GROWTH THEORY: An introductory lecture

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Keynes’ General Theory published in 1936 explained the principle of effective demand, in which exogenous investment determines saving as a function of income endogenously. Capacity utilization is determined by demand, saving is determined by income. Greater utilization of existing capacities causes more output, and if it can be sold in a large enough market leads to income, and more saving from income. Summed up in Diagram1)

We may write, $Y = zY^*$, where $z = 1$ for normal capacity utilization.

![Diagram 1](image-url)
Harrod, who had collaborated in developing the ideas of the General Theory (1936), soon (1939) publish a paper showing how to generalize the principle of effective demand over time. Modern Growth Theory was born.

Harrod made a distinction between two rates of growth: one in which investment is exogenous but saving is endogenously determined as in diagram 1 through capacity utilization. He called it the actual rate of growth \((Ga)\). The other growth rate at which planned investment of firms match the planned saving plan of households. Both are decided independently but match one another exactly with all expectations satisfied. This he called the warranted rate of growth \((Gw)\). At \(Gw\), the market size expands at exactly the rate at which firms expand their output with normal capacity utilization. This condition was made explicit by Domar (1946). He talked only about the warranted rate of growth, and although we call it the Harrod-Domar model, he makes no distinction between the actual and the warranted rate. Note later neo-classical generic models (Solow and Swan, both published in 1956) either ignore or confuse this essential Keynesian distinction arising from the principle of effective demand that planned investment (or expenditure), and planned saving usually diverge and are equated through income/capacity utilization adjustment (as in Diagram 1).
Keynesian, Post-Keynesian and Kaleckian Growth Models

Harrod made use of a growth equation which also becomes an identity ex post. Thus note,

\[ G_w = \left( \frac{S}{Y} \right) \left( \frac{Y^*}{K} \right), \]  
because planned \( I = \) planned \( S \) implying market is expanding at the rate businessmen expected at normal capacity utilization. Their sales expectations are exactly realized. {One often write it as \( g = \frac{s}{v} \)}

\[ G_a = \left( \frac{S}{Y} \right) \left( \frac{zY^*}{K} \right), \]  
implying actual \( I = \) actual \( S \) ex post, because capacity utilization \( z \) has adjusted appropriately. Thus \( z \) appears in \( G_a \), it is implicit in \( G_w \) at \( z=1 \).

If \( G_a > G_w \), it means \( z > 1 \) i.e capacity utilization is above normal. Businessmen would take that signal, and invest more to expand capacity to bring back capacity to normal utilization at \( z=1 \). So \( G_a \) would be even higher in the next period pushing the economy farther away from the equality between the actual and the warranted rate. The free enterprise capitalist economy is seen on an unstable knife edge growth path. This was the problem Harrod posed.
Later Keynesian writers, most of whom were also in Keynes’s original circle at the
time of writing the General Theory argued that Harrod might have overstated the
case. Joan Robinson (1956,1962) provided the simplest and clearest model  By
postulating that investors decide on the rate of capital accumulation depending not
only on the rate of capacity utilization(z) , but on the overall profit rate(r ) they
expect on their invested capital (K); r= P/K where P=total profit. However,note:
\[
(P/K) = (P/Y) (Y/Y^*)(Y^*/K) = h.z.a
\]
where (P/Y)= h=share of profit in income, (Y/Y^*)=z= degree of capacity utilization
and (Y^*/K)=a= technologically given ratio of the value of normal capacity output Y*
to accountants’ book value of capital K of firms. **Thus, given technology (a) profit
rate(r) depends both on profit share (h) and on capacity utilization (z).** Making the
simplifying assumption that all saving comes from profit, \( S=sP, \) and \( S=I, \) (ex post
when unplanned inventory accumulation is also counted as profit by accountants in ex
post statistics), we get
\[
sP/K = sr =I/K = Ga, \text{ the actual rate of growth, and } r= \text{ the realized rate of profit.}
\]
Since investors respond to the expected rate of profit \( \rho, \) the warranted rate which
Robinson interpreted more directly as the businessmen’s desired rate of growth
depends on \( \rho \), i.e.
\[
Gw = F(\rho),F'(\rho) > 0; \text{ Gw increases as the expected profit rate increases.}
\]
Because the expected rate of profit depends on the actual rate of profit, under static
expectation (‘business as usual’), \( r= \rho, \) and we get
Ga is the ray through origin with slope $s$; Gw is a curve and increasing at a decreasing rate starting somewhere above the interest rate $i$. 

