On Harrodian Instability: Two Stabilizing Mechanisms May Be Jointly Destabilizing

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Abstract

In economic discussions of dynamic stability, negative feedback effects are contrasted with positive feedbacks. With more complex relationships, stable and unstable submodels are set up, the intuition being that stability in an integrated model would be determined by the stronger forces. Accordingly, a combination of two stabilizing mechanisms will normally be expected to reinforce stability. The present paper gives a counterexample to this intuition. It considers two approaches that have been put forward in the literature to tame Harrodian instability: one by monetary policy acting through (indirect) interest rate effects, and the other by an autonomously growing, non-capacity creating component of aggregate demand. While each of the two mechanisms in isolation stabilizes the steady state if it is sufficiently strong, their interaction will necessarily render it unstable.

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1 Introduction

In order to understand the stability properties of dynamic system systems, one seeks to identify negative and positive feedback loops. The argument is that an equilibrium position will be stable if, in some sense, the stabilizing forces dominate the

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destabilizing mechanisms. Otherwise instability will prevail. A similar reasoning applies to more complex relationships, when one can single out a stable and an unstable submodel (or several of them). According to that intuition, a model made up of two stable submodels will all the more be expected to be stable. Unfortunately, things are not always so straightforward. The present contribution puts forward a simple example where the interaction of two stabilizing models will actually generate instability.

As an introduction to possible complications, we may refer to a two-dimensional Metzlerian model concentrating on the effects of goods market disequilibrium and the firms' decisions to buffer it by an active inventory management. A central parameter in this context is the stock adjustment speed (SAS), which governs how strongly the firms' inventory investment reacts to deviations of the stock of inventories from their desired level. In the two-dimensional world this parameter is unambiguously destabilizing. That is, the steady state is stable if SAS is small enough, and it is unstable for all values of SAS beyond a certain threshold (Franke, 1996; Chiarella et al., 2005, Chapters 7.2.1 and 7.2.2).

This clear characterization loses its validity when the Metzlerian dynamics is integrated into a richer framework, such as the six-dimensional so-called Keynes-Metzler-Goodwin model studied by Chiarella et al. (2005). Depending on other parameters in the full model, low values of SAS are then no longer sufficient for stability. There may in fact be no values of that parameter at all that could stabilize the economy; or a reswitching phenomenon may be obtained, in that the steady state is unstable for low values of SAS, stable over an intermediate range, and unstable again for sufficiently high values of SAS (Chiarella et al., 2005, Chapter 7.2.3). These observations may serve as a first note of caution against premature stability conclusions in higher-dimensional systems.

In this paper we want to make an even starker point. The full model is made up of two small-scale models and we abstract from other influences. It will then be demonstrated that if for each of the two small models a stability condition is assumed, the steady state of the integrated model is necessarily unstable.[1]

Our point of departure is the neo-Kaleckian baseline model of growth and distribution that faces the Harrodian instability problem when the normal rate of capacity utilization is conceived of as a fixed magnitude. Possible stabilization mechanisms have here been a much debated issue lately; see Hein et al. (2011) or Lavoie (2014, Chapter 6.5) for an overview of a wide array of proposals in the literature. We consider two approaches for our purposes. The first one assumes direct

[1] It may be added that also the reverse phenomenon is possible. Franke and Nesemann (1999) consider two learning rules each of which is unstable. However, a suitably weighted combination of the two is able to bring about stability.

and indirect interest rate effects in aggregate demand, an ordinary Phillips curve, and monetary policy with a standard Taylor rule. Suitably geared, the interest rate adjustments may overcome the process of cumulative causation that the Harrodian forces set in motion. The second approach is based on a non-capacity creating component of demand that autonomously grows at a constant rate. Its role is thus akin to that of an 'automatic stabilizer'.

The first model can be reduced to one dimension, the second adds another dimension. Putting the two together does not increase the number of dimensions. However, their interaction brings an additional feedback loop into existence. As already announced, it is not only destabilizing but also turns out to dominate the two original negative feedback effects.

The remainder of the paper is organized as follows. The next section recapitulates the Harrodian instability problem in the canonical Kaleckian framework. Section 3 integrates several interest rate effects and monetary policy into it. Alternatively, Section 4 introduces an autonomous demand component. Section 5 combines these two models and works out the resulting instability. A phase diagram analysis is here particularly illuminating. Section 6 concludes.

