The Sraffian Supermultiplier as an Alternative Closure to Heterodox Growth Theory

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07/10/2015

ABSTRACT

The paper shows that the Sraffian Supermultiplier model provides an alternative closure for the analysis of the relationships between economic growth, income distribution, capacity utilization and effective demand in heterodox growth models. The new closure comes from the variability of the ratio of the average to the marginal propensity to save (herein called fraction), which is entailed by the assumption of the existence of (an independently growing) autonomous expenditures that do not generate capacity for the private sector. This variability allows the marginal propensity to invest to determine the saving ratio without the need of changes in income distribution. If it is also assumed that changes in the marginal propensity to invest are induced by the competitive need to gradually adjust capacity to demand, this adjustment by means of endogenous changes in the fraction provides a closure that allows us to reconcile demand led growth, exogenous distribution and a tendency towards normal capacity utilization, even across steady states. A comparative analysis points out the distinctive features of this new clousure by contrasting its main results to the ones obtained with the closures associated to both the Cambridge and the Kaleckian growth models.

Key words: Effective Demand; Economic Growth; Income Distribution.

JELCode: E11; E12; O41

Introduction

The paper has two purposes. The first is to show how the Sraffian Supermultiplier model provides an alternative closure for the analysis of the relationships between economic growth, income distribution, capacity utilization and effective demand in heterodox growth models. This new closure comes from the variability of ratio of the average to the marginal propensity to save, which is entailed by the assumption of the existence of (independently growing) autonomous expenditures that do not generate capacity for the private sector. This allows the marginal propensity to invest to determine the saving ratio without the need of changes in income distribution (and more generally, in the marginal propensity to save). If it is also assumed that changes in the marginal propensity to invest are induced by the competitive need to gradually adjust capacity to demand, this adjustment by means of endogenous changes in the ratio of the average to the marginal propensity to save (the fraction) provides a closure that allows us to reconcile demand led growth, exogenous distribution and a tendency towards normal capacity utilization, even across steady states.

The second purpose of the paper is to compare and constrast this new closure with the closures associated with both the Cambridge and the neo-Kaleckian growth models. The closure provided by the Cambridge model involves the endogenous determination of income distribution, while the one associated with the neo-Kaleckian model operates through the determination of an equilibrium value for the degree of capacity utilization. The comparative analysis aims to establish the key distinctive features of the supermultiplier growth model with its alternative closure.

The rest of the paper is organized as follows. Section 1 first presents the Sraffian Supermultiplier model and how its closure operates, as capacity adjusts to demand and then shows the results of permanent effects of changes in the rate of growth of autonomous consumption and in income distribution in the model. Section 2 compares the Sraffian supermultiplier model with the Cambridge and the neo-Kaleckian growth models. Section 3 contains some brief final remarks.

1. The Sraffian Supermultiplier Growth Model

1.1. Basic assumptions and relations

We shall present the Sraffian supermultiplier growth model in its simplest possible form in order to facilitate the identification of the most relevant properties of the model and the comparison with alternative heterodox growth models. Hence, we assume a closed capitalist economy without a government sector. The only method of production in use requires a fixed combination of a homogeneous labor input with homogeneous fixed capital to produce a single output. Natural resources are supposed to be abundant, constant returns to scale prevails and there is no technological progress. We also assume that growth is not constrained by labor scarcity. Moreover, all variables are measured in real terms and output, income, profits, investment and savings are all presented in gross terms. The formal analysis will use continuous time for mathematical convenience.

In this very simple analytical context, the level of capacity output of the economy depends on level of the capital stock and on the technical capital/output ratio according to the following expression:

$$Y_{Kt} = \left(\frac{1}{v}\right) K_t \qquad (1)$$

where Y_{Kt} is the level of capacity output, K_t is the level of capital stock installed in the economy and v is the technical capital/output ratio. Since v is given, then the rate of growth of capacity output is equal to the rate of capital accumulation

$$g_{Kt} = \left(\frac{I_t/Y_t}{v}\right)u_t - \delta \qquad (2)$$

where g_{Kt} is rate of capital accumulation, $u_t = Y_t/Y_{Kt}$(with $0 \leq u_t \leq 1$) is the actual degree of capacity utilization defined as the ratio of the current level of aggregate output (Y_t) to the current level of capacity output;I_t/Y_t is the investment share in aggregate output defined as the ratio of gross aggregate investment (I_t) to the level of gross aggregate output; and δ is the exogenously given depreciation (and replacement) rate of the capital stock.[1] According to equation (2) the rate of capital accumulation depends on the actual degree of capacity utilization and on the investment share of output.[2] Given the capital/output ratio, the change in the actual degree of capacity utilization through time is then described by the difference between the rate of growth of output and the rate of capital accumulation following the differential equation below:

$$\dot{u} = u_t(g_t - g_{Kt}) \qquad (3)$$

whereg_t is the rate of growth of aggregate output.

