Growth and Crisis under Finance Dominated Capitalism : A stock-flow consistent approach.

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$$W + W + W^1 + C = Q^1 = R^1 \qquad \ldots(1)$$

$$W + R + (W + R) + W^1 + R^1 + C = R^1 + R + R + R^1$$

$$= R = I + B + F \qquad \ldots(2)$$

$$I + B + F = 7\, R^1 + R + R + R^1\, 7 - C = R - C = S$$

$$= S \qquad \ldots(3)$$

$$I + F = S = sR = s(P + \Pi),\ 1 > s > 0 \qquad \ldots(4)$$

$$I - sP = s\Pi - F \qquad \ldots(5)$$

$$T - C^{\ 1} = I - sP \qquad \ldots(5a)$$

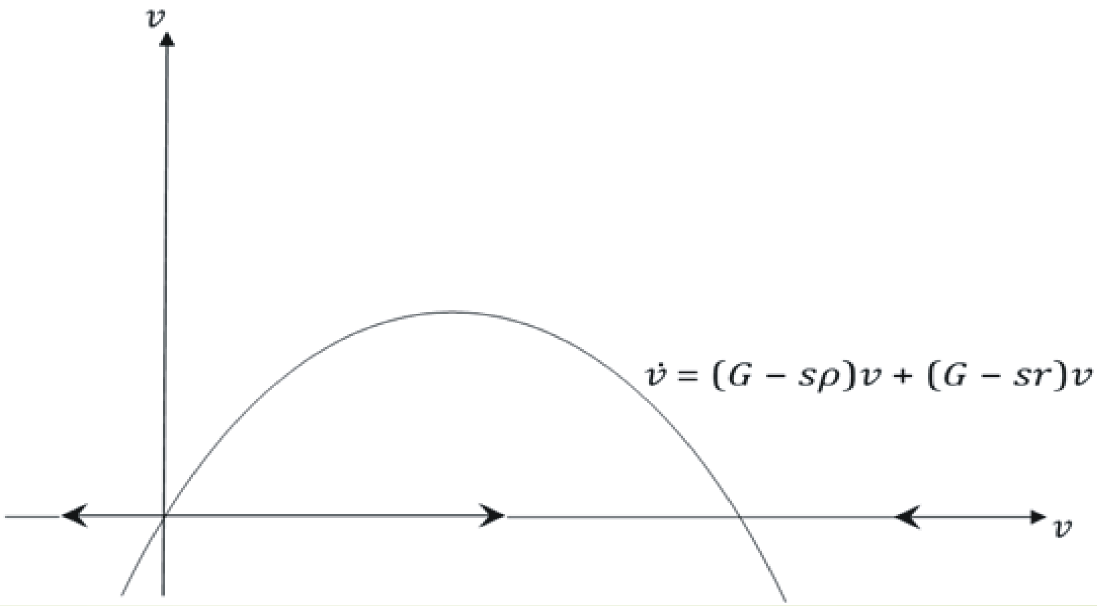
$$(g - sr) = (s\rho - G)v \qquad \dots(6)$$

$$v = \frac{g - sr}{s\rho - G} \qquad \dots(7)$$

$$\frac{\dot{v}}{v} = G - g, \qquad v \neq 0 \qquad \dots(8)$$

$$\dot{v} = (G - s\rho)v \; + (G - sr)v \qquad \dots(9)$$

$$v_7 = \frac{g - sr}{s\rho - G}, \qquad where\ g - sr > 0 \;\; and\ s\rho - G > 0 \qquad \dots(10)$$



