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# Stock-Flow Consistent Input–Output Models as a Bridge Between Post-Keynesian and Ecological Economics

Matthew Berg (The New School for Social Research)

Brian Hartley (The New School for Social Research)

Oliver Richters (International Economics, Oldenburg University)

E-Mail: [sfcio@oliver-richters.de](mailto:sfcio@oliver-richters.de)

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## Abstract

Navigating the transformation towards a sustainable economy requires a macroeconomic theory capable of robustly dealing with issues of energy use and emissions, but much of economic theory underemphasizes the importance of energy and natural resources. This paper suggests one alternative route to an ecological macroeconomic model with Keynesian features.

Post-Keynesian economics and ecological economics share substantial common ground, similarities include consumption and production theory, and the importance of instability, irreversibility and uncertainty. However, Post-Keynesians have heretofore tended to neglect the ecological dimension of the economy. Keynesian macroeconomic theory places great emphasis on the determination of a level of effective demand commensurate with key economic policy goals, but the ecological implications of those economic policy goals have often been neglected.

This paper proposes a synthesis of post-Keynesian Stock-Flow Consistent (SFC) models and Input–Output (IO) models that investigate sectoral interdependencies within the real economy. We present a conceptual model to simultaneously study monetary flows through the financial system, flows of produced goods and services through the real economy, and flows of physical materials through the natural environment. We analyze the model using concepts from dynamical system theory and apply it to energy related problems.

*Keywords:* Ecological Macroeconomics, Post-Keynesian Economics, Econophysics, Input-Output Analysis, Stock-Flow Consistent Models, Stationary Economy, Rebound Effects

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# 1 Introduction

One of the key issues faced by modern society is navigating the transformation towards a sustainable economy that respects ‘planetary boundaries’ (Rockström et al., 2009). This transformation could be facilitated by a macroeconomic theory capable of robustly dealing with issues of energy use and emissions, but much of economic theory underemphasizes the importance of energy and natural resources (Kümmel, 2011). In addition, many general equilibrium models abstract from institutional details of money creation and monetary flows which play a central role in real-world macroeconomic dynamics. This paper suggests one alternative route to an ecological macroeconomic model with Keynesian features, in which finance plays a central role.

One effort to explicitly represent the dynamics of debt, finance, and other monetary factors has been the post-Keynesian stock-flow consistent (SFC) approach. At the same time, input–output (IO) models have been widely used to investigate sectoral interdependencies within the real economy, while environmentally extended input–output models have been used to analyze the relationship between the economy and ecological subsystems. However, the role of monetary dynamics has been left relatively unexplored in IO models (Caiani et al., 2014). This paper proposes a synthesis of elements from both SFC and IO models with insights from ecological economics to provide an avenue for investigating the interrelations between the monetary economy and the physical environment.

The introduction into stock-flow consistent (SFC) models, input–output (IO) analysis and ecological macroeconomics and the theoretical foundations of common ground between these approaches, the model structure and preliminary analysis was published in Berg et al. (2015). We use the conceptual model presented there to study a generalized version of the Sraffian maximum rate of profit, the impact of rebound effects and to contribute to the discussion of whether a stationary economy is compatible with positive interest rates. The stability analysis is taken from the Master’s thesis of Oliver Richters in the Physics program of Oldenburg University under supervision of Prof. Dr. Ulrike Feudel in the *Theoretical Physics / Complex Systems* group. A brief conclusion assesses the relevance of the contribution and potential extensions.

## 2 Framework and Methods

### 2.1 The Macroeconomic Significance of Energy

Several studies underline that the wealth of industrial nations has grown thanks to energy services over the last several centuries and in particular since the beginning of the industrial revolution (Kander et al., 2013; Kümmel, 2011; Wrigley, 2010). Nevertheless, the significance of resources and energy has generally been downplayed in most modern theories (Binswanger and Ledergerber, 1974; Kümmel, 2011). In contrast, some pre-classical, physiocratic, and early nineteenth century classical economists were aware of the physical side of economic activity, similar to the contemporary *ecological economists* (Christensen, 1989; Cleveland, 1987; Røpke, 2004). The latter object to most economic models because of their focus on the circular flow of exchange value (i.e. money), rather than on the physical throughput of natural resources from which all goods and services are ultimately derived (R. U. Ayres, 1978; Cleveland, 1999; Daly, 1985; Frondel and Schmidt, 2004; Georgescu-Roegen, 1971). Most economists interpret energy services as enhanced labor or capital productivity associated with technological progress (Kümmel, 2011, p. 52), which is considered to be the biggest contribution to economic growth (Solow, 1956; Blanchard and Illing, 2014, pp. 321ff). But ascribing growth to this ‘amorphous force that can increase productive power without limit’ (Gowdy et al., 2009, p. 206) has been criticized for leaving the ‘main factor in economic growth unexplained’ (Solow, 1994).

Some economists have considered energy  $E$  as a factor of production, sometimes in combination with materials  $M$ , but have underestimated the importance of these factors. The responsiveness of output to a marginal change of one production factor in the neoclassical approach is given by its output elasticity  $E_{y,x}$ , the point elasticity of output  $y$  of an entity with respect to a production factor  $x$  (Kümmel and Lindenberger, 2014):

$$E_{y,x} = \frac{x}{y} \frac{\partial y}{\partial x}. \quad (1)$$

The theory assumes that in equilibrium, this should be identical to the cost share of the production factor. Energy costs represent about five percent of production costs;

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consequently, the output elasticity of energy has been estimated to be 0.05. As this is low compared to labor with 0.7 or capital with 0.25 during recent decades in OECD countries, energy has been left out of most economic models (Gowdy et al., 2009, p. 207; Manne, 1978), see Kümmel (2011, pp. 180–212) for a longer discussion. Cost share and output elasticity are not necessarily equal once a third factor is added that is not independent of the other two (R. U. Ayres and Warr, 2005, p. 16). This is the case here, since ‘capital in the absence of energy is functionally inert’, and technical engineering constraints limit substitution (Kümmel, 2011, p. 195). Based on a general equilibrium framework extended by incorporating energy as a production factor, Kümmel uses non-linear optimization with generalized shadow prices on real data to calculate time-averaged output elasticities of 0.37 for capital, 0.11 for routine labor, and 0.52 for energy, while the remaining residual of 0.12 is ascribed to the residuum called creativity (ibid., pp. 180, 212). Similar values are found by R. U. Ayres, L. W. Ayres, et al. (2003) and R. U. Ayres and Warr (2005). Using these elasticities, energy accounts for most of the growth attributed to technological progress (Kümmel, 2011, p. 221). This indicates that postulating an identity between factor costs and output elasticities is flawed, and the neglect of energy is without solid foundation.

The significance of these findings is underlined by the International Monetary Fund, which investigates the impact of lower oil supply in its World Economic Outlook, stating that ‘if the contribution of oil to output proved much larger than its cost share, the effects could be dramatic, suggesting a need for urgent policy action’ (International Monetary Fund, 2011, p. 109). Given the naturally constrained supply of fossil fuels, the connection between energy and the economy must be understood in order to avert potential challenges to the modern global industrial system, which currently depends categorically on fossil fuels and other non-renewable energy sources (Heinberg, 2007). A declining capacity to extract energy has sometimes been an important trigger of societal collapse (Homer-Dixon, 2006, p. 36; Tainter, 1988, pp. 91–122). This not only has historical implications, but could also potentially impact theoretical accounts of modern business cycles, as every US recession since World War II was accompanied by rising energy prices (Hamilton, 1983, 2013; Murphy and Hall, 2011). In Berg et al. (2015), we suggest that this could have been caused by a decline in effective demand due to higher energy prices. Other studies have underlined the contemporary significance of energy in terms of the ‘Energy Returned on Energy Invested’ (EROI), which is the usable energy acquired divided by the amount of energy expended to extract and process that energy resource (Cleveland et al., 1984). It is an open question whether unconventional oil fields allow for an extraction velocity comparable to conventional fields, and at lower EROI, economic growth will become ‘harder to achieve and come at an increasingly higher financial, energetic and environmental cost’

(Murphy, 2014).

In order for economic activity to be environmentally sustainable, such that it ‘meets the needs of the present without compromising the ability of future generations to meet their own needs’ (World Commission on Environment and Development, 1987), it must be the case that the ecosystem can absorb waste and recycle the inputs which are required for physical production (Daly, 1992, p. 186). Therefore, the physical and environmental sustainability of the economy can best be analyzed by considering the economy as an open subsystem of the larger but finite physical ecosystem, as energy usage entails heat and particle emissions (Kümmel, 2011). While energy conservation may provide a partial solution to this problem, there are inescapable thermodynamic limits to energy efficiency which may limit decoupling of resource use and economic growth. Furthermore, energy use may not necessarily decline even if energy conservation measures render such a decline technically feasible because of rebound effects (Kümmel, 2011; Madlener and Alcott, 2009), studied and described in section 4.3. In a sustainable economy, energy available for production will not be limited by the availability of energy *as such*, but rather by the *capacity to extract renewable resources* due to the fact that the buildup of the capital stock requires energy input (Dale et al., 2012a,b). For these reasons, some ecological economists argue that the necessity of adapting to planetary boundaries and resource extraction limits may decrease energy supply, and the constraint of this main driver of economic growth may render a stationary economy or economic degrowth unavoidable (Jackson, 2009; Kallis et al., 2012; Pueyo, 2014).

