since in the case under analysis the rate of interest is greater than the trend rate of growth of output, the situation before the change in the equilibrium is one of increasing indebtedness because we suppose that the debt ratio is above the prevailing equilibrium before the change.

[51] Since in the circumstance under investigation we have $r > g_z$, a value of the debt ratio at the moment of change below the new equilibrium implies that at this ratio we have $\dot{b} < 0$ because we know that at the equilibrium point we have $\dot{b} = 0$ and in the neighborhood of the equilibrium we have $d\dot{b}/db = r - g_z < 0$. On the other hand, if the debt ratio at the moment of change in the equilibrium (let us denote it $\tilde{b}$) happens to be exactly equal to the new equilibrium debt ratio we have $\dot{b} = 0$ and $\tilde{b} = b^* = PDR^*/(g_z - r)$. From these two cases and from the assumption that we are dealing with positive net debt positions, it follows directly that the rate of growth of the debt ratio is non-positive. That is, from equation (15''), the trend (i.e., compatible with the equilibrium of the supermultiplier subsystem of our model) rate of growth of the debt ratio is $g_{bt} = PDR^*/b_t + r - g_z \leq 0$. Therefore, since the rate of growth of the debt ratio without a change in the equilibrium would be positive, it follows that the rate of growth of the debt ratio after the change in equilibrium is lower than the one we would have without change for all moments of time after the change taking into account only positive net debt positions.

[52] The exception refers to an increase in the rate of interest in the particular case of the distributive closure based on the monetary theory of distribution. In this case we have a positive relationship between the rate of interest and the equilibrium debt ratio (i.e., $\partial b^*/\partial r > 0$). As we are supposing that the value of the debt ratio at the moment of the change is above the new equilibrium debt ratio it follows that government indebtedness increases continuously. Given the value of the primary deficit (surplus) ratio, this implies that $\lim_{b_t \to \infty} g'_{bt} = \lim_{b_t \to \infty} [PDR^{*\prime}/b_t + r' - g_z] = r' - g_z > r - g_z = \lim_{b_t \to \infty} [PDR^*/b_t + r - g_z] = \lim_{b_t \to \infty} g_{bt}$, where the superscript "′" is used to denote the value of the variable after the change in the equilibrium. Note, however, that the initial impact of the increase in the rate of interest over the rate of growth may be negative, null or positive depending on the value of the debt ratio prevailing at the moment of the change in the equilibrium. Indeed, since the new equilibrium debt ratio is above the old one then at the new equilibrium the rate of growth of the

have two sorts of changes reducing the average pace of growth of the debt ratio. On the one hand, we have the modifications in the share of government expenditures on total autonomous demand or in the tax rates that cause an increase in the equilibrium debt ratio. These changes affect the rate of growth of the debt ratio by reducing the primary deficit ratio (i.e., by raising the equilibrium primary surplus ratio), which causes a temporary decrease of the rate of growth of the debt ratio. The reduction is temporary because, as the debt ratio continues to increase, the contribution of the primary surplus (or deficit) ratio to the rate of growth of the debt ratio tend to disappear and the latter tends to be explained only by the (positive) difference between the rate of interest and the trend rate of growth of output (i.e., by the term $r - g_z$).[53] On the other hand, we have those changes in the trend rate of growth of output or the rate of interest, which lead to an increase in the equilibrium debt ratio. An increase in the trend rate of growth of output causes a reduction in the average rate of growth of the debt ratio by raising the primary surplus ratio and by diminishing the term $r - g_z$.[54] By its turn, apart from the exception mentioned above (i.e., when $\partial b^*/\partial r > 0$), a decrease in the rate of interest reduces the growth rate of the debt ratio by diminishing the term $r - g_z$. The latter contribution to the fall in the rate of growth of indebtedness is reinforced when the reduction in the rate of interest also leads to the decrease of the primary deficit ratio (i.e., when we have $\partial PDR^*/\partial r > 0$) and is partially counterbalanced in the opposite case (i.e. when $\partial PDR^*/\partial r < 0$ but $\partial b^*/\partial r < 0$).[55]