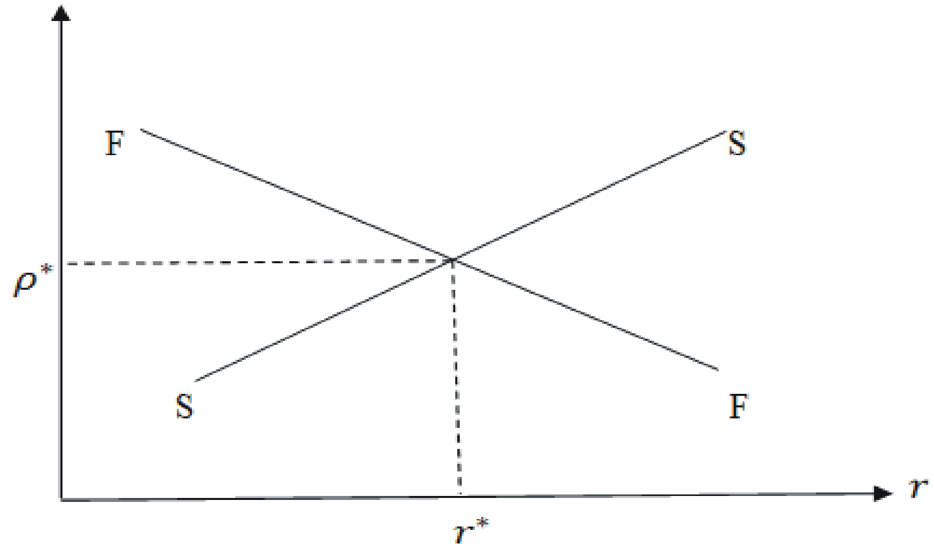


Diagram 2

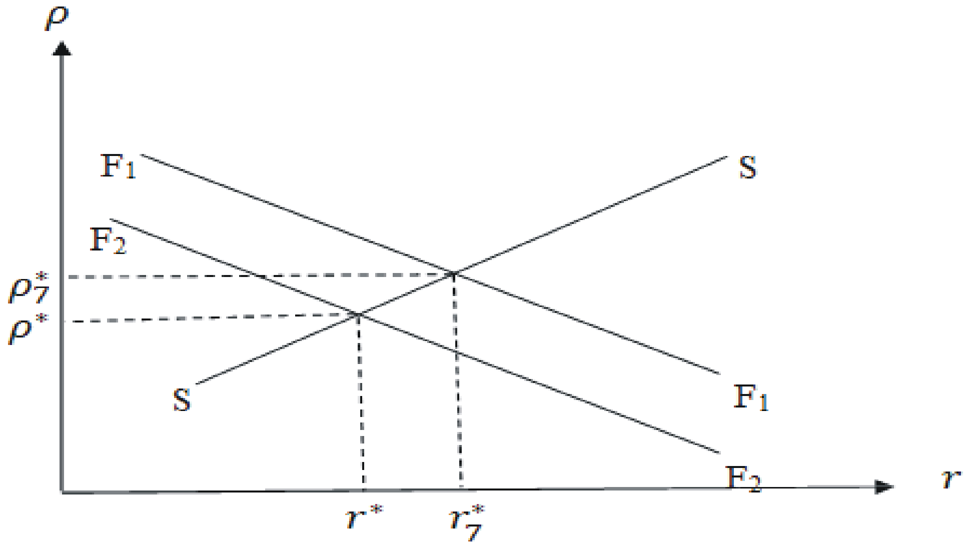


Diagram 3

$$\rho > \frac{1}{s} g > r \qquad \ldots (11)$$

$$G = G(\rho), \qquad G' > 0 \qquad \ldots (12)$$

$$g = g(r), \qquad g' > 0 \qquad \ldots (13)$$

$$\dot{v} = 0 \ \ at \ \ v = \bar{v}(\neq 0) \qquad \ldots (14)$$

$$With \quad G = g \qquad \ldots (15)$$

at all points in time, (12) (13) and(15) yield on total differentiation, the stock equilibrium condition as,

$$\frac{d\rho}{dr} = \frac{g'(r)}{G'(\rho)} > 0 \qquad \ldots (16)$$

With $v = \bar{v}$ holding at all points of time. In view of (12) and (13), the flow equilibrium condition (6) becomes,