## 2.2 Stock-Flow Consistent (SFC) Models

The post-Keynesian approach underlines the significance of a *monetary economy* (Godley and Lavoie, 2007; Graziani, 2003) and objects the neutrality of money used in most neoclassical general equilibrium models (Blanchard et al., 2014). One effort to explicitly represent the dynamics of debt, finance, and other monetary factors has been the post-Keynesian stock-flow consistent (SFC) approach. SFC models are a class of structural macroeconomic models grounded by a detailed and careful articulation of accounting relationships. They are constructed by tabulating the balance sheets and transactions of the different sectors.

Though post-Keynesian authors criticized the aggregation procedures of neoclassical authors, most SFC models are formulated as sectoral models, similar to a mean field approach, but the structure of the models allows for disaggregation. Godley, Lavoie, and a number of other authors expanded this approach into a family of applied macroeconomic

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models that respect accounting identities and are closed with behavioral assumptions based on post-Keynesian theory (Godley et al., 2007; Lavoie and Godley, 2001; Zezza and Dos Santos, 2004). Perhaps the single most important advantage of the SFC approach is that it enables the modeler to easily create scalable representations of institutional structures with an explicit monetary dimension. Post-Keynesian authors view production as a discrete and sequential technically determined process with limited possibility for immediate substitution, similar to input–output (IO) models.

## 2.3 Input–Output (IO) Models

Input–output models provide a detailed treatment of production and of the flow of real goods and services through the economy, and are commonly applied to analyze interactions and feedback effects between mutually interdependent industrial sectors. IO tables provide a static snapshot view of the economy, assuming constant returns to scale. They are displayed in matrix notation (‘Leontief matrix’), where each column represents inputs to a specific sector, while each row shows the output from a given sector to the rest of the economy. For an economy with  $n$  sectors, a  $n \times n$  matrix  $\mathbf{a}$  is used, where  $a_{ij} \geq 0$  is a flow of inputs produced by sector  $i$  to sector  $j$  in order to produce one unit of output  $j$ .

## 2.4 Common Ground

Post-Keynesian and ecological economists criticize different aspects of general equilibrium theory, where the aggregate behavior of a market is studied assuming that the behavior of the economy can be inferred from individual, rational decisions that are taken in isolation. Through an intertemporal optimizing procedure, a general equilibrium is determined, and alternative models have been considered ‘not scientific’ (Kirman, 2011, p. 12). One should point out that the use of the term ‘equilibrium’ in economics may be misleading to physicists, because the analysis does not look at a ‘rest point’ of a dynamical system, but it is a static description ‘of an allocation of resources to the individual consumers and firms, from which nobody, given the constraints imposed by the system, would have any interest in deviating’ (ibid., p. 7). Out-of-equilibrium dynamics in macroeconomic models based on general equilibrium are therefore only considered in linear order in a neighborhood around the equilibrium, and it cannot be determined why and how the model economy settles at a specific fixed point. Unfortunately, it has been proven that given some heterogeneity in preferences and endowment among agents, multiple equilibria exist (Debreu, 1974; Mantel, 1974; Sonnenschein, 1972), which has been ‘solved’ by introducing the representative agent

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abstracting from heterogeneity (Kirman, 2011, p. 16). The economist neglect ‘how the interactions between the individuals determine the state of the economy and, in particular, whether they would produce an equilibrium’ (ibid., pp. 13f).

Unfortunately, the different critiques from the ecological and monetary perspective remain largely unconnected. Keynesian macroeconomic theory places great emphasis on the determination of a level of effective demand commensurate with key economic policy goals, but the ecological implications of those economic policy goals have often been neglected. Therefore, Mearman (2005) concluded that ‘post-Keynesians need to embrace the environment’ in order to underline the relevance of their work. In contrast, most ecological economists abstract from the influences of the monetary side of the economy, though some analyses of the monetary dimension of sustainability have been conducted by Tokic (2012), Binswanger (2013) and Wenzlaff et al. (2014). But outside this work, some misunderstandings appear, such as a common claim that a zero interest rate is a stability condition for a stationary economy (Farley et al., 2013; Löhr, 2012). We will review this argument in section 4.2. Issues such as monetary policy and interest rates can be most fruitfully discussed within a framework of ecological macroeconomics which is cognizant of the implications of financial flows of funds for the economy (Jackson et al., 2014).

Gowdy (1991), Kronenberg (2010a), and the contributors of the book edited by Holt et al. (2009) have explicitly argued that post-Keynesian economics and ecological economics share substantial common ground, and are ripe for a synthesis. Despite the need for new analytical tools to explore this relationship, relatively little concrete work to that end has thus far been completed (Rezai et al., 2013), notable exceptions include the work of Kemp-Benedict (2013), Kronenberg (2010b), the work in progress by Dafermos et al. (2014), and the WWWforEurope project (Jackson et al., 2014). However, some previous attempts to integrate post-Keynesian and ecological economics are not SFC. Similarities have been recognized in terms of consumption, production theory, cumulative causation (path dependency), and the irreversibility of historical time (Holt et al., 2009; Kronenberg, 2010a; Lavoie, 2006). Both post-Keynesian and ecological economists emphasize the significance of fundamental ‘Knightian’ uncertainty, as opposed to computable probabilistic risk (Godley et al., 2007; Knight, 1921; Kolstad, 1996; Radner, 1968), and replace the axiom of perfect rationality and optimizing agents by ‘reasonable rationality’: Agents ‘follow norms and targets, and act in line with these and the expectations that they may hold about the future’ (Godley et al., 2007, p. 16). Both schools reject neoclassical aggregate production functions, but view production as a discrete and sequential technically determined process with limited substitution – because they either ask for compatibility with the laws of nature or because of the aggregation fallacies underlined by the Capital

Controversies (Kronenberg, 2010a). This allows for an integration of input–output models into post-Keynesian SFC models. Some indications on how agent-based models (ABM) may be integrated to incorporate interaction and heterogeneity explicitly will be given in chapter 5.1.

By combining SFC models and IO models, financial flows of funds can be integrated with flows of real goods and services. Lawrence Klein, who developed large scale macroeconomic models typified by the FRB-MIT-Penn model, has noted the natural synergies between the National Income and Product accounts, the IO accounts, and the FF accounts (Klein, 2003). The approach of combining both SFC and IO models with ecological macroeconomics affords one method to unite those accounts, as suggested by Klein, and to simultaneously model monetary flows through the financial system, flows of produced goods and services through the real economy, and flows of physical materials through the natural environment. Models of this type may provide additional tools to aid macroeconomists, ecological economists, and physicists in the task of understanding the economy and the physical environment as one united and complexly interrelated system, rather than as a colloidal agglomeration of artificially separated analytical domains. These modes of analysis are required to study pressing problems such as climate change, which are neither purely economic, nor purely environmental, nor purely physical, but rather are all of the above (Rezai et al., 2013). The following chapter presents the methodology and structure of a conceptual stock-flow consistent input–output model.

It is surprising that although SFC models are dynamical systems, very few concepts of dynamical system theory are applied, even though this approach allows to study general properties of the models such as stability or long-term development reducing the need for extensive simulations. This paper covers a very small fraction of the concepts established by the dynamical system community, but suggests that a more extended use may be helpful for rigorous analysis of macroeconomic models. Whether economic models should use continuous or discrete time has been subject to discussions, but Tobin (1982) argued that ‘either representation of time in economic dynamics is an unrealistic abstraction’. Godley, one of the fathers of the SFC approach, ‘preferred to work in discrete time, responding to the way the data are presented’ (Taylor, 2008). As the SFC model used is formulated in the tradition of Godley et al. (2007), the following model is described in discrete time.

### 3 SFCIO models—Methodology

This section introduces a conceptual baseline model that could serve as a point of common ground between the SFC, IO, and ecological macroeconomics approaches. A SFC model of a closed economy is coupled with an IO model. The model used is represented in discrete time  $t$  and includes multiple ( $n$ ) industry sectors, a household sector, and a government/banking system sector. However, the household sector and the government and banking system sector are both consolidated.