2 Harrodian instability in the Kaleckian baseline model

Let us start out from the problem of Harrodian instability as it is laid out by, for example, Hein et al. (2011, Section 2). To keep things simple, it is considered for a closed one-good economy without taxation, government spending and capital depreciation, where labour is supposed to be in perfectly elastic supply. Rather than follow the practice of referring to capacity utilization as a measure of economic activity, it simplifies the notation if we directly work with the output-capital ratio u in this respect, which is the utilization rate of the capital stock in place. For short, it will be called 'utilization', too. The Kaleckian IS part of the model is thus constituted by the following four basic equations:

$$r = hu \tag{1}$$
$$g^s = sr \tag{2}$$
$$g^i = a + \gamma(u - u^n) \tag{3}$$
$$g^i = g^s \tag{4}$$

In the first equation, r is the rate of profit and h the (fixed) share of profits in total income. The function g^s in (2) represents the saving in the economy normalized by the (replacement value of the) capital stock: the firms' profits are all paid out to the rentiers who save a constant fraction s of them, while the workers consume all of their wages.

The third equation specifies the investment function, i.e. the planned growth rate of the capital stock. Within the short period it is based on a trend rate of growth a, or on the rate at which sales are expected to grow on average in the near future. Frequently, this yardstick is also introduced as the firms' "animal spirits" (therefore the symbol 'a'). More demurely, we may refer to a as the general business *sentiment*, a term that still preserves the psychological and somewhat diffuse character of this type of expectations. The second component of investment in (3) features u^n as a normal rate of capital utilization. In contrast to another branch of Kaleckian modelling, it is here treated as a fixed magnitude, exogenously determined by technological and institutional factors. In case of overutilization $u > u^n$ (or underutilization $u < u^n$) the firms seek to close this gap relatively fast by increasing the capital stock at a proportionately higher (lower) rate than a.

Equation (4) sets up the temporary IS equilibrium, where market clearing is brought about by quantity variations. Utilization is then given by[2]

$$u = u(a) = \frac{a - \gamma u^n}{sh - \gamma} \tag{5}$$

Stability of the underlying ultra short-run quantity adjustment process requires the denominator to be positive, that is, investment must not be too sensitive to changes in utilization. This is the so-called Keynesian stability condition, which we will not call into question. Regarding the numerator it will be noted that in a long-run equilibrium (identified by variables with a superscript 'o'), where firms operate at desired utilization $u = u^o = u^n$, the trend growth rate will be $a = a^o = g^i = g^s = shu^n$. Hence in fact $u(a^o) = (sh-\gamma)u^n/(sh-\gamma) = u^n$. Certainly, $u(a) > 0$ for all a not too much smaller than a^o.

What happens if the economy is off the balanced growth, when $a \neq a^o$ in (5)? The utilization gap tells the firms that they have misread the economic situation, that they have under- or overestimated the current growth of demand. They consequently revise their expectations, which means that the sentiment a is a dynamic variable. Specifically, a is supposed to rise (fall) if u happens to be above (below) its normal level u^n. Translating this idea into formal language, it will be convenient to work in continuous time. Thus, denoting the speed of these adjustments by $\eta_u > 0$, we have,[3]

$$\dot{a} = \eta_u [u(a) - u^n] \tag{6}$$

[2] Since no other situations than IS are considered, we abstain from earmarking these values by an asterisk or a subscript like 'IS'.

[3] A dot above a dynamic variable x designates its derivative with respect to time, $\dot{x} = dx/dt$.

The process can only come to a halt if normal utilization is achieved, which as has just been seen is the case for $a = a^o$. If the economy is in a steady state and some news induces the firms to become more optimistic about their future sales, that is, if the sentiment variable jumps to some value $a > a^o$, the correspondingly higher investment raises aggregate demand and the multiplier causes $u(a) > u^o = u^n$. The positive utilization gap leads to a further increase in a, which in turn widens the utilization gap, which in turn increases a, etc. Hence we have a process of cumulative causation that drives the economy more and more away from its long-run equilibrium. The mathematical argument is, of course, $d\dot{a}/da = \eta_u \, du/da > 0$ in (6). Equations (1) – (6) are a particularly simple way to formalize the Harrodian instability problem and prepare the ground for a rigorous discussion on how it may be overcome.

