

# Firm Beliefs and Long-run Demand Effects in a Labor-Constrained Model of Growth and Distribution

Daniele Tavani\*, Luke Petach†

May 24, 2019

## Abstract

One of the most debated questions in alternative macroeconomics regards whether demand policies have permanent or merely transitory effects. While demand matters in the long run in (neo-) Kaleckian economics, both economists operating within other Keynesian traditions (e.g. Skott, 1989) as well as Classical economists (Duménil and Lévy, 1999) argue that in the long-run output growth is constrained by an exogenous, ‘natural’ rate. This paper attempts to bridge the gap by analyzing the role of firm beliefs about the state of the economy in a labor-constrained growth and distribution model based on Kaldor (1956) and Goodwin (1967). The main innovation is the inclusion of beliefs about economic activity in an explicitly dynamic choice of capacity utilization at the firm level. We show that: (i) the relevance of such beliefs generates an inefficiently low utilization rate and labor share in equilibrium; but (ii) the efficient utilization rate can be implemented through fiscal policy. Under exogenous technical change, (iii) the inefficiency does not affect the equilibrium employment rate and growth rate, but expansionary fiscal policy has positive level effects on both GDP and the labor share. However, (iv) with endogenous technical change à la Verdoorn (1949), fiscal policy has also temporary growth effects. Finally, (v) the fact that the choice of utilization responds to income shares has a stabilizing effect on growth cycles, even under exogenous technical change, that is analogous to factor substitution.

---

**JEL Codes:** D25, E12, E22, E25, E62.

**Keywords:** Beliefs, Capacity Utilization, Factor Shares, Growth Cycles.

---

\*Corresponding Author. Department of Economics, Colorado State University. 1771 Campus Delivery, Fort Collins, CO 80523-1771. Email: Daniele.Tavani@Colostate.edu.

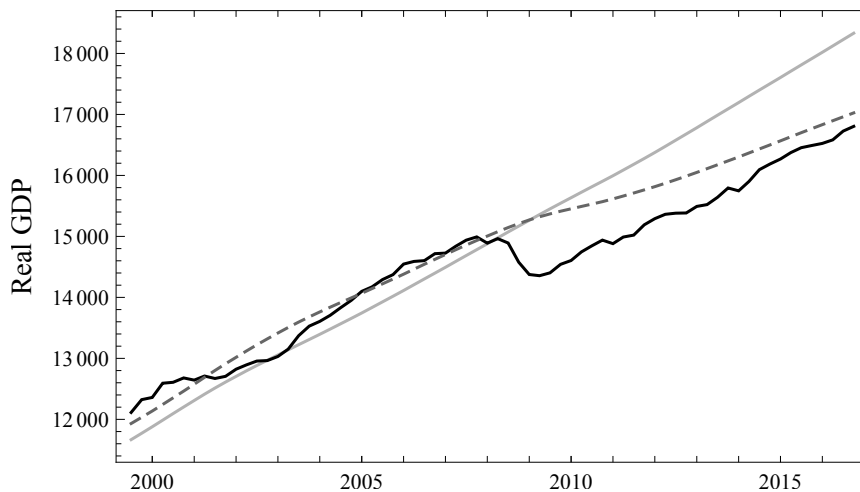
†Jack C. Massey College of Business, Belmont University. Email: Luke.Petach@Belmont.edu

# 1 Introduction

One of the most debated questions in alternative macroeconomics regards whether demand policies have permanent or merely transitory effects. While Kaleckian economists have argued that demand matters even in the long run, both economists operating within other Keynesian traditions—such as Harrodians (e.g. Skott, 1989, 2010, 2012, 2017)—as well as Classical economists (Duménil and Lévy, 1999) argue that long-run output growth is constrained by the so-called natural rate. Moreover, it is argued, the long-run rate of utilization of installed capacity, which proxies for effective demand in these frameworks, must necessarily be exogenous. With fully adjusted, exogenous utilization and growth fixed at the natural rate, many of the central results of canonical Kaleckian models appear to be in jeopardy: the paradox of thrift, the paradox of costs, and the relevance of effective demand in the long-run may no longer hold. The latter is especially concerning given its centrality for (post-) Keynesian economics.

This is not just an academic dispute: considerations about lackluster aggregate demand have been especially relevant in the aftermath of the Great Recession, and the sluggish recovery that followed in most of the world’s advanced economies. Figure 1 plots real GDP in the United States against the Congressional Budget Office estimates of potential output in 2007 and the revised CBO estimates in 2017. The downward revision of potential output points to the presence of considerable slack capacity in the US economy over a decade after the Great Recession ended (Fatas, 2019).

Figure 1: US Real GDP (billions of chained US dollars, black) vs. the CBO estimates of potential GDP in 2007 (gray) and the CBO estimates of potential GDP in 2017 (dashed). Source: Congressional Budget Office.



Such observations point to the question: if one takes the objections of the Harrodian

and Classical perspectives seriously, is it necessary to abandon hope for the possibility of a meaningful role for effective demand in the medium-to-long run? Our answer is no. While the Classical demarcation limiting the effectiveness of demand policy to the short-run—which has led Duménil and Lévy (1999) to a description of the world as “short-run Keynesian, long-run Classical”—may be analytically useful, it is unlikely to hold true in actually existing capitalist economies. Michl (2017), then, points out that a key task for alternative economic theories is to engage in what theologians call “irenic,” or the process of reconciling conflicting doctrines, in order to paint a more realistic picture of the space capitalist economies actually inhabit—a space which undoubtedly combines elements of both the Classical and Keynesian visions (p.74).

Our task in this paper is thus to develop a micro-to-macro, fully Classical model featuring an explicit dynamic choice of utilization at the firm level, in order to illustrate a mechanism that may give rise to lackluster economic performance as an equilibrium outcome. Our argument is that if firms formulate beliefs about the state of the economy when selecting their own level of capacity utilization, then demand policies can certainly affect the level of economic activity in the long-run—despite growth being fixed at the natural rate. The inclusion of beliefs in our model harkens back to the central role that expectations play in the process of employment determination envisioned by Keynes (1936) in *The General Theory*: “[T]oday’s employment can be correctly described as being governed by today’s expectations taken in conjunction with today’s capital equipment” (p.50). Despite the central role afforded to the process of expectations-formation by Keynes (1936), relatively little effort has been put forth in formalizing beliefs in (post-) Keynesian models with an explicit microeconomic structure. Borrowing insights from the literature on coordination games (Cooper, 1999), we will show that firm-level beliefs about economic activity can be captured in a simple way by including (expectations about) the aggregate utilization rate in the economy as a signal for the current level of effective demand. To gain intuition, consider the following scenario. Suppose an individual firm, operating at its desired utilization, expects other firms to increase production because the economy is picking up steam. If such beliefs about other firms’ behavior did not have an effect on the firm’s own choice, then it must be true that the firm has already maximized its profits and has no incentive to deviate from the corresponding desired rate of utilization. But this cannot be the case, because the firm can in fact increase its profits by simply utilizing more its capacity in light of its beliefs about the economy. Thus, it must be that beliefs enter the choice of utilization at the firm level.

