



# A demand-driven model of the natural rate of growth

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**Abstract:** Supermultiplier models including a Harrodian investment behavior of firms have been recently brought to the Post-Keynesian debate. Here, we draw on Allain (2019) in which the supermultiplier effect is due to formal and informal redistributive devices enabling everyone in the population (employed workers but also unemployed and inactive) to satisfy their primary needs. Introducing technical progress in the model, which is our main innovation, we show that primary needs give rise to a non-generating capacity autonomous component of aggregate demand whose rate of growth is given by the natural rate of growth. The three main outcomes of the model are that 1) Harrodian knife-edge instability can be tamed despite firms' attempts to adjust their actual rate of capacity utilization, 2) the long-run rate of growth of the economy converges towards the natural rate of growth, 3) the rate of employment stabilizes, but at a level that can differ from full employment. The two first outcomes being close to those of the Solow growth model, but having been obtained in an aggregate demand model, we also propose a detailed comparison between the two approaches.

**Keywords:** Aggregate demand, Long-run, Harrodian instability, Natural rate of growth, Supermultiplier model.

**JEL codes:** E12, E25, E62

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## 1. Introduction

Many growth models have been recently developed by combining a supermultiplier effect with a Harrodian investment behavior of firms. On the one hand, the supermultiplier effect results from the introduction of a non-generating capacity autonomous component growing at an exogenous rate in a Post Keynesian model. As shown first in Serrano (1995a, 1995b) and Bortis (1997), the main outcome is the convergence of both the rate of capital accumulation and the rate of economic growth towards the rate of growth of the autonomous component. On the other hand, the Harrodian investment behavior corresponds to entrepreneurs' attempts to adjust their investment in order to fill the gap between the actual and normal rate of capacity utilization. According to Harrod (1939), such behavior leads to knife-edge instability. However, as shown in Allain (2015), Freitas and Serrano (2015), Lavoie (2016) and many others, the Harrodian instability can be tamed by the stabilizing properties of the supermultiplier effect.<sup>2</sup> The main outcome is then that the actual rate of capacity utilization converges towards its normal level in the long run.

Moreover, although the long-run rates of capacity utilization and economic growth are now exogenously given, the main Keynesian message is safeguarded because any shock on aggregate demand generates effects in the same direction on the levels of both capacity, output and employment.

One of the reasons explaining the variety of models in this new strand of research is the diversity of the aggregate demand components that can generate a supermultiplier effect: public spending (Allain, 2015), autonomous consumption financed by credit (Freitas and Serrano, 2015), capitalists' consumption (Lavoie, 2016), consumption out of wealth (Brochier and Macedo e Silva, 2019), exports (Nah and Lavoie, 2017), etc. This great diversity raises two complementary issues. First, all these components cannot play the role of autonomous non-capacity creating expenditure at same time and permanently, hence the need of a reflection on the choice of the pertinent components and their

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<sup>2</sup> This outcome is conditioned by the value of the models' parameters: the accelerator effect resulting from entrepreneurs' attempts to adjust their investment must be lower than the supermultiplier effect generated by the autonomous expenditure component.

articulation. This issue is not considered in this article but warrants more attention in further researches.<sup>3</sup>

Second, since it focuses on the long run, this new strand of research must consider the second Harrodian instability problem regarding employment dynamic: the long-run rate of economic growth, and therefore the rate of growth of the autonomous component, must be equal to the natural rate of growth for the rate of employment to stabilize. This is not the case for the autonomous components mentioned above, except possibly for public spending if the government's objective is the employment rate stabilization.

The aim of this article is to propose a solution to this second issue. We draw on Allain (2019) which considers that consumption financed by the wage bill can include an autonomous consumption component. Indeed, through formal and informal distribution devices, the wage bill finances most, if not all, consumption of unemployed workers as well as inactive people (pensioners, breadwinners' relatives...)<sup>4</sup> This wage bill redistribution, however, does not constitute a sufficient condition to the emergence of an autonomous consumption component. For this to happen, the redistribution must be equal to the average propensity to consume out of wages. This is the case, for instance, when the size of a family gets bigger after the birth of a child: the family modifies the distribution of its income among its members to provide the child with a standard of living at least equal to the subsistence level. In other words, the average propensity to consume is reduced so that each member can satisfy its primary needs.<sup>5</sup> As showed in Allain (2019), this kind of behavior generates, at the aggregate level, an autonomous consumption component whose rate of growth is exogenously given by the rate of growth of population.<sup>6</sup> This rate of growth then attracts to it the rate of economic growth (depending on the model's

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<sup>3</sup> Fiebiger (2017) as Fiebiger and Lavoie (2017) use the term of semi-autonomous expenditures. However, they do not analyze the articulation between different kind of semi-autonomous expenditures. Part of Nikiforos' (2018) criticisms of supermultiplier models is based on the ground that autonomous expenditures are unlikely autonomous in the long run.

<sup>4</sup> Please just consider that there were just 32 million people in work in 2018 in Great Britain for almost 65 million inhabitants.

<sup>5</sup> Let us note that this set of assumptions is fully consistent with the principles of satiability, separability and subordination of needs that can be at the root of the Post-Keynesian consumer choice theory (see Lavoie, 2004, 2014).

<sup>6</sup> An underlying assumption is that demographic growth remains exogenous in the long run. We therefore do not retain the assumption adopted by some Malthusian, Marxian or Sraffian economists who claim that population is endogenous with respect to output growth (through migrations, for instance).

parameters). Consequently, the rate of employment stabilizes at a level that can differ from full employment, provided that there is no technical progress.

In this article, we go one step further by including technical progress in the model. The first consequence of this inclusion is destabilizing: if there are gains in labor productivity while the output increases only at the rate of demographic growth, there will be a decline in the rate of employment period after period. However, we must also consider that what is regarded as subsistence level of consumption (or material primary needs) varies from one period to another, so that the level of resources needed to achieve this level increases as societies become wealthier. The crucial consequence is that the dynamic of autonomous consumption component now corresponds to the natural rate of growth, that is the sum of the rate of growth of population and the rate of technical progress. Accordingly, this is the rate of growth at which economic growth will stabilize (if the supermultiplier effect remains higher than the accelerator effect).

The article is organized as follows. The consumption function that incorporates an autonomous component growing at the natural rate of growth is elaborated in Section 2. In Section 3, this consumption function is included in a basic neo-Kaleckian model<sup>7</sup> and the properties of the short-run equilibrium are analyzed. ‘Short run’ here means that we temporarily assume both that the autonomous consumption remains constant over time and that entrepreneurs do not adopt the Harrodian investment behavior. This temporary assumption are removed in Sections 4 and 5 which are respectively devoted to the algebraic and economic analyses of the long-run equilibrium. In addition to exogenous population growth, we assume a Harrod-neutral technical progress that is partly endogenous: it depends positively on capital accumulation (according to the Kaldor-Verdoorn’s law) and negatively on the profit share (according to Marx’s intuition that a decrease in the profit share encourages firms to improve labor productivity). The firms’ Harrodian investment behavior is also included. It is shown that the system converges

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<sup>7</sup> A distinction can be made between the Sraffian supermultiplier models proposed by Serrano (1995a, 1995b), Bortis (1997), Freitas and Serrano (2015), Serrano and Freitas (2017) or Brochier and Macedo e Silva (2019) and the neo-Kaleckian supermultiplier models proposed by Allain (2015, 2019), Lavoie (2016), Nah and Lavoie (2017). The main difference regards the specification of the investment function: Sraffians assume that investment depends on a propensity to invest the aggregate income while neo-Kaleckians assume that investment depends on both the entrepreneurs’ expectation of the secular rate of growth and the gap between the actual and the normal rate of capacity utilization. This latter approach is adopted here.

towards its long-run equilibrium depending on the value of some model's parameters. In the long-run equilibrium, the rate of capacity utilization is equal to its normal level while the rate of capacity accumulation corresponds to the natural rate of growth. Accordingly, the rate of employment stabilizes, but at a level that depends on the parameters. An important feature of our model is that it replicates the central result of Solow's model: in both cases, the long-run rates of economic growth converges towards the natural rate of growth. This outcome, however, is not generated by the same mechanisms since Solow's model is supply-driven while our model is demand-driven. Moreover, some crucial implications diverge, especially regarding employment: Solow built a full employment model while our model is consistent with a positive, stabilized rate of involuntary unemployment in the long run. Section 6 is devoted to this comparison between the two approaches. Section 7 concludes.