3 Interest rate effects and monetary policy

Interest rates have always played an important role in macroeconomics, where an interest rate reaction function of the central bank has emerged more and more as a device that may possibly stabilize the economy. Usually these functions are specified as some sort of a Taylor rule. Accordingly, the central bank raises the interest rate above its perceived 'natural' value in times of overutilization, and it also raises it more than one-to-one if inflation rises above a certain target rate. This idea works in a straightforward way if investment (or some component of demand) is postulated to be inversely related to the interest rate.[4]

While an interest rate inverse IS curve is a common pedagogical device, Franke (2015) proposes a framework that can better take account of the policy transmission mechanisms in the real world, which are typically of a more indirect nature and take some time to become effective. Here we adopt the specifications from this work that in the end will allow us to reduce the dynamics to a one-dimensional differential equation in the sentiment variable a again. Thus, instead of directly impacting on investment, the interest rate is assumed to affect the firms' expectations; and furthermore not their level but their rate of change. Another feature is that the firms do not react to the nominal interest rate itself but to the difference between their profitability and the real rate of interest. Regarding the latter we will also set up an elementary inflation theory.

[4]The basic components have found their way into the textbooks; see Romer (2000; 2005) or Taylor (2000; 2001, Chapters 24&25). The appendix in Lavoie and Kriesler (2007, p. 404) provides a succinct compilation of several (very) elementary specifications for post-Keynesian modelling. The intellectual copyright is usually given to Duménil and Lévy (1999); see the discussion in Lavoie (2014, Section 6.5.4).

Formally, let i be the nominal rate of interest, π the inflation rate, and ρ the profit differential, $\rho = r - (i - \pi)$. The difference ρ can be interpreted as a risk premium for investment in real capital as opposed to a financial investment in (almost) risk-free bonds. Here the firms have an idea of what provides a decent risk premium $\rho^\star$ for them and against which they evaluate the current risk premium. It may be mentioned that in this way another Kaleckian concept has been introduced into the model, because an influence of ρ on investment can be traced back to Kalecki's (1937) 'principle of increasing risk'.[5]

In this wider framework the firms may revise their expectations even though utilization is normal, namely, if their profitability is not in accord with the benchmark premium $\rho^\star$. For example, a situation $\rho > \rho^\star$ provides better prospects for their investment and so makes them more optimistic. This means, the sentiment variable is supposed to respond to the utilization gap as well as to the gap of the profit differential $\rho - \rho^\star$. With another reaction coefficient $\eta_\rho > 0$ to measure its influence, eq. (6) is extended and reads now,[6]

$$\dot{a} \;=\; \eta_u\,(u - u^n) \;+\; \eta_\rho\,[r - (i - \pi) - \rho^\star] \tag{7}$$

Concerning the precise specification of monetary policy it is convenient to work with the original Taylor rule (Taylor, 1993, p. 202). Accordingly, the interest rate is directly related to the utilization gap $(u - u^n)$ and, with respect to the central bank's inflation target $\pi^\star$, to the inflation gap $(\pi - \pi^\star)$.[7] Introducing the policy coefficients μ_π and μ_u and anchoring the nominal interest rate on $i^\star$, the sum of $\pi^\star$ and a "natural" real rate rate of interest that the central bank may perceive, the interest rate is set at

$$i \;=\; i^\star \;+\; \mu_\pi\,(\pi - \pi^\star) \;+\; \mu_u\,(u - u^n) \tag{8}$$

Certainly, the central bank is supposed to obey the so-called Taylor principle, according to which it increases the interest rate more than one-to-one in response to a rise in inflation. Hence, $\mu_\pi > 1$.

[5] To be precise, generally a distinction may be made between the central bank's short-term rate of interest, an interest rate on government or corporate bonds, and possibly also a loan rate. However, we need not make these rates explicit if we hypothesize constant markups on the short-term interest rate. The differences between the rates can then be viewed as being incorporated into the benchmark terms (such as $\rho^\star$) in our adjustment functions.

[6] A broader conceptual discussion of eq. (7) is given in Franke (2015).

[7] Franke (2015, Section 5.2) also discusses the consequences when the central bank has another idea of the normal output-capital ratio than the firms.

A most straightforward way to close the model is a price Phillips curve with full central bank credibility:

$$\pi \;=\; \pi^\star + \beta(u-u^n) \tag{9}$$

Equation (9) is compatible with several structural interpretations. If one likes, it can be viewed as being derived from a wage Philiips curve relationship and a constant markup on unit labour cost in the firms' pricing policy. Conceptually, the first term on the right-hand side represents expected inflation, where the private sector believes in a quick return of inflation back on target. In principle, also a zero slope coefficient $\beta = 0$ is admissible in (9).[8]

