Notes on accumulation and utilization of capital:  
some theoretical issues

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[Very preliminary draft]

Abstract
The paper discusses some issues related to the triangle between capital accumulation,  
distribution and capacity utilization. First, it explains why utilization is a crucial vari- 
able for the various theories of growth and distribution, and more precisely, with regards  
to their ability to combine an autonomous role for demand (along Keynesian lines) and  
an institutionally determined distribution (along classical lines). Second, it responds to  
some recent criticism by Girardi and Pariboni (2019) and I explain that their interpre- 
tation of the model in Nikiforos (2013) is misguided, and that the results of the model  
can be extended to the case of a monopolist. Third, it provides some concrete examples  
on why demand is a determinant for the long run rate of utilization of capital. Finally, it  
argues that when it comes to the normal rate of utilization it is the expected growth rate  
of demand that matters, and not the level of demand.

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1 Introduction

The rate of capacity utilization is a variable that the students of economics do not meet—if they ever do—until late in their graduate studies. The standard micro and macroeconomic theory implies either full utilization of capital or at least utilization being exogenous and constant. Nevertheless, utilization rate is a variable that plays a central role in the various theories of growth and distribution.

In particular the rate of capacity utilization is the connecting link between capital accumulation and distribution of income; a link that determines if accumulation can be determined from the demand side (along Keynesian lines) and if distribution is determined by institutions and social norms (as classical political economy maintains). The important question is if actual utilization is equal to normal utilization in the long run, and to what extent normal utilization is exogenous-to-demand or not. The next section provides a detailed account of these issues. With reference to the Cambridge equation it is explained that if utilization is exogenous, demand-led accumulation implies endogenous distribution along neo-Keynesian lines (Kaldor 1955; Robinson 1962). Alternatively, classical distribution implies a saving-led accumulation (Duménil and Lévy 1999; Foley and Michl 1999). On the other hand, some Sraffian economists (notably Garegnani 1992) argued that although there is an exogenous normal level of utilization, the economy will not gravitate towards it even in the long run, hence the Keynesian approach can be combined with the classical theory of distribution. The more recent literature on the so-called Sraffian Supermultiplier (Serrano 1995) can be understood along these lines as an effort to combine the Keynesian approach, the classical theory of distribution, and normal utilization in the long run; this is achieved by introducing the so-called autonomous expenditure. Finally, Kaleckian economists have suggested that normal utilization is endogenous to demand in the long run. In such a way, the Kaleckian model has three desirable properties in the long run: i) demand matters, ii) distribution is determined based on institutions and social norms along classical lines, and iii) there is path-dependence.

An important related question is how can the endogeneity to demand justified at the micro level. In a well known paper Kurz (1986) demonstrated that normal utilization depends only on cost and technological factors. In a more recent paper (Nikiforos 2013) I showed that the model proposed by Kurz can lead to an endogenous utilization if we account for possible returns to scale. More precisely, the paper demonstrates that a profit-maximizing firm will tend to increase its normal rate of utilization when demand increases if the rate of returns to scale decreases as output increases. This model is summarized in section 3.

Section 4 responds to some recent criticism by Girardi and Pariboni (2019). I explain that
their interpretation of the model is misguided, and also that it is relatively straightforward to extend the results to a monopolist who does not face a fixed level of demand.

In the following section, I provide some concrete examples related to the three main sources of returns to scale: indivisibilities, the division of labor and the three dimensional nature of space. These examples are intuitive and can provide some context to the preceding theoretical discussion.

Finally, another important question is related to the transmission mechanism between the micro and the macro levels. More precisely, how is normal utilization at the micro level being a positive function of demand combined with a macro adjustment that posits that normal utilization changes in the face of discrepancies between normal and actual utilization? In another recent paper (Nikiforos 2016b) I suggested that this could be done if we assume that under normal conditions the increase in demand is accommodated by more firms entering the market, while in period of deviations of the actual from the expected growth rate, excess demand is covered by existing firms who as a result raise their normal utilization rate. In section 6, I explain that the assumptions of this mechanism are unnecessarily strict. When a firm makes its investment decisions it takes into account not the level of demand per se, but rather the expected future flows of demand. Therefore, it is the expected growth rate and not the expected level of demand that enters this decision. In such a way it is more straightforward to reconcile a normal utilization rate that responds to discrepancies between the actual and the warranted growth rate. As a postscript to this introduction, I should also note that a companion paper (Nikiforos 2019a) discusses some empirical issues related to the utilization and accumulation of capital.

2 Utilization in various theories of distribution and growth

The role and the importance of capacity utilization can be understood with reference to the so-called Cambridge equation:

$$g = s_c \pi u \rho \quad [= s_c r]$$

where $g$ is the accumulation rate, $s_c$ is the saving rate of capitalists, $u$ is the rate of utilization and $\rho$ is the potential output to capital stock ratio [and $r$ is the rate of profit]. If one assumes that the saving rate is determined by preferences and norms that do not change over time (so that $s$ can be treated as constant), and that there is not much room for substitution (so that also $\rho$ is constant), there remain three variables that can be potentially determined within the
Regarding the rate of accumulation, there are two major approaches for its determination. According to classical political economy, and also neoclassical economics, accumulation is constrained by available savings. In terms of equation (1), this implies that $g$ will be determined endogenously by the variables on the right hand side of the equation. On the other hand, according to Keynesian theory investment has an autonomous role; those who make the investment decisions are different from those who save, hence there is no reason to expect that in general investment will automatically be generated from saving. Kaldor (1955, 95) called this—“the hypothesis that investment...can be treated as an independent variable”—the “Keynesian hypothesis.” In terms of the Cambridge equation this means that $g$ is exogenous, and either $\pi$ or $u$ will have to adjust endogenously.

