Testing an Extended version of Goodwin's Growth-Cycle Model to the US Economy

Ricardo Azevedo Araujo

Department of Economics, University of Brasilia, Brazil

Helmar Nunes Moreira

Department of Mathematics, University of Brasilia, Brazil

Marwil J. Dávila-Fernández

PhD Candidate, Department of Economics, University of Siena, Italy

Abstract

In this paper, we present an extension of the Goodwin growth-cycle model that considers the rate of capacity utilization as a new variable in the adapted Lotka-Volterra system of differential equations to study distributive cycles. With this approach, we intend to clarify the connections between demand, labour market and capital accumulation in a model where a cyclical pattern is generated amongst the employment rate, the profit share, and the rate of capacity utilization. The obtained model is then studied from a qualitative viewpoint and conditions to generate a limit cycle are studied. Finally, the model is tested for the US economy with quarterly data from 1967 to 2016.

Keywords: growth-cycle model, distributive cycles, limit cycles.

JEL classification: E32, E64.

1. Introduction

It is fair to say that the effort to understand business cycles within a growth framework is widespread in Economics nowadays. Theories such as the real business cycles (RBC) [see Kykland and Prescott (1982) and Long and Plosser (1983)] and the dynamic stochastic general equilibrium (DSGE) [Blanchard e Kiyotaki (1987) and Rotemberg and Woodford (1997)], which are the benchmark of mainstream macroeconomics, are built on a Solovian background [Solow (1956)] with a long run perspective. In these frameworks, a representative agent chooses not only the consumption but also the amount of labour to be supplied to maximize the lifetime utility, and fluctuations arise as the optimal response of the agent to exogenous shocks, which may be either productivity or demand shocks.

But in fact, the idea that growth and fluctuations are intertwined phenomena can be traced back to the seminal work of Richard Goodwin [Goodwin (1967)] who developed a growth-cycle model where profits squeezed out of the income of the workforce are invested, thus determining the pace of capital accumulation. His class struggle model, which blends aspects of the Harrod–Domar growth model with the Phillips curve, led him to his profound insight that the trend and the cycle are indissolubly fused and to the important conclusion that distributional conflict produces endogenous cycles [see Harcourt (2015)]. In this vein, Goodwin presents a theory of economic fluctuations whereby the economic variables interact with each other in a cyclical and endogenous way, with the cycle emerging from the dynamic interaction of deterministic variables of the model, and not as the outcome of the exogenous aleatory shocks.

Formally the growth-cycle framework consists of two simultaneous non-linear dynamic equations for the employment rate and the wage share. It can be interpreted as a Lotka-Volterra's predator-prey model stated in a Marxian form, in which the wage share is viewed as the predator and the employment rate as the prey. Although providing a compelling explanation for economic cycles, for some authors such as Tavani and Zamparelli (2014) and Harvie (2000) the Goodwin model seems to be too stylized to explain long-run shifts in the growth cycle. In order to furnish the model with more realism, it has been extended in a number of directions.

Initial extensions included Desai (1973) on inflation, Wolfstetter (1982) on fiscal policy, Van der Ploeg (1983) on its relation with neoclassical growth, and Choi (1995) on efficiency wages. Modifications to study qualitatively different dynamics were developed by Pohjola (1981) in a discrete-time version of the model and Sportelli (1995) in a Kolmogoroff generalization. Sato (1985) extended the model to a two-sector economy while Skott (1989) introduced Keynesian effective demand aspects.

The study of the role played by induced technical change in the growth-cycle dynamics was initiated by Shah and Desai (1981) and further elaborated by Van der Ploeg (1987). Recent contributions include Foley (2003) and Julius (2006). Velupillai (2006) also presents a disequilibrium macrodynamic model with a technical progress function. An entrepreneur state that invests in infrastructure capital and finance publicly funded research is modeled by Tavani and Zamparelli (2017) within the growth-cycle approach.

In order to better understand the relation between distributive cycles and the Minskyan financial instability hypothesis, Keen (1995, 2013) has built a family of Goodwin-Minksy models, which are shown to be structurally unstable. In the same way, even though with different features, Sordi and Vercelli (2006, 2012, 2014) present a series of growth-cycle models exploring the dynamic non-linear interactions between financial and distributive variables. Stockhammer and Michell (2017) on the other hand have tried to put forward the concept of pseudo-Goodwin cycles, understood as a distributive cycle which could, in fact, be generated by Minskyan dynamics.

Mariolis (2013) and Rodousakis (2015) in turn investigate the stability properties of the growth-cycle model within a Kaleckian accumulation function. Distributive cycles with Kaldorian and Kaleckian features are also part of the contributions of Arnim and Barrales (2015). In what concerns the direct address of Harrodian instability and the Keynesian effective demand principle, Sportelli (2000) and Schoder (2014) provide interesting exercises generating limit cycles.

Sasaki (2013) also blends a Goodwinian and Marxian macrodynamic disequilibrium model of growth and fluctuations with Kaleckian features. The departing point of his analysis rests on the observation that investment in the Goodwin model has the same passive role played by it in the Neo-classical growth model since it is determined by savings. Sasaki then presents an extension to Goodwin's formulation in which an autonomous investment function is presented in the lines of the post-

Keynesian growth model. Then the author is able to obtain a system of three variables, namely employment rate, profit share and rate of capacity utilization thus showing the existence of a limit cycle amongst these variables.

Last but not least, Chiarella et al (2005) have been engaged in a major research program on (disequilibrium) macroeconomic analysis that has resulted in a massive series of papers and books with several other contributors including Asada et al (2003, 2011) in which they have proposed what they called the Keynes-Meltzer-Goodwin approach. A similar effort can be found in Flaschel (2015) with the extension of Goodwin's distributive cycle by introducing both effective demand forces and endogenous innovation, providing a platform of what Goodwin himself described as the Marx-Keynes-Schumpeter approach. Flaschel et. al. (2008), for instance, have also proven the existence of limit cycles in three-dimensional extensions of the Goodwin (1967) model by using the Hopf bifurcation theorem, by considering the capital-output ratio as an endogenous variable.

Here we also present an extension of Goodwin's framework by introducing the rate of capacity utilization as a third variable in the model. But in order to avoid the criticisms made against the Kaleckian investment functions [see Blecker (2016) and Skott (2012)], we have endogenized the rate of capacity utilization through Skott's (1989) formulation of an output expansion function. In this vein, the differential equation for the rate of capacity utilization is derived as being proportional to the difference between the output expansion function and the rate of capital accumulation. If the output expansion function is higher than capital accumulation the rate of capacity utilization experiences positive variation, while the reverse is true for the case in which the output expansion function is lower than capital accumulation.

With this approach, we believe we introduce a channel in the model that makes it more inclusive of the demand side insofar as the output expansion registers as a function of the profit share and of the employment rate. Although the obtained model departs significantly from Sasaki (2013), by using the Hopf bifurcation theorem, we also have found a stable equilibrium that may degenerate into a limit cycle if the growth rate of population increases to a sufficient degree. In order to illustrate this result, we carry out the analysis both in terms of general functions as well as in terms of a particular example. With this approach, we intend to clarify the connections between demand, the labour market and capital accumulation in a model where a cyclical pattern is generated amongst these variables. Finally, the model is tested for the US economy.

In this paper, we have adopted a time series approach similar to Jump (2016) and Barbosa-Filho (2016) in order to test the extended model to the US economy. Jump (2016) after running a trivariate VAR model has adopted a Granger non-causality test to identify evidence of cyclical pattern amongst the unemployment rate, labour share and GDP. By previously establishing patterns related to the Granger causality that would support the existence of cyclical pattern amongst these variables, Jump (2016, p. 3) has concluded that "the evidence presented is not unambiguously consistent with a strong profit squeeze mechanism in the USA and UK during the period of study."

Barbosa-Filho (2016, p. 13) has focused on the impulse-response function for establishing the pattern for the US economy has concluded that the "rate of employment goes down after an exogenous increase in the wage share, as well as that the wage share goes up after an exogenous increase in the rate of employment. These results are characteristic of a Marxian profit-led economy (...)." Although of interest, the conclusion that the economy is a Marxian profit-led economy does not confirm unambiguously the existence of distributive cycles for the US economy insofar as to establish this result other econometric tests, such as causality should have been adopted.

In order to test for the US economy by using quarterly data from 1967 to 2016, we have adopted a VAR methodology. Besides, the main results extracted from the generalized impulse-response functions provided by the VAR model are: a positive profit share innovation affects positively the employment rate but has a negative effect on the rate of capacity utilization, suggesting a profit-led pattern. We also performed the traditional Granger causality test which yielded significant results.

This paper is organized as follows. Besides this introductory section, section 2 presents an extension to the Goodwin model by including the rate of capacity utilization as a new variable. Section 3 studies the stability and proves the existence of a limit cycle and simulations are presented that allow us to confirm the existence of a limit cycle from a numerical viewpoint. Section 4 presents the econometric estimations of the model for the US economy and section 5 concludes.

2. On the Existence and Stability of Equilibrium in an extended version of the Goodwin Model

In the original Goodwin (1967) model the endogenous variables are the wage share ω and the employment rate, e. Here following Skott (1989) we write the system[1] in terms of the profit share, h, and the employment rate, e. As in the original model, let us consider a closed economy without government activity that uses capital, K, and labour, L, to produce output, Y, by using a fixed coefficient technology, given by:

$$Y = min\left[\frac{K}{v}, \frac{L}{l}\right] \tag{1}$$

where v is the capital-output ratio, and l is the labour-output ratio. Following Goodwin, the capital-output ratio is constant but departing from his formulation we assume, for the sake of simplicity only, that the labour-output ratio is also constant. Let N be the available workforce and assume that N grows at an exogenous rate η.