The extension of the baseline model concerns the dynamic adjustments of the sentiment variable a only. Hence the temporary equilibrium part remains unaltered and the IS solution $u = u(a)$ from (5) is maintained. The profit differential ρ in (7) can be expressed as a function of the output-capital ratio. Substituting (1), (8) and (9) in $\rho = r-(i-\pi)$ yields

$$\begin{aligned} \rho-\rho^\star \;&=\; \rho(u)-\rho^\star \\ &=\; hu \;-\; (i^\star-\pi^\star) \;-\; (\mu_\pi-1)(\pi-\pi^\star) \;-\; \mu_u(u-u^n) \;-\; \rho^\star \\ &=\; hu \;-\; (i^\star-\pi^\star+\rho^\star) \;-\; [(\mu_\pi-1)\beta + \mu_u]\,(u-u^n) \end{aligned} \tag{10}$$

Thus, in obvious notation, the law of motion (7) for the business sentiment can be written as

$$\dot a \;=\; \eta_u\,[u(a)-u^n] \;+\; \eta_\rho\,\{\rho[u(a)]-\rho^\star\} \tag{11}$$

From the last row in (10) it can be seen that a state of rest with $\dot a = 0$ in (11) and output at normal utilization ($u = u^n$) is possible if (and only if) the central bank sets its targets such that the real rate of interest $(i^\star-\pi^\star)$ equals the 'natural' rate of profit $r^n = hu^n$ minus the normal risk premium $\rho^\star$. To ease the presentation let this consistency condition be satisfied.[9]

The second term in (11) has a negative feedback on a and could thus counteract the destabilizing Harrodian forces from the first term if the derivative $d\rho/du$ in (10) is negative, that is, if

$$h \;<\; \mu_u \;+\; (\mu_\pi-1)\,\beta \tag{12}$$

A little back-of-the-envelope calculation in Franke (2015, Section 5.3) has shown that this necessary condition for a stable steady state can be safely taken for granted;

[8] More flexible versions to determine inflation are investigated in Franke (2015). It is an intricate issue what then turns out to favour or disfavour stability.

[9] Franke (2015, Section 5.2) contains a discussion of additional adjustment forces when the condition is violated. Nevertheless, stability itself is not affected by this problem.

even in the presence of a horizontal Phillips curve (when $\beta = 0$). On this basis it can be concluded that stability is ensured if (and only if) the negative derivative of the second term in (11) dominates the positive derivative of the first term. This necessary and sufficient condition for stability may be noted down as Assumption 1:

Assumption 1

$$[\mu_u + (\mu_\pi - 1)\beta - h]\,\eta_\rho \;>\; \eta_u$$

Regarding the coefficients, the inequality can be read or rearranged in several ways. Emphasizing the role of monetary policy we can summarize that, with a positive profitability motive of the firms ($\eta_\rho > 0$), the central bank can always stabilize the economy by sufficiently strong reactions to the output gap (μ_u high enough) and/or sufficiently strong reactions to the inflation gap (μ_π high enough, provided $\beta > 0$). While the transmission mechanisms of monetary policy are less direct than in the standard textbook arguments, they are still effective and can overcome the basic Harrodian instability.[10]

4 Autonomous expenditures

Another mechanism to put a curb on Harrodian instability that has recently received some attention is tied to an exogenous growth component in the economy. It was originally proposed by Serrano (1995) and has become known under the name of the Sraffian supermultiplier. Serrano's main intention was a demonstration that some Keynesian results will still hold when in the long run the actual rate of utilization is brought back to its normal level. The crucial point of his approach is that although income distribution, utilization and the marginal propensity to save may all be assumed to be constant, saving can nevertheless adjust endogenously to investment when a part of aggregate demand grows autonomously and does not create additional capacity.

Serrano also believed that as long as the firms' demand expectations are not systematically biased, utilization will indeed return to normal. This statement was, however, not supported by a formal reasoning and so remained dubious in the profession. It took almost twenty years until this issue was resolved in a satisfactory manner by Allain (2013, 2015).

The basic idea is straightforward. Suppose there is a part Z of demand that grows at a constant rate g_z. *Ceteris paribus*, IS utilization will be above (below)

[10] The simpler treatment of Lavoie (2014), which likewise arrives at a one-dimensional differential equation for the firms' 'animal spirits', leads to an unambiguously stable steady state (see eq. (6.48) on p. 397).

normal if the ratio $z = Z/K$ is above (below) a certain level z^o. Consequently, the capital growth rate will then be above (below) its long-run equilibrium value, which is given by g_z. Hence, if $z > z^o$, the capital stock grows faster than the autonomous demand Z, which means $z = Z/K$ declines, and continues to decline, until it reaches z^o (the same in the opposite direction if initially $z < z^o$). It remains to establish a condition under which this stabilizing feedback can also dominate the Harrodian forces in the economy.

