

# Top Income Shares and Aggregate Wealth-Income Ratio in a Two-Class Corporate Economy \*

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## Abstract

This paper examines some determinants of top income shares and the aggregate wealth-income ratio in the United States. The paper, first, points out the difficulties in Piketty's neo-classical version of explanation of US income inequality, which stresses the effect of the rising aggregate wealth-income ratio and high elasticity of factor substitution. Second, the analysis, based on a Cambridge two-class model along the lines of Kaldor (1955/56, 1966) and Pasinetti (1962), highlights the role of financialization in increasing inequality. Third, the analysis suggests that the rise in the aggregate wealth-income ratio from 1980 to 2007 in the US is explained mostly by asset price inflation, not by technical relations. Finally, the analysis examines the effects of the slowdown in capital accumulation on income distribution and wealth-income ratios, which are very different from those in Piketty's *Capital in the twenty first century*.

**keyword** top income share, wealth-income ratio, financialization, top management pay, stock-flow consistency

**JEL classification** E12, E21, E25, E44

## 1 Introduction

This paper attempts to offer an explanation of the increase in top income shares and the rise in the wealth-income ratio in the United States since the early 1980s, during which there was no sustained increase in the reproducible capital-output ratio. The analysis is based on a Cambridge two-class model developed in Ryoo (2016b), which extends Kaldor (1955/56, 1966) and Pasinetti (1962) along the lines of Skott (1981, 1989) and Skott and Ryoo (2008). Some empirical components of this paper are motivated by recent findings of Piketty and Saez

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(2003), Piketty (2014) and Saez and Zucman (2014) regarding income and wealth distribution in the U.S.<sup>1</sup>

The paper starts to point out the difficulties of Piketty's neo-classical version of explanation of US income inequality which is based on the combination of the rising aggregate wealth-income ratio and high elasticity of factor substitution (section 2). The paper next presents an alternative model (section 3). The model economy assumes the capital-output ratio is exogenous and the long-run growth rate is fixed by the exogenous natural growth rate. The model consists of the corporate sector and the household sector. The household sector is further divided into capitalists (the top) and workers (the rest). There are two assets (stocks and deposits) which are available for both the top and the rest. Both classes earn labor income, but the labor income of the top is largely top managerial pay which I treat as a deduction from broadly defined profits. The analysis in this paper presumes that capitalist households tend to capture around the same share of wealth and income as the top one percent of wealth holders and income earners in the US economy.<sup>2</sup> Based on this theoretical framework, the analysis identifies a number of factors that may have raised the top income share in the U.S since the 1980s. These include several developments associated with financialization such as increases in the dividend payout and stock buybacks, increasing indebtedness of lower-income households, and asset bubbles (section 4). A further analysis shows that the rise in the aggregate wealth-income ratio from 1980 to 2007 is explained mostly by the rising wealth-income ratios of the top and the bottom, which themselves were heavily influenced by asset price inflation (section 5).

Throughout this paper, I also examine the effects of the slowdown in capital accumulation on income distribution and the wealth-income ratio. The analytic results are very different from Piketty's arguments in his *Capital in the twenty first century*. In this Cambridge two class economy, the fall in the growth rate of capital tends to reduce the top income shares, which decreases the aggregate wealth-income ratio.

It may be worth mentioning that a Kaldorian framework in this paper is fundamentally different from both Piketty's neo-classical and popular Kaleckian frameworks when it comes to the adjustment mechanism of investment-saving equilibrium relation. In both neo-classical and Kaleckian models, the long-run adjustment of the output-capital ratio brings investment in line with saving. In contrast, the endogenous adjustment of the profit share clears the goods market in the Kaldorian framework. I will return to this issue in conclusion.

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<sup>1</sup>James Galbraith (1998, 2012) has documented the increasing trend in income inequality, independently of Piketty and his colleagues. A referee informed me that the analysis in this paper shares Galbraith's emphasis on the role of finance and asset price inflation in increasing inequality.

<sup>2</sup>A caveat need be added. Wealth and income are distinct concepts and there is no reason why the top one percent of wealth holders are identical to that of income earners. Mapping from empirical data on wealth and income into the class structure requires both sophisticated theoretical criteria and careful empirical work. A recent empirical study by Mohun (2016) estimates the class structure of the US economy. Mohun identifies the capitalist class as the group of individuals whose non-labor income is high enough to afford a standard of living that allow non-participation in the labor market. The size of the capitalist class identified this way varies over time but is close to one percent of tax units.

## 2 Implications of some empirical observations for neoclassical growth models

### 2.1 The long-run trend of the growth rate of capital, the capital-output ratio and the profit share

The growth rate of capital, the capital-output ratio and the profit share are among the most important variables in various models of growth and distribution. Therefore it is worth looking at some empirical observations of those variables. The data presented below are largely based on the Penn World Table 8.0 (PWT) which has been widely used in contemporary growth studies.

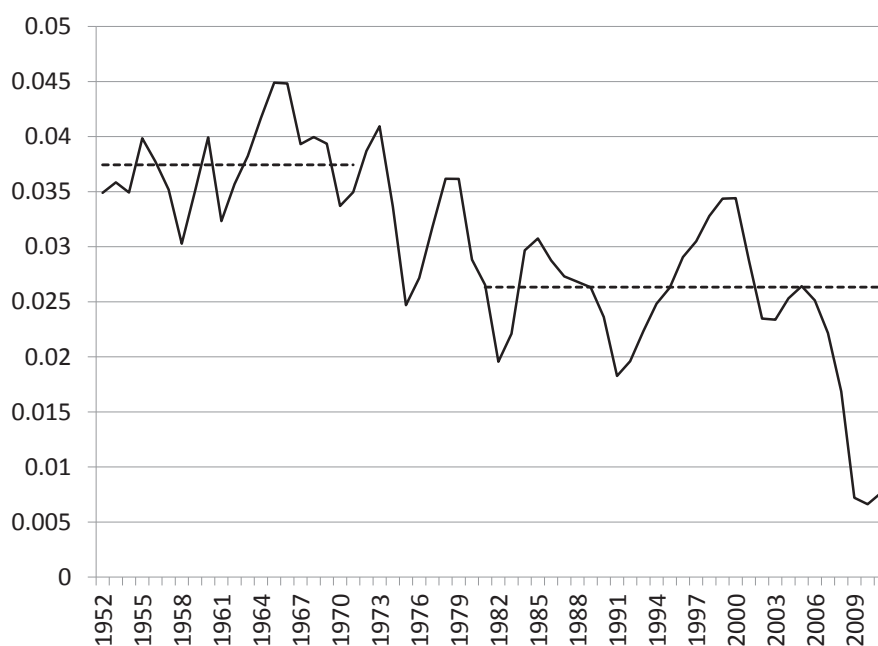


Figure 1: The growth rate of capital in the US (1952-2011)

Sources: Author's computations from the Penn World Table 8.0. The growth rate of capital is computed as a log-difference of the variable 'rkna' (real capital stock) in PWT.

Perhaps the least controversial among those three variables is the movement of the growth rate of capital, when it comes to the fact itself. Most would agree that there has been a decline in the long-run growth rate since the collapse of the 'Golden Age of Capitalism.' According to the Penn World Table, for instance, the annual average growth rate of capital in the U.S. was 3.8% in 1952-1970 and 2.6% in 1981-2007, respectively (see Figure 1). The interpretation of the fact, however, would be theory-dependent. Some theories may see the observed decline in the growth rate as a result of an exogenous decrease in the natural growth rate (the sum of the rates of the labor force growth and Harrod-neutral technical progress), while others may

see it as an induced outcome.<sup>3</sup>

Turning to the measure of distributive shares at the functional level, it has been widely recognized that the labor share has declined since the early 1980s in many OECD countries. This appears to be also true for the U.S. Defining the profit share as one minus the labor share estimated by PWT, the profit share had increased by 4 percentage points from 1982 to 2010 (Figure 2). The moderate increase in the conventionally measured profit share may underestimate the actual increase in ‘surplus’ in the US economy. It has been argued that the sharp increase in the labor income of the top income group – such as CEO pay, bonuses and salaries of investment bankers and corporate lawyers – has been a characteristic feature of the US economy since the 1980s. Such wages and salaries at the top end are registered as labor income in normal accounting practices. However, there is a long-standing perspective that treats ‘wages’ at the top end as part of profits rather than wages,<sup>4</sup> e.g. Marx (1984)[Ch.23], Kalecki (1938)[p.97] and Minsky (1986)[p.154]. If the concept of profits is expanded to include the labor income at the top end of personal income distribution, the rise in the resulting measure of the profit share in the US economy becomes more pronounced. The thick solid line in Figure 2 shows the movement of the expanded profit share by including in it the labor income of the top one percent income group. The expanded profit share had risen by 8 percentage points from 38% in 1970 to 46% in 2010.

The increasing gap between the conventionally measured and the expanded profit shares corresponds to the increasing importance of labor income received by the top one percent income earners since the early 1980s, as documented by Piketty and Saez. It has been pointed out, however, that the major increase in the top income share is largely explained by the tax effects in the 1980s (see Galbraith (2012)[pp.149-150], for instance). More specifically, the Tax Reform Act of 1986 substantially broadened the definition of taxable income for top earners. A referee also made me aware that the tax cuts of the early Reagan administration incentivized ‘businesses to shift effective compensation from the form of company-provided perks, previously treated as business expenses, directly into the private income’ of managers, owners, top employees and rich professionals. These tax effects reflect changes in accounting, not in the structure of the economy itself. The expanded profit share measured by the thick solid line in Figure 2 thus needs to be adjusted for these tax effects. The lower bound of the adjusted measure can be obtained by assuming that the major increase in the top income share from 1981 to 1988 is *entirely* due to these tax effects (see the dotted line in Figure 2). After this adjustment, the resulting measure of the expanded profit share had risen from 38% in 1970 to 44.5% in 2010 by 6.5 percentage points, which is still greater than the increase in the conventionally measured profit share.

Another important observation from the Penn World Table, finally, is that there is no appreciable upward trend in the capital-output ratio in the US since the 1980s. Figure 3 shows that the capital-output ratio based on PWT had actually fallen from 3.5 in 1982 to

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<sup>3</sup>The average growth rate of population in the U.S. has decreased from 1.49% in 1950-1970 to 0.99% in 1971-2007 (Penn world table). The reduction in population growth is about half of the decline in the long-run average growth rates of output and capital. Piketty and Zucman (2014) argue that variations in income growth rates are mostly driven by changes in population growth while the productivity growth is approximately the same across the advanced countries.

