Real Exchange Rate Policy Dilemmas in Developing Economies

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Abstract

This paper discusses some of the inter-temporal labor market issues relating to an economy where policy makers pursue real undervaluation in order to achieve rapid increases in the capital stock. Policy makers face a trade-off between achieving a capital stock target over time on the one hand and boosting real wages and output in the short run, on the other. The optimal path under a policy with “unambitious” aims will resemble the typical electoral business cycle trajectory. By contrast, achieving relatively ambitious capital stock targets requires choosing an undervalued level of the RER at the beginning of the planning horizon and gradually increasing the undervaluation thereafter as wages rise. Focusing too much on short-run wage and output expansion leads to real exchange rate volatility, and undermines the ability to meet the capital stock target in a timely manner. Changing the weights assigned to different objectives and pursuing pro-active investment and labor market policies affect the required initial level of real undervaluation and its optimal trajectory over time.

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1 Introduction

Several recent studies have shown the benefits of real exchange rate undervaluation in developing economies. Periods of sustained output growth and capital accumulation tend, on average, to be associated with real undervaluation. While the jury is still out regarding the relative importance of the underlying causes and mechanisms that lead from undervaluation to growth, the finding raises another highly pertinent question. Insofar as policymakers have some influence over the real exchange rate, what prevents them from pursuing real undervaluation in a sustained fashion? In this paper I explore, in a simple manner, interesting labor market issues that arise naturally when intertemporal considerations are incorporated into the execution of real exchange rate policies. In doing so, I contribute to two bodies of literature, one that studies the effect of sustained real exchange rate movements on growth and another that explores the political economy of exchange rate management.

Like most economic policy choices, real exchange rate/monetary policies create winners and losers and potentially generates tensions between short-run versus long-run consequences. Take first the evidence on long-term benefits suggested by recent empirical studies. Based on a panel study involving 188 countries and the time period 1950-2004, Rodrik [2008] finds that real exchange rate undervaluations are positively associated with better growth performance. This effect is particularly significant and robust for developing countries. Razmi et al. [2012], who analyze the relationship between real undervaluation and investment – based on a similarly large panel dataset – find a robust positive association, which again is statistically significant only for developing countries. Unlike the Rodrik model, which emphasizes the presence of market imperfections to a greater degree in the non-tradable sectors of developing economies, the latter study develops a theoretical model where the presence of underemployment in developing countries plays a key role in explaining the main empirical finding. Other evidence suggests that sustained episodes of accelerated investment and growth are often preceded by real undervaluations. To the extent that either less than full employment and/or external economies of scale create room for endogenous growth, a well thought out real exchange rate policy could help accelerate movement towards industrialization.

For real exchange rate policy to serve as a development policy tool, it appears crucial that the relative price signal be stable and reliable. Given its size and topical importance, China presents a pertinent case study here. There has been significant debate in recent years about whether or not China has passed the “Lewis” turning point. What is much less controversial is the presence of a large pool of underemployed labor, especially in the rural and semi-urban areas, on the eve of the economic reforms that took off in earnest in the early 1980s: a pool that is now considerably smaller after years of rapid capital accumulation and industrialization. Figure 1 below displays the time plots of real exchange rate misalignment, \( \text{Misalignment} \)1 and changes in the (log) of the level of capital stock per person, \( GROWTH_{KL} \) for the period 1973-2014. The former series, sourced from the recently developed CEPII \( \text{EQCHANGE} \) database, shows deviation of the real exchange rate

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1Other (mostly empirical) studies that have found a positive effect of real undervaluation on long-run growth include Levy-Yeyati and Sturzenegger [2007], Polterovich and Popov [2002], Berg et al. [2008], and Gala [2008] among others.

2See for Hausmann et al. [2005], Libman et al. [2017], and Freund and Pierola [2008] for investigations of growth, investment, and export surges, respectively.

3See, for example, Meiyan [2010] and Minami and Ma [2010].

4This series is based on time-varying weights constructed using 186 trading partners and 5 year averages. See Couharde et al. [2017] for details. A second broader series, based in addition to the Balassa-Samuelson effect, on terms of trade and net foreign asset data, yields a very similar picture but is available beginning only in 1981. The figure shows the negative of the values in the database so as to make upward movements correspond to real depre-
from a level calculated after adjusting for the Balassa-Samuelson effect. The latter is calculated from the capital stock data provided by the Penn World Table 9.0 database.\footnote{See Feenstra et al. [2015] for a description. Consistent with the real exchange rate variable, the numbers are 5 year (geometric) averages, i.e., individual points represent averages for five-year windows, using the expression, $GROWTH_{KL} = (k_{t+5} - k_t)^{0.2} - 1$.}

Figure 1 illuminates something striking; ignoring the brief dip in the late 1980’s, the significant and sustained increase in growth of capital stock per person that began in the late 1970’s is closely shadowed by a sustained increase in the misalignment index (from an overvaluation of close to 8 percent to an undervaluation of close to 10 percent). While China is an illustrative case, existing literature, some of it cited earlier, has highlighted the significance of sustained devaluations in other contexts.

Figure 1: China: Exchange rate misalignment, $Misalignment1$ (left scale) and change in capital stock per person, $Growth_{KL}$ (right scale), 1973-2014. Sources: CEPII EQCHANGE and Penn World Table, 9.0, respectively.

Assuming then that policy makers are aware of the benefits of stable and undervalued exchange rates, and are generally well-intentioned, what inhibits successful implementation of enabling macroeconomic policies? Several factors come to mind including the credibility of policy, fear of inflation, balance sheet effects, limited ability to control the real exchange rate, and a desire for greater monetary policy autonomy, among others. As pointed out by Rapetti [2012] and Dornbusch [1996] in the Latin American context, political economic considerations assume increased salience here. For example, part of the reason why policy makers cannot sustain real undervaluations surely has to do with distributional concerns. In the presence of nominal rigidities, large devaluations are likely to reduce the purchasing power of wages in terms of tradables.\footnote{A more subtle effect is detected by Cravino and Levchenko [2015], who find that devaluations in Mexico raise the price of tradables consumed by poorer households more than those consumed by rich households.} This, in turn, may limit the options that policy makers can pursue, especially since, as Edwards [1996] observes, the “political ramifications of a major economic disturbance, such as the abandonment of a promised parity, will
tend to be more pronounced in countries with a higher degree of structural political instability. Distributional consequences of relative price changes are likely to be a factor limiting the use of real exchange rate policy in the service of capital accumulation.

