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Implementation of Optimal Monetary Policy in a New Keynesian Model under Heterogeneous Expectations

Master Thesis

by

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Non-technical summary

How should a central bank set interest rates optimally given that the individuals populating the economy hold different views about the future? To answer this question, a rule is derived that specifies how the interest rate set by the central bank should react to changes in (expectations of) certain macroeconomic variables as for instance the GDP, inflation and consumption. To derive such a rule mathematically, one has to start from several assumptions about the economy in general and the central bank policy in particular.

Following Branch and McGough (2009), it is assumed that the population can be divided into rational and bounded rational individuals. Rational individuals are able to make fully optimal decisions as they know the true laws of how the economy evolves. However, there are shocks to the economy that cannot be foreseen by any individual and which are zero on average. Hence, expectations of rational individuals about the future are correct on average but can be wrong in particular cases. Thus, rational individual's expectations are not systematically biased. In contrast, bounded rational individuals only know the value of, say, GDP of the last period and project it into the future. Hence, their views about the future are systematically biased. Note that rational and bounded rational individuals only differ by their expectation schemes.

Further, the central bank is assumed to know in which economy they operate. In particular, it knows how individuals form their expectations. Since expectations are influenced by the economic environment and in turn expectations shape the economic environment, they are of utmost importance for central bank policy considerations.

Additionally, the central bank is assumed maximize social welfare. Social welfare is defined by the aggregate well-being of the individuals populating the economy. The central bank's aim is formalized into a clearly defined mathematical objective. One possible definition of this objective is to assume that it is sufficient to base social welfare *losses* on the volatility of inflation and GDP alone (Gasteiger, 2014, 2017). This definition can be seen as implicitly assuming that the central bank is concerned with rational individuals in their objective only. Such an objective is called *conventional*.

However, in Di Bartolomeo et al. (2016) the central bank bases its objective on the *heterogeneity* in individual expectations explicitly. This assumption implies an additional dimension of optimal central bank policy, i.e. consumption inequality. Hence, the central bank additionally aims at minimizing the inequality of consumption between the different types of individuals. This inequality is caused by the different expectation schemes. Thus, the central bank ultimately tries to stabilize the different expectations to reduce consumption inequality. The primary goal of this thesis is to derive an interest rate rule for the latter objective, i.e. where the central bank recognizes the heterogeneity in expectations explicitly.

The decision making process of the central bank is assumed to be either discretionary or committing. Discretionary policy means that the central bank has the discretion to update its decision in every period. However, in the commitment case the central bank specifies a certain rule to which it commits in the future. This commitment is then internalized in the expectation formation of rational agents. Thus, the advantage of commitment is that (rational) expectations can be manipulated directly.

However, the first result of this thesis is that under commitment there exists, almost certainly, no operational interest rate rule. The reason lies in the complexity of the policy problem. Still, a corresponding rule can be derived for the discretionary case (second result). This rule accounts for the backward-looking behavior of bounded rational individuals and the consumption inequality dimension. Additionally, the rule implies that the central banks acts more aggressively with respect to terms that are introduced by the bounded rational agents if their influence on the economy increases. This is either when their relative fraction of the whole population increases or when their expectations are trend-setting. Expectations are called trend-setting if bounded rational individuals expect the value of, say, GDP to be even higher in the future compared to last periods value, which represents an amplifying element.

The performance of the different policies, i.e. the policy under the conventional objective and under the objective that recognizes heterogeneity in expectations explicitly, can be assessed by comparing the true welfare losses they generate. True welfare losses are defined by the cumulated losses in well-being of the heterogeneous individuals following an unforeseen macroeconomic shock. However, losses measured by the conventional objective only recognizes rational individuals.

The third result is that a central bank policy under the conventional objective yields slight welfare improvements compared to an objective that accounts for heterogeneous expectations explicitly in most cases. At least this policy does not seem to generate significant welfare losses in any case even though it neglects heterogeneous expectations in its objective. A possible explanation is given by the fact that under the conventional objective the volatility of GDP is substantially reduced, which results in welfare improvements. Hence, a central bank that bases its objective on heterogeneous expectations might overreact to an economic shock that causes larger welfare losses (which is broadly in line with Berardi (2009)).

Nevertheless, the *true* welfare losses are based on both rational and bounded rational individuals. Thus, if welfare losses are measured by the conventional objective, the central bank will substantially underestimate the welfare consequences of its policy (fourth result). Further, the objective based on heterogeneous expectations implies lower consumption inequality in all cases. This is intuitive as this policy considers consumption inequality as a welfare component (fifth result).

The bottom line is that the policy under the objective that recognizes individual heterogeneity has the disadvantage that it can only be implemented via an interest rate rule under discretion but not under commitment. In addition, it yields *slight* welfare losses relative to a policy under the conventional objective in most cases. Hence, a central bank that employs the conventional objective might indeed yield higher overall welfare - but at the cost of higher consumption inequality.

Abstract

This thesis derives an optimal interest rate reaction function from a New Keynesian model with heterogeneous expectations and a fully model-consistent welfare criterion under discretion. The reaction function incorporates the heterogeneity in expectations and a consumption inequality dimension implied by the model-consistent welfare criterion. However, a monetary policy derived from a microfounded objective that only recognizes variations in inflation and the output gap might yield slightly lower welfare losses. Nevertheless, consumption inequality is lower across all considered parameter values in case of the model-consistent objective.

1 Introduction

The conduct of monetary policy plays a superordinate role in New Keynesian (NK) models since the interest rate is *the* measure by which agents decide upon consumption and investment. Further, the central bank is assumed to be able to control the interest rate in such models.[1] Naturally, it is of great interest how the central bank should set the interest rate optimally given its knowledge about the economic structure and in particular the behavior and expectation formation of individuals. Expectations are crucial since they are shaped by the economic environment that naturally includes central bank policy. In turn agents' expectations influence the economic environment as their economic decisions hinge on their expectations. Therefore, for a central bank that seeks to exert influence on expectations, it is of vital importance that assumptions at the core of the economic model used to design optimal central bank policy are accurate. Assumptions should be accurate in the sense that, if they are not realistic and thus chosen to achieve a certain degree of abstraction, they do not bias the outcome significantly.

However, optimal monetary policy is usually studied within a framework that assumes agents to form their expectations following the *rational expectations* (RE) hypothesis (see for instance Clarida et al. (1999)). Yet, data favors *heterogeneous expectations* (HE) with a certain degree of *bounded rationality* (Branch, 2004, 2007; Pfajfar and Santoro, 2010). Starting from this observation, several NK models including heterogeneous expectations with bounded rationality were designed (Branch and McGough, 2009, 2010; De Grauwe, 2011; Massaro, 2013).

[1] Of course, in reality there is not a single interest rate. The central bank can however control the overnight interbank lending rate by which private banks lend to each other and borrow liquidity in order to meet their reserve requirements. This interbank lending rate influences a wide range of other interest rates. For the purpose of abstraction it is therefore assumed that there is only a single interest rate that can be controlled by the central bank. Further, monetary policy in this context is defined as conventional, i.e. interest rate targeting. Unconventional measures like large-scale asset purchase programs and alike are not considered.

One rather simple approach represents the model constructed by Branch and McGough (2009) that includes two different representative agents. Both agents are assumed to optimize their utility given their *subjective* expectations. Thus, the two representative agents differ only by their expectation formation schemes. A certain (exogenously given) fraction, α, of the population is rational in the sense that it knows the true *aggregate* model while the other fraction, $1-\alpha$, is *backward-looking* and thereby has *biased expectations.* Optimal monetary policy in this particular setting is investigated by Gasteiger (2014), Gasteiger (2017) and Di Bartolomeo et al. (2016). Gasteiger (2014, 2017) bases the analysis on the *conventional* objective that incorporates only weighted variations in inflation and output gap. This objective can be seen as implicitly assuming RE. On the other hand, Di Bartolomeo et al. (2016) derive a fully *model-consistent* loss function that incorporates heterogeneous expectations explicitly. The latter gives rise to a third dimension of optimal monetary policy, namely consumption inequality. Consumption inequality arises because of the different forecast paths of the two agent types. Consequently, the central bank tries to stabilize the expectation paths in order to minimize consumption inequality. Although Di Bartolomeo et al. (2016) investigate optimal monetary policy under this particular loss function, they do not consider the *implementation* of it. Thus, they do not derive corresponding *reaction functions.* This master thesis tries to fill this gap.

It will be shown that, most likely, no operational reaction function in the case of *commitment* exists.[2] This is because the optimization problem technically leads to a second order difference equation in one of the Lagrange multipliers to which the solution is fairly complicated and exponentially depends on time. If there exists no such implementation strategy, this would cast doubt on the usefulness of the model-consistent loss function. This entails and confirms Gasteiger (2017)'s claim that the model-consistent loss function has some practical disadvantages. For instance, its complexity potentially impairs the ability of thorough communication, which is crucial if the central bank tries to influence expectations.

However, a suitable reaction function under discretion can be derived. This function recognizes the heterogeneity in expectations, the inertia caused by the backward-looking agents and the consumption inequality dimension introduced by the model-consistent loss function. In particular, it incorporates rational disaggregate consumption and two-period-ahead $(t+2)$ terms. Both can be explained by the introduction of the consumption-inequality dimension. Specifically, the $t+2$ terms appear because the expectations of the two types of agents in period $t+1$ about realizations of aggregate variables in period $t+2$ diverge transitorily (which have to be *stabilized* by virtue of the consumption-inequality dimension). Further, the reaction function has the desir-

[2] Commitment means that the central bank commits to a certain policy from a point in time t to infinity and thereby influences rational expectations directly.

able property that it encompasses the correct reaction function under RE as a special case. Nevertheless, there is a drop of bitterness as well: the simulation of the system in Dynare including the aggregate equations, the disaggregate rational consumption Euler equation and the reaction function under discretion yields *indeterminacy*. It can however be explained that this is most likely not due to the reaction function itself.

Another question at hand is whether the conventional objective as in Gasteiger (2014, 2017) or the fully model-consistent objective as in Di Bartolomeo et al. (2016) has more desirable properties. The conventional objective only involves inflation and output gap variations and can be seen as implicitly assuming RE. Besides the fact that there is, most likely, no operational reaction function under commitment in case of the model-consistent loss, there is further support for the conventional objective. The welfare analysis shows that the true losses, i.e. the losses that recognize the heterogeneity among agents, might indeed be slightly lower under the (microfounded) conventional objective than under the true model-consistent loss function. This holds for cases where the influence of bounded rational agents on the economy is relatively high. Thus, a central bank that hires a central banker whose preferences are such that the (microfounded) conventional objective is implied would yield welfare improvements in these cases.[3] This is at least in part driven by the fact that the policy under the conventional objective, although leading to a slight increase in the long-run variance of inflation, yields a substantial reduction in the variance of the output gap. Hence, a central bank that ignores bounded rational agents in their objective might yield welfare gains in some cases since it might overreact to exogenous shocks otherwise (Berardi, 2009). Still, there are two apparent disadvantages from applying the conventional objective. First, the model-consistent objective naturally yields lower consumption inequality. Second, the true welfare losses are significantly underestimated regardless of the chosen policy, which also confirms the claims made by Gasteiger (2017).

There are several reasons for why this thesis is of practical relevance. The incorporation of heterogeneous expectations into NK models is justified not merely by theoretical interest, but in particular since it is backed up by empirical evidence. The investigation of optimal monetary policy then becomes important if heterogeneous expectations alter the optimal policy. As for instance Gasteiger (2014, 2017) shows, the introduction of heterogeneous expectations is indeed important for optimal policy. This is because policies that are derived under RE imply indeterminacy for certain parameter constellations. Thus, heterogeneous expectations and bounded rationality

[3]This rather unhandy expression is necessary if one assumes that preferences are carved in stone. Hence, if one wants to recommend some other loss function that implies changes in the parameters, which are expressed in terms of central bank perferences, the central bank needs to hire a central banker that meets these perferences. If in the following expressions like ”the conventional objective should therefore be preferred” are used, it is done for simplicity and one has to keep in mind the unhandy expression used above.

can indeed be a stability issue if not thoroughly included in policy considerations. The most elementary target of monetary policy is to ensure a stable economy by setting nominal interest rates accordingly. Further, policy makers often view the trade off between inflation and the output gap as a policy menu (Gasteiger, 2017). The conventional objective represents this trade off and therefore pictures actual practice well. The relevant question then is if it should be recommended to change the policy mandate of central banks to incorporate heterogeneous expectations as represented by the model-consistent loss function. In the case where the latter should be preferred, the corresponding implementation strategy is derived, which substantiates its practical relevance.

The thesis is organized as follows. In section 2 the relevant literature is summed up and discussed. Section 3 introduces the model including its assumptions and basic properties. Subsequently, the notion of optimal monetary policy, the different objective functions and advantages and disadvantages of certain types of reaction functions are discussed in section 4. In section 5 the policy problem is introduced followed by the derivation of the reaction function and its properties. The corresponding welfare analysis including an investigation of consumption inequality is presented in section 6. Finally, the thesis is concluded in section 7. The technical appendix in section 8 shows all relevant technical descriptions and derivations if not already presented in the main text, supplementary graphs and the underlying Matlab code.

2 Related literature

This section briefly summarizes the relevant literature. It will be elaborated on some details of the literature in the following sections if necessary.

2.1 Bounded rationality, heterogeneous expectations and empirical evidence

The model under scrutiny is developed in Branch and McGough (2009). The authors slightly deviate from the canonical New Keynesian model by allowing for two types of different forecasting rules: rational expectations and a static backward-looking rule. It is important to note that the share of agents that use a specific forecast rule is exogenously given. Hence, agents do not have the ability to switch forecasting rules. Due to the introduction of backward-looking agents the model can account for persistence in macroeconomic aggregates, which is otherwise generated by *ad hoc* assuming persistence in the shock process of the canonical model. For instance, Fuhrer (2017) identifies slowly moving expectations as a source of persistence by incorporating survey data on inflation expectations into a NK model. In Branch and McGough (2009) all agents in the economy are assumed to make rational decisions given their forecasts. Thus, agents are assumed to satisfy their individual Euler equation given their subjective expectations. Agents that use the backward-looking rule are boundedly rational because their forecasts are systematically biased. In order to facilitate aggregation, i.e. obtaining the same functional forms as in the canonical model, some rather strong assumptions regarding the expectation formation have to be made. For instance, it is assumed that agents, irrespective of the type, do not know about the heterogeneity amongst them. Hence, higher-order beliefs are practically assumed away. A detailed introduction and discussion of these assumption can be found in section 3.1.

The work of Massaro (2013) is closely related as he also derives the New IS and NK Phillips curve under heterogeneous expectations. In contrast to Branch and McGough (2009), the agents in Massaro (2013) base their decision on an infinite-horizon forecast following Preston and Parker (2005). In a model with fully rational expectations only the one-period-ahead forecast matters, because agents already incorporate every available information in an optimal way in forecasting the next period. This implies an infinite horizon by the forward-looking structure of the model. Consequently, agents that are boundedly rational have to forecast over an infinite horizon since their forecast one period ahead is not optimal. As a result, the private sector equations depend on long-horizon forecasts as well. Thus, Massaro (2013) does not have to impose such restrictive assumptions on expectations as Branch and McGough (2009) in order to facilitate aggregation. However, this comes at the cost of not obtaining the same functional forms as in the canonical model, which impairs comparability.

There is already much further literature that incorporates heterogeneous expectations into macroeconomic models. Brock and Hommes (1997), for instance, introduce a dynamic predictor selection into the cobweb model.[4] This means that agents can choose from a finite set of forecasting rules with different forecasting performances. The cost of using a certain rule thereby increases with its performance. Branch and McGough (2010) extend their 2009-model by allowing agents to switch forecasting rules and thereby endogenize the share of agent types as well.[5] It should be noted that the dynamic predictor selection can generate complex non-linear dynamics. Further mentionable work is also done by De Grauwe (2011) who introduces herding behavior by assuming heterogeneous and bounded rational agents with cross-correlations in beliefs.

The introduction of heterogeneous expectations is backed-up by sufficient empirical evidence already. For instance, Branch (2004) sets up a model in which agents choose rationally from a set of costly forecast instruments that vary in the degree of sophistication and thereby lead to bounded rationality. This model is tested in a statistical discrete choice setting, which shows, first, that US inflation expectation data favors heterogeneous expectations over RE and, second, that there is a dynamic predictor selection as described above. Heterogeneity in expectations is also confirmed by his later work (Branch, 2007). In this work he compares different models of expectations heterogeneity including sticky information (Branch, 2007).[6] Further support is provided by Pfajfar and Santoro (2010). They identify three types of agents in the distribution of US inflation expectations that differ by their forecasting behavior: static backward-looking (as in Branch and McGough (2009)), near rational and adaptive learning behavior. Also, Mankiw et al. (2003) find disagreement in inflation expectations which can mostly be captured by a model of sticky information. This is also in line with Carroll (2003). Thus, incorporating HE into macroeconomic models is of practical relevance and more than being merely a theorist's "what if" thought experiment.

Motivated by data (Campbell and Mankiw, 1989), another mentionable strand of the literature on bounded rationality introduces "rule-of-thumb" agents that consume all of their period income (Krusell and Smith (1996); Mankiw (2000); Amato and Laubach (2003); Gali et al. (2004)). To put it in a nutshell, inertia in the policy design is optimal in an economy with "rule-of-thumb" consumers (Amato and Laubach, 2003), while the Taylor principle is not sufficient to guarantee determinacy in such a framework (Gali et al., 2004).

[4]For the cobweb model consider for instance Ezekiel (1938).

[5]One may also consider Branch and Evans (2006).

[6]Sticky information is comparable to sticky prices where agents are able to update their information set in the current period with an exogenously given probability.

2.2 Optimal monetary policy under bounded rationality and heterogeneous expectations

This thesis directly builds upon Di Bartolomeo et al. (2016) who derive a model-consistent loss function as a second order approximation of heterogeneous household utility to investigate fully optimal monetary policy. As a result, the central bank should consider consumption inequality between the different types of agents in its policy objective in addition to output gap variability and price dispersion. As different consumption paths are due to different expectations only, the central bank ultimately tries to stabilize expectations. Optimal monetary policy is simulated under discretion and timeless commitment. Discretionary optimal monetary policy thereby yields welfare gains over a simple, non-optimal Taylor rule across all considered parameter values. Likewise, timeless commitment is strictly preferable over discretion in terms of welfare losses. However, the authors do not consider the implementation of optimal monetary policy. This thesis tries to fill this gap.

