Behavioral Macroeconomics Workshop

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Motivation

- Prior to the 2007-08 crisis, mainstream macroeconomics seemed to have reached a consensus built around the theoretical foundations of the Real Business Cycle (RBC) school enhanced with "New Keynesian" nominal rigidities: the New Neoclassical Synthesis (NNS).
- The economics profession became over-confident in this New Neoclassical Synthesis, which became "the only game in town".
- The occurrence of the 2007-08 global financial crisis was a wake-up call which highlighted the need for alternative theoretical approaches.
- The "behavioral macroeconomics" field has exploded ever since.

Behavioral Economics and Behavioral Macroeconomics

Definition (Mullainathan & Thaler, 2000)

“Behavioral Economics is the combination of psychology and economics than investigates what happens in markets in which some agents display human limitations and complications” [like]

- The influence of emotions, perceptions and beliefs
- Bounded rationality and the use of heuristics
- Social norms (fairness, reciprocity, social status, etc.)

“Behavioral economics emerged in reaction to the notion, held by many neoclassical economists, that social and behavioral science should avoid reference to entities (like cognitive and affective states) that cannot be observed.” (Agner & Loewenstein, 2007, p.7)

While behavioral economics was for a long time mostly popular in the fields of experimental economics and microeconomics, its relevance for a better understanding of macroeconomic phenomena such as (see Akerlof, 2001)

- The existence of involuntary unemployment
- The impact of monetary policy on output and employment
- The inflation dynamics and the inflation-unemployment trade-off
- The prevalence of undersaving for retirement
- The excessive asset price volatility

and for the design of economic policies has been recognized in recent times.

Outline of this Workshop

1. The New Neoclassical Synthesis Approach
   - Assumptions
   - Implications
   - Criticisms
2. Behavioral Macroeconomics
   - Origins and Alternative Approaches
   - The Heterogenous Expectations Approach
   - The Aggregate Sentiments Approach
   - The Learning Approach
3. Concluding Remarks

The New Neoclassical Synthesis Approach

New Neoclassical Synthesis models are based on three building blocks:

1. Intertemporal optimization
2. Rational expectations
3. Imperfect competition and nominal rigidities

- While (1) and (2) stem from the RBC modeling paradigm, (3) results from the intent of "New Keynesians" to introduce a role for monetary policy at least in the short run through the assumption of imperfectly flexible prices and/or wages.

NNS Building Block 1: Intertemporal Optimization

- Most decisions of the economic agents have an important forward-looking component.
- In the undergraduate macro IS-LM model, for instance, private consumption is assumed to be a linear function of current income,

$$C_t = cY_t, \quad 0 < c < 1$$

- What if we knew that $Y_{t+1} > Y_t$? How would this affect private consumption today?

In the NNS framework households aim to maximize the expected present discount value of lifetime utility, i.e.

$$U_t = E_t \sum_{s=0}^{\infty} \beta^s u(c_{t+s}), \tag{1}$$

where c_t represents goods consumption, $u(\cdot)$ is an instantaneous utility which satisfies $u'(c_t) > 0$ and $u''(c_t) < 0$, $0 < \beta < 1$ is the subjective discount factor, and E_t is the mathematical expectations operator.

Households seek to maximize their objective functions subject to the budget constraints

$$a_{t+s+1} + c_{t+s} = x_{t+s} + a_{t+s} + r_{t+s} a_{t+s}, \quad s = 0, 1, \dots, \tag{2}$$

where x_t is the exogenous period income, a_{t+1} is the household's end-of-period net assets, and r_t is the exogenous interest rate.

The corresponding is Lagrangian function for this intertemporal optimization problem is given by

$$\mathcal{L}_t = E_t \sum_{s=0}^{\infty} \{\beta^s u(c_{t+s}) + \lambda_{t+s}[x_{t+s} + (1 + r_{t+s})a_{t+s} - c_{t+s} - a_{t+s+1}]\} \tag{3}$$

The corresponding first-order conditions (FOC) are

$$\begin{aligned}
\frac{\partial \mathcal{L}_t}{\partial c_{t+s}} &= \beta^s u'(c_{t+s}) - \lambda_{t+s} = 0, \quad s \geq 0, \\
\frac{\partial \mathcal{L}_t}{\partial a_{t+s}} &= \lambda_{t+s}(1 + r_{t+s}) - \lambda_{t+s-1} = 0, \quad s \geq 1, \\
\frac{\partial \mathcal{L}_t}{\partial a_{t+s}} &= x_{t+s} + (1 + r_{t+s})a_{t+s} - c_{t+s} - a_{t+s+1}, \quad s \geq 0.
\end{aligned}$$

The consumption Euler-equation is

$$u'(c_t) = \beta(1 + r_t)u'(c_{t+1}). \tag{4}$$

Interpretation of the Consumption Euler Equation

- The consumption Euler equation

$$u'(c_t) = \beta(1+r)u'(c_{t+1})$$

  can be seen as an intertemporal arbitrage condition, saying that at the optimum the representative consumer must be indifferent between consuming a marginal unit of c, yielding extra utility $u'(c_t)$, or, alternatively, investing this unit and consuming the return one period later, yielding extra utility $\beta(1+r)u'(c_{t+1})$
- The discount factor β ensures that consumption today and tomorrow will be comparable in terms of utility

Some Remarks

- The optimal path of consumption $\{c_t, c_{t+1}, \ldots\}$ depends on the household's lifetime or permanent income (Friedman' 1957 Permanent Income Hypothesis, or PIH).
- This optimal consumption path is achievable only if agents are not credit-constrained.
- Does an infinite planing horizon make sense economically, or is it just a mathematical modeling device?
- The realization of the optimal path is only feasible if the *expectations* concerning the households' complete lifetime income – and its financial solvency – is consistent with the Data-Generating Process and its common knowledge.

NNS Building Block 2: Rational Expectations

- The expectations formation process is intrinsically linked to
  - the information set available to the economic agents at the time when expectations are formed, and
  - the economic agents' degree of rationality and thus their capability to process that information.
  - In earlier days, expectations were considered only implicitly, or at most as modelled in an adaptive manner.
- Due to the inflation shocks in the 1970s and the rapid increase in inflation, adaptive expectations specifications became increasingly unrealistic, as they implied that agents were computing "wrong" expectations *in a systematic manner*.