The model simultaneously tracks the values of all flows of goods and services through the economy in both nominal terms (measured in terms of money-values) and in real terms (measured in terms of physical units of the heterogeneous real physical output of industry  $i$ ). In order to more easily identify which variables are in real terms and which are in nominal terms, all nominal variables are written in capital letters.

The flows are displayed in the transaction matrix in table 2. Adherence to the accounting constraints imposed by the balance sheet in table 1 and the transaction-flow matrix in table 2 guarantees the consistency of the model, and can be verified by checking that all the columns and rows of the matrices sum to the appropriate values, which in the case of financial assets sum to zero (Godley et al., 2007, p 27). All parameters are summarized in table 3.

We use a simplified model with only two sectors, but as shown in the flow diagram in figure 1, a variety of financial flows and physical flows are included in even a simple model with two sectors. All monetary payments (solid lines) flow from one sector to another and accumulate to the corresponding stock, providing consistency between stocks and flows. For a detailed description of the equations, see Berg et al. (2015).

Money flows from households to the government in the form of taxes  $T$ . Money flows from both the production sector and the energy industry to households in the form of wages  $\text{III}_{p/e}$  and distributed profits  $\text{II}_{p/e}$ . In turn, households spend their money on both production goods and energy goods, which creates flows of money  $C_{p/e}$  back to the production and energy sectors and corresponding flows of real goods and services back to the households. The government likewise buys both production goods and energy goods, which creates similar flows of both real goods and services and of money  $G_{p/e}$  between

Figure 1: Stocks of sectors and flow chart of money, energy, and materials,  $h$ : households,  $p$ : production sector,  $e$ : energy sector,  $g$ : government/banking system sector. For each sector, a balance sheet is shown in the form of a T-account. **Stocks:**  $M_h$ : money stock of households.  $M_g$ : money issued by government/banks.  $L_{p/e}$ : loans of production sector/energy sector.  $L_g$ : loans made by government/banks.  $\psi_{p/e}$ : physical inventories of industry sectors. **Money flows:**  $C_{p/e}$ : consumption of households.  $G_{p/e}$ : consumption of government.  $E_{ep}$ : money paid by production sector for energy.  $E_{pe}$ : money paid by energy sector for intermediate goods.  $\Pi_{p/e}$ : wage bill paid by production/energy sector.  $\Pi_{p/e}$ : distribution of profits.  $T$ : taxes.  $r_M M_h$ : interest payment to households.  $r_L L_{p/e}$ : interest payments by production/energy sector. **Energy/material flows:** Energy: energy extracted from the environment. Heat: heat emissions. Resources: extracted from the environment. Waste: emitted to the environment; not treated explicitly in the model, but implied.

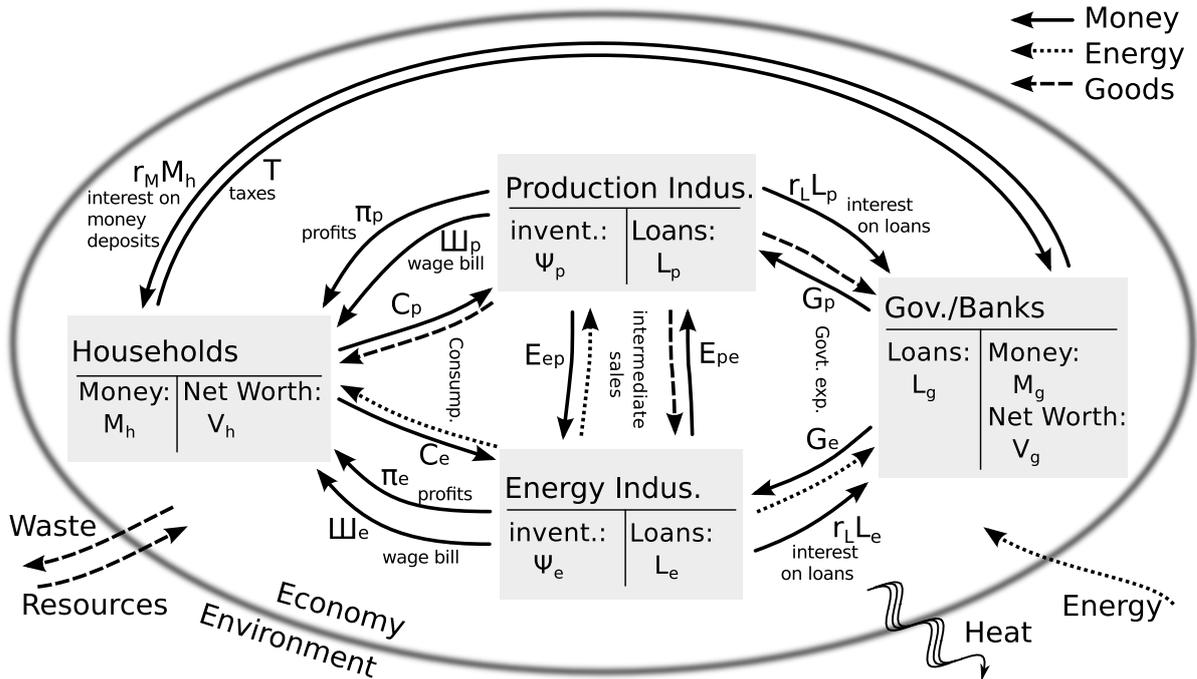


Table 1: Balance sheet matrix in nominal terms. The money deposits of households  $M_h$  are equivalent to the money issued by the government  $M_g$ , because we assume that the industry sector does not hold money deposits. All sums over  $i$  are preceded over the  $n$  industry sectors. For a more detailed description, see Berg et al. (2015).

	Households	Government	Industry $i \in \{1, \dots, n\}$	$\Sigma$
<b>Money Deposits</b>	$+M_h$	$-M_g$		0
<b>Loans</b>		$+L_g$	$-L_i$	0
<b>Inventories</b>			$+\Psi_i$	$+\sum_i \Psi_i$
<b>Net worth</b>	$-V_h$	$-V_g$	0	$-V_h - V_g$
$\Sigma$	0	0	0	0

the government and both of the two industries. The production industry buys energy goods as intermediate inputs, which creates flows of energy goods from the energy industry to the production industry and a corresponding flow of money  $E_{ep}$  from the production industry to the energy industry. The inverse is true for purchases of production goods as intermediate inputs by the energy industry  $E_{pe}$ . Finally, as physical raw materials are used in production, and as some raw materials are expended as waste, there are flows of physical materials between the human economy and the natural environment. Likewise, energy flows into the economy from the natural world, while heat is emitted by the economy into the natural environment. These economy-nature interactions are not *explicitly* considered in the model, but rather are simply implied.

The flow diagram in figure 1 also shows the balance sheets of each of the four sectors (the households sector, the government sector, the production goods sector, and the energy goods sector) in the form of T-accounts. Assets are shown on the left side of the T-account, while liabilities and net worth are shown on the right side of the T-account. In accounting, a fundamental equation known as the *balance sheet equation* states that the left side of the T-account is by definition always equal to the right side of the T-account. This is a symmetry principle, and is why balance sheets are called ‘balance’ sheets. We distinguish two types of stocks of financial assets: money deposits and loans. Loans appear on the asset side of the government/Banking system sector’s balance sheet and on the liability side of industry  $i$ ’s balance sheet. Money deposits, on the other hand, appear on the liability side of the government/Banking system sector’s balance sheet, and on the asset side of the household sector’s balance sheet. In the balance sheet perspective, the government sector holds assets of loans  $L_g$  on the left side of its T-Account, while it has liabilities of money

Table 2: The Transaction matrix tabulates all flows of funds within one time period. The fact that the columns sum to zero represents a sector's budget constraints, while the fact that rows sum to zero represents the fact that each financial transaction has a counterparty. Positive values indicate inflows, while negative values indicate outflows. For a more detailed description, see Berg et al. (2015).

	Households		Industry $i$		Government	$\Sigma$
	Current account	Capital account	Current account	Capital account		
Government spending					$-\sum_i G_i$	0
Taxes	$-T$	$+G_i$			$+T$	0
Consumption	$-\sum_i C_i$	$+C_i$				0
Wage bill	$+\sum_i III_i$	$-III_i$				0
Intermediate purchases		$\sum_i E_{ij} - \sum_j E_{ij}$				0
Profits	$+\sum_i \Pi_i$	$-\Pi_i$				0
Interest on money deposits	$+r_M M_{h(t-1)}$				$-r_M M_{g(t-1)}$	0
Interest on loans		$-r_L L_{i(t-1)}$			$+\sum_i r_L L_{i(t-1)}$	0
$\Delta$ Money deposits	$-\Delta M_h$				$+\Delta M_g$	0
$\Delta$ Loans		$+\Delta \Psi_i$			$-\sum_i \Delta L_i$	0
$\Delta$ Inventory Value						0
$\Sigma$	0	0	0	0	0	0

deposits  $M_g$  on the right side. The difference between its assets and liabilities determines its net worth  $V_g$ , also shown on the right side of the T-account.