Beqiraj et al. (2017) derive a model-consistent loss function similar to Di Bartolomeo et al. (2016) in the Massaro (2013) framework. Hence, bounded-rational agents are assumed to be backward-looking similar to the naive case of Branch and McGough (2009). However, instead of relying on short horizon forecasts, agents depend their decisions on forecasts over an infinite horizon to satisfy their intertemporal budget constraint. The model-consistent loss is then compared to the same objective excluding consumption inequality and to an objective that depends on inflation and output gap variations only. The latter is conceptual similar to the microfounded conventional objective employed here. As a result, the consumption inequality is minimized by the policy under the model-consistent loss and yields the lowest welfare costs as expected. Further, the inflation targeting objective performs worst since it neglects consumption inequality and proxies price dispersion with inflation. Matching price dispersion with inflation is not accurate under heterogeneous expectations since price dispersion has a more complex structure comparable to Di Bartolomeo et al. (2016). In contrast to Di Bartolomeo et al. (2016), the authors do only consider discretionary optimal monetary policy and neglect the commitment case. However, they do not investigate the corresponding implementation as well. Additionally, the authors estimate a share of 82 percent of rational agents using US data.

In contrast to Di Bartolomeo et al. (2016) and Beqiraj et al. (2017), Gasteiger (2014, 2017) assumes the conventional objective *ad hoc* to study optimal monetary policy in the Branch and McGough (2009) framework. His main contribution is to show that expectations-based reaction functions perform exceptionally well, i.e. yield determinacy for a large part of the parameter space. However, the expectations-based property of the reaction function does not guarantee determinacy throughout the considered parameter space as long as the conventional objective is used. This calls for an analysis of an expectations-based reaction function under the model consistent objective following Di Bartolomeo et al. (2016).

While Gasteiger (2014, 2017) and Di Bartolomeo et al. (2016) investigate optimal monetary policy in an economy with *heterogeneous* expectations, there is already much literature on the *homogeneous* expectations case. Homogeneous expectations can either take the form of *rational* expectations or *bounded-rational* expectations, which are certainly useful benchmarks. Thus, the work of Gasteiger (2014, 2017) and Di Bartolomeo et al. (2016) can be seen as an intermediate case. Optimal monetary policy under rational expectations is well known and extensively studied (see for instance Clarida et al. (1999); Woodford (1999); McCallum (1999)). Further, Kydland and Prescott (1977) already find that commitment to a specific policy rule is preferable over discretionary monetary policy if the model agents are forward looking.

On the other hand, Bullard and Mitra (2002), Evans and Honkapohja (2003a,b, 2006) and Duffy and Xiao (2007) investigate optimal monetary policy under homogeneous adaptive learning of agents. Agents thereby make biased forecasts but learn over time, i.e. update their parameters via recursive least squares. The convergence of the learning process to yield a determinate and stable equilibrium critically depends on the design of monetary policy. Evans and Honkapohja (2003a,b) therefore emphasize the importance of expectations-based reaction functions and discard fundamentals-based reaction functions, i.e. interest rate rules incorporating only shocks and predetermined variables. This is, because slight deviations of inflation expectations from the rational benchmark induce divergence from the rational expectations equilibrium that is not offset by the fundamentals-based reaction function. This is in line with Bullard and Mitra (2002) who recommend policy functions that bring about learnable rational expectations equilibria. However, Duffy and Xiao (2007) find that fundamentals-based reaction functions can yield stability and determinacy if the central bank considers also interest rate stabilization in their objective function. The implementation of the mostly preferred expectations-based reaction functions then surely depends on the observability of the expectation formation process. Evans and Honkapohja (2006) further show the importance of commitment in the case of learning agents and thereby present a further case where commitment is preferable over discretion.

Finally, Berardi (2009) constructs a model with two types of agents as well. One type is rational while the other one is uninformed in the sense that it does not know the state of the economy when forecasting. Additionally, the bounded rational agent follows a learning scheme. Thus, the model is different from Branch and McGough (2009) in the way bounded rationality is introduced. His main finding is that the central bank should base its policy solely on the rational expectations, because incorporating the biased beliefs of uniformed agents would lead to a policy that would overreact to shocks.

3 A New Keynesian model with heterogeneous expectations

This section briefly presents a New Keynesian model with heterogeneous expectations following Branch and McGough (2009). The authors ask under which assumptions it would be possible to obtain the same functional form of the New IS and the NK Phillips curve as under homogeneous rational expectations. In particular, under which conditions can the rational expectations operator simply be replaced by a linear combination of heterogeneous expectations? In identifying these assumptions, Branch and McGough (2009) find that these are in part very restrictive. They conclude that "[...]imposing heterogeneous expectations at an aggregate level may be ill-advised" (Branch and McGough, 2009, p. 1036). This is important to emphasize since the work of Gasteiger (2014) and Di Bartolomeo et al. (2016) are based upon Branch and McGough (2009), i.e. they impose these restrictive assumptions in order to analyze optimal central bank policy. Thus, the identification of the *optimal* monetary policy by Gasteiger (2014) and Di Bartolomeo et al. (2016) does require the assumptions to be accurate in the sense that if they are imposed, they do not bias the results significantly. However, as will be discussed below, this will not necessarily be satisfied, which also holds for this work as it extends Di Bartolomeo et al. (2016).

Since there is no government (apart from a central bank), no foreign trade and no capital, the aggregate income identity is

$$Y_t = C_t \quad \forall\ t \tag{1}$$

with Y_t being aggregate output and C_t aggregate consumption. Consequently, aggregate savings have to be zero since saving on aggregate in a closed economy is possible only in the form of capital goods, which are absent here. Saving in the form of net financial assets (for instance bonds) by individual agents is of course possible. However, since every agent's financial asset is another agent's liability, net financial assets are zero on aggregate in a closed economy. Hence, aggregate bond holdings will be zero in any period. Note that (1) is interpreted as an equilibrium condition in the context of this model.

3.1 Assumptions on heterogeneous expectations

As mentioned above, some assumptions have to be imposed in order to facilitate aggregation with the result that the same functional forms as under RE can be obtained. Thus, it is important to specifically name and discuss these assumptions. This section again closely follows Branch and McGough (2009). The assumptions on the subjective expectations operator E_t^τ of type τ are as follows:

A1 *Expectations operators fix observables.*

This assumption simply means that the expectation of a known value is the known value itself.

A2 *If x is a variable forecasted by agents and has steady state $\bar{x}$, then $E^{\tau}\bar{x} = E^{\tau'}\bar{x} = \bar{x}$, $\tau \neq \tau'$.*

In steady state different type of agents forecast the steady state correctly. This is of course intuitive for rational agents since they know the true economic structure, i.e. they know when the system comes to rest after all shock realizations are zero. In particular, rational agents know the true probability distribution of the exogenous shocks, i.e. that the shocks are normally distributed around a zero mean. Backward-looking agents will base their forecast in steady state on the steady state itself. To be precise, the steady state of the model variables is zero since they are expressed in terms of percent deviations from the steady state level and disturbances are not persistent. Hence, backward-looking agents predict the steady state, too, especially in the case where the forecasting rule is such that the percent deviation from steady state is multiplied by some parameter.

The following two assumptions impose certain linearity properties (α and β being some fixed parameters for now).

A3 *If x, $x+y$ and αx are variables forecasted by agents, then $E_t^{\tau}(x+y) = E_t^{\tau}(x) + E_t^{\tau}(y)$ and $E_t^{\tau}(\alpha x) = \alpha E_t^{\tau}(x)$.*

A4 *If for all $k \geq 0$, x_{t+k} and $\sum_k \beta^{t+k} x_{t+k}$ are forecasted by agents, then*

$$E_t^{\tau}\left(\sum_{k\geq 0} \beta^{t+k} x_{t+k}\right) = \sum_{k\geq 0} \beta^{t+k} E_t^{\tau}(x_{t+k}).$$

Linearity in expectations is a reasonable assumption in a linear model world.

It is also intuitive to assume that the agents do not expect to alter their expectation in a systematic way. The law of iterated expectations (LIE) at the *individual* level manifests this notion:

A5 *E_t^{τ} satisfies the law of iterated expectations: If x is a variable forecasted by agents at time t and time $t+k$, then $E_t^{\tau} E_{t+k}^{\tau}(x) = E_t^{\tau}(x)$.*

The next two assumptions are the most restrictive ones and therefore require a more thorough treatment.

A6 *If x is variable forecasted by agents at time t and time $t+k$, then $E_t^{\tau} E_{t+k}^{\tau'}(x_{t+k}) = E_t^{\tau}(x_{t+k})$, $\tau' \neq \tau$.*

Put simply, this means that agents of type τ do not know how agents of type τ' form their expectations. In fact, they do not know about heterogeneity in expectations. Consequently, agents expect other agents to expect the same independent of type. This assumption is especially restrictive regarding the rationality of "rational" agents. The theory of RE assumes that agents process *all* available information, which includes the expectation formation of other agents. However, rational agents know about the aggregate equations and therefore the macroeconomic persistence, even though they do not explicitly know where it stems from. Thus, this assumption places a particular structure on higher-order beliefs (Branch and McGough, 2009, p. 1038):

> [...] this structure is *necessary* for the aggregate expectations operator to satisfy LIE, and is thus *necessary* for an aggregation result.

However, (unrestricted) higher-order beliefs might be very important. For instance, Kurz (2008) compares the explanation of diverse beliefs by asymmetric private information to subjective heterogeneous beliefs in an asset pricing model where the distribution of beliefs is *observable*. The former is analogous to **A6** since the information about the expectation formation process is private in the Branch and McGough (2009) model. Kurz (2008) sets up a model with heterogeneous subjective beliefs in which endogenous variables depend on the distribution of beliefs. Thus, when traders want to forecast these endogenous variables they have to forecast the distribution of personal beliefs of other agents. Kurz (2008) then discards the private information approach, because of widely implausible assumptions and the impossibility of empirical validation in many respects. Further, Amato and Shin (2006) find in a model of monopolistic competition that if competitiveness increases, firms increase the weights on higher-order beliefs in their price setting rule. This leads to a detachment of equilibrium prices from the underlying cost conditions. Since monopolistic competition is the model of choice of the firm sector in NK models, this result can be more or less directly translated.

Therefore, one must conclude that not placing **A6** on higher-order expectations might indeed alter inflation and output dynamics. In fact, Branch and McGough (2009) show explicitly how the aggregate IS equation depends on higher-order beliefs if **A6** is not assumed. Further, for cases where higher-order beliefs are treated seriously, Amato and Shin (2006, p. 213) also emphasize that "[m]onetary policy must rely on less informative signals of the underlying cost conditions". Also, Woodford (2002) stresses that higher-order beliefs might be a source of inflation inertia in response to monetary policy shocks. This results from uncertainty that is generated by agents forming expectations about expectations that are formed about expectations (...).

The findings of the cited literature show that placing **A6** to satisfy the LIE on aggregate is everything but an innocuous assumption. The following assumption is not less restrictive.

A7 *All agents have common expectations on expected differences in limiting wealth.*

In case **A7** is not assumed, Branch and McGough (2009) show that limiting wealth, i.e. wealth for $t \to \infty$, is part of the IS equation as well and thereby affect macroeconomic outcomes.[7] However, if **A7** is imposed the respective terms regarding limiting wealth cancel out. **A6** imposes that the different types of agents do not know about the heterogeneity in expectations. Yet, **A7** imposes that agents do form expectations about the *differences* between types in limiting wealth without knowing the *difference* in expectation formation between the two types of agents. To take it even further, even though the two types of agents differ by their expectation formation, **A7** assumes that agents somehow have the *same* expectations regarding the differences in limiting wealth. This is quite unintuitive, when one fraction of agents has biased and the other fraction has unbiased beliefs.

3.2 The model

The following description of the model is based on Di Bartolomeo et al. (2016) and Branch and McGough (2009).

The economy is populated by a continuum of utility maximizing households that both produce a differentiated good as a monopolist, thereby can set prices, and consume a composite good. Each household i maximizes an infinite sum of discounted period utility out of consuming the composite good C_t^i and from producing the differentiated good $Y_t(i)$ according to

$$E_0^\tau \sum_{t=0}^{\infty} \beta^t [u(C_t^i) - v(Y_t(i))]. \tag{2}$$

$\beta < 1$ thereby represents the subjective discount factor, E_0^τ is the expectation in period zero of type τ and $u(\cdot)$ and $v(\cdot)$ are the respective functional forms of (dis-)utility. The real budget constraint of a household i is

$$C_t^i + B_t^i = \frac{1 + i_{t-1}}{\Pi_t} B_{t-1}^i + \zeta^\tau \tag{3}$$

where B_t^i is a one-period bond that pays $1 + i_t$ in $t + 1$, $\Pi_t = \frac{P_t}{P_{t-1}}$ is gross inflation in t and ζ^τ is the real income of type τ. By maximizing (2) under the constraint (3), a *representative* utility maximizing household of type τ will choose its consumption path according to the intertemporal Euler equation[8]

$$u_c(C_t^\tau) = \beta E_t^\tau \left[u_c(C_{t+1}^\tau) \frac{1 + i_t}{\Pi_{t+1}} \right]. \tag{4}$$

[7] To follow the argument mathematically, please consider Branch and McGough (2009).

[8] u_c stands for the first partial derivative of the utility function w.r.t. consumption.

To model the different household types as representative agents, it is assumed that households have perfect consumption insurance where a benevolent planer collects and redistributes average real income to each household within the group.

A Calvo-type mechanism is assumed to generate price stickiness where a fixed fraction of firms cannot reset their prices in a given period (Calvo, 1983).[9] Price dispersion arises, because, first, optimal prices are different *between* expectation types since they depend on *expected* future marginal costs and, second, they differ *within* each type due to the fact that only a fraction of firms can reset prices.

As already outlined, there are two types of agents in this model: rational (R) and bounded rational agents (B) so that $\tau \in \{R, B\}$. Their corresponding shares of the whole population are α and $1 - \alpha$, respectively, which are exogenously given. Thus, expectation schemes are fixed, i.e. there is no dynamic predictor selection in this setting. This assumption is fairly restrictive as well since US data on inflation expectations indicate that agents do indeed select forecast models in a dynamic way (Branch, 2004).

Bounded rational agents form their expectations about some variable x following $E_t^B x_t = \theta x_{t-1}$ where $\theta < 1$ stands for adaptive, $\theta > 1$ trend-setting and $\theta = 1$ naive expectations. Since it is assumed that bounded rational agents do not observe the current state of the economy when forecasting, one can write the rule by the LIE as $E_t^B x_{t+1} = \theta^2 x_{t-1}$.

Excluding the interest rate rule, which is going to be derived in section 5, the model can be summarized as

$$y_t = \hat{E}_t y_{t+1} - \sigma(i_t - \hat{E}_t \pi_{t+1}) \tag{5}$$

$$\pi_t = \beta \hat{E}_t \pi_{t+1} + \kappa y_t + e_t \tag{6}$$

$$\hat{E}_t y_{t+1} = \alpha E_t y_{t+1} + (1 - \alpha)\theta^2 y_{t-1} \tag{7}$$

$$\hat{E}_t \pi_{t+1} = \alpha E_t \pi_{t+1} + (1 - \alpha)\theta^2 \pi_{t-1} \tag{8}$$

where all lower case letters are in log-deviations from steady state.

Equation (5) represents the New IS curve, where y_t is the output gap, σ is the intertemporal elasticity of substitution[10], i_t is the nominal interest rate in period t and $\hat{E}_t \pi_{t+1}$ expected inflation in period $t + 1$. Equation (6) shows the NK Phillips curve, where e_t is a cost-push shock[11] and where the parameter κ measures the responsiveness

[9]Thus, the above described insurance mechanism is a hedge against the risk of the Calvo lottery.

[10]Which is the inverse of the coefficient of relative risk aversion, i.e. it measures the sensitivity of the output gap to changes in the real interest rate. The higher the intertemporal elasticity of substitution (the lower the coefficient of relative risk aversion), the more are agents willing to shift consumption across time.

[11]It can, for instance, be seen as exogenous variations in marginal costs.

of inflation with respect to changes in aggregate demand. κ in turn is given by

$$\kappa = \frac{(1-\xi_p)(1-\beta\xi_p)(\eta+\sigma^{-1})}{\xi_p(1+\epsilon\eta)}, \tag{9}$$

where ξ_p is the share of firms that cannot reset their price in a given period, η is the elasticity of marginal disutility of producing output and ϵ is the elasticity of demand for a differentiated good. Note that the stickier the prices (the higher ξ_p), the smaller is κ and therefore the smaller are the effects of changes in the output gap on inflation (and vice versa).

Both model equations have a forward-looking nature that stems from the Euler equation and the price setting problem of firms, respectively. Equations (7) and (8) explicitly show the heterogeneity in expectations.

The parameters for the baseline model are chosen to be the same as in Di Bartolomeo et al. (2016), i.e. the model is calibrated for the US economy following Rotemberg and Woodford (1997) with the time unit being one quarter.

Table 1: Baseline calibration

$\alpha = 0.7$	$\theta = 1$	$\beta = 0.99$	$\sigma = 6.25$	$\epsilon = 7.84$	$\eta = 0.47$	$\xi_p = 0.66$

3.3 Model properties: persistence and amplification

The model described by equations (5)-(8) plus some rule that specifies monetary policy has some interesting properties compared to the standard NK model. First, the model introduces an inherent persistence following a transitory cost-push shock. If a transitory cost-push shock, i.e. a shock with zero persistence, hits the economy described by a standard, entirely forward-looking NK model, all model variables will be back in steady state in the period after the shock hit. This mechanic stems from the fact that all agents make one-period-ahead rational forecasts, i.e. they know that the shock will not persist, and decide accordingly. Note, that monetary policy inertia can introduce persistence into an otherwise canonical New Keynesian model.

If a transitory cost-push shock hits the economy that includes backward-looking agents, it will take some time until model variables come back to steady state. This is caused by the forecast behavior of the backward-looking agents that depends on lagged values of the respective variables. Therefore, their beliefs are biased. To be specific, these agents will not alter their forecast in the period the shock realizes. Only in the following period their expectations and thereby their decisions react. Note that in this moment the shock realization is already zero again. Thus, backward-looking agents make their production/consumption decision based on the non-zero shock realization when its *direct* effect has already vanished. Hence, private sector equations are in part backward-looking, which introduces persistent behavior of model variables.