Aggregate income in the model is distributed as wages and gross profits. We shall assume that besides the single technique in use, income distribution (either the normal real wage or the normal rate of profits) is also given exogenously along classical (Sraffian) lines. We accordingly assume that there is free competition and that output (but not capacity) adjusts fairly quickly to effective demand such that that market prices are equal to normal prices that yield a uniform rate of profits on capital using the dominant technique when the actual degree of capacity utilization u_t is equal to the normal or planned degree μ.[3] Note that we can make the assumption of normal prices at this stage of our analysis even when dealing with situations in which the actual degree of capacity utilization can be quite different from the normal or planned degree; because under classical competition, individual firms do not have the power to sustain higher-than-normal prices when the actual degree of capacity utilization of a particular firm is below (or very much above) the normal level and their actual unit costs are higher than normal.[4] Indeed, at higher than normal prices other firms already in the market may be operating at the planned degree of utilization and can easily increase their market shares by undercutting the firms that have raised prices above the normal price. Moreover, these higher prices may also attract new entrants to the market which would also be able to operate their appropriately sized new capacities at the planned degree of utilization and reap higher than normal profits by undercutting incumbent firms that have raised prices to pass along their higher than normal actual average costs to market prices.Thus, both actual competition of existing firms as well as potential competition of new entrants would ensure that effective demand will be met at the normal price even if the actual degree of capacity utilization is quite different from the normal or planned degree.[5]

Since we are assuming that output adapts quickly to demand, when normal prices prevail aggregate demand determines the level of aggregate output. Effective demand is composed of aggregate

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[1]The equation of capital accumulation is derived from the equation $I_t = \dot{K} + \delta K_t$ that defines the level of aggregate gross fixed investment. Dividing both sides of the equation by K_t we obtain $I_t/K_t = g_{Kt} + \delta$. Solving this last equation for the rate of capital accumulation gives us $g_{Kt} = (I_t/K_t) - \delta = (I_t/Y_t)(Y_t/Y_{Kt})(Y_{Kt}/K_t) - \delta = ((I_t/Y_t)/v)u_t - \delta$.

[2]The logic underlying this equation appears to have first been outlined by Garegnani (1962). In his SVIMEZ report, Garegnani argued that what raises the rate of growth of capacity output is a rise in the investment share in *capacity* output rather than in *actual* output. He also noted that these two ratios can differ substantially from one another if the actual degree of capacity utilization can diverge from the normal or planned degree.

[3]Following Ciccone (1986, 1987), we interpret the normal or planned degree of capacity utilization as determined, among other things, by the historically 'normal' ratio of average to peak demand. This latter ratio, because it is presumably based on the observation of the actual cyclical pattern of the market over a very long period of time, is assumed to be unaffected by current oscillations of demand. See also Steindl (1952) for a seminal discussion of the existence and role of planned spare capacity in the operation of capitalist firms.

[4] If the actual degree of utilization is below the normal degree, fixed cost per unit of output will be higher than normal. If the degree of utilization is just above the normal degree unit costs will keep falling (giving rise, at normal prices, to extra profits) until at capacity utilization rates substantially above normal, the cost will begin to rise due to the extra expenses involved in operating capacity way above its cost minimizing range (Ciccone 1987).

[5] Normal price is thus a kind of entry-preventing 'limit price' in the language of the old industrial organization literature. Some Sraffians make the same argument in terms of a presumption of a uniformity of expected rates of profit on new investment (Garegnani 1992; Ciccone 1986, 2011).

consumption and investment. We suppose that aggregate consumption has an induced component and an autonomous one. The former is related to the purchasing power introduced to the economy by the production decisions of capitalist firms when they pay wages. Given consumption habits (i.e., a given marginal propensity to consume out of wages, c_w) and the wage share of output (ω), we assume that there is a positive relationship between induced consumption and aggregate output regulated by the marginal propensity to consume out of income $c = c_w \omega$ (with $0 < c < 1$, since $0 < c_w \leq 1$ and $0 < \omega < 1$). In order to simplify the analysis of the model, we suppose that all wages are expended in consumption (i.e. $c_w = 1$ and, hence, $c = \omega$). On the other hand, the autonomous component Z_t is that part of aggregate consumption financed by credit and, therefore, unrelated to the current level of output resulting from firms' production decisions. We assume that autonomous consumption grows at a exogenously given rate $g_Z > 0$.