Weak substitution effects on the supply and demand sides, at least in the short run, create an additional headache for policy makers seeking to employ price signals to confront short-run cyclical issues. The failure to change economic behavior rapidly implies that devaluations could have contractionary effects on output and employment on impact. Indeed, the temptation to appreciate the real exchange rate for short-run gains may be rather strong in these circumstances.

In sum, numerous hurdles complicate the use of real exchange rate policy as an instrument of long-run development policy. Most importantly from the perspective of this paper, these complications often involve intertemporal trade-offs in the labor market. As argued below, this makes public confidence in the ability of the policy regime to facilitate economic growth, and a widely-shared belief in the long-run benefits of such an outcome, keys to successful management of the real exchange rate.

This paper utilizes a simple optimal control framework to examine some of the dilemmas posed by the use of the real exchange rate as a development policy tool. The model employs a “dependent economy” framework, enabling me to incorporate trade-offs between capital accumulation and tradable sector employment on the one hand, and real wages and non-tradable employment on the other. Seen from another angle, the model analyzes the intertemporal balancing act between the short- and longer runs that many policy makers face. The analysis suggests that ambitious capital stock targets imply an initial undervaluation followed by continuous and increasing undervaluation as wages rise along the optimal path. The greater the wage sensitivity to falling underemployment, the larger the set of feasible capital stock targets. The shorter the amount of time available to reach the target, the greater the initial undervaluation. Changes in policy parameters and labor market shocks change the path and nature of exchange rate policy. For example, a weaker consensus in favor of tradable sector expansion increases the initial level of real undervaluation required to an extent that may render successful implementation politically beyond reach. A lower social discount rate has qualitatively similar effects.

By now, a substantial body of literature has emerged that explores the political economy of exchange rate policy. However, unlike the present paper, this literature mostly focuses on issues of exchange rate regime choice, international and national coordination, and short-run issues such as those pertaining to business and electoral cycles. I contribute to the literature by zooming in narrowly on labor market trade-offs and long-run growth issues in the context of a developing economy with significant un(der)employment.

The implied time path of the real exchange rate,

\[ \text{Path of Real Exchange Rate} \]

\[ \text{Path of Real Exchange Rate} \]

See Alesina and Perotti [1996] for evidence regarding the negative impact of inequality-fueled instability on investment.

See, for example, Krugman and Taylor [1978] for a formalized presentation of the contractionary devaluation argument initially made by Díaz-Alejandro [1963]. Lizondo and Montiel [1989] provide an extended overview.

Klein and Marion [1997] investigate the durations of exchange rate pegs when policy makers weigh the political costs of correcting misalignments against the economic benefits of doing so. Dornbusch [1996] considers the political factors that prevent authorities from correcting overvaluations even when the path to currency crises becomes increasingly clear. Leblang [1999] explores the connections between the nature of the political system and the choice of exchange rate regime, while Stein and Streb [2004] consider the timing of devaluations relative to the election cycle. Frieden et al. [2001] is an example of the body of papers that conclude that governments tend to maintain appreciated exchange rates up until after the elections in order to keep purchasing power high. See Frieden and Brox [2006] for a survey of the literature on the political economy of exchange rate regimes and policies.

One interesting contrast between the present paper and the electoral cycle literature is that the latter often
as a result, differs sharply from the former body of literature. For sufficiently short planning horizons and relatively unambitious capital stock targets, however, I show that the optimal trajectory emerging from the present set-up resembles that of the electoral cycle literature.

Before I conclude this section, it may be pertinent to make a couple of observations about the nature of real depreciations. As noted by Cooper [1971](p. 3) in his classic study of large exchange rate changes in developing countries, devaluations are “one of the most dramatic – even traumatic – measures of economic policy that a government may undertake.” The policy analyzed here, by contrast, is in the nature of a controlled, forethought path of the real exchange rate. It is assumed that the monetary authorities have sufficient reserves to defend any preferred level of the nominal exchange rate. My assumption of continuously balanced trade ensures that capital account problems do not force the authorities’ hand. Large devaluations at any given point in time originate from calculated measures rather than from panic in the face of sudden stops, financial contagion, or terms of trade shocks.

On a final note, one would be remiss in not (briefly) addressing the elephant in the room: can authorities typically influence the path of the real exchange rate over time? The textbook answer depends, of course, on various factors such as the degree of capital account openness, the monetary/exchange rate regime, and the secular stance of fiscal policy. Much of the recent microfounded macroeconomic theory has seen the real exchange rate as an endogenous variable, its value being determined in a general equilibrium set-up by ‘deeper’ parameters such as preferences, factor endowments, and productivity. Empirical literature indicates, however, that the real exchange rate tracks the nominal exchange rate rather closely over time which, in turn, suggests that controlling the latter may effectively influence the former as well. Governments have a variety of policy options including monetary and fiscal policy, capital controls, saving incentives, and foreign exchange reserve management, and the evidence suggests that at least some governments have successfully employed these instruments to influence real exchange rates. Indeed, as the discussion above and the analysis below suggests, the degree to which policy can influence the real exchange rate in a developing economy may itself be a function of the intertemporal dilemmas involved. In any event, my approach here is to explore the path of optimal policy if policy makers could implement it when faced with intertemporal labor market trade-offs. This implicitly requires some kind of societal consensus about assignment of policy weights if disruptive social conflict is to be avoided.

The next section develops the basic framework and investigates its properties. Section 3 carries out thought experiments that explore the costs and benefits of assigning different weights to different labor market measures. Section 4 concludes.

2 Formal Setup and Simple Analytics

This section develops the formal framework and highlights some of its implications.

assumes a positive discount rate, that is, future values of the variables of interest get more weight (since they lie closer in time to the next elections). My long-run developmental focus here implies the opposite assumption.

11Steinberg and Malhotra [2014] calculate that, between 1973-2006, military dictators lost power during 17 percent of their forty-eight devaluation episodes and democratic leaders did so in 38 percent of their seventy-nine devaluations.

12See, for example, the “fear of floating” literature originating with Calvo and Reinhart [2002], who showed that, following the Asian crises, developing countries have systematically intervened in the foreign exchange market to manage the behavior of exchange rates. See also Guzman et al. [2017] for a discussion of the various options available to policy makers to influence the real exchange rate.
2.1 The setting

The formal set-up here consists of a small open economy that produces a tradable good and a non-tradable one. The two are represented by the subscripts $T$ and $N$ when associated with the relevant variables. The foreign currency price of the tradable good is determined in the world market exogenously to the small economy and we assume it to be fixed at 1. The framework incorporates an independent investment function, non-tradable output that is determined by demand, and contractionary devaluations in the short-run.