In addition, the backward-looking nature of the forecasting rule of the bounded rational agents induces an amplification mechanism (Gasteiger, 2017). This is caused by the interaction of the two expectation types. Recall **A6** from Branch and McGough (2009), i.e. rational agents do not directly know about the expectation formation of bounded rational agents. However, rational agents have knowledge about the aggregate model equations, i.e. they know about the lagged behavior of the model variables. In solving the model, rational agents can decide upon the law of motions. These are called the *perceived law of motions* (PLM, perceived by the rational agents). As described above, if a transitory cost-push shock drives up inflation, rational agents know by their PLM about the lagged behavior of inflation. Therefore, their expectations of tomorrow's inflation is above zero, $E_t^R \pi_{t+1} > 0$, in contrast to the fully rational case. If $E_t^R \pi_{t+1} > 0$, the real interest rate of rational agents, $i_t - E_t^R \pi_{t+1}$, decreases *ceteris paribus*. Consequently, aggregate demand increases and thereby inflation rises even further *ceteris paribus*. It is therefore necessary for the central bank to incorporate this knowledge into its policy making and to exert influence on backward-looking agents' expectations even under discretion as will be explained in more detail in section 4.1.1. Otherwise, the central bank's policy would not be efficient and credible as rational agents would know about this inconsistency. Of course, a "leaning-against-the-wind" policy can mitigate this effect by raising the nominal interest rate accordingly, but as long as a fraction of backward-looking agents exist, the amplification effect cannot be fully muted. This leads to higher welfare losses in an economy with heterogeneous expectations compared to fully rational agents even under optimal monetary policy (Gasteiger, 2017).

4 Optimal monetary policy

This section gives an idea of optimal monetary policy. In particular, the discretion and commitment regime, advantages and drawbacks of the different objectives and reaction functions and the model-consistent loss function will be discussed.

4.1 The concept of optimal monetary policy and the conventional objective

Optimal monetary policy means that the central bank is committed to maximize social welfare. More precisely, it aims at maximizing the expected utility of households (Gasteiger, 2014).

The following equation is known as the *conventional* objective as employed by Gasteiger (2014, 2017):

$$E_t \sum_{s=0}^{\infty} \beta^s \frac{1}{2}[\pi_{t+s}^2 + \omega y_{t+s}^2] \tag{10}$$

This loss function is conventional in the sense that it can be seen as a second order approximation of household utility under RE for a cashless economy (Benigno and Woodford, 2003). It takes into account that agents suffer from inflation and output gap variations, which have to be minimized. The parameter ω is thereby the weight that the central bank assigns to output gap variations and is typically a function of deep model parameters. Indeed, further below, ω will be microfounded for the rational expectations case, i.e. when $\alpha = 1$. Variations in output should be low, because agents want to smooth consumption over time. Further, the inflation term proxies price dispersion, which is accurate in the context of RE and sticky prices.[12] Only when inflation arises while prices are sticky, price dispersion can occur. The story behind the inflation term in a model with intermediate goods firms and final good firms would be as follows[13]: inflation causes price dispersion, which induces dispersion in intermediate goods, because firms with lower prices produce more and firms with higher prices produce less than efficient. Because of imperfect substitution between inputs, dispersion yields losses in final output. Thus, price dispersion represents a *symptom of inefficiency* introduced by the Calvo (1983) mechanism. Consequently, this inefficiency should be minimized. In a yeoman-farmer economy there is not such an intuitive narrative. Hence, one has to be satisfied with the fact that the losses in final output are due to a constant-elasticity of substitution aggregator.

Also, (10) shows that variations of inflation and the output gap should be minimized *jointly*. However, the model equations imply a *trade off* between the both, which is

[12] It is however not accurate in the context of HE as will be shown later.

[13] Using this model context to illustrate the point is more intuitive than the more abstract yeoman-farmer framework.

typically described by a Taylor (1979) curve. Bringing down inflation in the aftermath of a cost-push shock comes at the cost of depressing output via an increase of the real interest rate. Hence, reducing the variations in inflation causes the variations of the output gap to increase (and vice versa). Following Gasteiger (2017), this trade off is typically seen as a policy menu by policy makers.

Gasteiger (2014, 2017) assumes the conventional objective in an *ad hoc* manner. However, using (10) in the context of heterogeneous expectations might be paradoxical if one aims at achieving optimality since it only recognizes the losses of the rational share of the population. To make the analysis more stringent, it is a natural way to extent Gasteiger (2014, 2017)'s analysis by deriving the welfare function from household utility incorporating heterogeneous expectations *explicitly* as done by Di Bartolomeo et al. (2016). However, there are also convincing reasons to use the conventional objective even in a HE setting as will be discussed below.

4.1.1 Discretionary optimal monetary policy

The conventional view on discretionary policy is that the central bank re-optimizes its actions every period and *takes the private sector expectations as given*, i.e. it does not seek to *choose* (rational) expectations explicitly. Such a policy is naturally time consistent. Time consistency means that there is no incentive for the central bank to deviate from its policy at any point in time. This ensures *credibility* as rational agents know that the central bank has no incentive to deviate from its announced policy. One advantage is the relatively simple computation.

Consequently, one might be tempted to formulate the optimization problem by assuming the central bank to minimize the *instantaneous* loss L_t subject to the private sector equations with respect to the instantaneous variables. This however would be flawed since the central bank would even ignore the knowledge about the amplification mechanism induced by the interaction of the backward-looking behavior of bounded-rational agents and rational agents described above. Neglecting this knowledge, which the central bank is assumed to have, would cast doubt on the credibility of such a policy. Inconsistency would arise, because there is an incentive for the central bank to internalize the knowledge about the backward-looking agents in its decision making. This inconsistency would be recognized by the rational agents (Gasteiger, 2014). Hence, it is required that the central bank minimizes an infinite sum of losses L_t subject to the private sector equations. Still, rational expectations are not chosen explicitly.

However, discretionary policy is a "[...]game against rational agents" (Kydland and Prescott, 1977, p.473). As long as (some) agents are rational, a superior equilibrium relative to discretion can be achieved by committing to a certain rule that influences rational expectations directly. Nevertheless, following Beqiraj et al. (2017), discretionary policy acts like a commitment in a setting with backward-looking agents as assumed here. When the central bank chooses a policy in t, the backward-looking agents' expectations in $t+1$ are fixed, which in turn influences inflation and output gap in $t+1$.

A central bank that conducts optimal monetary policy sets interest rates such that the particular loss function is minimized by taking into account the economic structure. To get an idea of the procedure, it will be useful to derive the rational benchmark for discretionary monetary policy under the general form of (10).[14] Minimizing (10) under discretion subject to the NK Phillips curve under RE yields the targeting rule

$$y_t = -\frac{\kappa}{\omega}\pi_t. \tag{11}$$

(11) states that the central bank should contract output following an increase in inflation depending on the degree of price stickiness implied by κ and the weight it places on the output gap, ω. Combining (11) with the Phillips and New IS curve under RE gives

$$i_t = \frac{1}{\sigma}E_t y_{t+1} + \left[1 + \frac{\kappa\beta}{\sigma(\kappa^2+\omega)}\right] E_t\pi_{t+1} + \left[\frac{\kappa}{\sigma(\kappa^2+\omega)}\right] e_t. \tag{12}$$

Assuming $\omega = 0.1$ (Gasteiger, 2014), the central bank positively reacts to expected steady state deviations in inflation by more than one-to-one and the expected output gap by less than one-to-one. Note however that the central bank employs discretionary policy, i.e. it does not explicitly try to influence expectations as can be seen in the derivations. Also, note that (12) is not of the same form as in Clarida et al. (1999). In Clarida et al. (1999) (11) is combined with the New IS curve only. However, here (11) is first combined with the Phillips curve and then with the New IS curve in order to ensure comparability to reaction functions derived below.

4.1.2 Commitment from a timeless perspective

There seems to be a consensus in the literature that policy under commitment outperforms discretionary policy in terms of welfare. This is because the central bank directly manipulates (rational) expectations that matter for current realizations of inflation and output. In general, such a policy is time *inconsistent*, which stems from the "[...] separate treatment of the initial conditions and policy in the longer term" (Blake et al., 2001, p.2). To see this, consider the two optimality conditions for monetary policy derived from a NK model under homogeneous rational expectations and

[14] The derivation is outlined in the appendix 8.1.1.

the conventional objective (see Clarida et al. (1999))

$$y_t - y_{t-1} = -\frac{\kappa}{\omega}\pi_t \text{ for } t \geq 1, \tag{13}$$

$$y_0 = -\frac{\kappa}{\omega}\pi_0. \tag{14}$$

In period $t = 0$, when the policy is chosen, the central bank should control the level of output gap as in (11) while for future periods it should control the *change* in the output gap sticking to (13). Note that (13) introduces persistence into the otherwise standard NK model. Thus, if the central bank chooses an optimal policy in $t = 0$, it should behave according to (13) in $t = 1$. However, if the central bank would re-optimize in $t = 1$, it would be *optimal* to behave according to (14) and *not* to (13). Thus, there is a clear-cut incentive for the central bank to deviate from its committed policy. If a central bank seeks to influence expectations, *credibility* of the central bank's decision is of vital importance. Obviously, under time inconsistency rational agents will recognize this incentive problem and act differently than intended by the monetary policy authority.

A possible solution to this problem as a central bank is to ignore (14) and follow (13) right from the start. In fact, this solution is known as commitment from a *timeless perspective*, which is proposed by Benigno and Woodford (2003).[15] Commitment from a timeless perspective is also employed by Di Bartolomeo et al. (2016) and Gasteiger (2014, 2017).

4.2 The model-consistent objective function

The goal of Di Bartolomeo et al. (2016) and this thesis is to study the optimal equilibrium under heterogeneous expectations and thus the optimal monetary policy. Hence, the central bank aims at maximizing household utility (2) subject to the private sector equations (5)-(8). Since the computation of the exact non-linear policy problem can be quite cumbersome, Benigno and Woodford (2003, 2012) and Woodford (2003) propose and extensively discuss the linear-quadratic (LQ) approach to optimal policy. Hence, the exact problem is approximated by a quadratic objective function and linear constraints. This method yields a *local linear approximation* of the optimal monetary policy. Using the LQ-approach it is straightforward to check whether the second-order conditions for optimality are satisfied by simply checking for concavity of the quadratic objective (Benigno and Woodford, 2012). In addition, comparability with already existing research is easy since most researchers use a quadratic loss function (Woodford, 2003). Note that a local linear approximation is valid only for sufficiently small shocks,

[15] See also Benigno and Woodford (2012) and Woodford (2003)

which also holds for the log-linearized model equations.[16]

Consequently, in contrast to Gasteiger (2014, 2017), Di Bartolomeo et al. (2016) employ the LQ-approach and derive the central bank's objective function as a quadratic approximation to the (heterogeneous) household utility. Hence, the resulting loss function recognizes the heterogeneity in expectations explicitly. The authors obtain the following instantaneous utility-based loss function

$$L_t = \frac{1}{2}\left[\left(\eta + \frac{1}{\sigma}\right) y_t^2 + (\epsilon^2\eta) var_i(p_t(i)) + \frac{1}{\sigma} var_i(c_t(i))\right]. \tag{15}$$

Recall that agents are inclined to smooth consumption over time. Agents suffer from variations in the output gap since it is a weighted average of disaggregate consumption in this context (first term). Therefore, it is intuitive that the effect of output gap variability on the loss is increasing with higher risk aversion, $\frac{1}{\sigma}$, of agents and decreasing with a higher intertemporal elasticity of substitution, σ.[17] The second term indicates that welfare is decreasing with price dispersion, $var_i(p_t(i))$ with $p_t(i)$ being the price of the differentiated good in t produced by agent i.[18] It is unconventional in the sense that it cannot be proxied by inflation alone. Price dispersion results in a lower than efficient output due to a CES aggregator as explained above. Additionally to being depend on η, its weight depends on the price elasticity of demand for differentiated goods, ϵ. This is intuitive since production naturally depends on the reagibility of demand with respect to changes in prices that cause price dispersion across agents i due to sticky prices. The third term introduces a loss in welfare because of consumption inequality, which is measured by the cross-sectional variance, var_i, of disaggregate consumption, $c_t(i)$. Inequality is generated by the different forecast paths since agents only differ by expectation types. Different forecasts in the group specific output result in different consumption decisions. Thus, aggregate output under heterogeneous expectations differs from the output under RE, which represents a further inefficiency. Consequently, consumption inequality yields welfare losses.

The loss (15) can be rewritten in order to emphasize heterogeneity in beliefs ac-

[16]Of course, it can be questioned especially in the light of the recent Great Recession whether shocks of interest are still sufficiently small. For now, the work focuses on "normal" times in the sense that shocks are small enough for the first order approximation to be accurate.

[17]In addition, the term depends on the elasticity of marginal disutility of producing output, η.

[18]Recall that there is price dispersion across and within agent groups τ. Therefore, the index i has to be used.

cording to

$$L_t = \underbrace{\frac{1}{2}\left[\left(\eta + \frac{1}{\sigma}\right) y_t^2 + (\epsilon^2\eta) var_i^0(p_t(i))\right]}_{L_0} + \underbrace{\frac{\epsilon^2\eta}{2}[var_i(p_t(i)) - var_i^0(p_t(i))]}_{L_1} + \underbrace{\frac{1}{2\sigma} var_i(c_t(i))}_{L_2} \tag{16}$$

where L_0 is the loss associated with output gap variability y_t^2 and the part of price dispersion which does *not* stem from heterogeneous beliefs (var_i^0 stands for the homogeneous expectation case), L_1 the loss due to the part in price dispersion that is attributed to heterogeneity in beliefs and L_2 the loss resulting from consumption inequality.

The variances in (15), which are consistent with the New IS and NK Phillips curve (5) and (6), are

$$var_i(p_t(i)) = \delta\pi_t^2 + \frac{\delta\xi_p(1-\alpha)}{\alpha}\left[\pi_t - \beta\theta^2\pi_{t-1} - \kappa\frac{c_t^B + \eta\sigma y_t}{1+\eta\sigma}\right]^2 \tag{17}$$

$$var_i(c_t(i)) = \alpha(1-\alpha)(c_t^R - c_t^B)^2 \tag{18}$$

where $\delta = \frac{\xi_p}{(1-\beta\xi_p)(1-\xi_p)}$ is a measure of price stickiness and c_t^τ with $\tau \in \{R, B\}$ is disaggregate consumption of rational and bounded ratonal agents, respectively (Di Bartolomeo et al., 2016). The price dispersion term (17) has some interesting properties. First, in case of homogeneous rational expectations, $\alpha = 1$, it is equal to $\delta\pi_t^2$.[19] Thus, price dispersion occurs if there is non-zero inflation and prices are sticky (as δ depends on ξ_p). Note that if prices were flexible, δ would be zero and price dispersion would be absent. Thus, under RE and sticky prices inflation is sufficient to proxy price dispersion. Second, under heterogeneous expectations, i.e for $0 < \alpha < 1$, optimal prices are different *between* expectation types due to different expectations regarding future marginal costs. Hence, price dispersion has a far more complex structure as it depends on lagged inflation introduced by the price-setting behavior of backward-looking agents, on consumption of backward-looking agents and on the output gap. Consequently, inflation alone is not a good proxy for price dispersion (Beqiraj et al., 2017).

Now, recall that due to the amplification mechanism described above inflation will be larger on impact, which requires a larger contraction of the output gap by the central bank to stabilize inflation. This effect causes both the first term in (15) to increase more relative to the RE case and the price dispersion term (17) to increase more relative to price dispersion proxied by inflation alone. Thus, the appearance of the output gap in (17) shows that the trade off between output and inflation deteriorates even more in

[19]i.e. $var_i^0(p_t(i)) = \delta\pi_t^2$ in (16).

terms of welfare losses relative to losses measured by the conventional objective. The deterioration of this trade off due to HE is confirmed by Gasteiger (2017), who compares the welfare losses under heterogeneity and full rationality employing the conventional objective.[20] However, at some point in Gasteiger (2017) the trade off recovers, i.e. for a very low fraction of rational agents. This is because the share of rational agents is so low that the channel through which the amplification mechanism works tightens. Note that the price dispersion (17) increases with the fraction of backward-looking agents $1-\alpha$.

Further, consumption inequality takes the form as in (18). Obviously, (18) becomes minimal for $c_t^R = c_t^B$, which requires the central bank to stabilize the variability of expectations between the two types since their decision making only differs by their forecasting rules.[21] Further, it is trivial to say that the cross-sectional consumption variance vanishes for $\alpha = 1$ (Di Bartolomeo et al., 2016).

In the rational case, i.e for $\alpha = 1$, (15) reduces to

$$L_t^{\alpha=1} = \frac{1}{2}\left[\left(\eta + \frac{1}{\sigma}\right) y_t^2 + \epsilon^2 \eta \delta \pi_t^2\right]. \tag{19}$$

Note that (19) is of a similar form as the the conventional objective (10) and the inflation targeting objective in Beqiraj et al. (2017). It is also equal to L_0 in (16). From now on, (19) will be referred to as the *microfounded* conventional objective. Minimizing (19) subject to the NK Phillips curve under RE gives a microfounded version of the targeting rule resulting from the conventional objective

$$\pi_t = -\frac{\sigma\eta + 1}{\kappa \epsilon^2 \eta \delta \sigma} y_t \tag{20}$$

where choosing

$$\omega = \frac{\sigma\eta + 1}{\epsilon^2 \eta \delta \sigma} \tag{21}$$

yields

$$\pi_t = -\frac{\omega}{\kappa} y_t. \tag{22}$$

The corresponding interest rate rule is

$$i_t = \frac{1}{\sigma} E_t y_{t+1} + \left[1 + \frac{\beta \delta \epsilon^2 \eta \kappa}{1 + \eta\sigma + \delta \epsilon^2 \eta \kappa^2 \sigma}\right] E_t \pi_{t+1} + \frac{\delta \epsilon^2 \eta \kappa}{1 + \eta\sigma + \delta \epsilon^2 \eta \kappa^2 \sigma} e_t, \tag{23}$$

which is equal to (12) for ω chosen as in (21). It will be shown later that (23) is a special case of the reaction function under heterogeneous expectations.

[20] Thus, his analysis does not include the additional effect due to the price dispersion channel.

[21] For $\alpha = 0.5$ dispersion in consumption is highest ceteris paribus.