## **2. Income distribution, wages redistribution and the consumption function**

In Post-Keynesian models, consumption basically depends on both national income and income distribution. Occasionally, an autonomous component is included but the presence of this component, as well as its dynamics over time, lack of theoretical justifications. Our main objective here is to provide such justifications and to show that, starting from intuitive assumptions, the rate of growth of this autonomous component can be equal to the natural rate of growth.

First, for sake of simplicity and without any loss of generality, we assume that capitalist do not consume so that profits are fully saved.

Second, as in Allain (2019), we assume that the wage bill received by  $L$  workers enables the consumption of all the rest of the population,  $N$ : employed workers, but also unemployed and inactive population (breadwinner's dependents, pensioners, etc.). Actually, there are many redistributive devices enabling inactive people to get a share of the wage income: formal devices through the pension or unemployment benefits systems, but also informal devices through the spontaneous redistribution within families between breadwinners and their dependents. Noting  $e = L/N$  the rate of employment, the inverse of  $e$  can be viewed as a measure of the dependency ratio of this economy. The greater or lesser generosity of the redistribution system will be taken into account by the introduction of an exogenous parameter,  $\alpha$ .

Third, material primary needs (food, clothing, shelter, etc.) are explicitly introduced in the analysis, which is fully consistent with the Post-Keynesian consumer choice theory reviewed by Lavoie (2004, 2014). The underlying assumptions are the principles of satiability, separability and subordination of needs: consumers make an allocation of their budget among needs relatively independently of what happens to the other needs (separability); there exist a pyramid (or hierarchy) of needs such as primary needs have to be fulfilled first (subordination); when a need is satisfied (or when a threshold level for that need has been reached), individuals start attending to subsequent needs.<sup>8</sup> A logical consequence of what precedes is that individual do not save if their material primary needs are not met. This gives theoretical foundations to Keynes's assumption of an increasing average propensities to save at the individual level: 'For the satisfaction of the immediate primary needs of a man and his family is usually a stronger motive than the motives towards accumulation, which only acquire effective sway when a margin of comfort has been attained. These reasons will lead, as a rule, to a *greater proportion* of income being saved as real income increases' (Keynes, 1936, p. 97, italics in original).<sup>9</sup> In the following model, this will be implemented by considering that the post-redistribution wage income share devoted to primary needs corresponds to a lump sum expenditure and that people start saving only on the part of their extra post-redistribution income. In addition, the marginal propensity to save out of the wages is assumed to be constant ( $0 \leq s < 1$ ).

Fourth, we assume that primary needs rest on both biological and sociological grounds. Because of this sociological dimension, we assume that material primary needs correspond to a fraction  $\beta$  of the real wage  $\omega$ .<sup>10</sup> An important consequence is that individual expenses to satisfy these needs increase with respect of the rise in the real wage resulting from labor productivity gains (see equation (2) below).<sup>11</sup> This is the main innovation compared to Allain (2019).

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<sup>8</sup> See also Drakopoulos (1999). About the inclusion of material primary needs in recent mainstream models of consumption, see, for instance, Chattopadhyay, Majumder and Coondoo (2009) and Kemp-Benedict (2013).

<sup>9</sup> About Keynes and the theory of consumption behavior, see Drakopoulos (1992).

<sup>10</sup> Such assumption is consistent with poverty definitions in relative terms. See Callan and Nollan (1991) or Foster (1998), among others, for debates about the poverty definitions.

<sup>11</sup> The underlying argument is that the amount of resources needed to achieve material primary needs increases as societies become wealthier. Therefore, the importance of those needs in consumption should not be confined to developing economies. For example, according to Jackson

Combining these assumptions, aggregate consumption in real terms corresponds to:

$$C = \alpha\beta\omega N + (1 - s)(\omega L - \alpha\beta\omega N), \quad (1)$$

where the first term on the right side corresponds to the aggregate consumption of material primary needs whereas the second term corresponds to the satisfaction of secondary needs. We do not assume that the (formal and informal) wage bill redistribution is calculated according to basic needs. However, we make a distinction between two situations: if  $\alpha = 1$ , the generosity of the system of redistribution enables everyone to satisfy their primary needs (and possibly beyond), which is not the case if  $\alpha < 1$  meaning that some people receive a post-redistribution income that is lower than  $\beta\omega$ . The average propensity to consume will be higher in the former case than in the latter. Note that for equation (1) to make sense, the wage bill must be larger than wage redistribution, that is  $\omega L > \alpha\beta\omega N$  or, simplifying,  $e > \alpha\beta$ . This condition is assumed to be fulfilled thereafter.

It may also be noted that, although the redistributive rule is not made explicit in the model, our specification is consistent with what enable most formal redistribution devices (pensions, unemployment benefits...): higher real wages cause more redistribution because of the presence of indexation rules; likewise, most unemployed or inactive people result in more redistribution. The same ought to be true in the case of informal redistribution: dependents benefit from a rise in their breadwinner's real wage; and more dependent leads to more redistribution.

We also have to keep in mind that the generosity of the redistribution system results from a social compromise that evolves over time. It can be questioned in many circumstances, especially when there is a decrease in the rate of employment (surge in unemployment, aging of the population, immigration, etc.).

Noting respectively  $\pi$  and  $y$  the profit share and labor productivity, the real wage corresponds to:

$$\omega = (1 - \pi)y \quad (2)$$

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and Marks (1999) on UK data, consumers devoted about half of their budget in 1954 and nearly one-third in 1994.

Substituting  $\omega$  and  $L = eN$  in (1), the consumption function in real terms can be rewritten as follows:

$$C = (1 - s)(1 - \pi)Y + s\alpha\beta(1 - \pi)yN, \quad (3)$$

where  $Y$  represents the national income. The first term on the right side of the equation corresponds to induced consumption whereas the second term corresponds to autonomous consumption.<sup>12</sup> As emphasized in Allain (2019), autonomous consumption is made ‘effective’ because the redistribution scheme is generous enough to make ‘effective’ the lump sum expenditure corresponding to (at least a part of) primary needs. It is worth noting that, while the induced component varies with the actual rate of growth of the economy, the autonomous component varies with the natural rate of growth. Actually, at the aggregate level, the increase in primary needs depends on the rate of demographic growth ( $\hat{N} = n$ ) because the more people there are, the more mouths to feed and bodies to clothe and shelter. However, because of their social dimension, the increase of primary needs also depends on the growth rate of the real wage, which is tied to the growth rate of labor productivity ( $\hat{\omega} = \hat{y}$  because of equation (2)).<sup>13</sup> Finally, the growth

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<sup>12</sup> ‘Autonomous’ must be opposed to the consumption which is ‘induced’ because related to aggregate income. As it will be made clear below, ‘autonomous’ does not mean either ‘invariant’. Indeed, the amount of autonomous consumption will be subject to exogenous (e.g. demographic growth) as well as endogenous (technical progress) variations.

Moreover, autonomous consumption differs from the amount of aggregate consumption of material primary needs (first term of equation 1). Autonomous consumption depends on both primary needs and the propensity to save. First, in the absence of primary needs ( $\beta = 0$ ) or wage redistribution ( $\alpha = 0$ ), consumption would be fully endogenous, depending on the wage bill and the propensity to consume  $1 - s$ . However, if the (marginal) propensity to save was zero, consumption would be equal to the wage bill: a part of the bill would be devoted to primary needs whereas the other part would be devoted to other needs. Finally, autonomous consumption corresponds to the part of the aggregate consumption of primary needs which would have been saved if  $\alpha = \beta = 0$ .