Our focus is on political economic considerations in the labor market. Specifically, policy makers have a capital stock target. Worker preferences dictate that some sort of societal consensus be developed on how to achieve the target while balancing labor market considerations. The policy makers, therefore, have an objective function that assigns weights $\phi_1$, $\phi_2$, and $\phi_3$ respectively, to:

1. the real wage in terms of tradables, i.e., the real wage in terms of non-tradables ($w$) divided by the real exchange rate ($q$),
2. non-tradable sector employment ($L_N$), and
3. tradable sector employment ($L_T$).

The real exchange rate here is defined as the price of tradables relative to non-tradables ($q = eP_T/P_N = e/P_N$). Denoting the discount factor by $\rho$, the policy makers are, therefore, interested in executing a real exchange rate profile that maximizes

$$
\max_q \int_0^T \left[ \phi_1 \ln \left( \frac{w}{q} \right) + \phi_2 \ln (L_N) + \phi_3 \ln (L_T) \right] e^{-\rho t} dt \tag{1}
$$

subject to a constraint that is yet to be defined. Tradable output, $Y_T$, is limited by the level of the capital stock through a Leontief technology.

$$
Y_T = \min(K, \frac{L_T}{a}) \tag{2}
$$

where $K$ is the level of the capital stock in the tradable sector and $a$ denotes the unit labor coefficient (i.e., the reciprocal of average and marginal labor productivity). Since labor is the only factor of production in the non-tradable sector, and we simplify by ignoring the extraction of rents from a fixed factor of production (i.e., land), therefore, by definition:

$$
Y_N = wL_N \tag{3}
$$

The wage function is strictly increasing and convex in $K$. Capital accumulation increases demand for labor, and hence puts upward pressure on wages. The intensity of this pressure increases as the pool of underemployed/unemployed labor shrinks.

$$
w = w(K); \quad w', w'' > 0 \tag{4}
$$

As shown in the Appendix, the convexity of the wage function also helps satisfy the Mangasarian sufficient conditions for a global optimum. Ignoring capital depreciation, the differential equation governing the evolution of the capital stock is specified as:

$$
K = g(r - \tau)K; \quad g' > 0, g'' = 0 \tag{5}
$$

where $\tau$ is the minimum level of the profit rate below which investment turns negative, $g(0) = 0$, $g' > 0$, and $g'' = 0$, and dots over variables denote time derivatives. The assumed linearity of
the investment function with respect to profit rate differentials simplifies the analysis considerably. The capital goods are internationally tradable.

\[ r = \frac{eY_T - WaK}{eK} = 1 - \frac{wa}{q} \quad (6) \]

The non-tradable market-clearing condition is simply given by:

\[ Y_N = C_N \quad (7) \]

Substituting from equation (3):

\[ wL_N = C_N \quad (8) \]

With trade assumed to be continuously balanced, consumption of tradables is simply the difference between tradable output and investment.

\[ C_T = Y_T - \dot{K} \quad (9) \]

The assumption of balanced trade is admittedly unrealistic for the short-run, and relaxing it will make a qualitative difference to our analysis. For example, any increase in investment will not then come at the expense of tradable (and, given the specific form of preferences assumed below, non-tradable) consumption. However, trade balance considerations are tangential to the core of the present analysis and the simplification seems worth the cost in terms of generality. Moreover, balanced trade is a reasonable assumption over an extended period of time, especially for a developing economy. From equations (5), (6), and (9),

\[ C_T = \left[ 1 - g \left( 1 - \frac{wa}{q} - r \right) \right] K \quad (10) \]

Consumer optimization, based on a simple additively separable log linear specification,\(^\text{13}\) implies that

\[ q = \frac{C_N}{C_T} \quad (11) \]

which, after substitution from eqs. (3), (11) and (8), yields the expression for employment in the non-tradable sector:

\[ L_N = \frac{1 - g \left( 1 - \frac{wa}{q} - r \right)}{w(K)} qK \quad (12) \]

Two crucial derivatives will prove informative in the subsequent analysis:

\[ \frac{dL_N}{dK} = \frac{q}{w} \left[ (1 - g) \left( 1 - \frac{w'K}{w} \right) + \frac{w'g'K}{q} \right] > 0 \quad (13) \]

\[ \frac{dL_N}{dq} = \left( 1 - g - \frac{g'w}{q} \right) \frac{K}{w} < 0 \quad (14) \]

\(^\text{13}\)Specifically, \( \max U = \ln C_T + \ln C_N \) s.t. \( qC_T + C_N = Y \); where \( Y \) is total national income.
An increase in the capital stock raises employment in the tradable sector and hence demand for non-tradables, increasing employment. It also, however, raises the real wage, which reduces output for a given level of demand. A sufficient (but not necessary) condition for non-tradable employment to increase – a plausible hypothesis – is that the real wage rise less than proportionately in response to capital stock increases.

A real depreciation (rise in $q$) has opposing effects on non-tradable employment. On the one hand, $L_N$ rises due to substitution towards non-tradables. On the other hand, real income falls in terms of tradables, causing lower demand for both goods. In line with Krugman and Taylor [1978] and other literature, I assume that the short-run effect of a depreciation is contractionary, i.e., non-tradable employment declines. In formal terms, this requires that, $1 - g < g' \frac{wa}{q}$. I assume this inequality to be strictly satisfied from here on.

Another derivative underlines the demand-side nature of non-tradable output.

\[ \frac{dL_N}{dw} = - \left( 1 - g - g' \frac{wa}{q} \right) \frac{qK}{w^2} = \frac{ag'K - L_N}{w} > 0 \quad (15) \]

where the rightmost expression derives from equation (12). A higher real wage (in terms of non-tradables) increases consumption at the expense of investment. This has a salutary effect on non-tradable employment and output.

Using information from (2), (4), and (12), we can now express the policy maker’s problem in a more detailed manner. The initial capital stock (at $t = 0$) is $K_0$ which is strictly less then the target stock, $K_T$. Formally, policy makers maximize the present value of a weighted measure of the labor market subject to the evolution of the capital stock.

\[ \max_q \int_0^T \left\{ \phi_1 \ln \left( \frac{w}{q} \right) + \phi_2 \ln \left( \frac{1 - g \left( 1 - \frac{wa}{q} - \tau \right)}{w(K)} \frac{qK}{qK} \right) + \phi_3 \ln (aK) \right\} e^{-\rho t} dt \quad (16a) \]

\[ \text{s.t.} \quad \dot{K} = g \left( 1 - \frac{wa}{q} - \tau \right) K \quad (16b) \]

\[ K(0) = K_0, \quad K(T) = K_T, \quad \lim_{t \to T} K(t) = K_T \quad (16c) \]

Planners face the problem of devising a policy path $\{q^*(t)\}$ that maximizes the labor market functional subject to the constraints captured by equations (16b) and (16c). The presence of two boundary value conditions renders an additional transversality condition moot.