4.3 The conventional vs. the model-consistent objective

Gasteiger (2017) makes a case of defending the use of equation (10) against the microfounded and fully model-consistent loss function derived by Di Bartolomeo et al. (2016). He makes four strong points in favor of (10). First, policy makers in practice often see the trade off between inflation and the output gap as a policy menu, which is well described by (10). Thus, central banker's decision making in practice seems to resemble more the conventional objective than the model-consistent one, which makes the analysis more realistic. Second, since the model-consistent objective depends on ten terms in total including disaggregate consumption of rational agents, it raises the problem of effective communication and transparency in practice. This is a problem for a central bank that tries to influence expectations by its policy rule. Further, it is an empirical question if the additional eight terms are significantly different from zero. The baseline calibration suggests indeed that they might in part be relatively small. Also, recall that agents do not explicitly know of the expectation formation of the other agents (**A6**). This assumption is necessary to facilitate aggregation since it ensures the LIE to hold on aggregate. Hence, assuming that the central bank communicates such a loss function would be inconsistent with **A6** since it indicates that expectations heterogeneity is present. In section 5.2, a reaction function based on this particular loss function is going to be derived. Hence, this problem also applies to the communication of such a reaction function. Third, since the model-consistent loss function depends on disaggregate consumption, it raises the question of whether such data is available (which includes data on expectations). If there is no data, a policy that is based on disaggregate consumption will not be feasible. Fourth, in case that there is uncertainty about the most appropriate model of the economy the model-consistent loss function is not suitable as it is applicable only to this particular model.

However, (10) has some serious drawbacks as well. First, as already mentioned, it corresponds to a second order approximation of household utility under RE. This is grossly inconsistent with the claim that incorporating heterogeneous expectations into optimal monetary policy is important. Therefore, true *optimality* cannot be ensured. Second, (10) understates the true losses measured by the model-consistent loss as will be shown below. Hence, an evaluation of the "optimal" policy is impaired. Third, as Gasteiger (2017) shows, implementation strategies derived from (10) do not guarantee determinacy throughout the considered parameter space.

As a result, this calls for an investigation of optimal monetary policy under the true, model-consistent loss function.

4.4 Fundamentals- vs. expectations-based reaction functions

Another question that might arise in the context of monetary policy, is whether a fundamentals- or expectations based reaction function should be used.

There almost seems to be a consensus in the literature that fundamentals-based reaction functions yield indeterminacy in case that agents' expectations slightly deviate from the rational benchmark (Evans and Honkapohja, 2003a,b). Fundamentals-based reaction functions solely depend on shocks and predetermined variables and incorporate no expectation terms. If agents' inflation expectations are above the rational benchmark, their (subjective) real interest rate decreases and the output gap increases accordingly. Thus, inflation actually increases via the Phillips curve, which confirms expectations. Consequently, if agents learn, they revise their expectations with respect to inflation and output gap upwards over time. Since expectations are not considered by fundamentals-based policy, there is no offsetting mechanism. As a result, a slight deviation from the rational expectations equilibrium induces a cumulative process away from it, which leads to *indeterminacy*. Therefore, Evans and Honkapohja (2003a,b) recommend the use of expectations-based reaction functions. However, Duffy and Xiao (2007) find that fundamentals-based reaction functions can in fact perform considerably well under learning if the central bank considers interest rate smoothing in their objective function. In light of the findings of Evans and Honkapohja (2003a,b) and since Gasteiger (2014), Gasteiger (2017) and Di Bartolomeo et al. (2016) use explicitly or implicitly expectations-based reaction function, they are employed in this work as well - at least for the sake of comparability.

5 Implementation of optimal monetary policy under heterogeneous expectations

An implementation strategy of optimal monetary policy specifies how a central bank should optimally set interest rates responding to (expectations of) model variables and exogenous shocks. The resulting equation is an interest rate reaction function. After having introduced the underlying model and the corresponding loss function, it is now possible to derive such a reaction function that minimizes the associated welfare losses.

5.1 The policy problem

Following Di Bartolomeo et al. (2016), the private sector equations can be summarized as

$$y_t = \alpha E_t y_{t+1} + (1-\alpha)\theta^2 y_{t-1} - \sigma(i_t - \alpha E_t \pi_{t+1} - (1-\alpha)\theta^2 \pi_{t-1}) \tag{24}$$

$$\pi_t = \alpha\beta E_t \pi_{t+1} + (1-\alpha)\beta\theta^2 \pi_{t-1} + \kappa y_t + e_t \tag{25}$$

$$c_t^\tau = E_t^\tau c_{t+1}^\tau - \sigma(i_t - E_t^\tau \pi_{t+1}) \text{ with } \tau \in \{R, B\} \tag{26}$$

where (24) and (25) are the New IS and Phillips curve (5) and (6) combined with heterogeneous expectations (7) and (8). (26) is the log-linearized consumption Euler equation for the respective expectation type $\tau \in \{R, B\}$. Plugging in (17) and (18) into (15) and using $y_t = \alpha c_t^R + (1-\alpha)c_t^B$, the policy problem under heterogeneous expectations takes the following form:[22]

$$\min \sum_{t=0}^{\infty} \beta^t \left[\frac{\sigma\eta + 1}{\sigma} y_t^2 + \frac{\alpha(y_t - c_t^R)^2}{(1-\alpha)\sigma} + \epsilon^2 \eta\delta \left\{ \pi_t^2 + \frac{\xi_p(1-\alpha)}{\alpha} \left[\pi_t - \beta\theta^2 \pi_{t-1} - \kappa y_t - \frac{\alpha\kappa(y_t - c_t^R)}{(1+\eta\sigma)(1-\alpha)} \right]^2 \right\} \right] \tag{27}$$

s.t.

$$y_t = \alpha E_t y_{t+1} + (1-\alpha)\theta^2 y_{t-1} - \sigma[i_t - \alpha E_t \pi_{t+1} - (1-\alpha)\theta^2 \pi_{t-1}] \tag{28}$$

$$\pi_t = \alpha\beta E_t \pi_{t+1} + (1-\alpha)\beta\theta^2 \pi_{t-1} + \kappa y_t + e_t \tag{29}$$

$$c_t^R = E_t c_{t+1}^R - \sigma(i_t - E_t \pi_{t+1}). \tag{30}$$

[22]Please consult the appendix of Di Bartolomeo et al. (2016) for further reading.

5.2 The reaction function

5.2.1 Commitment

The importance of commitment cannot be understated as all the relevant literature finds that there are gains from commitment under RE, homogeneous learning and heterogeneous expectations (Kydland and Prescott, 1977; Clarida et al., 1999; Evans and Honkapohja, 2006; Di Bartolomeo et al., 2016; Gasteiger, 2017). Especially within this model context, Gasteiger (2017) finds that the ability of the central bank to *directly* manipulate RE mitigates the amplification mechanism. Recall that the amplification mechanism works through the rational agents' PLM, which can be influenced by the central bank to increase macroeconomic performance if it commits to a certain policy.

When Di Bartolomeo et al. (2016) analyze fully model-consistent optimal monetary policy under commitment, it is implicitly assumed that there exists an implementation strategy that is credible and operational. However, they do not consider such. If there was no suitable reaction function one could question the importance of their findings. The policy problem under the model-consistent loss function can be found in the appendix 8.3.2. As outlined there, there is, almost certainly, no operational reaction function under commitment. This is because the derivation involves a solution of a second order difference equation of one of the Lagrange multipliers that exponentially depends on time. This implies, first, that Di Bartolomeo et al. (2016)'s findings for commitment might not be as important as there is no operational implementation and, second, that there is less support for the use of the model-consistent loss function. Hence, this strong result is more supportive for the use of the conventional loss function as in Gasteiger (2014, 2017).

5.2.2 Discretion

For comparison, it is useful to start with the reaction function under heterogeneous expectations and the conventional objective as in Gasteiger (2014, 2017). Minimizing the loss function (10) subject to the private sector equations under discretion and combining the resulting target rule with the New IS and Phillips curve gives:[23]

$$i_t = \gamma_1 y_{t-1} + \gamma_2 E_t y_{t+1} + \gamma_3 \pi_{t-1} + \gamma_4 E_t \pi_{t+1} + \gamma_5 e_t \tag{31}$$

[23]Derivations can be found in the appendix 8.2.

with

$$\gamma_1 = (1-\alpha)\left[\frac{\theta^2}{\sigma}\right] \tag{32}$$

$$\gamma_2 = \left[\alpha - (1-\alpha)\frac{\beta^2\theta^2}{\kappa^2+\omega}\right]\frac{1}{\sigma} \tag{33}$$

$$\gamma_3 = (1-\alpha)\left[\frac{\theta^2(\beta\kappa+\sigma(\kappa^2+\omega))}{\sigma(\kappa^2+\omega)}\right] \tag{34}$$

$$\gamma_4 = \alpha\left[1+\frac{\kappa\beta}{\sigma(\kappa^2+\omega)}\right] \tag{35}$$

$$\gamma_5 = \frac{\kappa}{\sigma(\kappa^2+\omega)}. \tag{36}$$

It can be seen easily that for $\alpha = 1$ equation (31) reduces to the reaction function under RE (12). Hence, the reaction function under RE is a special case of (31).

Further, choosing $\omega = \frac{\sigma\eta+1}{\epsilon^2\eta\delta\sigma}$ yields the reaction function under discretion and heterogeneous expectations derived from the microfounded conventional objective (19) with coefficients

$$\gamma_1 = (1-\alpha)\left[\frac{\theta^2}{\sigma}\right] \tag{37}$$

$$\gamma_2 = \alpha\frac{1}{\sigma} - (1-\alpha)\left[\frac{\beta^2\delta\epsilon^2\eta\theta}{1+\eta\sigma+\delta\epsilon^2\eta\kappa^2\sigma}\right] \tag{38}$$

$$\gamma_3 = (1-\alpha)\left[\frac{\theta^2(1+\eta(\sigma+\delta\epsilon^2\kappa(\beta+\kappa\sigma)))}{1+\eta\sigma+\delta\epsilon^2\eta\kappa^2\sigma}\right] \tag{39}$$

$$\gamma_4 = \alpha\left[1+\frac{\beta\delta\epsilon^2\eta\kappa}{1+\eta\sigma+\delta\epsilon^2\eta\kappa^2\sigma}\right] \tag{40}$$

$$\gamma_5 = \frac{\delta\epsilon^2\eta\kappa}{1+\eta\sigma+\delta\epsilon^2\eta\kappa^2\sigma}. \tag{41}$$

Now, for $\alpha = 1$, (31) reduces to the reaction function (23).

Recall that backward-looking agents are the source of an amplification mechanism in which rational agents expect a positive inflation deviation in $t+1$ through their PLM following a transitory cost-push shock. This positive expectation feeds back into current inflation, which is therefore higher relative to the fully rational case. Thus, it would be inefficient and inconsistent for the central bank to neglect that knowledge even under discretion. Consequently, (31) introduces backward-looking terms under discretion.

To derive the fully model-consistent reaction function, the policy problem (27)-(30) has to be solved under discretion. Thus, one has to satisfy *three* constraints, i.e. all private sector equations including the rational consumption Euler equation. However, to derive (31) one needs a single constraint only, i.e. the NK Phillips curve. This and the fact that the loss function has *eight* more terms compared to the conventional objective makes the optimization problem considerably more cumbersome. Following the usual steps, one can obtain the reaction function:[24]

$$
\begin{aligned}
i_t =& \Omega_1 y_{t-1} + \Omega_2 E_t y_{t+1} + \Omega_3 E_t y_{t+2} + \Omega_4 \pi_{t-1} + \Omega_5 E_t \pi_{t+1} + \Omega_6 E_t \pi_{t+2} \\
&+ \Omega_7 E_t c^R_{t+1} + \Omega_8 E_t c^R_{t+2} + \Omega_9 e_t.
\end{aligned} \tag{42}
$$

Equation (42) shows at least two remarkable results. First, it shows that the central bank should respond to rational agents' consumption reflecting that it should care about consumption inequality as stated by the loss function (15). Second, the consumption inequality dimension also leads to the fact that the central bank should react to the rational forecasts two periods ahead. Hence, (42) has four additional terms compared to (31).

To understand the introduction of the two-period-ahead ($t+2$) terms, consider the inflation expectations of rational and backward-looking agents in figure 1. Recall that a transitory cost-push shock hits the economy in $t = 1$.[25] As can be seen in Figure 2, inflation increases by almost one percent relative to the steady state, which is not anticipated by both agents, i.e. $E^R_{t-1}\pi_t = E^B_{t-1}\pi_t = 0$.[26] Because of increasing inflation expectations by the rational agents, the central bank depresses the output gap by raising the nominal interest accordingly.[27] Since all subsequent shock realizations are zero and rational agents know the true structural equations, they have *de facto* perfect foresight. Thus, rational agents' expectations in $t = 1$ about inflation in $t+1$ will be correct, i.e. $E^R_t \pi_{t+1} = \pi_{t+1}$. However, the (naive) expectations of bounded rational agents in t about inflation in $t+1$ will be zero. This is because backward-looking agents base their prediction on the previous period where all model variables were in steady state, i.e. $E^B_t \pi_{t+1} = \pi_{t-1} = 0$. In period $t+1$ backward-looking agents will expect $E^B_{t+1}\pi_{t+2} = \pi_t$, which means that they expect inflation to *increase* drastically in $t+2$. On the contrary, rational agents correctly expect inflation to *decrease* further as the output gap is still negative. Thus, the different expectations in $t+1$ about $t+2$ go in *different* directions. Now, recall that the central bank is committed to reduce consumption inequality. As consumption inequality is only driven by the different forecast paths, the central bank tries to stabilize them. Therefore, it should set interest

[24]Derivations and parameters can be found in the appendix 8.3.3. Note that the parameters in the appendix are written in terms of targeting rule parameters, *not* in terms of deep model parameters. The reason is that it would yield in part far too big equations to be useful to interpret.

[25]To be precise, the shock term to the Phillips curve takes on the value one in period $t = 1$ and is zero for all precedent and subsequent periods.

[26]Note that the simulation results are taken from the Söderlind (1999) routine provided by Di Bartolomeo et al. (2016). Thus, it does not explicitly involve the reaction function (42) out of reasons discussed in section 5.3.

[27]Note that the output gap drops by nine percent! This rather extreme drop is due to the combination of this particular parameterization, the amplification mechanism described above and the relatively low weights on output gap variations in the loss and reaction function.

rates so as to align the two expectation types. Since they go in different directions in $t+1$, $t+2$ terms have to be present in the reaction function. Note that expectations in $t+2$ about $t+3$ are *revised downwards by both* rational and backward-looking agents. Consequently, no further terms have to be introduced to align expectations. Also, note that the transitory divergence of the different expectations are inherent to the expectation mechanics and cannot be avoided. Hence, the central bank aims at minimizing the difference by setting nominal interest rates accordingly.

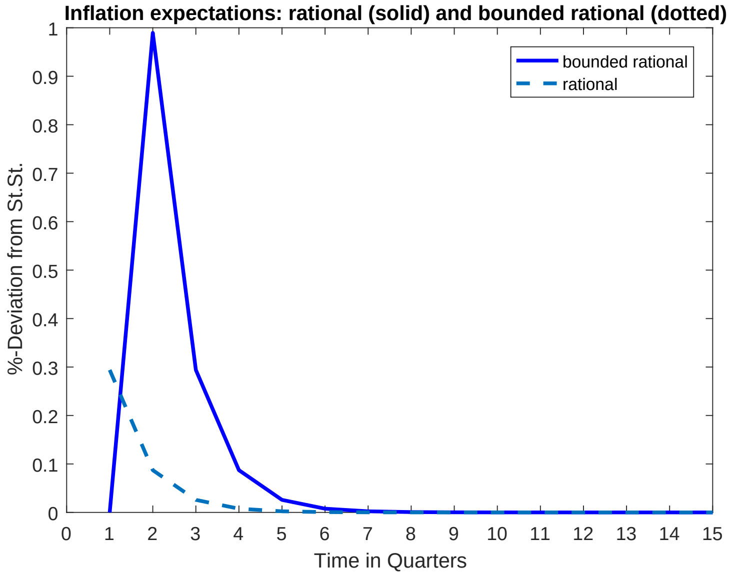

Figure 1: Inflation expectations of the two agent types

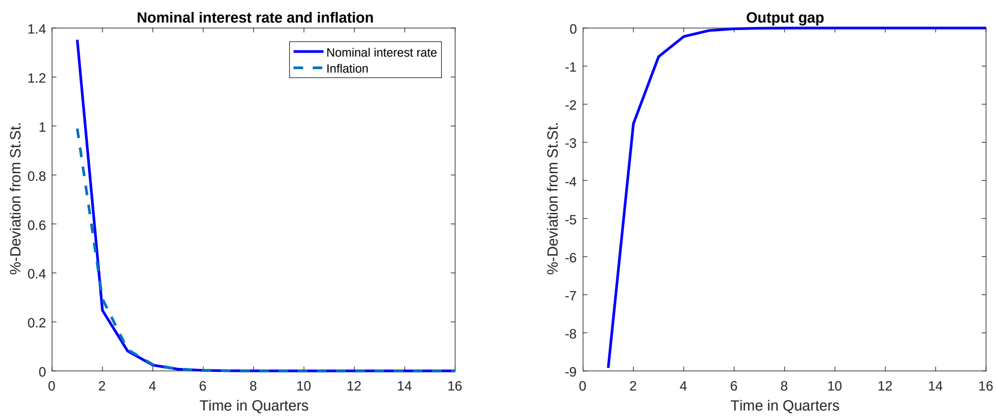

Figure 2: Impulse responses of the nominal interest rate, inflation and the output gap following a cost-push shock

To get an idea about the importance of the different terms in the reaction function and how they depend on heterogeneous expectations, one can compute the coefficients under different values for θ and α. Table 2 shows the parameters values under baseline calibration with $\theta = 1$, $\theta = 0.8$ and $\theta = 1.2$.[28]

[28]All values in this and the following tables in this section are rounded to the third decimal place.