<sup>4</sup>Mohun (2006) provides an empirical analysis along the lines.

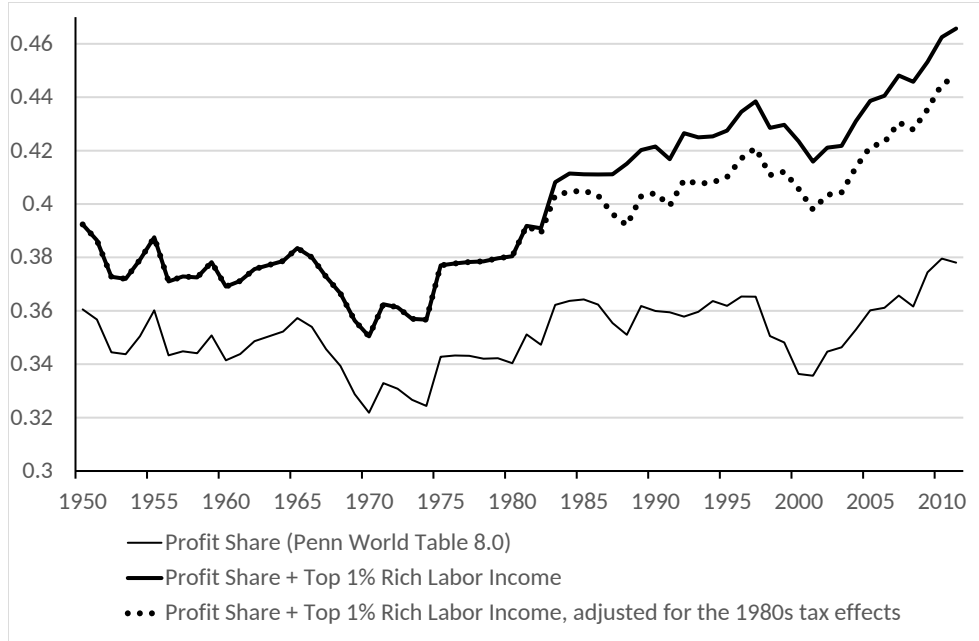


Figure 2: Profit shares in the US

Notes: The thin solid line plots the values of  $(1-\text{labor share})$  in The Penn World Table 8.0. The thick solid line is constructed by including labor income of the top 1% income earners in the measure of profits. The dotted line adjusts the thick solid line for the 1980s tax effects. The data on labor income of the top 1% income earners and total income from the tax returns data are from Piketty and Saez (2003)[updated data, 2013]

3.0 in 2007, where capital is measured by the perpetual inventory method and intends to capture the size of productive assets.<sup>5</sup> In contrast, the wealth-income ratio measured by Saez and Zucman (2014), whether or not it includes housing wealth, exhibits a marked increase since the 80s, interrupted by significant downturns in 2000 and 2007, where wealth includes all marketable wealth net of liabilities (housing, stocks, fixed income assets, pension wealth, net of mortgage and non-mortgage debt). The general trend of the wealth-income ratio by Saez and Zucman is similar to that in Piketty (2014) and that based on the Fed flow of funds data.

As is well-known, Piketty (2014) and Piketty and Zucman (2014) used the textbook Solow growth model to explain the observed increase in the capital income share. In this neo-classical account, the increase in the wealth-income ratio is seen as a key driving force behind the rise in the capital share. Critical in the argument is Piketty’s interchangeable use of the terms, ‘capital’ and ‘wealth’.<sup>6</sup> Identifying the observed increase in the wealth-income ratio with the increase in the capital-output ratio in the aggregate production function, Piketty suggests that the observed increase in the capital share is caused by the increase in ‘the

<sup>5</sup>The decline in the capital-output ratio during this period is also found in Franke (2016).

<sup>6</sup>Piketty defines capital as “the total market value of everything owned by the residents and government of a given country at a given point in time, provided that it can be traded on some market (Piketty, 2014) [p.48]

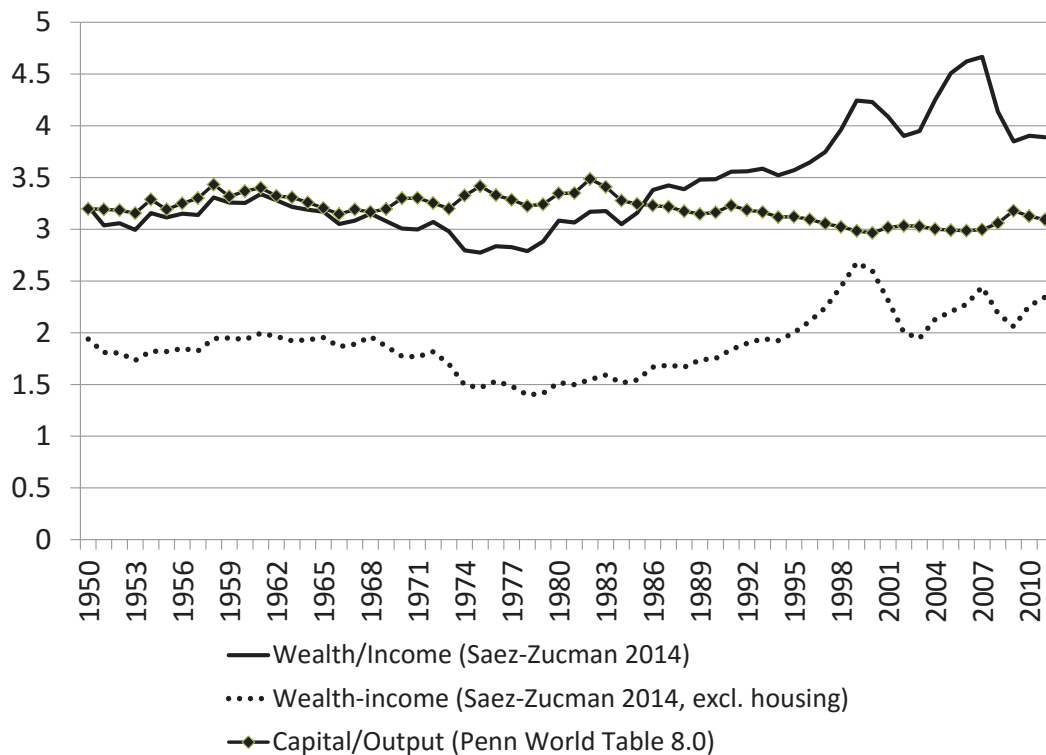


Figure 3: The wealth-income ratios in the US

Sources: The Penn World Table 8.0. Saez and Zucman (2014). Author’s computations

capital-output ratio’ given the assumption that the aggregate production has an elasticity of factor substitution far greater than unity. Piketty’s conflation of capital and wealth has been criticized on various grounds (Galbraith, 2014, Rowthorn, 2014, Jones, 2015, Stiglitz, 2015, Weil, 2015). The different movement of the capital-output ratio and the wealth-income ratio shown in Figure 3 poses a challenge to Piketty’s neoclassical version of explaining the rise of inequality in functional income distribution.

## 2.2 Implications for baseline neoclassical theories

The observed movement – the rise in the profit share and the fall in both the growth rate and the capital-output ratio in the US economy since the 1980s, assuming it is an accurate description – has two implications for growth models.<sup>7</sup> One is specific for simple neoclassical type of models such as the Solow and the Ramsey models, and the other is general for any model of growth and distribution.

First, the increase in the capital share and the fall in the capital-output ratio has impli-

<sup>7</sup>A referee questioned the robustness of the observation that the capital-output ratio had fallen from the early 1980s to 2007. As acknowledged by the referee, however, the general arguments in this section are valid as far as the capital-output ratio is non-increasing.

cations for the degree of factor substitution between capital and labor. Factor substitution between capital and labor is a key mechanism to bring savings into line with investment in neoclassical growth theories. Under the assumption of perfect competition, the capital share is negatively (positively) related to the capital-output ratio if and only if the elasticity of substitution between labor and capital is less (greater) than one. The Cobb-Douglas production function is the borderline case with unit elasticity where the capital share is invariant to changes in the capital-output ratio. Formally, assuming that the aggregate production function takes a CES form of capital  $K$  and labor  $L$  with the elasticity of substitution  $\sigma$ ,

$$Y = [bK^{\frac{\sigma-1}{\sigma}} + (1-b)L^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}}, \quad 0 < b < 1, \quad \sigma \geq 0, \quad (1)$$

the return on capital – the interest rate  $r$  – is equal to the marginal productivity of capital under perfect competition

$$r = \frac{\partial Y}{\partial K} = bK^{-\frac{1}{\sigma}} [bK^{\frac{\sigma-1}{\sigma}} + (1-b)L^{\frac{\sigma-1}{\sigma}}]^{\frac{1}{\sigma-1}} = b \left( \frac{K}{Y} \right)^{-\frac{1}{\sigma}} \quad (2)$$

and the capital share is given by

$$\text{capital share} = \frac{rK}{Y} = b \left( \frac{K}{Y} \right)^{\frac{\sigma-1}{\sigma}}. \quad (3)$$

The fall in  $K/Y$  and the increase in the capital share requires the elasticity of substitution between labor and capital to be *less than unity*, and this suggests that any explanation to reconcile the neoclassical framework to the observed fall in  $K/Y$  would be very different from what Piketty offered under the assumption of the elasticity greater than one.

The second implication of the stylized facts is related to the investment-saving relation in the steady state. Consider the textbook Solow-Swan model, where the saving rate  $s$  is taken as exogenous and, along with full employment assumption, the labor force grows at a constant rate  $n$ . Steady growth requirements are given by  $g = n$  and  $\hat{Y} = \hat{K}$ . The equilibrium capital-capital ratio is determined by

$$\frac{s}{n} = \left( \frac{K}{Y} \right)^* \quad (4)$$

Based on this well-known equality, Piketty argues that the increase in the ‘capital-output ratio’ has been caused by a reduction in the growth rate. However, the increase in the capital-output ratio in the US, which should be distinguished from the wealth-income ratio, is far from being clear and the data from the Penn World Table suggests otherwise, as already pointed out. If the capital-output ratio has actually fallen while the long-run growth rate has declined, the investment-saving relation becomes inconsistent with the assumption of the constancy of the average saving rate: the average saving rate must have fallen faster than the fall in the growth rate for the capital-output ratio to decline.<sup>8</sup> More modestly, even if one

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<sup>8</sup>The same implication applies to the Ramsey model that endogenizes the saving rate. Assuming that the instantaneous utility function of the representative household is logarithmic with standard exponential

reads the observed trend of  $K/Y$  as approximately constant, rather than falling, the constant average saving rate is still inconsistent with the decline in the natural growth rate under the investment-saving equilibrium relation.