<sup>13</sup> An important outcome of our assumptions is that, everything else being equal, an increase in the wage bill resulting from labor productivity gains does not affect the average propensity to save (thanks to the increase in autonomous consumption in the same proportion). Conversely, omitting the productivity gains would result in an increase in the average propensity to save. In the long run, data does not exhibit any positive trend the aggregate saving rates. On the contrary, the trend since 1970 is rather negative in many OECD countries such as U.S.A. (see Guidolin and La Jeunesse (2007) for instance), Germany, Japan, U.K., etc. (see OECD (2019), Saving rate (indicator). doi: 10.1787/ff2e64d4-en; accessed on 19 June 2019). Piketty (2011) even displays a remarkable stability of the private savings rate in France around 8-10% since 1820. Of course, the long-run dynamic of the saving rate can be explained in different ways. However, while remaining cautious, we can say that it is better to index autonomous consumption on real wages (and therefore on labor productivity) rather than not doing so.



rate of autonomous consumption is determined by the natural rate of growth, that is the sum of the growth rates of population and labor productivity ( $n + \hat{y}$ ).

### 3. The short-run equilibrium

In the short-run, we adopt the basic neoclassical framework of a closed economy without government. We assume a production function with fixed technical coefficients, that is:

$$Y = \min\left(yL, \frac{1}{v}K\right) = yL = \frac{u}{v}K, \quad (4)$$

where  $K$  represents the capital stock and  $v$  the exogenous capital coefficient. Assuming no capital or labor shortage, a change in the level of production requires a change in both the level of labor and the rate of capacity utilization,  $u$ .

We also assume that the price setting rule corresponds to an exogenous mark-up over the unit cost. Accordingly, the profit share  $\pi$  is exogenous, too.

In addition, the net rate of capital accumulation is assumed to depend on the rate of growth expected by entrepreneurs,  $\gamma$ , and on the gap between the actual and the natural rate of capacity utilization,  $u_n$ . We thus have:

$$g^i = \frac{I}{K} = \gamma + \gamma_u(u - u_n), \quad (5)$$

where  $\gamma_u > 0$ .

From (3) and assuming capital depreciation at an exogenous rate,  $\delta$ , the saving function in terms of capital stock corresponds to:

$$g^s = \frac{S}{K} = \frac{[\pi + s(1 - \pi)]}{v}u - [s\alpha\beta(1 - \pi)y\eta + \delta]. \quad (6)$$

where  $\eta = N/K$  represents the population-to-capital ratio.

In the short run,  $\eta$ ,  $y$  and  $\gamma$  are assumed to be exogenously given. At the goods market equilibrium,  $g^i = g^s$ . The resulting short-run equilibrium rate of capacity utilization is thus:

$$u^* = \frac{v[\gamma + \delta - \gamma_u u_n + (1 - \pi)s\alpha\beta y\eta]}{\pi + s(1 - \pi) - v\gamma_u} \quad (7)$$

The denominator of equation (7) is supposed to be negative for the Keynesian stability condition to be fulfilled.<sup>14</sup> Accordingly, the numerator must also be positive for the rate of capacity utilization to be positive. In addition, the equilibrium rate of capital accumulation is given by:

$$g^* = \gamma + \gamma_u(u^* - u_n), \quad (8)$$

while the equilibrium rate of employment corresponds to:

$$e^* = \frac{u^*}{\nu y \eta}. \quad (9)$$

Finally, the necessary condition  $\alpha < e^* / \beta$  imposes that:

$$y \eta < \frac{\gamma + \delta - \gamma_u u_n}{\alpha \beta (\pi - \nu \gamma_u)}. \quad (10)$$

In other words, the redistribution of the wage income from employed workers to unemployed and inactive people enables to meet the primary needs of everyone only if two conditions are combined together. First, the population must not be too important in terms of production capacity (a low level of  $\eta$ ). Second, labor productivity  $y$  must not be too high either for two reasons: because a high level in  $y$  implies a low level in the rate of employment therefore a high level in the dependency ratio ( $1/e$ ); and also because a high level in  $y$  implies a high level of individual primary needs ( $\beta\omega$ ), which depends on real wages that themselves depend on labor productivity (equation 2).

The comparative static outcomes are summarized in Table 1. Without surprise, we find the usual outcomes of neo-Kaleckian models. The economy is both stagnationist and wage-led because a rise in  $\pi$  results in a decrease in both  $u^*$  and  $g^*$ . The paradox of thrift occurs because a rise in the marginal propensity to save implies a decline in economic activity. Finally, an increase in  $\gamma$  (improvement in animal spirits) results in a rise in  $u^*$ ,  $g^*$  and  $e^*$ .

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<sup>14</sup> In other words, the model satisfies  $dg^s/du > dg^i/du$ . Please note that the intercept of the  $g^s$  curve being negative, the intercept of the  $g^i$  curve can also be negative, provided that  $-[s\alpha\beta(1-\pi)y\eta + \delta] < \gamma + \delta - \gamma_u u_n$ .

<b>Table 1.</b> Comparative static assuming that $\alpha < e^*/\beta$ . Short-run equilibrium: $\eta = N/K$ , $y = Y/L$ and $\gamma$ are exogenous.										
	$s$	$\pi$	$\alpha\beta^{(a)}$	$y$	$\eta$	$\nu$	$\delta$	$\gamma$	$\gamma_u$	$u_n$
$u^*$	-	-	+	+	+	+	+	+	+ / -	-
$g^*$	-	-	+	+	+	+	+	+	+ / -	-
$e^*$	-	-	+	+ / -	+ / -	+	+	+	+ / -	-
(a) The two parameters $\alpha$ and $\beta$ are formally interchangeable in the model. The qualitative consequences of a change in either $\alpha$ , or $\beta$ or $\alpha\beta$ are identical.										

Regarding the autonomous consumption component, an increase either in the fraction of the real wages considered as primary needs ( $\beta$ ) or in the generosity of the redistribution system ( $\alpha$ ) causes an increase in aggregate demand then a rise in both  $u^*$ ,  $g^*$  and  $e^*$ . Likewise, an increase in labor productivity ( $y$ ) or in the population-to-capital ratio ( $\eta$ ) implies an increase in both  $u^*$  and  $g^*$ : with a higher  $\eta$ , a larger share of the capital stock has to be devoted to the primary needs; the same is true if labor productivity increases because it increases real wages (see equation 2), which have a positive effect on primary needs. The resulting growth on economic activity generates a positive effect on the rate of employment. However, this effect can be offset by a negative effect: everything else being equal, a larger population results in a lower rate of employment, which requires more redistribution between employed workers and unemployed or inactive people; likewise, higher labor productivity makes it possible to produce the same output with less labor. It can be shown that the negative effect dominates if  $\gamma + \delta - \gamma_u u_n > 0$ : a rise in either  $\eta$  or  $y$  therefore causes a decrease in  $e^*$ . The opposite consequences occur if  $\gamma + \delta - \gamma_u u_n < 0$ .

#### **4. The long-run equilibrium: algebraic analysis**

In the previous section, the value of some parameters was assumed to be given while these parameters are subject to long-run dynamics.<sup>15</sup> This is the case for the population-

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<sup>15</sup> In the past, this type of models was presented by distinguishing a medium-run analysis (introducing the supermultiplier effects) with the long-run analysis (adding the entrepreneurs' Harrodian investment behavior). Such distinction is useful for pedagogical purpose as it allows a sequential analysis of mechanisms operating in opposite directions. However, this distinction can lead to the mistaken belief that the mechanisms themselves operate sequentially while they are

to-capital ratio ( $\eta$ ) which varies according to the difference between the rate of growth of population ( $n$ ) and the rate of capital accumulation:<sup>16</sup>

$$\hat{\eta} = n - g, \quad (11)$$

In addition, we assume Harrod-neutral technical progress resting on the following labor productivity growth function:<sup>17</sup>

$$\hat{y} = \lambda + \lambda_g g - \lambda_\pi \pi. \quad (12)$$

Exogenous productivity gains are encapsulated in the  $\lambda$  parameter, which can be positive or negative. The second term,  $\lambda_g g$  where  $0 \leq \lambda_g < 1$ , refers to the Kaldorn-Verdoorn's law according to which labor productivity increases with capital accumulation, particularly because technical progress is capital embodied. The last term,  $-\lambda_\pi \pi$  where  $\lambda_\pi \geq 0$ , is based on Marx's intuition that a decrease in the profit share encourages firms to improve labor productivity in order to restore their profits.