Table 2: Comparison of reaction coefficient values w.r.t. θ

	Adaptive ($\theta = 0.8$)	Naive ($\theta = 1$)	Explorative ($\theta = 1.2$)
y_{t-1}	0.031	0.048	0.069
$E_t y_{t+1}$	0.109	0.108	0.106
$E_t y_{t+2}$	-0.003	-0.007	-0.015
π_{t-1}	0.370	0.609	0.954
$E_t \pi_{t+1}$	1.208	1.199	1.220
$E_t \pi_{t+2}$	0.005	0.012	0.024
$E_t c^R_{t+1}$	-0.020	-0.030	-0.042
$E_t c^R_{t+2}$	0.003	0.007	0.014
e_t	0.935	1.040	1.221

First, consider naive expectations, $\theta = 1$ (second column, baseline). Note that all terms introduced by the consumption inequality dimension ($t + 2$ and consumption terms) have fairly small values. Further, the central bank's reaction with respect to inflation deviations are positive throughout. In particular, it should respond to rational inflation expectations one period ahead more than one-to-one. However, the reaction coefficients for the output gap are relatively small in general due to the low weight assigned to the output gap variations in the loss function.[29] Due to the persistence introduced by the forecasting behavior of bounded-rational agents, the central bank should also respond to $t - 1$ realizations of inflation and the output gap as before. These coefficients have the same sign as for the rational forecasts for $t + 1$ and are numerically roughly the half of the respective rational forecast coefficient, which can be explained by the relative fractions of the expectation types.

Second, consider adaptive expectations, $\theta = 0.8$. Since $\theta < 1$, this represents is an inherently *stabilizing force*. This is because backward-looking agents expect the systems' reaction to a transitory cost-push shock to be less strong in the future. It is therefore intuitive that the central bank has to act less aggressive on realizations of the lagged output gap, lagged inflation, the $t + 2$ and consumption terms. This is intuitive as the first two terms are due to the backward-looking agents directly and the latter two because of the expectation stabilization aim of the central bank. In contrast, the very same reaction coefficients get numerically bigger if backward-looking agents' expectations are trend-setting, i.e. $\theta > 1$. This introduces an *additional amplifying force*. Thus, the amplification of inflation today is not only due to the mere existence of the backward-looking terms in the rational agents' PLM, but also because these terms are *further* amplified by $\theta > 1$. Hence, the central bank has to act more hawkish. Effects of these parameter constellations on social welfare can be found section 6.

[29]For a rewritten form of loss function, consider the appendix 8.3.1. If calculated under baseline calibration, the coefficient of the output gap variation, Γ_1, is roughly 1, which is very small compared to the coefficient assigned to inflation variation, which is roughly 207.

Table 3: Comparison of reaction coefficient values w.r.t. α

	$\alpha \to 1$ (RE)	$\alpha = 0.5$	$\alpha = 0.7$	$\alpha = 0.9$
y_{t-1}	0	0.080	0.048	0.016
$E_t y_{t+1}$	0.160	0.045	0.108	0.172
$E_t y_{t+2}$	0	-0.009	-0.007	-0.003
π_{t-1}	0	1.071	0.609	0.191
$E_t \pi_{t+1}$	1.851	0.697	1.199	1.648
$E_t \pi_{t+2}$	0	0.033	0.012	0.001
$E_t c^R_{t+1}$	0	-0.021	-0.030	-0.040
$E_t c^R_{t+2}$	0	0.008	0.007	0.003
e_t	0.859	1.154	1.040	0.921

Now, consider the effect of variations in the share of rational agents, α, on the reaction coefficients in table 3. Starting from the baseline calibration, $\alpha = 0.7$, it can be seen that the weights on the lagged output gap and lagged inflation, the $t+2$ and consumption terms become smaller if α increases. This is intuitive, because the fraction of backward-looking agents becomes smaller. Hence, the effects of the amplification mechanism caused by the backward-looking behavior described above and the transitory divergent expectations that have to be stabilized become less influential. On the other hand, by the same logic the respective coefficients become considerably bigger if α decreases.

It can be shown that for the rational limit, $\alpha \to 1$, equation (42) takes the following form:

$$i_t^{\alpha\to 1} = \Omega_2^{\alpha\to 1} E_t y_{t+1} + \Omega_5^{\alpha\to 1} E_t \pi_{t+1} + \Omega_7^{\alpha\to 1} E_t c^R_{t+1} + \Omega_9^{\alpha\to 1} e_t \tag{43}$$

where $\Omega_i^{\alpha\to 1}$ stands for the respective parameter i in the case of the rational limit.[30] Note that all terms associated with heterogeneous expectations and, in particular, the consumption inequality dimension vanish. Note also that in the rational limit $y_t = c_t^R$ has to hold. Therefore, (43) can be rewritten as

$$i_t^{\alpha\to 1} = \Omega_5^{\alpha\to 1} E_t \pi_{t+1} + (\Omega_2^{\alpha\to 1} + \Omega_7^{\alpha\to 1}) E_t y_{t+1} + \Omega_9^{\alpha\to 1} e_t \tag{44}$$

with

$$\Omega_5^{\alpha\to 1} = 1 + \frac{\beta\delta\epsilon^2\eta\kappa}{1 + \eta\sigma + \delta\epsilon^2\eta\kappa^2\sigma} \tag{45}$$

$$\Omega_2^{\alpha\to 1} + \Omega_7^{\alpha\to 1} = \frac{1}{\sigma} \tag{46}$$

$$\Omega_9^{\alpha\to 1} = \frac{\delta\epsilon^2\eta\kappa}{1 + \eta\sigma + \delta\epsilon^2\eta\kappa^2\sigma}, \tag{47}$$

[30] Limits were calculated with Wolfram Mathematica.

which is naturally equal to the interest rate rule under rational expectations (23). The coefficients imply a positive reaction of the nominal interest rate with respect to the expected output gap, the cost-push shock and expected inflation. In particular, the central bank should react more than one-to-one with respect to expected inflation. Also, the coefficients do not depend on α and θ anymore. The values of the reaction coefficients under baseline calibration can be found in table 3.

5.3 Indeterminacy due to the reaction function?

A system of equations is said to be indeterminate if it has more than one solution. In the special case of a linear system indeterminacy means that there are infinitely many solutions. This corresponds to infinitely many equilibria in this particular case. In turn, if such a model is said to be determinate, it has a unique rational expectations equilibrium (see for instance Benhabib and Farmer (1999)).

The fully model-consistent reaction function derived in section 5.2 seems to make sense. It was shown that it encompasses the correct reaction function under rationality as a special case. Further, the appearance of the $t + 2$ and consumption terms have been explained in detail. At the same time, the movements of the reaction coefficients with respect to the forecast parameter of the backward-looking agents, θ, and the share of rational agents, α, follow an intuitive pattern. However, simulating the economy including the reaction function (42) explicitly using Dynare yields *indeterminacy*. Thus, there exist infinitely many equilibria, which is frequently associated with high volatility in macroeconomic aggregates (Chari et al., 1998; Clarida et al., 2000). To be precise, the New IS and the Phillips curve, the consumption Euler equation of the rational agents and monetary policy specified by (42) were used. There are in principle two possible reasons for this result. First, the reaction function (42) has bad properties as it induces the system to be indeterminate. Second, something else is wrong. The latter opens the door for many potential errors.

To minimize the possibility of computational errors, every step of the derivations was calculated additionally by Wolfram Mathematica. Further, it has been checked whether there might be errors in the derivation of the reaction function from the very start. Therefore, the first order conditions (FOCs) derived from the policy problem both under discretion and commitment to describe the optimal policy were used to simulate the economy.[31] The simulation with the FOCs under commitment reproduced the impulse responses of the output gap, inflation and consumption of Di Bartolomeo et al. (2016).[32] Thus, it can be assumed that the FOCs under commitment are correct. Since the central bank does not aim at influencing rational expectations under discre-

[31] See Appendix 8.3.3 and 8.3.2.

[32] Note, that the indeterminacy problem does not arise in this case. However, the impulse response of the policy rate is not equal to the impulse response presented in Di Bartolomeo et al. (2016).

tion directly, the respective terms can be dropped in the FOCs. Hence, the FOCs under commitment "include" the FOCs under discretion. It can therefore be assumed that the FOCs under discretion are correct as well. However, simulating the same economy with the FOCs under discretion, where the the only difference is that they do not contain the respective terms for manipulating rational expectations, yields *indeterminacy*. As a result, a computational error seems less likely.

Besides the possibility that the reaction function has bad properties, there might be another one. The derivation of the New IS curve involves several assumptions as discussed in section 3.1. It can be shown that if one simulates the model as in Gasteiger (2014, 2017), it yields indeterminacy as soon as one adds the consumption Euler equation of the rational agents. Thus, it is a guess that, when simulating the *aggregate* equations together with *disaggregate* consumption, the assumptions necessary for aggregation and the respective budget constraints become important somehow. Since the reaction function (42) requires the consumption Euler equation to be used, this constitutes a serious problem. However, because of time restrictions this problem cannot be solved within this master thesis. This issue is left for future research.

The bottom line is that, unfortunately, within this work it cannot finally be clarified whether the reaction function (42) is actually a suitable implementation strategy or not. However, since the reaction function under discretion does not seem to be the source of indeterminacy it is likely that an implementation of optimal monetary policy under the model-consistent objective is found.

6 Welfare implications

The relative performance of the policies derived from the microfounded conventional objective (31) and from the model-consistent loss (42) can be measured by the *true losses* they generate if a cost-push shock hits the economy. True losses are measured by the fully model-consistent objective that recognizes the heterogeneity in expectations. Simulating the model under the respective monetary policy "regime" yields the paths of inflation, the output gap and consumption of rational agents. To compute the losses, the respective long-run (auto/co-)variances can be derived from the solution provided by the Söderlind (1999) algorithm and then can be plugged into the loss function (15).

As explained in section 4.3, the conventional objective potentially underestimates the true welfare losses induced by the respective policy. The reason is that the conventional objective represents only the losses of the rational agents, i.e. only a share α of the population. Since the backward-looking agents are boundedly rational and therefore make systematic forecast errors, one can expect their losses to be relatively higher. Consequently, a weighted average of the losses of the two types, i.e. the model-consistent loss, has to be higher as well.

Both the relative performance of the reaction functions and the possibility that the conventional objective might underestimate the true welfare losses will be investigated in this section. Since an operational reaction function under commitment is, almost certainly, not available, the welfare implications in this section are restricted to the discretionary case.

6.1 The state-space model and long-run (auto/co-)variances

The model described by (5)-(8) and the shock processes can be put in state-space form with appropriate matrices **A** and **B** as

$$X_{t+1} = \mathbf{A}X_t + \mathbf{B}i_t + \begin{bmatrix} \upsilon_{t+1} \\ 0_{n_2 \times 1} \end{bmatrix} \tag{48}$$

with $X_t = [e_t \; d_t \; \pi_{t-1} \; y_{t-1} \; c_t^R \; \pi_t \; y_t]'$ and $d_t = 0 \forall t$ being the shock to the New IS curve.[33] To apply the Söderlind (1999) algorithm, one has to distinguish $n_1 = 4$ predetermined variables and shocks $x_{1,t} = [e_t \; d_t \; \pi_{t-1} \; y_{t-1}]'$ and $n_2 = 3$ nonpredetermined variables $x_{2,t} = [c_t^R \; \pi_t \; y_t]'$. Then, with υ_{t+1} being a n_1x1 vector of innovations to the $x_{1,t}$ process, (48) can be written as

$$\begin{bmatrix} x_{1,t+1} \\ E_t x_{2,t+1} \end{bmatrix} = \mathbf{A} \begin{bmatrix} x_{1,t} \\ x_{2,t} \end{bmatrix} + \mathbf{B}i_t + \begin{bmatrix} \upsilon_{t+1} \\ 0_{n_2 \times 1} \end{bmatrix}. \tag{49}$$

[33] A full description of the state-space form, variance-covariances and auto-covariances and the policy problem can be found in the appendix 8.4.

Rewriting the loss function with weighting matrix $\mathbf{Q}$ yields

$$L_t = \beta^t X_t' \mathbf{Q} X_t. \tag{50}$$

The matrix $\mathbf{Q}$ specifies the particular form of the loss function. Thus, by choosing $\mathbf{Q}$ accordingly it is possible to compute the long-run variances resulting from the respective policy regime. The solution algorithm developed by Söderlind (1999) then provides optimal paths for i_t under discretion and commitment based on (50).[34] Using the algorithms solution one can obtain equations

$$x_{1,t+1} = \mathbf{G}x_{1,t} + \upsilon_{t+1} \tag{51}$$

$$x_{2,t} = \mathbf{C}x_{1,t} \tag{52}$$

$$i_t = -\mathbf{F}_{1,t}x_{1,t} \tag{53}$$

under discretion with appropriate solution matrices $\mathbf{G}$, $\mathbf{C}$ and $\mathbf{F}_{1,t}$ (Söderlind, 1999). The vectorized variance-covariance matrix Λ_1 for $x_{1,t}$ is then given by

$$vec(\Lambda_1) = (\mathbf{I} - (\mathbf{G} \otimes \mathbf{G}))^{-1} vec(\Sigma) \tag{54}$$

with $\Sigma = Var(\upsilon_{t+1})$, from which one can retrieve the variance-covariance matrix Λ_1. The variance- covariance matrix for $x_{2,t}$ is given by

$$\Lambda_2 = \mathbf{C}\Lambda_1\mathbf{C'} \tag{55}$$

with $\Lambda_2 = Var(x_{2,t})$. Likewise the auto-covariances can be computed as

$$\Lambda_{+1,1} = \mathbf{G}\Lambda_1 \tag{56}$$

$$\Lambda_{1,2} = \Lambda_1\mathbf{C'} \tag{57}$$

with $\Lambda_{+1,1} = Cov(x_{t+1}, x_{1,t})$ and $\Lambda_{1,2} = Cov(x_{1,t}, x_{2,t})$.[35]

6.2 Welfare losses and implications under discretion

Welfare losses can be computed by plugging the respective entries from Λ_1, Λ_2, $\Lambda_{+1,1}$ and $\Lambda_{1,2}$ derived under the different policies into the model-consistent loss, L_t, and the micro-founded conventional objective, $L_t^{\alpha=1}$. The resulting losses under baseline

[34]The algorithm computes the paths of the model variables by solving the policy problems directly. There is no reaction function involved explicitly. Hence, the optimization problems in the commitment case could in principle be solved and the respective impulse responses be computed, even if there is no operational reaction function under commitment.

[35]Please consult the appendix 8.4 for details.

calibration can be found in table 4.[36] The columns show the respective losses, while the rows indicate from which loss function the respective policy is derived. Note that the losses are only the instantaneous ones.[37]

Table 4: Instantaneous losses under the respective loss function

	loss	
policy	L	$L^{\alpha=1}$
L_t	357.73	248.54
$L_t^{\alpha=1}$	357.47	247.87

Gasteiger (2017) states that a potential drawback of the conventional objective is that it might understate true welfare losses. In fact, the computations as shown in table 4 corroborate this. To see that, consider the policy derived from the conventional objective, $L_t^{\alpha=1}$ (second row). This policy corresponds to reaction function (31). Obviously, the true loss L is substantially higher than the loss $L^{\alpha=1}$ under the policy derived from $L_t^{\alpha=1}$. Hence, this result clearly confirms the claim of Gasteiger (2017). Recall that backward-looking agents will suffer from larger losses than rational agents because their biased beliefs result in non-optimal decisions. Thus, the true loss, L, must be considerably larger. This also reflects the higher influence of the trade off between the output gap and inflation on the true loss, L, through the price dispersion channel as described in section 4.2. The same holds true if the policy is derived from the fully model-consistent loss function, L_t. Therefore, a central bank that employs the conventional objective might significantly underestimate the welfare consequences in a world with heterogeneous expectations. Despite the numerous practical advantages that come with the conventional objective, this puts a warning sign on its usage.

Now consider the first column of table 4, which shows the true losses L under the different polices. The true loss of the fully model-consistent policy implied by L_t is indeed slightly *higher* than the true loss of the policy derived from the microfounded conventional objective, $L_t^{\alpha=1}$.[38] This is a striking result. One might be tempted to conclude that the policy derived from L_t is not minimizing L_t. However, this would be mistaken: L_t implies that the central bank has certain preferences represented by the coefficients in the objective. Since the central bank is committed to maximize social welfare, its preferences are functions of deep model parameters. Thus, the coefficients cannot be chosen arbitrarily. Nevertheless, implementing the policy under $L_t^{\alpha=1}$ would yield slightly lower welfare costs. Therefore, a central bank that hires a central banker

[36]Note that the losses computed here are numerically different from Di Bartolomeo et al. (2016). However, they are in the same dimension.

[37]The cumulative losses are given by $\frac{1}{1-\beta}L$ with L being the instantaneous loss, i.e. L_t based on the long-run variances. However, these would just be the values in table 4 multiplied by $\frac{1}{1-\beta} = 100$.

[38]Of course, it can be questioned if this difference would be empirically significant. However, this result is further substantiated by the computations under different parameterizations below.

with preferences represented by $L_t^{\alpha=1}$ would yield welfare gains compared to the policy implied by L_t.

An explanation for this result can be given by considering table 5. The conventional policy menu is typically given by the trade off between inflation and the output gap (Gasteiger, 2017). The policy derived from the true loss, L_t, yields slightly less volatility in inflation compared to the policy derived from $L_t^{\alpha=1}$ under baseline calibration. Yet, the variance of the output gap is considerably higher in case the policy is derived from L_t. Thus, hiring a central banker that is willing to let inflation be slightly more volatile can yield a substantial reduction in the variance of the output gap (second column). It is important to note that the result regarding the output gap holds for any value of θ and α considered.[39] As the contribution of inflation and output gap variations to the whole loss L is relatively high (96 percent[40]), this is a possible explanation.

Table 5: Long-run variances under the respective loss function

	policy	
variance	L_t	$L_t^{\alpha=1}$
$Var(\pi)$	1.16	1.18
$Var(y)$	95.70	89.71

Further, this finding is broadly in line with Berardi (2009). Berardi (2009) also introduces two different types of agents into an otherwise standard NK model. However, his model is different from Branch and McGough (2009) in the way he introduces bounded rationality. The PLM of bounded rational agents only depends on the vector of exogenous shocks. Additionally, the bounded rational agents update their coefficient matrix in the PLM via recursive least squares. The rational agents use the correct specified model and thereby recognize the inertia introduced by the commitment regime of the central bank. Berardi (2009) finds that the central bank should base its policy on the rational expectations only. As a result, inflation increases slightly more on impact following a cost-push shock, but with a significantly lower reduction in the output gap necessary. This leads to an overall improvement of the policy trade off relative to a policy that is based on average expectations. Berardi (2009, p.98) concludes:

> "Those expectations, in fact, though affecting the economy, do not give a precise picture of the future states of the system, and call for a policy that

[39] However, there are exceptions for inflation. Consider table 10 in the appendix 8.5. For $\theta = 1.2$, $\alpha = 0.3$ and $\alpha = 0.5$, the variance in inflation is even lower for the policy implied by the conventional objective.