Let us consider that workers do not save and that the propensity save of capitalists is denoted by s. Assuming, for the sake of simplicity only, that there is no depreciation of the stock of capital, the change in the stock of capital is given by: $K' = s(Y - wL)$, where w is the wage and $' = \frac{d}{dt}$. By dividing both sides of this expression by K, it yields after some algebraic manipulation the growth rate of the stock of capital:

$$\hat{K} = \frac{s(1 - wl)}{v} \tag{2}$$

where the hat denotes the growth rate of a variable. By defining the employment rate as $e = \frac{L}{N}$, where N stands for total population, we obtain: $\hat{e} = \hat{L} - \hat{N} = \hat{L} - \eta$. The efficiency condition of the Leontief function requires that $\frac{K}{v} = \frac{L}{l}$, which implies that $\hat{K} = \hat{L}$ since v and l are assumed to be constant. We can substitute expression (2) into the growth rate of employment, which allows us to obtain after some algebraic manipulation:

[1] Our choice of writing the system in terms of the profit share instead of the wage share is related to the fact that the extension presented here is made by using the expansion output function due to Skott (1989), which registers as a function of the profit share and the employment rate.

$$\hat{e} = \frac{s(1-wl)}{v} - \eta \quad (3)$$

By considering that the wage share, ω, is given by $\omega = wl$ and that the profit share, denoted by h, sums up to unity, namely $\omega + h = 1$, we can rewrite the above expression as:

$$e' = e\left[\frac{s}{v}h - \eta\right] \quad (3)'$$

In order to take into account the labour market, Goodwin (1967) has considered a Marxian reserve army mechanism translated in terms of a Phillips Curve[2], in which the growth rate of real wages increases with the employment rate. Here let us adopt the following linear specification:

$$\hat{w} = \beta(e - e^o) \quad (4)$$

where e^o denotes the natural rate of employment. Such specification may be viewed as a simplified version of the expectations-augmented wage Phillips curve adopted by Chiarella et al. (2005). By following Goodwin (1967), we do not focus on the dynamics of prices and hence we assume that the expectations of inflation do not enter the expression (4), thus generating this simplified version of the wage Phillips curve. Since $h = 1 - wl$, by taking the derivative with respect to time yields: $h' = -lw'$. Then by substituting h' into expression (4) it yields after some algebraic manipulation:

$$h' = \beta(1-h)(e^o - e) \quad (5)$$

The system formed by expressions (3)' and (5) is akin to Goodwin's system. It is possible to show then that evaluating the system in steady-state yields an interior equilibrium[3] which is given by: $(e^*, h^*) = \left(e^o, \frac{\eta v}{s}\right)$. The Jacobian matrix for the system is given by:

---

[2] According to Franke et al. (2006, p.453) "'Goodwin' indicates that income distribution plays a crucial role in the dynamics of nominal and real variables. It is determined by the interplay of a wage as well as a price Phillips curve, and in turn impacts positively on aggregate demand via workers' consumption and negatively via profit-oriented investment."

[3] Another singular point of system formed by (3)' and (5) is given by $e = 0$ and $h = 1$. Although its mathematical relevance, from an economic viewpoint, the point (0,1) would correspond to the case in which there is no employed labour, with the income being paid entirely to profits. But if $L = 0$ then, from expression (1), $Y = 0$, which corresponds to the case of no production. The distributional issue in this case lose its meaning since the economy does not exist.

$$J(e,h)=\begin{bmatrix}\left(\frac{s}{v}\right)h-\eta & \frac{es}{v}\\ -\beta(1-h) & -\beta(e^o-e)\end{bmatrix} \qquad (6)$$

By evaluating the Jacobian at $(e^*,h^*)=\left(e^o,\frac{\eta v}{s}\right)$ it yields:

$$J\left(e^o,\frac{\eta v}{s}\right)=\begin{bmatrix}0 & \frac{e^o s}{v}\\ \beta\left(1-\frac{\eta v}{s}\right) & 0\end{bmatrix} \qquad (6)'$$

The trace and the determinant of the Jacobian at $(e^*,h^*)=\left(e^o,\frac{\eta v}{s}\right)$ are respectively given by: $trJ\left(e^o,\frac{\eta v}{s}\right)=0$ and $detJ\left(e^o,\frac{\eta v}{s}\right)=\beta\left(1-\frac{\eta v}{s}\right)\frac{e^o s}{v}$. The eigenvalues of the characteristic polynomial are given by: $\lambda_{1,2}=\pm\sqrt{-\beta\left(1-\frac{\eta v}{s}\right)\frac{e^o s}{v}}$.

Since $h^*=\frac{\eta v}{s}<1$, then the system has two pure imaginary roots, which gives rise to a non-hyperbolic system. In this case, we have to prove the following proposition to show that the system exhibits periodic orbits.

Proposition 1: The trajectories of the dynamical system formed by expressions (3)' and (5) are all closed orbits.

Proof: In order to prove that let us divide expression (5) by (3) to obtain:

$$\frac{dh}{de}=\frac{\beta(1-h)(e^o-e)}{e\left(\frac{s}{v}h-\eta\right)}$$

By separating variables, this expression may be rewritten as:

$$\frac{\left(\frac{s}{v}h-\eta\right)}{(1-h)}dh=\frac{(e^o-e)}{e}de$$

By integrating both sides, namely:

$$\int\frac{\left[\frac{s}{v}(h-1+1)-\eta\right]}{(1-h)}dh=\int e^o\frac{de}{e}-\int de$$

It yields after some algebraic manipulation:

$$\frac{s}{v}h+\left(\frac{s}{v}-\eta\right)\ln(1-h)=-e^o lne+e+lnC$$

where C is an arbitrary constant. After some algebraic manipulation the above expression may be written as: $e^{e^o}(1-h)^{\left(\frac{s}{v}-\eta\right)} \exp\left(\frac{s}{v}h - e\right) = C$. The function on the left hand side is therefore constant along the trajectories, which may give rise to closed orbits – see figure 1. □

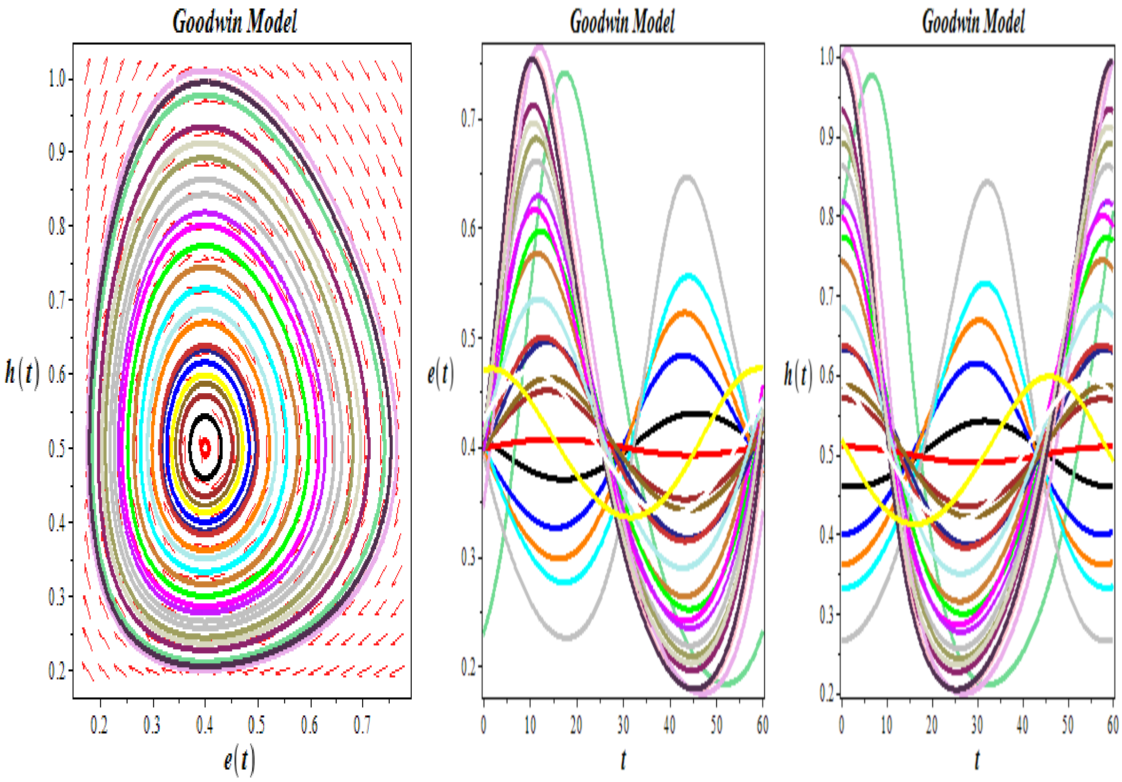

Figure 1. Periodic solutions to the Goodwin model (3)' – (5) in the phase plane with $e(t)$ and $h(t)$ as functions of time t, $0 \leq t \leq 60$ for $\frac{s}{v} = 0.2$, $\eta = 0.1$, $\beta = 0.3$ and $e^o = 0.4$. The initial conditions are given in Appendix and the equilibrium point $P^* = (0.4,\ 0.5)$ is a center.

The Goodwin cycle works as follows: an increase in the employment level leads to an increase in the wage share, which decreases the profit share and thus capital accumulation. The outcome of lower capital accumulation is a decrease in the output and consequently in the employment level, leading to a decrease in the wage share and an increase in the profit share. A higher profit share leads to faster capital accumulation and, hence, an increase in output growth and employment level. At this point the cycle restarts.

Although the story told by Goodwin (1967) is reasonable, the model is built under the concept of full capacity utilization. And as pointed out by Hein (2014, p. 248), "the rate of capacity utilization is the important indicator for the development of demand in relation to the capital stock in existence and thus becomes one of the major factors influencing investment decisions." Then, by introducing the possibility of under capacity utilization yields a version of the model that considers a more inclusive role for demand, with a less stylized explanation to the growth cycle insofar as in the real world the stock of capital is often underutilized. This view is according to Sasaki (2013), who writes the production function as:

$$Y = min\left[\frac{uK}{v}, \frac{L}{l}\right] \tag{7}$$

Where u, $0 < u < 1$, denotes the rate of capacity utilization. It is possible to show after some algebraic manipulation that the growth rate of the stock of capital is now given by:

$$\widehat{K} = \frac{su(1-wl)}{v} \tag{8}$$

By adopting the same procedure to arrive at expression (3) from expression (2), from expression (8) it is possible to show that in the presence of under capacity utilization expression (3)' should be replaced by:

$$e' = e\left[\frac{s}{v}hu - \eta\right] \tag{9}$$

In order to derive the dynamical path for the rate of capacity utilization, let us assume that $\widehat{Y}$ is given by the output expansion function: $\widehat{Y} = \varphi(e,h)$ [see e.g. Skott (1989)] with $\varphi_e(e,h) < 0$, $\varphi_h(e,h) > 0$. According to this specification, higher profitability stimulates the rate of expansion of output per unit of capital and higher employment rate will reduce this rate. Related to the profit share, the mechanism is straight insofar as a positive demand shock increases the profit share, and firms respond to this rise by increasing the growth rate of output. With respect to the employment rate, the rationale rests on the fact that there are adjustment costs with respect to output expansion and employment expansion. Skott and Ryoo (2008, p. 837) for instance, consider that "[h]igh rates of employment increase the costs of recruitment, and since the quit rate tends to rise when labour markets are tight, the gross recruitment needs associated with any given rate of expansion increase when low unemployment makes it difficult to attract new workers".