The consequence of these two first assumptions is that the rate of growth of autonomous consumption (the second term on the right side of equation 3) is given by the natural rate of growth ( $n + \hat{y}$ ). It is the presence of this non-generating capacity autonomous component that will result in a supermultiplier effect: both the rate of capital accumulation and the rate of economic growth will converge towards the natural rate of growth.<sup>18</sup>

Finally, we introduce a Harrodian investment behavior assuming that entrepreneurs adjust their expected rate of growth depending on the gap between their expectation and the actual rate of economic growth:

$$\dot{\gamma} = \psi(g - \gamma). \quad (13)$$

The  $\psi$  parameter measures the speed of adjustment of entrepreneur's expectations ( $\psi \in [0,1]$ ). As shown in Allain (2015, 2019), Freitas and Serrano (2015), Lavoie (2016) and

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contemporaries of each other. For this reason, we prefer to conduct a simultaneous analysis of these different effects in the article.

<sup>16</sup> A dot over a variable is used to indicate a rate of change ( $\dot{x} = dx/dt$ ) while a hat is used to indicate a growth rate ( $\hat{x} = \dot{x}/x$ ).

<sup>17</sup> See Cassetti (2003), Hein (2014, ch.8) and Lavoie (2014, ch.6) among others.

<sup>18</sup> Please note that the definition of the rate of growth of the non-generating capacity autonomous component is more complex than in most models where it is limited to one single, exogenous parameter. See Brochier and Macedo e Silva (2019) which proposes an endogenous definition of this rate of growth.

many others, the model suffers from Harrodian, knife-edge instability if the value of this parameter is too high: a fall in  $u$  results in a decrease of  $\gamma$  through equation (8), which induces another fall in  $u$ , *etc.* However, as will be re-established below, if  $\psi$  is low enough, the Harrodian instability can be tamed by the supermultiplier effect introduced above. The main outcome is then that the actual rate of capacity utilization converges towards its normal level in the long run.

The three following conditions must be fulfilled for the economy to be at a long-run equilibrium:

$$\begin{cases} u = u^* \\ \dot{\gamma} = 0 \\ g = n + \hat{y} \end{cases} \quad (14)$$

The first equation of system (14) remind that the long-run equilibrium also corresponds to a short-run equilibrium. Moreover, the long-run equilibrium rate of capacity utilization must be a position of rest although  $u^*$  in equation (7) is now subject to the dynamic of three parameters:  $\gamma$ ,  $y$  and  $\eta$ . In the long run, the entrepreneurs' expected secular rate of growth ( $\gamma$ ) must stabilize according to the second equation of system (14). However, the labor productivity parameter ( $y$ ) varies over time with technical progress (equation 12). As a consequence, for the product  $y\eta$  to stabilize in equation (7), the variations in  $y$  must be counterbalanced by those of the population-to-capital ratio. We therefore must have  $\hat{\eta} = -\hat{y}$  or, substituting with equation (11),  $g = n + \hat{y}$ , which is the third equation in (14).

The resulting long-run equilibrium is given as follows:

$$u^{**} = u_n, \quad (15)$$

$$\hat{y}^{**} = \frac{\lambda + \lambda_g n - \lambda_\pi \pi}{1 - \lambda_g}, \quad (16)$$

$$g^{**} = \gamma^{**} = n + \hat{y}^{**} = \frac{n + \lambda - \lambda_\pi \pi}{1 - \lambda_g}, \quad (17)$$

$$e^{**} = \frac{(1 - \pi) s \alpha \beta u_n}{[\pi + s(1 - \pi)] u_n - \nu(n + \hat{y}^{**} + \delta)}. \quad (18)$$

Please note that  $n + \hat{y}^{**} + \delta$  in the denominator of equation (18) corresponds to the gross natural rate of growth of the economy.

In addition, two necessary conditions must be fulfilled for this outcome to make sense:

$$\pi u_n < \nu(n + \hat{y}^{**} + \delta) < [\pi + s(1 - \pi)] u_n, \quad (20)$$

where the first inequality corresponds to the condition that  $\beta < e^{**}$  while the second one corresponds to the condition that (the denominator of)  $e^{**}$  is positive.

The local stability conditions depend on the dynamics of both  $\gamma$ ,  $y$  and  $\eta$ . In accordance with what was written above, the linear condition  $g = n + \hat{y}$  in system (14) is equivalent to the condition that  $\dot{y}\eta = 0$ . The resulting Jacobian matrix is given as follows:

$$J = \begin{pmatrix} \frac{\partial(y\eta)}{\partial(y\eta)} & \frac{\partial(y\eta)}{\partial\gamma} \\ \frac{\partial\gamma}{\partial(y\eta)} & \frac{\partial\gamma}{\partial\gamma} \end{pmatrix}. \quad (21)$$

After linearization, we get:

$$J = \frac{1}{D - v\gamma_u} \begin{pmatrix} -(1 - \lambda_g)\gamma_u(u_n D - vB^{**}) & -\frac{(Du_n - vB^{**})D(1 - \lambda_g)}{vA} \\ \psi\gamma_u vA & \psi\gamma_u v \end{pmatrix}, \quad (22)$$

with  $A = (1 - \pi)s\alpha\beta$ ,  $B^{**} = n + \hat{y}^{**} + \delta$  and  $D = \pi + s(1 - \pi)$ .

For the system to reach a long-run equilibrium, the determinant of the Jacobian matrix must be positive while its trace must be negative. The determinant is given as follows:

$$DET(J) = \frac{\psi\gamma_u v(1 - \lambda_g)(Du_n - vB^{**})\left(\frac{D}{v} - \gamma_u\right)}{(D - v\gamma_u)^2}. \quad (23)$$

It can be shown that the  $DET(J) > 0$  provided that both  $e^{**} > 0$  and the Keynesian stability condition holds so that  $D > v\gamma_u$ . The trace of  $J$  is given by:

$$TR(J) = -\frac{\gamma_u}{D}(Du_n - vB^{**} - \psi v). \quad (24)$$

Substituting (19) in (24) and rearranging, the condition for  $TR(J)$  to be negative is:

$$\psi < \frac{u_n}{v} [\pi + s(1 - \pi)] - (n + \hat{y}^{**} + \delta), \quad (25)$$

where the right member corresponds to the ratio of autonomous consumption relative to the capital stock.<sup>19</sup> That means that autonomous consumption (in terms of capital stock) must be higher than the speed of adjustment of entrepreneur's expectations for the multiplier effect to be higher than the accelerator effect and for the system to converge

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<sup>19</sup> Autonomous consumption in primary goods corresponds to the second term on the right side of equation (3). Dividing by  $K$ , substituting with equations (2), (9), (15) and (18) and rearranging leads to the right member of the condition (25).

towards its long-run equilibrium. Otherwise, the accelerator effect dominates and knife-edge Harrodian instability prevails.

Radical uncertainty and animal spirits provide arguments in favor of a relatively low level of the  $\psi$  parameter. According to Freitas and Serrano (2015, p. 13), a “drastic adjustment is highly unrealistic, both because of the durability of fixed capital (which means that producers want normal utilisation only on average over the life of equipment and not at every moment) and also because producers know that demand fluctuates a good deal and therefore do not interpret every fluctuation in demand as indicative of a lasting change in the trend of demand”. We adopt the same argument to assume that condition (25) holds.<sup>20</sup>

## 5. The long-run equilibrium: economic analysis

The main outcomes of the model can be summarized as follows. First, if condition (25) is fulfilled, then the system converges towards its steady-state equilibrium because the stabilizing supermultiplier effects resulting from the satisfaction of primary needs is larger than the destabilizing effect of the entrepreneurs’ Harrodian investment behavior (through the  $\psi$  parameter). Accordingly, firms reach their normal rate of capacity utilization,  $u_n$ , without generating knife-edge instability.