Such an autonomous component of demand can be government spending as in Allain (2013, 2015), exports, or a part of private consumption as in Lavoie (2016; 2014, Section 6.5.7) or Freitas and Serrano (2015). Here we adopt Lavoie's version since it is most parsimonious. Thus, let the consumption of the rentiers be given by a constant fraction $(1-s_r)$ of the profits huK they receive and additionally an autonomous part Z, which exogenously grows at a constant rate $g_z > 0$. Note that in contrast to eq. (2), s_r is not the the rentiers' average propensity to save but the marginal rate. Accordingly, putting $z := Z/K$, the saving function (2) is modified as

$$g^s \;=\; s_r hu \;-\; z \tag{13}$$

Maintaining the investment function (3), utilization in the temporary equilibrium (4) is now a function of the sentiment variable a and the expenditure ratio z,

$$u \;=\; u(a,z) \;=\; \frac{a + z - \gamma u^n}{s_r h - \gamma} \tag{14}$$

Regarding the sentiment dynamics, we return to the plain Harrodian adjustments of eq. (6). The law of motion of z derives from $\hat{z} = \hat{Z} - \hat{K} = g_z - [a + \gamma(u - u^n)]$. In this way the economy is described by the following two differential equations in a and z:

$$\begin{aligned} \dot{a} &= \eta_u\,[u(a,z) - u^n] \;=\; \{\eta_u/(s_r h - \gamma)\}\,(a + z - s_r h u^n) \\ \dot{z} &= z\{g_z \;-\; a \;-\; \gamma[u(a,z) - u^n]\} \end{aligned} \tag{15}$$

A non-degenerate long-run equilibrium $(a^o, z^o) > 0$ implies $u(a^o, z^o) = u^n$ (see the first equation), so that a^o is equal to the autonomous growth rate g_z (second equation). The value of z^o can subsequently be obtained from equating the right-hand side of (14) to u^n. Hence,

$$a^o \;=\; g_z\,, \qquad z^o \;=\; s_r hu^n - a^o \;=\; s_r r^n - g_z \tag{16}$$

As it should be, the entire economy grows at the rate $g^o = g_z$ in this state. Certainly, for this situation to be economically meaningful the exogenous growth rate

g_z must not exceed $s_r r^n$, which may thus be viewed as the economy's maximal rate of growth.[11]

The general stability argument mentioned above can be identified in the second equation of system (15). Fix, for the sake of the argument, the business sentiment at $a = a^o$ and consider $z > z^o$. Because of $\partial u/\partial z > 0$ in (14), this implies $u(a^o, z) > u^n$ and therefore $\dot{z} < 0$. This decline goes on until eventually the ratio z levels off at its equilibrium value z^o. This is what Lavoie (2014, 2016) calls the Serrano-Allain adjustment process.

The next question is whether this mechanisms can be strong enough, such as to counteract the unstable sentiment dynamics in the first equation of (15) where $\partial \dot{a}/\partial a > 0$. This is readily checked by setting up the Jacobian matrix J of (15),

$$\begin{aligned} J &= \begin{bmatrix} \eta_u u_a & \eta_u u_z \\ -z^o(1+\gamma u_a) & -z^o \gamma u_z \end{bmatrix} = \begin{bmatrix} + & + \\ - & - \end{bmatrix} \\ u_a = u_z &= 1/(s_r h - \gamma) > 0 \end{aligned} \tag{17}$$

A necessary and sufficient condition for the stability of the equilibrium position is a positive determinant and a negative trace. While the sign of the determinant cannot be inferred from the qualitative sign pattern alone, its explicit calculation yields a definite result,

$$\det J = \eta_u z^o u_a > 0 \tag{18}$$

Stability therefore boils down to the trace, which is negative if (and only if) the negative entry J_{22} dominates the positive entry J_{11}. Using (16), we may write down this condition as our second assumption.

Assumption 2

$$\gamma(s_r h u^n - g_z) > \eta_u$$

Stability of the steady state is thus not ensured but possible, namely, if the sentiment dynamics of the firms is not too fast; i.e., if the adjustment speed η_u is not too high. Also conducive to stability are a low autonomous consumption growth rate g_z or a high marginal propensity to save s_r. A similar reference to the firms' responsiveness γ in their investment decisions can be misleading because the Keynesian stability condition puts an upper-bound $s_r h$ on this coefficient. It may be noted that, in contrast to the stability condition of Assumption 1 in the previous section, very high values of η_u would make it impossible for the Serrano-Allain mechanism to

[11] Incidentally, the same constraint holds in Allain's (2015) model.

stabilize the economy. For the stability of system (11) high values of η_u could be neutralized by high values of μ_u or μ_π, and in this respect monetary policy can be said to have a stronger potential.