In general, the assumption of a downward shift in the saving function is necessary to make the IS relation consistent with non-increasing capital-output ratio and falling growth rate. This point also applies to various heterodox models of growth and distribution where the average saving rate is typically an increasing function of the profit share. The observed increase in the profit share must have increased  $s$ , but the combination of the fall in  $g$  and  $K/Y$  requires  $s$  to fall.<sup>9</sup>

### 3 A Cambridge two-class corporate economy

This section extends a basic Kaldorian model by introducing corporate firms' financial behavior, financial stocks and two social classes explicitly.<sup>10</sup> The general structure of the model is similar to that in Skott (1981, 1989) and Skott and Ryoo (2008), but the household sector is disaggregated into two social classes, 'capitalists' and 'workers'. Both capitalists and workers make wages, dividends on stocks and interest incomes on deposits, but wages of the capitalist households are modeled as an allocation of profits by corporations in the form of the compensation for top managers. The model assumes a fixed-coefficients production technology. The capital-output ratio is taken as exogenous on the grounds of limited factor substitutability and Harrodian investment behavior ( $Y/K = u$  where  $u$  is constant). The economy is mature and the long-run growth rate of capital and output  $g$  is determined by the (exogenous) natural growth rate  $n$ , i.e.,  $g = n$ .<sup>11</sup> A key Kaldorian feature of the model lies in the endogenous adjustment of the profit share to clear the product market.

The framework developed in this model has several motivations:

1. A careful modeling of the saving side requires the introduction of the corporate sector given that firms' financial practices such as the payout policy and the equity issue policy have important implications for savings and distribution.

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discounting, the Euler rule fixes the equilibrium interest rate by  $r^* = \rho + n$  where  $\rho$  is the discount rate. Using (2), we have

$$\left(\frac{K}{Y}\right)^* = \left(\frac{b}{\rho + n}\right)^\sigma$$

The capital-output ratio, *ceteris paribus*, is inversely related to  $n$ . The decline in the capital-output ratio is consistent with the fall in the growth rate only if there is a large increase in  $\rho$ , for a given value of  $b$ .

<sup>9</sup>The empirical observation of the rise in the profit share, the fall in  $g$  and non-increasing  $K/Y$  has an implication specific for popular Kaleckian models where capital accumulation depends positively on the profit share and the output-capital ratio. The observed decline in capital accumulation should be explained by a large downward shift in the accumulation function that more than offsets positive inducements to accumulation coming from the rise in the profit share. This point is independent of whether the economy is 'profit-led' or 'wage-led.'

<sup>10</sup>This section heavily draws on a Kaldorian model in Ryoo (2016b) and summarizes the features and the results of the model. The detailed analysis and arguments are found in the original paper.

<sup>11</sup>The assumption that the long-run growth rate equals the natural rate does *not* mean that the economy achieves full-employment in the long-run. The analysis in this paper assumes that the employment rate is determined through the interaction between the goods and the labor markets as in Skott (1989), where changes in aggregate demand have level effects on output and employment.



2. The identification of capital with wealth is questionable. It would be desirable to introduce financial stocks explicitly.
3. Given the importance of increasing inequality at the level of personal income distribution both in income and wealth, the model with heterogeneous households is necessary.
4. The model needs to address the increasing importance of managerial compensation in income distribution.

**Firms' financial decisions and managerial pay** The firms' budget constraint is written as

$$pI + W_w + W_c + Div + iM = pY + v\dot{N} + \dot{M} \quad (5)$$

where  $p$  is the price of output,  $I$  real investment,  $W_w$  wages to workers,  $W_c$  top managerial pay,  $Div$  dividends,  $i$  the nominal interest rate,  $M$  bank loans,  $Y$  real output,  $v$  the unit price of stocks, and  $N$  the number of stocks. A dot over a variable refers to the time derivative of the variable.

A broad definition of profits  $\Pi$  is introduced, which captures the excess of total revenue over wages to workers:

$$\Pi = pY - W_w \quad \text{or} \quad W_w = (1 - \pi)pY \quad (6)$$

where  $\pi$  is the profit share,  $\pi \equiv \Pi/(pY)$ . Firms pay a constant fraction  $\lambda$  of profits – net of depreciation and interest payments – to top managers as salaries and bonus.<sup>12</sup>

$$W_c = \lambda(\Pi - \delta pK - rM) \quad (7)$$

where  $r$  is the real interest rate.

The rest of surplus is broken into dividends and retained earnings

$$Div = (1 - s_f)(1 - \lambda)(\Pi - \delta pK - rM) \quad (8)$$

where  $s_f$  is the retention rate that is exogenously determined by the firms' retention policy.

Firms also make decision on equity financing policy. The equity issue policy is captured by an exogenous parameter,  $\hat{N}$ , the growth rate of the number of stocks.<sup>13</sup>

**Banks** The banking sector is modeled following a simple endogenous money story. Banks set the real interest rate on loans at  $r$  exogenously. Banks make loans to firms and accept deposits from households. Under simplifying assumptions,<sup>14</sup> 'loans create the same amount of deposits,' represented by the following balance sheet relation:

$$M = M_c + M_w \quad (9)$$

where  $M_c$  and  $M_w$  are the level of deposits held by capitalists and workers, respectively.

<sup>12</sup>This specification is distinguished from the typical approach in the recent post-Keynesian literature that assumes that managerial compensation is in a definite relation to wages for non-managerial labor through structurally determined 'wage' premium.

<sup>13</sup> $\hat{N}$  can be either positive or negative (or zero). A negative value of  $\hat{N}$  means the net acquisition of stocks by the firm sector from households (stock buybacks).

<sup>14</sup>I assume that the rate of interest on loans equals that on deposits, banks do not hold any other asset than loans, nobody holds cash, and banking does not incur costs other than the interest payments on depositors. Given these assumptions, banks do not make pure profits, their net worth equals zero.

**Capitalists and workers** The household sector is divided into the capitalists' and the workers' households. Their budget constraints are given by:

$$pC_j + v\dot{N}_j + \dot{M}_j = W_j + Div_j + iM_j, \quad j = c, w \quad (10)$$

The subscript  $j$  refers to the types of households ( $c$  for capitalists and  $w$  for workers).  $W_j$ ,  $Div_j$ , and  $iM_j$  are labor income, dividends and interest income, respectively. Note  $W_c$  stands for managerial compensation for capitalists.  $C_j$  is real consumption.  $v\dot{N}_j$  and  $\dot{M}_j$  are net acquisition of stocks and deposits by  $j$ -class.

Each class receives dividends from firms in proportion to their share of stock ownership. In equilibrium, the total number of shares will be equal to the sum of the shares held by capitalists and workers,  $N = N_c + N_w$ . The class-share of stocks is denoted as  $k_j \equiv N_j/N$  with  $k_c + k_w = 1$ . Dividends can be written as:

$$Div_j = k_j \cdot Div = k_j(1 - s_f)(1 - \lambda)(\Pi - \delta pK - rM), \quad j = c, w \quad (11)$$

As will be shown, the distribution of stocks  $k_j$  is endogenously determined jointly with the profit share  $\pi$  in the steady state.

Following the approach in Skott (1981, 1989) and Skott and Ryoo (2008), households' saving/portfolio behavior is specified in terms of their desired stock-flow ratios:

$$vN_j = \alpha_j p Y_j \quad (12)$$

$$M_j = \beta_j p Y_j, \quad j = c, w \quad (13)$$

where  $Y_j$ 's are capitalist and workers' real income and defined as

$$Y_j = (W_j + Div_j + rM_j)/p \quad (14)$$

$\alpha_j$ 's and  $\beta_j$ 's are the ratios of stock and deposit holdings to income that each class desires to achieve, respectively. I will assume that  $\alpha_j$ 's and  $\beta_j$ 's are exogenous in the benchmark model.<sup>15</sup> Appendix I analyzes three alternative specifications that endogenize  $\alpha_j$  and  $\beta_j$ , which do not change qualitative results.

Using (9), (11), (13), and (14), the households' income can be expressed as a function of the profit share and the share of stocks:

$$Y_c/K \equiv y_c(\pi_n, k_c) = \frac{a(k_c)(\pi_n - \beta_w r)u_n}{a(k_c)(\beta_c - \beta_w)r + (1 - \beta_c r)(1 - s_f^* \beta_w r)} \quad (15)$$

$$Y_w/K \equiv y_w(\pi_n, k_c) = \frac{[(1 - \beta_c r)(1 - s_f^* \pi_n) - a(k_c)(\pi_n - \beta_c r)]u_n}{a(k_c)(\beta_c - \beta_w)r + (1 - \beta_c r)(1 - s_f^* \beta_w r)} \quad (16)$$

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<sup>15</sup>The desired stock-flow ratios may depend on a number of variables such as the rates of return on various assets and the growth rate of incomes, but comparative statics are simplified if those ratios are taken as exogenous. The results with exogenous stock-flow ratios will carry over to the general case with variable ratios if the effects from induced changes in the ratios are relatively small. Skott (1981, 1989) introduced the stock-flow specification of saving/portfolio behavior. Skott and Ryoo (2008) and Ryoo and Skott (2008) applied this approach to the study of financialization.

where  $u_n \equiv u - \delta$ ,  $\pi_n \equiv (\pi u - \delta)/u_n$ ,  $a(k_c) \equiv \lambda + (1 - s_f)(1 - \lambda)k_c$ , and  $s_f^* \equiv s_f(1 - \lambda)$ .  $u_n$  is aggregate income net of depreciation measured in capital stock and  $\pi_n$  the share of net profits in net income.  $a(k_c)$  represents the share of the capitalists' claims in profits net of depreciation and interest payments.  $s_f^*$  represents the effective retention ratio which takes into account the deduction of top manager pay from profits. It can be readily shown that (15) is increasing in  $\pi_n$  and  $k_c$  and (16) decreasing in  $\pi_n$  and  $k_c$ : a rise in the profit share or a rise in the capitalists' share of stocks shifts personal income in favor of capitalists.