In turn, that means that if the rate of utilization is also determined exogenously, distribution has to become endogenous and bear the burden of adjustment. This is the essence of the neo-Keynesian theory of growth and distribution that was proposed in the late 1950s by Nicholas Kaldor (1955) and Joan Robinson (1962). For Kaldor, constant utilization was one of his six famous stylized facts: “the capital/output ratio [remains] virtually unchanged over longer periods” (Kaldor 1957, 592). On the other hand, Robinson justified it more on theoretical grounds, but with the same conclusion: “In long-run competitive equilibrium the relation of total income to the stock of capital is determined within certain limits by technical conditions” Robinson (1962, 11). Based on this, they argued that distribution becomes endogenous so that total savings adjust to the autonomous rate of accumulation. Robinson (1962, 11–12) continues the previous quote writing that: “Whatever the ratio of net investment to the value of the stock of capital may be, the level of prices must be such as to make the distribution of income such that net savings per unit of value of capital is equal to it. Thus, given the propensity to save from each type of income (the thriftiness conditions) the rate of profit is determined by the rate of accumulation of capital.” Similar results were also derived by Kaldor (1955) and then further elaborated by Pasinetti (1962).

From a history-of-economic-thought point of view, two points are worth mentioning. First, these ideas go back to von Mises (1953 [1912]), Schumpeter (2011 [1934]), who coined the related term “forced saving,” and Keynes of the Treatise (2013 [1930]). Second, as it is common in these cases different names have been tagged to this approach. Kaldor and Robinson called it the Keynesian theory. The term “neo-Keynesian” was coined by Sen (1963) in order to distinguish from Keynesian, because of the neo-Keynesian assumption of full employment (or constant utilization). This neo-Keynesian approach was the prevalent one in Cambridge (UK) in the 1960s and 1970s, hence sometimes in that period it was referred to
as the Cambridge theory of distribution. At the same time it was also given the most generic name “Post-Keynesian” (Eichner and Kregel 1975). Later, Garegnani (1992) called this the “First Keynesian Position.”

It is also important to note that both Kaldor and Robinson juxtaposed this theory of distribution to the classical theory of distribution. Kaldor (1955) in his “Alternative Theories of Distribution” first goes through and rejects the Ricardian and the Marxian theory—on the basis that they cannot explain the constancy of the income shares (probably the most famous of his stylized facts)—and then states his “Keynesian” theory. Robinson is also critical of the assumption of an exogenous real wage (or wage share) that is postulated by Marx and Ricardo, on the basis that workers and capitalists bargain over the nominal and not the real wage (see for example Robinson [1962, 7–17] or her foreword in Kregel [1973]).

Based on equation (1), it is not hard to see that the classical theory of distribution—which postulates that distribution is determined outside the economic sphere based on institutions and social norms—is incompatible with the Keynesian hypothesis and an exogenous-to-demand utilization. Or, equivalently, if one wants to combine an autonomous role for demand, and the classical theory of distribution, then utilization has to become endogenous. This is how the system closes in the so-called Kaleckian model of growth and distribution; but also in a large part of the macroeconomic approach of neo-Sraffians. Garegnani (1992) called this kind of closure the “Second Keynesian Position.”

Three more points should be made here. First, to make the taxonomy more complicated several Sraffian scholars have argued against endogenous utilization, while others—Sraffians and Kaleckians—have argued both for and against over time. Second, Joan Robinson in latter writings is in agreement with the classical theory of distribution. For example, in Robinson (1980, 117) she writes “As for what determines the share of wages, in any actual case, we must look for it where it is to be found, in the structure of society at large.” Third, given that Kaldor emphasized the constancy of the wage share as the main weakness of the classical theory, and the main advantage of his Keynesian theory, it is highly questionable that had he lived to see the decline in the wage share over the last three decades, he would still hold onto the same opinion (Nikiforos 2016a).

In any case, if the Second Keynesian Position is chosen, then the endogenization of the rate of utilization has to be justified. In more precise terms, one can define a “normal” rate of utilization which is the rate of utilization that maximizes firm’s profits or minimizes its unit costs. Following Kaldor and Robinson, this “normal” rate of utilization is determined by relative costs, and technology but not by demand. Kurz (1986), writing from a Sraffian vantage point, builds a formal model, where a firm can produce a certain level of output, by
utilizing a certain stock of capital for one or half of it for two shifts (low and high utilization respectively). At the same, labor is more expensive during the second shift. As a result, there is a trade-off between producing in one shift with a lower average cost of labor but higher average cost of capital or in two shifts with the higher average cost of labor and lower cost of capital. Hence, a firm that maximizes its profits will more likely employ two shifts, the higher the share of the cost of capital in production is; while the more expensive labor becomes in the second shift, the more likely it is that the firm will use only one shift. Importantly for our discussion, according to Kurz the choice of utilization is independent from demand. Fluctuations in demand, might cause fluctuations of the actual rate of utilization around the normal rate, however, the normal rate itself will not be affected.

The Kaleckian model has been thus criticized on these grounds because its equilibrium rate of utilization is not equal to this exogenous normal rate of utilization. It is not clear, the critique goes, why firms would not adjust their utilization rate to its normal rate in the long run, when they are able to adjust their capital stock. Hence, according to this critique, the conclusions of the model apply only in the short run but not in the long run. Given the discussion above, it is not hard to understand that the critique comes from two main directions. First, from a neo-Kaldorian perspective that accepts the Keynesian hypothesis, but favors endogenous endogenous distribution (Peter Skott has been the most relentless among these critics e.g. Auerbach and Skott 1988; Skott 2017; Skott and Zipperer 2012). Second, from a classical perspective which posits an exogenously given distribution. As it was mentioned above, such an approach to distribution together with exogenous utilization, necessitate the endogenization of the growth rate, which becomes determined by savings. In other words, this approach implies the abandonment of the Keynesian hypothesis in the long run. This is exactly the argument of Duménil and Lévy (1999), with the explicit title “Being Keynesian in the Short Term and Classical in the Long Term.” This critique, although has been addressed toward the Kaleckian model also holds for the Sraffian analyses, which following Garegnani (1962) adopt the Second Keynesian Position.

One answer to this criticism has been provided by Garegnani (1992), who defends the Second Keynesian Position by arguing that actual utilization does not have to be equal to the normal one, even in the long run. In particular, and with reference to the Cambridge equation, Garegnani argues, that if utilization on the right hand side of equation (1) is taken to refer to the normal rate of utilization, then the resulting growth rate on the left hand side is not the same as the actual rate of utilization. And vice versa, if the growth rate is taken as the actual one, then the resulting utilization is different from the normal one. However, although Garegnani is right that after a shock to demand actual and normal utilization will be different
in the traverse from one normal position to another, even on average, it is not clear why they will be different at the normal positions—if these positions are not affected by demand.