Following Petach and Tavani (2019), we operationalize this assumption extending the Keynesian notion of the user cost of capital—the opportunity cost incurred by firms when-

ever they choose to undertake production—to a more general adjustment cost of operating equipment. First, the concept of user cost as outlined by Keynes (1936) is forward-looking: “[I]t is the expected sacrifice of *future* benefit involved in present use which determines the amount of user cost, and it is the *marginal* amount of this sacrifice which... determines his (*sic*) scale of production” (p.70). In other words, the user-cost of capital does not merely depend on the firm’s current own-rate of utilization, but intertemporal considerations are at play when the choice of utilization is made. Thus, our rendition of the user cost has an individual component and it is increasing and convex in the firm’s own utilization rate and is embedded in a forward-looking optimization problem. Greenwood *et al.* (1988) have already argued that this individual component captures in a simple but faithful way the firm-level trade-offs pointed out by Keynes.

Second, we argue that the cost of utilizing equipment also entails an *outward-looking* component, which depends on the individual firm’s beliefs about the behavior of the aggregate economy, and motivates our extension of the user cost into a more general adjustment cost function. Thus, we introduce a dependence of the cost of utilization on the aggregate utilization rate in the firm’s sector—or in the economy in the representative firm case—in order to capture the individual firm’s incentives to adjust its production given changes in expectations. Specifically, we propose a specification of the adjustment cost function where the *marginal* adjustment cost is declining in the average rate of utilization in the economy. Under empirically-sound restrictions (see the results in Petach and Tavani , 2019), this formalization delivers strategic complementarities among firms: the desired rate of utilization at the firm level increases in aggregate utilization: it is endogenous to demand, in other words. This feature will produce an equilibrium utilization rate, and therefore an equilibrium level of economic activity, that is inefficiently low: firms will keep some spare capacity in order to react to the possibility of increased aggregate demand in the economy, while at the same time engaging in the accumulation process.

The result can then be used in order to draw policy implications. With exogenous technical change—the most common specification of technological progress in Harroddian models—the economy’s growth rate is exogenously fixed at the natural rate  $n$ . Accordingly, there are no growth effects of demand policies. However, underutilization implies that spending policies have positive *level* effects, both on output per worker and the share of labor. The latter will increase in response to spending policies that boost economic activity. Thus, the conclusions of this analysis are that demand matters even in the long run, both for the level of economic activity and the labor share.

In addition to analyzing the effects of demand policies on an economy’s balanced growth path, we also consider the implications of incorporating adjustment costs and a strategic

choice of capacity utilization for the Goodwin (1967) model, whose steady state is virtually identical to Kaldor (1956) but whose focus is on the endless distributive cycle between the employment rate and the labor share. The literature has identified four main channels through which the perpetual cycle can be resolved in the long run: (i) capital/labor substitution along a neoclassical production function (van der Ploeg, 1985); (ii) an endogenous labor-augmenting bias of technical change (Shah and Desai, 1981; Foley, 2003; Julius, 2005); (iii) endogenous labor-augmenting innovation financed out of profits (Tavani and Zamparelli, 2015), and (iv) differential tax rates on capital versus labor incomes in models with public infrastructure (Tavani and Zamparelli, 2018). In all of these cases, the symmetry in the bargaining positions between capital and labor, which is responsible for the endless Goodwin growth cycle (Shah and Desai, 1981), is broken in favor of the former. We show in this paper that the choice of utilization implied by our model acts in the same way, namely turning the resulting steady state from the Goodwin center to a stable focus. Essentially, the possibility of varying the rate of utilization is analogous to changing the technique of production in response to factor shares, which is at the heart of the stabilizing channels discussed in the literature.

Finally, we also analyze an extension of the model featuring endogenous technical change in the form of a Verdoorn law (Verdoorn, 1949): the growth rate of labor productivity depends on the accumulation rate. However, as it is well-known in the literature, a Verdoorn law delivers a *semi-endogenous* long-run growth rate: it is derived within the model but is constrained by exogenous parameters such as the growth rate of the labor force and the Verdoorn pass-through (see the discussion in Tavani and Zamparelli, 2017). Differently from the exogenous technical change model, however, expansionary fiscal policy that fosters utilization will have a temporary growth effect in addition to its permanent level effect. Therefore, this version of the model shows that demand policies translate meaningfully into changes in the growth rate via their impact on the accumulation rate along the transition to balanced growth. Thus, the paper offers one explanation for why temporary demand shocks have permanent level effects under exogenous growth, or temporary growth effects under endogenous labor-augmenting technical change, and offers an additional contribution to the analysis of hysteresis in Classical and Keynesian growth models by Setterfield (2018); Michl and Oliver (2019).

## 2 Basic Elements of the Model

Consider a competitive capitalist firm in a closed economy, and assume away the government for now. Let the firm's production possibilities be summarized by the Leontief

technology  $Y = \min\{uK, AL\}$  where:  $Y$  is the firm's output, homogeneous with capital stock  $K$  so that we can normalize its price to one;  $u$  denotes the rate of capacity utilization;  $L$  stands for labor;  $A$  is the current stock of labor-augmenting technologies, assumed to grow at the constant rate  $g_A > 0$  in the benchmark model, so that the analysis is as close as possible to baseline Harrodian models such as Skott (1989); and the long-run output/capital ratio  $B$  is normalized to one for simplicity. Time is continuous. Maximizing profits requires to set  $uK = AL$ , which solves for labor demand  $L = uK/A$ . If the firm pays the same real wage  $w$  to each worker, the share of wages in output will be equal to the unit labor cost:  $\omega \equiv w/A$ . Further, let the labor force  $N \geq L$  grow at the constant rate  $n > 0$ . Thus, the employment rate in the economy  $e$  will be equal to  $L/N = uK/(AN)$ .