The stock-flow ratios (12) and (13), together with the budget constraints (10), can be used to derive steady-growth consumption functions. On a steady growth path, the share of stock ownership  $k_j$  and the class-share of income remain constant, and therefore  $\hat{N}_c = \hat{N}_w = \hat{N}$  and  $\hat{Y}_c = \hat{Y}_w = n$ . Given these requirements, (12), (13), (10) and (14) yield steady-growth consumption functions for workers and capitalists:

$$C_j = (1 - s_j)Y_j, \quad \text{where } s_j \equiv \alpha_j \hat{N} + \beta_j n, \quad j = c, w \quad (17)$$

where  $\alpha_j \hat{N} + \beta_j n$  represents the ratio of newly acquired financial stocks to income and thus the average saving rate of class  $j$ . I will follow a standard assumption  $s_c > s_w$ .<sup>16</sup>

**Product market equilibrium** The equilibrium condition for the goods market is given by

$$C_c + C_w + I = Y \quad (18)$$

and, using (17) and (14), can be written as

$$(1 - s_c)y_c(\pi_n, k_c) + (1 - s_w)y_w(\pi_n, k_c) + n = u_n \quad (19)$$

where  $y_c(\pi_n, k_c)$  and  $y_w(\pi_n, k_c)$  are given by (15) and (16), respectively. Equation (19) can be used to determine the profit share for a given  $k_c$ . The stability condition for this Kaldorian distribution mechanism requires aggregate demand to be decreasing in the profit share. Formally,

$$(s_c - s_w)a(k_c) + (1 - s_w)(1 - \beta_c r)s_f^* > 0 \quad (20)$$

Under condition (20), (19) can be solved for the equilibrium profit share for any given level of  $k_c$ :

$$\pi_n^*(k_c) = \frac{(\iota - s_w)[a(k_c)(\beta_c - \beta_w)r + (1 - \beta_c r)(1 - s_f^* \beta_w r)]}{(s_c - s_w)a(k_c) + (1 - s_w)(1 - \beta_c r)s_f^*} + \beta_w r \quad (21)$$

where  $\iota$  is the share of net investment in net output,  $\iota \equiv n/u_n$ .

Equation (21) then is used to determine the division of income between capitalists and workers as a function of  $k_c$ . Substituting (21) back into (15) and (16) yields:

$$y_c(\pi_n^*(k_c), k_c) = \frac{a(k_c)(\iota - s_w)u_n}{(s_c - s_w)a(k_c) + (1 - s_w)(1 - \beta_c r)s_f^*} \quad (22)$$

$$y_w(\pi_n^*(k_c), k_c) = \frac{[a(k_c)(s_c - \iota) + (1 - \iota)(1 - \beta_c r)s_f^*]u_n}{(s_c - s_w)a(k_c) + (1 - s_w)(1 - \beta_c r)s_f^*} \quad (23)$$

---

<sup>16</sup>Note that  $s_c > s_w$ , a typical assumption in post-Keynesian/structuralist models, imposes restrictions over  $\alpha_j$ ,  $\beta_j$ ,  $\hat{N}$  and  $n$ .

The capitalists' income (22) will be strictly positive if and only if

$$\iota > s_w, \quad (24)$$

i.e., the workers' saving rate should not be too large.<sup>17</sup> The workers' income (23), on the other hand, will be positive for all possible states of stock ownership distribution if

$$a(1)(s_c - \iota) + (1 - \iota)(1 - \beta_c r)s_f^* > 0. \quad (25)$$

Once the steady state value of  $k_c$  is found, the model is fully solved.

**Steady state** Dividing (22) by (23), the ratio of the capitalists' income to the workers' is expressed as a function of  $k_c$ .

$$\frac{y_c}{y_w} \equiv x = \frac{a(k_c)(\iota - s_w)}{a(k_c)(s_c - \iota) + (1 - \iota)(1 - r\beta_c)s_f^*} \equiv F(k_c) \quad (26)$$

$x$ , the ratio of the capitalists' income to the workers', serves as a measure of income inequality in this model.

The capitalists' share of stocks  $k_c$ , on the other hand, is determined by income distribution between the two classes:

$$k_c = \frac{\alpha_c y_c}{\alpha_c y_c + \alpha_w y_w} = \frac{\frac{\alpha_c}{\alpha_w} x}{1 + \frac{\alpha_c}{\alpha_w} x} \equiv G(x) \quad (27)$$

The steady state requires the mutual consistency of (26) and (27), and it can be shown that there exists a unique steady state  $(x^*, k_c^*)$  such that  $x^* = F(k_c^*)$  and  $k_c^* = G(x^*)$  (Ryoo, 2016b).

## 4 Determinants of the capitalists' income share ('top income share')

The US economy has gone through major structural changes since the early 1980s. As for the changes that can be captured by the present model, Table 2 summarizes some of the stylized facts. The average growth rate of capital in the period after 1980 has been lower than the Golden age of capitalism in the 50s and the 60s. This may be interpreted as a permanent decrease in the natural rate of growth ( $n$ ) in this framework. The fraction of top managerial income in profits ( $\lambda$ ) has increased, whereas the corporate retention rate ( $s_f$ ) and the rate of net new equity issues ( $\hat{N}$ ) have fallen sharply. Table 1 shows main comparative statics.

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<sup>17</sup>The violation of (24) leads to too low a profit share, which makes the capitalists' income vanish. (24) is the familiar viability condition for a two-class economy in the Pasinetti model (Pasinetti, 1962, Samuelson and Modigliani, 1966).

Table 1: Comparative statics

	$n$	$s_f$	$\hat{N}$	$\lambda$	$\beta_w$
$y_c^*/y_w^*$	+	-	-	+	-
$k_c^*$	+	-	-	+	-

Table 2: Some stylized facts

	$s_f$	$\hat{N}$	$\lambda^*$	$n$
1952-1970	.56	.004	.074	.038
1981-2012	.33	-.019	.158	.024

Notes:  $s_f$  and  $\hat{N}$  are calculated from the FRB flow of funds accounts.  $\lambda$  is based on the FRB flow of funds accounts and Piketty and Saez (2003)[2013, updated]. (Profits in the formula of  $\lambda^*$  are gross profits including depreciation and before interest payments.  $\lambda$  in the theoretical model is the fraction of wages out of profits net of both depreciation and interest payments and its actual values are likely about twice  $\lambda^*$ ). The values of  $n$  are calculated from the Penn World Table 8.0.

#### 4.1 The rate of economic growth ( $n$ )

The reduction in the natural growth rate has an implication for aggregate demand and distributional consequences in the Kaldorian framework. A once-and-for-all decrease in the natural growth rate represents a permanent decline in the steady-growth level of investment demand relative to capital stock. The fall in investment demand will lower the profit share and the capitalist share of income, other things being equal. The size of the effect of a fall in  $n$  on  $\pi$  can be large. With the baseline values in Table 3, a fall in the growth rate from 3.8% to 2.4% reduces the expanded profit share ( $\pi$ ) by 7.85 percentage points and the conventionally measured profit share by 6.7 percentage points. The change in functional income distribution is associated with a decline in the capitalists' share of income and wealth. The capitalists' income share falls by 4.81 percentage points. In other words, the observed decline in the rate of capital accumulation represents a significant stagnationary pressure over aggregate demand, which might have reduced the profit share and top income shares in the absence of any offsetting effect. This result is in stark contrast to the argument in Piketty (2014) that slow growth causes high inequality.

#### 4.2 Distributional implications of financialization

The global economy saw profound changes in the financial sector and its increasing dominance over the other part of the economy. The implications of changes in the structure and the

Table 3: Comparative statics: numerical illustrations

	initial equilibrium	fall in $n$	rise in $\lambda$	fall in $s_f$	fall in $\hat{N}$	all changes combined
$\pi$	39.94%	32.09%	43.20%	49.15%	42.97%	50.00%
conventional $\pi$	36.73%	30.03%	35.97%	44.69%	39.33%	40.95%
$k$	36.33%	20.93%	52.93%	60.13%	42.25%	66.88%
$\frac{y_c}{y_c+y_w}$	9.40%	4.59%	16.97%	21.52%	11.74%	26.85%

Notes: The benchmark parameters are  $n = 0.038$ ,  $\lambda = 0.15$ ,  $s_f = 0.55$ ,  $\hat{N} = 0.004$ ,  $u = 0.3$ ,  $\delta = 0.05$ ,  $r = 0.03$ ,  $\alpha_c = 3.3$ ,  $\beta_c = 3.5$ ,  $\alpha_w = 0.6$  and  $\beta_w = 0.6$ . The alternative values for simulations are  $n = 0.024$ ,  $\lambda = 0.3$ ,  $s_f = 0.35$  and  $\hat{N} = -0.019$ . The conventional  $\pi$  refers to the measure of the profit share that results from deducting top management pay ( $W_c$ ) from the broadly defined profit, i.e.,

$$\frac{\Pi - \lambda(\Pi - \delta pK - rM)}{pY} = \frac{\pi^* u^* - \lambda(\pi^* u^* - \delta - rm^*)}{u^*}$$

behavior of the financial sector have been examined under the heading of financialization.<sup>18</sup> I will focus on a small subset of the developments associated with financialization which are closely related to the results from the model analyzed in this paper.

The retention rate of non-financial corporations in the US decreased significantly from 56% in 1952-1970 to 33% in 1981-2012: the U.S. corporations have paid out the greater portion of their earnings to their shareholders in the recent decades. This is captured by the reduction in  $s_f$  in the present model. The initial impact effect of a fall in  $s_f$  benefits both the capitalists' and workers' household by raising their dividend income, but because the fall in the retention rate represents an income transfer from the high saving sector (corporations) to the low saving sector (households), it tends to stimulate aggregate demand and increase the profit share. The increase in the profit share gives an additional benefit to the capitalists' households because of increased managerial compensation, at the expense of the share of wages of the workers' households. Induced changes in the distribution of stock holdings ( $k_c$ ) make the demand effect of a fall in  $s_f$  less certain, but it turns out that the capitalists' share of income increases unambiguously. The overall distributional effect of the fall in the retention rate is quantitatively very large. With the benchmark parameters (Table 3), the fall in  $s_f$  from 0.55 to 0.35 increases both the profit share and the capitalists' share of income significantly, by about 10 and 12 percentage points, respectively. This effect alone more than offsets the negative effect of the fall in the growth rate on the profit share and the top income share.