Rational Expectations in a Nutshell

- Agents with rational expectations (Muth 1961) use all the information available to them to form optimal forecasts.
- Information set available at the beginning of period t:

$$\Omega_t \equiv \{\underbrace{y_{t-1}, y_{t-2}, \ldots, i_{t-1}, i_{t-2}, \ldots \pi_{t-1}, \pi_{t-2}, \ldots, \pi^e_{t-1}, \pi^e_{t-2}, \ldots}_{(a)}; \underbrace{a, b}_{(b)}; \underbrace{\nu_t \sim N(0, \sigma^2)}_{(c)}\}$$

(a) Agents do not forget (relevant) past information.
(b) Agents know the parameters of the model.
(c) Agents know the stochastic process of the shocks.

- The DGP-consistent expected value of a variable x_t is denoted by $E(x_t \mid \Omega_t) \equiv E_t x_t$, where the expectations operator E_t indicates that the expectations are conditional upon information set Ω_t.
- If an agent have rational expectations, their subjective expectations w.r.t. x_t (x^e_t) coincide with the objective expected value $E_t x_t$ conditional on the information set of that agent.

Implications

- If all agents know about the financial solvency of the other agents and understand the functioning of the economy, the allocation of resources in the economy should be efficient.
- In other words: If all agents maximize their utility/profits in a consistent manner with their intertemporal budget constraints and possess similar/same information sets, allocations should be optimal and Ponzi games should be ruled out.
- The recent financial crisis can be traced back to
  - Overly optimistic expectations concerning future house prices (and the feasibility of Ponzi games)
  - Wrong assessment of loan risk by banks
  - Lack of proper understanding of the limits of securitization concerning the diversification of risk
- How consistent are these developments with the rational expectations assumption?

NNS Building Block 3: Nominal Rigidities

- While the intertemporal optimization framework and the rational expectations assumption of the RBC school were inherited by the NNS, the assumption of fully flexible prices which characterized the former was dropped and replaced by the incorporation of nominal rigidities.
- The reason for this is that monetary policy is essentially irrelevant for real economic activity under fully flexible prices.
- Through the introduction of imperfectly flexible prices or wages, monetary policy becomes again relevant, at least in the short run.
- The assumption of imperfect competition in the goods markets delivers a "microfoundation" for price setting power by firms.

The Workhorse NNS ("New Keynesian") Model

The workhorse NNS ("New Keynesian") model (see e.g. Galí, 2008) consists of the following three equations (in log-linearized form):

$$
\begin{aligned}
y_t &= E_t(y_{t+1}) - a_2(i_t - E_t(\pi_{t+1}) - r_t^*) + \varepsilon_t^{IS} && (5)\\
\pi_t &= \beta E_t(\pi_{t+1}) + \kappa y_t + \varepsilon_t^{AS} && (6)\\
i_t &= r_t^* + \theta_\pi \pi_t + \theta_y y_t + \varepsilon_t^{MP} && (7)
\end{aligned}
$$

where

- y_t is the percent deviation of current output from the flexible price (potential) output level
- i_t is the short-term rate of interest
- π_t is the price inflation rate
- r_t^* is the "natural" or "equilibrium" rate of interest
- ε_t^{IS}, ε_t^{AS} and ε_t^{MP} are exogenous shocks

This model can be written in matrix notation as follows

$$\begin{bmatrix} 1 & -\kappa & 0 \\ 0 & 1 & -a_2 \\ -\phi_y & -\phi_\pi & 1 \end{bmatrix} \begin{bmatrix} \pi_t \\ y_t \\ i_t \end{bmatrix} = \begin{bmatrix} \beta & 0 & 0 \\ -a_2 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} E_t(\pi_{t+1}) \\ E_t(y_{t+1}) \\ E_t(i_{t+1}) \end{bmatrix} + \begin{bmatrix} \varepsilon_t^{AS} \\ \varepsilon_t^{IS} \\ \varepsilon_t^{MP} \end{bmatrix}$$

Condition for (saddle-path) stability: $\phi_\pi > 1$ (Taylor Principle) $\Longrightarrow$ Blanchard-Kahn (1980) conditions!

Shortcomings

- Immediate and short-lived real effects of monetary policy shocks
- Complete lack of persistence
- Dynamic inconsistence with inflation dynamics
- Poor fit to empirical data

Behavioral Extensions: Consumption

Due to the underlying PIH in the Euler-equation which is the basis of the NNS-IS-equation, private consumption as predicted by the NNS model exhibits "excess smoothness" (Campbell and Deaton, 1989).

Some possible behavioral explanations for the "excessive sensitivity" to current income:

- Habit formation $\Longrightarrow$ eq.(5) features a lagged output term
- "Rule-of-thumb" consumption with liquidity constraints
- "Catching-up with the Joneses" behavior

Behavioral Extensions: Inflation

Since the baseline New Keynesian Phillips Curve is derived from the forward-looking profit maximization problem of monopolistic firms under staggered price setting, the predicted current inflation is independent from lagged inflation, and features thus no autocorrelation.

Some possible behavioral explanations for the observed inflation persistence:

- Indexation (of wage contracts or prices)
- Sticky information
- "Prospect-theory" perception
- Rational inattention

Bounded Rationality and Macroeconomic Models

- Economic agents do not have full information
- Acquisition and processing of information is costly
- Humans have a limited knowledge and cognitive skills
- Economic agents do not optimize in the NNS sense, but use instead
    - Near-optimal solutions
    - Own heuristics or rules-of-thumb
    - Imitation and social learning
    - Norms
    - Heterogenous expectations
    - Aggregate Sentiments
    - Learning

The Heterogenous Expectations Approach

- Seminal theoretical paper: Beja & Goldman (1980, Journal of Finance)
- Empirical papers: Frankel & Froot (1990 AER), Allen and Taylor (1992 JIMF), and Manzan & Westerhoff (2007 JEBO), among others.
- However: Most of these models focus solely on the financial markets!
- In recent times, heterogeneous expectations are being incorporated in a new generation of macroeconomic models, see e.g. Branch & McGough (2009, JEDC), Proaño (2011, JEBO), De Grauwe (2012, JEBO), Proaño (2013, MS), etc.
- In this models, the importance of rule-of-thumb behavior for macroeconomic stability is investigated.

Key Features

- Agents use different types of forecasting rules based on heuristics.
- For the sake of simplicity, most models in the behavioral finance literature assume two types of forecasting rules: a fundamentalist rule and a chartist rule (see e.g. Brock and Hommes, 1997, or De Grauwe and Grimaldi, 2006)
- The "market" mood, or the aggregate expectations, are a weighted average of the two forecasts, where the relative weight is itself a function of the respective profitability or the accuracy of the two rules.