Based on the set-up here, the current value Hamiltonian takes the form:

\[ H \equiv \{ \phi_1 \ln \left( \frac{w}{q} \right) + \phi_2 \ln \left( \frac{1 - g \left( 1 - \frac{wa}{q} - \tau \right)}{w(K)} \frac{qK}{qK} \right) + \phi_3 \ln (aK) \} + \lambda g \left( 1 - \frac{wa}{q} - \tau \right) K \quad (17) \]

where $\lambda$ is the costate variable representing the shadow value of capital.

### 2.2 Broad implications

As alluded to earlier, changes in the real exchange rate create trade-offs. Perhaps a figure will help illustrate the nature of the problem. Figure 2 is drawn under the assumption that the shadow price
of capital is non-negative. As we see below, this shadow value does indeed turn out to be positive. The costs faced by policy makers in the form of lower real wages and nontradable employment are falling in the real exchange rate. The benefits, on the other hand, are increasing in $q$. Both functions are drawn as concave (see the appendix for an exploration of the sufficiency conditions). The Hamiltonian, $H$, is drawn at each given instant for the corresponding value of $q$.

![Figure 2: Illustration of policy makers’ trade-offs](image)

Employing Pontryagin’s maximum principle, the solution for the shadow value in terms of the real exchange rate can be derived from the first order condition, which yields:

$$
\lambda = \frac{\phi_1 - \phi_2}{\frac{wa}{q}} + \frac{\phi_2 g'}{1-g} \frac{g'}{K} \tag{18}
$$

A sufficient (but not necessary) condition for the shadow value to be positive is that $1 - g < g' \frac{wa}{q}$, which is the same condition as that required for real devaluations to be contractionary in the short run.

It may be useful here to say a few words about the intuition underlying the condition for $\lambda$ to be positive. As illustrated earlier by Figure 2, the marginal cost of a rise in $q$ (a real devaluation) appears in the form of lower real wages and reduced non-tradable employment. This is countered by the marginal benefit of increased investment. If $\phi_2 = 0$, that is, policy makers assign no weight to non-tradable employment, then a real depreciation creates an immediate trade-off between the other two objectives, i.e., boosting the real wage versus increasing accumulation. The shadow value of capital must be positive to encourage accumulation. If, on the other hand, $\phi_1 = 0$, that is, policy makers assign no importance to the real wage, then the trade off between this variable and accumulation is ameliorated, but, as long as $1 - g < g' \frac{wa}{q}$, another trade-off re-appears – that between non-tradable sector employment and accumulation. A real depreciation reduces the former
but raises the latter. Again, the shadow value of capital must be positive. In brief, as long as real depreciation has a non-negative cost, $\lambda$ must be positive.

The real exchange rate varies positively with the shadow value of capital. Mathematically,

$$ dq = \frac{wg'}{\phi_1 + \phi_2 \left( \frac{q'}{1-g} + \frac{w}{q} \right)^2} - 1 \quad K > 0 \quad (19) $$

The expression on the right hand side is positive since we have already assumed $\frac{q'}{1-g} + \frac{w}{q} > 1$ in line with the contractionary short-run effects of a real devaluation. Put succinctly,

$$ q = q(\lambda); \; q' > 0 \quad (20) $$

The marginal cost to benefit ratio is increasing in $q$. This implies that a rise in the real exchange rate should be accompanied by a rise in the shadow value of capital. Keeping this in mind will help guide intuition through the thought experiments in later sections.

Applying standard techniques, we can now derive an expression for the optimal motion of $\lambda$.

$$ \lambda = - (\phi_1 - \phi_2) \frac{w'}{w} - \frac{\phi_2 + \phi_3}{K} - \frac{\phi_2}{1-g} \frac{q'}{q} + \lambda \left( \frac{w'g'}{q} \left( K - g + \rho \right) \right) \quad (21) $$

Let’s for a moment ignore the second term (i.e., $(\phi_2 + \phi_3)/K$) – which appears due to the non-linearity of the function log specification – and the subjective discount rate. Notice then, in the interest of building intuition, that assuming a constant real wage $w$, will make the right hand side vanish around the steady state. Rising wages in response to capital accumulation and the decline in underemployment thereof play a central role in driving the dynamics of our system. The appendix discusses the case where wages are constant in a bit more detail.

We have arrived at a system of two differential equations, (5) and (21). The Jacobian determinant of the system is given by:

$$ J = \begin{vmatrix} K_K & K_\lambda \\ \lambda_K & \lambda_\lambda \end{vmatrix} \quad (22) $$

where, evaluated at the steady state,

$$ K_K = - \frac{w'}{w} g'(1-\tau)K < 0 \quad (23a) $$

$$ K_\lambda = g'(1-\tau) \frac{q'}{q} K > 0 \quad (23b) $$

$$ \lambda_K = - \frac{\phi_1 - \phi_2}{w} \left[ w' - \frac{\left( w' - \frac{w'}{w} \right)^2}{w} \right] + \phi_2 + \phi_3 \frac{w'g'}{q} \left[ w' - \frac{\left( w' - \frac{w'}{w} \right)^2 wq'}{wq} \right] + \lambda \frac{w'g'}{q} \left( 2 + \frac{w''}{w} K \right) \quad (23c) $$

$$ \lambda_\lambda = \left( \frac{w'g'}{q} K + \rho \right) + \frac{w'g'}{q} \left( 1 - \frac{wq'}{q} - \phi_1 - \phi_2 \right) \left( 1 + \frac{w'}{w} K \right) \left( \frac{q'}{q} \right) \quad (23d) $$

The expressions for marginal cost ($MC$) and marginal benefit ($MB$) are given respectively by:

$$ MC = \frac{2w}{q} - \frac{wq'}{q} \left( 1 - \frac{w'}{1-g} \frac{w}{q} \right) \quad \text{and} \quad MB = \frac{2w}{q} \frac{q'}{q} K. $$
The first two partial differentials (i.e., $\dot{K}$ and $\dot{\lambda}$) are easily and unambiguously signed. The sign for $\dot{\lambda}$ is slightly more involved. The terms in the square brackets are both negative, owing to the convexity of the $w(.)$ function. Thus, a sufficient, but by no means necessary, condition for this partial to be positive is that $\phi_1 \geq \phi_2$. In intuitive terms, the contribution of capital to labor market conditions weakens as the economy builds up its capital stock and wages rise steeply. This feature drives, in our later analysis, two important results. First, it places a limit on how ambitious policy makers can be while setting the capital stock target. Second, it helps ensure that the real exchange rate is depreciating as the economy approaches the target.