Concerning the other component of aggregate demand, we suppose that capitalist firms undertake all investment expenditures in the economy (i.e., we abstract from residential investment). Further, we assume that the capital stock adjustment principle explains the behavior of aggregate investment. According to this principle, inter-capitalist competition influences the process of investment leading to the tendency towards the adjustment of productive capacity to meet demand at a price that cover the production expenses and allows, at least, the obtainment of a minimum required profitability. Thus, the capital stock adjustment principle conceives the demand for capital goods as a derived demand with the objective of generating capacity to meet the profitable (or effective) demand.[6]

Here we use a very simple (and yet not unreasonable) investment function that is compatible with the capital stock adjustment principle. Hence, the level of aggregate real invetment is determined according to the following expression

$$I_t = h_t Y_t \qquad (4)$$

where h_t (with $0 \leq h_t < 1$) is the marginal propensity to invest. The latter variable changes endogenously in response to deviations of the actual degree of capacity utilization from its normal level as follows

$$\dot{h} = h_t \gamma (u_t - \mu) \qquad (5)$$

where γ is a parameter that measures the reaction of the growth rate of the marginal propensity to invest to the deviation of the actual degree of utilization u_t from the normal or planned level μ. We suppose that the normal degree of capacity utilization has a positive but lower than one value (i.e. $0 < \mu < 1$) since we assume that, under the pressure of competition, firms try to maintain margins of planned spare capacity to avoid the risk of loosing market shares for not being able the supply their markets when they are booming. On the other hand, we assume that the parameter γ has a small positive value (i.e. $\gamma > 0$). The reason for that is that it makes no sense for firms to attempt to adjust the whole of the stock of fixed capital fully to every single fluctuation in demand for two main reasons. The first is that firms want normal utilization to prevail on average over the life time of the productive equipment and not in every single period of use. The second is that firms also know that demand does fluctuate and some of the fluctuations are temporary, while others are not (though it is not easy to distinguish the two quickly). Indeed, it is not plausible for firms to build new capacity for a temporary increase in demand when that can be easily met by changes in the actual degree of utilization of the already existing capacity (see references on footnotes 3 and 5). As we shall see below, the value of the latter parameter has an important role in the analysis of the stability of the equilibrium of the supermultipler growth model. Relatively high values of γ imply a drastic reaction of investment to deviations between u and μ, which, depending on the value other parameters and exogenous variables, leads to the instability of the model's equilibrium. In opposition, a relatively low value of γ implies a gradual response of investment to deviations between u and μ and a corresponding tendency for the equilibrium path of the model to be stable.

Together equations (4) and (5) imply that investment grows according to the relation below

$$g_{It} = g_t + \gamma (u_t - \mu) \qquad (6).$$

[6] See Matthews (1959) for a more detailed account of the capital stock adjustment principle.

The above equation represents the idea that the process of inter-capitalist competition leads to a tendency for the growth rate of aggregate investment to be higher than the rate of growth of demand (and hence of output) whenever the actual degree of capacity utilization is above its normal or planned level and *vice-versa*. The pressure exerted by competition would ensure that firms as a whole would be compelled to invest in order to ensure that they can meet future peaks of demand when the degree of utilization is above the normal (or planned) degree and the margins of spare capacity are getting too low. Conversely, firms would not want to keep accumulating costly unneeded spare capacity when the actual degree of capacity utilization remains below the profitable normal or planned level.

Given the assumptions above, the level of aggregate demand determines a positive level of aggregate output provided that we assume that the marginal propensity to spend $c + h_t$ is strictly lower than one and that there is a positive level of autonomous consumption (or, more generaly, a positive level of autonomous non-capacity generating expenditures).[7] The demand-determined level of output is:

$$Y_t = \left(\frac{1}{s - h_t}\right) Z_t \tag{7}$$

where $s = 1 - \omega$ is the given aggregate marginal propensity to save. The term within the parenthesis is the supermultiplier that captures the effects on the level of output associated with both induced consumption and investment.[8]

The equilibrium between aggregate output and demand implies the equality between the saving ratio and the investment share of output (i.e. the marginal propensity to invest). Since, as we saw, the latter is liable to changes in response to deviations between actual and normal degrees of capacity utilization, then the maintainance of the aggregate output and demand equilibrium requires the endogenous determination of the saving ratio. In the supermultiplier model the endogenous determination of the saving ratio follows from the assumption that there exists a positive level of autonomous consumption. Indeed, given $Z_t > 0$ the marginal propensity to invest is equal to and determines the saving ratio as follows:

$$\frac{S_t}{Y_t} = s - \frac{Z_t}{Y_t} = sf_t = \frac{I_t}{Y_t} = h_t \tag{8}$$

where f_t is what is called 'the fraction' in Serrano (1995b), the ratio of the average to the marginal propensities to save $f_t = (S_t/Y_t)/s = I_t/(I_t + Z_t)$.[9] With positive levels of autonomous consumption, it follows that $f_t < 1$ and $S_t/Y_t < s$. Therefore, the given marginal propensity to save defines only an upper limit to the value of the saving ratio of the economy corresponding to a given marginal propensity to save (i.e. a given level of income distribution). Hence, in the case under analysis, the saving ratio depends not only on the marginal propensity to save but also on the proportion between autonomous consumption and investment. Thus, an increase (decrease) in the levels of aggregate investment in relation to autonomous consumption leads to an increase (decrease) in the saving ratio. Consequently, the existence of a positive level of autonomous consumption is sufficient to make the saving ratio an endogenous variable. Therefore, for a given level of income distribution (and consumption habits) and, hence, a given marginal propensity to save, the marginal propensity to invest h_t (equal to the investment share of output) uniquely determines the saving ratio of the economy. The endogenous determination of the saving ratio with a given level of income distribution is the distinctive feature of the closure provided by the supermultiplier growth model.