A Baseline Behavioral Macroeconomic Model

De Grauwe (2012) proposes the following behavioral macroeconomic model consisting of the following equations

$$\begin{aligned} y_t &= a_1 \tilde{E}_t y_{t+1} + (1-a_1)y_{t-1} + a_2(r_t - \tilde{E}_t \pi_{t+1}) + \varepsilon_t \\ \pi_t &= b_1 \tilde{E}_t \pi_{t+1} + (1-b_1)\pi_{t-1} + b_2 y_t + \eta_t \\ i_t &= c_1(\pi_t - \pi^*) + c_2 y_t + c_3 r_{t-1} + u_t \end{aligned}$$

with

$$\begin{aligned} \tilde{E}_t y_{t+1} &= \omega_t^f \tilde{E}_t^f y_{t+1} + (1-\omega_t^f)\tilde{E}^e y_{t+1} \quad \text{with } \tilde{E}^f y_{t+1} = 0,\ E^e y_{t+1} = y_{t-1} \\ \tilde{E}_t \pi_{t+1} &= \beta_t^f \tilde{E}_t^{tar} \pi_{t+1} + (1-\beta_t^f)\tilde{E}^{exp}\pi_{t+1} \text{ with } \tilde{E}^{tar}\pi_{t+1} = \pi^*,\ E^{exp}\pi_{t+1} = \pi_{t-1} \end{aligned}$$

and

$$\omega_t^f = \frac{\exp(\gamma \Psi_t^f)}{\exp(\gamma \Psi_t^f) + \exp(\gamma \Psi_t^c)} \quad \text{and } \beta_t^{tar} = \frac{\exp(\gamma \Psi_t^{tar})}{\exp(\gamma \Psi_t^{tar}) + \exp(\gamma \Psi_t^{exp})}$$

where U_t^i is a measure of the performance of the i forecasting rule.

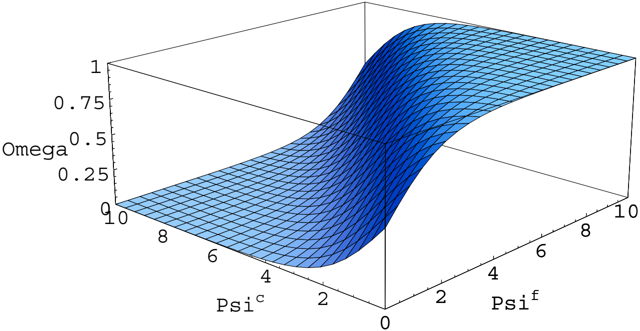
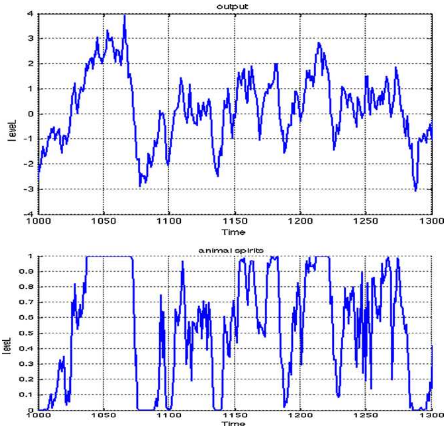


Figure: The ω_t function



De Grauwe shows that the inclusion of behavioral expectations in a standard NNS model allows to generate, among other things

- endogenous waves of optimism and pessimism
- fluctuations in output and inflation which are not normally distributed, including fat tails.

A Baseline Behavioral Small-Open Economy Model

- Proaño (2013) analyzes a small open economy described by a backward-looking AD equation, a standard Phillips Curve and a Taylor interest rate rule.
- The FX market is populated by agents which use either a *fundamentalist* or a *chartist* rule for their exchange rate forecasts.
- *Sequence of Events*
  - At the beginning of a period t, the FX market participants form their forecasts of the nominal exchange rate at $t+1$ on the basis of the information set containing macroeconomic data generated up to $t-1$.
  - Independently, the domestic monetary authorities set the nominal interest rate on the basis of the same information.
  - The actual nominal exchange rate is determined by a no-arbitrage condition which holds at the aggregate ex-ante level.
  - Finally, the real variables output and inflation in the domestic economy are determined.

The FX Market

According to the "fundamentalist" forecasting rule, the expected log nominal exchange rate at $t+1$ is given by

$$E_t^f s_{t+1} = s_{t-1} + \beta_s^f (f_{t-1} - s_{t-1}), \tag{8}$$

where f_{t-1} represents the (log) fundamental nominal exchange rate at time $t-1$ and $\beta_s^f > 0$ a scaling factor linked with the speed of adjustment of the log nominal exchange rate towards its long-run equilibrium level f

$$f_t = p_t - p_t^* \tag{9}$$

Inserting this expression in eq.(8) yields

$$\begin{aligned} E_t^f s_{t+1} &= s_{t-1} + \beta_s^f (p_{t-1} - p_{t-1}^* - s_{t-1}) \\ &= s_{t-1} - \beta_s^f (\eta_{t-1}) \end{aligned} \tag{10}$$

where η_t is the log of the real exchange rate $\mathcal{N} = SP^*/P$ at time t and $\eta_o = 0$ its PPP-consistent level.

By contrast, according to the "chartist" forecasting rule, it is assumed that the respecting expected log nominal exchange rate at $t+1$ is given by

$$E_t^c s_{t+1} = s_{t-1} + \beta_s^c \Delta s_{t-1}, \tag{11}$$

The last-period earnings of investing one unit of domestic currency in the foreign currency depend of course on whether a nominal appreciation ($\Delta s_{t-1} < 0$) or a depreciation ($\Delta s_{t-1} > 0$) took actually place between the periods $t-2$ and $t-1$, that is

$$\psi_{t-1}^j = [S_{t-1}(1+i_{t-1}^*) - (1+i_{t-1})S_{t-2}] \ \text{sgn}\ [E_{t-2}^j \Delta s_{t-1}] \quad j = c, f \tag{12}$$

with

$$\text{sgn}[\, E_{t-2}^j \Delta s_{t-1} \,] = \begin{cases} 1 & \text{for } E_{t-2}^j \Delta s_{t-1} > 0 \\ 0 & \text{for } E_{t-2}^j \Delta s_{t-1} = 0 \\ -1 & \text{for } E_{t-2}^j \Delta s_{t-1} < 0 \end{cases}$$

At every t, the share of FX traders using the fundamentalist forecasting rule (the so-called “market mood”) is determined by

$$\omega_t = \frac{\exp[\gamma(\psi^f_{t-1} - \sigma^2_{f,t-1})]}{\exp[\gamma(\psi^f_{t-1} - \sigma^2_{f,t-1})] + \exp[\gamma(\psi^c_{t-1} - \sigma^2_{c,t-1})]} \tag{13}$$

with

$$\lim_{\psi^f_{t-1}\to\infty} \omega_t = 1 \quad \text{and} \quad \lim_{\psi^f_{t-1}\to 0} \omega_t = 0,$$

and

$$\sigma^2_{j,t-1} = (E^j_{t-2}S_{t-1} - S_{t-1})^2 \quad j = c, f,$$

being the last period’s squared forecast error of the behavioral forecasting rule j, and γ measuring the sensitivity with which traders revise their choice of the forecasting rules (a higher γ implying a stronger reaction to the profitabilities differentials between the two rules).