This difficulty has led some other Sraffian authors to the formulation of the so-called Supermultiplier model (Serrano 1995). The supermultiplier is able to combine the Second Keynesian Position with exogenous-to-demand normal utilization by introducing the so-called “autonomous expenditure,” expenditure which is independent of income and other economic variables like autonomous consumption, residential investment, government expenditure and exports. In the long run the system converges to a balanced-growth path driven by the growth rate of autonomous expenditure. This growth rate plays the role that the natural growth rate plays in Kaldorian (or neoclassical) models. 1

A third answer is the endogenization of the normal rate of utilization itself. From a Kaleckian point of view this possibility was first suggested by Amadeo (1986, 155), who writes that “if the equilibrium degree is systematically different from the planned degree of utilization, entrepreneurs will eventually revise their plans, thus altering the planned degree. If, for instance, the equilibrium degree of utilization is smaller than the planned degree \((u^* < u_n)\), it is possible that entrepreneurs will reduce \(u_n\).” In formal terms this adjustment process is the following. Assume as standard version of the Kaleckian model with the following investment and saving functions:

\[
g^i = g^i[\gamma, \pi, (u - u_n)]
\]

\[
g^s = g^s(s_c, s_w, \pi, u)
\]

where \(g^i\) and \(g^s\) are investment and saving normalized to capital stock, \(\gamma\) is a term that captures expectations about the growth rate, \(\pi\) is the profit share, \(u - u_d\) is the deviation of the actual from desired/normal rate of capacity utilization, and \(s_c\) and \(s_w\) are the saving rates of capitalists and workers respectively. All partial derivatives are positive.

We can also define the warranted growth rate \((\gamma_w)\), as the growth rate where actual and normal utilization are equal. The means that we could rewrite equation (2) as:

\[
g^i = g^i[\gamma_w, (u - u_d)].
\]

If profitability did not have any effect on investment the warranted rate, thus defined, would

1. Recent contributions along these lines include Freitas and Serrano (2015), Allain (2015), and Lavoie (2016). For a critical discussion see Nikiforos (2018).
be equal to $\gamma$. Alternatively, if the investment function was linear ($g^i = \gamma + \beta_1 \pi + \beta_2 [u - u_n]$), the warranted growth rate would be equal to $\gamma$ adjusted for the effect of profitability: $\gamma_w = \gamma + \beta_1 \pi$.

Assuming a classical exogenous distribution of income the short run equilibria of utilization and the growth rate are

$$u^* = u^*(s_c, s_w, \gamma, \pi) \quad (5)$$

$$g^* = g^*(s_c, s_w, \gamma, \pi) \quad (6)$$

The partial derivatives of the $u^*$ and $g^*$ with respect to the saving propensities is negative—the so-called paradox of thrift—with respect to $\gamma$ is positive, while with respect to $\pi$ depends on the relative magnitude of the propensities to invest and save out of profits ($\partial g^i / \partial \pi$ and $\partial g^* / \partial \pi$)—this is the well known distinction between wage- and profit-led growth. This equilibrium rate of utilization will generally be different from the normal utilization. The adjustment process Amadeo is talking about can be formalized as:

$$\dot{u}_n = \mu (u^* - u_n) \quad (7)$$

where $\mu$ is a positive constant, and the dot stands for the time derivative.

The adjustment of normal utilization can be combined with a Harrodian endogenous adjustment of the warranted growth rate:\[2\]

$$\dot{\gamma} = \theta (g^* - \gamma_w) \quad (8)$$

The system of equations (7) and (8) is able to combine the Keynesian hypothesis with a classical theory of distribution, and a normal utilization equal to the actual one. Another important property of this system is path dependence. Short run shocks carry over to the long run growth rates. This is an important difference compared to Kaldorian and supermultiplier-type models where the growth of the system in the long run depends solely on the natural growth rate or the exogenous growth rate of autonomous demand. In these models path dependence can come about only through endogenizing technical change (or the rate of growth of population or labor force participation in the case of the Kaldorian model).

However, the difficulty remains; why would normal utilization be endogenous? In particular two questions need to be answered. First, why would a firm adjust its normal rate of

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2. The first ones to propose this were Lavoie (1995, 1996) and Dutt (1997).
utilization in the face of changes in demand. In other words, how is the micro-level analysis different from that of Kurz (1986)? And second, how do the micro-level results carry over to the macro level? Related to the first question, in a series of articles, Lavoie (1995, 1996), Lavoie, Rodríguez, and Seccareccia (2004), and Hein, Lavoie, and Treeck (2012) argue that the normal rate of utilization is just a convention. Deviations of the actual rate from this conventional normal rate will cause the conventions to shift and the normal rate to move towards the endogenous. This rationale is not convincing because, as the discussion of Kurz’s model implied and as the theory of utilization makes clear, the choice of the rate of utilization is similar to the choice of the technique of production and there is nothing conventional about it.3

I provide another justification in Nikiforos (2013). It is demonstrated that Kurz’s model is extended to take into account economies of scale, then it is straightforward to show that at the micro-level the increase in demand will lead to changes in the cost-minimizing rate of utilization. The sufficient condition for this to happen is that the rate of returns to scale decreases as the scale of production increases. As I argue in this paper, the theory of production justifies this kind of behavior of economies of scale.4 In a companion paper (Nikiforos 2016b, 2019b), I provide a mechanism that links the changes at the micro-level to a macro adjustment like that of equation 7. Section 4 of the present paper generalizes this mechanism.

A few final points are worth being made before moving on. An empirical discussion of the rate of utilization is beyond the scope of this paper. An extensive discussion is provided in Nikiforos (2016b), who shows that the endogeneity of the rate of utilization is supported by the data. However, one anecdotal piece of evidence is interesting. There are two main books in the literature on utilization: The economics of capital utilisation by Robin Marris (1964) and Capital Utilization by Roger Betancourt and Christopher Clague (1981). Both books, in their second page(!) mention the results of answers to entrepreneurs to questionnaires on the the main factors that determine their decision about the utilization of their capital. In the words of Marris:

“in business inquiries, one of the commonest reasons given for working shifts or not (as the case may be) relates to demand [emphasis added].”