By also considering that the growth rate of the stock of capital is given by expression (8), and substituting this expression and the output expansion function into $\hat{u} = \widehat{Y} - \widehat{K}$ we obtain the third differential equation of our system, namely:

$$u' = u\left[\varphi(e,h) - \left(\frac{s}{v}\right)hu\right] \tag{10}$$

Expression (10) shows on one hand that if the capital accumulation grows at a lower rate than the output then the rate of capacity utilization will increase since there will be a pressure on the use of existing stock of capital. On the other hand, if the capital accumulation grows at a higher pace than the output expansion the rate of capacity utilization will decrease meaning that the underutilization of the stock of capital will prevail. Let us now consider the following dynamical model of three autonomous nonlinear differential equations formed by expressions (5), (9) and (10).

In steady state the relevant solution $P^* = (e^*,\ h^*,\ u^*)$ (with economic meaning[4]) may be obtained from considering $e' = h' = u' = 0$. From expression (5), $e^* = e^o$, since $h < 1$. Then, from expression (10), the value of h^*, if exists, is given implicitly by $\varphi(e^o, h^*) = \eta$. The value of u^* is then given by expression (9) according to: $u^* = \frac{v\eta}{sh^*}$. By using the to the Routh-Hurwitz criterion, we can proof the following:

Proposition 2: The singular point $P^* = (e^*,\ h^*,\ u^*)$ of system formed by expressions (5), (9) and (10) is asymptotically stable if $\varphi_h(e^o, h^*) > max\left\{\frac{\eta}{h^*}, \eta - \frac{\eta}{\beta}\frac{h^*}{1-h^*}\varphi_e(e^o, h^*)\right\}$.

Proof. In order to proof this fact let us consider the Jacobian matrix calculated in $P^* = (e^*,\ h^*,\ u^*)$, which is given by:

$$J = \begin{bmatrix} j_{11} & j_{12} & j_{13} \\ j_{21} & j_{22} & j_{23} \\ j_{31} & j_{32} & j_{33} \end{bmatrix} \tag{11}$$

Where: $j_{11} = 0, j_{12} = \left(\frac{s}{v}\right)e^o u^o, j_{13} = \left(\frac{s}{v}\right)e^o h^*, j_{21} = -\beta(1-h^*),\ j_{22} = 0,\ j_{23} = 0$, $j_{31} = u^*\varphi_e(e^o, h^*), j_{32} = u^*\left(-\left(\frac{s}{v}\right)u^* - \varphi_h(e^o, h^*)\right),\quad j_{33} = -\eta$. In order to prove the stability we need to bear in mind that the Routh-Hurwitz criterion states that

[4] An alternative singular point for system (5), (9) and (10) would be given by: $e = 0$, $h = 1$, and $u = \frac{v}{s}\varphi(0,1)$. But we have excluded this possibility by following the same rationale of the two-dimensional system (see footnote 5). Although this is a singular point of the system, this point is irrelevant from an economic viewpoint since the employed labour would be equal to zero and hence the production would be also equal to zero.

$P^* = (e^*,\ h^*,\ u^*)$ is asymptotically stable if and only if $S_1(P^*) < 0$, $S_2(P^*) > 0$, $S_3(P^*) < 0$, and $E(P^*) = S_1\, S_2 - \ S_3 > 0$, where: $S_1,\ S_2,$ and S_3 are the coefficients of the characteristic polynomial of matrix J, which is is given by:

$$\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0 \tag{12}$$

where:

$$S_1(P^*) = -\eta \tag{13}$$

which is negative since $\eta > 0$.

$$S_2(P^*) = \left(\frac{s}{v}\right)\beta e^o u^*(1-h^*) - \eta e^o\ \varphi_e(e^o, h^*) \tag{14}$$

which is negative since the first term on the right hand side is positive since all the parameters are positive and we assume that $0 < h < 1$. The sign of the second term on the right hand side is negative since $\varphi_e(e^o, h^*) < 0$. Hence we conclude that $S_2(P^*) > 0$.

$$S_3(P^*) = -\beta\eta e^o(1-h^*)\ \varphi_h(e^o, h^*) \tag{15}$$

The sign of $S_3(P^*)$ is negative since $\varphi_h(e^o, h^*) > 0$, $0 < h < 1$, and the other parameters are positive. Besides:

$$E(e^o, \eta) = \eta e^o\left(\eta\varphi_e(e^o, h^*) + \beta\left(\varphi_h(e^o, h^*) - \left(\frac{s}{v}\right)u^*\right)(1-h^*)\right) \tag{16}$$

By substituting the steady state value for u^* in expression (16) it yields:

$$E(e^o, \eta) = \eta e^o\left(\eta\varphi_e(e^o, h^*) + \beta\left(\varphi_h(e^o, h^*) - \frac{\eta}{h^*}\right)(1-h^*)\right) \qquad (16)’$$

In order to guarantee that $E(e^o, \eta) > 0$, it is necessary then to assume that: (i) $\varphi_h(e^o, h^*) > \frac{\eta}{h^*}$ and (ii) $\varphi_h(e^o, h^*) > \eta - \frac{\eta}{\beta}\frac{h^*}{1-h^*}\varphi_e(e^o, h^*)$. We can summarize this information by stating that $\varphi_h(e^o, h^*) > max\left\{\frac{\eta}{h^*}, \eta - \frac{\eta}{\beta}\frac{h^*}{1-h^*}\varphi_e(e^o, h^*)\right\}$ is necessary for guaranteeing the local stability of $P^* = (e^*,\ h^*,\ u^*)$. Then we can conclude that the interior equilibrium $P^* = (e^*,\ h^*,\ u^*)$ given by $e^* = e^o$, $\varphi(e^*, h^*) = \eta$ and $u^* = \frac{v\eta}{sh^*}$ is asymptotically stable. □

This result shows that the introduction of the rate of capacity utility as a new variable in the Goodwin framework through the output expansion function changes the dynamical property of the system from a center to a stable equilibrium. However, it is possible to show that the extended model may exhibit endogenous cycles insofar as such equilibrium degenerate into a limit cycle if the growth rate of population increases to a sufficient degree.

The static comparative analysis shows us that by taking the derivative of $u^* = \frac{v\eta}{sh}$ with respect to the profit-share it yields: $\frac{\partial u^*}{\partial h} = -\frac{v\eta}{sh^2} < 0$, which characterizes a profit-led regime. Let us now then study the possibility of the existence of a limit cycle in the system formed by expressions (5), (9) and (10) by using the Hopf bifurcation analysis, what will be accomplished in the next section.

3. Limit Cycle and Numerical Simulations

In order to study the existence of limit cycles let us adopt the Hopf bifurcation theorem for the system formed by expressions (5), (9) and (10) by using $\eta > 0$ as the bifurcation parameter. Note that for $\eta = \eta^*$, the Jacobian matrix $J = J(P^*)$ has a pair of complex eigenvalues with zero real part if and only if: $E = S_1S_2 - S_3 = 0$ or, equivalently:

$$(\lambda^2 + S_2)(\lambda - S_1) = 0 \tag{17}$$

where: $\lambda_{1,2} = \pm i\sqrt{S_2} = \pm i\omega_o$, and $\lambda_3 = S_1$. The roots are in general of the form: $\lambda_1(\eta) = u(\eta) + iv(\eta)$, $\lambda_2(\eta) = u(\eta) - iv(\eta)$ and $\lambda_3(\eta) = -S_1(\eta)$. To apply the Hopf bifurcation theorem at $\eta = \eta^*$, we need to verify if the transversality condition holds [see Chiarella, et al. (2005, p. 99)], namely: $\left[\frac{d\,Re(\lambda_i)(\eta)}{d\eta}\right]_{\eta=\eta^*} \neq 0,\ \ i = 1,2$. The existence of the parameter η^* considered above is not difficult to prove, which is the content of:

Proposition 3: There exists a one-parameter family of periodic solutions at $\eta = \eta^*$, bifurcating from the equilibrium point $P^* = (e^*,\ h^*,\ u^*)$ with period T, where $T \to T_0$ as $\eta \to \eta^*$where $T_0 = 2\pi/\sqrt{S_2}$, with with S_2 given by $S_1(\eta^*)S_2(\eta^*) - S_3(\eta^*) = 0$.

Proof. To apply the Hopf bifurcation theorem we need verify the transversality condition:

$$\frac{d}{d\eta}Re\big(\lambda_i(\eta)\big)\,|_{\eta=\eta^*}=\frac{du}{d\eta}\,|_{\eta=\eta^*}\neq 0 \qquad (18)$$

Substituting $\lambda_i(\eta)=u(\eta)\pm iv(\eta)$ into $E=0$ and calculating the derivative, we obtain after some algebraic manipulation the following linear system:

$$A(\eta)u'(\eta)-B(\eta)v'(\eta)=-C(\eta)$$

$$B(\eta)u'(\eta)+A(\eta)v'(\eta)=-D(\eta)$$

Where: $A(\eta)=3u^2(\eta)+2S_1(\eta)\,u(\eta)+S_2(\eta)-3v^2(\eta)$,

$B(\eta)=6u(\eta)v(\eta)+2S_1(\eta)v(\eta)$,

$C(\eta)=S_1'(\eta)u^2(\eta)+S_2'(\eta)u(\eta)+S_3'(\eta)-S_1'(\eta)v^2(\eta)$,

$D(\eta)=2S_1'(\eta)u(\eta)v(\eta)+S_2'(\eta)v(\eta)$.

Since: $A(\eta^*)C(\eta^*)+B(\eta^*)D(\eta^*)\neq 0$,we have:

$$\frac{d}{d\eta}Re\big(\lambda_i(\eta)\big)\,|_{\eta=\eta^*}=\frac{du}{d\eta}\,|_{\eta=\eta^*}=-\left[\frac{AC+BD}{A^2+B^2}\right],$$

that is:

$$\frac{d}{d\eta}Re(\lambda_i(\eta))\,|_{\eta=\eta^*}=\frac{(S_1S_2'+S_2S_1')|_{\eta=\eta^*}\;-\;(S_3')|_{\eta=\eta^*}}{2(S_1^2+S_2)|_{\eta=\eta^*}}\neq 0, \qquad \text{and} \qquad \lambda_3(\eta^*)=-S_1(\eta^*)\neq 0.$$

Therefore, the transversality condition holds. This implies that the Hopf bifurcation occurs at $\eta=\eta^*$ and is non-degenerate. □

The rationale here is that a decrease in the rate of population growth can lead to cyclical fluctuations between employment level, the profit share and the rate of capacity utilization. If population growth decreases, the employment level tends to increase. From expression (5), an increase in e leads to a decrease in the profit share. The effect of a decrease in the profit share in the rate of capacity utilization depends on the magnitude of the first and second terms of the right hand side of expression (10). On one hand if $\varphi(e,h)$ decreases faster than $\left(\frac{s}{v}\right)hu$ when h decreases, we conclude that the rate of capacity utilization decreases, which characterizes this economy as profit-led. On the other hand, if $\varphi(e,h)$ decreases slower than $\left(\frac{s}{v}\right)hu$ when h decreases, the rate of capacity utilization increases, and we face a wage-led regime.