Our main hypothesis is that operating capital equipment entails an *adjustment cost*  $\lambda$ , which would not be incurred if machinery remained idle (Keynes, 1936). First, and to capture the user cost, we assume that the adjustment cost increases more than proportionally with the firm's own utilization rate  $u$ : denoting partial derivatives by subscripts,  $\lambda_u > 0, \lambda_{uu} > 0$ . Greenwood *et al.* (1988) have noted that this specification formalizes the Keynesian effects played by the 'marginal efficiency of investment' and the Keynesian notion of user cost for the individual firm. Second, we capture the strategic nature of the choice of utilization by postulating that the adjustment cost also responds to the firm's beliefs about the utilization chosen by other firms as captured by  $\tilde{u}$ , the average utilization rate. We assume that  $\tilde{u}$  has a negative impact on the *marginal* adjustment cost:  $\lambda_{u\tilde{u}} < 0$ . This assumption implies that the marginal benefit of increasing own utilization increases in the firm's beliefs about aggregate utilization, and is required to generate a strategic complementarity, which is central in our contribution. Petach and Tavani (2019) have offered strong and robust empirical support for this assumption using a panel of state-by-sector data for the United States. To sharpen our conclusions, we specify a log-linear user cost function as in Petach and Tavani (2019):

$$\lambda(u; \tilde{u}) = \beta u^{\frac{1}{\beta}} \tilde{u}^{-\frac{\gamma}{\beta}}, \beta \in (0, 1), \gamma \in [0, 1 - \beta) \quad (1)$$

The size of the parameter  $\gamma$  determines the extent to which beliefs about other firms' behavior are relevant for the firm choice. The special case  $\gamma = 0$  corresponds to the isolated firms case where the choices made by other firms are irrelevant, while  $\gamma \neq 0$  implies a strategic environment in which beliefs matter. Further, under  $0 < \gamma < 1 - \beta$ , the choice of utilization generates a (weak) strategic complementarity, which will result in a unique non-trivial long-run equilibrium utilization in the model. The estimates in Petach and Tavani (2019) strongly support the above parametric restriction that  $0 < \gamma < 1 - \beta$ .

In line with the basic Classical and Post Keynesian literature, we assume that only profit-earning (capitalist) households save in order to accumulate capital stock. However, while in most of the literature capitalists save a constant fraction of their profit incomes at all times, we follow Foley, Michl and Tavani (2019) in assuming that capitalist households are forward-looking in their consumption, accumulation, and utilization decisions. This assumption is made here in order to rule out any potential ‘inefficiency’ result implied by a limited planning horizon, or by ‘rule of thumb’ behavior such as saving the same fraction of income at all times.<sup>1</sup> Here, the capitalist household discounts the future at a constant rate  $\rho > 0$ , derives instantaneous logarithmic utility from its per-period consumption flow, denoted by  $c(t)$ , and has perfect foresight on the entire planning horizon  $t \in [s, \infty)$ . Finally, assume that there is no independent investment demand function, so that capitalist savings are immediately invested at all times. The accumulation constraint, omitting the time-dependence for notational simplicity, is:

$$\dot{K} = (1 - \omega)uK - c - \lambda(u; \tilde{u})K \quad (2)$$

Note that increasing the own rate of utilization raises the capitalist’s revenues  $(1 - \omega)uK$ , but also increases the adjustment cost  $\lambda(u; \tilde{u})$  given average utilization. As shown in Appendix A, the solution to a simple control problem delivers the firm-level choice of capacity utilization as a decreasing function of the labor share while increasing in average utilization as per equation (3) below. The firm-level choice of utilization is equivalent to a best-response function in the game-theoretic sense, or a reaction function in Cournot-style models.

$$u(\omega; \tilde{u}) = (1 - \omega)^{\frac{\beta}{1-\beta}} \tilde{u}^{\frac{\gamma}{1-\beta}} \quad (3)$$

Figure 2 displays the behavior of the best-response function in relation to the aggregate utilization rate. The intuition for the inverse dependence on the labor share is that an increase in the latter reduces revenues everything else equal: the firm can then cut back on its utilization in order to reduce the user cost of capital. On the other hand, an increase in aggregate utilization reduces the own marginal adjustment cost everything else equal, thus rising the marginal profits that can be made by utilizing more a firm’s own installed capacity: the firm can increase utilization up to the point where the marginal benefit of doing so—given by  $(1 - \omega)K$ —is equal to the marginal user cost  $\lambda_u(u; \tilde{u})K$  for a given average utilization rate. Finally, given that the firm’s beliefs about aggregate utilization represent expectations about the overall economic activity in the economy, the dependence

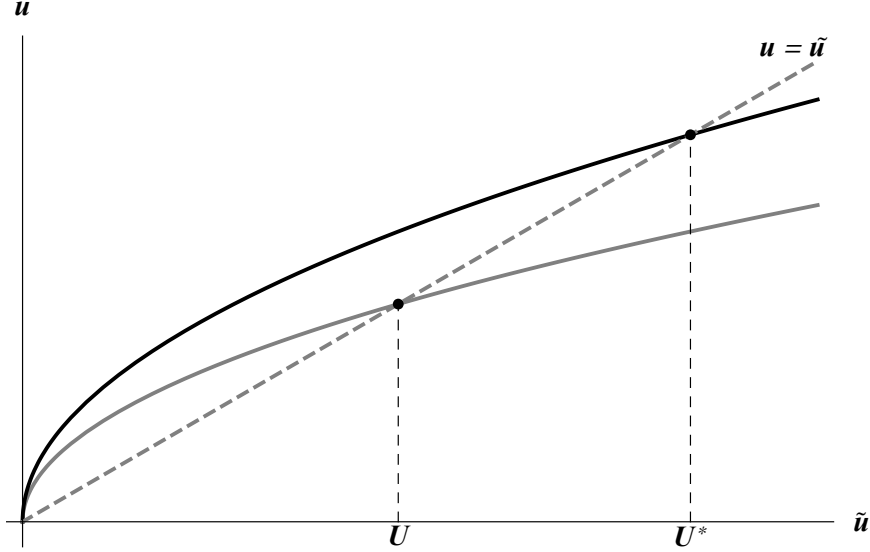
---

<sup>1</sup>Similar reasoning applies to the assumption of competitive firms: the results of the model do not depend on slow price adjustments, as Skott (2017) has argued to be the case in neo-Kaleckian growth models.

on average utilization can be thought of as capturing the endogeneity of the firm’s desired rate of utilization to demand. Appendix A also shows that the growth rate of consumption for the typical capitalist household—which describes the household-level saving rule—satisfies:

$$g_c \equiv \frac{\dot{c}}{c} = (1 - \omega)^{\frac{1}{1-\beta}} \tilde{u}^{\frac{\gamma}{1-\beta}} - \rho \quad (4)$$

Figure 2: Equilibrium vs. efficient utilization rates.



### 3 Equilibrium Utilization and Accumulation

An equilibrium growth path is a sequence of quantities  $\{c(t), u(t), \tilde{u}(t), k(t)\}_{t \in [s, \infty)}$  that solve the capitalist’s control problem given the path for the labor share  $\{\omega(t)\}_{t \in [s, \infty)}$  and such that  $u(t) = \tilde{u}(t)$  for all  $t$ . The latter requirement requires firms to best-respond to other firms’ choices, similarly to the notion of a Nash equilibrium. Omitting the time notation for simplicity, the equilibrium rate of utilization is easily found as

$$u(\omega) = (1 - \omega)^{\frac{\beta}{1-\beta-\gamma}} \quad (5)$$

and is inversely related to the labor share, since  $1 - \beta - \gamma > 0$  by assumption. The intuition is simple: a higher wage share lowers the firm’s profits and thus the resources available for accumulation. But the firm can offset the higher wage costs by utilizing less its plants, thus reducing the user cost.