[7] In fact if the marginal propensity to spend were equal to one and there were no autonomous demand we would have Say's Law and if the marginal propensity to spend was lower than one but with no autonomous demand the economy would collapse and output would fall to zero (see Serrano 1995a; López and Assous 2010, chapter 2).

[8] On the marginal propensity to consume, see Serrano (1995a); on the (not fully adjusted) long-period supermultiplier with a given marginal propensity to invest, see Cesaratto, Serrano and Stirati (2003) and Freitas and Serrano (2015). Note that equation (7) can determine the level of output only if $u_t \leq 1$. So for all$t \geq 0$ we must have$Y_{Kt} > Y_t = [1/s - h_t]Z_t$ and, therefore, $Z_t \leq [(s - h_t)/v]K_t$. That is, the level of output determined by effective demand is restricted by the level of capacity output for given values of s, *h* and *v*.

[9] According to (8), if there were no autonomous consumption (i.e., $Z_t = 0$) then $f_t = 1$ and $S_t/Y_t = s$. That is, the marginal propensity to save determines the savings ratio. Note also that, in this extreme case the equilibrium between aggregate demand and aggregate output with a given income distribution also implies the endogenous determination of the investment share of output by the marginal propensity to save.

From our assumptions, we can also obtain the following equation for the growth rate of aggregate output/demand:[10]

$$g_t = g_Z + \frac{h_t \gamma (u_t - \mu)}{s - h_t} \tag{9}$$

Equation (9) shows that when actual and normal degrees of capacity utilization are different, the rate of growth of output and demand is determined by the rate of expansion of autonomous consumption plus the rate of change of the supermultiplier given by the second term on the right-hand side of the equation.

Let us now substitute equations (9) and (2) into equation (3). From the combination of the resulting equation and equation (5) we obtain a system of two first order nonlinear differential equations in two variables, the share of investment in output *h* and the actual degree of capacity utilization *u*, which we present below:

$$\dot{h} = h_t \gamma (u_t - \mu) \tag{5}$$

$$\dot{u} = u_t \left(g_Z + \frac{h_t \gamma (u_t - \mu)}{s - h_t} - \left(\frac{h_t}{v}\right) u_t + \delta \right) \tag{10}$$

We will use the above system to develop our presentation of the supermultiplier growth model's main implications for the analysis of the process of economic growth.

1.2. The fully adjusted equilibrium

The equality between actual and normal degrees of capacity utilization characterizes the fully adjusted position of the economy according to the supermultiplier model. Setting $u_t = u^* = \mu$ in equation (5) we obtain $\dot{h} = 0$, and since μ is constant we also have $\dot{u} = 0$. Thus, if $u_t = u^* = \mu$ then $\dot{h} = \dot{u} = 0$ and we can verify that the fully adjusted position is an equilibrium of the supermultiplier model represented by the system comprising equations (5) and (10).

From equations (6) and (9) we can also verify that in the fully adjusted position of the supermultiplier model the growth rates of output, demand and investment are equal to the growth rate of autonomous consumption. Moreover, since $\dot{u} = 0$ and $\mu > 0$ then, from equation (3), the rate of capital accumulation is equal to the growth rate of output. Hence, in the fully adjusted equilibrium we have $g_K^* = g_I^* = g^* = g_Z$. That is, the growth rate of autonomous consumption determines the equilibrium growth rates of the capital stock, investment and output/demand. The model generates an equilibrium path where economic growth is consumption-led and normal utilization of productive capacity prevails.

The last result shows that, according to the model, the growth of autonomous consumption drives the pace of capital accumulation and, therefore, the growth of productive capacity. Thus, it is able to represent a sustainable process of demand-led growth. Such a result is compatible with the maintenance of a normal (or planned) degree of capacity utilization throughout the equilibrium path (and a tendency of the economy to converge towards this path) because the investment share of output (i.e., the marginal propensity to invest) assumes a required value h^* given by:

$$h^* = \frac{v}{\mu}(g_Z + \delta) \tag{11}$$

The required investment share is uniquely determined by the rate of growth of autonomous consumption, the technical capital/output ratio, the normal degree of capacity utilization and the rate of depreciation.