On the basis of the expressions for $E_t^f s_{t+1}$ and $E_t^c s_{t+1}$ given by eqs. (8), (10), and (13), respectively, the market expectation of the log nominal exchange rate at $t+1$ is simply the weighted average of the two expected nominal exchange rates, that is

$$\begin{aligned} E_t^m s_{t+1} &= \omega_t E_t^f s_{t+1} + (1-\omega_t) E_t^c s_{t+1} \\ &= s_{t-1} - \omega_t \eta_{t-1} + (1-\omega_t)\Delta s_{t-1}. \end{aligned} \tag{14}$$

with ω_t given by eq.(13).

According to UIP:

$$s_t = i_t^* - i_t + E_t^m s_{t+1}. \tag{15}$$

or

$$s_t = i_t^* - i_t + s_{t-1} - \omega_t \eta_{t-1} + (1 - \omega_t)\Delta s_{t-1}. \tag{16}$$

By subtracting s_{t-1} from both sides, we obtain the following behaviorally founded law of motion for the log nominal exchange rate

$$\begin{aligned} \Delta s_t &= i_t^* - i_t + E_t^m s_{t+1} \\ &= i_t^* - i_t - \omega_t \eta_{t-1} + (1 - \omega_t)\Delta s_{t-1}. \end{aligned} \tag{17}$$

The Macroeconomy

- Standard open-economy AD-relationship

$$y_t = \alpha_y y_{t-1} - \alpha_{yr}(i_{t-1} - \pi_t - (i_o - \pi_o)) + \alpha_{y\eta}\eta_{t-1} \tag{18}$$

- Standard backward-looking Phillips Curve

$$\pi_t = \alpha_{\pi y} y_{t-1} + \alpha_\pi \pi_{t-1} \tag{19}$$

- The log real exchange rate:

$$\begin{aligned} \Delta\eta_t &= \Delta s_t + \bar{\pi}^* - \pi_t \\ &= i_t^* - i_t - \omega_t \beta_s^f \eta_{t-1} + (1-\omega_t)\beta_s^c(\Delta s_{t-1}) + \bar{\pi}^* - \pi_t. \end{aligned} \tag{20}$$

with

$$\omega_t = \frac{\exp[\gamma(\psi_{t-1}^f - \sigma_{f,t-1}^2)]}{\exp[\gamma(\psi_{t-1}^f - \sigma_{f,t-1}^2)] + \exp[\gamma(\psi_{t-1}^c - \sigma_{c,t-1}^2)]}$$

Monetary Policy

Concerning monetary policy, the following general specification for the domestic nominal interest rate

$$i_t \quad = \quad \phi_i i_{t-1} + (1-\phi_i)\left[i_o + \phi_\pi(\pi^c_{t-1} - \pi_o) + \phi_y\, y_{t-1} + \phi_s \Delta s_{t-1}\right] \qquad (21)$$

is formulated, with

$$\pi^c_t = (1-\xi)\pi_t + \xi\pi^m_t = (1-\xi)\pi_t + \xi(\pi^*_t + \Delta s_t),$$

defining CPI inflation, $\pi^m_t = \pi^* + \Delta s_t$ being the domestic-currency inflation of foreign goods and ξ being the share of imported goods in the CPI basket.

A monetary policy rule with a PPI inflation target, an output gap target and an interest rate smoothing term can be obtained by setting $\xi = 0$ and $\phi_s = 0$, so that

$$i_t \quad = \quad \phi_i i_{t-1} + (1-\phi_i)\left[i_o + \phi_\pi(\pi_{t-1} - \pi_o) + \phi_y\, y_{t-1}\right].$$

In contrast, a flexible CPI inflation targeting without interest rate smoothing results when $\phi_i = 0$ and $\phi_s = 0$, namely

$$i_t = i_o + \phi_\pi(\pi^c_{t-1} - \pi_o) + \phi_y\, y_{t-1},$$

and a strict nominal exchange rate targeting without interest rate smoothing can be expressed by $\phi_\pi = 0$, $\phi_i = 0$ and $\phi_y = 0$ as

$$i_t = i_o + \phi_s \Delta s_{t-1}.$$

Dynamic Adjustments

In the following we discuss the simulation results based on the following parameter values:

Table: Parameter Values

Output Gap	Phillips Curve	Monetary Policy	FX Markets
$\alpha_y = 0.9$	$\alpha_{\pi y} = 0.24$	$\phi_i = 0.7$	$\beta_s^f = 1/6$
$\alpha_{yr} = 0.1$	$\alpha_\pi = 0.8$	$\phi_\pi = 1.5$	$\beta_s^c = 1.25$
$\alpha_{y\eta} = 0.01$		$\phi_y = 0.5$	$\gamma = 10$

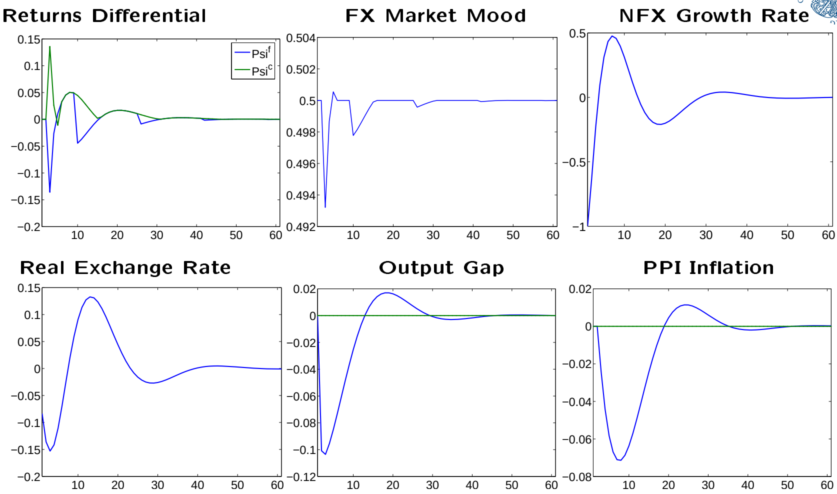


Figure: Dynamic responses of FX markets and real economy to a one-time domestic monetary policy shock