We can then obtain the equilibrium growth rate by using the condition  $u = \tilde{u}$  and imposing balanced growth so that consumption and capital stock grow at the same rate:



$g_c = g_K = g$ . We have:

$$g = (1 - \beta)(1 - \omega)^{\frac{1-\gamma}{1-\beta-\gamma}} - \rho \quad (6)$$

where the first term in the right-hand-side of (6) is the profit rate net of the adjustment cost. Further, because of the equilibrium condition  $u = \tilde{u}$ , the equilibrium adjustment cost  $\lambda(u; u)$  reduces to the pure user cost of capital, i.e. depreciation.

## 4 The Dynamical System

First, consider the economy's share of labor  $\omega = w/A$ . Its growth rate will be given by the difference between the growth rate of the real wage and the growth rate of labor productivity. Following Goodwin (1967), assume that real wages obey a version of the Phillips curve  $\dot{w}/w = f(e)$  such that  $f_e > 0, f_{ee} > 0$ . In particular, we impose  $f(e) = \eta e^{1/\delta}, \eta > 0, \delta \in (0, 1)$ . Hence, the labor share evolves according to:

$$\frac{\dot{\omega}}{\omega} = f(e) - g_A \quad (7)$$

Next, consider the employment rate  $e = uK/(AN)$ . Logarithmic differentiation gives:

$$\frac{\dot{e}}{e} = g_K + g_u - (g_A + n)$$

Differentiating (5), we obtain the growth rate of utilization as:

$$\frac{\dot{u}}{u} \equiv g_u = -\frac{\beta}{1 - \beta - \gamma} \left( \frac{\omega}{1 - \omega} \right) \frac{\dot{\omega}}{\omega} \quad (8)$$

so that, using the real-wage Phillips curve (7) and the accumulation rate from (6), we find:

$$\dot{e} = \left\{ (1 - \beta)(1 - \omega)^{\frac{1-\gamma}{1-\beta-\gamma}} - \frac{\beta}{1 - \beta - \gamma} \left( \frac{\omega}{1 - \omega} \right) [f(e) - g_A] - (\rho + g_A + n) \right\} e \quad (9)$$

### 4.1 Steady State

A steady state is a pair  $(\omega_{ss}, e_{ss})$  so that  $\dot{\omega} = \dot{e} = 0$ . Notice that the steady state value of utilization is uniquely pinned down by the steady state labor share, given the equilibrium condition  $u = \tilde{u}$  that delivers equation (5) above. The steady state value of the employment rate is readily found by setting  $\dot{\omega} = 0$  in (7):

$$e_{ss} = \left( \frac{g_A}{\eta} \right)^\delta \quad (E)$$

The steady state employment rate is fully exogenous because of the exogenous nature of technological change in the model. On the other hand, the steady state value for the labor share can be solved for in closed form once noting that  $f(e_{ss}) = g_A$  in setting the right-hand side of equation (9) equal to zero. By so doing, we find:

$$1 - \omega_{ss} = \left( \frac{\rho + n + g_A}{1 - \beta} \right)^{\frac{1-\beta-\gamma}{1-\gamma}} \quad (\Omega)$$

We can now state a first result of this analysis, namely the dependence of steady state utilization on the capitalist saving rate, as a proposition.

**Proposition 1** *The steady state desired utilization rate  $u_{ss}$  increases in the discount rate  $\rho$ . Since the latter reduces savings in the economy, the paradox of thrift holds in this model.*

To prove this result, use (5) to solve for the steady state utilization rate as:

$$u_{ss} = \left( \frac{\rho + n + g_A}{1 - \beta} \right)^{\frac{\beta}{1-\gamma}} \quad (10)$$

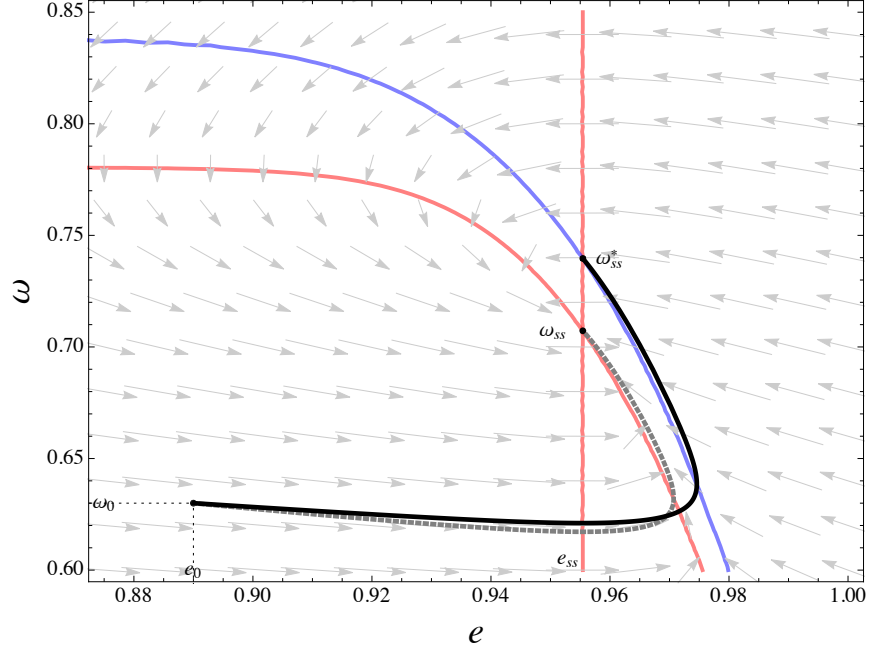
and  $\partial u_{ss}/\partial \rho > 0$ . The result is remarkable, because even without an explicit investment function, this model delivers a version of the Keynesian paradox of thrift. A higher value of the discount rate  $\rho$  reduces the long-run capitalists' saving: the corresponding increase in long-run aggregate consumption can be met by increasing utilization and therefore the steady-state level of economic activity.

## 4.2 Stability Analysis

Appendix C shows that, unlike the original Goodwin (1967) long-run, the steady state of this model is locally stable. Thus, the explicit choice of capacity utilization is an additional way of breaking the symmetric bargaining positions between capital and labor, resolving the distributive conflict in the long run (Shah and Desai, 1981; van der Ploeg, 1985). Figure 3 shows the  $\dot{\omega} = 0$  nullcline, which is vertical corresponding to the steady state employment rate  $e_{ss}$  and is the same at both the equilibrium and the efficient solution; and plots the  $\dot{e} = 0$  nullcline in red. The latter is downward sloping, and so will be the nullcline corresponding to the efficient growth path, as differentiation of the expression in braces in (9) and (13) will show. Notice also that convergence to the steady state is not monotonic: the employment rate, which is the forward-looking variable in this model, overshoots before converging to its long-run value. In this respect, explicit demand shocks (see below) will determine some short-run cyclical behavior of the system before a new steady state is achieved. A

time-series plot is displayed in Figure 4.