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[10] The equation is deduced as follows. Taking the time derivatives of the endogenous variables involved in expression $Y_t = Z_t + cY_t + h_t Y_t$ and dividing both sides of the resulting equation by the level of aggregate output, we obtain $g_t = cg_t + h_t g_t + \dot{h} + (Z_t/Y_t) g_Z$. If $Z_t/Y_t = s - h_t > 0$, then we can solve the last equation for the rate of growth of aggregate output and demand, obtaining $g_t = g_Z + \dot{h}/(s - h_t)$. Finally, we can substitute the right-hand side of equation (5) in the second term on the right-hand side of the last equation, which gives us equation (9).

The maintenance of the equilibrium between aggregate output and demand in the fully adjusted position of the model requires the endogenous determination of the saving ratio (or average propensity to save) by the required level of the investment share of output. As we mentioned above, the endogeneity of the saving ratio in the model is a consequence of the hypothesis of the existence of a positive level of autonomous consumption. Actually, the latter hypothesis makes it possible for the fraction $f_t = h_t/s$ to change its value according to the modifications of the investment share of output. As a result, $Z_t/Y_t = s(1-f_t) = s - h_t$, the ratio of autonomous consumption to aggregate output can change, making the saving ratio an endogenous variable and allowing its adjustment to the investment share of output. In fact, along the equilibrium path of the model, once the investment share is determined by equation (11) we can obtain the equilibrium values of the fraction, of the aggregate autonomous consumption/output ratio and, accordingly, of the equilibrium value of the saving ratio as follows

$$f^* = \frac{h^*}{s} = \frac{\frac{v}{\mu}(g_Z + \delta)}{s} \tag{12}$$

$$\left(\frac{Z_t}{Y_t}\right)^* = s(1-f^*) = s - h^* = s - \frac{v}{\mu}(g_Z + \delta) \tag{13}$$

and

$$\left(\frac{S_t}{Y_t}\right)^* = s - (Z_t/Y_t)^* = sf^* = h^* = \frac{v}{\mu}(g_Z + \delta) \tag{14}$$

The endogenous determination of the saving ratio with a given level of income distribution associated with the supermultiplier growth model supplies the heterodox literature on economic growth with an alternative closure for the analysis of the relationships between economic growth, income distribution, capacity utilization and effective demand. Indeed, substituting the expression for the saving ratio in equation (8) into equation (2) we obtain

$$g_{Kt} = \left(\frac{sf_t}{v}\right)u_t - \delta \qquad (2').$$

Now, we just saw that in the fully adjusted equilibrium of the supermultiplier growth model normal capacity utilization prevails. Thus, the fraction f is the only variable that can be adjusted in order to reconcile the rate of capital accumulation with the exogenously given rate of growth of autonomous consumption. Therefore, for the fully adjusted equilibrium to be possible, the fraction f must assume the value given by equation (12) above, which represents the closure provided by theSraffian supermultiplier model.

Finally, by introducing the required level of the investment share h^* in equation (7) we obtain the fully adjusted (final equilibrium) level of output:

$$\frac{\mu K^*}{v} = Y_t^* = Y_t = \left(\frac{1}{s - \frac{v}{\mu}(g_Z + \delta)}\right)Z_t \tag{15}$$

Equation (15) shows that for each t along the equilibrium path, the level of autonomous consumption and the fully adjusted (final equilibrium) level of the supermultiplier (the term within the parenthesis) determine the levels of output of the fully adjusted positions (final equilibria) towards which the economy slowly gravitates.[11] Thus in the fully adjusted positions of the supermultiplier growth model not only aggregate output but also the levels of capacity output and the capital stock adjust to the levels of aggregate demand.

1.3. The stability of the fully adjusted equibrium

[11] In terms of the old long-period method the fully adjusted position is thus a classical 'secular' rather than a long-period position (Ciccone 1986) just as a Solow steady state is a secular not a long-period neoclassical equilibrium path. In the terms of the recent neo-Kaleckian literature the fully adjusted equilibrium is a type of 'final equilibrium' (see Lavoie 2014).

Let us now look at the stability of the fully adjusted equilibrium.The adjustment process of capacity to demand that characterizes the convergence to the fully adjusted equilibrium requires the fulfillment of two conditions. First, the investment share of output must be susceptible to changes via the influence of the competitive process over capitalist investment decisions. Such changes are possible because of the existence of an autonomous consumption component of aggregate demand that allows the concomitant adjustment of the saving ratio. In fact, from equations (6) and (9) we can obtain the following relations:

$$g_{It} \gtreqless g_t \gtreqless g_Z \text{ as } u_t \gtreqless \mu \qquad (16)$$

According to them, if the actual degree of capacity utilization is above (below) the normal level the marginal propensity to invest increases (decreases). At the same time, the saving ratio ($S_t/Y_t = sf_t$) also increases (decreases) because the growth rate of investment is higher (lower) than the growth rate of autonomous consumption and, therefore, the fraction $f_t = I_t/(I_t + Z_t)$ increases (decreases) while the marginal propensity to save is constant. These changes in the investment share of output in response to deviations from the normal degree of capacity utilization are necessary for the adjustment of capacity to demand and the corresponding tendency towards a fully adjusted position of the model. However, they are not sufficient to assure such a result since the intensity of the adjustment must also be considered. The latter point leads us to the condition discussed below.