In addition to stocks of financial assets (money deposits and loans), stocks of real assets also appear on balance sheets. A heterogeneous vector of inventories consisting of all the unsold output of each industry  $i$  at the end of each period constitutes the real assets of the model economy. These inventories are denoted by  $\psi_i$ , each held on the balance sheet of the corresponding industry sector, and are valued at unit costs. The monetary value of the stock of Inventories at unit costs is signified by  $\Psi_i$ . The production goods industry holds assets of production good inventories  $\Psi_p$  on the left, counterbalanced by loans  $L_p$  on the right. The energy goods industry similarly holds assets of energy good inventories  $\Psi_e$  on the left, counterbalanced by loans  $L_e$  on the right. It is assumed as a simplification that industries do not hold stockpiles of cash, and instead distribute all excess cash holdings at the end of each period to their owners in the household sector, keeping their net worth at zero. Since real assets can change in value, maintaining the symmetry principle requires that loans adjust in response to a change in the value of a real asset. Since for every financial asset in the economy there is a corresponding financial liability, the net worth of the model as a whole consists only of the monetary values of real assets (inventories), because all financial assets and financial liabilities must necessarily sum to zero.

The very existence of stocks introduces historical time and a certain path dependence into the model. Even though the model may asymptotically converge to a steady state if all exogenous parameters are undisturbed, the model will follow a different traverse path for every possible set of stocks. Moreover, not all conceivable sets of stocks are in fact possible; only some sets of stocks are consistent with the model's accounting. Thus, depending upon the set of stocks with which the model economy has been endowed by the past, the model will follow a different trajectory forwards into the future. A detailed description of the equations can be found in Berg et al. (2015).

Usually, SFC models contain implicit functions and are typically solved numerically by iterative techniques (Caverzasi and Godin, 2013; Godley et al., 2007), but in this case, the time step evolution can be solved explicitly, though because of the number of variables, the calculations will be performed numerically. All relevant parameters are put together in table 3. It can be stated that the model dynamics can be decomposed into two procedures: The price adjustment mechanism and the dynamics of the economy. It is crucial to point out that the pricing depends only on the input–output matrices, the wages and the markup (and later the interest costs). These factors are independent on the actual scale or dynamics of the rest of the economy.

The pricing equation in matrix form, assuming that markup  $\phi_i$ , wages per unit  $\omega_i \lambda_i$  and

Table 3: Parameter names and their values used.  $\omega_i$  and  $\lambda_i$  were merged into one single parameter.

parameter name	model presented
Consumpt. func. parameters: $\alpha_1, \alpha_2$	$\alpha_1 = 0.8, \alpha_2 = 0.2$
Input-Output matrix: $\mathbf{a} = (a_{ij})$	$\mathbf{a} = \begin{bmatrix} 0.48 & 0.60 \\ 0.02 & 0.15 \end{bmatrix}$
Price matrix: $\mathbf{P} = \mathbf{diag}(P_i)$	$\mathbf{P} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
Inventory adjustment: $\beta$	$\beta = 0.75$
Expected sales adjustment: $\gamma$	$\gamma = 0.5$
Government spending: $\mathbf{G}$	$G_p = 46.6, G_e = 0$
Consumption per sector: $\mathbf{C}^0$	$C_p^0 = 0.961, C_e^0 = 0.039$
Individual markups: $\phi$	$\phi_p = 0.3333, \phi_e = 0.1364$
Interest rates: $r_M, r_L$	$r_M = 0.04, r_L = 0.05$
Tax rate: $\theta$	$\theta = 0.48$
Inventory to expected sales ratio: $\sigma^\top$	$\sigma^\top = 0.5$
Labor demand per output unit: $\lambda$	$\omega_p \lambda_p = 0.25; \omega_e \lambda_e = 0.13$
Wages per labor unit: $\omega$	

input–output coefficients  $a_{ki}$  are constant over time, is given by:

$$\mathbf{P}_{(t)} = \mathbf{P}_{(t-1)} \mathbf{a} (\mathbf{1} + \mathbf{diag}(\phi_i)) + \mathbf{diag}((1 + \phi_i) \lambda_i \omega_i). \quad (2)$$

If prices are calculated, the evolution of the other economic variables is defined by a non-homogeneous first-order matrix difference equation with a mapping matrix  $\mathcal{M}$ , see Berg et al. (2015). The economy reaches a general stationary stock-flow equilibrium if all stocks and all flows remain constant over time, and therefore inflows equal outflows. The unique stock-flow equilibrium is a stable fixed point if the absolute values of all eigenvalues of matrix  $\mathcal{M}$  are smaller than 1.

## 4 Stability Analysis and Applications

An important part of this paper that goes beyond Berg et al. (2015) is a rigorous stability analysis of the model by means of dynamical system theory. As shown in Berg et al. (ibid.), the dynamics can be decomposed into the pricing process and the rest of the economic process. This is caused by the somewhat unrealistic assumption of an exogenous markup. If the markup were endogenized and various economic forces such as competition, market power as well as the effect of unemployment on worker's bargaining power were taken into account endogenously, the dynamics could not be broken up into two pieces this way.

The matrix of the time evolution has two eigenvalues that are 0, two pairs of complex eigenvalues, and one real eigenvalue  $> 0$ . The matrix is therefore singular and not invertible. The zero eigenvalues are caused by the adaption process of expected sales. The linear subspace of the corresponding stable manifold exists for all parameter values and is given by

$$\left(\mu_1\beta, \mu_2\beta, \mu_1(1-\beta), \mu_2(1-\beta), 0, 0, 0\right)^T \text{ for } \mu_1, \mu_2 \in \mathbb{R}. \quad (3)$$

The real, positive eigenvalue is studied in section 4.2 in the context of an instability caused by the interaction between the interest rate and the propensity to consume. The complex eigenvalues result in inventory oscillations not treated here. Studying the stability of the pricing process yields a generalized version of the Sraffian maximum rate of profit in section 4.1. In the following, if no other values are indicated, the parameters from table 3 are used.

### 4.1 Price Interdependence and the Sraffian Maximum Rate of Profit

In the model, pricing is determined by a sector-specific markup  $\phi_i$  on unit costs which consist of the unit wage bill  $\omega_i\lambda_i$  and the intermediate purchases  $\sum_k P_{k(t-1)}a_{ki(t-1)}$ . In matrix form, assuming that  $\phi_i$ ,  $\omega_i$ ,  $\lambda_i$  and  $a_{ki}$  are constant over time, the price evolution

yields:

$$\mathbf{P}_{(t)} = \mathbf{P}_{(t-1)} \mathbf{a} (\mathbf{1} + \mathbf{diag}(\phi_i)) + \mathbf{diag}((1 + \phi_i) \lambda_i \omega_i). \quad (4)$$

Thus if the absolute value of one of the eigenvalues of matrix  $\mathbf{a}(\mathbf{1} + \mathbf{diag}(\phi_i))$  is greater than 1, prices do not converge to stable values, but rather explode. The passing of the threshold corresponds to a bifurcation, the bounded price equilibrium loses its stability, and the price trajectory is unboundedly going to infinity.

In the general case with  $n$  sectors, this would correspond to a  $n-1$ -dimensional stability hyperspace in  $n$ -dimensional space. In the case of  $n=2$  sectors  $p$  and  $e$ , we can calculate the maximum markup in one sector dependent on the markup in the other sector and draw a one-dimensional stability frontier, as shown in figure 2:

$$\phi_p^{max} = \frac{1 - (1 + \phi_e) a_{ee}}{a_{pp} - (1 + \phi_e)(a_{pp} a_{ee} - a_{ep} a_{pe})} - 1, \quad (5)$$

$$\frac{\partial \phi_p^{max}}{\partial \phi_e} = \frac{-a_{ep} a_{pe}}{[(1 + \phi_e)(a_{pp} a_{ee} - a_{ep} a_{pe}) - a_{pp}]^2} \leq 0. \quad (6)$$

If the sectors are not interconnected and  $a_{ep} a_{pe} = 0$ ,  $\phi_p^{max} = a_{pp}^{-1} - 1$  is independent of  $\phi_e$ . In all other cases, the value of  $\phi_p^{max}$  is maximized if  $\phi_e = 0$  and then yields:

$$\phi_p^{max} = \frac{1}{a_{pp} + \frac{a_{ep} a_{pe}}{1 - a_{ee}}} - 1. \quad (7)$$

We can now study the impact of this price inflation on the economy. If the nominal payment  $G$  of the government is fixed, then ever increasing prices will drive down the real production of the economy, while at the same time the nominal wealth of households and government debt grow without limit. If government expenditures are price adjusted, the economy stabilizes at a lower real level, see figure 3. In both cases, the profit share approaches 1.