$$\frac{d\rho}{dr} = \frac{g'(r) - s}{[s - G'(\rho)]\bar{v}}$$

If *both* the real and the financial sector satisfy the Keynesian stability condition *in isolation* then $(g'(r) - s) < 0$and $(s - G'(\rho)) > 0$. In this case

$$\frac{d\rho}{dr} = \frac{g'(r) - s}{[s - G'(\rho)]\bar{v}} < 0 \qquad \ldots (17)$$

$$G = G(r, \rho), \qquad G_\rho > 0 \qquad \ldots (18)$$

$$g = g(r, \rho), \qquad g_r > 0 \qquad \ldots (19)$$

$$\frac{d\rho}{dr} = Slope\ SS = \frac{g_r - G_r}{G_\rho - g_\rho}, \quad G_\rho \neq g_\rho \qquad \ldots (20)$$

The stability of equilibrium is easily analysed by postulating that the rate of change in the profit rate of the real sector is positively related to the excess demand for consumption and investment goods, so that

$$\dot{r} = \varphi(g - sr - s\rho\bar{v} + G\bar{v}), \varphi(0) = 0, \varphi' > 0$$

The necessary and sufficient condition for local stability at the equilibrium point yields,

$$\frac{d}{dr}(g - sr - s\rho\bar{v} + G\bar{v}) < 0$$

$$\text{i.e. } g_r + g_\rho \frac{d\rho}{dr} - s - s\bar{v}\frac{d\rho}{dr} + G_r\bar{v} + G_\rho\bar{v}\frac{d\rho}{dr} < 0$$

For given value of $\bar{v}$, if the indirect or external effect g_ρ is either negative or positive but sufficiently small, $\bar{v}(s - G_\rho) - g_\rho > 0$, and the stability condition reduces to , Assuming, to isolate the flow adjustment case, that stock equilibrium is always maintained, the derivative on the left hand side of the above inequality is equal to the slope of the SS curve at the equilibrium point and we obtain the simpler condition for stability,

$$Slope\ SS > Slope\ FF$$

In the opposite case it can be proved in a similar manner that, for a sufficiently large positive external effect of the financial profit rate on the growth rate of the real sector implying, $\bar{v}(s - G_\rho) - g_\rho < 0$, the necessary and sufficient condition for stability of equilibrium becomes

$$Slope\ SS < Slope\ FF$$

Abrupt Change and Financial Catastrophe.

In conformity with traditional growth theory the analysis so far explored the possibilities of stock and flow equilibrium in a financialized economy with (section 5) or without (section 4) independent investment functions. Its purpose has been to examine the stability ,i.e. adjustment paths to equilibrium (section 4), or a comparison of the properties of stable equilibria under parametric perturbation(section 5). However, real life experiences that financialized economies typified by behaviours of the stock markets, are prone to sudden, abrupt changes including occasional catastrophic booms and bursts finds no place in such analysis. This section explores how the basic structure of the model may be modified to accommodate such occurrences.

Let, x=(ρ/r) and h=(ρW/R) and (1-h)= (rK/R) to yield,

$x.v = m = (h/1-h) = h + h^2 + \ldots,\ 1 > h > 0.$ (31)

We simplify by assuming the investment function $G(r, \rho)$ is homogeneous of degree one and linear, represented by[1],

$G = b_1 r + b_2 \rho$, with $G(0,0) = 0$, $b_1, b_2 > 0$ (32)

Using equations 1 and 2 above with equation 9 in the text from section (4), we obtain on simplification a quadratic equation,

$(dv/dt) = r \{ v^2 - [(s-b_1)+(s-b_2)m]v(b_1)^{-1} + b_2 m(b_1)^{-1} = 0$ (33)

turning point behavior of stock price v in the the stock market may be represented by the differential equation,

$(dm/dv) = a_1 - a_2 v$, $a_1, a_2 > 0$ (34),

implying m increases with v until threshold value (a_1/a_2) and then decreases.