[40] Consider the rewritten loss function in the appendix 8.3.1 based on the long-run (auto/co-)variances. This number was computed by comparing the contribution to the whole loss of the first two terms regarding the variations of inflation and of the output gap relative to the remaining terms under baseline calibration. In fact, the value of the sum of the first two terms are fairly close to the value of overall welfare across all considered parameter values.

would overreact to an external shock."

Although Berardi (2009)'s work has some differences to the model here, it might give a direction for why the economy is better off if the central bank neglects the knowledge about the backward-looking agents in their objective function.

Further, to obtain some insights about the influence of heterogeneous expectations on welfare, true losses are computed for the respective policy with respect to variations in θ and α.

Table 6: True instantaneous losses, L, with variations in θ

	forecasting coefficient		
policy	$\theta = 0.8$	$\theta = 1$	$\theta = 1.2$
L_t	264.23	357.73	571.31
$L_t^{\alpha=1}$	264.49	357.47	553.78

In the second column of table 6 one can find the true losses under baseline calibration, i.e. for naive expectations. The first and third column show a change in θ for the adaptive and explorative case, respectively. As one can see, the welfare losses under adaptive expectations, $\theta = 0.8$, are considerably smaller and for explorative expectations, $\theta = 1.2$, considerably higher compared to the baseline case. This is an intuitive result since for $\theta < 1$ the amplification mechanism is mitigated and for $\theta > 1$ it is further amplified. Recall that rational agents expect $E_t\pi_{t+1} > 0$ following a transitory cost-push shock since they recognize the persistence induced by the backward-looking agents. For $\theta < 1$ rational agents have smaller inflation expectations on impact, because backward-looking agents will have lower inflation expectations compared to the naive case. Therefore, the amplification mechanism is less strong and the welfare costs are lower. In contrast, explorative or trend-setting expectations, $\theta > 1$, imply inflation expectations of backward-looking agents to be even higher than their observed lagged realization of inflation. Thus, rational agents expect by their PLM that inflation will be even higher today. Hence, the amplification mechanism is amplified further and the associated welfare costs increase. The increase in welfare costs from $\theta = 1$ to $\theta = 1.2$ are significantly stronger than the increase from $\theta = 0.8$ to $\theta = 1$. Thus, the increase in the welfare loss is non-linear in θ. These results carry over to the case where the losses are calculated by $L_t^{\alpha=1}$. Note, that this result holds even though the optimal policy is employed. Hence, the amplification mechanism cannot be fully muted by the monetary policy authority as long as a fraction of backward-looking agents are present. Further, for explorative and naive expectations, the true losses implied by the conventional objective are lower compared to the model-consistent loss as well. However, in the case of adaptive expectations, $\theta = 0.8$, the model-consistent loss yields slightly lower welfare costs. Note that the larger θ gets, the better performs the conventional objective compared to the model-consistent one.

Similar results can be obtained when variations in α are explored. Consider table 7 where the third column shows the baseline case. In the remaining columns one can find the variations in α. Apparently, this relationship is non-linear as well. The intuition of the results is the same as for variations in θ, because the fraction of the backward-looking agents decreases with α, which causes the amplification mechanism to be less influential (and vice versa). Hence, for larger fractions of bounded rational agents, the welfare losses increase substantially. Also, it seems that the conventional objective performs relatively better in terms of true losses the higher the share of bounded rational agents is.

The fact that the model-consistent loss function performs relatively poorer the more heterogeneous expectations have influence on the macroeconomy points to the conclusion of Berardi (2009). Hence, if bounded-rational agents are present and have a certain degree of influence on economic performance, it seems to be preferable to base the objective function on rational agents alone.

Table 7: True instantaneous losses, L, with variations in α

	share of rational agents			
policy	$\alpha = 0.3$	$\alpha = 0.5$	$\alpha = 0.7$	$\alpha = 0.9$
L_t	865.43	579.17	357.73	197.85
$L_t^{\alpha=1}$	855.23	570.38	357.47	198.54

Note that for almost all considered values of θ and α, the welfare losses under the conventional objective are slightly lower.

The bottom line is that, surprisingly, the microfounded conventional objective performs slightly better in terms of true overall welfare compared to the model-consistent one if bounded rational agents have a certain degree of influence. At least it does not seem that the microfounded conventional objective yields significant welfare losses in any way under reasonable parameter values.

6.3 Consumption inequality

The novelty introduced by the model-consistent loss function is the consumption inequality dimension. This section provides a brief discussion on this topic.

Beqiraj et al. (2017) investigate the consumption inequality component in the Massaro (2013) framework. As outlined in the literature review, the authors derive a model-consistent welfare function and compare it, among others, to a micro-founded inflation targeting objective, which depends on inflation and output gap only. Thus, their inflation-targeting objective is similar to the micro-founded conventional objective, $L_t^{\alpha=1}$, here. As a result, consumption inequality is slightly lower for the policy implied by the model-consistent objective since it explicitly accounts for the cross-sectional variance in consumption.

Since the Söderlind (1999) algorithm delivers the path for c_t^R, consumption deviations for the backward-looking agents, c_t^B, can be calculated via the market-clearing condition as

$$c_t^B = \frac{1}{1-\alpha} y_t - \frac{\alpha}{1-\alpha} c_t^R. \tag{58}$$

Consumption inequality in period t is then defined by the cross-sectional variance in consumption

$$var_i(c_t(i)) = \alpha(1-\alpha)(c_t^R - c_t^B)^2 \tag{59}$$

as in the model-consistent loss function (15). Inserting (58) into (59) and multiplying out yields

$$var_i(c_t(i)) = \frac{\alpha}{1-\alpha} \left[(c_t^R)^2 - 2c_t^R y_t + (y_t)^2 \right]. \tag{60}$$

Now, the respective long-run (co-)variances from section 6.1 can be used to the compute consumption inequality, $var_i(c(i))$.

Note that both consumption terms are in percent deviations from steady state. Recall that in steady state expectations of different household types are the same. Hence, consumption inequality between households is zero since everyone consumes the same amount.[41] Then, if a cost-push shock hits the economy and drives up inflation, the consumption terms of different types start to evolve differently as follows: The central bank will raise the nominal interest rate to generate a recession. However, rational agents' inflation expectations will increase due the perceived persistence, while expectations of backward-looking agents will remain zero. Thus, the (subjective) real rate of backward-looking agents increases substantially more compared to the (subjective) real rate of rational agents. Consequently, consumption will drop more in the case of backward-looking agents (figure 3), which creates inequality between household types. In case the percent deviations of consumption from steady state would be equal across types, the consumption level of each single household would evolve equally as well. Hence, no inequality would emerge and (59) would be zero (and unequal zero otherwise).

To understand how consumption inequality evolves with respect to heterogeneous expectations, consumption inequality as defined by (60) is computed for different values of θ and α. In tables 8 and 9 one can find the consumption inequality values based

[41] Of course, the consumption level of the aggregate household sectors are different since they have an unequal share of the total population. However, by virtue of perfect consumption insurance every single household (and the representative agents) must have the same consumption level across types as long as the economy is in the steady state.

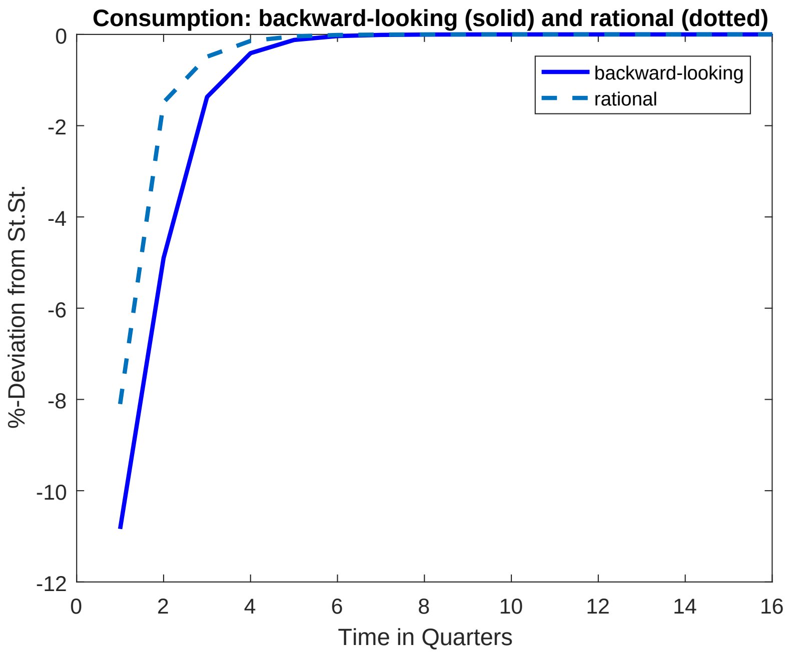

Figure 3: Consumption of the two agent types

on the long-run (co-)variances for the different parameterization. An intuitive picture

Table 8: $var_i(c(i))$ with variations in θ

	forecasting coefficient		
policy	$\theta = 0.8$	$\theta = 1$	$\theta = 1.2$
L_t	3.34	10.23	68.53
$L_t^{\alpha=1}$	3.47	10.40	76.93

emerges: first, without distinguishing between the two policies, consumption inequality decreases (increases substantially) for adaptive (trend-setting) expectations relative to the naive case. Hence, consumption inequality hinges on the forecasting behavior of backward-looking agents, i.e. if their behavior incorporates a stabilizing ($\theta < 1$) or a further amplifying ($\theta > 1$) element. Also, consumption inequality increases substantially with middle-sized ($\alpha = 0.5$) or lower shares ($\alpha = 0.3$) of rational agents. Hence, inequality positively depends on the degree of influence of backward-looking agents the macroeconomy.

Second, the policy implied by the model-consistent objective yields the lowest consumption inequality across all considered parameter values as in Beqiraj et al. (2017). This result is not surprising as the central bank is additionally committed to minimize consumption inequality.

It should be noted that inequality in this context is a short-term phenomenon as the model is build to describe business cycle fluctuations. Hence, the model is not suitable for investigating longer-term inequality.

Table 9: $var_i(c(i))$ with variations in α

policy	share of rational agents			
	$\alpha = 0.3$	$\alpha = 0.5$	$\alpha = 0.7$	$\alpha = 0.9$
L_t	68.01	32.02	10.23	3.36
$L_t^{\alpha=1}$	72.95	33.04	10.40	3.60

The above analysis shows that the conventional objective performs slightly better in terms of overall welfare whereas the model-consistent loss function yields lower consumption inequality across the considered parameter values. Hence, hiring a central banker with preferences represented by the microfounded conventional objective would yield slight welfare improvements compared to the model-consistent loss function - but at the cost of rising inequality.

7 Conclusion and outlook

The recent literature has shown that incorporating heterogeneous expectations into macroeconomic models is important for both the aggregate dynamics and for monetary policy considerations (Branch and McGough, 2009; Gasteiger, 2014; Di Bartolomeo et al., 2016; Gasteiger, 2017). It has also shown that the result that commitment yields welfare gains over discretion under RE carries over to the homogeneous bounded rational and the heterogeneous expectations case (Evans and Honkapohja, 2006; Di Bartolomeo et al., 2016; Gasteiger, 2017). In contrast to the recent literature that employs the conventional objective to investigate optimal monetary policy, Di Bartolomeo et al. (2016) derive a fully model-consistent loss function. This welfare criterion recognizes heterogeneous expectations explicitly and implies a third dimension of monetary policy, namely consumption inequality. Yet, Gasteiger (2017) argues that the model-consistent loss function entails some practical disadvantages, as for instance its complexity, and consequently favors the conventional objective. Although Di Bartolomeo et al. (2016) investigate optimal monetary policy under the model-consistent objective, they do not consider the implementation of it. Hence, they do not derive interest rate reaction functions. The primary objective of this thesis was to derive such an implementation.

It was shown that there, almost certainly, exists no operational reaction function under commitment. This impressively extends Gasteiger (2017)'s claim that the model-consistent loss function entails some practical problems. Further, this result cannot be understated as the literature throughout finds that commitment is preferable over discretion. Thus, if the model-consistent objective is the very reason for why there is no operational reaction function under commitment, its advantage, i.e. the ability to derive *fully* optimal monetary policy, reduces substantially.

However, an implementation strategy of discretionary optimal monetary policy seems to exist. Due to the central bank's additional aim to minimize consumption inequality, disaggregate consumption and two-period-ahead ($t+2$) terms appear in the corresponding reaction function. In particular, the $t+2$ terms are due to the fact that expectations of the different agent types diverge transitorily, which have to be stabilized by virtue of the consumption inequality dimension. The reaction function thereby meets several desirable properties: first, it encompasses the reaction function under full rationality as a special case. Second, its coefficients move with the share of rational agents and the forecast parameter of backward-looking agents corresponding to the intuition. For instance, the reaction coefficients for lagged inflation and the lagged output gap increase with the share of bounded rational agents. This is intuitive as the central bank tries to exert influence on the backward-looking expectations of bounded rational agents even under discretion. However, by now the simulation of the system including the reaction function yields *indeterminacy*. Although, it was argued that it is most likely not due to reaction function but due to the *disaggregate* consumption Euler equation of the rational agents, this still states a serious problem.

Subsequently, the relative performance of optimal discretionary monetary policy derived from the microfounded conventional and the model-consistent objective was assessed by comparing the *true* losses they generate. It was shown that the policy derived from the conventional objective yields slightly less true welfare losses compared to the policy implied by the model-consistent objective in some cases. In these cases bounded rational agents have a certain degree of influence on the economy. Hence, a central bank that hires a central banker with preferences such that the conventional objective is implied would then attain higher overall welfare. However, it was shown that the conventional objective significantly underestimates true welfare losses. Thus, neglecting heterogeneous expectations in assessing the welfare consequences of some policy might lead to substantially biased judgments. Additionally, it was shown that consumption inequality is naturally lower under the model-consistent loss function across all considered parameter values.

Consequently, these advantages and disadvantages have to be kept in mind when the modeler decides on which assumptions to make on central bank preferences. As both objective functions have their own (dis-)advantages there is no clear-cut solution. However, the inability to derive a meaningful and operational reaction function under commitment from the model-consistent loss function and the claims made by Gasteiger (2017) seem to weigh particularly strong. Therefore, the use of the conventional objective as in Gasteiger (2014, 2017) seems more reasonable. From this, one can then deduce whether or not it would be advisable to recommend central banks to change their mandates accordingly. However, this is a question of central bank *practice* and is therefore not within the scope of this thesis.

As already indicated, the work conducted in this thesis is not the end of the road. Future research should be dedicated to find a solution for the indeterminacy problem. Once such a solution is found, the model under discretion including the reaction function can further be investigated. In addition, one could dig deeper into the question why ignoring heterogeneity in the loss function might be actually beneficial in terms of true welfare losses. Berardi (2009) already gives an indication: it might be preferable to ignore expectations that give an imprecise picture of the state of the economy since monetary policy would overreact to an exogenous shock. Further, future research can incorporate potential mismeasurement of the share of rational agents by the central bank as in Gasteiger (2017) into this particular context. In terms of the underlying model, the respective shares of the different agent types can be endogenized as in Branch and McGough (2010) and the implications for monetary policy be studied. However, since the central bank is committed to minimize consumption inequality, it will be considerably harder to stabilize expectations as the central bank has to internalize also the switching behavior of agents.

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8 Appendix

8.1 The rational case

8.1.1 The conventional objective

Defining $f_t = E_t y_{t+1}$ and $g_t = E_t \pi_{t+1}$, the policy problem under discretion and rationality takes the following form:

$$\begin{aligned}\mathcal{L} = &\frac{1}{2}[\pi_t^2 + \omega y_t^2] \\ &+ \lambda_t[\pi_t - \beta g_t - \kappa y_t - e_t].\end{aligned} \tag{61}$$

Note that the New IS Curve is not binding in this scenario. Once the optimal paths for the output gap and inflation are known, the IS Curve delivers the interest rate. Computing the first order conditions, setting then equal to zero and eliminating the Lagrange multiplier yields the target rule

$$\pi_t = -\frac{\omega}{\kappa} y_t. \tag{62}$$

Setting the target rule equal to the Phillips-Curve and solving for y_t gives

$$y_t = -\frac{\kappa}{\omega + \kappa^2}[\beta E_t \pi_{t+1} + e_t]. \tag{63}$$

Again, setting (63) equal to the New IS Curve and solving for i_t yields the reaction function under rationality (12).

8.1.2 The microfounded conventional objective

The policy problem under discretion and rationality for the loss function following Di Bartolomeo et al. (2016) takes the following form:

$$\begin{aligned}\mathcal{L} = &\frac{1}{2}\left[\frac{\sigma\eta + 1}{\sigma} y_t^2 + \epsilon^2 \eta \delta \pi_t^2\right] \\ &+ \lambda_t[\pi_t - \beta g_t - \kappa y_t - e_t].\end{aligned} \tag{64}$$

Computing the first order conditions, setting then equal to zero and eliminating the Lagrange multiplier yields the target rule

$$\pi_t = -\frac{\sigma\eta + 1}{\kappa \epsilon^2 \eta \delta \sigma} y_t. \tag{65}$$

Setting the target rule equal to the Phillips curve and solving for y_t gives

$$y_t = -\frac{1}{\kappa + \frac{\sigma\eta+1}{\kappa\epsilon^2\eta\delta\sigma}}[\beta E_t \pi_{t+1} + e_t]. \tag{66}$$

Again, setting (66) equal to the New IS-Curve and solving for i_t yields the reaction function under rationality (23).