Second, the long-run rate of growth ( $g^{**}$ ) converges towards the natural rate of growth (equation 17). This implies among other things that the rate of employment also stabilizes in the long run. Therefore, the two Harrodian instability problems are solved simultaneously. Another important implication is that the rate of growth of income per capita is given by the gains in labor productivity.<sup>21</sup> Note in addition that the natural rate of growth is partly endogenous since labor productivity growth now depends positively on demographic growth and negatively on the profit share (equation 16).

Third, these above outcomes do not imply that economic shocks have no effect on the long-run equilibrium. Actually, shocks that have only a temporary effect on either  $u^{**}$  or

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<sup>20</sup> Condition (25) is at the heart of a lively debate between proponents and opponents of the type of approach adopted here. On the opponents’ side, Skott (2017) argues that the adjustment resulting from this condition is too slow for the model to be of practical significance. See Nikiforos (2018), Franke (2019) and Skott (2019) for other criticisms and Lavoie (2017) for a response to Skott (2017).

<sup>21</sup> At the long-run equilibrium, the income per capita is  $Y/N = ye^{**}$  whose rate of growth is equal to  $\hat{y}^{**}$ .

$g^{**}$  can have permanent effect on the level of some important variables. That is especially the case for the output and the capital stock, as well as for the level of labor productivity. This is also the case for the level of the rate of employment (equation 18), implying that the long-run steady-state equilibrium does not correspond to a full-employment equilibrium.

Convergence towards the normal rate of utilization of capacity  $u_n$  is a common result of models that, as here, combine a supermultiplier effect with entrepreneurs adjusting their expectations. However, the mechanism leading to this outcome must be made explicit, especially since stabilization seems at first sight incompatible with the continuous increase in labor productivity that destabilizes the short-term equilibrium (see equation 7 in which  $y$  increases at every period). To understand the underlying mechanisms, let us assume that the economy has reached a long-run equilibrium at a given point of time, which means among other things that  $g = n + \hat{y}$  (see equation 17). This equilibrium is instantaneously jeopardized because of labor productivity gains (assuming that  $\hat{y} > 0$ ). Actually, as stated above in the short-run, a rise in  $y$  results in an increase in  $u^*$ : productivity gains have a positive impact on real wages, which in turn affect primary needs, which has a positive effect on economic activity. However, this mechanism is counterbalanced by an inverse mechanism: because of equation (17), the capital stock is growing faster than the population, therefore a decrease in the  $\eta$  parameter, which results in a decrease in  $u^*$ : a lower share of the capital stock has to be devoted to satisfying primary needs. Accordingly, the product  $y\eta$  in equation (7) remains stable despite the presence of productivity gains. Likewise, the balance between the two opposite effects on  $y$  and  $\eta$  leads to a stable rate of employment (equation 18): in short, the possibility to produce the same output with less labor is counterbalanced by the need of a greater redistribution between employed workers and unemployed or inactive people.

The long-run comparative static analysis provides a more in-depth understanding of the mechanisms and properties of the model (see Table 2).



<b>Table 2.</b> Comparative static assuming that $\alpha\beta < e^{**}$ . Long-run equilibrium: $\eta = N/K$ , $y = Y/L$ and $\gamma$ are endogenous.											
	$s$	$\pi$	$\alpha\beta$	$\nu$	$\delta$	$\gamma_u$	$u_n$	$\lambda$	$\lambda_g$	$\lambda_\pi$	$n$
$u^{**}$	0	0	0	0	0	0	+	0	0	0	0
$g^{**} = \gamma^{**}$	0	-	0	0	0	0	0	+	+	-	+
$\hat{y}(= -\hat{\eta})$	0	-	0	0	0	0	0	+	+	-	+
$e^{**}$	-	-	+	+	+	0	-	+	+	-	+

*Animal spirits.* Starting from a long-run equilibrium position, assume an increase in the value of the  $\gamma$  parameter because of entrepreneurs' optimistic expectations. This change results in an increase in both  $u^*$ ,  $g^*$  and  $e^*$  in the short run (see Table 1). In a longer run, however, the increase in  $g^*$  generates two opposite effects. First, it encourages a further upward adjustment in entrepreneurs' expectations ( $\dot{\gamma} > 0$ ), which drives the system in positive knife-edge instability. Second, the increase in  $g^*$  also results in a decrease in the population-to-capital ratio, which makes it possible to devote a lower share of the capital stock to the satisfaction of primary needs, a shift that hinders economic activity and growth. Eventually, the two opposite effects neutralize each other so that the two Harrodian instability problems are solved simultaneously: knife-edge instability is tamed as the rate of capacity utilization converges back towards its normal value  $u_n$ , while both the rate of accumulation and entrepreneurs' expectation converge back towards the natural rate of growth.

However, it must be emphasized that this return to the long run equilibrium does not mean that nothing has changed in the economy: the transitory increases in both  $u^*$  and  $g^*$  positively affects the levels of both the output and capital stock, which remain permanently higher than they would have been if entrepreneurs had not first form optimistic expectations. The dynamic of these variables in level is therefore subject to path dependency in the long run. This is also the case for labor productivity: the transitory increase in  $g^*$  causes a temporary rise in productivity gains (see equation 12). Accordingly, labor productivity remains permanently higher in the long-run. This explains why the long-run rate of employment ( $e^{**}$ ) is not affected despite a permanent increase in the production per capita.

*Paradox of thrift.* The same outcomes as above result from an decrease in the marginal propensity to save that can be caused by either a decrease in the propensity to save out of wages ( $s$ ), an increase in the share of the real wage devoted to primary needs ( $\beta$ ) or an increase in the generosity of the redistribution system ( $\alpha$ ): the short-run surge in both  $u^*$  and  $g^*$  is offset by a further deterioration caused by the decrease in the relative weight of the primary needs in aggregate demand; the rate of capacity utilization and rate of accumulation are not affected in the long run. Therefore, the paradox of thrift no longer holds in the strict sense.

However, as pointed in other articles, a weaker version of the paradox still applies since the levels of both output and capital stock are permanently affected. In addition, the long-run employment rate ( $e^{**}$ ) also stabilizes at a higher level than in the initial equilibrium. Actually, the redirection of a part of the wage bill towards consumption to the detriment of savings supports aggregate demand, which causes a permanent increase in the level of employment.

*Paradox of costs.* A decrease in the profit share ( $\pi$ ) leads to temporary increases in  $u^*$  and  $g^*$ . The innovation is now that firms have more incentives to improve labor productivity, which results in a higher long-run rate of growth of labor productivity (equation 16). Therefore, the final value of the natural rate of growth ( $g^{**}$ ) is higher than in the initial equilibrium. Likewise, the level of variables such as  $Y$ ,  $K$  and  $y$  are permanently positively affected, as is the rate of employment  $e^{**}$ .

These outcomes can be considered as a weak version of the paradox of costs. The strict version is, however, no longer fulfilled because a decrease in  $\pi$  implies a decrease in the long-run rate of profit,  $r^{**} = u_n \pi / \nu$ . Capitalists objectives are therefore clearly conflicting with those of workers or of the 'economic system' itself. This outcome differs from those of the neo-Kaleckian basic models in which capitalists are subject to a fallacy of composition since a decrease in  $\pi$  causes an increase in their rate of profit.

*Demographic growth and technical progress.* Although a rise in the population growth rate ( $n$ ) has no effect on the long-run rate of capacity utilization, it results in an increase in the natural rate of growth, both because  $n$  is a component of  $g^{**}$  and because of higher productivity gains because of the Kaldor-Verdoorn law (equation 16). In addition, despite higher levels of population and labor productivity, the rate of employment stabilizes at a

higher level than in the initial situation because the positive multiplier effect of basic needs on economic activity.

Likewise, a rise in the pace of exogenous technical progress growth ( $\lambda$ ) leads to a higher rate of growth of labor productivity and therefore to a higher natural rate of growth. It also causes an increase in the long-run rate of employment because of two opposite effects. First, in accordance with the usual outcome of Post-Kaleckian models, greater technical progress makes it possible to produce a given level of effective demand with less labor. However, this effect is more than offset by the positive impacts of technical progress on, successively, the real wage (equation 2), autonomous consumption (primary needs) and effective demand. Eventually,  $e^{**}$  increases because technical progress sustains the standard of living of breadwinners, and therefore the wage redistribution towards their dependents and other non-workers.