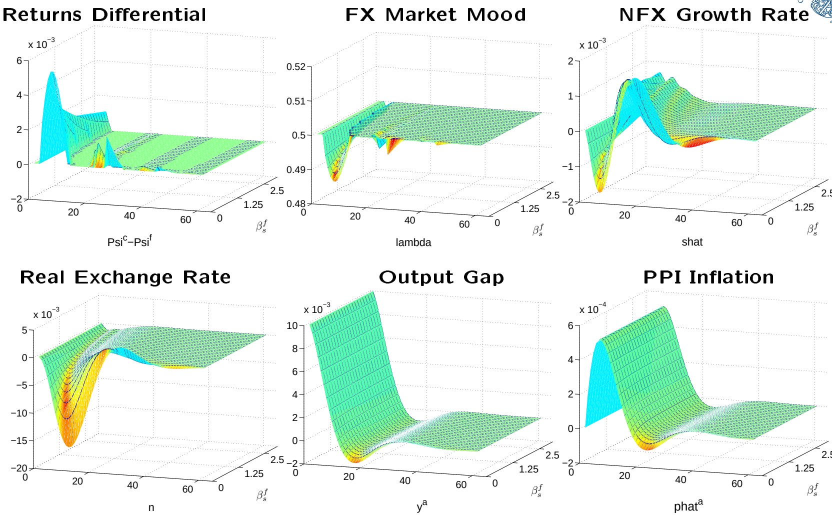


Figure: Dynamic responses of FX markets and real economy to a one-time aggregate demand shock for varying values of $\beta_s^f \in (0, 1.5)$ (with $\beta_s^c = 0.5$)

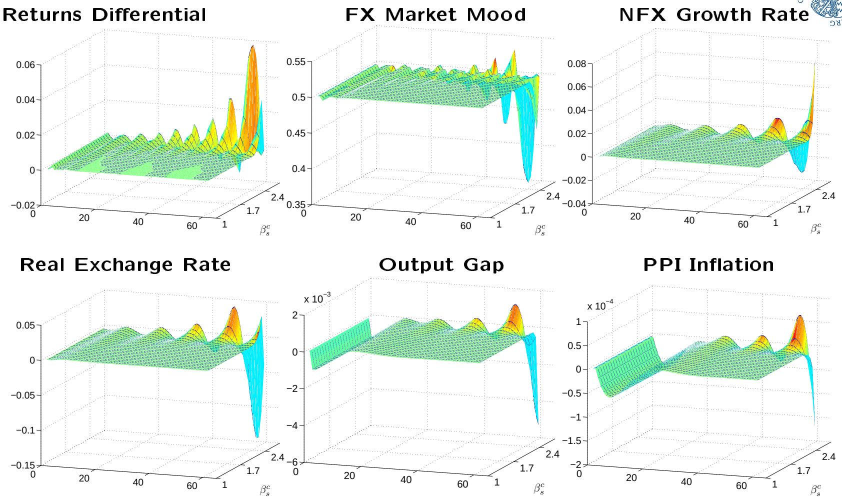


Figure: Dynamic responses of FX markets and real economy to a one-time domestic monetary policy shock for varying values of $\beta_s^c \in (0, 2.4)$ (with $\beta_s^f = 0.5$)

Key Insights

- Chartist expectations rules are destabilizing for the whole macroeconomy, and not only for the financial markets
- Fundamentalist expectations rules stabilizing
- The relative importance of both types of rules depend on their *profitability*
- Further analysis: performance of alternative monetary policy rules under behavioral forecasting rules (Proaño, 2013)

The Aggregate Sentiments Approach

- **Main Idea:** Economic fundamentals influence the agents' sentiments (optimism/pessimism) in a nonlinear and probabilistic manner.
- This *transition probability approach* has its origin in statistical mechanics and was first applied in social science by Weidlich and Haag (1983). The first contribution adapting this approach in the context of financial markets is Lux (1995, 1998).
- Recently, Franke (2012) put it forward into a business cycle model where the business cycle fluctuations are driven by the investors state-of-confidence in the economy.

Basic Features of the Franke (2012) Model

Aggregate Sentiments Dynamics

Consider an economy with $2N$ heterogeneous firms. Each firm has a binary set of choices. The firm can either be optimistic $(+)$ or pessimistic $(-)$.

$$n_t^+ + n_t^- = 2N \tag{22}$$

The *aggregate sentiment* can be explicitly defined

$$x_t = \frac{n_t^+ - n_t^-}{2N}, \qquad \text{where} \quad x_t \in [-1, 1] \tag{23}$$

The respective population share of the optimists and pessimists can be expressed in terms of the sentiment x_t as

$$\frac{n_t^+}{2N} = \frac{1}{2}(1 + x_t) \qquad \text{and} \qquad \frac{n_t^-}{2N} = \frac{1}{2}(1 - x_t) \tag{24}$$

The Concept of Transition Probabilities

Changes in the two groups are determined by their share multiplied by the transition probabilities

- $\Delta t p_t^{+-}$: transition probability of a single firm to switch from optimism (at t) to pessimism (at $t + \Delta t$)
- $\Delta t p_t^{-+}$: transition probability for a single firm to switch from pessimism to optimism for the time interval $[t, t + \Delta t)$

The effects determining the evolution of the transition probabilities are summarized in a *switching index function* $s_t(\cdot)$.