On integration (4) yields ,

$$m = a_1 v - a_2 v^2/2 + a_3 , \; a_3 > 0 \qquad (35).$$

Note, $a_3 > 0$ on the assumption that m>o and v> 0 (i.e. W>0) for the model to avoid degeneracy , and permit mathematical operations like logarithmic differentiation on v, although both m and v can be arbitrarily close to zero.

Inserting (35) in (33), we obtain on simplification a cubic equation,

$(dv/dt) = (r)[v^3+H_2v^2+H_1v+H_0]= 0$, (36)

In (6), given Keynesian stability conditions, $H_2 =-\{(s-b_1)+ (s-b_2)\}a_1+b_1(a_1-2)]/[(s-b_2)a_2]$, which is negative for $a_1>2$ as a sufficient condition , while the necessary and sufficient condition is, $s>(b_1 /a_1)+ (b_2/2)$ implying for, $1>s$, $2>[a_1 b_2 /(a_1-b_1)]$; $H_1=[2(b_1-s)+ (b_2-s)a_3-a_1b_2]/ [(s-b_2)a_2]$ whic is unambiguously negative; and, $H_0 = 2a_3 b_2 /[(s-b_2)a_2]$ which is unambiguously positive . Therefore we have,

$H_1 <0$, $H_0 >0$ and H_2 is of undetermined sign unless $a_1>2$ (as a sufficient condition) (37)

Given $r\neq o$, the cubic equation permits one real and a pair of conjugate complex roots if $M^3+N^2> 0$; all roots are real and at least two equal if $M^3+N^2= 0$; all roots are real if $M^3+N^2 <0$ where,

$M=(1/3) H_1-(1/9) H_2^2$ and,

$N= (1/6)(H_2 H_1-3H_0)-(1/27)H_2^2$ (Abramowitz and Stegun, 1965 p 17). Although the behavior of v can be analyzed directly from studying the roots of (6) to show the possibility of sudden transition (Nicolis and Prigogine, 1978, pp168-17),it would be more convenient to study it geometrically(Strogatz,1994, pp.69-73).

We reduce (6) to the standard, 'depressed' form by eliminating the quadratic term. By substituting, $y=[v-(H_2/3)]$, we obtain on simplification,

$(dy/dt) = f(y) = y^3 - py - q = 0$, where, $p=[(H_2^{2)}/3 - H_1]$ and, $q=[H_2 H_1/3) - (2 H_2^3/27) - (H_0)]$ (38)

From (38), $p>0$ and, $q<0$ for $H_1<0$, $H_2>o$ requiring $a_1>2$, and $H_0>0$. However , if H_2 is negative requiring $a_1<2$, and the cubic power dominating other terms, q is positive. However , if H_2 is negative which requires $a_1<2$, the term with cubic power to dominate other terms for making q positive .

Equation (38) is indeed the canonical case of 'cusp catastrophe' with discriminant $D = 4p^3 + 27q^2$, and its sign determining the nature of the roots, projected with fold in three dimensional geometry((Poston and Stewart, 1978, pp78-83,174-176).

For a geometric view of the problem, note that for $f(y)=0$, the roots of the equation is given by ,

$$y^3 - py = q \qquad (39).$$

The curve has a maximum positive value, $(2/3p.(p/3)^{1/2}$ at the negative value of $y = -(p/3)^{1/2}$,and an equal opposite minimum value at the same positive value of $y=+(p/3)^{1/2}$. Thus the left hand side of (9) defines curve is symmetrical S-shaped curve passing through the origin. While $p>0$, q can be positive of negative, and the right hand side of (9) defines line parallel to the horizontal axis with a positive or negative intercept, The intersection points give the roots of the equation (Diagram 5 here).