Within the Goodwin literature in which the profit squeeze is the outcome, the profit-led regime is the most probable outcome, and it can be obtained by considering that $\varphi_h(e,h)$ is sufficiently large[5] to guarantee that $\varphi(e,h)$ decreases faster than $\left(\frac{s}{v}\right)hu$ when h decreases. But in the end, the prevalence of wage-led or profit-led regimes remains an empirical question that will be addressed in the next section.

Besides, the character of the occurring Hopf-bifurcation is however difficult to determine in the third dimension and thus must be a matter of numerical simulations of the model[6]. We now present a numerical simulation to verify the local asymptotically stability of the positive equilibrium point and to describe how a phase portrait changes as a parameter changes.

Let $(s/v) = 0.6, \beta = 0.5,$ and $\varphi(e,h) = e^2 - h^2 - 6e + 8h$, then system then system formed formed by expressions (5), (9) and (10) becomes:

$$e' = e\,(\,0.6\,hu - \eta) = F(e,h,u), \qquad (5)'$$

$$h' = 0.5\,(1-h)(e^o - e) = G(e,h,u), \qquad (9)'$$

$$u' = u(\,e^2 - h^2 - 6e + 8h - 0.6\,hu\,) = H(e,h,u), \qquad (10)'$$

where $\frac{\partial\varphi}{\partial e} = 2e - 4 < 0;\ \ \frac{\partial\varphi}{\partial h} = 4 - 2h > 0,\ \ 0 < e,h < 1.$

In steady state the relevant solution $P^* = (e^*,\ h^*,\ u^*)$ (with economic meaning) may be obtained by equations $F(e,h,u) = 0,\ G(e,h,u) = 0,\ H(e,h,u) = 0.$

From the second equation , $e^* = e^o$, since $h < 1$. From the third equation we obtain the value of $h^* = 4 - \sqrt{(e^o)^2 - 6e^o + (16 - \eta)}$. The value of u^* is then given by $u^* = \left(\frac{5}{3}\right)\frac{\eta}{h^*}$. We verify the situation of a Hopf bifurcation for the system (5)', (9)' and (10)' in the special formulation of this section. Here, we consider: $\eta \in [\eta_1, \eta_2] = [0.2866238100,\ \ 0.2866238200]$.

[5] Note that this conditions is consistent with the proposition 2, in which $\varphi_h(e,h)$ has to be sufficiently large to guarantee the stability of the system.

[6] This approach is according to Chiarella et al. (2005, p. 99) who consider that "the local analysis – along with a Hopf bifurcation result – serves us as a first piece of information about the economy's general potential for oscillatory behaviour. It thus provides a firm basis for a subsequent global analysis, which from three dimensions on will soon have to resort to computer simulations."

Here, the bifurcation diagram in the $(e^o, \eta) -$ plane of system (5)', (9)' and (10)' for $\frac{s}{v} = 0.6, \beta = 0.5$, is given by the equation of Andronov-Hopf :

$$E(e^o, \eta) = \eta e^o \left(\eta \varphi_e(e^o, h^*) + \beta \left(\varphi_h(e^o, h^*) - \left(\frac{s}{v}\right) u^*\right)(1 - h^*)\right) = 0.$$

Hence: $\eta(2e^o - 6) + 0.5(8 - 2h^* - 0.6u^*)(1 - h^*) = 0$, with $h^* = 2 - \sqrt{(e^o)^2 - 6e^o + (16 - \eta)}$ and $u^* = \left(\frac{5}{3}\right)\frac{\eta}{h^*}$

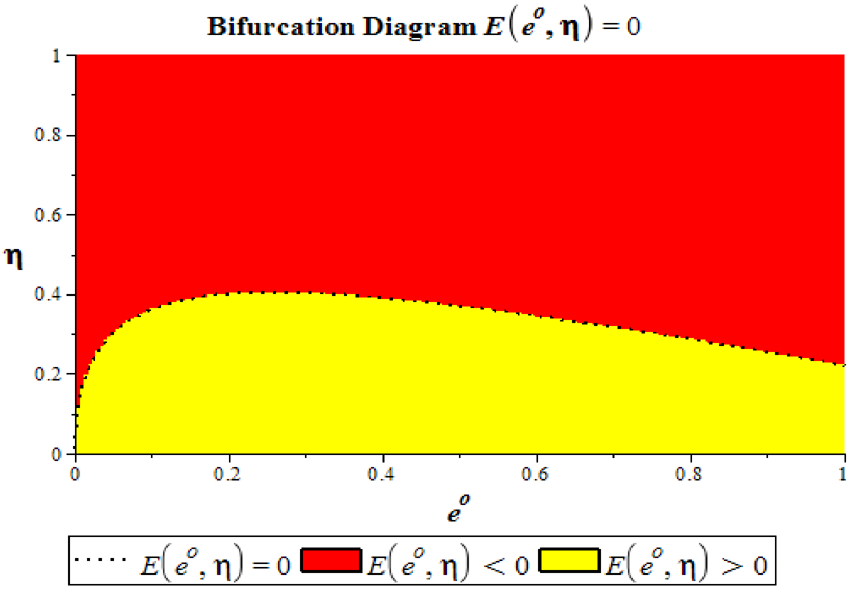

Figure 2. Bifurcation diagram $E(\eta. e^o) = 0$ for system (5)', (9)' and (10)' when $\frac{s}{v} = 0.6$; $\beta = 0.5$. If $E(\eta, e^o) > 0$, it follows that $P^*$is stable; if $E(\eta, e^o) < 0$, then P^* is unstable. The Hopf bifurcation parameter is given by $\eta = \eta^* \approx 0.28662381848812924231 4$ if $e^o = 0.8$.

We observe that:

(i) If $\eta_1 = 0.28662381$, then system (5)', (9)' and (10)' has a positive equilibrium: $P^* = (0.8, 0.6009742293, 0.7948865804)$, which is a saddle spiral with *unstable focus*, with eigenvalues: $\lambda_{2,3} = 4.28828389464897 \times 10^{-9} \pm 1.04165208629811\, i$, $\lambda_1 = -0.286623818876568$. Here: $\lambda_1 < 0$, $Re\, \lambda_{2,3} > 0$; with characteristic polynomial: $det(J, \lambda) = \lambda^3 + S_1\lambda^2 + S_2\lambda + S_3$; where $S_1 = 0.2866238103 > 0, S_2 = 1.085039066 > 0$, and $S_3 = 0.3109980416 > 0$; Besides: $E(\,0.8,\ \eta_1) = -1.02\,(10)^{-8} < 0$.

(ii) If $\eta_2 = 0.28662382$, then system (5)', (9)' and (10)' has a positive equilibrium: $P^* = (0.8,\ 0.6009742308,\ 0.7948866061)$ which is a spiral *stable focus*, with eigenvalues: $\lambda_{2,3} = -8.22123091825944 \times 10^{-10} \pm 1.04165210291311\, i$, $\lambda_1 = -0.286623818455754$. Here: $\lambda_1 < 0$, $Re\, \lambda_{2,3} < 0$; with characteristic polynomial: $det(J, \lambda) = \lambda^3 + S_1\lambda^2 + S_2\lambda + S_3$; where $S_1 = 0.2866238201 > 0, S_2 = 1.085039104 > 0$, $S_3 = 0.3109980509 > 0$; and $E(\,0.8,\ \eta_2) = 9 \times\ 10^{-10}$.

If we decrease the values of η from η_2 to η_1 , by the Intermediate Value Theorem, applied to the function $E(0.8,\ \eta)$, there exists at least one $\eta = \eta^* \epsilon [\eta_1, \eta_2]$ such that $E(0.8,\ \eta^*) = 0$, that is, the complex eigenvalues are purely imaginary. It is possible to show that the Hopf bifurcation parameter that satisfies $E(\,0.8,\ \eta^*\,) = 0$ is given by $\eta = \eta^* = 0.2866238184881 29242314$. To apply the Hopf bifurcation theorem, we need to verify the transversality condition, namely,

$$\frac{d}{d\eta} Re(\lambda_i(\,\eta))\ |_{\eta=\eta^*} = \frac{(S_1 S_2' + S_2 S_1')|_{\eta=\eta^*}\ -\ (S_3')|_{\eta=\eta^*}}{2(S_1^2 + S_2)|_{\eta=\eta^*}} \neq 0 \qquad (18)'$$

where: $S_1(e^o, \eta) = -\eta$, $S_2(e^o, \eta) = e^o\left(6\eta - \beta\eta - 2\eta e^o + \frac{\beta\eta}{h^*}\right)$, $S_3(e^o, \eta) = -2\beta e^o \eta(1 - h^*)(4 - h^*)$, $S_1'(e^o, \eta) = -1$, $S_2'(e^o, \eta) = e^o(\,-\beta - 2e^0 + 6 + \frac{\beta}{4 - \sqrt{e^{o2} - 6e^o + 15.71337618}} - \frac{0.1433119092\beta}{\left(4 - \sqrt{e^{o2} - 6e^o + 15.71337618}\right)^2 \sqrt{e^{o2} - 6e^o + 15.71337618}}$ and $S_3'(e^o, \eta) = e^o(h^2 - (5 + 2\eta)h + (4 + 5\eta)$. Calculating the derivative at $\eta = \eta^* = 0.2866238184881 29242314$, we have:

$$\frac{d}{d\eta} Re\big(\lambda_i(\,\eta)\big)\ |_{\eta=\eta^*} = -1.2582351131 \neq 0$$

So it is clear that system enters into a Hopf bifurcation for decreasing η, and $e(t)$, $h(t)$ and $u(t)$ show oscillations when $\eta = \eta^* = 0.2866238184881 29242314$. The oscillatory coexistence of $e(t), h(t)$ and $u(t)$, are illustrated in figures 3-10, with positive equilibrium point: $P^* = (0.8,\ 0.6009742305,\ 0.7948866023)$.