And in the words, of Betancourt and Clague:

3. For a more detailed discussion of this issue see Nikiforos (2016b, section 3.3)
4. Two recent papers (Tavani and Petach 2018; Franke 2018) also argue in favor of an endogenous rate of utilization. Their provide a game-theoretic justification and emphasize the role of strategic complementarities in the formation of the desired utilization on behalf of the firms. Their argument can be thought of as complementary to ours.
“interviews have shown that that when factory managers have been asked why they are operating only one shift one of the most frequent answer given is that the firm would not be able to sell its product [emphasis added].”

Given these answers it is surprising that so many economists, who otherwise emphasize the importance of demand, are willing to go to such great lengths and argue in favor of independence of demand from such a central economic variable.

Finally, to complete the comparative theoretical discussion of this section, it should be stressed that capacity utilization is also of interest for neoclassical economists but for different reasons. With regards to the Cambridge equation, it is well-known that neoclassical model assumed that distribution and capital intensity are determined in such a way that the labor market clears, while utilization is fixed. As a result, accumulation is determined endogenously by the availability of savings. What is more interesting for neoclassical economists though is the role of utilization in growth accounting and the Real Business Cycle theory. Changes of utilization over time or over the course of the cycle affect the magnitude of total factor productivity, with obvious consequences. This sort of concerns have motivated some interesting empirical work on the subject from neoclassical economists.

3 A benchmark model for the firm

As it was explained above, the belief that utilization is exogenous originates at the firm level. If then, one rejects that normal utilization is purely a convention it is not clear why a profit-maximizing firm will change its normal utilization rate in the face of changes in demand. From a theoretical perspective this view has been reinforced by the conclusions of Kurz (1986), who shows that it is only technological and cost factors that determine utilization. This section presents a model from Nikiforos (2013), which follows Betancourt and Clague (1981) and extends the analysis of Kurz and shows that if economies of scale are taken into account utilization becomes endogenous to demand.

Assume that when an entrepreneur decides to invest they expect that the demand for the product of their firm will be $Q$. There is only one technique of production available, which requires labor and capital as inputs. For the production of $Q$, it takes a certain amount of capital service ($K^3$) and labor services ($L^3$). Also assume that the services of capital and labor are proportional to the stock of capital ($K$) and the number of workers ($L$). The utilization of capital depends on the time the capital is used. The firm can employ a single-shift system (a workweek of 40 hours) or a double-shift system (a workweek of 80 hours). In the single-shift
system capital, $K^1$, and labor, $L^1$, are combined to produce $Q$. In the double shift system, half of the amount of capital ($K^2 = K^1 / 2$) is combined with an amount of labor $L^{21}$ in the first shift and $L^{22}$ in the second to produce $Q$ (The first number of the superscript refers to the system of operation [single or double shift] and the second—if there—to the shift within each system). The amount of labor in each of the two shifts is equal, so $L^{21} = L^{22} = L^2 = L^1 / 2$. At the same time, the cost of labor for the second shift is higher; firms have to pay a utilization differential, $\frac{w_2}{w_1} = 1 + a$, where $w_1$ and $w_2$ are the wage for working in the morning and evening shift and $a > 0$. Finally, $r$ is the unit cost of capital.

The model so far is the same with the one of Kurz (1986). Additionally, assume that the production under the single shift system leads to some economies of scale relative the double shift system, and that these economies of scale depend on the level of the production of the firm. If we denote these economies of scale as $\zeta(Q^e)$, the total cost of production under the first system will be:

$$C^1 = \frac{(rK^1 + w_1L^1)}{\zeta(Q)}$$

while under the double shift system the cost of production is:

$$C^2 = rK^2 + w_1L^{21} + w_2L^{22} = rK^2 + (2 + a)w_1L^2$$

The firm will choose the system of production that maximizes its profits (or minimizes its costs). The ratio of the cost of the double shift system over the cost of the single shift system is:

$$\Lambda = \left[\pi + (2 + \alpha)\psi\right] \frac{\zeta(Q)}{2}$$

where $\pi$ is the share of capital cost and $\psi = 1 - \pi$ is the share of wage cost to the total cost of production under the single shift system respectively. Hence, the double shift system will be chosen as long as $\Lambda < 1$.

Based on equation (11) it is easy to see that (i) since $\partial \Lambda / \partial \pi < 0$ the more capital intensive the technique of production is, the more the firm will tend to utilize it; and (ii) since $\partial \Lambda / \partial \alpha > 0$ the larger the utilization differential the more the firm will tend to use a single shift system.

Finally, since $\partial \Lambda / \partial Q = (\pi + a)\psi \frac{\zeta'(Q)}{2} \neq 0$ utilization depends on demand. Assuming that $\zeta'(Q) < 0$ then $\partial \Lambda / \partial Q < 0$. In other words, the entrepreneur will tend to choose a double shift system of operation over a single shift system of operation as the demand for the product of their firm increases, if the degree of the returns of scale decreases as the scale of
production increases. This result can be extended to a technology with more than one technique of production and an infinite continuum of techniques of production, as long as these techniques are assumed to be “well behaved” (Nikiforos 2013, section 6).

Based on the theory of the economies of scale, it can be shown that this condition (the degree of the returns of scale decreases as the scale of production increases) can be justified. The most common reason for returns to scale is indivisibilities. In a famous passage, Kaldor (1934) writes that “it appears methodologically convenient to treat all cases of large-scale economies under the heading indivisibility.” More than twenty years later, Koopmans (1957, p.152) agrees: “I have not found one example of increasing returns to scale in which there is not some indivisible commodity in the surrounding circumstances.” The benefits of indivisibilities are exhausted as production increases and thus the degree of the returns to scale is decreasing. Therefore, the firm will tend to utilize its capital more as the demand for its product increases. Other factors that lead to economies of scale can also present the same behavior.