Figure 3: Phase Diagram: equilibrium nullclines (red) and efficient  $\dot{e} = 0$  nullcline (blue), and dynamic trajectories converging to the equilibrium (dotted gray) as opposed to the efficient (solid black) steady state. The vector field plot refers to the equilibrium path.



## 5 Welfare and Policy

An interesting feature of this economy is that so long as a (weak) strategic complementarity is present—that is so long as  $0 < \gamma < 1 - \beta$ —it will generally operate with excess capacity in equilibrium, while at the same time accumulating capital stock. To see this, let us introduce a benevolent planner solving the accumulation problem under the additional constraint that  $u = \tilde{u}$  at all times. Appendix B proves the following result.

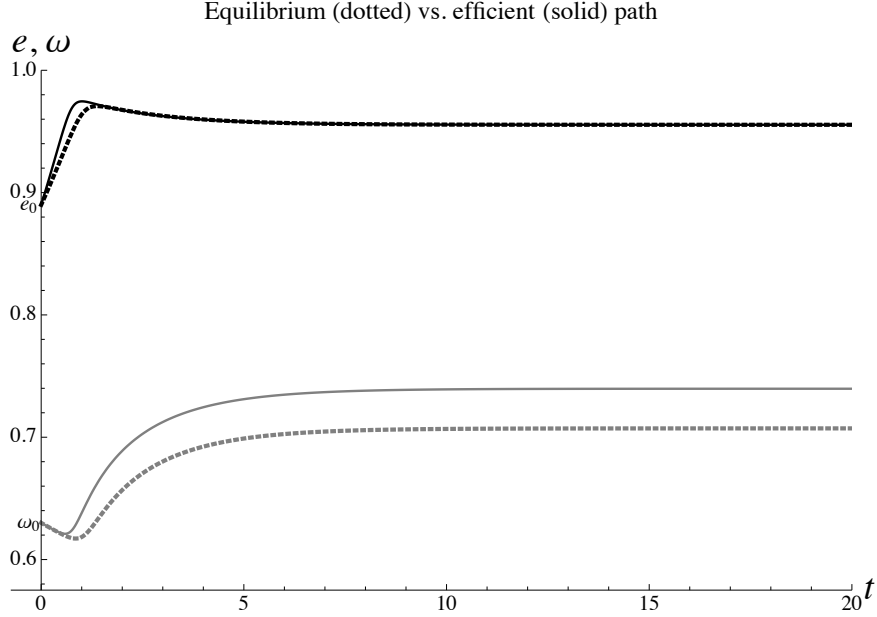
**Proposition 2** *Let  $\gamma \in (0, 1 - \beta)$ . Then, the efficient choice of utilization is:*

$$u^*(\omega) = \left( \frac{1 - \omega}{1 - \gamma} \right)^{\frac{\beta}{1 - \beta - \gamma}} \quad (11)$$

*and is strictly greater than its decentralized counterpart (5).*

Notice that the parameter  $\gamma$  generates a multiplier effect: the larger the extent to which firm beliefs matter, the higher the socially-coordinated utilization rate relative to the equilibrium rate. Absent a role for beliefs,  $\gamma = 0$ : the equilibrium rate and the socially-coordinated rate

Figure 4: Equilibrium vs. efficient trajectories for the employment rate and the labor share.



coincide. This result is important for two reasons: first, as long as firms operate within a strategic context, they will find it profit-maximizing to keep accumulating capital stock even though their capacity is not fully utilized. Second, the result shows that the individual component of the user cost function is not sufficient to generate underutilization: the presence of a strategic environment where firms' decisions are affected by other firms' choices is necessary for the economy to operate below full capacity in equilibrium. Figure 2 shows the equilibrium as opposed to the efficient utilization rate in this model. Using (11), we can then solve for the efficient accumulation rate as

$$g^* = \left( \frac{1 - \omega}{1 - \gamma} \right)^{\frac{1-\gamma}{1-\beta-\gamma}} (1 - \beta - \gamma) - \rho \quad (12)$$

which can then be used in order to track the evolution of employment along the efficient growth path. The law of motion for the labor share does not change: therefore, the efficient steady state value for the employment rate is still given by equation (E) above. Instead, the evolution of employment over time modifies to:<sup>2</sup>

$$\frac{\dot{e}}{e} = (1 - \beta - \gamma) \left( \frac{1 - \omega}{1 - \gamma} \right)^{\frac{1-\gamma}{1-\beta-\gamma}} - \frac{\beta}{1 - \beta - \gamma} \left( \frac{\omega}{1 - \omega} \right) [f(e) - g_A] - (\rho + g_A + n) \quad (13)$$

<sup>2</sup>Interestingly, the growth rate of utilization obtained log-differentiating (11) is identical to the corresponding equilibrium growth rate as per equation (8)

Proceeding as before, then, we find the steady state labor share at the efficient growth path as the solution to:

$$1 - \omega_{ss}^* = (1 - \gamma) \left( \frac{\rho + g_A + n}{1 - \beta - \gamma} \right)^{\frac{1 - \beta - \gamma}{1 - \gamma}} \quad (\Omega^*)$$

The Appendix, then, proves the following result.

**Proposition 3** *Let  $1 - \beta > \gamma > 0$ . Then,  $\omega_{ss}^* > \omega_{ss}$ .*

Petach and Tavani (2019) have estimated equation (3) through a series of empirical models using state-by-sector US data with the aim of assessing the extent of strategic complementarities. Those results provide strong evidence regarding the empirical bite of our argument. The point estimates for the main parameters of interest,  $\gamma$  and  $\beta$ , are highly significant and robust to different model specifications, and fully in line with the theoretical restrictions needed for Proposition 3 to hold. Thus, the analysis provides strong reasons to believe that demand policies can be used in order to improve the workers' distributional position in the economy. We carry a simple exercise to this aim in the next Section.

## 5.1 Decentralization

Let us introduce a government authority that subsidizes the user cost at a rate  $s$ , and—for simplicity—taxes capitalist income lump-sum by an amount  $\tau$  while running a balanced budget:  $\tau = s\lambda(\cdot)K$  at all times.<sup>3</sup> The capitalist households' budget constraint modifies as follows:

$$\dot{K} = (1 - \omega)uK - \tau - c - \lambda(u, \tilde{u})K(1 - s) \quad (14)$$

Solving the corresponding optimal control problem, we have the following result, proven in the Appendix.