Finally, $\dot{\lambda}$ is ambiguously signed. The term in the first parentheses is positive and that in the second parentheses is ambiguous but very likely positive. The last term too is ambiguously signed, although non-positive if we assume that that $\phi_1 \geq \phi_2$. On the whole the right hand side could be positive or negative, although, as long as $\phi_1 \approx \phi_2$, the former is more likely.

If $\dot{\lambda}$ is positive, the path that leads to the steady state is unambiguously unique, i.e., we get a saddle path solution. The stable arm is negatively-sloped (see the Appendix for a derivation of the slope). In the following sub-sections, we will analyze this case. Assuming that $\dot{\lambda}$ is negative, on the other hand, yields qualitatively similar results and adds little to the analysis when we analyze the saddle path case. This case is discussed in the Appendix.

Our non-linear system can only be analyzed qualitatively (the Appendix derives explicit time paths for a simplified version of the model here). Figure 3 captures our dynamic set-up with the help of a phase diagram. The $K = 0$ is upward-sloping. An increase in the capital stock raises the real wage and reduces profitability. A higher shadow price of capital is required to maintain a given level of investment. The other isocline is downward-sloping since raising the capital stock or the shadow value of capital has the same (positive) effect on the rate of change of $\lambda$. The negatively-sloped stable arm in the figure is represented by the curve labeled SS. This, of course, corresponds to the unique (saddle) path to the steady state.

Since the link from the real exchange rate to investment is the main focus, it may be useful to make the use of the terms over- and undervaluation more precise. For any given level of capital stock, I will treat the corresponding level of $\lambda$ on the stable arm, i.e., on the unique solution path, as the benchmark. I will refer to the real exchange rate as:

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$^{15}$It is positive as long as the elasticity of investment with respect to the real wage is less than one, i.e., as long as investment responds less than proportionally to a change in the profit rate.
“highly overvalued” if \( \lambda \) is at a level below both isoclines,

“slightly overvalued” if \( \lambda \) is above the \( \lambda = 0 \) isocline but below the stable arm,

“slightly undervalued” if \( \lambda \) is above the \( \lambda = 0 \) isocline and the stable arm but below the \( \dot{K} = 0 \) isocline, and

“highly undervalued” if \( \lambda \) is at a level above both isoclines.

2.3 Policy sans targets

Let’s begin the analysis by momentarily deviating from equation (16c) and assuming that policy makers do not have a capital stock target. All that matters is the societal consensus that has assigned weights to labor market objectives. In a setting where policy makers forego specific development goals, following the unique solution path that takes the economy to the steady state asymptotically will involve choosing the point on the stable manifold that corresponds to \( K_0 \).

This means accumulating capital with continuous, if gradual, appreciation over time. The secular movement of the exchange rate resembles that under the Balassa-Samuelson effect although the underlying mechanism differs (there is no productivity growth here).

Given a finite time horizon, so that \( t \in [0, T] \), and beginning at a capital stock such as \( K_0 \), policy makers might choose a real exchange rate at the level corresponding to point A in Figure 4. Investment is negative at this highly overvalued exchange rate, and at any non-infinite terminal time horizon \( T \), the capital stock is optimally run down to a value where high exchange rate appreciation has reduced the shadow value of capital to zero.
Alternatively, given a longer planning horizon, they may pick a higher real exchange rate at a point like B. A slightly overvalued beginning exchange rate at that point leads to a similar end result with the economy consuming away its capital stock after initially experiencing some accumulation and employment generation. The real exchange rate appreciates along the entire path as policy makers, faced with a declining capital stock, lean towards keeping the real wage high and boosting non-tradable employment at each instant. Notice, however, that keeping non-tradable employment high through real appreciation will eventually become unsustainable as the capital stock declines, pulling $L_N$ down with it. Excessive macroeconomic populism may run into hurdles even if the economy is not hit by a crisis of confidence.

2.4 Setting goals

Now let’s return to the more interesting case, as captured by equation (16c), where policy makers pursue a capital stock target. This case, of course, is relevant to capital scarce developing countries.
with long-term development aspirations. Suppose, as a first pass, that the capital stock target to be reached at time \( T \) is a relatively unambitious one such as \( K_T' \). This target can be reached regardless of the degree of real undervaluation, and indeed even with an initially overvalued real exchange rate, as long as the degree of overvaluation is small. The optimal paths do, however, depend on the initial level of \( q \) chosen. When starting with a slightly overvalued exchange rate, low wages still allow for accumulation, even though \( q \) will have to be lowered to keep real wages and non-tradable employment relatively high. Initial overvaluation beyond that at a point like \( C \), will have to be guarded against even though that point is still in the slightly overvalued zone. The time path here is similar to one that would emerge from the typical electoral business cycle framework. Election-focused politicians may be perfectly fine with less ambitious targets.

Next, what if the policy makers have a much more ambitious target such as \( K_T'' \)? To answer this question, notice that there is no entry from the northwestern quadrant into the northeastern one. Too ambitious a capital stock target is unattainable, regardless of the initial level of \( \lambda \) chosen via the choice of \( q \). There are limits to real exchange rate policy as long as wages are sensitive to the level of employment. One interesting implication, among others, is that an economy that starts with a high initial level of capital stock (and low level of underemployment) may not be able to employ real exchange rate undervaluation as a successful tool for capital accumulation. This is consistent with the findings of the literature cited in Section 1. Alternatively, meeting highly ambitious targets may require wage insensitivity to employment that is that is so high as to rupture any existing social consensus, even if it successfully generates employment.\(^{16}\)

Finally, let’s examine the case where the capital stock target, \( K_T'' \), is in the “Goldilocks range,” neither too unambitious, nor beyond reach. This range is defined by the feature that the targeted level of capital stock is adequately high so that its shadow value is rising in this range (although not high enough for investment to be profitable).\(^{17}\) Now meeting the target requires, as a necessary condition, that the chosen real exchange rate level corresponding to \( K_0 \) be in the undervalued range. Moreover, it must remain undervalued throughout the time interval \([0, T]\) to compensate for the rising real wage (in terms of non-tradables) along the way. If initially highly overvalued, the optimal path involves further real depreciation over time. If the time available is sufficient so that the initial undervaluation puts the exchange rate into the slightly undervalued zone,\(^{18}\) policy will require initial appreciation followed by depreciation all the way through as real wages rise. The shorter the policy makers’ time horizon, the higher the initial level of undervaluation. For instance, starting at a point like \( G \) gets the economy there faster than from starting points \( E \) or \( F \). It may also, however, be the hardest path to achieve for political economy reasons. Steinberg and Malhotra [2014](p. 507) note that “all segments of society initially oppose a depreciation;” continuously suppressing the real wage in terms of tradables is unlikely to be the most popular recipe for economic development, even though it is offset by rising wages in terms of non-tradables.