- An increase of s is supposed to increase the probability that a pessimist becomes optimistic and to decrease the probability that an optimist becomes a pessimist.
- Thus, the specification reads

$$\begin{aligned} p^{-+} &= p^{-+}(s_t) = \nu \exp(s_t) \\ p^{+-} &= p^{+-}(s_t) = \nu \exp(-s_t) \end{aligned} \tag{25}$$

Accordingly,

$$\left(\frac{n_t^+}{2N}\right)\uparrow = \Delta t\,\nu\exp(-s_t)\frac{(1+x_t)}{2} \quad \text{and}$$

$$\left(\frac{n_t^-}{2N}\right)\uparrow = \Delta t\,\nu\exp(s_t)\frac{(1-x_t)}{2}$$

The evolution of the aggregate sentiment is thus given by

$$x_{t+\Delta t} = x_t + \Delta t\left[(1-x_t)\nu\exp(s_t) - (1+x_t)\nu\exp(-s_t)\right] \tag{26}$$

or in continuous time (going to the limit $\Delta t \to 0$)

$$\begin{aligned} \dot{x} &= \nu[(1-x)\exp(s) - (1+x)\exp(-s)] \\ &= 2\nu[\tanh(s) - x]\cosh(s) \end{aligned}$$

with $\tanh(s) = \sinh(s)/\cosh(s)$, $\sinh(s) = (\exp(s) - \exp(-s))/2$ and $\cosh(s) = (\exp(s) + \exp(-s))/2$.

The Concept of Transition Probabilities

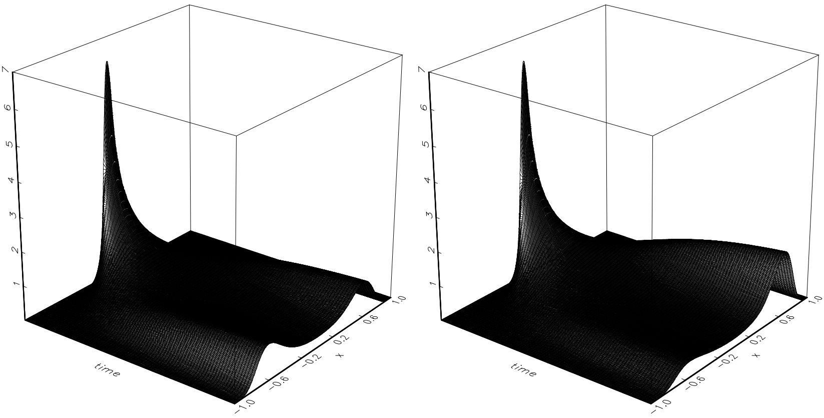


Figure: Transient dynamics of the sentiment index dynamics

Factors Influencing the Aggregate Sentiment

- The switching index function captures a *self-reference* effect (herding), i.e. positive feedback of x on itself.
- It also reflects macroeconomic fundamentals.
- Within the Franke (2012) model, the hetero-reference effects are:
  - a positive effect of the state of the business cycle y
  - a negative effect of the real rate of interest $i - \pi - r^o$
  - a negative effect of financial distress d (is assumed to be exogenous)
- Therefore, the switching index function becomes

$$s = \phi_x x + \phi_y y - \phi_i(i - \pi - r^o) - d \tag{27}$$

The Macroeconomic Framework

The structure of macroeconomic framework corresponds to the new macroeconomic consensus, i.e. it consists of three main building blocks:

- one determining the output gap y
- one for the rate of inflation π
- another for the nominal interest rate i

Goods Markets

Investor choose between two different investment opportunities: g_{max} if the investor is optimistic ($x = +1$), and g_{min} if $x = -1$. Aggregate capital accumulation becomes

$$g(x) = \frac{g_{max}(1+x)}{2} + \frac{g_{min}(1-x)}{2} = g^o + \beta_{gx} x \tag{28}$$

where $g^o = (g_{max} + g_{min})/2$ and $\beta_{gx} = (g_{max} - g_{min})/2$.
Defining the *output-capital ratio* or *capital utilization* as

$$u = Y/K = \frac{1}{\sigma} g(x) + \beta_u \tag{29}$$

where β_u a constant composite parameter and σ is the households saving propensity out of disposable income, the output gap (%-deviation from normal output) can be easily obtained by $y := (u - u^o)/u^o$ with $u^o = (g^o/\sigma) + \beta_u$. Hence

$$y = \eta x \qquad \text{with } \eta := \beta_{gx}/(\sigma u^o). \tag{30}$$

The Dynamics of the Inflation Rate

- For the dynamics of inflation, let us consider the following Phillips Curve relationship

$$\pi = \pi^c + \kappa y. \tag{31}$$

The term π^c stands for the general inflation climate rather than expected inflation.

- The (adaptive) inflation climate (AIC) adjusts according to

$$\dot{\pi}^c = \alpha[\gamma\pi^\star + (1-\gamma)\pi - \pi^c] \tag{32}$$

with the adjustment speed parameter α, the target rate of inflation $\pi^\star$ faced by the central bank and the central banks credibility γ.

The Taylor Rule

Consider that the central bank governs the nominal interest rate according to a Taylor-type Rule.

$$i = i^o + \mu_\pi(\pi - \pi^\star) + \mu_y y. \tag{33}$$

Concerning the numerical values of the Taylor Rule parameters consider that the famous Taylor Principle $\mu_\pi > 1$ holds.

The Full Model in Its Two-Dimensional Canonical Form

The entire system can be represented in its reduced form as

$$\dot{x} = 2\nu[\tanh(s) - x]\cosh(s) \tag{34}$$

$$\dot{\pi}^c = \alpha[\gamma\pi^\star(1-\gamma)(\pi^c + \kappa\eta x) - \pi^c] \tag{35}$$

$$s = Ax - \phi_i(\mu_\pi - 1)(\pi^c - \pi^\star) - d \tag{36}$$

$$A = \phi_x + \phi_y\eta - \phi_i\eta[(\mu_{pi} - 1)\kappa + \mu_y] \tag{37}$$

The Isoclines

In order to study the dynamics of the system, the isoclines of both differential equations should be derived. Thus,

$$\begin{aligned} \dot{\pi}^c &= 0 \\ \pi^c &= \pi^\star + \frac{(1-\gamma)\kappa\eta}{\gamma} x \end{aligned} \tag{38}$$

and (by make use of arctanh$(x) = (1/2) \cdot \ln[(1+x)/(1-x)]$)

$$\begin{aligned} \dot{x} &= 0 \\ \pi^c &= \pi^\star + \frac{1}{\phi_i(\mu_\pi - 1)} \left[Ax - \frac{1}{2} \ln\left(\frac{1+x}{1-x}\right) - d \right] \end{aligned} \tag{39}$$

For a better evaluation of the shape of the $(\dot{x} = 0)$-isocline, it is useful to compute the corresponding partial derivative. It is given by