This corresponds to the Sraffian maximum rate of profit, ‘the rate of profits as it would be if the whole of the national income went to profits’ (Sraffa, 1960, p. 19). This means that if the markup is set higher than the maximum real rate of profit, the price system will adjust so that the whole of the national income goes to profits as defined by Sraffa. In Sraffa’s case, the profit  $\phi_{max}$  was identical for all sectors and the maximum rate of profit was given by

$$\phi_{max} = (1/\lambda_a^{max}) - 1, \quad (8)$$

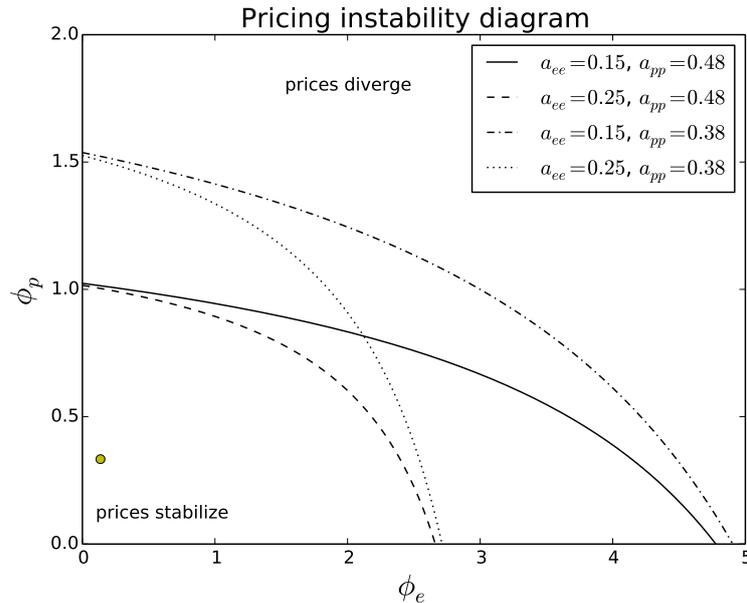


Figure 2: Pricing instability for  $n = 2$  sectors: The lines correspond to the stability frontier of different input–output matrices, given by the eigenvalues of the matrix in equation 4 passing the unit circle. The black dot indicates the position of the markups given in table 3 estimated for the German economy from (Statistisches Bundesamt, 2010b), which is well within the stable region for the input–output coefficients used that are indicated by the solid line.

with  $\lambda_a^{max}$  being the maximum of the moduli of the eigenvalues of the input–output matrix  $\mathbf{a}$  (Eatwell, 1975). The price instability is a generalized version of the Sraffian maximum rate of profit in the case of different markups in each sector: It’s about finding a *vector* of heterogeneous maximum rates of profit, while in Sraffa’s original formulation, the vector is assumed to be  $\phi_{max}$  times an all-ones vector.

Though this generalization is very straightforward, a literature review did not yield any publication mentioning this result. The work of Sraffa incorporates a uniform profit rate condition, because competition is assumed to equalize profits (Lawlor, 1994). Therefore, work in the tradition of Sraffa mostly assume identical markup in each sector of the economy, which may be part of the reason why heterogenous markups have not been more prominent in Sraffian literature.

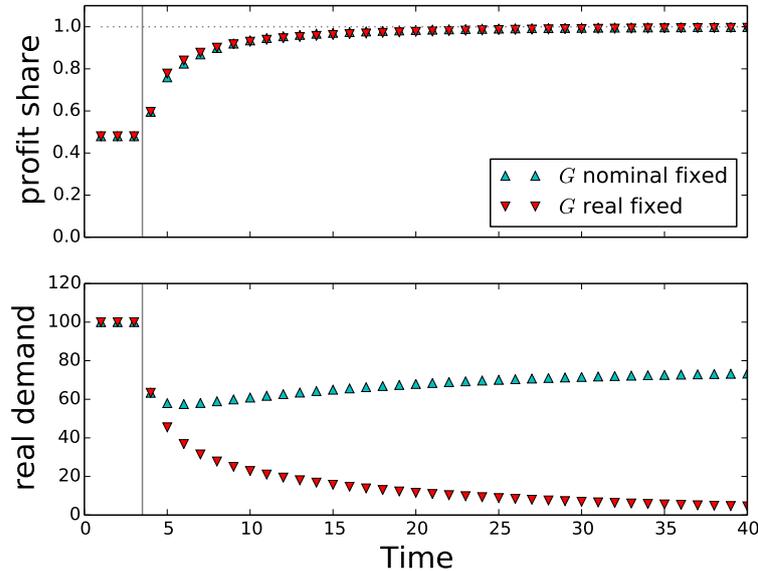


Figure 3: Time evolution profit share and real demand in the case of markups above the generalized Sraffian maximum rate of profit, therefore outside the stability range of figure 2. The values depicted are  $\phi_p = 1.1$ ,  $\phi_e = 1.0$ , while  $\gamma = 0.80$  and  $\beta = 0.02$  used in order to smoothen the graph. The graphs depicts two behavioral assumptions, keeping government expenditures  $G$  fixed in nominal terms with real output of the economy declining to zero, and price adjusted expenditures resulting in a stabilization of real output at a lower equilibrium value. In both cases, the profit share converges to 1 as predicted.

## 4.2 The Instability of a Stationary Economy with Positive Interest Rates

Within ecological economics, several authors propose a non-growing economy as a solution to environmental problems (Daly, 1991; Jackson, 2009; Kallis et al., 2012; Pueyo, 2014). In recent publications, it has been claimed that this is incompatible with positive interest rates (Farley et al., 2013; Löhr, 2012). It is argued that positive interest rates imply that in a non-growing economy, the stock of debts will rise, and it is argued that such an increase would be unsustainable. Using our model, we show that an equilibrium state of a stationary economy is possible, even with positive interest rates. To facilitate the analysis, we assume that  $P_p = P_e = 1$ . The stability of the stock-flow equilibrium is graphed in the parameter space of interest rates  $r_{M/L}$ , consumption parameters  $\alpha_{1/2}$ , and for different tax rates  $\theta$  in figures 4 and 5. The stability frontiers depicted correspond to the real non-zero eigenvalue of matrix  $\mathcal{M}$  passing the unit circle at 1. It is a bifurcation, where the bounded fixed point

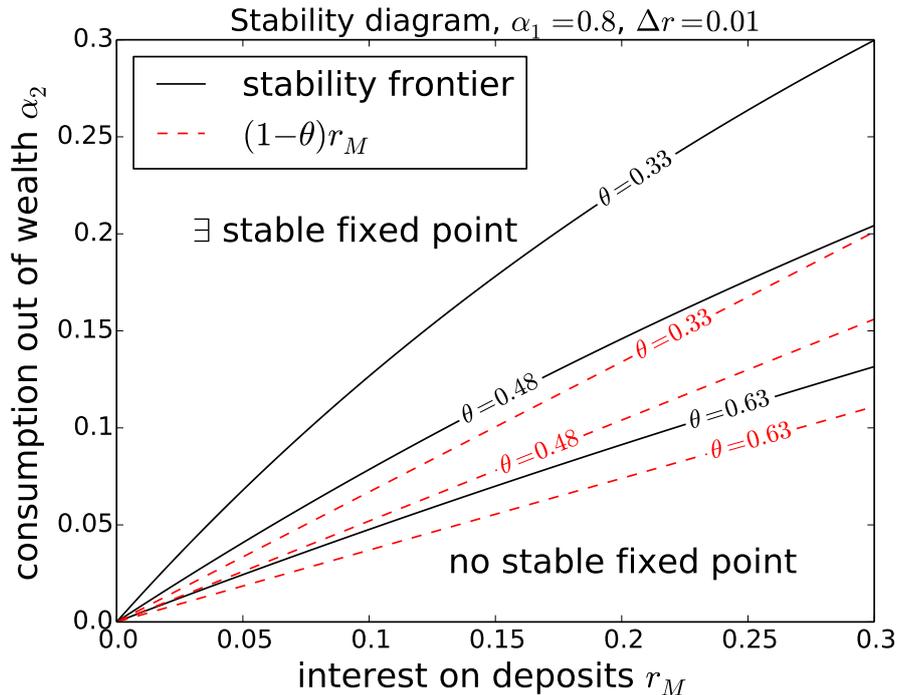


Figure 4: Stability diagram for the interdependence of interest rate  $r_M$  and consumption parameter  $\alpha_2$ , including the influence of the tax rate  $\theta$ . For different tax rates  $\theta$ , we check whether a stable stock-flow equilibrium exists. For a given interest rate  $r_M$ , there exists a minimum consumption out of wealth  $\alpha_2$  for which the model is stable, given by equation 10. An increase in the tax rate reduces this threshold. If consumption out of wealth is smaller than interest income after taxes (as indicated by the dashed lines), the fixed point will definitely be unstable, as inflows to households are always bigger than outflows for  $\alpha_1 < 1$ .

loses its stability, and the time evolution start to be divergent, while the fixed point of the map becomes undefined if the eigenvalue is 1 and changes sign at the bifurcation point.