The second condition is that the marginal propensity to invest changes *gradually* in response to deviations of the actual degree capacity utilization from its planned level. The latter is required to assure that the value of the marginal propensity to spend remains lower than one throughout the process of convergence to the fully adjusted position.The reason for that is quite simple. Given that empirically it is plausible that normal technical capital/output ratios for fixed capital tend to be greater than one, an immediate and full reaction of induced investment that tried to adjust capacity drastically to any current deviation from normal utilization would certainly lead to a marginal propensity to invest greater than one. Since the marginal propensity to consume is positive, the overall marginal propensity to spend in the vicinity of the fully adjusted position would be greater than one, which would lead to the instability of the adjustment process. Such drastic adjustment however is highly unrealistic, both because of the durability of fixed capital (which means that producers want normal utilization only on average over the life of equipment and not at every moment) and also because producers know that demand fluctuates a good deal and, therefore, do not interpret every fluctuation in demand as indicative of a lasting change in the trend of demand.[12]

The precise economic meaning of this second condition is that the local stability of the fully adjusted equilibrium requires that the aggregate marginal propensity to spend in the neighborhood of the equilibrium must be lower than one (see Freitas & Serrano (2015) for the proof). Here we assume that such a stability condition is met and, therefore, we have:

$$c + \frac{v}{\mu}(g_Z + \delta) + \gamma v < 1 \qquad (17)$$

We can interpret (17) as an expanded marginal propensity to spend that besides the final equilibrium propensity to spend ($c + \frac{v}{\mu}(g_Z + \delta)$) includes also a term (i.e., γv) related to the behavior of induced investment outside the fully adjusted position. The extra adjustment term captures the fact that the investment function of the model is grounded in the capital stock adjustment principle (a type of flexible accelerator investment function) and that outside the fully adjusted trend path there must be room not only for the induced gross investment necessary for the economy to grow at its final equilibrium rate g_Z, but also for the extra induced investment responsible for adjusting capacity to demand. Thus, *ceteris paribus*, for a sufficiently low value of the reaction parameter γ the fully adjusted equilibrium described above is stable.

[12] This is the pattern of behavior underpinning the flexible accelerator model of investment.

1.4. Effects of changes in the rate of growth of autonomous consumption and in income distribution

We can now analyze the impact on the fully adjusted (final equilibrium) positions of changes in the growth rate of autonomous consumption (i.e., g_Z) and in income distribution (i.e. the wage shareω), assuming that the fully adjusted equilibrium is stable both before and after the change.

As we saw above, according to the supermultiplier growth model, the rate of growth of autonomous consumption determines the trend rate of growth of output/demand (i.e., g^*). So an increase (a decrease) in the rate of growth of autonomous consumption causes an increase (a decrease) in the trend rate of growth of output/demand. Therefore changes in g_Z have a *growth effect* on aggregate output. Note also that, since there is a tendency towards normal capacity utilization, then changes in g_Z have also a *growth effect* on capacity output. We also have seen that g_Z determines the value of the rate capital accumulation in equilibrium (i.e., g_K^*). Hence a permanent rise (fall) in g_Z causes a permanent increase (decrease) in g_K^*. Since, according to the model, there is a tendency for the economy to converge towards an equilibrium with normal capacity utilization, this effect of g_Z on the pace of capital accumulation g_K^* occurs through the effect of g_Z on the equilibrium level of the investment share of output (i.e., h^*). In fact, from Equation (11) we can see that the there is a positive relationship between g_Z and h^*, such that a permanent increase (fall) in g_Z has a positive (negative) *level effect* on h^*. Moreover, from Equations (12) to (14) we can observe that a rise (fall) in g_Z has, on the one hand, a positive (negative) *level effect* on the equilibrium value of the fraction (i.e., f^*) and on the equilibrium level of the saving ratio (i.e. $(S_t/Y_t)^* = sf^*$) and, on the other, a negative (positive) *level effect* on the equilibrium level of the autonomous consumption to output ratio (i.e., $(Z_t/Y_t)^* = s(1-f^*)$).Therefore, the supermultiplier growth model implies that there is a positive causal relationship running from the trend rate of growth of output/demand to the the investment share of output and the saving ratio. As argued above such a relationship is essential to the obtainment of the distinctive results of the model, according to which an economy can be demand-led, while it mantains a tendency of the actual degree of capacity utilization to converge towards its normal level with a given income distribution.