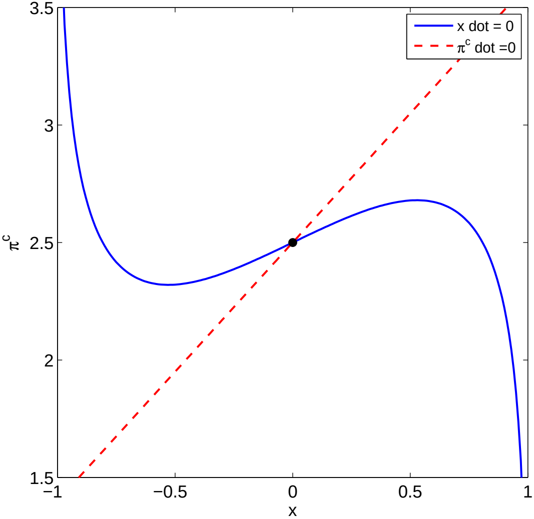
$$\frac{\partial \pi^c|_{\dot{x}=0}}{\partial x} = \frac{1}{\phi_i(\mu_\pi - 1)} \left[A - \frac{1}{1 - x^2} \right] \tag{40}$$

From this expression, it can be inferred that

- the isocline is strictly decreasing $\forall A < 1$
- it has a positive slope over an intermediate range of x
- the isocline decreases when x gets close to the ± 1 boundaries

Hence, the properties give rise for a S-shaped isocline quit similar to the assumed shape of the isoclines in the Kaldor model.

Figure: The Isoclines of the Franke Model



Local Stability

In order to assess the stability of the system, make use of the Jacobian. It can be represented by

$$J = \begin{pmatrix} \frac{2\nu[A - \cosh^2(s)]}{\cosh^2(s)} & \frac{-2\nu\phi_i(\mu_\pi - 1)}{\cosh^2(s)} \\ \alpha(1-\gamma)\kappa\eta & -\alpha\gamma \end{pmatrix} = \begin{pmatrix} ? & - \\ + & - \end{pmatrix} \tag{41}$$

From the sign pattern in (41) one directly obtains a sufficient condition for local stability, which says that in a stationary point (x, π^c) the inequality $A < \cosh^2[s(x, \pi^c, d)]$ is satisfied; this yields a negative sign in the upper-diagonal entry and thus a negative trace and a positive determinant. Accordingly,

- the isocline is downward sloping if the upper-left element is negative
- in a locally stable point of rest $(x^o, \pi^{c,\star}\ )$ the $(\dot{x} = 0)$-isocline is downward sloping (it crosses the $(\pi^c = 0)$-isocline from above)

Figure: The Emergence of a Stable Limit Cycle

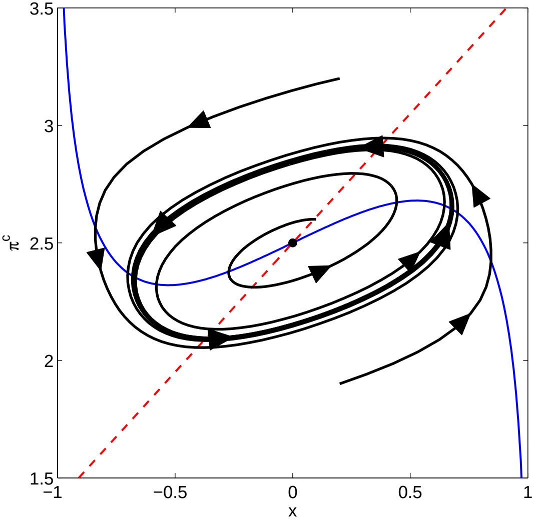
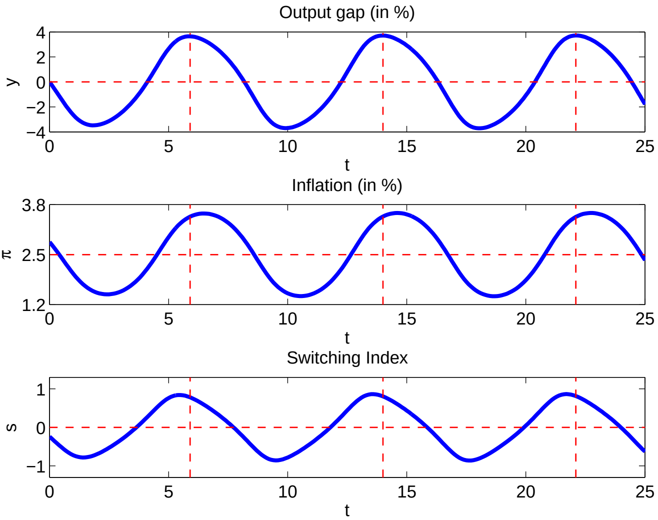
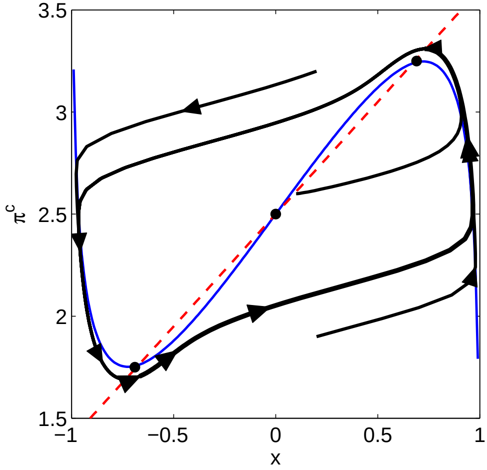


Figure: Model Generated Time Series'



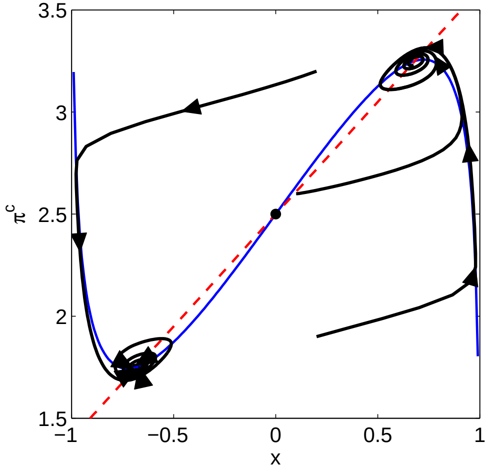
The Emergence of Multiple Equilibria

Figure: Multiple Repelling Equilibria ($\phi_x = 7.41$)



The Emergence of Multiple Equilibria

Figure: Multiple Equilibria ($\phi_x = 7.42$)