8.2 Heterogeneous expectations and the conventional loss function

Defining $f_{t+s} = E_t y_{t+s+1}$ and $g_{t+s} = E_t \pi_{t+s+1}$, the policy problem under discretion, the conventional objective and heterogeneous expectations as in Gasteiger (2014) takes the following form:

$$\begin{aligned}\mathcal{L} = E_t \sum_{s=0}^{\infty} \beta^s \frac{1}{2}[\pi_{t+s}^2 + \omega y_{t+s}^2] \qquad\qquad (67)\\ + \lambda_{t+s}[\pi_{t+s} - \alpha\beta g_{t+s} - (1-\alpha)\beta\theta^2\pi_{t+s-1} - \kappa y_{t+s} - e_{t+s}]\end{aligned}$$

$$\frac{\partial \mathcal{L}}{\partial \pi_{t+s}} : \; E_t\{\beta^s[\pi_{t+s} + \lambda_{t+s}] - \beta^{s+1}[(1-\alpha)\beta^2\theta^2\lambda_{t+s+1}]\} \overset{!}{=} 0 \tag{68}$$

$$\frac{\partial \mathcal{L}}{\partial y_{t+s}} : \; E_t\{\beta^s[\omega y_{t+s} - \kappa\lambda_{t+s}]\} \overset{!}{=} 0. \tag{69}$$

Since the central bank re-optimizes every period, the index s can be dropped:

$$\pi_t = -\lambda_t + (1-\alpha)\beta^2\theta^2 E_t\lambda_{t+1} \tag{70}$$

$$\lambda_t = \frac{\omega}{\kappa} y_t. \tag{71}$$

The corresponding target rule is

$$\pi_t = -\frac{\omega}{\kappa}[y_t - (1-\alpha)\beta^2\theta^2 E_t y_{t+1}]. \tag{72}$$

It can be seen already that (72) reduces to (62) for $\alpha = 1$. Combining with the Phillips-Curve gives

$$y_t = -\frac{\kappa}{\kappa^2 + \omega}\left[-\frac{(1-\alpha)\beta^2\theta^2\omega}{\kappa} y_{t+1} + \alpha\beta\pi_{t+1} + (1-\alpha)\beta\theta^2\pi_{t-1} + e_t\right]. \tag{73}$$

Setting (73) equal to the New IS-Curve and solving for i_t yields the reaction function (31). Choosing $\omega = \frac{\sigma\eta+1}{\epsilon^2\eta\delta\sigma}$ yields the reaction function under discretion corresponding to the model-consistent loss function under RE (19).

8.3 Heterogeneous expectations and the model-consistent loss function

8.3.1 Rewriting the loss function

The period loss function L_t is given by

$$L_t = \frac{\sigma\eta + 1}{\sigma} y_t^2 + \frac{\alpha(y_t - c_t^R)^2}{(1-\alpha)\sigma} + \epsilon^2\eta\delta \left\{ \pi_t^2 + \frac{\xi_p(1-\alpha)}{\alpha} \left[\pi_t - \beta\theta^2\pi_{t-1} - \kappa y_t - \frac{\alpha\kappa(y_t - c_t^R)}{(1+\eta\sigma)(1-\alpha)} \right]^2 \right\} \tag{74}$$

By multiplying out, L_t can be re-written as

$$L_t = \Gamma_1 y_t^2 + \Gamma_2 \pi_t^2 + \Gamma_3 \pi_{t-1}^2 + \Gamma_4 (c_t^R)^2 + \Gamma_5 y_t c_t^R + \Gamma_6 \pi_t c_t^R + \Gamma_7 \pi_{t-1} c_t^R + \Gamma_8 \pi_t \pi_{t-1} + \Gamma_9 \pi_t y_t + \Gamma_{10} \pi_{t-1} y_t \tag{75}$$

with

$$\Gamma_1 = \frac{((\alpha-1)\eta\sigma - 1)(\alpha(\eta^2\sigma^2(\delta\epsilon^2\kappa^2\xi_p - 1) - 1 - 2\eta\sigma) - \delta\epsilon^2\eta\kappa^2\xi_p\sigma(1+\eta\sigma))}{(1-\alpha)\alpha\sigma(1+\eta\sigma)^2} \tag{76}$$

$$\Gamma_2 = \frac{\delta\epsilon^2\eta(\alpha + \xi_p - \alpha\xi_p)}{\alpha} \tag{77}$$

$$\Gamma_3 = \frac{(1-\alpha)\beta^2\delta\epsilon^2\eta\theta^4\xi_p}{\alpha} \tag{78}$$

$$\Gamma_4 = \frac{\alpha(1 + \eta\sigma(2 + \delta\epsilon^2\kappa^2\xi_p) + \eta^2\sigma^2)}{(1-\alpha)\sigma(1+\eta\sigma)^2} \tag{79}$$

$$\Gamma_5 = \frac{2(\alpha + 2\alpha\eta\sigma + \alpha\eta^2\sigma^2(1 - \delta\epsilon^2\kappa^2\xi_p) + \delta\epsilon^2\eta\kappa^2\xi_p\sigma(1+\eta\sigma))}{(\alpha-1)\sigma(1+\eta\sigma)^2} \tag{80}$$

$$\Gamma_6 = \frac{2\delta\epsilon^2\eta\kappa\xi_p}{1+\eta\sigma} \tag{81}$$

$$\Gamma_7 = -\frac{2\beta\delta\epsilon^2\eta\theta^2\kappa\xi_p}{1+\eta\sigma} \tag{82}$$

$$\Gamma_8 = \frac{2(\alpha-1)\beta\delta\epsilon^2\eta\theta^2\xi_p}{\alpha} \tag{83}$$

$$\Gamma_9 = \frac{2\delta\epsilon^2\eta\kappa\xi_p((\alpha-1)\eta\sigma - 1)}{\alpha + \alpha\eta\sigma} \tag{84}$$

$$\Gamma_{10} = \frac{2\beta\delta\epsilon^2\eta\theta^2\kappa\xi_p(1 + \eta\sigma(1-\alpha))}{\alpha + \alpha\eta\sigma}, \tag{85}$$

which is far easier to handle than (74).

8.3.2 Timeless commitment

The policy problem under commitment takes the following form:

$$\mathcal{L} = E_t \sum_{s=0}^{\infty} \beta^s \Bigg[\Gamma_1 y_{t+s}^2 + \Gamma_2 \pi_{t+s}^2 + \Gamma_3 \pi_{t+s-1}^2 + \Gamma_4 (c_{t+s}^R)^2$$
$$+\Gamma_5 y_{t+s} c_{t+s}^R + \Gamma_6 \pi_{t+s} c_{t+s}^R + \Gamma_7 \pi_{t+s-1} c_{t+s}^R + \Gamma_8 \pi_{t+s}\pi_{t+s-1} + \Gamma_9 \pi_{t+s} y_{t+s} + \Gamma_{10} \pi_{t+s-1} y_{t+s}$$
$$+ \lambda_{1,t+s}[y_{t+s} - \alpha E_t y_{t+s+1} - (1-\alpha)\theta^2 y_{t+s-1} + \sigma[i_{t+s} - \alpha E_t \pi_{t+s+1} - (1-\alpha)\theta^2 \pi_{t+s-1}]]$$
$$+ \lambda_{2,t+s}[\pi_{t+s} - \alpha\beta E_t \pi_{t+s+1} - (1-\alpha)\beta\theta^2 \pi_{t+s-1} - \kappa y_{t+s} - e_{t+s}]$$
$$+\lambda_{3,t+s}[c_{t+s}^R - E_t c_{t+s+1}^R + \sigma(i_{t+s} - E_t \pi_{t+s+1})]\Bigg]. \tag{86}$$

The first order conditions are

$$\frac{\partial \mathcal{L}}{\partial y_{t+s}} : E_t \Big\{ \beta^s [2\Gamma_1 y_{t+s} + \Gamma_5 c_{t+s}^R + \Gamma_9 \pi_{t+s} + \Gamma_{10} \pi_{t+s-1} + \lambda_{1,t+s} - \kappa \lambda_{2,t+s}] \tag{87}$$
$$- \beta^{s+1}(1-\alpha)\theta^2 \lambda_{1,t+s+1} - \beta^{s-1} \alpha \lambda_{1,t+s-1} \Big\} \stackrel{!}{=} 0$$

$$\frac{\partial \mathcal{L}}{\partial \pi_{t+s}} : \ E_t \Big\{ \beta^s [2\Gamma_2 \pi_{t+s} + \Gamma_6 c_{t+s}^R + \Gamma_8 \pi_{t+s-1} + \Gamma_9 y_{t+s} + \lambda_{2,t+s}]$$
$$+ \beta^{s+1}[2\Gamma_3 \pi_{t+s} + \Gamma_7 c_{t+s+1} + \Gamma_8 \pi_{t+s+1} + \Gamma_{10} y_{t+s+1} - (1-\alpha)\theta^2 \sigma \lambda_{1,t+s+1} \tag{88}$$
$$- (1-\alpha)\beta\theta^2 \lambda_{2,t+s+1}] - \beta^{s-1}[\alpha\sigma\lambda_{1,t+s-1} + \alpha\beta\lambda_{2,t+s-1} + \sigma\ \lambda_{3,t+s-1}] \Big\} \stackrel{!}{=} 0$$

$$\frac{\partial \mathcal{L}}{\partial c_{t+s}^R} : \ E_t \Big\{ \beta^s [2\Gamma_4 c_{t+s}^R + \Gamma_5 y_{t+s} + \Gamma_6 \pi_{t+s} + \Gamma_7 \pi_{t+s-1} \tag{89}$$
$$+ \lambda_{3,t+s}] - \beta^{s-1} \lambda_{3,t+s-1} \Big\} \stackrel{!}{=} 0$$

$$\frac{\partial \mathcal{L}}{\partial i_{t+s}} : \ E_t \Big\{ \beta^s \sigma \lambda_{1,t+s} + \beta^s \sigma \lambda_{3,t+s} \Big\} \stackrel{!}{=} 0. \tag{90}$$

Again, the index s can be dropped assuming commitment from a timeless perspective. Using $\lambda_{3,t} = -\lambda_{1,t}$ the FOCs can equivalently be written as

$$2\Gamma_1 y_t + \Gamma_5 c_t^R + \Gamma_9 \pi_t + \Gamma_{10}\pi_{t-1} + \lambda_{1,t} - \kappa\lambda_{2,t} - (1-\alpha)\beta\theta^2\lambda_{1,t+1} - \beta^{-1}\alpha\lambda_{1,t-1} \overset{!}{=} 0 \tag{91}$$

$$\begin{aligned} &2\Gamma_2\pi_t + \Gamma_6 c_t^R + \Gamma_8\pi_{t-1} + \Gamma_9 y_t + \lambda_{2,t} \\ &+ 2\Gamma_3\beta\pi_t + \Gamma_7\beta c_{t+1} + \Gamma_8\beta\pi_{t+1} + \Gamma_{10}\beta y_{t+1} - (1-\alpha)\beta\theta^2\sigma\lambda_{1,t+1} \\ &- (1-\alpha)\beta^2\theta^2\lambda_{2,t+1} + \beta^{-1}(1-\alpha)\sigma\lambda_{1,t-1} - \alpha\lambda_{2,t-1} \overset{!}{=} 0 \end{aligned} \tag{92}$$

$$2\Gamma_4 c_t^R + \Gamma_5 y_t + \Gamma_6\pi_t + \Gamma_7\pi_{t-1} - \lambda_{1,t} + \beta^{-1}\lambda_{1,t-1} \overset{!}{=} 0. \tag{93}$$

(93) can be used to replace $\lambda_{1,t-1}$ and $\lambda_{1,t+1}$ with $\lambda_{1,t}$ in (91). Then, solving (91) for $\lambda_{1,t}$ and inserting in (92) yields a second order difference equation in $\lambda_{2,t}$. A solution to this equation can in principle be substituted back into the difference equation, which would give a targeting rule. However, this solution is fairly complicated and relatively big in which some parameter terms exponentially depend on time. The solution is available upon request.[42] The resulting targeting rule and, hence, a reaction function would also be of such a complicated form where parameters exponentially depend on time. Thus, there will, almost certainly, be no operational reaction function.

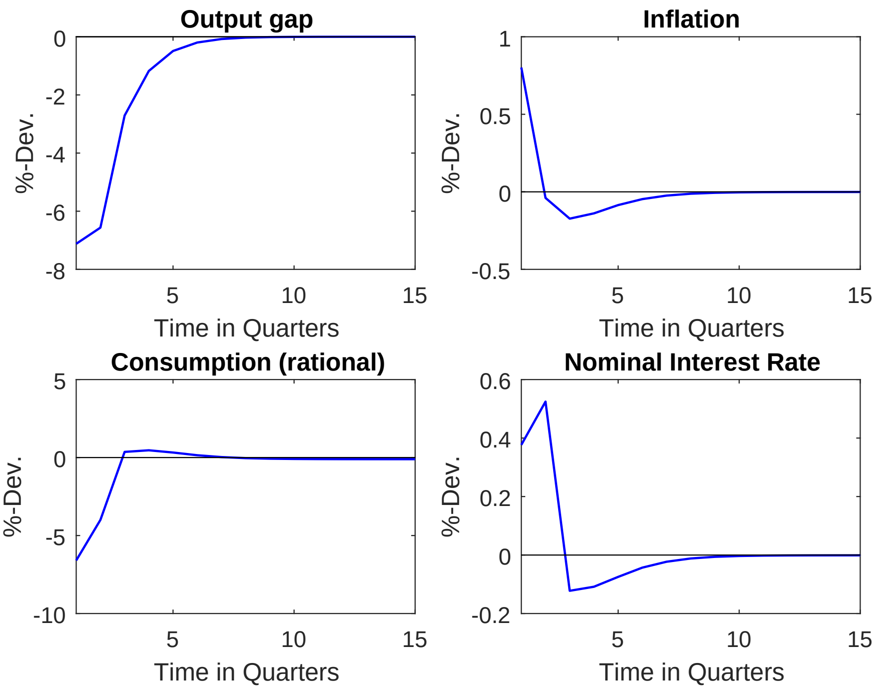

Figure 4: Impulse responses of the simulated FOCs under commitment

[42]Please send an e-mail to tim.hagenhoff@hotmail.de

8.3.3 Discretion

Defining $f_{t+s} = E_t y_{t+s+1}$, $g_{t+s} = E_t \pi_{t+s+1}$ and $h_{t+s} = E_t c^R_{t+s+1}$, the policy problem is

$$\mathcal{L} = E_t \sum_{s=0}^{\infty} \beta^s \Bigg[\Gamma_1 y^2_{t+s} + \Gamma_2 \pi^2_{t+s} + \Gamma_3 \pi^2_{t+s-1} + \Gamma_4 (c^R_{t+s})^2$$
$$+\Gamma_5 y_{t+s} c^R_{t+s} + \Gamma_6 \pi_{t+s} c^R_{t+s} + \Gamma_7 \pi_{t+s-1} c^R_{t+s} + \Gamma_8 \pi_{t+s} \pi_{t+s-1} + \Gamma_9 \pi_{t+s} y_{t+s} + \Gamma_{10} \pi_{t+s-1} y_{t+s}$$
$$+ \lambda_{1,t+s}[y_{t+s} - \alpha f_{t+s} - (1-\alpha)\theta^2 y_{t+s-1} + \sigma[i_{t+s} - \alpha g_{t+s} - (1-\alpha)\theta^2 \pi_{t+s-1}]]$$
$$+ \lambda_{2,t+s}[\pi_{t+s} - \alpha\beta g_{t+s} - (1-\alpha)\beta\theta^2 \pi_{t+s-1} - \kappa y_{t+s} - e_{t+s}]$$
$$+\lambda_{3,t+s}[c^R_{t+s} - h_{t+s} + \sigma(i_{t+s} - g_{t+s})]\Bigg]. \tag{94}$$

The first order conditions are

$$\frac{\partial \mathcal{L}}{\partial y_{t+s}} : E_t \Big\{ \beta^s [2\Gamma_1 y_{t+s} + \Gamma_5 c^R_{t+s} + \Gamma_9 \pi_{t+s} + \Gamma_{10} \pi_{t+s-1} + \lambda_{1,t+s} - \kappa \lambda_{2,t+s}] \tag{95}$$
$$- \beta^{s+1}(1-\alpha)\theta^2 \lambda_{1,t+s+1} \Big\} \stackrel{!}{=} 0$$

$$\frac{\partial \mathcal{L}}{\partial \pi_{t+s}} : \ E_t \Big\{ \beta^s [2\Gamma_2 \pi_{t+s} + \Gamma_6 c^R_{t+s} + \Gamma_8 \pi_{t+s-1} + \Gamma_9 y_{t+s} + \lambda_{2,t+s}]$$
$$+ \beta^{s+1}[2\Gamma_3 \pi_{t+s} + \Gamma_7 c_{t+s+1} + \Gamma_8 \pi_{t+s+1} + \Gamma_{10} y_{t+s+1} - (1-\alpha)\theta^2 \sigma \lambda_{1,t+s+1} \tag{96}$$
$$- (1-\alpha)\beta\theta^2 \lambda_{2,t+s+1}] \Big\} \stackrel{!}{=} 0$$

$$\frac{\partial \mathcal{L}}{\partial c^R_{t+s}} : \ E_t \Big\{ \beta^s [2\Gamma_4 c^R_{t+s} + \Gamma_5 y_{t+s} + \Gamma_6 \pi_{t+s} + \Gamma_7 \pi_{t+s-1} + \lambda_{3,t+s}] \Big\} \stackrel{!}{=} 0$$

$$\frac{\partial \mathcal{L}}{\partial i_{t+s}} : \ E_t \Big\{ \beta^s \sigma \lambda_{1,t+s} + \beta^s \sigma \lambda_{3,t+s} \Big\} \stackrel{!}{=} 0. \tag{97}$$

Since the central bank re-optimizes every period, the index s can be dropped. Using $\lambda_{3,t} = -\lambda_{1,t}$ the FOCs can equivalently be written as

$$2\Gamma_1 y_t + \Gamma_5 c_t^R + \Gamma_9 \pi_t + \Gamma_{10}\pi_{t-1} + \lambda_{1,t} - \kappa\lambda_{2,t} - (1-\alpha)\beta\theta^2 E_t\lambda_{1,t+1} \overset{!}{=} 0 \tag{98}$$

$$\begin{aligned} &2\Gamma_2\pi_t + \Gamma_6 c_t^R + \Gamma_8\pi_{t-1} + \Gamma_9 y_t + \lambda_{2,t} + 2\Gamma_3\beta\pi_t + \Gamma_7\beta E_t c_{t+1}^R + \Gamma_8\beta E_t\pi_{t+1} \\ &+ \Gamma_{10}\beta E_t y_{t+1} - (1-\alpha)\beta\theta^2\sigma E_t\lambda_{1,t+1} - (1-\alpha)\beta^2\theta^2 E_t\lambda_{2,t+1} \overset{!}{=} 0 \end{aligned} \tag{99}$$

$$2\Gamma_4 c_t^R + \Gamma_5 y_t + \Gamma_6\pi_t + \Gamma_7\pi_{t-1} - \lambda_{1,t} \overset{!}{=} 0. \tag{100}$$