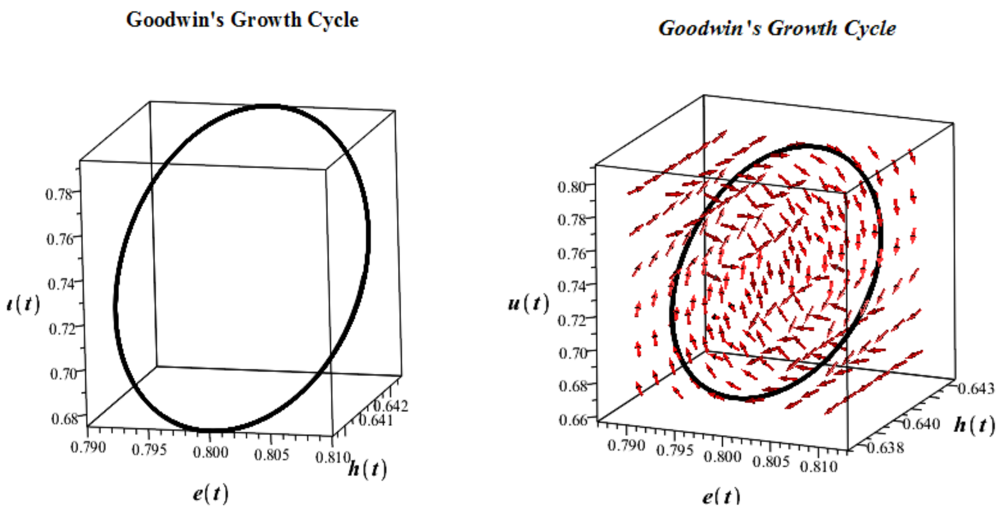

Figure 3. The periodic solution for system (5)', (9)' and (10)' in the space (e, h, u), where $\beta = 0.5$, $e^o = 0.8, t = 0..100$. The initial conditions are $(e(0),\ h(0), u(0)) = (0.81,\ 0.6404823688,$ $0.732516671)$, with Hopf bifurcation parameter $\eta = \ \eta^* = 0.28662381848812924231 4$.

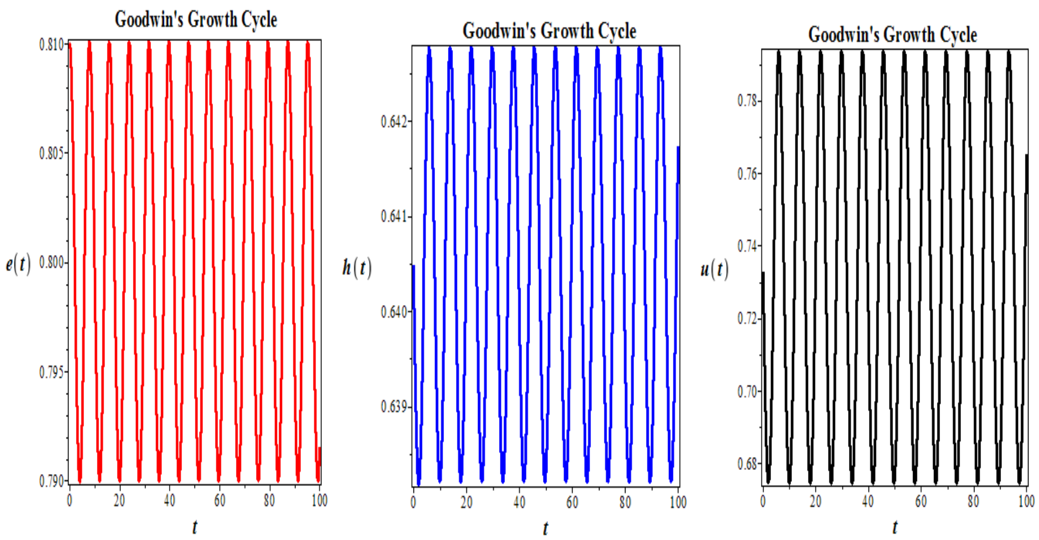

Figure 4. Time series solutions in the planes $(t, e(t)), (t, h(t))$ and $(t, u(t))$, respectively, where $\beta = 0.5$, $e^o = 0.8, t = 0..100$. The initial conditions are $(e(0), h(0), u(0)) = (0.81, 0.6404823688,$ $0.732516671)$, with Hopf bifurcation parameter $\eta = \ \eta^* = 0.286623818488129242314$.

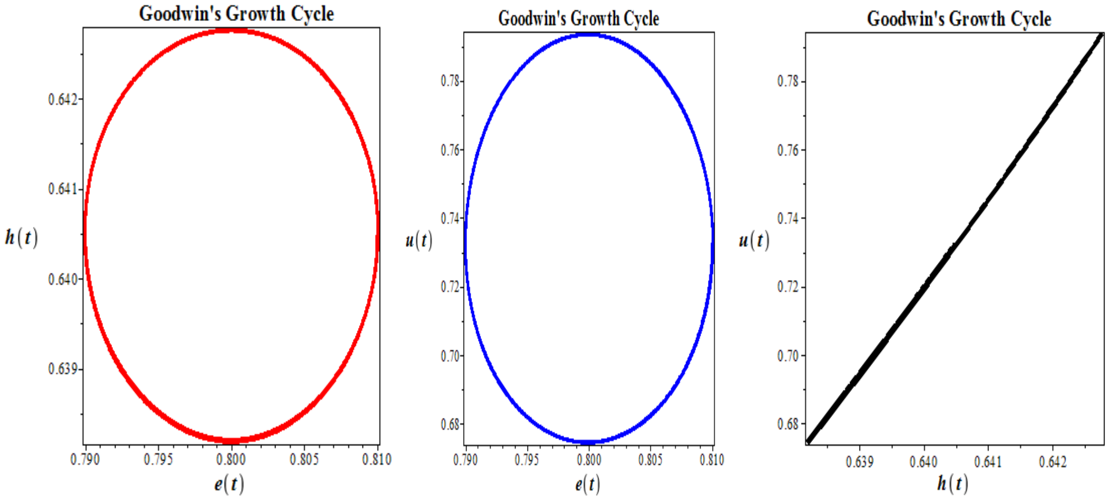

Figure 5. The $eh-, eu-$, and $hu-$ projections of the solution, where $\beta = 0.5,\ e^{o} = 0.8, t = 0..100$. The initial conditions are $(e(0),\ h(0), u(0)) = (0.81,\ 0.6404823688,\ 0.732516671)$, with Hopf bifurcation parameter $\eta = \ \eta^{*} = 0.286623818488129242314$.

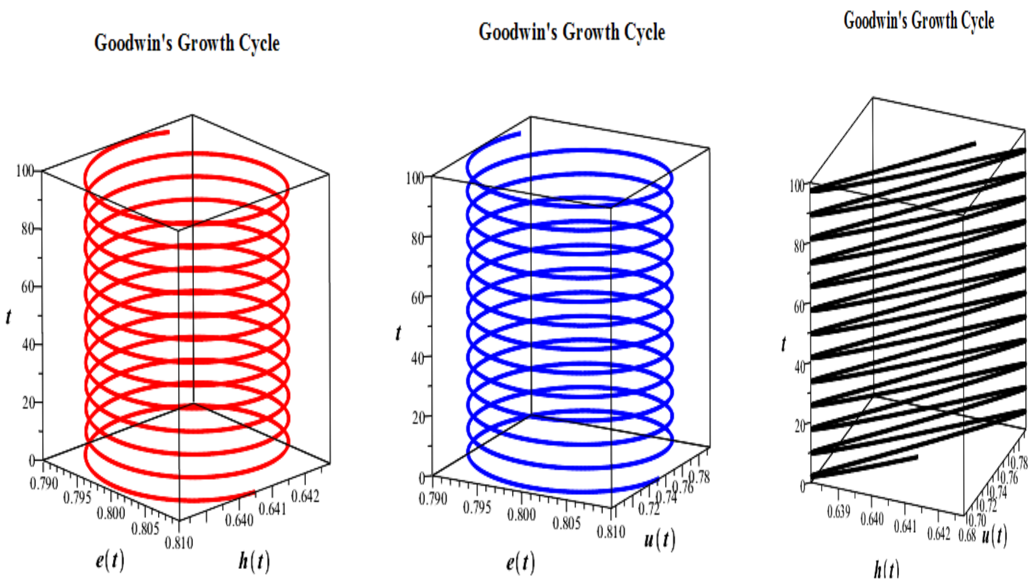

Figure 6. The solutions in the $eht-$, $eut-, hut-$ spaces, respectively, where $\beta = 0.5,\ e^{o} = 0.8, t = 0..100$. The initial conditions are $(e(0), h(0), u(0)) = (0.81, 0.6404823688,\ 0.732516671)$, with Hopf bifurcation parameter $\eta = \ \eta^{*} = 0.286623818488129242314$.

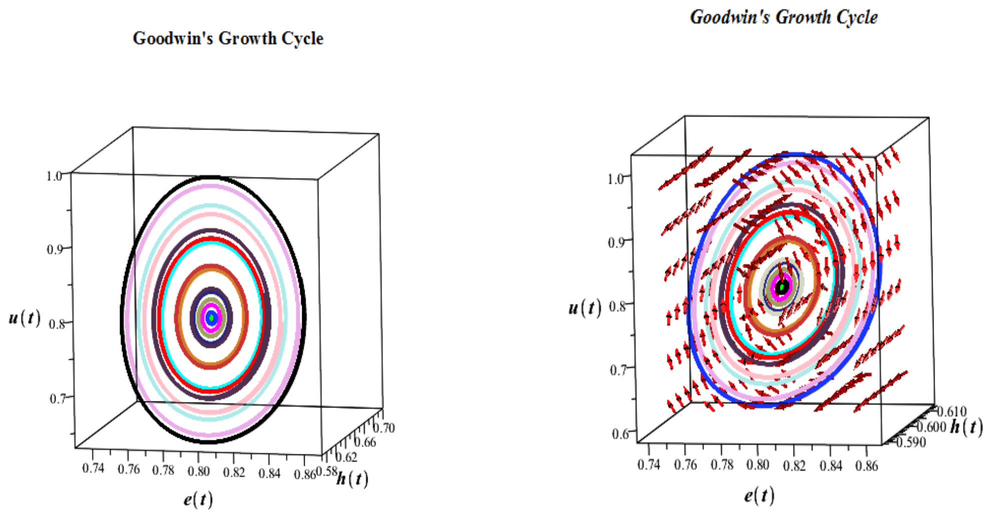

Figure 7. Several periodic solutions in the space (e, h, u), when $\beta = 0.5$, $e^o = 0.8, t = 0..100$ with Hopf bifurcation parameter $\eta = \eta^* = 0.28662381848812924231 4$.