Now let us look at the effects of changes in income distribution. First, our analysis of the steady state of the model has shown that the equilibrium rates of growth of output/demand and of the capital stock are independent from the value of the wage share. Hence, in the supermultiplier model there is no direct relationship between income distribution and the equilibrium rate of growth of output and demand. Next, since, as we just saw, a change in income distribution does not have a permanent growth effect on output, then equation (11) shows that such a changedoes not have a permanent effect on the equilibrium value of the investment share of output either.

Nevertheless, it should be noted that changes in income distribution do have a *level effect* over the equilibrium values of all non-stationary variables of the simplified supermultiplier growth model here presented. This is so because a change in income distribution affects, by means of its influence over the marginal propensity to save, the equilibrium value of the supermultiplier and, hence, the equilibrium value of aggregate output. Thus, the model is subject to wage led output level effects. Indeed, as can be easily verified, an increase (decrease) in the wage share reduces (raises) the marginal propensity to save, which yields an increase (a decrease) in the equilibrium value of the supermultiplier and, accordingly, an increase (a decrease) in the equilibrium level of output and capacity.[13]

On the other hand, we know that, according to the supermultplier growth model, the investment share of output determines the saving ratio. Thus, modifications in income distribution do not affects the saving ratio also. Note, however, that changes in income distribution do affect the marginal propensity to save. In fact, since the latter variable is equal to the profit share (i.e. $s = 1 - \omega$), an increase (decrease) in the wage share would reduce (raise) the marginal propensity to save (i.e. $ds = -d\omega$). But making use of equations (12) and (14) we can verify how this latter result is reconciled with the invariability of the saving ratio in relation to an income distribution change. Thus, given the value of the investment share of output, an increase (a decrease) in the marginal propensity to save leads to a fall (an increase) in the

[13] Note that, in this sense, the Supermultiplier growth model allows the extension of the validity of the paradox of thrift concerning the level of output to the context of analysis of the economic growth process.

fraction that exactly compensates it (since $f^* = h^*/s$). This explains why the saving ratio is unaffected by changes in income distribution.

2. A Comparison with other heterodox models[14]

In this section, we shall compare the main results obtained in our discussion of the Sraffian supermultiplier growth model with alternative heterodox growth models that also deal with the relationship between economic growth, income distribution and effective demand. More specifically, we will compare the supermultiplier, Cambridge and neo-Kaleckian growth models trying to point out their similarities and, more importantly, their main differences. The comparison will be made within the same simplified analytical context used in the discussion of the supermultiplier model. Thus, the following assumptions will be maintained: we have a closed capitalist economy without government; aggregate income is distributed in the form of wages and profits; the method of production in use requires a fixed combination of homogeneous labor input with homogeneous fixed capital to produce a single output; there is constant returns to scale and no technological progress; and there exists a permanent surplus of labor. Moreover, as occurred in the case of the supermultipier model, the alternative growth models will be presented in their simplest form in order to facilitate the comparisons and to draw our attention to the different theoretical closures provided by each model.

2.1. Cambridge growth models

Let us start with Cambridge growth models. Maintaining the hypothesis of permanent labor surplus, the version of the Cambridge growth model presented here[15] supposes that the level of aggregate output is determined by the level of capacity output. So, contrary to the supermultiplier growth model, the equilibrium level of aggregate output is not determined by effective demand, but by the supply constraint associated with full utilization of productive capacity. That is, we have

$$Y_t = Y_{Kt} = \frac{1}{v} K_t$$

and thus

$$u^* = 1$$

Also differently from the supermultiplier growth model, there is no autonomous consumption and, additionally to the consumption induced by the wage bill, there is also a component of aggregate consumption induced by total current profits. We retain the assumption that the propensity to consume out of wages c_w is equal to one (i.e., $c_w = 1$) and suppose that the propensity to consume out of profits c_π is a positive constant and has a value lower than one (i.e., $0 < c_\pi < 1$). Thus the consumption function is given by the following expression

$$C_t = \big(\omega_t + c_\pi(1 - \omega_t)\big) Y_t$$

where $\omega_t + c_\pi(1 - \omega_t)$ is the marginal propensity to consume, which is equal to the average propensity to consume since there is no autonomous component in the consumption function. Moreover, in contrast to the supermultiplier growth model, aggregate investment is an autonomous expenditure in this version of the Cambridge model, and we suppose, for the sake of simplicity, that investment expands at an

[14]This section is based on and confirms the main findings contained in the more general comparative analysis presented in Serrano (1995b, chapter 3). See also Serrano and Freitas (2007), for an early discussion of the main differences between the supermultiplier, Cambridge and neo-Kaleckian growth models containing similar results to the ones here obtained.