As seen from the diagram, the equation can have one or three real roots, and at the critical point of bifurcation the horizontal line is tangent to the curve (i.e.two roots identical), and we can divide the parameter space (p,q) in terms of whether it permits one or three roots (fixed points). Within a critical interval value of q spanning a positive to negative range, the dynamical system can have three real roots. As can be seen from our previous discussion from equations (38) and (39), a configuration of parametric values would be consistent with the possibility of three real roots. In all such cases the dynamical system defining the behaviour of the stock price can undergo sudden and discrete 'catastrophic' change in stock prices. This can be brought out by visualizing two disjoint loci of stable equilibrium points of unbroken curves on the (q,v) plane that are separated by unstable points shown bya broken curve.Noting that, $y=(v-H_2/3)$ which means a shift of the curve and rotating Diagram 5 clockwise 90^0, the connection between diadgams 5 and 6 can be geometrically made more transparent. (Diagram 6 here). It should also be noted that, while the model shows the possibility of abrupt change in stock price, it cannot predict how long a given state would last, and linger on points of the stable locus

From (21), the right hand side ratio is simply the slope of the FF curve at the equilibrium point. Given that $g - sr > 0$, from (21)−(23) it follows that this is equal to the value of $Slope\ FF$ at the equilibrium point. Assuming, to isolate the flow adjustment case, that stock equilibrium is always maintained, the derivative on the left hand side of the above inequality is equal to the slope of the SS curve at the equilibrium point and we obtain the simpler condition for stability,

$Slope\ SS > Slope\ FF$

In the opposite case it can be proved in a similar manner that, for a sufficiently large positive external effect of the financial profit rate on the growth rate of the real sector implying $\bar{s} - G - g < 0$, the necessary and sufficient condition for stability of equilibrium becomes