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debt ratio would be equal to zero (i.e., $\dot{b}' = 0$ at $b^{*\prime}$) while, at the same debt ratio, the rate of growth of the debt ratio that would prevail if nothing had changed would be positive (i.e., $\dot{b}' > 0$ at $b^{*\prime}$). However, since we know that $\partial\dot{b}'/\partial b' = r' - g_z > r - g_z = \partial\dot{b}/\partial b$ we must have (by continuity) a value of the debt ratio at which $\dot{b}' = \dot{b} > 0$. Let us call this specific value of the debt ratio $\bar{b}$. Hence, for values of the debt ratio at the moment of the change in the equilibrium $\tilde{b}$ such that $b^{*\prime} < \tilde{b} < \bar{b}$ the change in the equilibrium causes a fall of the rate of growth of the debt ratio at the time of the modification. After that moment the rate of growth of the debt ratio will tend to increase at a faster pace than the one that we would have in the old trajectory and, eventually, the rate of growth of the debt ratio in the new trajectory becomes higher than the one in the old trajectory. On the other hand, for values of debt ratio at moment of the change in the equilibrium such that $\tilde{b} \geq \bar{b}$ there is no reduction in the rate of growth of the debt ratio at the moment of change and there is a tendency to have $g_b' > g_b$ on average for all moments of time after the change in the equilibrium.

[53] At the moment of the equilibrium change the rate of growth of the debt ratio falls since the primary deficit ratio decreases (or the primary surplus ratio increases) for given values of the debt ratio, the rate of interest and the trend rate of growth of output, that is we have $g_b' = PDR^{*\prime}/\tilde{b} + r - g_z < PDR^*/\tilde{b} + r - g_z = g_b$ where $PDR^{*\prime} < PDR^* < 0$ and $\tilde{b}$ is the debt ratio at moment of the change in the equilibrium. After that, as b_t increases the negative contribution of the primary surplus ratio to the rate of growth of the debt ratio tends to zero, implying that the rate of growth of the debt ratio tends to converge, from below, to the same rate it would converge in the old trajectory, that is we have $\lim_{b_t\to\infty} g_{bt}' = \lim_{b_t\to\infty} g_{bt} = r - g_z$ according as $\lim_{b_t\to\infty} PDR^*/b_t = \lim_{b_t\to\infty} PDR^{*\prime}/b_t = 0$. Therefore, the average rate of growth of the debt ratio tends to be smaller in the new trajectory than in the old one after the increase in the equilibrium value of the debt ratio.

[54] In this case, at the moment of the change in the equilibrium, for any $\tilde{b} > b^{*\prime}$, we have $g_b' = PDR^{*\prime}/\tilde{b} + r - g_z' < PDR^*/\tilde{b} + r - g_z = g_b$ since $PDR^{*\prime} < PDR^* < 0$ and $g_z' > g_z$. From then on, we have $\lim_{b_t\to\infty} g_{bt}' = r - g_z' < r - g_z = \lim_{b_t\to\infty} g_{bt}$. Thus, we have a lower average rate of growth of the debt ratio in the new trajectory when compared to what we would have in the old trajectory after the increase in the equilibrium level of the debt ratio.

[55] Here a fall in the rate of interest causes an increase of the equilibrium debt ratio (i.e., $\partial b^*/\partial r < 0$), which, as we saw, is always the case if $\partial PDR^*/\partial r > 0$ and also obtained under certain conditions if $\partial \mathrm{PDR}^*/\partial \mathrm{r} < 0$ (see note 47 above). In the first case (i.e., $\partial \mathrm{b}^*/\partial \mathrm{r} < 0$ and $\partial \mathrm{PDR}^*/\partial \mathrm{r} > 0$), since the value of the new equilibrium debt ratio is higher than the old one (i.e., $b^{*\prime} > b^*$) and $\partial\dot{b}'/\partial b = r' - g_z < r - g_z = \partial\dot{b}/\partial b$ then for any initial value of the debt ratio after the reduction of the interest rate such as $\tilde{b} > b^{*\prime}$ we have $g_b' = PDR^{*\prime}/\tilde{b} + r' - g_z < PDR^*/\tilde{b} + r - g_z = g_b$ because $PDR^{*\prime} < PDR^* < 0$ and $r' < r$. Moreover, besides the rate of growth of the debt ratio being lower at the moment of the change in the equilibrium, the average rate of growth of the debt ratio in new trajectory after the change is also lower than we would obtain in the old trajectory because $\lim_{b_t\to\infty} g_{bt}' = r' - g_z < r - g_z = \lim_{b_t\to\infty} g_{bt}$. Finally, in the second case (i.e., $\partial b^*/\partial r < 0$ *and* $\partial PDR^*/\partial r < 0$) although a reduction in the rate of interest causes an increase in the equilibrium

These results are important for the discussion of the effects of changes in fiscal policy instruments over a process of increasing government indebtedness according to the model. In fact, they show that the latter process can in principle be controlled by the management of fiscal policy instruments if public authorities deem it necessary to do so.[56] For instance, such control can be made by the manipulation of the composition of autonomous demand and/or tax rates that increases by a sufficient amount the primary surplus ratio. It should be remarked that independently of the intensity of these changes the unstable nature of the equilibrium debt ratio cannot be modified by them. However, in the case in which the debt ratio is controlled by the management of the rate of interest and/or the trend rate of growth of output, a change from a situation of unstable debt ratio equilibrium towards a stable one is possible. Indeed, provided they are feasible, a sufficient reduction in the rate of interest (e.g., cheap money policies) and/or a sufficient rise in the trend rate of growth of output led by an increase in the rate of growth of government autonomous demand can change the situation. It follows that after such change the equilibrium value of the debt ratio would be stable and the process of increasing government indebtedness would be contained even in an economy with positive primary deficit and debt ratios.