An extensive theoretical discussion is provided in Nikiforos (2013, section 7); some more concrete examples will be given in the next section. For the moment it suffices to say that the conclusion is intuitive. The inputs of production are not perfectly divisible. Therefore, the firms will necessary underutilize some of them. Hence, the necessary condition to increase the utilization of their resources is the increase for the demand of the product of the firm. Georgescu-Roegen (1969, 1970, 1972) makes a similar argument. He argues that during the production of any good there are inevitably some idle resources and the degree of this idleness can only be reduced if the demand for the output of the firm increases. According to him, the Industrial Revolution was brought about by the increase in demand, which allowed to move from artisanal production to the factory system (with high utilization of capital). The examples provided below are close to his reasoning.

4 A recent critique

In a recent paper Girardi and Pariboni (2019, 349–50) have criticized this model on two main grounds. First, because it assumes a very “particular” and “arbitrary” behavior for the returns to scale. And, second, because returns to scale imply that firms have market power, which are not price-takers and should not take the level of demand as given. This is a good place to address these two issues. As I will show below the first critique comes from a misunderstanding of what my model says, while it is also straightforward to extend its conclusions for a
monopolist who faces a downward-sloping demand curve. I thus conclude that their critique is not valid.\footnote{Girardi and Pariboni (2019, 350) also point that the proposed model does not survive the Cambridge capital critique. This is true but there is nothing new here. I explicitly mention that with respect to the use of a production function (Nikiforos 2013, 521), while (Kurz 1986, 47) had earlier argued that we can treat the two systems of production as separate techniques, and therefore we cannot exclude the possibility of reswitching as the wage rate changes. Note however, that this issue is not central for the conclusion regarding the effects of demand. The existence of returns to scale mean that the wage-profit rate schedule of the single-shift system will tend to move outwards, and therefore all other things equal, it will be more profitable relative to the double-shift system for all levels of the wage. If the rate of the returns to scale decreases this advantage will decrease as well.}

4.1 “Particular” and “arbitrary” behavior of the returns to scale

In Girardi and Pariboni’s own words (2019, 349):

Nikiforos (2013) models “increasing returns at a diminishing rate” in a very particular way. In his model, the double-shift system displays constant returns to scale. The single-shift system displays decreasing returns to scale: for a firm adopting the single-shift system, as Q increases the average cost of production increases (because $\zeta < 0$) ... This is a highly non-standard definition of increasing returns to scale. However, the resulting positive relation between quantity produced and desired rate of utilization depends entirely on, and follows quite directly from, this arbitrary setup.

Based on this, they continue:

Another problem also arises. Given the specification of the cost functions assumed ... the single-shift system implies lower unit costs for low levels of Q ... It is not clear, then, why entrepreneurs should not decide to build multiple small plants, each one producing a small Q and adopting a single-shift system.

In other words, Girardi and Pariboni understand the term $\zeta$ to apply individually to the single shift of production (which thus displays decreasing returns to scale since $\zeta' < 0$), while at the same time the double shift system displays constant returns to scale (see equations [9] and [10]). This would indeed be a particular, arbitrary and highly nonstandard treatment of economies of scale.

However, this is a careless—and in fact particular and arbitrary—reading of the model. It is clear, and it is stated explicitly in the original paper, that the term $\zeta$ does not refer to the individual systems of production, but rather to “economies of scale [of the single shift
system] vis-à-vis the double shift system” (Nikiforos 2013, 524, emphasis in the original). In equations (9) and (10) it would be equivalent to multiply the cost of production of the double shift system with $\zeta$. Economies of scale are not related to the system of production per se, but rather to the scale of production, and thus can be present in both systems. However, because by definition the scale of production in the single shift system is higher than in the double (since the same amount of output is produced simultaneously and not in two pieces) the single shift system has some cost advantages over the double shift one.

Moreover, Girardi and Pariboni’s interpretation is hard to sustain if one reads the treatment of the issue at hand in the case of infinite techniques of production (section 5.3). As it is explained there, a homothetic production function is assumed in order to isolate the role of the economies of scale in the choice of the system of production. The economies of scale are captured there with the positive monotonic transformation of the homogeneous production function, and they are clearly related to the scale of production and not with individual systems of production.

Finally, from a theoretical—and practical—point of view since economies of scale are due to indivisibilities (as it was explained above and in more details in Nikiforos [2013, section 7]), their return will turn to decrease as the scale of production increases. In the context of the present model, this implies that as the demand for the product of a firm increases, the firm will tend to increase the utilization of its capital. Hence, the treatment of economies of scale is neither particular of arbitrary.

4.2 Profit maximization

The case of a monopolist, who can choose what quantity to produce is slightly more complicated. However, it is not difficult to show that demand plays a role in a way similar to the case of a firm that takes demand as given. The issue at hand has already been analyzed by Betancourt and Clague (1975; 1981, ch. 3). The exposition below follows their analysis and generalizes their main analytical result.

As usual the goal of the firm is assumed to be the maximization of profits. Hence the firm will choose the double shift system over the single shift system if the total profits of the former are higher than those of the latter:

$$ T\mathcal{P}^2 > T\mathcal{P}^1 \iff TR(Q^2) - C^2(Q^2) > TR(Q^1) - C^1(Q^1) $$

(12)
where the superscript denotes the system of production, \( T \mathcal{P}^i \) total profits, \( TR \) total revenues, and \( Q^i \) the optimal level of output for each system. If output is fixed this condition coincides with the cost minimization condition since total revenues will be the same for both systems. However, in the case of a monopolist the optimal output level of the two systems of production is different. Therefore, even if the relative cost of one system of production is higher, the difference might be more than compensated by higher total revenues.

**figure 1 around here**

As was explained in the previous section, if the technology of production exhibits returns to scale with a decreasing return, the relative cost of the double shift system will decrease as output increases. This implies that the marginal cost of the double shift system decreases faster than the marginal cost of the single shift system. In this case the marginal cost of the double shift system cuts that of the single shift system from above as in figure 1.\(^6\)

The marginal cost of the two systems is the same for level of output \( \bar{Q} \). At that level of output the cost of the double shift system is higher than the single shift system.\(^7\) This also implies that for \( Q < \bar{Q} \) the cost ratio is greater than one (\( \Lambda = C^2/C^1 > 1 \)).