Eliminating the Lagrange multipliers yields the reduced-form FOC

$$\begin{aligned} &\Delta_1\pi_t + \Delta_2 E_t\pi_{t+1} + \Delta_3 E_t\pi_{t+2} + \Delta_4\pi_{t-1} + \Delta_5 y_t + \Delta_6 E_t y_{t+1} + \Delta_7 E_t y_{t+2} \\ &+ \Delta_8 c_t^R + \Delta_9 E_t c_{t+1}^R + \Delta_{10} E_t c_{t+2}^R \overset{!}{=} 0 \end{aligned} \tag{101}$$

with

$$\Delta_1 = -\frac{\Gamma_6 + \Gamma_9 + 2\Gamma_2\kappa + 2\beta\Gamma_3\kappa - (1-\alpha)\beta\theta^2(\Gamma_7 + \beta(\Gamma_{10} + \Gamma_7) + \Gamma_7\kappa\sigma)}{\kappa} \tag{102}$$

$$\Delta_2 = \frac{(1-\alpha)\beta\theta^2(\beta(\Gamma_9 - (1-\alpha)\beta\Gamma_7\theta^2) + \Gamma_6(1+\beta+\kappa\sigma))}{\kappa} - \beta\Gamma_8 \tag{103}$$

$$\Delta_3 = -\frac{(\alpha-1)^2\beta^3\theta^4\Gamma_6}{\kappa} \tag{104}$$

$$\Delta_4 = -\frac{\Gamma_{10} + \Gamma_7 + \Gamma_8\kappa}{\kappa} \tag{105}$$

$$\Delta_5 = -\frac{2\Gamma_1 + \Gamma_5 + \Gamma_9\kappa}{\kappa} \tag{106}$$

$$\Delta_6 = -\frac{\beta((\alpha-1)\beta(2\Gamma_1 + \Gamma_5)\theta^2 + \Gamma_{10}\kappa + (\alpha-1)\Gamma_5\theta^2(1+\kappa\sigma))}{\kappa} \tag{107}$$

$$\Delta_7 = -\frac{(\alpha-1)^2\beta^3\theta^4\Gamma_5}{\kappa} \tag{108}$$

$$\Delta_8 = -\frac{2\Gamma_4 + \Gamma_5 + \Gamma_6\kappa}{\kappa} \tag{109}$$

$$\Delta_9 = -\frac{\beta((\alpha-1)\beta\Gamma_5\theta^2 + \Gamma_7\kappa + 2(\alpha-1)\Gamma_4\theta^2(1+\beta+\kappa\sigma))}{\kappa} \tag{110}$$

$$\Delta_{10} = -\frac{2(\alpha-1)^2\beta^3\theta^4\Gamma_4}{\kappa}. \tag{111}$$

Solving (101) for π_t and setting it equal to the NK Phillips curve yields

$$\begin{aligned} y_t = &-\frac{1}{\Delta_5 + \Delta_1\kappa}(\Delta_6 y_{t+1} + \Delta_7 y_{t+2} + (\Delta_2 + \alpha\beta\Delta_1)\pi_{t+1} + \Delta_3\pi_{t+2} + (\Delta_4 + (1-\alpha)\beta\theta^2\Delta_1)\pi_{t-1} \\ &+ \Delta_8 c_t^R + \Delta_9 c_{t+1}^R + \Delta_{10} c_{t+2}^R + \Delta_1 e_t). \end{aligned} \tag{112}$$

Setting (112) equal to the New IS curve and solving for i_t gives the central bank's reaction function under discretion

$$\begin{aligned} i_t =& \Omega_1 y_{t-1} + \Omega_2 E_t y_{t+1} + \Omega_3 E_t y_{t+2} + \Omega_4 \pi_{t-1} + \Omega_5 E_t \pi_{t+1} + \Omega_6 E_t \pi_{t+2} \\ &+ \Omega_7 E_t c^R_{t+1} + \Omega_8 E_t c^R_{t+2} + \Omega_9 e_t \end{aligned} \tag{113}$$

with

$$\Omega_1 = \frac{(1-\alpha)\theta^2}{\sigma} \tag{114}$$

$$\Omega_2 = \frac{\alpha}{\sigma} + \frac{\Delta_6}{(\Delta_5 + \Delta_1 \kappa)\sigma} \tag{115}$$

$$\Omega_3 = \frac{\Delta_7}{(\Delta_5 + \Delta_1 \kappa)\sigma} \tag{116}$$

$$\Omega_4 = \frac{\Delta_4 + (1-\alpha)\beta\theta^2\Delta_1}{(\Delta_5 + \Delta_1 \kappa)\sigma} + (1-\alpha)\theta^2 \tag{117}$$

$$\Omega_5 = \frac{\alpha\beta\Delta_1 + \Delta_2}{(\Delta_5 + \Delta_1 \kappa)\sigma} + \alpha \tag{118}$$

$$\Omega_6 = \frac{\Delta_3}{(\Delta_5 + \Delta_1 \kappa)\sigma} \tag{119}$$

$$\Omega_7 = \frac{\Delta_9}{(\Delta_5 + \Delta_1 \kappa)\sigma} \tag{120}$$

$$\Omega_8 = \frac{\Delta_{10}}{(\Delta_5 + \Delta_1 \kappa)\sigma} \tag{121}$$

$$\Omega_9 = \frac{\Delta_1}{(\Delta_5 + \Delta_1 \kappa)\sigma}. \tag{122}$$

The Ω-coefficients are expressed in terms of the targeting rule coefficients for simplicity. Writing them in terms of the deep model parameters would yield in part some very big and complicated expressions.

8.4 Computing long-run variances

8.4.1 State-space model

The model can be written in state-space form according to

$$X_{t+1} = \mathbf{A} X_t + \mathbf{B} i_t + \begin{bmatrix} \upsilon_{t+1} \\ 0_{n_2 \times 1} \end{bmatrix} \tag{123}$$

with $X_t = [e_t \; d_t \; \pi_{t-1} \; y_{t-1} \; c_t^R \; \pi_t \; y_t]'$, υ_{t+1} being a n_1x1 vector of innovations to the $x_{1,t}$ process with covariance matrix Σ and matrices given as follows:

$$\mathbf{A} = \begin{bmatrix} \rho_e & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \rho_d & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ \sigma(\alpha\beta)^{-1} & -1 & (1-\alpha)\theta^2\sigma\alpha^{-1} & 0 & 1 & -\sigma(\alpha\beta)^{-1} & \sigma\kappa(\alpha\beta)^{-1} \\ -(\alpha\beta)^{-1} & 0 & -(1-\alpha)\theta^2\alpha^{-1} & 0 & 0 & (\alpha\beta)^{-1} & -\kappa(\alpha\beta)^{-1} \\ \sigma(\alpha\beta)^{-1} & -\alpha^{-1} & 0 & -(1-\alpha)\theta^2\alpha^{-1} & 0 & -\sigma(\alpha\beta)^{-1} & (\beta+\sigma\kappa)/(\alpha\beta) \end{bmatrix} \tag{124}$$

$$\mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \sigma \\ 0 \\ \sigma\alpha^{-1} \end{bmatrix} \tag{125}$$

where ρ_e and ρ_d are the AR-parameters in the respective shock processes. Since only transitory shocks are of interest, both AR-parameters are set to zero. Also, note that $d_t = 0 \; \forall \; t$. The respective loss function displaying cumulative losses, L^{cum} is given by

$$L^{cum} = \sum_{t=0}^{\infty} \beta^t X_t' \mathbf{Q} X_t \tag{126}$$

with weighting matrices

$$\mathbf{Q} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \Gamma_3 & 0 & \Gamma_7/2 & \Gamma_8/2 & \Gamma_{10}/2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \Gamma_7/2 & 0 & \Gamma_4 & \Gamma_6/2 & \Gamma_5/2 \\ 0 & 0 & \Gamma_8/2 & 0 & \Gamma_6/2 & \Gamma_2 & \Gamma_9/2 \\ 0 & 0 & \Gamma_{10}/2 & 0 & \Gamma_5/2 & \Gamma_9/2 & \Gamma_1 \end{bmatrix} \tag{127}$$

in the fully model-consistent case and

$$\mathbf{Q} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \Gamma_2^R & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \Gamma_1^R \end{bmatrix} \tag{128}$$

with $\Gamma_1^R = \frac{\sigma\eta+1}{\sigma}$ and $\Gamma_2^R = \epsilon^2\eta\delta$ in the model-consistent case with rational expectations, respectively.

8.4.2 Discretion

Following Söderlind (1999), one can write down the following equations using the solution for the optimal interest rate path

$$x_{1,t+1} = (\mathbf{A}_{11} + \mathbf{A}_{12}\mathbf{C} - \mathbf{B}_1\mathbf{F}_1)x_{1,t} + \upsilon_{t+1} \tag{129}$$

$$x_{2,t} = \mathbf{C}x_{1,t} \tag{130}$$

with $\mathbf{C}$, $\mathbf{B}_1$ and $-\mathbf{F_1}$ being certain solutions taken from the algorithm and $\mathbf{A}_{11}$, $\mathbf{A}_{12}$ and $\mathbf{B}_1$ submatrices from the respective ones in (123). Since (129) is a stable process one can calculate variances as follows:

$$\begin{aligned} \Lambda_1 &= Var(x_{1,t+1}) = E(x_{1,t+1}x'_{1,t+1}) \\ &= \mathbf{G}E(x_{1,t}x_{1,t})\mathbf{G}' + E(\upsilon_{t+1}\upsilon'_{t+1}) \\ &= \mathbf{G}\Lambda_1\mathbf{G}' + \mathbf{\Sigma} \end{aligned} \tag{131}$$

$$\begin{aligned} \Lambda_2 &= Var(x_{2,t}) = Var(\mathbf{C}x_{1t}) \\ &= \mathbf{C}E(x_{1,t}x'_{1,t})\mathbf{C}' = \mathbf{C}\Lambda_1\mathbf{C}' \end{aligned} \tag{132}$$

with $\mathbf{\Sigma} = E(\upsilon_{t+1}\upsilon'_{t+1})$ and $\mathbf{G} = \mathbf{A}_{11} + \mathbf{A}_{12}\mathbf{C} - \mathbf{B}_1\mathbf{F}_1$. Using vectorization, i.e. $vec(ABC) = (C' \otimes A)vec(B)$ and $vec(A + B) = vec(A) + vec(B)$ following Lütkepohl (2005), the variance of (131) can be written as follows:

$$\begin{aligned} vec(\Lambda_1) &= vec(\mathbf{G}\Lambda_1\mathbf{G}') + vec(\Sigma) \\ &= (\mathbf{G} \otimes \mathbf{G})vec(\Lambda_1) + vec(\Sigma) \\ &= [\mathbf{I} - (\mathbf{G} \otimes \mathbf{G})]^{-1}vec(\Sigma). \end{aligned} \tag{133}$$

Still, to compute the true losses one needs the auto-covariances for one lag distance, which can be computed as follows:

$$\begin{aligned}\Lambda_{+1,1} &= Cov(x_{t+1}, x_{1,t}) = E(x_{t+1}x'_{1,t}) \\ &= E([\mathbf{G}x_{1,t} + \upsilon_{t+1}]x'_{1,t}) \\ &= E(\mathbf{G}x_{1,t}x'_{1,t} + \upsilon_{t+1}x'_{1,t}) \\ &= \mathbf{G}E(x_{1,t}x'_{1,t}) = \mathbf{G}\Lambda_1\end{aligned} \tag{134}$$

$$\begin{aligned}\Lambda_{1,2} &= Cov(x_{1,t}, x_{2,t}) \\ &= E([x_{1,t} + \upsilon_{t+1}]x'_{2,t}) \\ &= E(x_{1,t}(\mathbf{C}x_{1,t})') \\ &= E(x_{1,t}x'_{1,t})\mathbf{C}' = \Lambda_1\mathbf{C'}.\end{aligned} \tag{135}$$

Since $\beta < 1$ cumulative true losses L^{cum} can be computed as

$$L^{cum} = \frac{1}{1-\beta}L \tag{136}$$

with L given by (75) incorporating the long-run variances.

8.5 Tables

Table 10: Long-run variance of inflation, $Var(\pi)$, and output gap, $Var(y)$

	$Var(\pi)$			$Var(\pi)$			
policy	$\theta = 0.8$	$\theta = 1$	$\theta = 1.2$	$\alpha = 0.3$	$\alpha = 0.5$	$\alpha = 0.7$	$\alpha = 0.9$
L_t	0.96	1.16	1.58	1.69	1.50	1.16	0.86
$L_t^{\alpha=1}$	0.97	1.18	1.50	1.64	1.48	1.18	0.87
	$Var(y)$			$Var(y)$			
policy	$\theta = 0.8$	$\theta = 1$	$\theta = 1.2$	$\alpha = 0.3$	$\alpha = 0.5$	$\alpha = 0.7$	$\alpha = 0.9$
L_t	54.97	95.7	218.13	352.23	204.14	95.71	41.82
$L_t^{\alpha=1}$	52.21	89.71	213.76	351.02	198.00	89.71	40.05

8.6 Matlab code

The following Matlab code is an excerpt of the full code. The full code is based in the file provided by Di Bartolomeo et al. (2016). Listing 1 shows additions made to the code for this thesis. Note that in the Di Bartolomeo et al. (2016) code σ is defined as the *inverse* of the intertemporal elasticity of substitution and not the intertemporal elasticity itself. The additions made to the code account for that. The full code reproduces all figures except for figure 4. Figure 4 can be reproduced by running the Dynare code for the commitment case. Tables can be reproduced by changing the parameters and Q-matrix accordingly and rerunning the code except tables 2 and 3. Tables 2 and 3 can be reproduced by running the Dynare code for the reaction function under discretion. However, limits were calculated using Wolfram Mathematica.

Listing 1: Excerpts from matlab code

```
% weight for x(t)^2 for alpha = 1
gamma1R = (eta + sigma);

% weight for pi(t)^2 for alpha = 1
gamma2R = (epsilon^2*eta*delta);

% Q-Matrix in loss function
% model-consistent loss
Q = [ 0 0 0 0 0 0 0;
      0 0 0 0 0 0 0;
      0 0 gamma3 0 (gamma7)/2 (gamma8)/2 (gamma10)/2;
      0 0 0 0 0 0 0;
      0 0 (gamma7)/2 0 gamma4 (gamma6)/2 (gamma5)/2;
```

```
      0 0 (gamma8)/2 0 (gamma6)/2 gamma2 (gamma9)/2;
      0 0 (gamma10)/2 0 (gamma5)/2 (gamma9)/2 gamma1];

% microfounded conventional objective
%     Q = [0 0 0 0 0 0 0;
%       0 0 0 0 0 0 0;
%       0 0 0 0 0 0 0;
%       0 0 0 0 0 0 0;
%       0 0 0 0 0 0 0;
%       0 0 0 0 0 gamma2R 0;
%       0 0 0 0 0 0 gamma1R];

% Section 5.2.2
% Figure 1: expectation paths
EBinfl = [0;y_Disc(1:14,2)]
ERinfl = y_Disc(2:16,2)

figure(1)
plot(tid(1:15), EBinfl ,'b', tid(1:15) , ERinfl, '--','
   LineWidth',2)
title('Inflation expectations: rational (solid) and bounded
    rational (dotted)')
legend('bounded rational','rational')
xlabel('Time in Quarters')
ylabel('%-Deviation from St.St.')

% Figure 2: interest rate and inflation
figure(2)
subplot(1,2,1)
plot(tid,uu,'b',tid,y_Disc(:,2),'--','LineWidth',2)
title('Nominal interest rate and inflation')
legend('Nominal interest rate','Inflation')
xlabel('Time in Quarters')
ylabel('%-Deviation from St.St.')

% output gap
subplot(1,2,2)
plot(tid,y_Disc(:,[3]),'b','LineWidth',2)
title('Output gap')
```

```
xlabel('Time in Quarters')
ylabel('%-Deviation from St.St.')

% Section 6.1
% Long-run (auto/co-)variances
% See equation (1.27) and below in the Soederlind (1999)
   paper
G = A(1:n1,1:n1) + A(1:n1,n1+1:n1+n2) * C_Disc + B(1:n1,1)
   * F;
Id = eye(size(G,1)^2);
Sigma = eye(n1);

% variance-covariances
vec_lambda1 = (Id - kron(G,G)) \ vec(Sigma);
lambda1 = vec2mat(vec_lambda1,4) % (n1xn1)
lambda2 = C_Disc * lambda1 * C_Disc' % (n2xn2)

% auto-covariances
lambda_lead11 = G * lambda1 % (n1xn1)
lambda12 = lambda1 * C_Disc' % (n1xn2)

% Section 6.2
% welfare loss, model-consistent
% not multiplied by 1/2
Lt = gamma1*lambda2(3,3)+gamma2*lambda2(2,2)+gamma3*lambda1
   (3,3)+gamma4*lambda2(1,1)+gamma5*lambda2(1,3)+gamma6*
   lambda2(1,2)+gamma7*lambda12(3,1)+gamma8*lambda_lead11
   (3,3)+gamma9*lambda2(2,3)+gamma10*lambda12(3,3)

% welfare loss, microfounded conventional objective
% not multiplied by 1/2
LRt = gamma1R*lambda2(3,3)+gamma2R*lambda2(2,2)

% Section 6.3
% impulse response of consumption of backward-looking
   agents
cb = (1/(1-alpha))*y_Disc(:,3) - (alpha/(1-alpha))*y_Disc
   (:,1)
```

```
% consumption inequality based on long-run (co-)variances
cieq = (alpha/(1-alpha))*(lambda2(1,1)-2*lambda2(1,3)+
   lambda2(3,3))

% Figure 3: plot consumption impulse responses
figure(3)
plot(tid, cb, 'b',tid,y_Disc(:,1),'--', 'LineWidth',2)
title('Consumption: backward-looking (solid) and rational (
   dotted)')
legend('backward-looking','rational')
xlabel('Time in Quarters')
ylabel('%-Deviation from St.St.')
```

Eidesstattliche Erklärung

Ich versichere, dass ich die Masterarbeitselbstständig verfasst habe. Andere als die angegebenen Hilfsmittel und Quellen wurden nicht benutzt. Die Arbeit hat keiner anderen Prüfungsbehörde vorgelegen. Es ist mir bekannt, dass ich bei Verwendung von Inhalten aus dem Internet diese zu kennzeichnen habe und einen Ausdruck davon mit Datum und Internet-Adresse (URL) als Anhang der Masterarbeit beizufügen habe.

Ort, Datum   Unterschrift