4. Conclusion

The main results of our analysis can be summarized as follows. In the case in which the rate of interest is smaller than the trend rate of growth of output, concerning the changes in the pattern of autonomous demand, fiscal policy using the rate of growth of government expenditures as an instrument may have a steady-state growth effect (i.e. an effect on g_Z). Indeed, if this kind of policy has a growth effect it modifies the investment-output ratio in the same direction and, concomitantly, the ratio of autonomous demand to output in the opposite direction. Moreover, if the composition of autonomous demand (i.e., σ) does not change, the government share of output, the primary deficit ratio (given the tax burden) and debt ratio also changes in the opposite direction of the equilibrium rate of growth of output. Secondly, a pure change in the government share of autonomous demand (i.e., in σ)

value of the primary deficit ratio (i.e., $PDR^* < PDR^{*\prime} < 0$), the pure financial effect of the interest rate change dominates, implying that $b^{*\prime} > b^*$. The latter situation together with the fact that $\partial \dot{b}'/\partial b = r' - g_z < r - g_z = \partial \dot{b}/\partial b$ implies, by its turn, that for any $\tilde{b}$ at the moment of the change such as $\tilde{b} > b^{*\prime}$ we have $g_b' = PDR^{*\prime}/\tilde{b} + r' - g_z < PDR^*/\tilde{b} + r - g_z = g_b$ at the moment of the change. Further, since we know that after the change in the equilibrium we have $\lim_{b_t \to \infty} g_{bt}' = r' - g_z < r - g_z = \lim_{b_t \to \infty} g_{bt}$, hence the average rate of growth of the debt ratio after the change in the equilibrium is also lower in the new trajectory than it would be in the old one.

[56] Although we may disagree from a normative point of view with the control of the debt ratio as a fiscal policy objective, from a positive point of view we should be able to analyze such kind of policy and their effects over the economy since they are adopted by various governments in reality. Moreover, from a normative viewpoint, debt ratio control policies may be desirable in order to avoid a perverse redistribution of income promoted by a high government debt service burden, particularly when the rate of interest is high and the unemployment rate is low.

does not affect the pace of economic growth, the investment-output ratio and the autonomous demand share of output. It does not have a level effect either, since there is no change in the steady-state value of the supermultiplier. In fact, as we saw, such change leads to a modification in the primary government deficit (given the tax burden) and debt ratios in the same direction. Finally, we verified that in the general case involving changes in both the rate of growth of autonomous demand and its composition (i.e., changes in g_Z and σ), the impacts of the two sources of changes over the primary deficit and debt ratios have different signs. We showed that if g_Z and σ change in different directions we have unambiguous impacts on the primary deficit and debt ratios. Conversely, if they change in the same direction, the final impacts on the primary deficit and debt ratios depend on the relative strength of the two sources of change. Our analysis of the general case also showed that the debt paradox and the self-defeating austerity scenarios are possibilities but not necessary outcomes of Supermultiplier models with a government sector.

Regarding the changes in the tax rates over wages and profits, pure changes in the tax burden (i.e., changes with $d\tau = dt_k - dt_w = 0$) do not have steady-state growth effects nor affect the investment to output ratio. However, this kind of change leads to a modification in the marginal propensity to consume out of total income in the opposite direction and to a change in the autonomous demand to output ratio in the same direction, which causes a negative output level effect. Moreover, as we saw, these changes cause modifications in the primary deficit and debt ratios in the opposite direction. On the other hand, a change in the tax rate over profit in relation to the tax rate on wages that does not lead to a change in the tax burden (i.e., $d\tau \neq 0$ and $dt = 0$) has no growth effect and does not affect the investment-output ratio. This kind of change causes a redistribution of after-tax income between wages and profits, which, by its turn, has an effect over the marginal propensity to consume out of total income and, therefore, causes an output level effect. It follows, as we saw, that these changes lead to modifications in the autonomous demand to output ratio and in the primary deficit and debt ratios in the opposite direction. Finally, we verified that in the general case (i.e., $d\tau \neq 0$ and $dt \neq 0$) changes in the taxes rates over wages and profits causes modifications in the tax burden and in the after tax distribution of income between wages and profits. We showed that these combined changes neither have steady-state growth effects nor affect the steady-state investment-output ratio. In fact, we argued that they have an output level effect only when they involve changes in the tax rate on wages. The direction of the effect over the primary deficit and debt ratios is unambiguous when the tax burden t and the relative tax rates $\tau = t_k - t_w$ change in the same direction and ambiguous otherwise.