Complementing the purely numerical results published in Berg et al. (2015), an analytical solution for the stability frontier can be calculated: If one the eigenvalues of the  $7 \times 7$  matrix  $\mathcal{M}$  is 1, one can replace the time evolution with this  $3 \times 3$  matrix equation of sales  $s'$  and the money stock of households  $M'_h$ :<sup>1</sup>

$$\mathbf{X}'_{(t)} = \begin{pmatrix} s'_{p(t)} \\ s'_{e(t)} \\ M'_{h(t)} \end{pmatrix} = \mathcal{M}' \cdot \mathbf{X}'_{(t-1)} + \begin{pmatrix} +G_p \\ +G_e \\ 0 \end{pmatrix}, \quad \text{with } \mathcal{M}' = \quad (9)$$

<sup>1</sup> $Z_{ij} = a_{ij} + \alpha_1(1 - \theta)\omega_j\lambda_j C_i^0$  with  $i, j \in p, e; Z_p = (1 - \theta)(1 - Z_{pp} - Z_{ep} - \theta\omega_p\lambda_p); Z_e = (1 - \theta)(P_{e(t)} - a_{pe}P_{p(t)} - a_{ee}P_{e(t)} - \alpha_1\omega_e\lambda_e)$

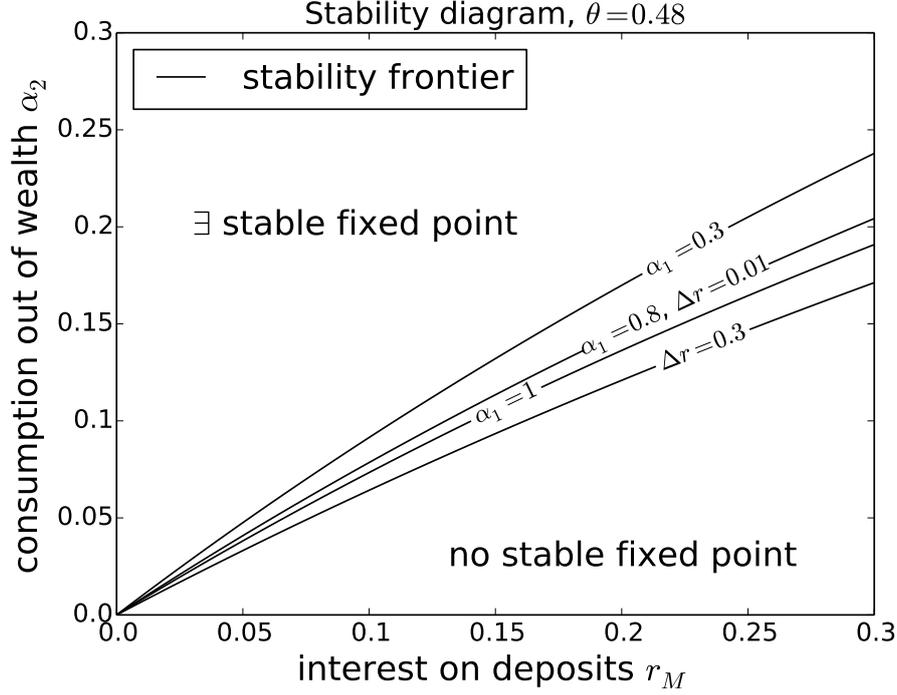


Figure 5: Stability diagram for the influence of interest rate spread and consumption out of wages. The impact of the interest rate spread  $\Delta r = r_L - r_M$  and the consumption parameter  $\alpha_1$  are depicted,  $\alpha_1 = 0.8$  and  $\Delta r = 0.01$  serves as a benchmark; only changes of these parameters are indicated. A higher interest rate spread shifts the stability lines down slightly. A higher consumption out of wages  $\alpha_1$  increases the size of the stable region.

$$\begin{bmatrix} Z_{pp} & Z_{pe} & \alpha_2 C_p^0 \\ Z_{ep} & Z_{ee} & \alpha_2 C_e^0 \\ Z_p + \sigma^\top r_L \frac{\theta-1}{1+\phi_p} & Z_e + \sigma^\top r_L \frac{\theta-1}{1+\phi_e} & 1+r_M(1-\theta)-\alpha_2 \end{bmatrix}.$$

One root of the eigenvalue polynomial of this matrix has to be 1 following our assumption, and the equation can then be solved for  $\alpha_2$  to determine the minimal consumption rate out of wealth:

$$\alpha_2 = \frac{r_M [(1 - Z_{pp})(1 - Z_{ee}) - Z_{ep}Z_{pe}]}{\frac{\theta}{1-\theta} [(1 - Z_{pp})(1 - Z_{ee}) - Z_{ep}Z_{pe}] + (Z_{ep}C_p^0 + (1 - Z_{pp})C_e^0) \left( \frac{\sigma^\top r_L}{1+\phi_e} + \alpha_1 \theta \omega_e \lambda_e \right) + (Z_{pe}C_e^0 + (1 - Z_{ee})C_p^0) \left( \frac{\sigma^\top r_L}{1+\phi_p} + \alpha_1 \theta \omega_p \lambda_p \right)}. \quad (10)$$

The term

$$(1 - Z_{pp})(1 - Z_{ee}) - Z_{ep}Z_{pe} \quad (11)$$

corresponds to the determinant of  $\mathbf{1} - \mathbf{Z}$  which is negative only if one of the eigenvalues of  $\mathbf{Z}$  is bigger than 1. This cannot be the case, as  $a_{ii} + a_{ij} + \omega_i \lambda_i$  must be always smaller than 1 in order to guarantee a positive markup. As  $0 < Z_{ij} < 1$  and all other parameters are positive,  $\alpha_2$  is always a well-defined non-negative number. The result is independent on government expenditures  $\mathbf{G}$ .

In the special case of  $a_{pp} = a_{ee}$ ,  $a_{ep} = a_{pe}$ ,  $\omega_p \lambda_p = \omega_e \lambda_e = \omega \lambda$  and therefore  $\phi_p = \phi_e = \phi$ , thus in case of symmetrical production conditions, even with  $C_p^0 \neq C_e^0$  as the two commodities are structurally identical, this simplifies to:

$$\alpha_2 = r_M(1 - \theta) \frac{\phi + \omega \lambda(1 + \phi)(1 - \alpha_1(1 - \theta))}{\theta(\phi + \omega \lambda(1 + \phi)) + \sigma^\top r_L(1 - \theta)}. \quad (12)$$

We can see that  $\alpha_2$  is proportional to the interest rate on money deposits  $r_M$ , therefore higher interest rates require higher consumption out of wealth.

If no stable fixed point exists, we see an exponential increase of private money deposits and a corresponding growth in public debt, illustrating the accounting principle that all financial assets have symmetrical financial liabilities. Flows of interest payments from the government accumulate and increase the money stock  $M_h$  held by the household sector. But if consumption out of wealth  $\alpha_2$  is high enough to counteract the interest and profit payments, households increase their consumption as their stock of wealth increases. The fixed point is stable which enables the economy to remain in stock-flow equilibrium, even though interest rates are positive. It is then not the case that the interest payments drive down government net worth. This shows that the stability of a non-growing economy is indeed a question of the interplay of interest payments and the propensity to save, as suggested by Wenzlaff et al. (2014).

For  $\alpha_2 = 0.2$ ,  $r_M = 0.05$ ,  $r_L = 0.04$  and  $\theta = 0.48$  as in table 3 and for a nominal GDP of  $d_p P_p + d_e P_e = 100$ , this is realized with  $M_h \approx 162.9$ ,  $V_g \approx -86.1$ ,  $L_p \approx 73.7$  and  $L_e \approx 3.1$ . In this state, the industry sectors realize positive profits ( $\Pi_p \approx 45.4$ ,  $\Pi_e \approx 0.7$  per period) which are distributed to the households, the tax income ( $T \approx 49.3$ ) and interest income ( $r_L L_g \approx 3.8$ ) of the government equals the government expenditures ( $G \approx 46.6$ ) and interest costs ( $r_M M_h \approx 6.5$ ), and the total income of the households ( $Y \approx 102.7$ ) equals taxes and consumption ( $C_p \approx 51.3$ ,  $C_e \approx 2.1$ ). Once equilibrium is reached, no sector accumulates any additional stocks, and all income is consumed or distributed, which allows for a stationary economy. Though the model shows that positive interest rates do not necessarily imply exponential growth of liabilities, this result crucially depends on consumption decisions by households.