These comparative static results have implications for debates involving Post-Keynesian economists. We focus below on secular stagnation, welfare state and counter-cyclical policies.

*Secular stagnation.* The above model could be a basis for pursuing the debate on secular stagnation,<sup>22</sup> especially since it seems at first glance to take the opposite direction to Keynesians' expectations. Indeed, in our model, secular stagnation results from either slow demographic growth or slow labor productivity gains. This seems closer to Gordon (2012, 2014) than to Summers (2014) or Krugman (2014); and this seems very far from Hein (2016). However, in our model, secular stagnation unambiguously results from a too low growth in aggregate demand. In fact, if labor productivity gains ( $\hat{y}^{**}$ ) generate economic growth, it is not because they make it possible to produce more with fewer workers but because they lead to an increase in real wages, which drives the autonomous consumption relative to basic needs. Although the mechanism differs radically, the cause is essentially the same: a weak growth in labor productivity can be explained first and foremost by the weakness of exogenous technical progress.<sup>23</sup> The sluggishness can be reinforced by the cumulative causation of the Kaldorn-Verdoorn's law, but also by the secular increase in the profit share that has been recently documented in ILO et al. (2015)

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<sup>22</sup> See Hein (2016) for a Post-Keynesian (more precisely, a Post-Steindlian) perspective.

<sup>23</sup> As pointed by Gordon (2012, 2014), the growth in labor productivity was much stronger during the 'thirty glorious' than since the eighties.

and IMF (2017).<sup>24</sup> Otherwise speaking, the positive effects of labor productivity gains on economic growth are lower if they are captured by capitalists through a rise in their earnings.

Likewise, if demographic growth ( $n$ ) generates economic growth, it is not because it increases labor supply in a full employment model<sup>25</sup> but because it leads to an increase in the autonomous consumption relative to basic needs: more population corresponds to more mouths to feed and bodies to clothe and shelter. It is worth noting that the level of income per capita is also impacted even in the absence of productivity growth. Indeed, in the long run, we have:

$$\frac{Y}{N} = \frac{Y}{L} \frac{L}{N} = ye^{**}, \quad (26)$$

where  $y$  remains constant if there is no productivity gains (i.e., if  $\lambda = \lambda_g = \lambda_\pi = 0$ ). However, as seen above, an increase in  $n$  corresponds to in a higher equilibrium rate of employment ( $e^{**}$ ). The underlying causality is here as follows: a higher rate of growth of the population results in a higher multiplier effect of primary needs on economic activity, which both causes a rise in income per capita and makes it necessary to increase the employment rate. Conversely, of course, a decrease in  $n$  results in a decrease in both the natural rate of growth,<sup>26</sup> the income per capital and the rate of employment.

*Redistribution, welfare state and automatic stabilizers.* Our model reaffirms the positive role of the welfare state in a capitalist economy, a point already emphasized in Allain (2019). However, the magnitude of the wage bill redistribution is not limited to formal mechanisms such as pension or unemployment benefits systems. Informal channels, especially within families, can also generate very high redistribution.

Note that the autonomous consumption stemming from redistribution plays the role of automatic stabilizer. Indeed, starting from equation (3), it is possible to show that the rate of capacity utilization elasticity of consumption (in terms of capital) is lower than unity, that is:

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<sup>24</sup> See equation (16).

<sup>25</sup> According to Gordon (2012, 2014), the slowdown in the increase of 'effective' labor supply is due to the fact that both the female employment rate and the level of education have reach a ceiling in recent decades.

<sup>26</sup> Partly because of cumulative causation (see equation 16).

$$\frac{d\left(\frac{C}{K}\right)}{d\hat{u}} = \frac{\frac{(1-s)(1-\pi)u}{v}}{\frac{(1-s)(1-\pi)u}{v} + s\alpha\beta(1-\pi)\gamma\eta} < 1. \quad (27)$$

In other words, because of autonomous consumption (the second term in the denominator), any deviation in the rate of capacity utilization causes a deviation in  $C/K$  in the same direction but of a lower magnitude. Autonomous consumption thus dampens the impact of activity shocks on household consumption expenditure.

The generosity of the redistribution system is measured by the  $\alpha$  parameter. As already mentioned, less generosity (i.e. a decrease in the level of  $\alpha$ ) do not have long-run consequences on either the rate of capacity utilization or the rate of capital accumulation. However, it results in lower levels of aggregate income, income per capita, capital stock, and employment rate.

Consequently, the model can shed new light on some important societal questions such as immigration. An afflux of migrants corresponds to a rise of the rate of growth of population. According to our model, the economic outcomes will depend on the impact on the redistribution system. If migrants benefit from the redistribution system as other people, immigration will have positive effects on economic growth as well as on income per capita and the rate of employment.<sup>27</sup> Conversely, if migrants do not benefit from the redistribution system or if immigration involves a decrease in the system generosity, then both income per capita and the rate of employment can be subject to a decrease although the rate of economic growth converges towards the higher level of the natural rate in the long run.

*Economic policies.* Finally, although the government and the central bank are not introduced into the model, it seems in first sight that expansionary or counter-cyclical policies can be envisaged. If policymakers have in mind this economic model, they believe that the actual rate of capacity utilization converges towards its normal level. Therefore, they probably do not react to the differences between these two rates unless these gaps become too large or if convergence is too slow.<sup>28</sup>

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<sup>27</sup> This is the necessary condition to make their consumption demand in primary needs 'effective'.

<sup>28</sup> The slowness of the convergence is a criticism addressed by Skott (2017, 2019). See Lavoie (2017) for a response.

In addition, policymakers would believe that the only ways to increase the long-run rate of economic growth involve stimulating population (birth policy, health policy, etc.) and technical progress (research and development, education, etc.). Note that the latter can be indirectly improved by reducing the profit share, which can be implemented through adequate tax, social and labor market policies.

However, more in line with the Keynesian tradition, the use of fiscal and monetary policies could have permanent positive impacts on crucial variables such as per capita income and employment rate (although their positive impact on both the rate of capacity utilization and the rate of capital accumulation is only temporary).<sup>29</sup>

## 6. Comparison with the Solow model of economic growth

The model we presented above shares a crucial feature with the Solow model of economic growth (1956): in both cases, the long-run rates of economic growth and capital accumulation converge towards the natural rate of growth. However, the underlying mechanisms are in total opposition to each other since our model is demand-driven while Solow model is supply-driven. The purpose of this section is to point out the main oppositions between the two approaches. We assume that the reader is sufficiently familiar with the Solow model, so that its detailed presentation is unnecessary.

Nevertheless, let us remind the fundamental differential equation of Solow model, that is:

$$\dot{k} = \sigma f(k) - (n + \hat{y} + \delta)k \quad (26)$$

where  $k$  represents the capital intensity or, more precisely, the capital per 'effective unit of labor' ( $k = K/AL$  with  $A = A_0 e^{\hat{y}t}$  corresponding to a efficiency labor parameter). In addition,  $\sigma$  corresponds to the average propensity to save and  $f(k)$  to the production function. To simplify the comparison between the two models, it is assumed in this section that technical progress is both labor-augmenting and exogenous, which consists in

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<sup>29</sup> Franke (2019) criticizes the supermultiplier models by showing that their combination with another stabilizing mechanism (for instance, a monetary policy based on Taylor rule) results in a destabilizing dynamics. However, Franke assumes that the aim of the Taylor rule is to make  $u$  converging towards  $u_n$ . From my point of view, this should not be a priority for policymakers who should instead worry about the employment rate and per capita income.

assuming that  $\lambda_g = \lambda_\pi = 0$  from equation (12) so that  $\hat{y} = \lambda$ .<sup>30</sup> The two other parameters ( $n$  and  $\delta$ ) are defined as before.