5 Combining the two stabilizing mechanisms

A discussion of a number of mechanisms that may possibly tame Harrodian instability, such as Hein et al. (2011) or Lavoie (2014, Chapters 6.4 and 6.5), does not view them as mutually exclusive. It rather seems to suggest that a combination of them should only reinforce stability, according to the motto 'the more the better'. We want to test this intuition with the present two stabilizing devices, which is easily done. It suffices to take system (15) and extend the adjustments of the business sentiment in the first equation with the profitability motive along the formulation of eq. (11). Using (10) and expressing the profit differential ρ as a function of IS utilization $u = u(a,z)$, the economy made up of the Serrano-Allain mechanism and the interest rate effects remains two-dimensional:

$$\begin{aligned} \dot{a} &= \eta_u\,[u(a,z) - u^n] \;+\; \eta_\rho\,\{\rho[u(a,z)] - \rho^\star\} \\ &= \{\widetilde{\eta}/(s_r h - \gamma)\}\,(a+z) \;+\; \text{const.} \\ \dot{z} &= z\{g_z \,-\, a \,-\, \gamma[u(a,z) - u^n]\} \\ \widetilde{\eta} &= \eta_u \;-\; [(\mu_\pi - 1)\,\beta + \mu_u - h]\,\eta_\rho \;<\; 0 \qquad \text{(by Assumption 1)} \end{aligned} \tag{19}$$

Though somewhat clumsy, let us refer to (15) as the Harrod-Serrano-Allain model and its enriched version of eq. (19) as the Harrod-Taylor-Serrano-Allain model (because of the added Taylor rule governing the interest rate effects). Before executing the algebra of the stability analysis of (19), it is useful to compare the two models in a less technical way by studying their phase diagrams.[12] They are constituted by the isoclines $\dot{a} = 0$ and $\dot{z} = 0$ in the (z,a)-plane, i.e., the geometric locus of the pairs (z,a) bringing about $\dot{a} = 0$ and $\dot{z} = 0$, respectively. The steady state position where all motion ceases is defined by their point of intersection.

The second differential equations in (15) and (19) are identical, hence the $\dot{z}$-isoclines are the same in the two models. With their negative slope, they are given by the flatter straight lines in panel (a) and (b) in Figure 1. Above (below) that locus, the own-derivative of z becomes negative (positive). Accordingly, as indicated by the horizontal arrows in the lower-right corner, the variable z tends to move towards the $\dot{z}$-isocline. Certainly, this is just the geometric representation of the Serrano-Allain mechanism.

[12] It goes without saying that the same steady state is presupposed, which formally the constant term in the first equation of (19) takes care of.