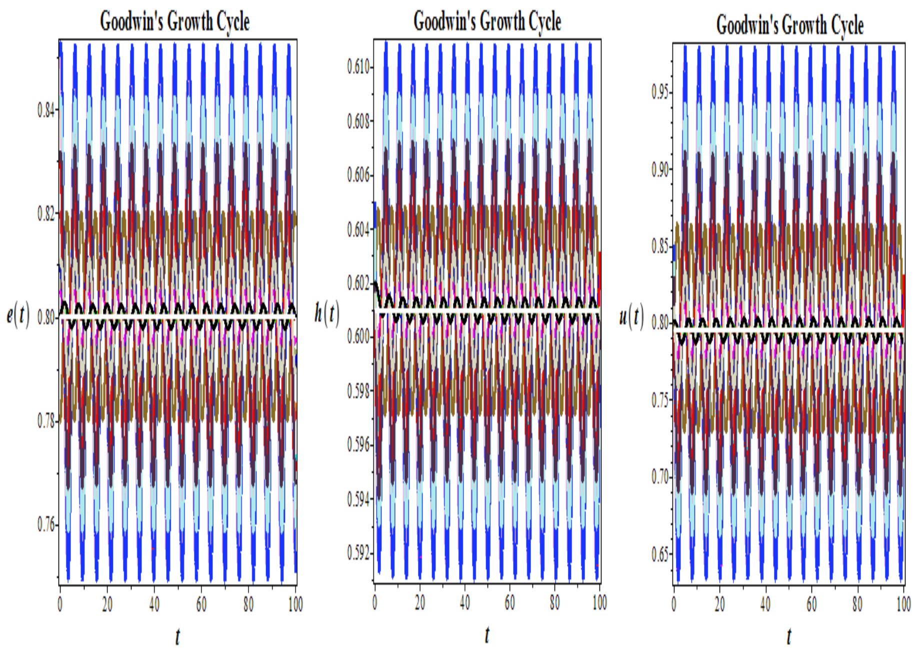

Figure 8. Time series solutions in the planes $(t, e(t)), (t, h(t))$ and $(t, u(t))$, respectively, $\beta = 0.5$, $e^o = 0.8, t = 0..100$ with Hopf bifurcation parameter $\eta = \eta^* = 0.286623818488129242314$.

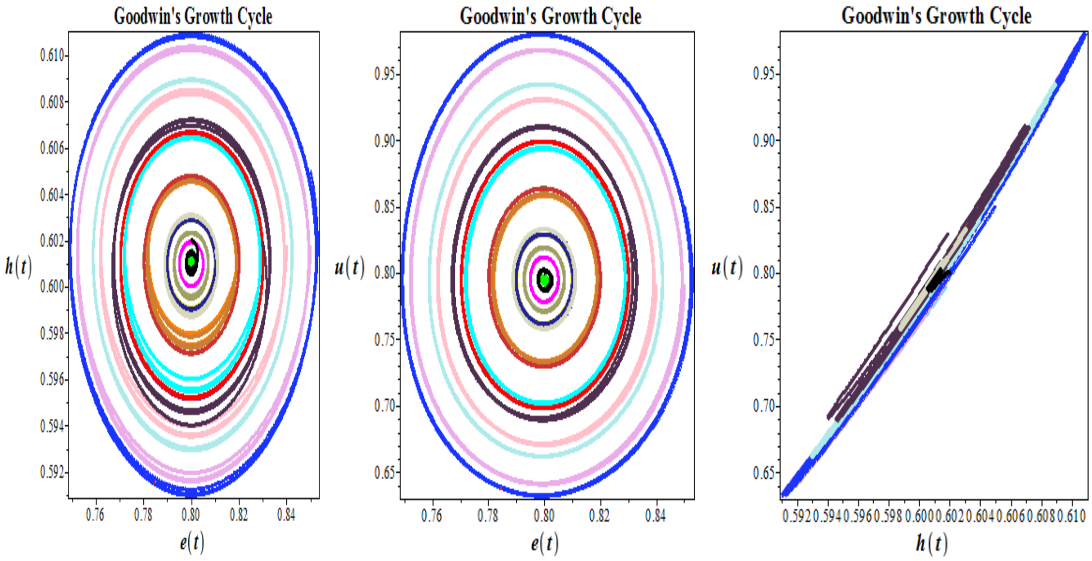

Figure 9. The $eh-, eu-$, and $hu-$ projections of the solutions, when $\beta = 0.5$, $e^{o} = 0.8, t = 0..100$ with Hopf bifurcation parameter $\eta = \eta^{*} = 0.28662381848812924 2314$.

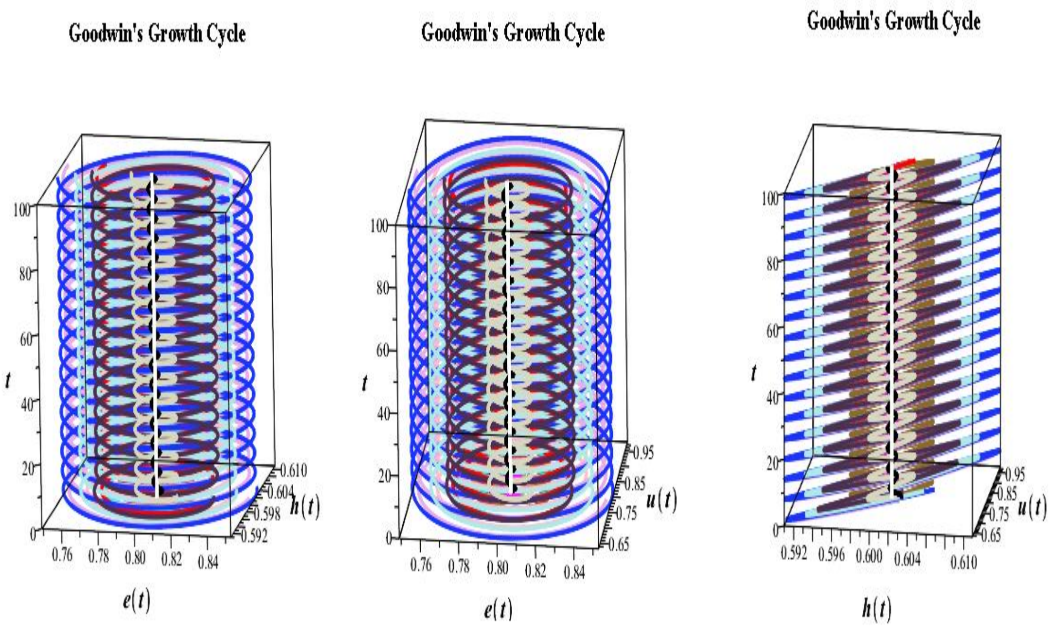

Figure 10. Several solutions in the $eht-$, $eut-, hut-$ spaces, respectively, when $\beta = 0.5$, $e^{o} = 0.8, t = 0..100$, with Hopf bifurcation parameter $\eta = \eta^{*} = 0.286623818488129242314$.

4. Testing the Model for the US economy

The theory of distributive endogenous cycles has been examined empirically in a significant number of studies. Qualitative support can be found in Desai (1984), Harvie (2000), Mohun and Veneziani (2008), and Zipperer and Skott (2011), among others. On the one hand, Goldstein (1999), Barbosa-Filho and Taylor (2006), Tarrassow (2010), Moura Jr. and Ribeiro (2013), Basu, Chen and Oh (2013), Kiefer and Rada (2015) and Barbosa-Filho (2016) provide parametric quantitative evidence. On the other hand, non-parametric treatments have also been recently explored by Kauermann, Teuber and Flaschel (2012).

The present paper adopts a country-specific time-series approach to the problem. We closely follow the methodology used by Barbosa-Filho (2016) and Jump (2016). Taking as departure point the model developed in the previous sections, we address the relation between the employment level, the profit-share and the rate of capacity utilization for the US economy. The main innovation of this section exactly relies in confronting these three series altogether.

From an empirical point of view, distributive cycles can be interpreted as the short-run dynamics that generate the long-run trend as result of non-linear interactions. Therefore, we detrend time series using the traditional Hodrick-Prescott filter before estimating a Vector Auto Regressive (VAR) model. The respective impulse response functions are studied. Finally, Granger causality tests allow us to confront the requirements of the theory, given the signs of the point estimates agreeing with it.

Data

Our dataset is quarterly and comprehend the period between 1967 and 2016. Employment rate series (from now on just employment) were obtained from the U.S. Bureau of Labour Statistics. The profit-share was computed using data from the Bureau of Economic Analysis as the opposite of the compensation of employees as percentage of the net domestic income at production prices. Finally, we use the capacity utilization index provided by the Board of Governors of the Federal Reserve System. For the VAR estimations data was converted to logarithmic form.

We detrended our time series using the traditional Hodrick-Prescott filter with a smoothing parameter of 1600 as depicted in figure 1. Values of cyclical deviations of the trend are showed on the left axis while values of the trend itself are on the right.

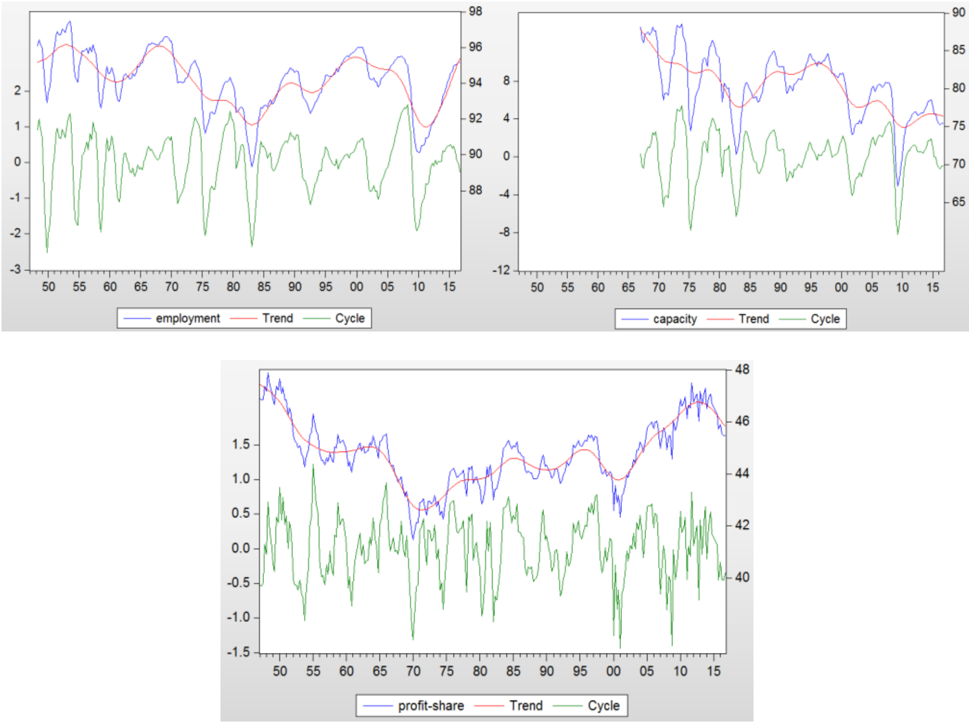

Figure 11: Trend and Cycle of Employment, Profit-share and Capacity utilization

Employment rates fluctuate around 95% with a peak of 97.5% during the fifties and a valley of 90% in 1983 and after the 2007 financial crisis. Capacity utilization on the other hand presents a declining trend going from 88% in the beginning of the series to 75% by the end of the period. Finally, the profit share exhibits a U-shaped trajectory. It declines from 48% in the late forties to 43% just before the oil shocks, and recovers to 47% in 2010.