$Slope\ SS < Slope\ FF$

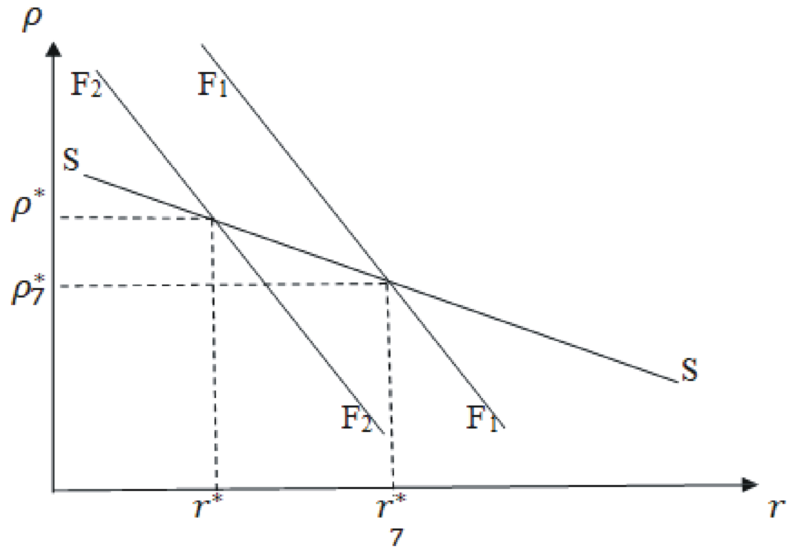


Diagram 4

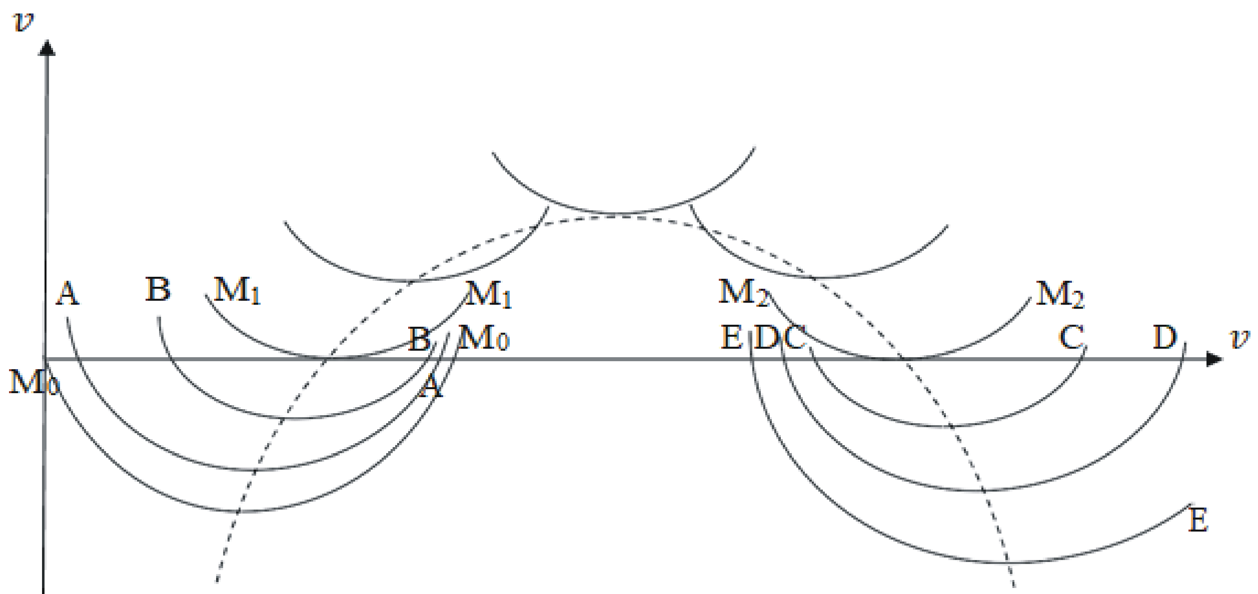


Diagram 5