On the other hand we can define a level of output \( Q^* \) where the cost of the two systems if the same (\( C^2(Q^*) = C^1(Q^*) \)). Obviously for levels of output above \( Q^* \) the cost ratio is lower than one (\( \Lambda < 1 \)).

Figure 1 also shows that for area on the left of \( \bar{Q} \) since the marginal cost of the double shift system is above the marginal cost of the single shift system and the demand curve is downward sloping the optimal output for the double shift system is lower than the optimal output of the single shift system (\( Q^2 < Q^1 \)). Vice versa in the area on the right of \( \bar{Q} \) we have \( Q^2 > Q^1 \). In turn, since demand for a monopolist is elastic, this implies that total revenue for the double shift system is lower than total revenue of the single shift system on the left of \( \bar{Q} \) and higher on the right (\( TR(Q^2) \leq TR(Q^1) \) for \( Q \leq \bar{Q} \)).

Overall then, when \( Q < \bar{Q} \) the single shift system is undoubtedly the most profitable, since both total revenue is higher and cost is lower compared to the double shift system. On the other hand, for the same reasons the double shift system is undoubtedly the most profitable for \( Q > Q^* \). For \( Q \in (\bar{Q}, Q^*) \) it is not clear what system is most profitable. Within this range

\(^6\) Of course it is also likely that the two curves never cross. For example, if the utilization differential is very big the marginal cost of the double shift system will likely always be higher than that of the single shift system.

\(^7\) The reason for that is simple. By definition \( C^2(Q) = \Lambda(Q) \cdot C^1(Q) \), if we take the derivative with respect to \( Q \) we get \( MC^2(Q) = \Lambda'(Q) \cdot C^1(Q) + \Lambda(Q) \cdot MC^1(Q) \). Since at \( \bar{Q} \) the marginal cost of the two systems is the same we can write \( MC^1(\bar{Q}) = \frac{\Lambda'(\bar{Q}) \cdot C^1(\bar{Q})}{\Lambda(\bar{Q})} \). Since the fraction on the left hand side of this equation is positive and \( \Lambda'(\bar{Q}) \) is negative, \( \Lambda(\bar{Q}) \) is positive.
of output, the single shift system has lower total cost but also lower total revenue compared to the double shift system. Based on this it is clear that as demand increases (a rightward shift of the demand curve in figure 1), the double shift system will tend to become more profitable.

In fact, it can be established analytically that the profitability of the double shift system relative to the single shift system will tend to increase as demand increases as long as $Q > \bar{Q}$.

Assume an elastic demand curve for the product of the firm, $P = P(A, Q)$, where $P$ is the price level and $A$ is a shift variable, so that increases in $A$ lead to increases in demand ($\partial P / \partial A > 0$).

The difference between the total profits of the two systems will be equal to $T P^2 - T P^1 = [TR(A, Q^2) - C^2(Q^2)] - [TR(A, Q^1) - C^1(Q^1)]$. As a result the effect of a change in $A$ on the profitability differential will be:

$$d[T P^2 - T P^1] = \frac{\partial [TR(A, Q^2) - TR(A, Q^1)]}{\partial A} * dA + [MR(Q^2) - MC^2(Q^2)] * dQ^2 + [MR(Q^1) - MC^1(Q^1)] * dQ^1$$

The terms in the square brackets of the second line of 13 are equal to zero. Therefore we can rewrite this equation as $d[T P^2 - T P^1] = \frac{\partial [TR(A, Q^2) - TR(A, Q^1)]}{\partial A} * dA$. Thus, the sufficient condition for an increase in demand ($dA$) to have a positive effect on the difference between the total profits of the two systems is: $\partial [TR(A, Q^2) - TR(A, Q^1)] / \partial A > 0$. Given the demand function $P = P(A, Q)$, we can rewrite this as

$$\frac{\partial P(A, Q^2)}{\partial A} * Q^2 - \frac{\partial P(A, Q^1)}{\partial A} * Q^1 > 0$$

Or, equivalently, as

$$Q^2 / Q^1 > \frac{\partial P(A, Q^1)}{\partial A} / \frac{\partial P(A, Q^2)}{\partial A}$$

In the region on the right of $\bar{Q}$, optimal output for the double shift system is higher than the single shift system, therefore the left hand side of inequality (14') is higher than one.

Although, we cannot generalize the results for any functional form of demand, we can take two generic functional forms that are common in the literature. First, we can use an additive demand function of the form $P(A, Q) = \phi(A) + \chi(Q)$ with $\phi(A) > 0$, $\phi'(A) > 0$ and $\chi'(Q) < 0$. The usual textbook linear demand function is a special form of this function. In this case $\partial P / \partial A = \phi'(A)$ and is independent from the level of output. Hence the term on the right hand side of (14') is equal to one, and therefore the inequality condition is satisfied.

We can also examine a multiplicative demand function of the form $P(A, Q) = \phi(A) * \chi(Q)$
with $\phi(A) > 0$, $\phi'(A) > 0$, $\chi(Q) > 0$ and $\chi'(Q) < 0$. The usual constant elasticity of demand function is a form of this. In this case $\partial P/\partial A = \phi'(A) \times \chi(Q)$. Based on this we can rewrite the left hand side of inequality (14) as $\phi'(A) \left[ \chi(Q^2) \times Q^2 - \chi(Q^1) \times Q^1 \right]$ which is equal to $\left[ \phi'(A)/\phi(A) \right] \times \left[ TR(A,Q^2) - TR(A,Q^1) \right]$. This inequality is positive on for $Q > \bar{Q}$ because $Q^2 > Q^1$ and demand is elastic.

This discussion shows that the positive effects of demand on utilization can be generalized to the case of a profit-maximizing monopolist who can choose what quantity to produce.

5 Some examples

As it was explained above and in more details in Nikiforos [2013, section 7] the theory of production identifies indivisibilities as the main source of returns to scale. In addition two more potential sources economies of scale are the division of labor and the three dimensional nature of space. This section provides some intuitive examples for each of this category, with an emphasis on indivisibilities since they are the most important source of returns to scale. These examples can provide some context to the preceding theoretical discussion.