Diagram 2

Ga, Gw

$i$

$r^*$

$r$
The corresponding differential equation for those mathematically inclined, 
\[ \frac{dr}{dt} = m(F(r) - sr), \]
where \( m \) is an arbitrary positive constant speed of adjustment.

In a more general case, the expected profit rate \( \rho \) is a function of the actual profit rate \( r \), i.e. \( \rho = f(r) \), with \( f'(r) > 0 \).

By substitution it is easy to show that the equilibrium at \( r^* \) remains stable so long as business expectations are not violently optimistic i.e. elasticity of expectation of \( \rho \) with respect to \( r \) is moderate.

Three comments are in order on this generic Post Keynesian growth model.

1. It is based on a mechanism similar to Kalecki’s model of business cycles (1933;1937) in so far as the reinforcing mechanism of either upswing or downswing is concerned: Higher(lower) investment causes higher(lower) profit through higher capacity utilization (like in Keynes’s theory) which in turn causes higher (lower) profit. Turning points of the cycle are explained by the depressing effect of capital stock which is accumulated investment.
2. Pasinetti (1962) provided a generalization of equilibrium along steady state growth path at \( r=r^* \) where, \( Gw=Ga=sr \). He showed that, even if the only other class in the economy namely workers save a fraction (lower than capitalists) of their income from wage and property acquired through their saving, and earns the same profit rate on their accumulated wealth, in this property owning democracy workers saving propensity would not matter. The economy would behave in steady state as if only capitalists saved!

3. Note that both profit share (\( h \)) and capacity utilization (\( z \)) enter in determining the profit rate at technologically given output capital ratio (\( a \)), as stated before, \( r= hza \). We consider more general investment functions where \( h \) and \( z \) are independent arguments rather than related in a specific way (rectangular hyperbola). In this case a greater sensitivity of investment to profit share \( h \) would lead to profit-led growth; whereas greater sensitivity of investment to capacity utilization \( z \) due to greater purchasing power from higher wage share would lead to wage-led growth. Interestingly, two opposite politics emerging from the same model based on the principle of effective demand (Bhaduri and Marglin, 1990)
Neo Classical Models.
While Keynesian models are investment driven, neo-classical models are savings driven because they assume all savings are automatically invested in the same period (Say’s Law).
Neo-classical economists do not ask the question asked Harrod because they do not consider the possibility of insufficient or excessive effective demand affecting capacity utilization and output. Without distinguishing between actual and warranted rate, they ask another question, also formulated by Harrod whether the natural rate of growth would equal the warranted rate. The natural rate of growth, $G_n = \text{labourforce growth (n)} + \text{growth of labour productivity (p)}$. Since by definition, $Y = sY = sF(K,L)$. Assuming constant returns to scale, $y = s f(k)$, and $(dk/dt)/k = (I/K) - n$, where $k = (K/L)$, we can draw a production function per worker in diagram 3. Note $sf(k) =$ saving and investment per worker i.e. capital accumulation per worker. Assuming $p=0$ to keep the diagram simple, $n.k$ is the rate of capital accumulation per worker needed to match the saving driven capital accumulation $s f(k)$, per worker. Hence solving the equation $sf(k) = nk$ at $(K/L)^*$ gives the equilibrium at $(K/L)^*$. 

Diagram 3
For the more mathematically inclined, note logarithmic differentiation of \((K/L) = k\) yields
\[
\frac{dk}{dt}/k = \frac{dK}{dt}/K - \frac{dL}{dt}/L = sLf(k)/K = \frac{sf(k)}{k} - n
\]
This yields the differential equation,
\[
\frac{dk}{dt} = sf(k) - nk, \quad \text{has the non-trivial solution where } sf(k) \text{ intersects the ray through the origin } nk \text{ in the diagram.}
\]
One can now construct and compare the one variable phase diagrams of the two prototype models the post-Keynesia and the neo-classical to squeeze out more formal properties, but that is better left for another occasion.
Two obvious extensions of this supply-side saving driven neo-classical model are:
(1) disembodied technical progress when the production function shifts
(2) technical progress that augments labour force in efficiency units.
Both raise the long run steady state growth rate \((n)\) from the supply side in diagram 3.