**Proposition 4** *(i) The subsidy that decentralizes the efficient utilization rate and labor share is equal to  $\gamma$ , which governs the extent of strategic complementarities. (ii) The aggregate response to an increase in  $s$  is always greater than the individual firm's response. (iii) The resulting 'fiscal multiplier' is equal to  $\frac{1}{1 - \frac{\gamma}{1 - \beta}}$ .*

## 6 Extension: Endogenous Labor Productivity Growth

The reason why the baseline model delivers only level effects of demand policies has to be found in the assumption of exogenous technical change. Since the growth rate of labor pro-

---

<sup>3</sup>Setting  $\tau = 0$  amount to impose a deficit-financed user cost subsidy.

ductivity is fixed at its exogenous rate  $g_A$ , there are necessarily no growth effects of spending policies aimed at implementing the efficient rate of utilization. However, economists working within alternative traditions (Cornwall and Setterfield, 2002; Naastepad, 2006; Rada, 2007; Taylor et al., 2018) have long emphasized the possibility of demand shocks affecting labor productivity growth: here, we show that a simple extension of the model delivers transitory effects of demand shocks on the growth rate of labor productivity in addition to its permanent level effects. In line with the contributions mentioned above, assume that labor productivity growth follows a version of Verdoorn’s law (Verdoorn, 1949):<sup>4</sup>

$$g_A = \phi g_K, \phi \in (0, 1) \quad (15)$$

While labor productivity growth is endogenous, its long run value is anchored by the balanced growth condition  $g_A + n = g_K$ , so that, in the long run, we end up with a *semi-endogenous* growth rate: the growth rate is determined within the model but independent of policy (Jones, 1995, 1999). In fact, we have:

$$g_K = \frac{n}{1 - \phi}; \quad g_A = \frac{\phi n}{1 - \phi} \quad (16)$$

Importantly however, Verdoorn’s law creates transitory effects of demand policy on labor productivity growth. Both the efficient and the equilibrium accumulation paths converge to the same growth rate in (16); but the efficient path involves a growth rate of labor productivity that converges *from above* relative to the equilibrium path, as shown in Figure 5.<sup>5</sup> This result points to the additional welfare effects of demand policies—given by the integral of the distance between the two curves—along the transition to balanced growth. Thus, even if it only impacts the level of output in the long-run, demand policy may nonetheless have permanent positive effects on social welfare via its impact on productivity growth during the transition to the steady-state.

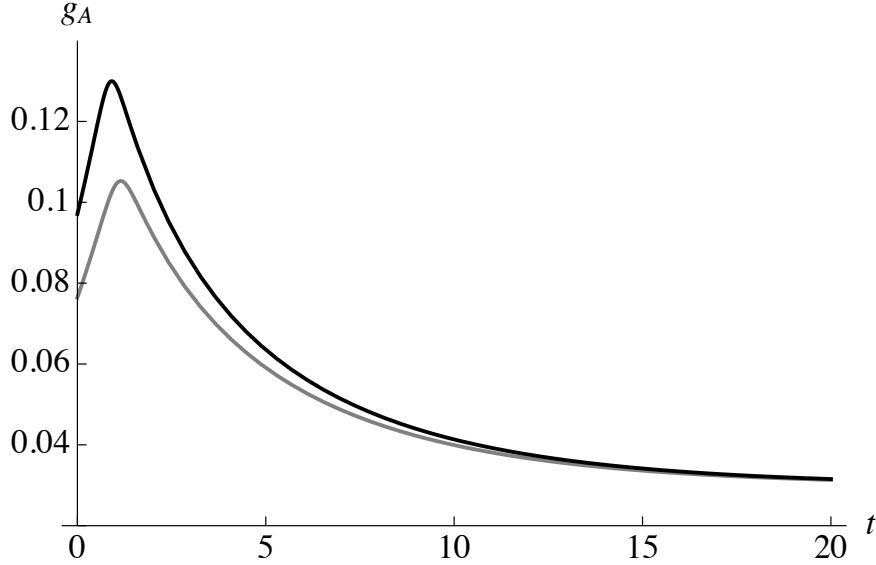
For completeness, let us evaluate the steady state of the extended model. Incorporating equation (15) into the dynamical system (7)-(9) and solving for the steady state income

---

<sup>4</sup>In the conventional AK-growth literature, the stock of labor-augmenting technologies depends on the aggregate capital stock:  $A = \zeta K^\phi, \zeta > 0, \phi \in (0, 1)$  through a learning-by-doing externality. Log-differentiation of this expression gives equation (15). See Romer (1987).

<sup>5</sup>The figure uses the same parameter calibration as the baseline model, plus a value of  $\phi$  that ensures a long-run growth rate of 3%. It is displayed for illustrative purposes, and it is not meant to argue that demand policies can boost labor productivity growth up to over 12% as a literal interpretation of the figure would suggest.

Figure 5: Equilibrium (gray) vs. efficient (black) path of labor productivity growth, given an initial condition on the labor share, with a Verdoorn effect.



shares and employment rate corresponding to an equilibrium path gives:

$$1 - \omega_{ss} = \left[ \frac{n + \rho(1 - \phi)}{(1 - \beta)(1 - \phi)} \right]^{\frac{1 - \beta - \gamma}{1 - \gamma}} \quad (17)$$

$$e_{ss} = \left[ \frac{\phi n}{\eta(1 - \phi)} \right]^{\delta} \quad (18)$$

The efficient path solves for the same long-run employment rate as (18), but delivers the following long-run distribution of income:

$$1 - \omega_{ss}^* = (1 - \gamma) \left[ \frac{n + \rho(1 - \phi)}{(1 - \beta - \gamma)(1 - \phi)} \right]^{\frac{1 - \beta - \gamma}{1 - \gamma}} \quad (19)$$

and Proposition 3 holds in this version of the model, too.<sup>6</sup>

## 7 Conclusion

In this paper we have offered one potential resolution to what Michl (2017) calls the “discord in the marriage of Classical and Keynesian economics that defines modern heterodox macroeconomics” (p.77). Namely, the discord caused by conflicting claims about the relationship of effective demand to long-run growth. Our solution relies on the introduction

<sup>6</sup>The proof goes along the same lines as Proposition 3.

of *beliefs* into the firm's choice of utilization in an otherwise Classical long-run model of growth and distribution. Firms' beliefs are modeled via an adjustment cost function, and in particular a declining marginal adjustment cost in the economy-wide rate of utilization. Under reasonable and empirically-supported restrictions on parameter values, we showed that this specification is sufficient to generate a desired utilization rate at the firm level that increases in aggregate utilization. As a result, we are able to carve out a role for demand policy in our model. Even though growth is fixed at the natural rate, spending policies will always have level effects on long-run output because equilibrium utilization is below the efficient level. We then provided a dynamic extension of the model to study the implications of a firm-level choice of utilization in the Goodwin (1967) growth cycle: the choice of utilization resolves the distributive conflict in favor of the capitalist class, altering the steady-state of the Goodwin model into a stable focus. However, the model exhibits overshooting of the employment rate before it reaches equilibrium, consistent with Keynes (1936). Finally, we have shown that with a Verdoorn (1949) effect on labor productivity growth, demand shocks can temporarily impact the rate of growth via their impact on the accumulation rate.

As a final observation, the focus of this paper is on the effectiveness of fiscal policy in labor-constrained economies. Our analysis points to the relevance of coordination failures in devising a role for spending policies that will increase economic activity and the labor share, despite equilibrium utilization being *profit-led* in the usual Post-Keynesian jargon. Even though our model is purely supply-side, we showed that: (i) the paradox of thrift holds; (ii) spending policies have generally multiplier effects, and (iii) are generally labor-friendly. Our conclusions will only be reinforced in an analysis that includes a demand-driven determination of economic activity.