As is well known, the Solow model is supply-driven because of the absence of an independent investment function. Accordingly, investment is always identical to savings, a property that makes sure Say's law fulfillment: whatever the level of aggregate supply, aggregate demand ( $C + I$ ) adjusts to provide outlet for production. The logical underlying mechanisms can be described as follows. At the beginning of every period, profit maximization and factor prices adjustment determines both factors' prices (paid at their marginal productivity), the combination of factors of production ( $k$ ) and the level of production. It is worth noting that both labor is at full employment and the rate of capacity utilization is equal to unity.

Then, households share the aggregate income between consumption and savings. Eventually, savings determines the amount of investment. In the following period, the mechanism resumes with the new capital stock and the new 'effective' labor supply, which depends on both demographic growth and the labor efficiency parameter ( $A$ ).

The comparison between the Solow model and our model is conducted assuming that the system is on its steady-state growth path before to be subject to a shift in an exogenous parameter. We first compare the consequences of a positive shock on the natural rate of growth ( $n + \hat{y}$ ). In the Solow model, this shift causes an excess in the supply of effective units of labor. It results in a decline in the wage per effective unit of labor relatively to the rental rate of capital. This is the reason why profit maximizing firms adjust the productive combination by reducing the  $k$  ratio. Moreover, this microeconomic outcome is fully consistent with macroeconomic shifts because savings (or investment) per capita is no longer high enough to maintain capital intensity at its previous level ( $\sigma f(k) < (n + \hat{y} + \delta)k$  in equation (26)). Therefore, capital intensity adjusts downward period after period until the economy reaches its new steady-state growth path corresponding to the new natural rate of growth. Eventually, the total income is greater than if the shock did not occur. In addition, labor (in effective units) has increased although the labor

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<sup>30</sup> See Solow et al. (1966) which introduces capital embodiment in the Solow model, a specification close to the Kaldor-Verdoorn's law.

market never left full employment. However, the income per unit of effective labor has decreased.<sup>31</sup>

The Post-Keynesian model presented above replicates some central outcomes of the Solow model, mainly the converges towards the natural rate of growth at the steady state equilibrium. However, crucial differences need to be emphasized. First, the adjustment mechanisms of the Post-Keynesian model are demand-driven. They depend on the changes in the relative weight of the primary needs in aggregate demand which increases because a rise in either  $n$  (more population means more primary needs to be satisfied) or  $\lambda$  (primary needs increase with labor productivity). Second, because of the rejection of factor substitution in favor of a production function with fixed technical coefficients, the long-run equilibrium capital per 'effective unit of labor' is exogenously given:  $k^{**} = v/u_n$ .<sup>32</sup> Third, the rate of capacity utilization deviates from  $u_n$  in the short run. However, it is brought back to  $u_n$  because of the combination of the supermultiplier effect and firms' Harrodian behavior: on the one hand, the rise in the natural rate of growth causes an increase in  $u^*$  that led  $g^*$  (supermultiplier effect); on the other hand, the increase in  $g^*$  encourages entrepreneurs to revise their growth expectations ( $\gamma$ ) upward (firms' Harrodian behavior). Fourth, as in the Solow model, the total income increases. Moreover, the income per capita ( $Y/N = ye^{**}$ ) now shows an unambiguous upsurge. Finally, the equilibrium rate of employment reaches a higher level, meaning that the rate of unemployment stabilizes at a lower level.

Now assume an increase in the propensity to save. The strict version of the paradox of thrift do not hold in these two models because neither the long-run rate of economic growth or the long-run rate of capacity utilization are affected. However, the short-run responses go in opposite directions. In the Solow model, because of the increase in  $\sigma$ , savings per capita is too high to maintain capital intensity at its previous level ( $\sigma f(k) > (n + \hat{y} + \delta)k$  in equation (26)). In parallel, the excess in capital causes an increase in the wage per effective unit of labor relatively to the rental rate of capital. Both argument encourages firms to increase the capital per 'effective unit of labor' ( $k$ ). The weak version

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<sup>31</sup> The decrease in the income per effective unit of labor corresponds to a fall in income per capita if the shift is due to an increase in  $n$  for a given value of  $\lambda$ . On the contrary, the income per capita increases if the shift is due to an increase in  $\lambda$  for a given value of  $n$ .

<sup>32</sup> This outcome directly derives from equation (4). Noting that  $A = y$  in our model, it can also be obtained by substituting (9) in  $K/AL = 1/y\eta e$ .



of the paradox of thrift does not occur because the total income as well as the income per capita enhance. In the Post-Keynesian model, conversely, the increase in the propensity to save out of wages ( $s$ ) reduces consumption, which leads to transient falls in both the rate of capacity utilization and the rate of capital accumulation. Although these two variables converge back to their steady-state values in the long run, both the total income and the income per capita are negatively impacted. The weak version of the paradox of thrift therefore occurs. In addition, the rate of employment (unemployment) stabilizes at a lower (higher) level than in the initial equilibrium.

## **7. Conclusion**

The intuition at the heart of our model is that population growth and technical progress lead to increases in autonomous consumption that stimulate both economic activity and growth. The underlying mechanisms do not involve aggregate supply but aggregate demand through formal and informal redistribution devices. Indeed, a growth in population generates greater redistribution to enable everybody to satisfy their primary needs. That is what happens when families welcome newborns, for example. At the macroeconomic level, households react by reducing their average propensity to save. In the long run, the rate of capital accumulation converges towards the new natural rate of growth because of the supermultiplier mechanisms due to the exogenous rate of growth of autonomous consumption. In parallel, instead of generating knife-edge instability, firms' attempts to adjust their rate of capacity stabilization at its normal level are successful.

Similarly, technical progress leads to a rise in autonomous consumption through the wage bill redistribution, provided both that it entails an increase in the real wage and the basic needs fulfilment is indexed to the level of the real wage. Under these conditions, technical progress causes a decrease in the average propensity to save, which generates the same macroeconomic adjustments as above.

Eventually, the long-run rate of economic growth converges towards the natural rate of growth. This outcome, which is a necessary condition to avoid the long-run rate of employment instability, is at the heart of the mainstream approach. We have shown in this article that it can also be generated by an aggregate demand model. This could be a

major step forward for Post-Keynesian approaches, especially since this result is fully consistent with the presence of a stable rate of unemployment.

In my opinion, the long-run adjustments described in this model are not exclusive from other mechanisms. Nikiforos (2018), for instance, defends the idea of an endogenous adjustment in the normal rate of capacity utilization.<sup>33</sup> This is evident in some industries where the time of use of equipment declined over time, for instance when the three shifts have been abandoned in favor of two or even one shift in many factories. However, one of the initial motivations of the 'first' supermultiplier models (Allain, 2015; Freitas and Serrano, 2015) was the belief that the endogeneity of the normal rate of capacity utilization could not be the only long-run adjustment mechanism of the system. This belief has not changed, but it might be interesting to take a closer look at the possibility of combining the two approaches.<sup>34</sup> The analysis could also be extended to take into account the important decline in the working time over the life cycle experienced in many countries.

As mentioned above, Harrodian instability can also be tamed by monetary policy. However, according to Franke (2019), the combination of this mechanism with the stabilizing properties of the supermultiplier should be destabilizing. This question must be reconsidered in future research, especially since Franke did not include the employment rate in his analysis. Consequently, he assumes that the two stabilizing mechanisms target the rate of capacity utilization. Such assumption is debatable. Indeed, while entrepreneurs have an objective in terms of capital utilization (which is formalized in the supermultiplier models), this is unlikely to be the case for central banks and government whose concerns, besides inflation, are more about unemployment and per capita income.

The study of the supermultiplier models must of course continue. The issues raised by critics, particularly those of Skott (2017, 2019) about the slowness of convergence mechanisms, need to be carefully considered.

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<sup>33</sup> This mechanism was put forward for a long time by many Post-Keynesian. Beside Nikiforos (2018), see also Hein, Lavoie, van Treeck (2012).

<sup>34</sup> In particular, this double convergence could improve the stability of the system, an issue insistently raised by Skott (2017, 2019).

In addition, the multiplicity of demand components that aspire to the role of autonomous component in the existing literature poses a problem: can all these components be autonomous?<sup>35</sup> Can they be autonomous at the same time? How can we combine them in the same analysis? These questions will be the subject of future research.