The rate of net new equity issues by non-financial corporations was small but positive, 0.004 on average in the 1952-1970, but the figures turned to large negative numbers in 1981-2012, -0.019 on average. The fall in  $\hat{N}$  means net acquisition of stocks by households decreases,

<sup>18</sup>There are a number of studies on financialization, including (Epstein, 2005, Stockhammer, 2004, Skott and Ryoo, 2008, Ryoo and Skott, 2008, Hein, 2012, Palley, 2013).

and thus it reduces the household saving rate and stimulates consumption demand. The profit share will increase and raise the capitalists' share of income similarly to the case of the fall in  $s_f$  (see Table 3 for numerical illustrations).

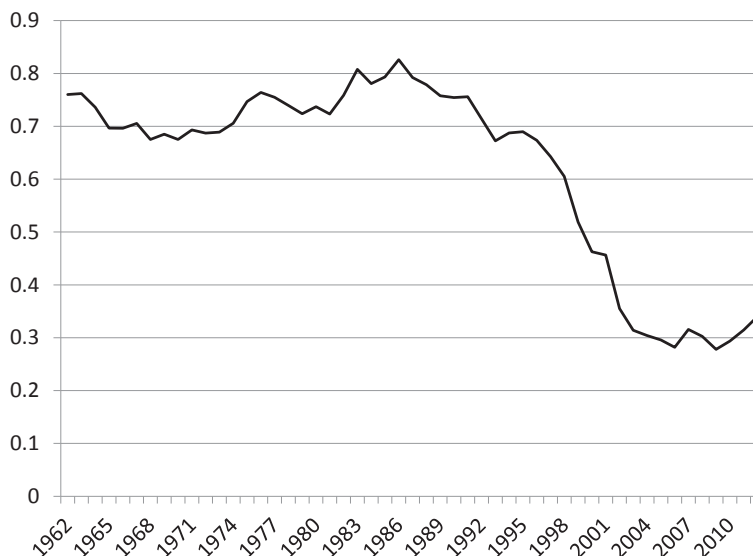


Figure 4: Ratio of nonequity-nonhousing net worth to income of the bottom 99% wealth group ( $\beta_w$ )

*Notes:* Author's calculations from Saez and Zucman (2014). I recategorized the five items in the net worth of the bottom 99% wealth (stocks, net housing, fixed income assets, enterprise income and pension wealth) in the Saez-Zucman data into the three items (corporate equity, *gross* housing, and the others). In doing so, I decomposed pension wealth into stocks and the other assets assuming the composition follows that of stocks and fixed income assets in the five-items classification. I also converted net housing (housing net of mortgages in the Saez-Zucman data) into gross housing by taking the mortgage component out and adding it to 'the others' (as a negative value). The chart plots the values of 'the others' and intends to capture the movement of  $\beta_w$  in this paper.

Increasing household indebtedness (esp. of lower income households) is another important development since the 1980s. Increasing household debt has received great attention recently especially as a consequence of increasing inequality, but it can also be a cause of increasing inequality. The model economy in this model highlights this aspect. High indebtedness of lower income households can be captured by a fall in  $\beta_w$  since  $\beta_w$  stands for fixed income assets net of debt. Figure 4, which is constructed using the Saez-Zucman data, shows the net worth of the bottom 99% wealth class excluding corporate equity and housing relative to their income had fallen dramatically since the early 1980s. This movement reflects the well-known increase in the debt-income ratio of the bottom 99% wealth class, which is mostly explained by increasing mortgage debt.<sup>19</sup> The decrease in  $\beta_w$  – the increase in the debt-income ratio

<sup>19</sup>A referee pointed out the importance of the 1986 Tax Reform Act for the rise in mortgage debt. Before 1986, all interest expenditure was tax-deductible, but, after the 1986 reform act, only mortgage interest was deductible. As a result, many types of lending and borrowing shifted into the mortgage rubric. In particular,

– has conflicting effects on aggregate demand and the profit share. On the one hand, it corresponds to a decrease in net interest income (an increase in debt servicing) of workers (low savers), but, a lower  $\beta_w$  means a fall in net acquisition of fixed income asset (or a rise in net borrowing) and reduces the saving rate of workers. Depending on the relative strength of the two effects, the profit share may rise or fall. The first effect, it turns out, remains primary when it comes to its effect on the distribution between workers and capitalists, regardless of how functional income distribution shifts. The effect of interest income transfer (away from the workers’ households) is dominant<sup>20</sup> and it unambiguously raises the top income share.

The positive effect of rising household debt (a fall in  $\beta_w$ ) on the top income share may indirectly capture the distributional effect of a housing market boom (which is typically shown in high housing/income ratios. See Figure 5). The increases in the housing/income ratio and the debt-income ratio tends to reinforce each other via the well-known interaction between asset price and credit supply, and the fall in  $\beta_w$  in the 2000s may be seen as a result of such interaction.<sup>21</sup>

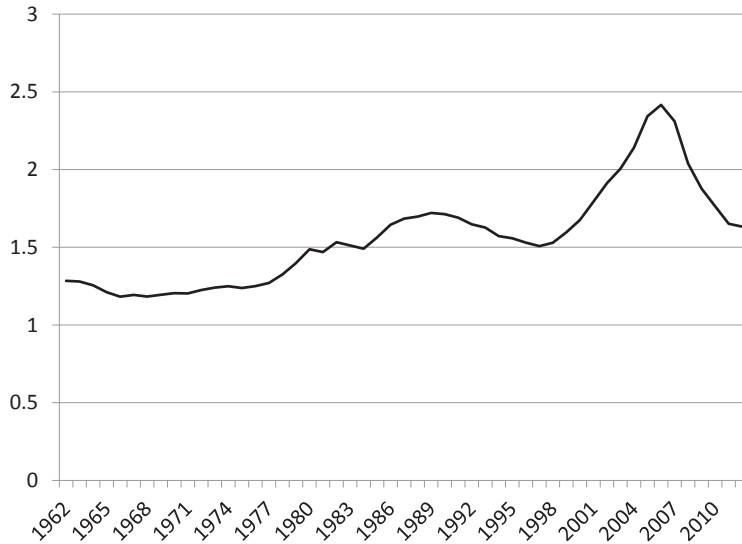


Figure 5: (Gross) Housing wealth-income ratio of the bottom 99% wealth group

Notes: Author’s calculations from Saez and Zucman (2014). See the notes in Figure 4 for the detail.

A stock market boom may be captured by a rise in  $\alpha_c$  and  $\alpha_w$ . The ratio of corporate equity to income, constructed from the Saez-Zucman data, increased from 1.92 in 1980 to

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small business liabilities were shifted onto personal balance sheets, and secured by personal housing. I am grateful to the referee for providing this detailed account.

<sup>20</sup>Because of this, even in the case where a fall in  $\beta_w$  has a positive effect on demand and the profit share, the magnitude is small in simulations for a range of parameter values.

<sup>21</sup>The analysis of macroeconomic interaction between housing prices and debt dynamics is complex and beyond the scope of this paper. See Ryoo (2016a) for an attempt to formalize such interaction along the Minsky-Kaldor lines.



4.33 in 2007 for the top 1% wealth class. During the same period, the equity-income ratio for the rest 99% group rose from 0.34 to 0.97 (Table 5). The rising trend in the equity-income ratios was particularly pronounced in the 1990s, as expected.

The impact effect of changes in  $\alpha_j$ 's on aggregate demand works through its impact on the personal saving rates (recall  $s_j = \alpha_j \hat{N} + \beta_j n$ ) and thus depends on the sign of the rate of net equity issues  $\hat{N}$ . If  $\hat{N} = 0$ , the effect of changes in  $\alpha_j$  on the saving rate is neutral and thus there will be no impact effect on aggregate demand and income distribution.<sup>22</sup> During the Golden Age,  $\alpha_j$ 's increased and drove a long stock market boom until the mid 60s, but since  $\hat{N}$  was close to zero on average, the increase in  $\alpha_j$ 's did not lead to any large movement in income distribution either at the functional or personal levels.

The situation was very different during the period from the early 1980s until 2007. There were massive scale stock buybacks ( $\hat{N} \approx -0.02$  on average). If  $\hat{N} < 0$ , an increase in  $\alpha_j$ 's reduces personal saving rates, which tends to stimulate aggregate demand and raise the profit share and the top income share.<sup>23</sup> Such distributional effects were strong especially during the 1990s' stock market boom.

### 4.3 The share of labor income of top one percent in total profits ( $\lambda$ )

Figure 6 shows that the ratio of labor income of the top one percent to the broadly defined profits varied little around 7% in the 50s and 60s, but continued to rise since then, reaching historical high at 20.6% in 2000.<sup>24</sup> Looking at the data this way highlights a significant change in the way how profits are allocated by firms in the past decades. This change is captured by an increase in  $\lambda$  in the present model.

How a rise in  $\lambda$  affects aggregate demand and the profit share in this model is somewhat complex because it represents both an income transfer from the corporate sector (high saving sector) to the capitalist households (low saving sector) and a transfer from the workers' households (low savers) to the capitalists' (high savers). When the first channel – a reduction in the effective retention rate  $s_f^* = s_f(1 - \lambda)$  – is stronger, the rise in  $\lambda$  can raise aggregate demand and the profit share by stimulating capitalists' consumption demand (otherwise that part of profits would have been saved in the form of retained earnings). The effect of a rise in  $\lambda$  on the profit share is ambiguous in general,<sup>25</sup> but the rise in  $\lambda$  unambiguously raises income and wealth inequality between workers and capitalists ( $x$  and  $k$ ): the positive impact of the increase in top management pay on the capitalists' share of income remains a first-order effect,

<sup>22</sup>There will be a change in the distribution of stocks ( $k_c$ ), which affects demand and the profit share. But this effect will be small compared to the case when  $\alpha_j$  is nonzero.

<sup>23</sup>The precise total effect on distributive shares again depends also on the endogenous adjustment of stock ownership distribution ( $k_c$ ). If  $\hat{N} < 0$  and both  $\alpha_c$  and  $\alpha_w$  increases, the total effect on the top income and wealth shares depends on the movement of  $\alpha_c/\alpha_w$ . If  $\alpha_c/\alpha_w$  has increased, the top income and wealth shares unambiguously increases. This result remains valid unless  $\alpha_c/\alpha_w$  falls too much. This case roughly captures what the US economy experienced from the early 1980s until 2007. See Ryoo (2016b) for the analytic details.

<sup>24</sup>As pointed by a referee, the sharp rise around 1986-1988 is mainly due to the effect of the 1986 Tax Reform Act, as discussed in section 2.1, and therefore the data since 1986 exaggerate the actual increase in the ratio of top labor income to profits by about three percentage points.