The achievability of the most direct route to capital stock expansion, in other words, will depend on factors such as the weight of tradables and non-tradables in the consumption basket, the political weight of the those who were previously part of the pool of un(der)employed but now have jobs versus those who always had access to jobs, and the interests of owners of capital who invest versus...
workers. More ambitious targets generate trickier trade-offs, and the trickier the trade-offs, the more likely any existing societal consensus is to pass the breaking point!

Now that we have briefly toured various time path scenarios with the help of Figure 4, a direct comparison of the setup here with the electoral business cycle literature helps understand the crucial role of the planning horizon. Given adequately unambitious capital stock targets, the optimal path is defined by a slight initial undervaluation followed by appreciation over time (see the trajectory beginning at point $D$). The path involves rising real wages, and expanding non-tradable employment along the way as the economy approaches the target. Lengthening the time horizon sufficiently to achieve a more ambitious target would, however, involve course reversal in the form of consistent and increasing undervaluation.

To summarize, given an adequately short time window, the optimal path in our set-up approaches that of the electoral cycle literature in the short run. Achieving more ambitious targets, on the other hand, will involve consistent and increasing undervaluation. The next section fleshes out these considerations in more detail.

3 Other Thought Experiments

So far we have assumed a societal consensus that allows policy makers to target a capital stock level and explored how changes in the planning horizon or target will define the path followed. What if the societal consensus shifts, changing policy maker weights along with it? This section considers the consequences of shifting policy concerns including: (1) a shift in preference from non-tradable to tradable sector employment (or from the level of real wages to tradable employment), (2) a rise in the real wage function at a given level of the capital stock, and (3) a decline in the rate of time preference.

3.1 Increased policy focus on tradable sector employment

Let’s explore a scenario where policy makers, perhaps motivated by a widely-felt need to modernize the economy, or to internalize externalities (not modeled by the present paper but see Rodrik [2008] and Rapetti [2012]), redistribute policy weights from non-tradable sector to tradable sector expansion. In the present set-up, suppose $d\phi_3 = -d\phi_2$ at time $t = 0$. The results are captured by Figure 5. The change shifts the $\lambda = 0$ isocline and the saddle path down and to the left. We can tell from equation (18) that the shadow value of capital that satisfies the first order condition declines, as does the corresponding real exchange rate (equation (20)). Intuitively, the change in weights has reduced the relative marginal cost of real undervaluation facing policy makers.

The altered policy emphasis makes it easier to pursue the capital stock target at the cost of employment in the short run. In terms of the figure, a point like C, which before the shift in emphasis would have fallen in the slightly undervalued range, is now located in the highly undervalued zone. The level of initial undervaluation required to get the direct path originating from point $D$ is now lower and corresponds to a point such as C. Stated differently, and starting at a level of $q$ captured by point C, a path, that before the change would have resembled the one starting at B, now

---

19 One should note here that tradable sector expansion in the present set-up does lead indirectly to non-tradable sector employment but only over time. The reference here is to the effect of real exchange rate policy, whereby undervaluation leads gradually to accumulation in the tradable sector while contracting the non-tradable sector on impact.

20 Although not of much interest to the analysis here, the steady state value of $K$ and $\lambda$ both decline.
resembles the more direct one starting at C. Counterintuitively, perhaps, the political economic conditions that caused the shift in emphasis toward tradable employment imply that, for a given time horizon \( T \), the level of initial undervaluation required is lower, as is the optimal trajectory of the real exchange rate. The lower initial value of \( q \) is followed by depreciation at an accelerated pace compared to the pre-shift path.

Conversely, if the focus shifts toward non-tradable employment, policy makers may not be able to achieve their target in the given time if the optimal path now involves real appreciation followed by depreciation. This circuitous path involves more exchange rate volatility. The solution, i.e., greater initial undervaluation to attain the target, may strain public patience, especially given the new consensus. Both routes involve balancing the kind of political economy complications discussed in Section 1. When it rains, it pours!

In sum, factors, such as some form of societal consensus, that enables policy makers to place more emphasis on tradable employment make it easier to reach a capital stock target in a given amount of time. Since \( q \) is lower than it would be in the absence of such a consensus, the real wage in terms of tradables will be higher along the path and at the terminal point. A similar decline in the real wage on impact results in a higher real wage over the long run.\(^{21}\)

\[ \dot{K} = 0 \]
\[ \dot{\lambda} = 0 \]

\( K_0 \)
\( K_T \)
SS
C
B
A

Figure 5: Shifting focus from non-tradable to tradable sector employment

It may be worth pointing out here that the analysis will be qualitatively unchanged if the shift in emphasis on tradable employment were to occur at the cost of the real wage in terms of tradables rather than that of non-tradable unemployment. In other words, a shock whereby \( d\phi_3 = -d\phi_1 \)

\(^{21}\) Notice that the value of \( q(T) \) is now lower for the corresponding value of \( K_T \). One can prove this by contradiction.
generates outcomes that are qualitatively similar.

3.2 Higher wages at given levels of capital stock

Now suppose that there is a rise in the \( w \)-function due to factors such as greater worker bargaining power, stronger unions, or reduced abundance of labor that raise the real wage for each given level of capital stock. How does this affect the optimal path that policy makers should pursue to reach the desired level of \( K \) at time \( T \)?