### 4.3 Energy in a SFCIO Model and Rebound Effects

As explained in section 2.1, energy plays a crucial role in the economic process. We apply our general framework to a model with two goods and sectors: energy and a multi-purpose consumer/industry good. A representation of the flows of money, goods, and energy is given in figure 1 on page 11.

The ‘physical quantities’ of the IO matrix  $\mathbf{a}$  are defined such that the prices are 1 monetary unit for all goods in the first period, but prices of these quantities vary over time. The parameters are matched to the situation of Germany around 2010. The IO parameters, the markups, the wage bill, and the consumption vectors are estimated from Statistisches Bundesamt (2010b): For each unit sold, the consumer/industry sector requires an input of 0.48 from its own sector and 0.02 from the energy sector and pays 0.25 units of wages. The energy sector requires 0.60 units from the industry sector and 0.15 units from the energy sector itself and pays 0.13 units of wages. Therefore, the markups on costs can be calculated as  $\phi_p = 0.3333$  and  $\phi_e = 0.1364$ . The tax rate of 0.48 is taken from Statistisches Bundesamt (2010a), the interest rates, accelerators, inventory to sales ratio and consumption parameters are set as rough estimates. All parameters used are displayed in table 3 on page 15.

William Stanley Jevons (1865) discovered that rising energy efficiency may not lead to a reduction in energy consumption because the improvements may encourage higher-than-otherwise levels of consumption at the economy-wide level (Brookes, 2000, p. 356). The reduction of energy consumption usually falls short of *engineering savings*, the theoretical quantity of energy saved after an increase in energy efficiency if the quantity of goods and services demanded or consumed were held constant. This effect is caused by behavioral or systemic responses known as ‘rebound effects’, tending to offset a portion of the beneficial effects of the new technology or other measures taken. If the engineering savings are 50%, but the reduction in energy consumption is only 40%, the rebound effect is given by  $1 - 0.4/0.5 = 20\%$  (Madlener et al., 2009; Sorrell and Dimitropoulos, 2008).

We can demonstrate the impact of an increased energy efficiency in our model by cutting by half the input–output parameters  $a_{ep}$  and  $a_{ee}$ . The *engineering savings* are therefore 50%. One could expect a halving of energy consumption, but the feedback effects in our model lead to an increase in real consumption. The prices in the production sector are reduced by 4.2%, while the energy price is reduced by 12.4%. The decreasing price of goods due to lower energy input leads to a *direct* rebound effect: The cheaper prices per unit (visible in figure 6) lead to higher demand: prices in the energy sector are reduced by 12.5%, leading to an increase in consumption of 14.2% keeping energy expenditures fixed.

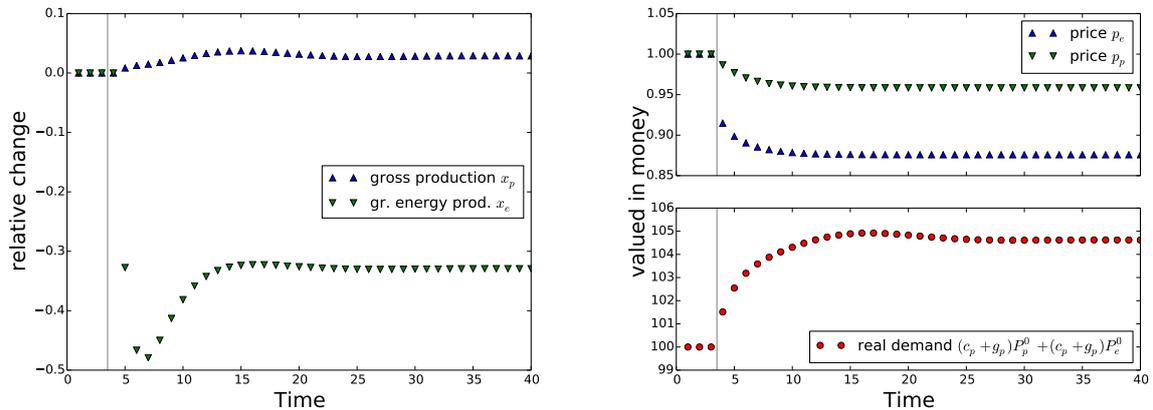


Figure 6: Rebound effect for a doubling of energy efficiency in both the production and the energy sector on their gross production. The left plot shows the relative change in production within the two sectors, showing a decline in the energy sector combined with an increase in the production sector. The right plot indicates the price shift and the time evolution of real demand, valued in prices of the first period.

The rest can be attributed to *indirect* and *economy-wide effects*, where the lower price of energy services leads to changes in the demand for other goods, services, and factors of production that also require energy for their provision. They are caused by systemic interlinkages between efficiency changes, prices, income and demand (Madlener et al., 2009; Sorrell et al., 2008).

If we look at the time evolution displayed in figure 6, we see that a temporary reduction in energy consumption close to the engineering savings of 50%, but the economy-wide feedback effects increase consumption subsequently, and the real demand is increased by around 4.6%. The production in the energy sector settles at a decline of 32.9%, while the output of the production sector is increased by 2.9%, as visible in figure 6. The size of the total rebound effect for this improvement in energy efficiency is therefore  $\frac{0.5-0.329}{0.5} = 34.2\%$ . This corresponds well to the total rebound effect of real economic systems that is estimated to be around 25% – 40% according to Madlener et al. (2009). Again, the numerical values should only be taken as an indicator of a reasonable behavior of the model economy.

# 5 Discussion and Conclusion

## 5.1 The Analytical Framework

The development of the stock-flow consistent input–output model integrated aspects from input–output analysis, post-Keynesian and ecological economics, and constitutes a synthesis of these fields. Several simplification help to keep the model tractable, but from the perspective of the different schools, this causes fundamental drawbacks concerning aggregation, substitution, and scale. Some possible extensions or variations are discussed.

From the perspective of post-Keynesian monetary economics, the complex structure of the banking system, the variety of financial assets and portfolio decisions, but also investment decisions involving long-lived fixed capital assets are missing in the model. But as the model is based on the post-Keynesian SFC approach, the extension to more complex balance sheet and transaction matrices is only a matter of increasing complexity, not of structural or theoretical problems. The integration of investment in fixed capital goods in the model is a necessary condition for developing a post-Keynesian ecological growth model.

From the perspective of ecological economics, the embeddedness of the economic system into the ecosystem and its reliance on energy, resources and space was not included properly into the model. In particular, an explicit treatment of scale is missing (Daly, 1992); the linear structure of the model economy does exactly the opposite. The implementation of a physical scale would have addressed the critique by Daly, and added a non-linearity to the model. This would have lead to a richer dynamical behavior, underlining the need for concepts of dynamical system theory instead of rather simple linear algebra. This linear approach also caused methodological problems, as diverging dynamics had to be treated. If one assumes that economic activity is indeed a dissipative process (Kümmel, 2011), one should think about modeling it as a dissipative dynamical system (Ott, 2002). In the model, the dissipation simply happens in the surrounding ecosystem, but the (arbitrary) extraction of energy is neither restricted nor explicitly modeled. It should be pointed out as well that a stable stationary economy given by a fixed point in monetary terms does not imply an equilibrium state with the environment. These problems were caused by sticking

closely to post-Keynesian demand side reasoning, making it difficult to address supply side constraints consistently.

While it has been underlined that arbitrary substitution is not a realistic assumption because of technological constraints, the constant IO model used here faces the inverse problem by disallowing any adaptive processes within the production sectors. Dynamic input–output (DIO) models that incorporate a feedback effect of investment on future production adjusting the Leontief technical coefficient matrix  $\mathbf{a}$  (Miller and Blair, 2009, pp. 639–42) may help to mitigate this simplification in future work.

Another issue not addressed in the model is aggregation. Though post-Keynesian authors reject the concept of a representative agent, there is not much difference between the assumption of a representative agent and the study of sectoral behavior as in the SFC model presented here. The reliance on a mean field approach excludes heterogeneity, self-organization and emergence (Kirman, 2011, p. 22). The field of complexity economics claims that the economy should be considered as a complex adaptive system, and focuses on interaction, interdependence, networks, trust, and contagion between economic agents. These complex phenomena may cause sudden, endogenously produced changes in system behavior (*ibid.*), resulting in ‘spontaneous emergence of extreme events in self-organizing systems’ (Sornette, 2009, p. xv). To relax the assumption of rationality and to consider interaction explicitly, agent-based models (ABM) have been proposed. They can implement locality and search costs, bounded rationality and heterogeneity among consumers, the possibility of coordination failures (Delli Gatti et al., 2011), and defaults and network effects (Battiston et al., 2007). Researches in econophysics have used them to explain distributions with fat tails and volatility clustering (Ballot et al., 2014; Feng et al., 2012). This enables the analysis of emergent disequilibrium dynamics created by the interactions of heterogeneous agents.