Assume an extreme case of indivisibility. For the production of a certain good, there is only one type of machine available, in only one size. The machine can be utilized in one or two shifts, that is for forty or eighty hours per weeks, which—since the maximum number of hours per week is 168—implies a utilization rate of 24% or 48% respectively. The machine combined with forty hours of labor time can produce 80 units of output per week if it is utilized in one shift (40 hours per week). If it is utilized for two shifts (80 hours per week) combined with eighty hours of labor time (forty in the first and another in the second) it can produce 160 units of output per week. Also assume that there is no utilization differential or this differential is small enough to make the second shift unprofitable.

In this case, and because of the extreme indivisibility, the utilization of capital will depend only on demand. What will be the rate of utilization of the capital of the individual firm as expected demand at the time of investment increases? The answer is depicted in figure 2. If expected demand is below 80, then the firm will utilize one unit of capital for one shift (a 24% rate of utilization); if demand exceeds 80, the rate of utilization will increase to two shifts (48% rate). If demand increases above 160, the firm will need two machines. It will now employ the first machine for two shifts and the second one for one (a 36%) rate of utilization. If demand increases above 240, the second machine will also be employed for a second shift (we are back at 48% rate of utilization). If demand increases above 320, the firm will need
three machines. The first two will be employed for two shifts and the third for one (a 40% rate of utilization). This process will continue as figure 2 shows. Overall, the increase in expected demand will be accompanied by an (non-monotonic) increase in the rate of utilization as the filtered series shows.

**figure 2 around here**

It is worth going one step further. The use of the 40-hour shift as the unit of time to measure the utilization of capital is related to social norms and regulations. In general, labor cannot be hired at an hourly rate; it has to be employed for the whole 40 hours of week. However, that does not mean that the utilization of capital is the same in the various horizontal segments of figure 2. The implications are presented in figure 3. Given the aforementioned specifications, each machine can produce 2 units of output per hour. Therefore, if the expected demand for the product of the firm is 10, the firm will invest in one machine that will be utilized for five hours per week, a rate of utilization around 3%. If demand is 20 utilization will be 10 hours per week or 6%. Eventually, when demand becomes 80, utilization will reach 24%, as it was the case in figure 3 for all levels of demand below 80. The level of utilization will keep increasing monotonically, until demand surpasses 160. Above this level of demand, the firm will invest in two machines. Utilization falls, and then starts increasing monotonically as demand increases. The result is again that utilization increases as demand increases, as the filtered line shows, but now in a saw-tooth fashion. The important point here is that the indivisibility of labor adds another layer of adjustment, that makes the system more elastic. More generally, the combination of several inputs of production, each of which is not completely divisible adds several potential layers that allow normal utilization to adjust to demand.

**figure 3 around here**

Figures 2 and 3 also show why modeling indivisibilities. The pervasive non-linearities that emerge are very difficult to be treated mathematically, but also logically.

Finally, it is worth examining what is happening at the macro level. Figure 4 depicts average demand and utilization in an economy of 1000 firms. Each of the firms has access as before to one type of machine. For each of the firms, if utilized for two shifts the machine can produce 160 units of output plus a random shock distributed according to a normal distribution. Figure 4a presents utilization when the unit of time is shifts, while figure 4 presents utilization at smaller time units (analogous to figure 2) and 3 respectively). The result now is a smooth monotonic increase in utilization at the macro level as average demand increases.
These examples obviously make some strong assumptions. In reality there are machines of different sizes and different speeds of operation. Nevertheless the varieties are always finite and therefore pervasive indivisibilities are always present and pervasive. Moreover, as it was mentioned returns to scale emerge not only from the indivisibility of an individual input in production, but also from the indivisibilities of other factors that are combined with it. Finally, the examples does take into account some features of the model (e.g., the utilization wage differential). Relaxing the assumptions or adding these features would not change the qualitative results of the simulations, and the positive effect of demand on utilization.

What about the division of labor? To begin with, as Edwards and Starr (1987) have pointed out labor specialization can be thought as a special case of indivisibilities. Nevertheless let’s consider a pin factory in Paris or Glasgow in the late 18th century. Adam Smith (1999) distinguishes between “eighteen distinct operations, which, in some manufactories, are all performed by distinct hands, though in others the same man will sometimes perform two or three of them ... One man draws out the wire; another straights it; a third cuts it; a fourth points it; a fifth grinds it at the top for receiving the head; to make the head requires two or three distinct operations; to put it on is a peculiar business; to whiten the pins is another; it is even a trade by itself to put them into the paper”). Let’s assume that demand for pins is low, so a small pin factory can produce the desired amount either by running two shifts, with one worker in each, or by running one shift with two workers. Because of the division of labor, employing two workers in one shift will lead to some economies of scale (this is the idea of \( \zeta \) in the model outlined above). If demand increases, the factory can produce the desired output by employing two workers in each shift, or four workers in one shift; again one shift provides some benefits compared to two shifts. If we keep running this thought experiment, given the technical specifications outlined by Smith, it is clear that as the demand for the product increases the benefits of the single shift will eventually diminish. If demand is higher than the demand that can be covered by employing eighteen workers in a single shift, the benefits of the single shift will disappear and the firm will start running a second shift.

Finally, let’s think about the three dimensional nature of space. The usual example in the literature is that of a cylinder (Koopmans 1957; Kaldor 1972; Eatwell 2008). Think of a firm whose production requires some input provided through a tube, or whose production produces output or waste that need to be disposed through a tube. If we denote the radius of the tube.

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8. The example of the pint factory first appeared in l’Art de l’Épinglier (The Art of the Pin-Maker) (de Réaumur and Perronet 1761). Adam Smith is believed to have borrowed his classic example in the first chapter of The Wealth of Nations (Smith 1999) from there.
of the tube as \( r \), the capacity of the tube varies with the area of its top \( (\pi r^2) \), while its cost with its diameter \( (2\pi r) \). This gives an advantage to a single shift production (with a big tube providing the inputs or disposing the waste) over a smaller tube. However, it is reasonable to assume that as the radius of the tube increases its plate will also need to increase in order to sustain the increased pressure of its content. As a result, the benefits of producing under a single shift will diminish as the scale of production increases.