## A The Capitalists' Optimization Problem

Suppose that the representative capitalist household has logarithmic preferences over consumption streams, and discounts the future at a constant rate  $\rho > 0$ . Then, the household



solves:

Given  $\{\tilde{u}(t), \omega(t)\}_{\forall t}$ ,

$$\begin{aligned} \text{Choose } \{c(t), u(t)\}_{t \in [s, \infty)} \quad & \text{to max } \int_s^\infty \exp\{-\rho(t-s)\} \ln c(t) dt \\ \text{s. t.} \quad & \dot{K}(t) = [1 - \omega(t)]u(t)K(t) - c(t) - \lambda[u(t); \tilde{u}(t)]K(t) \\ & K(s) \equiv K_s > 0, \text{ given} \\ & \lim_{t \rightarrow \infty} \exp\{-\rho(t-s)\}K(t) \geq 0 \end{aligned} \tag{20}$$

Observe first that the problem stated in (20) involves a strictly concave objective function to be maximized over a convex set. Thus, the standard first-order conditions on the associated current-value Hamiltonian

$$\mathcal{H} = \ln c + \mu[u(1 - \omega)K - c - \lambda(u; \tilde{u})K]$$

will be necessary and sufficient for an optimal control. They are:

$$c^{-1} = \mu \tag{21}$$

$$1 - \omega = \lambda_u(u, \tilde{u}) \tag{22}$$

$$\rho\mu - \dot{\mu} = \mu[(1 - \omega)u - \lambda(u; \tilde{u})] \tag{23}$$

$$\lim_{t \rightarrow \infty} \exp\{-\rho t\}\mu(t)k(t) = 0 \tag{24}$$

Solving (22) for the rate of utilization under the specific functional form (1) gives (3). To obtain the Euler equation for consumption, differentiate (21) with respect to time and use (21) and (23) to get:

$$g_c \equiv \frac{\dot{c}}{c} = (1 - \omega)u(\omega; \tilde{u}) - \{\lambda[u(\omega; \tilde{u})] + \rho\}.$$

Using both (3) and (1) while imposing a balanced growth path where consumption and capital stock grow at the same rate gives (4).

## B The Efficient Solution

A benevolent social planner solves the accumulation problem under the additional constraint that  $u = \tilde{u}$  at all times. Accordingly, the control problem (20) is solved under the

modified accumulation constraint

$$\dot{K} = u(1 - \omega)K - c - \beta u^{\frac{1-\gamma}{\beta}} K \quad (25)$$

The first-order condition on consumption is the same as (21) above. On the other hand, the choice of utilization and the costate equation satisfy the first-order conditions which, once again, are necessary and sufficient for an optimal control:

$$1 - \omega = (1 - \gamma)u^{\frac{1-\beta-\gamma}{\beta}} \quad (26)$$

$$\rho - \frac{\dot{\mu}}{\mu} = u(1 - \omega) - \beta u^{\frac{1-\gamma}{\beta}} \quad (27)$$

Solving equation (26) for utilization gives (11). To obtain the efficient accumulation rate (12), simply impose balanced growth ( $g_c = g_K$ ).

## C Stability Analysis

### C.1 Equilibrium Path

The Jacobian Matrix evaluated at a steady state has the following structure:

$$J(e_{ss}, \omega_{ss}) = \begin{bmatrix} -\frac{\beta}{1-\beta-\gamma} \frac{\omega_{ss}}{1-\omega_{ss}} f'(e_{ss}) e_{ss} & -\frac{(1-\beta)(1-\gamma)}{1-\beta-\gamma} (1 - \omega_{ss})^{\frac{\beta}{1-\beta-\gamma}} e_{ss} \\ (-) & (-) \\ f'(e_{ss}) \omega_{ss} & 0 \\ (+) & (0) \end{bmatrix}$$

Thus, it has a negative trace and a positive determinant. It follows that its eigenvalues are of the same sign and sum to a negative number, which can only occur if they both have uniformly negative real parts. We conclude that the steady state is locally stable.

### C.2 Efficient Path

The Jacobian Matrix evaluated at the efficient steady state is:

$$J(e_{ss}, \omega_{ss}^*) = \begin{bmatrix} -\frac{\beta}{1-\beta-\gamma} \frac{\omega_{ss}^*}{1-\omega_{ss}^*} f'(e_{ss}) e_{ss} & -(1 - \gamma) \left( \frac{1-\omega_{ss}^*}{1-\gamma} \right)^{\frac{\beta}{1-\beta-\gamma}} e_{ss} \\ (-) & (-) \\ f'(e_{ss}) \omega_{ss}^* & 0 \\ (+) & (0) \end{bmatrix}$$

again, with negative trace and positive determinant, so that the efficient steady state is locally stable, too.

## D Proofs

**Proposition 1.** Consider that that, using (5) and (11),

$$\frac{u^*}{u} = \left( \frac{1}{1-\gamma} \right)^{\frac{\beta}{1-\beta-\gamma}} > 1$$

since  $0 < \gamma < 1 - \beta$  by assumption.

**Proposition 2.** Showing that  $\omega^* > \omega$  is tantamount to showing that  $\ln(1-\omega) - \ln(1-\omega^*) > 0$ . We have that

$$\begin{aligned} D_\omega &\equiv \ln(1-\omega) - \ln(1-\omega^*) \\ &= \frac{1-\beta-\gamma}{1-\gamma} [\ln(1-\beta-\gamma) - \ln(1-\beta)] - \ln(1-\gamma) \end{aligned}$$

and

$$\frac{\partial D_\omega}{\partial \gamma} = -\frac{\beta}{(1-\gamma)^2} [\ln(1-\beta-\gamma) - \ln(1-\beta)]$$

Hence, the difference  $D_\omega$  increases in  $\gamma$  provided that the term in brackets is negative. This is certainly true under  $0 < \gamma < 1 - \beta$ , since  $\partial \ln(1-\beta-\gamma)/\partial \gamma < 0$ .

**Proposition 3.** First, observe that the first-order necessary condition for the choice of utilization with the tax and subsidy solves for the firm-level utilization as

$$u = \left( \frac{1-\omega}{1-s} \right)^{\frac{\beta}{1-\beta}} \tilde{u}^{\frac{\gamma}{1-\beta}} \quad (28)$$

Imposing the equilibrium condition  $u = \tilde{u}$ , we find

$$u^{subs} = \left( \frac{1-\omega}{1-s} \right)^{\frac{\beta}{1-\beta-\gamma}} \quad (29)$$

The comparison with equation (11) makes it clear that  $s = \gamma$  achieves the efficient utilization rate.