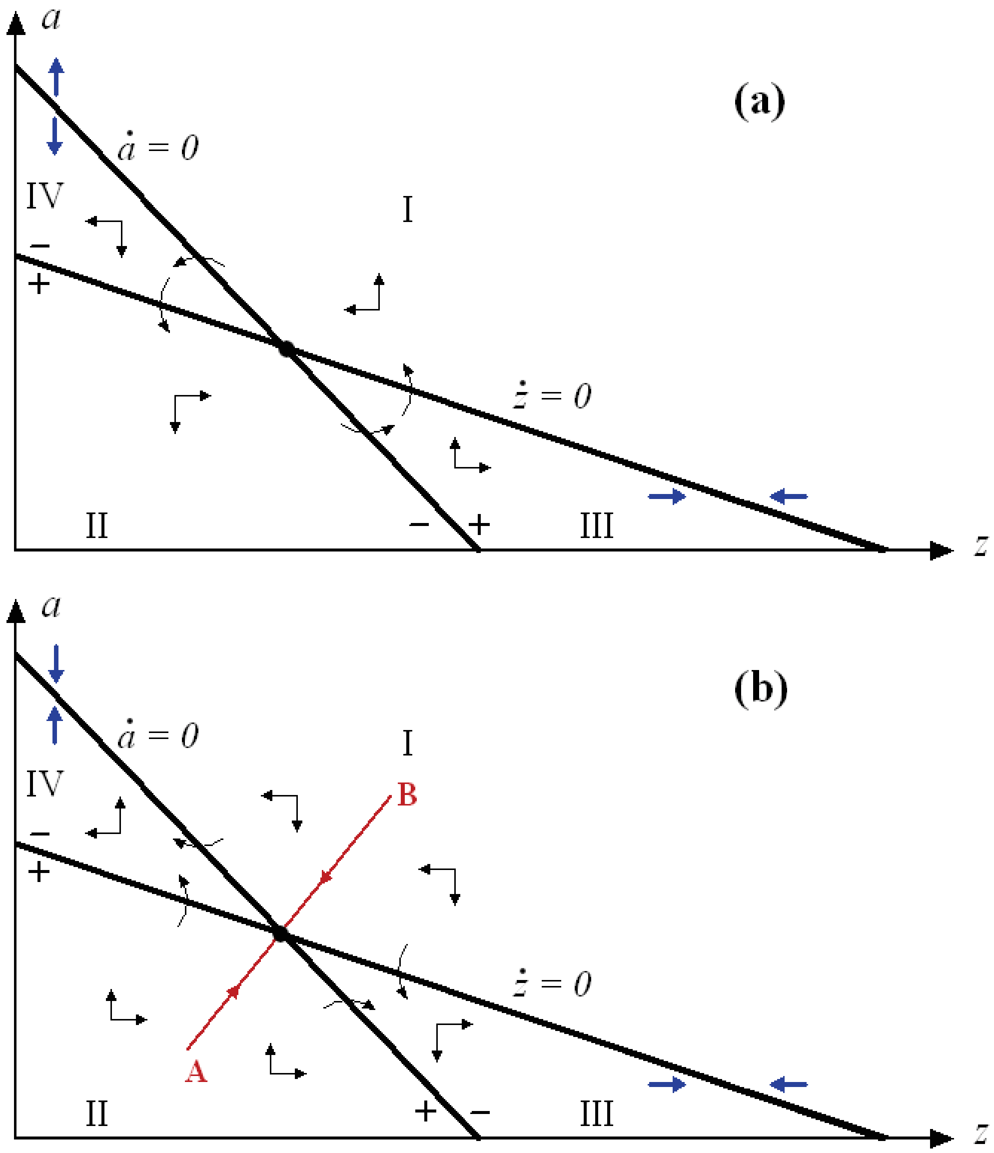

Figure 1: Phase diagrams of the Harrod-Serrano-Allain model (a) and the Harrod-Taylor-Serrano-Allain model (b).

The first differential equations in (15) and eq. (19) are not identical but still have the same formal structure. Thus, they give rise to the same straight line on which $\dot{a} = 0$; see the steeper line in Figure 1a and 1b. What is different are the adjustments of the sentiment variable a off that line. With the plain Harrodian forces in system (15), the variable moves vertically away from $\dot{a} = 0$ (as indicated by the two vertical arrows in the upper-left corner of Figure 1a). By contrast, with the stabilizing interest rates effects in (19), which render the composite coefficient $\widetilde{\eta}$ negative, also variable a tends to move in the direction of its isocline. Does this mean that with 'Taylor' added, the Harrod-Taylor-Serrano-Allain model is even 'more stable' than the plain version?

The two isoclines constitute four regions where depending on which side of which isocline (z,a) is lying, the variables are increasing or decreasing, such that together (z,a) may move north-west- south-west, etc. As it is drawn, Figure 1a suggests cyclical movements from region I to IV to II to III and then back to I again. Nonetheless, the system could theoretically also stay within region I or II forever. The diagram itself cannot tell us whether stability or instability prevails. Global convergence towards the point of intersection of the two isoclines can be concluded from Assumption 2 (a limited influence of the Harrodian forces). and the resulting negative trace in the Jacobian matrix (17).

Figure 1b demonstrates that the world changes if already 'Taylor' tames the Harrodian instability. Consider region I. With variable a tending towards the $\dot{a}$-iscoline, the pairs (z,a) move south-west instead of north-west. Again, the trajectories can move over into region IV, but now, if starting sufficiently to the right in region I, they may also change into region III. Intuitively, there must be a line separating these two cases, and on this line the system must directly converge to the equilibrium point. The dynamics starting in region II are a mirror image of these features.

The crucial point is that once in region IV (or III), the system can no longer leave it. Apparently, it diverges north-west (or south-east, respectively). Therefore, on the whole, the composite Harrod-Taylor-Serrano-Allain model exhibits saddle-point instability. If, and only if, the system starts somewhere on the line AB will it find its ways into the equilibrium, everywhere else it moves more and more away from it. Locally more stability (i.e. locally near the $\dot{a}$-iscoline) has turned into global divergence, except for the stable saddle path AB.