Recommended Readings

- Franke, R. (2012). Microfounded Animal Spirits in the New Macroeconomic Consensus. *Studies in Nonlinear Dynamics & Econometrics*, **16**(4), 1-41
- Lojak, B. (2016). Sentiment-Driven Investment, Nonlinear Corporate Debt Dynamics and Co-Existing Business Cycle Regimes. *BERG Working Paper*, University of Bamberg.
- Flaschel, P., Hartmann, F., Malikane, C. & C.R. Proaño (2015). A Behavioral Macroeconomic Model of Exchange Rate Fluctuations with Complex Market Expectations Formation. *Computational Economics*, **45**(4), 669-691, Apr.
- Lux, T. (1995). Herd Behaviour, Bubbles and Crashes. *Economic Journal*, **105**(431), 881-96, July.
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The Learning Approach

- **Main Idea:** Agents do not know the exact numerical values of the economy's parameters, but they have a good notion about the model's reduced form.
- On the basis of the observation of the interaction between their expectations and the actual economic outcomes, agents may learn about the true parameter values of the economy's model over time.
- A key issue in this strand of the literature is the question if a rational expectations equilibrium (REE) is learnable, i.e. whether agents's expectations and actions may converge to the rational expectations analogous over time.

A Simple Example

Consider the simple model given by the equations

$$y_t = a + by^e_{t,t-1} + cx_t \tag{42}$$

$$x_t = \rho x_{t-1} + v_t \tag{43}$$

where v_t is an i.i.d. random variable with zero mean and finite variance. Inserting the latter equation into the former yields the *actual law of motion (ALM)*

$$y_t = a + by^e_{t,t-1} + c(\rho x_{t-1} + v_t) \tag{44}$$

Setting $y^e_{t,t-1} = E_{t-1}(y_t)$ and taking expectations on the basis of the information set at $t-1$ yields

$$\begin{aligned} E_{t-1}(y_t) &= E_{t-1}(a + by^e_{t,t-1} + c(\rho x_{t-1} + v_t)) \\ &= a + bE_{t-1}(y_t) + c\rho x_{t-1} \\ &= \frac{a}{1-b} + \frac{c\rho}{1-b}x_{t-1} \qquad \text{where } E_{t-1}(E_{t-1}(y_{t,t-1})) = E_{t-1}(y_t). \end{aligned}$$

Inserting this expression into eq.(47) yields the law of motion of this simple system under rational expectations:

$$
\begin{aligned}
y_t &= a + b\left(\frac{a}{1-b} + \frac{c\rho}{1-b}x_{t-1}\right) + c(\rho x_{t-1} + v_t) \\
&= \frac{a-ab}{1-b} + \frac{ba}{1-b} + \frac{bc\rho}{1-b}x_{t-1} + \frac{(c\rho - cb\rho)}{1-b}x_{t-1} + cv_t \\
&= \frac{a}{1-b} + \frac{c\rho}{1-b}x_{t-1} + cv_t
\end{aligned}
$$

Now suppose that the agents do not have rational expectations, but they have only a vague idea of the true structure of the economy. For instance, suppose that the agents thinks that y_t is driven by a linear function of x_{t-1} and v_t. Their *perceived law of motion (PLM)* is thus

$$y_t = \alpha_n + \beta_n x_{t-1} + \gamma_n v_t \tag{45}$$

where the subscript n indexes the period over which expectations are revised.

Believing this to be the economy's structure, the agents form their expectations at time $t-1$ as

$$y^e_{t,t-1} = \alpha_n + \beta_n x_{t-1}.$$

Since the actual dynamics of the economy depend of the agents' expectations, the realization of y_t at date t is given by

$$\begin{aligned} y_t &= a + b y^e_{t,t-1} + c(\rho x_{t-1} + v_t) \\ &= a + b(\alpha_n + \beta_n x_{t-1}) + c\rho x_{t-1} + c v_t \\ &= (a + b\alpha_n) + (b\beta_n + c\rho) x_{t-1} + c v_t. \end{aligned}$$

After observing the realization of y_t, the agents will revise their perception of the economy to

$$y_t = \alpha_{n+1} + \beta_{n+1} x_{t-1} + \gamma_{n+1} v_t.$$

with $\alpha_{n+1} = a + b\alpha_n$, $\beta_{n+1} = c\rho + b\beta_n$ and $\gamma_{n+1} = c$

Question: Under what conditions will the parameters perceived by the agents $(\alpha_n, \beta_n, \gamma_n)$ converge to the parameters of the rational expectations solution $a/(1-b), c\rho/(1-b), c$?

- As long as $|b| < 1$, the learning process for α_n and β_n will eventually converge to the parameter values

$$\frac{a}{1-b}, \qquad \frac{c\rho}{1-b},$$

respectively, while from the third relationship the agents will learn c in just one learning period.

- By contrast, if $|b| > 1$, the two first learning process will diverge, causing the dynamics of the overall system to diverge.

Expectational Stability

- A key question in the learning approach is thus whether the dynamics of the economy may allow agents to update their expectations in such a way that, if enough time passes by, those expectations become rational in the Muth (1961) sense.
- The basic required concept is the mapping from the perceived law of motion (PLM) to the actual law of motion (ALM).
- If the interaction between the PLM and the ALM leads asymptotically to the emergence of a REE, then this REE will be expectationally stable or *E-stable*.
- **IMPORTANT:** Not every REE is e-stable, so that the explicit analysis of e-stability is key for a more realistic analysis of important issues such as the conduction of monetary policy.

Recommended Readings

- Evans, G. & Honkapohja, S. (2001), Learning and Expectations in Macroeconomics, MIT Press, Ch.1-2.
- Turnovsky (2000), Methods of Macroeconomic Dynamics, Second Edition. Ch. 3.7.
- Bask, M. & Proaño, C.R. (2016), Optimal Monetary Policy under Learning and Structural Uncertainty in a New Keynesian Model with a Cost Channel and Inflation Inertia. *Journal of Economic Dynamics and Control*, **69**, 112-126, Aug.

Concluding Remarks

- Agents have limited information sets and/or cognitive capabilities to process all the information available to them in reality
- They use rule-of-thumb rules instead, which can lead to market instability
- These issues need to be incorporated in macroeconomic models not as exceptions, but as the baseline scenario

Thank you for your attention

Questions for Group Discussion

- Is the New Neoclassical Synthesis an adequate baseline theory for behavioral economics, and for macroeconomics in general?
- Should we consider certain assumptions about economic behavior as general?
- Can or should a model be valid for all possible situations, or should we use different types of models, depending on the question at hand?
- Other suggestions?