To prove the second claim, differentiate equations (29) and (28) (after taking logs for

simplicity) with respect to the subsidy  $s$  to see that

$$\frac{\partial \ln u^{subs}}{\partial s} = \frac{\beta}{1 - \beta - \gamma} \frac{1}{1 - s} > \frac{\partial \ln u}{\partial s} = \frac{\beta}{1 - \beta} \frac{1}{1 - s} \iff \gamma \in (0, 1 - \beta).$$

The size of the fiscal multiplier  $m$  can be recovered by dividing the aggregate response by the individual response. We have that

$$m = \frac{1 - \beta}{1 - \beta - \gamma} = \frac{1}{1 - \frac{\gamma}{1 - \beta}}$$

### **Conflict of Interest**

The authors declare that they have no conflict of interest.

## **References**

- Auerbach, P., and Skott, P., 1988. ‘Concentration, Competition, and Distribution – A Critique of the Theories of Monopoly Capital.’ *International Review of Applied Economics* 2 (1): 42–61.
- Cooper, R., 1999. *Coordination Games*. Cambridge, UK: Cambridge University Press.
- Cornwall, J. and Setterfield, M. (2002) A neo-Kaldorian perspective on the rise and decline of the golden age. In M. Setterfield (ed.), *The Economics of Demand-Led Growth: Challenging the Supply Side Vision of the Long Run*, pp. 67–86. Cheltenham, UK: Edward Elgar.
- Duménil, G., and Levy, D., 1999. ‘Being Keynesian in the Short Term and Classical in the Long Term: the Traverse to Classical Long-Term Equilibrium.’ *Manchester School* 67 (6): 684–716.
- Fatas, A., 2019. ‘Fiscal Policy, Potential Output and the Shifting Goalposts.’ Paper presented at the Euro 20 academic conference organized by the IMF and Bank of Ireland. Available at: <https://faculty.insead.edu/fatas/euro20.pdf>.
- Foley, D. K., Michl, T. R., and Tavani, D., 2019. *Growth and Distribution, Second Edition* Cambridge, MA: Harvard University Press.
- Foley, D. K., 2003. ‘Endogenous Technical Change with Externalities in a Classical Growth Model.’ *Journal of Economic Behavior and Organization* 52: 167–189.

- Foley, D. K., 2016. 'Keynes' Microeconomics of Output and Labor Markets.' In Bernard, L., and Nyambuu, eds. *Dynamic Modeling, Empirical Macroeconomics, and Finance: Essays in Honor of Willi Semmler*, Springer, pp. 183–194.
- Goodwin, R., 1967. 'A Growth Cycle'. In: *Socialism, capitalism, and economic growth*. Cambridge, UK: Cambridge University Press.
- Greenwood, J., Hercowitz, Z., and Huffman, G., 1988. 'Investment, Capacity utilization, and the Real Business Cycle.' *American Economic Review* 78 (3): 402–417.
- Jones, C. I., 1995. R&D-Based Models of Economic Growth. *Journal of Political Economy* 103 (4): 759–784.
- Jones, C. I., 1999. Growth: With or Without Scale Effects? *American Economic Review (Papers and Proceedings)* 89 (2): 139–144.
- Julius, A. J., 2005. Steady State Growth and Distribution with an Endogenous Direction of Technical Change. *Metroeconomica* 56 (1): 101–125.
- Kaldor, N., 1956. 'Alternative theories of distribution.' *Review of Economic Studies* 23 (2): 83–100.
- Kaldor, N., 1961. 'Capital accumulation and economic growth.' In F. A. Lutz and D. C. Hague (eds.) *The Theory of Capital*. New York, NY: St. Martins Press.
- Kennedy, C. (1964) Induced bias in innovation and the theory of distribution. *Economic Journal* 74 (295): 541–547.
- Keynes, J. M., 1936. *The General Theory of Employment, Interest and Money*. London: MacMillan.
- Michl, T. R., 2009. *Capitalists, Workers, and Fiscal Policy*. Cambridge, MA: Harvard University Press.
- Michl, T. R., 2017. "Profit-led Growth and the Stock Market." *Review of Keynesian Economics*, 5 (1): 61–77.
- Michl, T. R., and Oliver, K. M. 2019. "Combating Hysteresis with Output Targeting." *Review of Keynesian Economics*, 7 (1): 6–27.
- Naastepad, C.W.M. (2006) Technology, demand and distribution: A cumulative growth model with an application to the Dutch productivity growth slowdown. *Cambridge Journal of Economics* 30: 403–434.

- Petach, L., and Tavani, D., 2019. 'No one is Alone: Strategic Complementarities, Capacity Utilization, Growth, and Income Distribution' (April 17, 2019) Available at SSRN: <https://ssrn.com/abstract=3123149>.
- van der Ploeg, R., 1985. Classical Growth Cycles. *Metroeconomica* 37 (2): 221-230.
- Rada, C. (2007), Stagnation or transformation of a dual economy through endogenous productivity growth. *Cambridge Journal of Economics*, 31: 711–740.
- Romer, P. (1987). Growth Based on Increasing Returns Due to Specialization. *American Economic Review* 77: 56–62.
- Setterfield, M. 2018. "Long-run Variation in Capacity Utilization in the Presence of a Fixed Normal Rate." *Cambridge Journal of Economics*, 43 (2): 443–463.
- Shah, and Desai, M., 1981. Growth Cycles with Induced Technical Change. *Economic Journal* 91: 1006–1010.
- Skott, P., 1989. *Conflict and Effective Demand in Economic Growth*. Cambridge, UK: Cambridge University Press.
- Skott, P. 2010. 'Growth, Instability and Cycles: Harrodian and Kaleckian Models of Accumulation and Income Distribution.' In M. Setterfield, Ed., *Handbook of Alternative Theories of Economic Growth*. London, UK: Edward Elgar.
- Skott, P., 2012. 'Theoretical and Empirical Shortcomings of the Kaleckian Investment Function.' *Metroeconomica* 63 (1): 109-138.
- Skott, P. 2017. "Weaknesses of Wage-led Growth." *Review of Keynesian Economics*, 5 (3): 336-359.
- Tavani, D., and Zamparelli, L., 2015. 'Endogenous Technical Change, Employment and Distribution in the Goodwin Model of the Growth Cycle'. *Studies in Nonlinear Dynamics and Econometrics* 19 (2): 209-226.
- Tavani, D., and Zamparelli, L., 2017. Endogenous Technical Change in Alternative Theories of Growth and Distribution. *Journal of Economic Surveys* 31 (5): 1272-1303.
- Tavani, D., and Zamparelli, L., 2017. 'Growth, Income Distribution, and the Entrepreneurial State.' *Journal of Evolutionary Economics* 2018. <https://doi.org/10.1007/s00191-018-0555-7>.

Taylor, L., Foley, D.K., Rezai, A., (2018) Demand drives growth all the way: Goodwin, Kaldor, Pasinetti and the Steady State. *Cambridge Journal of Economics*, bey045, <https://doi.org/10.1093/cje/bey045>.

Verdoorn, P.J. (1949) Fattori che regolano lo sviluppo della produttività del lavoro. *L'Industria* 1: 3-10.