Distributive cycles both in terms of employment and capacity utilization can be visualized in figure 2. Looking first to employment (left) the first cycle goes from 1948 until 1960 when there seems to be a structural break and a cycle with a new centre begins. It goes until 1977 when we have a new break. The third wave comprehends the period between 1977 and 1988. A fourth cycle can be identified until 2001. Between 2001 and 2007 employment and profit-share went in opposite directions but without a clear cyclical pattern. The last cycle initiates with the financial crisis in 2007 and continues nowadays. All of them are clockwise following what is expected from theory.

The picture does not change significantly if we look to the level of capacity utilization (right). A first clockwise cycle can be observed from 1967 to 1977. After what seems to have been a structural break, a new wave begins that goes until 1990. Two more cycles follow the first until 2005 and another one until 2016.

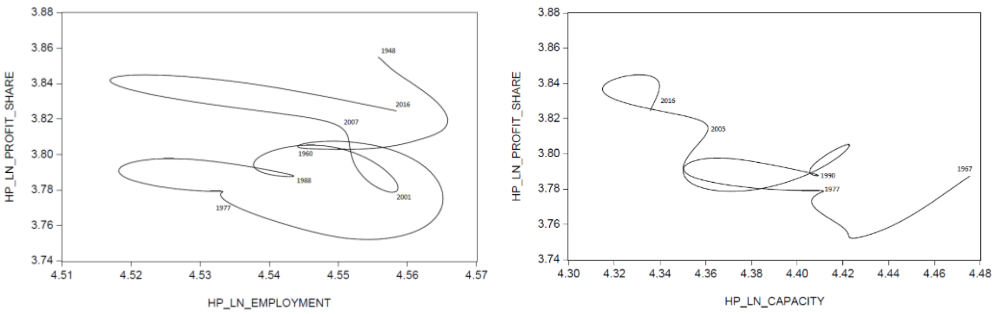

Figure 12: Visual distributive cycles

Impulse Response functions

The presence of structural breaks is tested due to the sequential Bai-Perron test. Several structural breaks are identified for the employment level (1958Q3, 1974Q4, 1987Q1, 2006Q4), the profit share (2005Q1) and the rate of capacity utilization (1974Q4, 1987Q3, 2001Q1). Therefore, we included dummy variables to capture the structural break effects. One dummy variable was assigned for each indicator. They assume value 1 for years with break and 0 for years with no break.

A VAR model with 2 lags is estimated following the Schwarz lag order selection criteria. The Schwarz criterion was preferred over the popular Akaike criteria insofar as usually it assigns a lower number of lags which in this case is desirable given the nature of the relation we are addressing[7]. Figure 3 presents the impulse response functions to study how the series react to shocks from other variables. Shocks to each error traces out cyclical oscillations in time paths of each of the three variables:

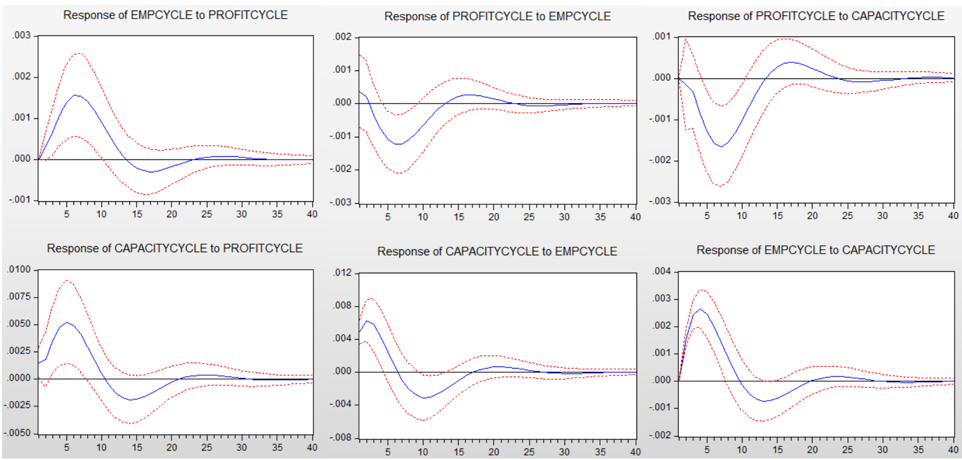

Figure 13: Impulse Response to Cholesky One S.D. Innovations, Monte Carlo: 10000 repetitions

[7] Schwarz criterion is strongly consistent while Akaike is not consistent, but it is generally more efficient. In other words, while the former will asymptotically deliver the correct model order the latter will deliver on average too large a model [Brooks (2014)].

The first column presents the response of employment and capacity utilization to an exogenous shock in the profit share. Both variables respond positively following from what is expected from the theoretical model. After 10 periods the response becomes slightly negative so we need to further investigate the net effect even though it seems to be clearly positive. The second column (centre) indicates that a shock in the employment level reduces the profit share as predicted by the predator-prey mechanism. The rate of capacity utilization initially increases but after 5 periods it starts to decrease and the net effect is not straightforward. Finally, a positive shock in the rate of capacity utilization (third column) strongly decreases the profit share. This result is similar to the predator-prey mechanism and is in line with the strong correlation between employment and capacity utilization. As a matter of fact, employment responds positively to a shock in the capacity utilization. Figure 4 presents the accumulated impulse response functions to further study the net effects on a certain variable of exogenous shocks in the other variables:

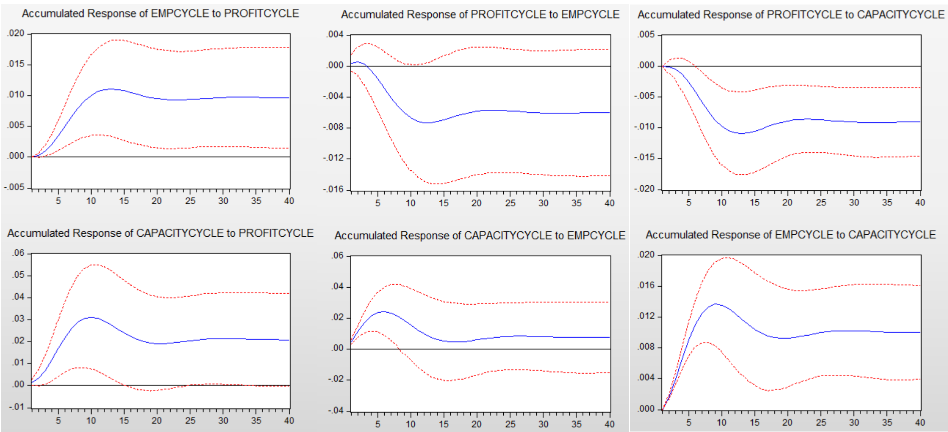

Figure 14: Accumulated Response to Cholesky One S.D. Innovations, Monte Carlo: 10000 repetitions

Results clearly indicate that while an increase in the profit-share increases employment an increase in the rate of employment reduces the profit share. This corresponds to the traditional distributive cycle mechanism. Similar dynamics are observed between the rate of capacity utilization and the profit-share. Finally, we also can state that employment and capacity utilization go always in the same direction. Convergence in all cases takes around 20 quarters.

Stability of the VAR system is verified checking if all eigenvalues lie inside the unit circle. To assess a valid inference and not spurious regressions, residuals are checked for serial correlation. We conclude that our estimates satisfy the stability condition and are consistent. Results of those tests are reported in the appendix (figure A1 and table A1). VAR coefficients are available under request.

Granger causality

Granger causality Wald test is reported in table 1. Results support the hypothesis that employment, profit-share and capacity utilization cause each other. For instance, the profit-share and capacity utilization jointly Granger cause the employment rate. On the other hand, employment and capacity utilization jointly cause the profit-share. Last but not east, the profit-share and employment cause capacity utilization. Table 1 reports the estimate p-values:

Dependent variable	Chi-sq	p-values
Employment	183.8564	0.0000
Profit-share	19.55965	0.0006
Capacity Utilization	12.30311	0.0152

Table 1: Causality tests

As discussed in the beginning of this section from an empirical point of view, distributive cycles can be interpreted as the short-run cycles that generate the long-run trend as result of non-linear interactions. Using quarterly data for the United States we do find some empirical support for the theoretical formulation developed in this paper.

5. Concluding Remarks

In this paper we have extended Goodwin's model to consider the rate of capacity utilization as a new variable in the system. The introduction of this variable allows us to better understand the connections amongst employment, profit share and rate of capacity utilization. The extended model displays a stable equilibrium growth path that can be destabilized into a limit cycle due to an increase in the growth rate of population. Finally the model is tested econometrically to the US economy. We have identified a cointegration relation amongst variables which means that they are related in the long run in a stable way but causality relations were found amongst them only the number of lags is increased. By using generalized impulse-response functions provided by the VAR model we have concluded that a positive profit share innovation affects positively the employment rate but has a positive effect on the rate of capacity utilization, suggesting a profit-led pattern. From these results we conclude that impulse response functions also give support to the theoretical model presented, especially to the profit-squeeze mechanism. In sum the results found support Goodwin's idea that cycles are an inherent characteristic of the US growth process in the last decades.