The algebraic counterpart of the geometric analysis of the Harrod-Taylor-Serrano-Allain model is given by the Jacobian matrix of (19), which reads

$$J = \begin{bmatrix} \widetilde{\eta}\, u_a & \widetilde{\eta}\, u_z \\ -z^o(1+\gamma u_a) & -z^o\,\gamma u_z \end{bmatrix} = \begin{bmatrix} - & - \\ - & - \end{bmatrix} \qquad (20)$$

$$u_a = u_z = 1/(s_r h - \gamma) > 0$$

In contrast to (17), its trace is now unambiguously negative. The sign pattern of J alone is not sufficient information about the sign of the determinant. Algebraically, however, a definitely negative sign and therefore saddle-point instability is obtained, just because of the negative coefficient $\widetilde{\eta}$,

$$\det J = \widetilde{\eta}\, z^o\, u_a < 0 \qquad (21)$$

Inequality (21) may appear a rather technical issue and one may wonder what economic mechanisms will bring about this instability instability. Suppose the economy is in a long-run equilibrium and some positive news incites the optimism of the firms, such that the business sentiment jumps up to a level $a > a^o$. This raises utilization and also speeds up the growth of the capital stock. Consequently, the expenditure ratio $z = Z/K$ falls. Formally, this effect is represented by entry $J_{21} = \partial\dot{z}/\partial a < 0$ in the Jacobian.

The decline of z has a negative effect on utilization. The Harrodian component alone in the sentiment adjustments (measured by the coefficient η_u) would make the firms more pessimistic. On the other hand, the profit differential ρ increases, because of $\partial\rho/\partial u = h - [(\mu_\pi - 1)\beta + \mu_u] < 0$ from (11) and (12). With Assumption 1, the latter is the dominating influence, so that actually the firms become more optimistic. Formally, this effect is represented by $-J_{12} = -\partial\dot{a}/\partial z > 0$.

Taken together, we have identified a positive feedback loop in the sentiment variable: a positive shock to a sets a chain of events in motion that increases a even further. While there are two more immediate effects with a stabilizing potential, $J_{11} = \partial\dot{a}/\partial a < 0$ and $J_{22} = \partial\dot{z}/\partial z < 0$, a purely verbal reasoning is not sufficient to reach a definite conclusion about the net effect. In the end we therefore have to resort to the mathematical result of inequality (21) for the determinant, which tells us that the destabilizing indirect effect $a\uparrow \Rightarrow z\downarrow \Rightarrow a\uparrow$ dominates the two stabilizing direct effects $a\uparrow \Rightarrow a\downarrow$ and $z\uparrow \Rightarrow z\downarrow$.

6 Conclusion

In discussions of economic stability it usually goes without saying that the joint effects of several stabilizing mechanisms should only reinforce stability. The present note provides a simple counterexample to this perception.

In the much debated issue of how Harrodian instability in the Kaleckian baseline model of growth and distribution could be possibly tamed, two topical mechanisms are considered: one where a few (indirect) interest rate effects enable the central bank to stabilize the steady state position, and one where an autonomously growing, non-capacity creating component of aggregate demand acts as an 'automatic stabilizer'. Marrying these two economies, however, brings a positive feedback loop into existence and even leads to saddle-point instability.

Would the finding become irrelevant if the two mechanisms are integrated into a still broader framework? Of course, this depends on the relative strength of the additional stabilizing and destabilizing forces thus brought into play, and how they will work out when they are all interacting. In a local analysis referring to the higher-dimensional Routh-Hurwitz stability conditions, the negative determinant of eq. (21) would still show up in one of the 2×2 principal minors of the new Jacobian (the sum of which should be positive for stability). It will exhibit the ‘wrong’ sign, and mathematically the sum of the other minors may or may not offset this destabilizing term.

Is our instability result too special? As long as not other mechanisms of similar complexity are combined and their joint effects are examined, nobody can tell. The present contribution just points out that one cannot blindly rely on a straightforward economic intuition that an increase in the number of stabilizing effects that one is able to identify will also increase the prospects of overall system stability. Furthermore, while in a comparison of the counteracting mechanisms a mathematical analysis allowed us to evaluate the net effect, we cannot be sure that other and more elaborate examples would give rise to a similarly definite message. Therefore, the richer and more realistic the modelling, the more a careful numerical investigation will be required. Our intuition, which is only able to capture some partial dynamics, is no more than a working hypothesis that needs to be checked by more sophisticated tools.

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