Perhaps surprisingly, the effect on the optimal path is ambiguous. The changes are illustrated below in Figure 6. Both isoclines shift leftwards; the \( K = 0 \) isocline because the level of the real exchange rate that ensures a given level of accumulation is now higher and the \( \lambda = 0 \) isocline because the contribution of a unit of capital to present and future labor market conditions is now lower, which requires a correspondingly lower rate of depreciation of the capital value. If the \( K = 0 \) isocline shifts sufficiently more, the required initial real depreciation and the optimal time path are now higher. An initial point such as C now behaves like an initial point like B. For planning horizon with a given fixed end point, the required initial level of real depreciation will have to be higher. The intuition is simple; a rise in the \( w \)-function acts as an increase in labor costs, and requires a depreciation to maintain a given level of investment. If, by way of contrast, the \( \lambda = 0 \) isocline shifts more, then the required level of initial depreciation, and the subsequent optimal time path are lower, i.e., a smaller initial depreciation is now needed to get to the target capital stock level at time \( T \).

The upshot is that an instantaneous rise in wages reduces both the marginal cost and the marginal benefit of real undervaluation. If the latter impact dominates, the shadow value of capital, and thus the initial real exchange rate, must now be higher to meet the target. Moreover, \( q \) must now accelerate at a rate higher than before along the optimal path.
3.3 An increase in the discount rate

Suppose finally that policy makers become more impatient in the sense that future labor market conditions get less weight. The analysis is logically akin to the case where the weight shifts toward tradable employment (Section 3.1). The optimal initial level of $q$ chosen is lower for a given target level of capital stock while the speed of adjustment at each level of $K$ increases. Since a higher real wage pays off dividends today while the benefits of capital accumulation only gradually evolve, the new optimal path involves starting with a higher real wage in terms of tradables and raising the real exchange rate faster. Again, Figure 5 illustrates this case. Any tensions that arise from the distributional effects of relative price changes have, like the proverbial can, been kicked down a (uphill?) road.

It may be interesting at this point to contrast the implications here with that of the electoral business cycle literature noted in Section 1. A plausible assumption in that literature is that of a negative social discount rate, that is, the farther off in the future an observation, the lower the weight it gets. Policy makers, therefore, care most about conditions at the last minute before the elections. Here the opposite holds. Policy makers care about the long-run, but given the discount rate, have to assign higher weights to current outcomes while planning. In contrast to the behavior emerging from the electoral cycle literature, there is no gradual appreciation after the initial undervaluation. Rather, rising wages mean that the real exchange rate has to keep depreciating to constrain unit labor costs in terms of tradables.
4 Concluding Remarks

A body of literature has attempted to model the interplay between various policy considerations as politicians try to maximize their votes. Confirming some of the predictions emerging from this literature, for example, Edwards [1993] studied the timing of 39 large devaluations (15% or more) by democratic regimes in developing countries, finding they tended to occur early on in the term. This literature has, however, focused mostly on short-run (business or electoral) cycle considerations. The focus of the present paper, by contrast, has been on the trade-offs between short-run rewards and long-run developmental benefits. For adequately unambitious policy targets the optimal path in our set-up approaches that emerging from electoral cycle considerations over short planning horizons, with a rising real wage, expanding non-tradable employment, and falling tradable inflation as the next election cycle approaches. The problem, however, to borrow a phrase from Dornbusch [1996], is that “bringing down inflation is not the end of the story; rather, it is the beginning of the next cycle.”

Taking as given the potential long-run benefits of real undervaluation, how should policy makers maximize these while allowing for rising wages along the way? The major lesson that emerges from our exercise is that a one-shot real undervaluation, although necessary is not sufficient. For the process to be sustainable, the initial undervaluation should indeed be sufficiently large and the target realistic. The more challenging bit comes later, however, as monetary and fiscal policies must be aligned in a way to increase the level of real undervaluation along the growth path as real wages rise and underemployment declines.

Why might some countries be better at successfully implementing relative price signals to pursue capital accumulation? The analysis here suggests that at least part of the explanation lies in the different weights that policy makers assign when planning over extended horizons. Policy design that takes place in an environment of an underlying social consensus which enables greater focus on tradable sector expansion lowers the initial real exchange rate undervaluation required while allowing policy makers to devalue more quickly along the way. A lower initial undervaluation, in turn, translates into a higher real wage at the beginning of the policy push. Virtue begets virtue. Perhaps counterintuitively, higher real wages at each level of capital stock may also lower the initial level of real undervaluation required along the optimal path if investment is relatively insensitive to the profit rate.

The analysis here abstracts away from a number of complications in order to maintain tractability. Wages in the two sectors are assumed to be identical. More plausibly, one would expect intersectoral wage differentials, especially if the traded sector consists of manufacturing and other modern industries where workers are likely to be more skilled. Labor market segmentation created by different skill intensities, worker bargaining power, and efficiency wage considerations are some of the considerations that we swept under the rug. Landlords, or owners of a fixed factor are likely to play a major role in the political economy of the typical developing economy and a more detailed analysis would benefit from incorporating this angle.\textsuperscript{22} We have also eschewed analysis of consumer welfare under different scenarios, which is beyond the scope of the paper. Finally, unlike Rodrik [2008] and Ros and Skott [1998], the tradable sector in our model does not exhibit any “special” characteristics such as external economies of scale. We simply assume the desirability of promoting tradable employment and focus on the underlying political economic factors that would inhibit or facilitate policies designed to achieve this goal in a given time frame. The analysis here is only relevant to the extent that there is underemployment. Once the modern tradable sector has sufficiently

\textsuperscript{22}See, for example, Razmi et al. [2012].
expanded to remove this feature, steady state growth considerations become salient.

The assessment of the appropriate level of the real exchange rate will depend on the urgency of the development targets, and identifying it accurately is obviously, and to put it mildly, a challenge. What I have attempted here is to highlight the likely constraints and their relationship to the shadow price of capital that would facilitate successful development strategies. The analysis here reinforces the empirical finding that tenure security facilitates exchange rate undervaluation by governments.23

5 Appendix

5.1 Sufficiency conditions

This sub-section considers the Mangasarian sufficiency conditions in the context of our baseline set-up of Section 2. Consider first the constraint:

\[ f = g \left( 1 - \frac{w}{q} - \bar{r} \right) K \]

It can be easily verified that \( f_{qq}, f_{KK} < 0 \). The remaining condition for \( f \) to be concave, that is,

\[ w'' \geq \frac{1}{2w} \left( w' - \frac{w}{K} \right)^2 \]

is always satisfied for any power function of the form \( w = K^n \), where \( n \geq 1 \).