Appendix

Figure A1 reports results of VAR unit circle stability:

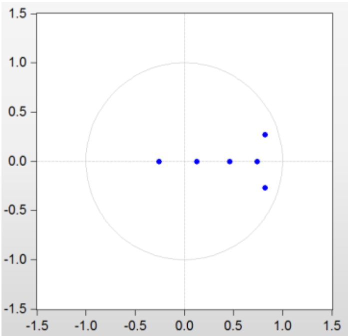

Figure A1: Inverse Roots of AR Characteristic Polynomial

Table A1 reports results for the tests of errors serial correlation:

Lags	LM-stat	Prob.
1	13.75538	0.1313
2	10.92809	0.2807
3	13.11365	0.1575

Table A1: VAR residual serial correlation LM test – Null hypothesis: No serial correlation

Thus, we cannot reject the null hypothesis of no serial correlation of errors.

References

Arnim, R. and Barrales, J. Demand-driven Goodwin cycles with Kaldorian and Kaleckian features. *Review of Keynesian Economics*, 3(3): 351 – 373, 2015.

Asada, T., Chiarella, C., Flaschel, P. and Franke, R. *Open Economy Macrodynamics: An Integrated Disequilibrium Approach*. Springer-Verlag, 2003.

Asada, T., Chiarella, C., Flaschel, P., Mouakil, T., Proaño, C. and Semmler, W. Stock-flow interactions, disequilibrium macroeconomics and the role of economic policy. *Journal of Economic Surveys*, 25(3): 569 – 599, 2011.

Barbosa-Filho, N. Elasticity of substitution and social conflict: a structuralist note on Piketty's *Capital in the Twenty-first* Century. *Cambridge Journal of Economics*, 40(4): 1167 – 1183, 2016.

Barbosa-Filho, N., Taylor, L. Distributive and demand cycles in the US economy: a structuralist Goodwin model. *Metroeconomica*, 57 (3): 389–411, 2006.

Basu, D.; Chen, Y. and Oh, J. Class struggle and economic fluctuations: VAR analysis of the post-war US economy. *International Review of Applied Economics*, 27(5): 575 – 596, 2013.

Blanchard, O. and Kiyotaki, N. Monopolistic Competition and the Effects of Aggregate Demand. *The American Economic Review*, 77(4): 647 – 666, 1987.

Blecker, R. Wage-led versus profit-led demand regimes: the long and the short of it. *Review of Keynesian Economics*, 4(4): 373–390, 2016.

Brooks, C. *Introductory Econometrics for Finance*. 3rd edition, Cambridge University Press, 2014

Chiarella, C., Flaschel, P. and Franke, R. *Foundations for a Disequilibrium Theory of the Business Cycle: Qualitative Analysis and Quantitative Assessment*. Cambridge University Press, 2005.

Choi H. Goodwin's growth cycle and the efficiency wage hypothesis. *Journal Economic Behavior & Organization*, 27:223–235, 1995.

Desai, M. An econometric model of the share of wage in national income: UK 1855-1965. In: Goodwin R., Vercelli, A. Kruger, M. (Ed.) *Nonlinear Models of Fluctuating Growth*. Berlin: Springer-Verlag, p. 253 –277, 1984.

Desai, M. Growth Cycles and Inflation in a Model of Class Struggle. *Journal of Economic Theory*, 6:527 –545, 1973.

Dixon, H. Equilibrium and Explanation. In: Creedy, J. (Ed.), *Foundations of Economic Thought*. Oxford: Basil Blackwell, 1990.

Flaschel, P. Goodwin's MKS system: a baseline macro model. *Cambridge Journal of Economics*, 39(6): 1591 – 1605, 2015.

Foley, D. Endogenous Technical Change with Externalities in a Classical Growth Model. *Journal of Economic Behavior & Organization*, 52: 167 – 189, 2003.

Franke, R., Flaschel, P. and Christian P. (2006). Wage–price dynamics and income distribution in a semi-structural Keynes–Goodwin model. *Structural Change and Economic Dynamics* 17, 452–465.

Goldstein, J. Predator-Prey model estimate of the cyclical profit squeeze. *Metroeconomica*, 50(2): 139 – 17, 1999.

Harcourt, G. Fusing indissolubly the cycle and the trend: Richard Goodwin's profound insight. *Cambridge Journal of Economics,* 39(6): 1569 –1578, 2015.

Harvie, D. Testing Goodwin: growth cycles in ten OECD countries. *Cambridge Journal of Economics*, 24: 349 –376, 2000.

Hein, E. (2014), *Distribution and Growth After Keynes: A Post-Keynesian Guide*, Cheltenham, UK, Edward Elgar.

Julius, A. Steady State Growth and Distribution with an Endogenous Direction of Technical Change. *Metroeconomica*, 56(1): 101-125, 2006.

Jump, R. An empirical analysis of the profit squeeze theory for the USA and UK. *FMM Conference*, 2016.

Kauermann, G.; Teuber, T. and Flaschel, P. Exploring US Business Cycles with Bivariate Loops Using Penalized Spline Regression. *Computational Economics,* 39: 409 – 427, 2012.

Keen, S. Finance and Economic Breakdown: Modeling Minsky's Financial Instability Hypothesis. *Journal of Post Keynesian Economics,* 17(4): 607-635, 1995.

Keen, S. A monetary Minsky model of the Great Moderation and the Great Recession. *Journal of Economics Behavior & Organization,* 86: 221-235, 2013.

Kiefer, D. and Rada, C. Profit maximizing goes global: the race to the bottom. *Cambridge Journal of Economics*, 39(5): 1333-1350, 2015.

Kydland, F. and Prescott, E. Time to Build and Aggregate Fluctuations. *Econometrica* , 50(6): 1345 – 1370, 1982.

Mariolis, T. Goodwin's Growth Cycle Model with the Bhaduri-Marglin Accumulation Function. *Evolutionary Institutional Economic Review*, 10(1): 131-144, 2013.

Mohun, S., Veneziani, R. Goodwin cycles and the U.S. economy 1948–2004. In: Flaschel, P., Landesmann, M. (Eds.), *Mathematical Economics and the Dynamics of Capitalism: Goodwin's legacy continued*. Routledge, New York, 2008.

Moura Jr. N. and Ribeiro, M. Testing the Goodwin growth-cycle macroeconomic dynamics in Brazil. *Phisica A*, 392: 2088-2103.

Long, J. and Plosser, C. Real Business Cycles. *Journal of Political Economy*, 91(1): 39 – 69, 1983.

Pohjola, M. Stable, cyclic and chaotic growth: the dynamics of a discrete-time version of Goodwin's growth cycle model. *Journal of Economics*, 41(1-2): 27-38, 1981.

Rodousakis, N. Goodwin's growth cycle model with the Bhaduri–Marglin accumulation function: a note on the C.E.S. case. *Evolutionary Institutional Economic Review*, 12(1): 145 – 171, 2015.

Rotemberg, J. and Woodford, M. An Optimization-Based Econometric Framework for the Evaluation of Monetary Policy in: *NBER Chapters, National Bureau of Economic Research, Inc*. 1997.

Sasaki, H. Cyclical growth in a Goodwin-Kalecki-Marx model. *Journal of Economics,* 108: 145 – 171, 2013.

Sato, Y. Marx-Goodwin growth cycles in a two-sector economy. *Journal of Economics*, 45(1):21–34, 1985.

Schoder, C. Instability, stationary utilization and effective demand: A structuralist model of endogenous cycles. *Structural Change and Economic Dynamics,* 30:10 – 29, 2014.

Shah, A. and Desai, M. Growth Cycles with Induced Technical Change. *Economic Journal,* 91(364): 1006 – 1010, 1981.

Skott, P. *Conflict and Effective Demand in Economic Growth*. Cambridge University Press, Cambridge, UK, 1989.

Skott, P. Effective Demand, Class Struggle and Cyclical Growth. *International Economic Review*, 30(1): 231 – 247, 1989.

Skott, P. Theoretical and empirical shortcomings of the Kaleckian investment function. *Metroeconomica* 63(1), 109–138, 2012.

Skott, P. and Ryoo, S. Macroeconomic implications of financialisation. *Cambridge Journal of Economics*: 32, 827–862, 2008

Solow, R. A Contribution to the Theory of Economic Growth. *Quarterly Journal of Economics*: 70, 65–94, 1956.

Sordi, S. and Vercelli, A. Financial fragility and economic fluctuations. *Journal of Economic Behavior & Organization*, 61:543 – 561, 2006.

Sordi, S. and Vercelli, A. Heterogeneous expectations and strong uncertainty in a minskyian model of financial fluctuations. *Journal of Economic Behavior & Organization*, 83:544 – 557, 2012.

Sordi, S. and Vercelli, A. Unemployment, income distribution and debt-financed investment in a growth cycle model. *Journal of Economic Dynamics and Control*, 48:325 – 348, 2014.

Sportelli, M. A Kolmogoroff generalized predator-prey model of Goodwin's growth cycle. *Journal of Economics*, 61(1):35–64, 1995.

Sportelli, M. Dynamic Complexity in a Keynesian Growth-Cycle Model Involving Harrod's Instability. *Journal of Economics,* 71(2): 167 – 198, 2000.

Stockhammer, E. and Michell, J. Pseud-Goodwin cycles in a Minsky model. *Cambridge Journal of Economics,* 41(1): 105 – 125, 2017.

Tavani, D. and Zamparelli, Distributive Conflict, Growth, and the 'Entrepreneu State'. *American Economic Association Annual Meeting*, 2017.

Tarassow, A. The empirical relevance of Goodwin's business cycle model for US economy. MPRA Series 21012. University Library of Munich, Munich, 2010.

Toda, H. and Yamamoto, T. Statistical inference in vector auto regressions with possibly integrated processes *Journal of Econometrics*, 66: 225 – 250, 1995.

Van der Ploeg, F. Predator-Prey and Neoclassical Models of Cyclical Growth, *Journal of Economics*, 43:235 –256, 1983.

Van der Ploeg, F. Growth cycles, induced technical change, and perpetual conflict over the distribution of income, *Journal of Macroeconomics,* 9(1):1–12, 1987.

Velupillai, V. A disequilibrium macrodynamic model of fluctuations. *Journal of Macroeconomics,* 28: 752 –767, 2006.

Wiggins, S.. *Introduction to applied Nonlinear Dynamical systems and Chaos*. Springer-Verlag, New York, 1996.

Wolfstetter, E. Fiscal Policy and the Classical Growth Cycle. *Journal of Economics,* 42(4): 375 – 393, 1982.

Zipperer, B., Skott, P. Cyclical patterns of employment, utilization, and profitability. *Journal of Post Keynesian Economics*, 34 (1): 25–58, 2011.