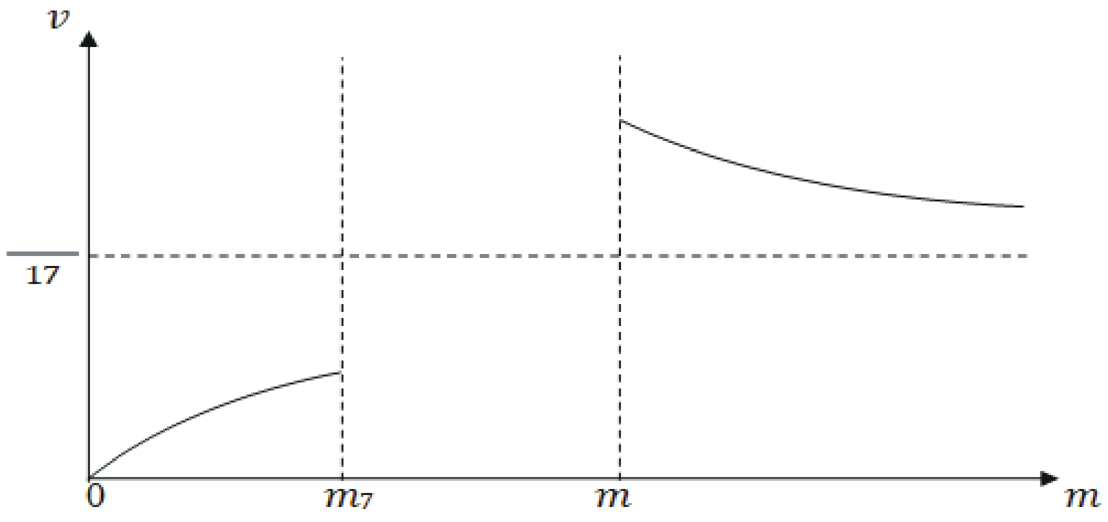


Diagram 6

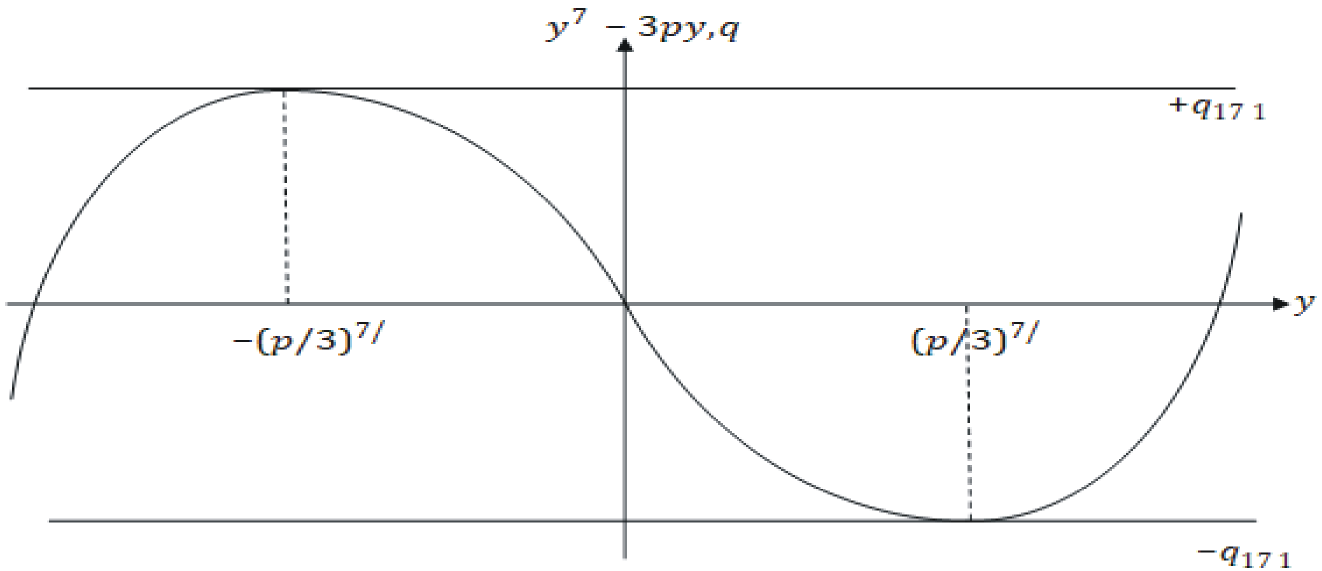


Diagram 7

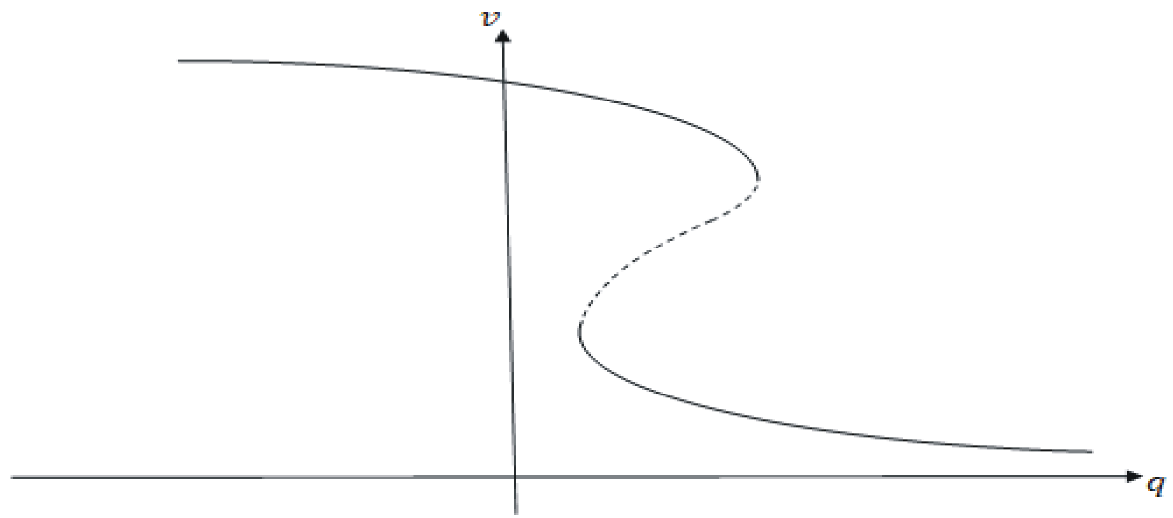


Diagram 8

Thank You