<sup>25</sup>The underlying mechanism is very similar to the ambiguous effect of a rise in top manager wages and their proportion in total employment on aggregate demand in the Kaleckian analysis of Dutt (2016), which also highlights the possibility of an expansionary demand effect of top management income.

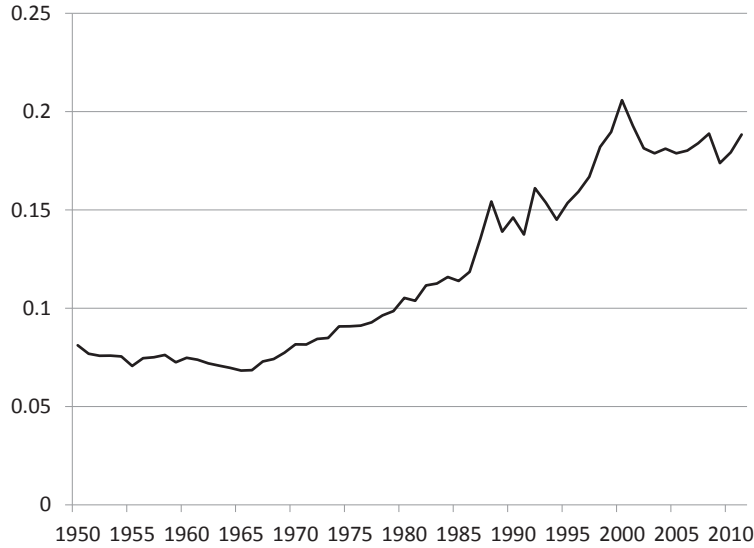


Figure 6: The ratio of the top 1% income earners' labor income to broadly defined profits in the US

Notes: The share of wages in broad profits is calculated by dividing the share of labor income of the top one percent in GDP by the broadly defined profit share. The figure may roughly correspond to  $W_c/\Pi$  in this paper. Author's calculations based on Penn World Table and Piketty and Saez (2003)[updated data, 2013].

regardless of the direction of induced change in the profit share. The impact effect can be reinforced by an increase in the profit share if the aforementioned income transfer away from the corporate sector – reduced effective retention rate – is strong. The simulation result in Table 3 illustrates such a case. With the benchmark parameters, a rise in  $\lambda$  from 0.15 to 0.30 raises the broadly defined profit share by 3.26 percentage points although the conventionally measured profit share somewhat declines. The first-order effect on the capitalists' share of income combined with the effect of the induced increase in the profit share produces a large increase in the capitalists' income share by nearly 8 percentage points. The mechanism behind the expansionary effect of a rise in corporate spending on top management pay is in line with the old wisdom of Keynes's widow's curse in his *Treatise on Money* or Kalecki's dictum 'capitalists earn what they spend.' Minsky also argued that corporate spending on the compensation of top managers is self-financing and its distributional effect tends to be self-reinforcing due to the essentially same mechanism as in this model(Minsky, 1986)[pp. 155-156].

By a change in  $\lambda$  the model intends to capture a structural change within the corporate sector that determines the size of profits allocated to top executives and managers, but note that increasing share of labor income of the top income class in national income is determined by macroeconomic forces. The share of top management pay in aggregate net income is represented by  $W_c/(pY - \delta pK)$ , which is endogenous in this model. It is certainly affected by the increase in  $\lambda$ . Using the same benchmark parameters, an increase in  $\lambda$  from 0.15 to 0.30

Table 4: The share of top management pay in national income: various determinants

	initial equilibrium	rise in $\lambda$	fall in $s_f$ and $\hat{N}$	fall in $n$	combined effect
$\frac{W_c}{pY - \delta pK}$	3.84%	8.67%	6.40%	2.47%	10.87%

*Notes:* The parameter values are the same as in Table 3.

causes the share of top management pay to increase from 3.84% to 8.67%. It is worth noting, however, that the share of top management pay in aggregate income may have been affected by several other factors that look seemingly unrelated to the way how the broadly defined profits are allocated as top management pay ( $\lambda$ ). For instance, a fall in  $s_f$  and  $\hat{N}$  can raise the profit share and the distributive share of top management income,<sup>26</sup> while the decline in the growth rate has the opposite effect. The quantitative effects of those factors are sizable in the present model, as shown in Table 4.

To summarize this section: the US economy has witnessed several developments which have crucial impacts on functional and personal income distribution. The slowdown in capital accumulation – low  $n$  – tends to decrease income and wealth inequality, but this effect appears to have been more than offset by those worsening inequality: the rise in  $\lambda$  and the decline in  $s_f$ ,  $\hat{N}$ , and  $\beta_w$ , along with a long-period of asset bubbles, in the US economy since the early 1980s.

## 5 The aggregate wealth-income ratio

The present model assumes that the capital-output ratio –  $1/u$  – is at the desired level, which is taken as exogenous to simplify the analysis. The aggregate wealth-income ratio, is however, distinguished from the capital-output ratio and determined endogenously. The aggregate wealth-income ratio is the weighted average of those of the top one percent and the rest with the weights being given by their respective income share. In the current setting,

$$\omega \equiv \frac{NW_c + NW_w}{pY_c + pY_w} = \sigma_c \omega_c + (1 - \sigma_c) \omega_w \quad (28)$$

where  $\sigma_c = \frac{y_c}{y_c + y_w} = \frac{x}{1+x}$  and  $\omega_j = \alpha_j + \beta_j$ . The assumption of exogenous  $\alpha_j$  and  $\beta_j$  means the wealth-income ratio for each class is exogenous. With constant  $\omega_j$ , the aggregate wealth-income ratio  $\omega$  is fully determined by the income distribution between capitalists and workers. Empirically, the wealth-income ratio of the rich is far greater than the poor (see Figure 7 and

<sup>26</sup>A stock market boom – a rise in  $\alpha_j$ 's – also raises the profit share and the share of top labor income when net issues of equity is negative.

Table 5):  $\omega_c \gg \omega_w$ . This means that any factor that raises income inequality in favor of the rich, *ceteris paribus*, leads to an increase in the aggregate wealth-income ratio.

Table 5: Wealth-income ratios and their components

	Top 1% income share	<b>total wealth/ income</b>	equity	housing	others	<b>1% wealth/ income</b>	equity	housing	others	<b>99% wealth/ income</b>	equity	housing	others
1962	10.59%	<b>3.28</b>	0.95	1.32	1.02	<b>9.18</b>	4.40	1.60	3.18	<b>2.58</b>	0.54	1.28	0.76
1980	9.14%	<b>3.08</b>	0.49	1.57	1.03	<b>8.21</b>	1.92	2.34	3.94	<b>2.57</b>	0.34	1.49	0.74
2007	16.82%	<b>4.66</b>	1.53	2.22	0.91	<b>9.97</b>	4.33	1.78	3.85	<b>3.59</b>	0.97	2.31	0.32

Sources: Authors' calculations from Saez and Zucman (2014). See the notes in Figure 4 for the detail.

**The wealth-income ratio and income inequality** The positive effect of higher income inequality on the aggregate wealth-income ratio appears to have been significant in the U.S. economy. The aggregate wealth-income ratio increased from 3.08 in 1980 to 4.66 in 2007 and the income share of the top one percent rose from 9.14% to 16.82% (Table 5). Using the data on the movements of individual wealth-income ratios, the total increase can be decomposed into the effect of increasing inequality for given wealth-income ratios of different groups and the effect of changes in the individual wealth-income ratios for given income inequality.

A quick computation from Table 5 shows that if the wealth-income ratios had never changed from 1980 for the top one percent and the rest (8.21 and 2.57, respectively), the aggregate wealth-income ratio would have increased from 3.08 to 3.52. This change, 0.44, amounts to about 28% of the total change in the aggregate wealth-income ratio. The increase in income inequality thus explains a significant part of the increase in the aggregate wealth-income ratio.

**Asset price inflation** A larger part of the rise in the wealth-income ratio, however, is explained by increases in individual wealth/income ratios. Table 5 shows that the wealth-income ratio increased for both classes. Even if the top one percent income share had been the same at 9.14% over the period 1980-2007, the aggregate wealth income ratio would have increased from 3.08 to 4.18.

Table 5 also shows the breakdown of the wealth-income ratios according to the asset types. At the aggregate level, both the corporate equity/income ratio and the housing/income ratio increased substantially from 0.48 to 1.53 and from 1.57 to 2.22 in period 1980-2007. This suggests that the increase in the aggregate wealth-income ratio reflects booms in stock and housing markets. The decline in the ratio of 'others' to income from 1.03 to 0.91 is largely explained by the fall in the same ratio of the bottom 99% wealth class. The driving force of the latter was the increase in the debt-income ratio of this class (both mortgage and non-mortgage debt).

The comparison of the movements in the wealth-income ratios before and after 1980 may help understand the driving forces of changes in the ratios both at the aggregate and the disaggregate levels. Table 6 shows how the growth rate of a wealth-income ratio is broken

Table 6: The decomposition of the change in the wealth-income ratios

	period	growth of wealth/income $\hat{\omega}_j$	capital gain induced growth $\hat{p}_j^a$	saving induced growth $\hat{A}_j$	income growth $\hat{Y}_j$
Aggregate	1950-1979	-0.55%	-0.79%	4.05%	3.80%
	1980-2007	1.72%	1.99%	2.71%	2.97%
Top 1%	1950-1979	-0.07%	-1.05%	3.72%	2.74%
	1980-2007	1.13%	1.68%	4.40%	4.95%
Bottom 99%	1950-1979	-0.74%	-0.73%	4.21%	3.94%
	1980-2007	1.42%	2.14%	1.96%	2.67%

Sources: Authors' calculations from Saez and Zucman (2014).

down into the three growth components, capital gain induced growth of wealth, saving induced growth of wealth and the growth of income, using the language of Piketty and Zucman (2014). The decomposition is based on a fairly straightforward identity<sup>27</sup>:

$$\hat{\omega}_j = \hat{p}_j^a + \hat{A}_j - \hat{Y}_j = \underbrace{\hat{p}_j^a}_{\text{capital gain induced}} + \underbrace{s_j/\omega_j}_{\text{saving induced}} - \underbrace{\hat{Y}_j}_{\text{income growth}} \quad (29)$$

where the net worth of class  $j$  is expressed by the product of the composite asset price index  $p_j^a$  and the quantity index of assets  $A_j$ , i.e.,  $NW_j = p_j^a A_j$ . Since the net acquisition of assets equals the flow of savings,  $p_j^a \dot{A}_j = S_j$ , dividing through by  $p_j^a A_j$  yields  $\hat{A}_j = s_j/\omega_j$ . During 1950-1979, both at the aggregate and disaggregate levels, the saving-induced growth of wealth is moderately higher than income growth, and the small positive gaps are outweighed by the negative growth of the asset price index, by slight margins. The wealth-income ratios in 1950 had returned to a somewhat lower level in 1979 (see Figure 7). The period of 1980-2007 saw a strong upward movement in the wealth-income ratio both at the top and the bottom ends. During this period, saving induced growth of wealth was lower than income growth at all levels. The wealth-income ratios might have fallen with saving-induced wealth accumulation alone, but the strong asset price inflation effect fueled the rise in wealth-income ratios, especially for the bottom 99% wealth class and at the aggregate level. The increase in the wealth-income ratios and the decline in personal saving rates since the 1980s (see Figure 8 and 9) thus can be explained by strong asset price inflation.