Finally, concerning the effects of a change in the rate of interest we showed that different results are obtained according to the specific closure adopted for the Classical/Sraffian theory of income distribution. As we saw, in the closure in which the wage share is explained by wage bargain and technical conditions and determines the rate of profit independently of the rate of interest, a change in the latter variable has no repercussion on the wage share. In this case, changes in the rate of interest have no steady-state growth effect nor an effect over the equilibrium investment-output ratio. In addition, such changes do not affect the marginal propensity to consume and, therefore, the equilibrium value of the supermultiplier. It follows, as we verified, that there is no output level effect and, therefore, the change in the rate of interest does not affect the autonomous demand to output and the primary deficit ratios. Finally, we showed that there is a positive relationship between the rate of interest and the debt ratio due to the temporary and pure financial effect of changes in the interest rate on the pace of debt accumulation. On the other hand, in the case of the monetary theory of distribution a change in the rate of interest has an effect over the wage share in the opposite direction. We showed that the ratio of autonomous demand to output and, given the composition of autonomous demand, the government expenditure share of output are positively related to the change in the interest rate. However, the effect of interest rate changes on the primary deficit ratio depends also on the transmission mechanism through which the redistribution of income affects the tax burden. In this connection, we verified that we have a negative relationship between the rate of interest and the equilibrium value of the primary deficit ratio for sufficiently and relatively high values of the tax rate on profits and a negative relationship in the opposite case. Lastly, since the equilibrium values of the debt and primary deficit ratios are positively related we showed that the effect a change in the rate of interest over the equilibrium debt ratio is negative for sufficiently and relatively high values of the tax rate on profits and non-negative otherwise, including the case in which the tax rate on profits has lower value than the tax rate on wages.

Next, we addressed the case in which the rate of interest is greater than the trend rate of growth of output. As we saw, in this case the equilibrium value of the debt ratio is unstable. We also verified that the effects of changes in exogenous variables are the same for all the endogenous variables with the exception of the impacts on the debt ratio, which was the focus of our analysis. In fact, we argued that all signs of the effects over the equilibrium debt ratio change in relation to the case in which the rate of interest is smaller than the trend rate of growth of output. In addition, we discussed two more general proposition based on the equilibrium properties of the model. First, we showed that those modifications in the exogenous variables that lead to a rise (a fall) of the equilibrium value of the debt ratio increase (decrease) the likelihood of a given debt ratio being on a region in which this variable does not grow indefinitely. Secondly, we argued that for positive values of the debt ratio strictly above the old equilibrium value at the moment of the change in the equilibrium, each modification in the exogenous variables that contribute to an increase of the equilibrium value of the debt ratio lead to a tendency towards a lower average rate of growth of the debt ratio after such change than the one we would have in the trajectory without any modification. These propositions show that, according to the model, a process of increasing government indebtedness can in principle be controlled by the management of fiscal policy instruments if this is the desire of public authorities. We argued, however, that the effects are different depending on the type of instrument applied. In the case of the manipulation of the composition of autonomous demand and/or the tax rates that increases by a sufficient amount the primary surplus ratio such a control can be obtained, but the unstable nature of the equilibrium debt ratio cannot be modified by them. The latter modification can be achieved if the debt ratio is controlled by the management of the rate of interest and/or the trend rate of growth of output, provided such management is feasible.

As already pointed out above, the type of static analysis undertaken limits our ability to discuss the economic effects and relative merits of different stylized versions of fiscal policy rules (and conventions) adopted by governments in practice, particularly in the case of the analysis in which the rate of interest has a value above the one of the trend rate of growth of output. Thus, the next step in our research program requires the adoption of a full dynamic (disequilibrium) approach to analyze more concrete fiscal policy issues in an appropriate way. Our analysis was also purposefully simplified in other dimensions. Therefore, it follows that some further research along the lines of the model seems to be necessary in order to investigate the effects of the introduction of some omitted variables over the conclusions so far obtained from the analysis of the model. This is particularly the case of the effects related to the introduction of the tax rate on interest income, indirect taxation, social transfers, and public investment. This will be the task for future research.

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