In a similar way, assume that production takes place in a rectangular space with length \( \alpha \) and width \( \beta \). The capacity of production is proportional to the area of the rectangular \( (\alpha \cdot \beta) \) while the cost of fencing the area is proportional to its perimeter \( (2\alpha + 2\beta) \). Again, ceteris paribus, this gives rise to economies of scale and an advantage to producing under a single shift system. Nevertheless, if we assume—as it is reasonable—that the average cost of the fencing increases as the perimeter increases (because it has to be taller, or because security requirements increase), then this advantage will also slowly diminish.

6 From micro to macro: Levels or growth rates?

The aforementioned discussion shows that demand indeed plays a role for the determination of the normal rate of utilization at the micro level. A remaining step is the specification of the linkages between the micro and the macro level. To a certain extent this is a technicality; if utilization is endogenous to demand at the micro level, then it should be at the macro level as well. It is hard to think why that would be otherwise.

The issue involves the familiar—albeit sometimes blurry—distinction between levels and growth rates. The question is the following. If normal utilization at the firm level is a function of the level of demand, how can this be reconciled with a macro relationship like that of equation (7), that is \( \dot{u}_n = \mu (u^* - u_n) \)?

A possible mechanism is suggested in Nikiforos (2016b, section 6). The basic idea is that under normal conditions (when output grows at the warranted rate), the increase in demand will be covered by the entrance of new firms in the market. As a result, the level of demand for the individual firm will not increase. The demand for the individual firm will increase only when the actual growth rate deviates from the warranted. It is then straightforward to arrive to equation (7). Some prima facie support to this mechanism is provided by the data. Indeed there is a strong positive correlation between the change in the number of firms and the change in real GDP, which means that a certain part of the increase in GDP over time is absorbed by increase in the number of firms. There is also a positive correlation between the change
in the size of the firm (measured as employment per firm, or employment per establishment) and deviation of real GDP from its trend (which can be interpreted as deviation of the actual growth rate from the warranted growth rate).

This mechanism has been criticized by Peter Skott (in several individual discussions and conferences panels) and more recently by Girardi and Pariboni (2019, 350–53) on the basis that it is not realistic to assume that demand of the average firm does not increase over time. Girardi and Pariboni point that US data show that output per firm increases over time.

Two answers can be given. First, the theoretical discussion of an abstract model always needs to be put in perspective. In particular arguing that normal utilization adjusts to changes in demand in the long run, does not mean that utilization is the only variable that adjusts in the long run. In reality, long run adjustment takes place through several variables, which for the sake of this analysis have been taken to be constant. The most important of these is technical change. Technical change can account for a large part of the increase in the output of the firm. This is why employment per firm—as opposed to output per firm—is a more appropriate measure for the size of the firm in this context; it is a measure that can partially control for the increase in labor productivity.

Second, if we think about the issue more carefully, the theoretical connection between the micro and the macro level can be made on less restrictive assumptions. The reason for that is simple. The discussion so far established that normal utilization is a positive function of the demand that the firm expects for its product at the time of investment. What is the nature of this expected demand at the time of investment? To answer this question we need to think about the investment decision of the firm. Probably more than any economic decision, the investment decision is at the same time both static and dynamic. Static because it is made and executed at a certain period in time. Dynamic because it is determined by more than any other economic decision by dynamic considerations. The theory of investment as it was elaborated by Keynes in *The General Theory* (2013 [1936], ch. 11) or as it was subsequently elaborated by Minsky (1986, ch. 8) makes clear that what matters for investment is not profitability per se, but rather the (discounted) expected future cash flows. In a similar fashion what matters for investment is not current or expected level of demand per se, but rather expected flows of demand over the lifetime of the invested capital.

In other words, when a firm makes an investment, it does not take into account a static demand (e.g. the demand expected in the next period), but rather the demand expected over an extended period of time. Note that as has been emphasized by Steindl (1952, 10) the

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9. This is something I emphasized in Nikiforos (2016b, 459).
reason for “not [being] possible for the producer to expand his capacity step by step as his market grows” is “the indivisibility and durability of plant equipment” (the same reason for the endogeneity of utilization). Hence, it is the growth of demand that is taken into account when the firm invests. Thus, the variable \( Q \) in the previous section—expected demand at the time of investment—depends on the various levels of future expected demands. If the system grows at the warranted rate, this is the growth rate that the firm will take into account, and upon which it will base its decision about its desired level of utilization. For rates of growth above or below this the firm will adjust its expected demand and normal utilization rate accordingly. This is a more general way to establish the linkages between the micro and the macro level.

7 Conclusion

The present paper discussed the relation between accumulation and utilization of capital. Four main themes were touched. First, it was discussed why utilization is so important in the context of alternative theories of growth and distribution. With reference to the Cambridge equation, it was shown that utilization plays a central role in the ability of a theory to combine an autonomous role for aggregate demand and a classical theory of distribution. Within this context it was shown how the neo-Keynesian, the classical, the Kaleckian, and the Sraffian approaches provide different answers based on the assumption they make about the long run rate of utilization.

It was argued that the Kaleckian closure with an endogenous normal rate of utilization has three desirable properties: (i) it allows for an autonomous role for demand, (ii) it is based on a theory of distribution where institutions and social norms play a crucial role, and (iii) there is path dependence in the long run, without having to reside on induced technical change to achieve it.

Second, responds to some point recently made by Girardi and Pariboni (2019). I explain that their interpretation of the model is wrong, and that it is relatively straightforward to extend the results to a monopolist who faces a negatively sloped demand curve.

Third, the paper provided some examples to show the pervasive role of indivisibilities and economies of scale in the determination of capacity utilization in the long run. These examples, clarify and put some context to the theoretical discussion.

Finally, the issue of linking the behavior of the firm with the macro changes in capacity utilization was revisited. It was explained that the when it comes to the investment decisions
of the firms it is the expected growth rate of demand that matters, and not the level of demand. One can talk about expected demand, as the geometric mean of expected demand flows based on the expected growth rate.
References


Figure 1: Profit maximization
Figure 2: Demand and Utilization at the firm level
Figure 3: Demand and Utilization

Figure 4: Average Demand and Utilization