<sup>27</sup>The decomposition equation (29) has an affinity with the one based on the definition of  $\alpha_j$  if stocks are the only household asset where the saving rate is given by  $s_j = \alpha_j \hat{N}$ :

$$\hat{\alpha}_j = \left( \frac{\hat{v}}{p} \right) + \hat{N} - \hat{Y}_j = \left( \frac{\hat{v}}{p} \right) + \frac{s_j}{\alpha_j} - \hat{Y}_j$$

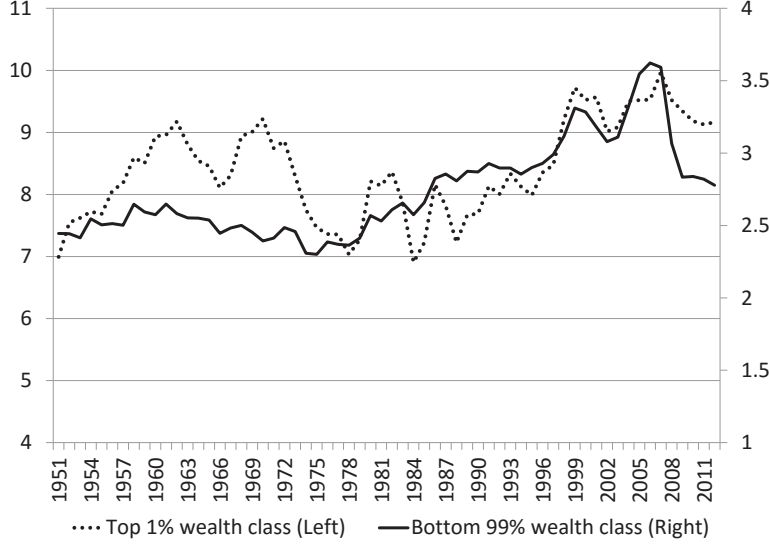


Figure 7: Wealth/income ratios by wealth class in the US

Sources: Author's computations from Saez and Zucman (2014).

**The effect of the growth rate on the aggregate wealth-income ratio** The relation between  $n$  and  $\omega$  may merit attention given Piketty's argument that slow growth caused high wealth-income ratio. In the present model, the fall in  $n$  unambiguously reduces the capitalists' income share, and under the assumption of the constancy of individual wealth-income ratios, *decreases* the aggregate wealth-income ratio. Thus the result is in contrast to Piketty's argument that 'capital is back because slow growth is back'. The result, however, may depend on the seemingly restrictive assumption that individual wealth-income ratios  $\omega_c$  and  $\omega_w$  are exogenous. Different specifications of consumption/portfolio behavior may make wealth/income ratios for each class endogenous and furthermore make individual wealth-income ratios decreasing in the growth rate. To check the robustness of the effect of changes in the growth rate on the aggregate wealth-income ratio, I consider three different specifications of consumption/portfolio behavior (Model II, III, IV) along with the benchmark model (Model I) that takes  $\alpha_j$  and  $\beta_j$  as exogenous.

Model II assumes exogenous ratios of financial stocks to *consumption* as in Skott (1981), i.e.,

$$vN_j = \tilde{\alpha}_j p C_j \quad \text{and} \quad M_j = \tilde{\beta}_j p C_j$$

Appendix I shows that  $\alpha_j$  and  $\beta_j$  become endogenous under this specification and the wealth-income ratio for each class is given by

$$\omega_j \equiv \alpha_j + \beta_j = \frac{\tilde{\alpha}_j + \tilde{\beta}_j}{1 + \tilde{\alpha}_j \hat{N} + \tilde{\beta}_j n} \quad (30)$$

Model III considers a fairly standard consumption specification as a function of income

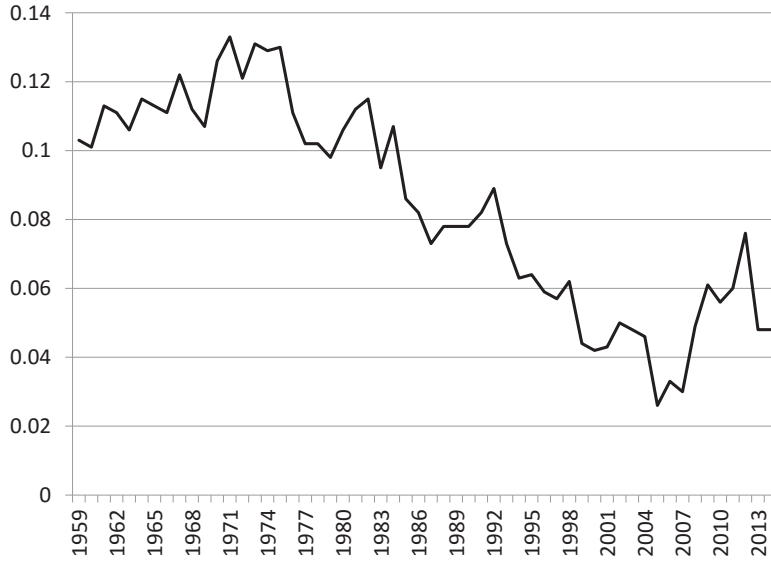


Figure 8: The personal saving rate in the US

Notes: US. Bureau of Economic Analysis, Personal Saving Rate [PSAVERT], retrieved from FRED, Federal Reserve Bank of St. Louis. <https://research.stlouisfed.org/fred2/series/PSAVERT/>, October 12, 2015.

and wealth, along with a constant portfolio  $\epsilon_j$  for each class:

$$pC_j = c_j pY_j + \nu_j (vN_j + M_j) \quad \text{and} \quad \epsilon_j = vN_j/M_j$$

Appendix I derives the implied values of wealth-income ratios:

$$\omega_j = \alpha_j + \beta_j = \frac{(1 + \epsilon_j)(1 - c_j)}{\epsilon_j \hat{N} + n + \nu_j(1 + \epsilon_j)} \quad (31)$$

Model IV assumes the constancy of average saving ratios  $(1 - \bar{c}_j)$  for each class by dropping the wealth-related term from (5).

$$pC_j = \bar{c}_j pY_j \quad \text{and} \quad \epsilon_j = vN_j/M_j$$

The implied values of wealth-income ratios are given by

$$\omega_j = \alpha_j + \beta_j = \frac{(1 + \epsilon_j)(1 - \bar{c}_j)}{\epsilon_j \hat{N} + n} \quad (32)$$

Model IV appears to be merely a special example of Model III when  $\nu_j = 0$ , but it turns out to be quantitatively very different from Model III as well as the other two. All specifications except Model I make the wealth-income ratio  $\omega_j$  decreasing in  $n$ . Thus one may suspect that the result that a fall in  $n$  reduces the aggregate wealth-income ratio in Model I may be reversed in the models with endogenous wealth-income ratios. Interestingly, the difference does not lie between Model I and the others but between Model IV and the others.



Figure 9: Saving rates of Top 1% and Bottom 99% wealth class in the US

Sources: Author's computations from Saez and Zucman (2014).

Table 7 examines the effect on the aggregate wealth-income ratio of a decline in the growth rate from 0.038 to 0.024 by looking at its determinants  $\sigma_c$ ,  $\omega_c$  and  $\omega_w$ .<sup>28</sup> The fall in  $n$  reduces the capitalists' share of income  $\sigma_c$  in all models. This is in line with the analysis in section 4.1. In Model I, the wealth-income ratios of both classes are assumed to be constant and therefore the decrease in the capitalists' income share reduces the aggregate wealth-income ratio. In Model II and III, the fall in the natural rate does affect and raise the wealth-income ratios for both classes, but the adjustment of the wealth-income ratios is modest in size and the aggregate wealth-income ratio still declines due to the primary income distribution effect. What happens to Model IV, however, is very different from the other three models, where the specification of the consumption function which only depends on income in the absence of the influence of wealth on consumption leads to drastic changes in wealth-income ratios ( $\omega_c$  raises from 6.8% to 10.228% and  $\omega_w$  raises from 1.2% to 1.8%). These effects dominates the income distribution effect, and, as a result, the aggregate wealth-income ratio increases in response to the fall in the growth rate. One may notice that the feature of Model IV is similar to Piketty's numerical exercises based on  $K/Y = s/n$ . A decline in  $n$  with a constant  $s$  produces an implausibly large movement of  $K/Y$  in the neoclassical framework, especially when the growth rate approaches zero.<sup>29</sup>

The analysis of the models with endogenous stock-flow ratios suggests that a fall in the growth rate decreases the aggregate wealth-income ratio under plausible specifications of consumption behavior. The main mechanism is the induced income distribution effect on the

<sup>28</sup>Appendix II shows a further decomposition of the effect of the growth rate on the wealth-income ratios.

<sup>29</sup>In Model IV, the overly sensitive response of stock-flow ratios to changes in the growth rate is more visible if the rate of equity issues is negative.