## 8. References

- Allain, O. 2015. Tackling the instability of growth: a Kaleckian-Harrodian model with an autonomous expenditure component, *Cambridge Journal of Economics*, vol. 39, no. 5, 1351-1371
- Allain, O. 2019. Demographic growth, Harrodian (in)stability and the supermultiplier, *Cambridge Journal of Economics*, vol. 43, no. 1, 85-106
- Bortis, H. 1997. *Institutions, behaviour and economic theory: a contribution to classical-Keynesian political economy*, New York, Cambridge University press
- Brochier, L., and Macedo e Silva, A. C. 2019. A supermultiplier Stock-Flow Consistent model: the “return” of the paradoxes of thrift and costs in the long run?, *Cambridge Journal of Economics*, vol. 43, no. 2, 413-442
- Callan, T., Nolan, B. 1991. Concepts of poverty and the poverty line, *Journal of Economic Surveys*, vol. 5, no. 3, 243-261
- Cassetti, M. 2003. Bargaining power, effective demand and technical progress: a Kaleckian model of growth, *Cambridge Journal of Economics*, vol. 27, no. 3, 449-464.
- Chattopadhyay, N., Majumder, A., Coondoo, D. 2009. Demand Threshold, Zero Expenditure and Hierarchical Model of Consumer Demand, *Metroeconomica*, vol. 60, no. 1, 91-118
- Drakopoulos, S. A. 1992. Keynes’ economic thought and the theory of consumer behaviour. *Scottish Journal of Political Economy*, vol. 39, no. 3, 318-336
- Drakopoulos, S. A. 1999. *Post-Keynesian choice theory*, in O’Hara, P. A. (ed.), *Encyclopedia of political economy* (vol. 2), London, Routledge

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<sup>35</sup> See one of Nikiforos’s (2018) criticisms.

- Fazzari, S.M., Ferri, P.E., Greenberg, E.G. and Variato, A.M. 2013. Aggregate demand, instability, and growth, *Review of Keynesian Economics*, vol. 1, no. 1, 1-21
- Fiebigler, B. 2018. Semi-autonomous household expenditures as the *causa causans* of postwar US business cycles: the stability and instability of Luxemburg-type external markets, *Cambridge Journal of Economics*, vol. 42, no. 1, 155-175
- Fiebigler, B., Lavoie, M. 2019. Trend and business cycles with *external markets*: Non-capacity generating semi-autonomous expenditures and effective demand, *Metroeconomica*, vol. 70, no. 2, 247-62
- Foster, J. E. 1998. Absolute versus relative poverty, *The American economic review*, vol. 88, no. 2, 335-341
- Franke, R. 2019. On Harrodian instability: two stabilizing mechanisms may be jointly destabilizing, *Review of Keynesian Economics*, vol. 7, no. 1, 43-56
- Freitas, F., Serrano, F. 2015. Growth rate and level effects, the stability of the adjustment of capacity to demand and the sraffian supermultiplier, *Review of Political Economy*, vol. 27, no. 3, 258-281
- Gordon, R. J. 2012. 'Is US economic growth over? Faltering innovation confronts the six headwinds', Centre for Economic Policy Research (CEPR), Policy Insight no. 63.
- Gordon, R. J. 2014. The turtle's progress: Secular stagnation meets the headwinds, in Teulings, C., Baldwin R. (eds), *Secular Stagnation: Facts, Causes and Cures*, London, Centre for Economic Policy Research (CEPR), 47-59
- Guidolin, M., La Jeunesse, E. A.. 2007. The decline in the US personal saving rate: Is it real and is it a puzzle? *Federal Reserve Bank of St. Louis Review*, vol. 89, no. 6, 491-514
- Harrod, R.F. 1939. An essay in dynamic theory, *The Economic Journal*, vol. 49, no. 193, 14-33
- Hein, E. 2014. *Distribution and growth after Keynes: A Post-Keynesian guide*, Aldershot, Edward Elgar Publishing
- Hein, E. 2016. Secular stagnation or stagnation policy? Steindl after Summers, *PSL Quarterly Review*, vol. 69, no. 276, 3-47

- Hein, E., Lavoie, M. and van Treeck, T. 2012. Harrodian instability and the 'normal rate' of capacity utilization in Kaleckian models of distribution and growth—a survey, *Metroeconomica*, vol. 63, no. 1, 139-169
- ILO, IMF, OECD, World Bank Group. 2015. 'Income inequality and labour income share in G20 countries: Trends, Impacts and Causes', report prepared for the G20 Labour and Employment Ministers' Meeting and Joint Meeting with G20 Finance Ministers (3-4 September 2015)
- International Monetary Fund (IMF) 2017. *World Economic Outlook: Gaining Momentum?* Washington, DC
- Jackson, T., Marks, N., 1999. Consumption, sustainable welfare and human needs with reference to UK expenditure patterns between 1954 and 1994, *Ecological Economics*, vol. 28, no. 3, 421-441
- Kemp-Benedict, E. 2013. Material needs and aggregate demand, *The Journal of Socio-Economics*, vol. 44, 16-26.
- Keynes, J.M. 1936 [1964]. *The General Theory of Employment, Interest and Money*, London, Macmillan
- Krugman, P. 2014. Four observations on secular stagnation, in Teulings, C., Baldwin R. (eds), *Secular Stagnation: Facts, Causes and Cures*, London, Centre for Economic Policy Research (CEPR), 61-68
- Lavoie, M. 2004. Post Keynesian consumer theory: Potential synergies with consumer research and economic psychology, *Journal of Economic Psychology*, vol. 25, no. 5, 639-649
- Lavoie, M. 2014. *Post-Keynesian Economics: New Foundations*, Cheltenham, Edward Elgar Publishing
- Lavoie, M. 2016. Convergence towards the normal rate of capacity utilization in neo-Kaleckian models: the role of non-capacity creating autonomous expenditures, *Metroeconomica*, vol. 67, no. 1, 172-201
- Lavoie, M. 2017. Prototypes, reality and the growth rate of autonomous consumption expenditures: a rejoinder, *Metroeconomica*, vol.68, no.1, 194-199

- Nah, W.J., Lavoie, M. 2017. Long-run convergence in a neo-Kaleckian open-economy model with autonomous export growth, *Journal of Post Keynesian Economics*, vol. 40, no. 2, 223-238
- Nikiforos, M. 2018. Some comments on the Sraffian Supermultiplier approach to growth and distribution, *Journal of Post Keynesian Economics*, vol. 41, no. 4 659-675
- Piketty, T. 2011. On the long-run evolution of inheritance: France 1820–2050, *The Quarterly Journal of Economics*, vol. 126, no. 3 1071-1131.
- Serrano, F. 1995a. Long period effective demand and the Sraffian supermultiplier, *Contributions to Political Economy*, no. 14, 67-90
- Serrano, F. 1995b. 'The Sraffian Supermultiplier', PhD Thesis, Faculty of Economics and Politics, University of Cambridge
- Serrano, F., Freitas, F. 2017. The Sraffian supermultiplier as an alternative closure for heterodox growth theory, *European Journal of Economics and Economic Policies: Intervention*, vol. 14, no. 1, 70-91
- Skott, P. 2017. Autonomous demand and the Harrodian criticisms of the Kaleckian model, *Metroeconomica*, vol. 68, no. 1, 185-193
- Skott, P. 2019, Autonomous demand, Harrodian instability and the supply side, *Metroeconomica*, vol. 70, no. 2, 233-246
- Solow R., Tobin, J., von Weizsacker, C., Yaari M. 1966. Neoclassical growth with fixed factor proportions, *Review of Economic Studies*, vol. 33, no. 2, 79-115
- Solow, R. M. 1956. A contribution to the theory of economic growth, *Quarterly Journal of Economics*, vol. 70, no. 1, 65-94
- Summers, L. 2014. U.S. Economic Prospects: Secular Stagnation, Hysteresis, and the Zero Lower Bound, *Business Economics*, vol. 49, no. 2, 65-73