As was pioneered by Bergmann (1974), ABMs can also integrate a SFC description of monetary stocks and flows, recently rediscovered including endogenous credit creation (Caiani et al., 2014; Dawid et al., 2012; Kinsella et al., 2011; Riccetti et al., 2014; Seppecher, 2012). Consequently, while the model presented in this paper is not an ABM, it is clear that ABMs offer SFC models a potential method to incorporate a greater degree of heterogeneity. Likewise, the SFC framework offers ABMs a way to implement financial macro constraints, which may help ABMs avert the common criticism that their results are driven too much by the choice of particular parameter values. These innovations may help to transform the SFC perspective from a ‘top-down’ approach into an agent-based or ‘fully-scalable’ mode of macroeconomic modeling (Caiani et al., 2014; Dawid et al., 2012). If IO models are also incorporated into the analysis, it would be possible (at least in theory) to trace

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the implications of the behavior of heterogeneous agents in financial markets on flows of physical materials through the economy as well as through the natural world. Until now, ABM have tended to disregard physical resource flows and energy and therefore miss the ‘minimum complexity of endogenous growth models’, as claimed by R. U. Ayres (2001).

The approach offers post-Keynesian economists a possible way to more explicitly incorporate production into their models, and offers ecological economists the opportunity to integrate monetary aspects of the economy into their reasoning. Modelers from econophysics or agent-based economics may profit from incorporating both production and the symmetry between financial assets and liabilities as an alternative to treating money as a conserved quantity. Though the baseline model proposed here does not capture the rich behavior possible from either approach, the method is designed to enable scaling to an arbitrary number of industries, and also to allow the incorporation of more realistic elements from other already-existing IO models, SFC models, and agent-based models, that may similarly be studied with methods of dynamical system theory.

## 5.2 Conclusion

The paper conceptualizes the synthesis of disparate insights which have heretofore been developed largely in isolation. This is intended to provide an avenue to study the economy and the environment as a unified macroeconomic-ecological system.

A conceptual macroeconomic stock-flow consistent input-output model is presented using mathematical concepts from discrete dynamical system theory. The model consists of a household sector and a consolidated government and banking system sector, along with several industrial sectors.

The stability analysis of the model revealed three instabilities that are all economically meaningful. Studying the price evolution yielded a generalization of the Sraffian maximum rate of profit to a multi-sectoral model with different markups per sector. If markups are high, prices do not converge but diverge, causing the profit share to converge to 1. If markups are below the stability frontier, prices converge to an equilibrium value. The stability frontier corresponding to a bifurcation could be calculated analytically.

If prices converge to an equilibrium value, the time evolution of the dynamical system can nevertheless be diverging, corresponding to a real eigenvalue of the mapping matrix bigger than 1. The parameter analysis revealed that this instability is caused by the interplay of consumption and interest rates, shedding light on the controversy about whether a non-growing economy is compatible with positive interest rates. The model economy was found to have a stable fixed point if consumption out of wealth is high enough to counteract

accumulation. This supports recent claims that the stability of a stationary economy with positive interest depends on consumption decisions. The bifurcation point for the corresponding consumption parameter could be derived analytically.

The role of energy use and the energy sector was specifically emphasized as one of the key linkages connecting the natural environment with the economy. The model was applied to rebound effects yielding an economy-wide effect within the range found by empirical studies. The numerical simulations show that the model can plausibly be applied to such types of problems. As only very few empirical data are investigated, conclusive results could not be expected and cannot be drawn.

This paper marks a small step towards conceptualizing a macroeconomic framework which is able to describe a monetary economy within its ecological surroundings. An empirical validation of the model is desirable, but was not performed in this paper. Aspects from complexity economics and econophysics could additionally be integrated, potentially leading to physical agent-based stock-flow consistent models with explicit treatment of environmental scale, energy use, monetary flows, and interaction effects. This could form a fruitful pluralistic and interdisciplinary research program for different schools of economic thought and the natural sciences. Connecting their insights may lead to a deeper understanding of the economy and help manage a transition towards an environmentally sustainable society.

## Appendix: Equation list of Model

$$V_h(t) = M_h(t), \quad (5)$$

$$V_g(t) = L_g(t) - M_g(t), \quad (6)$$

$$M_h(t) = M_g(t), \quad (7)$$

$$L_g(t) = \sum_i L_i(t), \quad (8)$$

$$L_i(t) = \Psi_{i(t)}, \quad \Delta L_i = L_i(t) - L_i(t-1) \quad (9)$$

$$V_h(t) + V_g(t) = \sum_i \Psi_{i(t)}, \quad (10)$$

$$C = \alpha_1(1 - \theta)\text{III} + \alpha_2 M_h(t-1), \quad (11)$$

$$C_i = C C_i^0 \leq C \quad \text{with } \sum_j C_j^0 = 1, \quad (12)$$

$$c_i = C_i/P_i \quad \forall i, \quad (13)$$

$$Y = \sum_i \text{III}_i + \sum_i \text{II}_i + r_M M_h(t-1), \quad (14)$$

$$M_h(t) = (1 + r_M)M_h(t-1) + \sum_i (\text{III}_i + \text{II}_i - C_i) - T, \quad (15)$$

$$T = \theta \cdot Y, \quad (16)$$

$$g_i = G_i/P_i \quad \forall i, \quad (17)$$

$$M_g(t) = (1 + r_M)M_g(t-1) + \sum_i (G_i + \Delta L_i - r_L L_i) - T, \quad (18)$$

$$\mathbf{a} = (a_{ij}), \quad (19)$$

$$\mathbf{P} = \mathbf{diag}(P_i) \quad \Leftrightarrow \quad P_{ij} = P_i \delta_{ij}, \quad (20)$$

$$\mathbf{A} = \mathbf{P} \cdot \mathbf{a} \quad \Leftrightarrow \quad A_{ij} = a_{ij} P_i, \quad (21)$$

$$\mathbf{s}(t) = \mathbf{c} + \boldsymbol{\xi} + \mathbf{g}, \quad (22)$$

$$\mathbf{s}_{(t)}^X = \beta \mathbf{s}_{(t-1)} + (1 - \beta) \mathbf{s}_{(t-1)}^X, \quad (23)$$

$$\boldsymbol{\psi}^\top = \sigma^\top \mathbf{s}_{(t)}^X, \quad (24)$$

$$\Delta \boldsymbol{\psi}^\top = \gamma [\boldsymbol{\psi}^\top - \boldsymbol{\psi}_{(t-1)}^\top], \quad (25)$$

$$\mathbf{x} = \mathbf{s}_{(t)}^X + \Delta \boldsymbol{\psi}^\top, \quad (26)$$

$$l_i = \lambda_i x_i, \quad (27)$$

$$\text{III} = \sum_i \text{III}_i = \sum_i \omega_i \lambda_i x_i, \quad (28)$$

$$\mathbf{d} = (\mathbf{1} - \mathbf{a}) \mathbf{x}, \quad (29)$$

$$\boldsymbol{\xi} = \mathbf{a} \cdot \mathbf{x}, \quad (30)$$

$$P_{i(t)} = (1 + \phi_{i(t)}) [\omega_{i(t-1)} \lambda_{i(t-1)} + \sum_k P_{k(t-1)} a_{ki(t-1)}], \quad (31)$$

$$\mathbf{E} = \mathbf{P} \mathbf{a} \mathbf{diag}(x_i) \quad \Leftrightarrow \quad E_{ij} = a_{ij} P_i x_j, \quad (32)$$

$$\boldsymbol{\psi}(t) = \boldsymbol{\psi}_{(t-1)} + \mathbf{x}(t) - \mathbf{s}(t), \quad (33)$$

$$L_{i(t)} = \Psi_{i(t)} = \psi_{i(t)} [\omega_{i(t-1)} \lambda_{i(t-1)} + \sum_k P_{k(t-1)} a_{ki(t-1)}], \quad (34)$$

$$\text{II}_i = C_i + G_i - \text{III}_i + \sum_j E_{ij} - \sum_j E_{ji} - r_L L_{i(t-1)} + \Delta \Psi_i. \quad (35)$$

All matrices are displayed as bold roman letters, vectors in bold italic characters.

$\mathbf{diag}(x_i)$  indicates a diagonal matrix with  $x_i$  on the diagonal.

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Analytical solutions were obtained using significant amounts of pen, paper, and eraser (Munroe, 2007). Numerical calculations and plots were performed using LibreOffice 4.2.8.2, Python 2.7.6, scipy 0.13.3, numpy 1.8.2, and matplotlib 1.3.1.