Next, consider the relevant component of the objective functional:

\[ F = \phi_1 \ln \left( \frac{w}{q} \right) + \phi_2 \ln \left[ \frac{1 - g \left( 1 - \frac{w}{q} - \bar{r} \right)}{w(K)qK} \right] + \phi_3 \ln (aK) \]

In this case,

\[ F_{KK} = \frac{(\phi_1 - \phi_2)}{w} \left[ w'' - \frac{(w')^2}{w} \right] - \frac{\phi_2 + \phi_3}{K^2} \]

\[ + \phi_2 \frac{g'}{(1-g)q} \left[ w'' - \frac{(w')^2}{w} \frac{w}{q} \frac{g'}{1-g} \right] \]

which is negative as long as \( \phi_1 \simeq \phi_2 \) (recall that \( w'' - \frac{(w')^2}{w} < 0 \) by definition, and the assumption of contractionary short-run devaluation implies that \( \frac{w}{q} \frac{g'}{1-g} > 0 \)). Notice that this is a sufficient, not a necessary condition.

\[ F_{qq} = \frac{\phi_1 - \phi_2}{q^2} + \frac{\phi_2}{q^3} \frac{wg'}{1-g} \left\{ 2 - \frac{w}{q} \frac{g'}{1-g} \right\} \]

23See, for example, Steinberg and Malhotra [2014].
which is negative as long as $\phi_1 \simeq \phi_2$ and $w q_1 \frac{g'}{1-g} > 2$. Satisfaction of the latter inequality requires that $(q/LN)(dL_N/dq) < -1$, or in other words, non-tradable employment decline more than proportionately in response to a real depreciation.

The remaining relevant expression that needs to be negative to make $F$ concave is rather involved. Again, assuming $\phi_1 \simeq \phi_2$ to avoid clutter, and simplifying:

$$FqqF_{KK} - (F_q)_{KK}^2 = \frac{\phi_2}{q^3 (1-g)} \left[ \left( -\frac{\phi_2 + \phi_3}{K} \right) + \frac{\phi_2 g'}{(1-g)q} \left[ w'' - \left( w'' - \frac{(w')^2}{w} \right) \left( 1 - \frac{w}{q (1-g)} \right) \right] \right]$$

The term in the first square brackets on the right hand side is unambiguously positive, given our assumption about the proportionate response of non-tradable employment to changes in $q$, while the term in the second square brackets is ambiguously signed. Concavity is satisfied if the two terms add up to a positive term, which is more likely the less convex the wage function.

5.2 Slope of the saddle path

Denoting the negative eigenvalue by $\delta_1$, the slope of the stable manifold is given by:

$$\left. \frac{d\lambda}{dK} \right|_{SS} = \frac{\delta_1 - \dot{K}_K}{K_\lambda}$$

Given that,

$$\delta_1 = \frac{\dot{K}_K + \dot{\lambda}_K - \sqrt{\dot{K}_K + \dot{\lambda}_K}^2 - 4(\dot{K}_K \dot{\lambda}_K - \dot{K}_K \dot{\lambda}_K)}{2}$$

allows us to derive the expression for the slope:

$$\left. \frac{d\lambda}{dK} \right|_{SS} = -\frac{(\dot{K}_K - \dot{\lambda}_K) + \sqrt{(\dot{K}_K - \dot{\lambda}_K)^2 + 4\dot{K}_K \dot{\lambda}_K}}{2K_\lambda}$$

(24)

which is negative since $\dot{K}_K < 0$ and $\dot{\lambda}_K > 0$.

5.3 The alternative case where $\dot{\lambda}_\lambda$ is negative.

In this case, as captured by Figure 7, a saddle path solution requires that the $\lambda = 0$ locus be steeper. The qualitative results are more or less similar to the case addressed in the main text, although now the optimal path of the real exchange rate that achieves the “Goldilocks” capital stock target ($K_T^{\prime\prime}$) invariably requires an initial real undervaluation, followed first by real appreciation and then by real appreciation as the target is approached. Unlike the main text, in other words, there is no optimal control path that involves continuous depreciation all the way. Thus, there is greater real exchange rate volatility. To understand the intuition, recall that this case involves greater policy emphasis on the real wage in terms of tradables, i.e., a high value for $\varphi_1$. This, in turn means that policy makers will optimally use real appreciation earlier to raise the real wage and then depreciate once capital accumulation has raised $w$ over time.
5.4 A simplified set-up

Assuming a constant real wage, $w$, and specifying $\phi_1 = \phi_2$, dramatically simplifies the analysis and allows us to derive more explicit expressions for the time paths in the neighborhood of the steady state.

Based on eqs. (23a) - (23d) of the main text, the explicit values of the eigenroots in this case are:

$$
\sigma_{1,2} = \frac{\rho \pm \sqrt{\rho^2 + 4 \frac{\sigma_1}{K} g'^q (1 - r) \phi_2}}{2}
$$

with $\sigma_1 < 0$ and $\sigma_2 > 0$. The time paths for the two variables of interest, after solving and incorporating the boundary conditions, are given by the following expressions:

$$
K(t) = A_1 e^{\sigma_1 t} + A_2 e^{\sigma_2 t}
$$

and,

$$
\lambda(t) = A_1 D e^{\sigma_1 t} + A_2 F e^{\sigma_2 t}
$$

where, $A_1 = \frac{K_T - K_0 e^{\sigma_2 T}}{e^{\sigma_1 T} - e^{\sigma_2 T}}$, $A_2 = \frac{K_0 e^{\sigma_1 T} - K_T}{e^{\sigma_1 T} - e^{\sigma_2 T}} > 0$, $D = \frac{\sigma_1}{g'^q (1 - r) K} < 0$, and $F = \frac{\sigma_2}{g'^q (1 - r) K} > 0$.

Since $A_2$ is positive, as $t$ increases, $\lambda$, and thus the real exchange rate, must be rising, even though initially it could decline. This is consistent with the qualitative results from the main text. Moreover, if $K_T$ sufficiently large, so that $K_T - K_0 e^{\sigma_2 T} > 0$, then the longer the planning horizon, i.e., the higher $T$ is, the greater the likelihood that the optimal path will involve a real appreciation followed by a real depreciation. Longer planning horizons generate more volatile control trajectories.
Figure 8 represents the phase diagram. The analysis is unchanged in qualitative terms, with one rather interesting exception. The $K = 0$ isocline is now horizontal implying that any terminal capital stock target can now be achieved with an appropriately chosen initial real exchange rate. Policy makers in other words, can set as high a target as they want, as long as they are willing to implement the corresponding real exchange rate policy. The constancy of the real wage makes it easier for the policy makers to be ambitious.

Figure 8: Phase diagram with a constant $w$

References