Table 7: The effect of a fall in the growth rate on the aggregate wealth-income ratio

	Baseline	Model I	Model II	Model III	Model IV
$\sigma_c$	9.4%	4.59%	4.67%	4.50%	4.14%
$\omega_c$	6.8	6.8	7.150	7.562	10.228
$\omega_w$	1.2	1.2	1.21	1.33	1.8
$\omega = \sigma_c\omega_c + (1-\sigma_c)\omega_w$	1.726	1.457	1.487	1.610	2.148

*notes:* The parameter values for Model II, III and IV are chosen to produce the same stock/income ratios and average saving rates as in Model I if  $n = 0.038$ . In other words, for all models,  $\alpha_c = 3.3$ ,  $\beta_c = 3.5$ ,  $\omega_c = 6.8$ ,  $s_c = 0.146$ ,  $\alpha_w = 0.6$ ,  $\beta_w = 0.6$ ,  $\omega_w = 1.2$ , and  $s_w = 0.0252$  when  $n = 0.038$ . The numbers in the table show the effect of a decrease in  $n$  to  $n = 0.024$ . For all cases,  $\hat{N} = 0.004$ ,  $s_f = 0.55$ ,  $\lambda = 0.15$ ,  $u = 0.3$ ,  $\delta = 0.05$ , and  $r = 0.03$ .

aggregate wealth-income ratio. In these Kaldorian models, a fall in the growth rate reduces the capitalists' share of income and thus tends to reduce the aggregate wealth-income ratio. Therefore, the causes of the observed increase in the aggregate wealth-income ratio since the 1980s must be found somewhere else than the fall in the growth rate. As Tables 5 and 6 suggest, the upward movement of the wealth-income ratios since the early 1980s is driven by asset price inflation for the most part and does not seem to be caused by the lower growth rate.

## 6 Conclusion

The perspective in this paper places a strong emphasis on financial factors as the determinants of income inequality. I have suggested that several financial changes – the decline in the retention rate, the rise in stock buybacks, increasing indebtedness of the working class and asset market booms – affect the level of aggregate demand, and such changes tend to shift the aggregate saving function downward. The macroeconomic effect of the decline in the saving ratio differs in competing models of growth and distribution. The investment-saving equilibrium is achieved through the adjustment of the output-capital ratio both in the Solow-Swan model which serves as a key theoretical framework in Piketty's neo-classical explanation and in Kaleckian models which are popular in the heterodox approach. The induced change in the output-capital ratio represents the process of factor substitution along the aggregate production function in the former and the long-run adjustment in the rate of capacity utilization in the latter. In contrast to these models, the Kaldorian model presented above takes the output-capital ratio as exogenous, where the investment-saving equilibrium is established through variations in income distribution (the profit share). The downward shift in the saving function has distributional implications in this Kaldorian model, and actually raises the profit share and income inequality between capitalists and workers.

Piketty often expresses his skepticism about the explanation of income inequality based

on marginal productivity in *Capital in the twenty first century*, .<sup>30</sup> This, however, should not mask the centrality of his neo-classical version of income distribution theory in the seminal book. The proposition that the low growth rate raises income inequality by increasing the wealth-income ratio occupies substantial part of the book and Piketty uses the neo-classical benchmark as *the* key theoretical framework to support the proposition.

In the Kaldorian model presented here, the effect of changes in the growth rate on income inequality is precisely opposite to that in the neo-classical framework. The observed decline in the growth rate tends to reduce income inequality in this Kaldorian framework. Moreover, Piketty's proposition that a decline in the growth rate raises the aggregate wealth-income ratio loses its significance here. I have distinguished financial wealth from reproducible capital stock, and taken the ratio of reproducible capital to income as given at the outset. It has been shown that a fall in the natural growth rate does not raise the aggregate financial wealth-income ratio under plausible specifications of consumption/saving behavior, and that the observed increase in the aggregate wealth-income ratio in the U.S. since the 1980s is explained mostly by asset price inflation.

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<sup>30</sup>In particular, see Piketty's discussion of increasing inequality of labor income in chapter 9 (Piketty, 2014).

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## Appendix I

**Model II: Constant ratios of financial stocks to consumption** The specification of consumption/portfolio behavior is given by

$$vN_j = \tilde{\alpha}_j pC_j \quad (33)$$

$$M_j = \tilde{\beta}_j pC_j \quad (34)$$

Combining these with the budget constraint, we get

$$pC_j + \tilde{\alpha}_j pC_j \hat{N} + \tilde{\beta}_j n pC_j = pY_j$$

Solving for  $pC_j$ ,

$$pC_j = \frac{pY_j}{1 + \tilde{\alpha}_j \hat{N} + \beta_j n}$$

Plugging back into (33) and (34) and dividing through by  $pY_j$ ,

$$\frac{vN_j}{pY_j} = \alpha_j = \frac{\tilde{\alpha}_j}{1 + \tilde{\alpha}_j \hat{N} + \beta_j n} \quad (35)$$

$$\frac{M_j}{pY_j} = \beta_j = \frac{\tilde{\beta}_j}{1 + \tilde{\alpha}_j \hat{N} + \beta_j n} \quad (36)$$

Substituting (35) and (36) in  $\omega_j = \alpha_j + \beta_j$  and  $s_j = \alpha_j \hat{N} + \beta_j n$ , we obtain

$$\omega_j = \frac{\tilde{\alpha}_j + \tilde{\beta}_j}{1 + \tilde{\alpha}_j \hat{N} + \beta_j n}$$

$$s_j = \alpha_j \hat{N} + \beta_j n = \frac{\tilde{\alpha}_j \hat{N} + \tilde{\beta}_j n}{1 + \tilde{\alpha}_j \hat{N} + \beta_j n}$$

**Model III: Consumption as a function of income and wealth**

$$pC_j = c_j pY_j + \nu_j (vN_j + M_j)$$

$$\epsilon_j = vN_j / M_j$$

Substituting  $vN_j = \epsilon_j M_j$  into the consumption function and the budget equation,

$$pC_j = c_j pY_j + \nu_j (\epsilon_j + 1) M_j \quad (37)$$

$$pC_j + vN_j \hat{N} + M_j n = pC_j + \epsilon_j M_j \hat{N} + M_j n = pY_j \quad (38)$$

Plugging (37) in (38),

$$c_j pY_j + \nu_j (\epsilon_j + 1) M_j + \epsilon_j M_j \hat{N} + M_j n = pY_j \quad (39)$$

Rearranging and dividing by  $pY_j$ ,

$$\frac{M_j}{pY_j} = \beta_j = \frac{1 - c_j}{\epsilon_j \hat{N} + n + \nu_j(1 + \epsilon_j)} \quad (40)$$

Because  $vN_j = \epsilon_j M_j$ ,  $\omega_j = \alpha_j + \beta_j$  and  $s_j = \alpha_j \hat{N} + \beta_j n$ , we have

$$\frac{vN_j}{pY_j} = \alpha_j = \frac{\epsilon_j(1 - c_j)}{\epsilon_j \hat{N} + n + \nu_j(1 + \epsilon_j)} \quad (41)$$

$$s_j = \frac{(\epsilon_j \hat{N} + n)(1 - c_j)}{\epsilon_j \hat{N} + n + \nu_j(1 + \epsilon_j)} \quad (42)$$

$$\omega_j = \frac{(\epsilon_j + 1)(1 - c_j)}{\epsilon_j \hat{N} + n + \nu_j(1 + \epsilon_j)} \quad (43)$$

**Model IV: constant average saving rates** Setting  $c_j = \bar{c}_j$  and  $\nu_j =$ , (41), (40), (43) and (42) are rewritten into

$$\frac{vN_j}{pY_j} = \alpha_j = \frac{\epsilon_j(1 - \bar{c}_j)}{\epsilon_j \hat{N} + n} \quad (44)$$

$$\frac{M_j}{pY_j} = \beta_j = \frac{1 - \bar{c}_j}{\epsilon_j \hat{N} + n} \quad (45)$$

$$s_j = \frac{(\epsilon_j \hat{N} + n)(1 - \bar{c}_j)}{\epsilon_j \hat{N} + n} \quad (46)$$

$$\omega_j = \frac{(\epsilon_j + 1)(1 - \bar{c}_j)}{\epsilon_j \hat{N} + n} \quad (47)$$

## Appendix II

Table 8 demonstrates the effect of a decline in the growth rate from 0.038 to 0.024 on various stock-flow ratios. The result shows that all  $\alpha_j$  and  $\beta_j$  (and their sum  $\omega_j$ ) increase in Model II, III and IV in response to the decrease in  $n$ . The increases in the ratios in Model IV, however, are very large relative to those in Model II and III.

Table 8: The effect of a fall in the growth rate on stock-flow ratios and average saving rates

	Model I	Model II	Model III	Model IV
specification	$vN_j = \alpha_j pY_j$ $M_j = \beta_j pY_j$	$vN_j = \tilde{\alpha}_j pC_j$ $M_j = \tilde{\beta}_j pC_j$	$pC_j = c_j pY_j + \nu_j (vN_j + M_j)$ $\epsilon_j = vN_j/M_j$	$pC_j = \bar{c}_j pY_j$ $\epsilon_j = vN_j/M_j$
parameters	$\alpha_c = 3.3,$ $\beta_c = 3.5,$ $\alpha_w = 0.6,$ $\beta_w = 0.6$	$\tilde{\alpha}_c = 3.365,$ $\tilde{\beta}_c = 4.099,$ $\tilde{\alpha}_w = 0.616,$ $\tilde{\beta}_w = 0.616$	$c_c = 0.514, \nu_c = 0.05,$ $\epsilon_c = 0.943$ $c_w = 0.915, \nu_w = 0.05,$ $\epsilon_w = 1.00$	$\bar{c}_c = 0.854,$ $\epsilon_c = 0.943$ $\bar{c}_w = 0.975,$ $\epsilon_w = 1.00$
$\alpha_c$	3.3	3.470	3.670	4.964
$\beta_c$	3.5	3.680	3.892	5.264
$\omega_c$	<b>6.8</b>	<b>7.150</b>	<b>7.562</b>	<b>10.228</b>
$s_c$	0.097	0.102	0.108	0.146
$\alpha_w$	0.6	0.605	0.665	0.9
$\beta_w$	0.6	0.605	0.665	0.9
$\omega_w$	<b>1.2</b>	<b>1.21</b>	<b>1.33</b>	<b>1.8</b>
$s_w$	0.0167	0.0170	0.0186	0.0252

*notes:* The parameter values for Model II, III and IV are chosen to produce the same stock/income ratios and average saving rates as in Model I if  $n = 0.038$ . In other words, for all models,  $\alpha_c = 3.3$ ,  $\beta_c = 3.5$ ,  $\omega_c = 6.8$ ,  $s_c = 0.146$ ,  $\alpha_w = 0.6$ ,  $\beta_w = 0.6$ ,  $\omega_w = 1.2$ , and  $s_w = 0.0252$  when  $n = 0.038$ . The numbers in the table show the effect of a decrease in  $n$  to  $n = 0.024$ . For all cases,  $\hat{N} = 0.004$ .