[15] See Robinson (1962). For similar formalizations of the Cambridge growth model see Dutt (1990 and 2011) and Lavoie (2014).

exogenously determined rate g_I>0. From these hypotheses we obtain the aggregate demand equation of the Cambridge model

$$D_t = \left(\omega_t + c_\pi(1-\omega_t)\right)Y_t + I_t$$

Since aggregate output is determined by capacity output, the equilibrium between aggregate demand and output requires that the former adjusts to the latter. In the Cambridge model such an adjustment involves a change in the marginal propensity to consume through the modification of income distribution (i.e., of the wage share). Thus, according to the model, a situation of excess aggregate demand (supply) raises (reduces) the general price level and, with a relatively rigid nominal wage, it causes a decrease (increase) in real wages. So, given labor productivity, the excess aggregate demand (supply) causes a reduction (an increase) in the wage share ω and, since $0 < c_\pi < c_w = 1$, it causes a decline (rise) in the marginal propensity to consume. Therefore, the adjustment implies a tendency for the establishment of an equilibrium between aggregate demand and supply at the level of output capacity (i.e. the level of potential output), which in equilibrium determines the level of aggregate demand.[16] At the same time, in equilibrium between aggregate demand and output capacity the model endogenously determines the income distribution between wages and profits. So, in the Cambridge model, the determination of a required level of income distribution allows the adjustment of aggregate demand to potential output, while in the supermultiplier growth model it is the appropriate change in the level of aggregate output that explains the adjustment of aggregate output to the level of aggregate demand.

Now, in equilibrium between aggregate output and aggregate demand, we have $Y_t = \left(\omega^* + c_\pi(1-\omega^*)\right)Y_t + I_t$ and, thus

$$S_t^* = s^* Y_t = s_\pi(1-\omega^*)Y_t = I_t$$

where $s_\pi = 1 - c_\pi$ is the marginal propensity to save out of total profits. The last equation shows that, according to the Cambridge model, aggregate investment determines aggregate (capacity) savings, although, as we saw above, the level of potential output determines the level of real aggregate demand. Furthermore, dividing both sides of the last equation by the level of aggregate output, we have an expression relating the saving ratio to the investment share of output as follows

$$\left(\frac{S_t}{Y_t}\right)^* = s^* = s_\pi(1-\omega^*) = \frac{I_t}{Y_t}$$

From the above equation we can see that in the Cambridge model the investment-output ratio determines the saving ratio $s^* = s_\pi(1-\omega^*)$, a result shared with the supermultiplier growth model. But, since in the Cambridge model there is no autonomous consumption component, the marginal and average propensities to save are equal to each other. Hence, the burden of the adjustment of the saving ratio to the investment share relies on required modifications in the *marginal* propensity to save and, therefore, on appropriate changes in income distribution.

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[16]Note however that the adjustment mechanism based on endogenous modifications of income distribution only guarantees the adjustment of aggregate demand to the level of potential output and not necessarily the adjustment of the degree of capacity utilization to its full capacity level. As Kaldor (1955-6) argued in his seminal discussion of such adjustment mechanism, the latter can be viewed as an alternative to the usual Keynesian adjustment based on variations of the level of aggregate output leading to the equilibrium between aggregate output and demand. In the present version of the Cambridge growth model, the maintenance of a full utilization of capacity resulting from the operation of the adjustment mechanism involving changes in income distributions is a consequence of the assumption that the binding supply constraint in the economy is the availability of capital. If the operative supply constraint was the full employment of the labor force, then the adjustment of aggregate demand to potential output based on endogenous changes in income distribution would not guarantee the full (or the normal) utilization of the available capital stock. In Kaldor's full employment growth models (c.f. Kaldor, 1957, 1958 and 1962) it is the investment function that is responsible for the adjustment of the degree of capacity utilization as it makes the level and growth of the capital stock adjust itself to the exogenous levels and rates of growth of full employment output. For an analysis of this role of the investment functions in Kaldor's full employment growth models see Freitas (2002, chapter 2; and 2009) and for a different point of view on induced investment in Kaldor see Palumbo (2009, pp. 343-345).

Let us now discuss the determination of the equilibrium level of the investment share of output. From the assumption of full capacity utilization, the growth rate of output is given by the rate of capital accumulation (i.e., $g_t = g_{Kt}$). Thus, if we have initially $g_{Kt} < g_I$ ($g_{Kt} > g_I$), then we also have $g_t < g_I$ ($g_t > g_I$). It follows that the investment share of output would increase (decrease) and, according to equation (2) and with the constant degree of capacity utilization, the rate of capital accumulation would increase (decrease). Eventually this type of adjustment leads to the convergence of the rate capital accumulation to the investment growth rate.[17] Therefore in the equilibrium path of the model we have

$$g^* = g_K^* = g_I$$

Substituting this last result in equation (2), solving for the investment share of output and recalling that $u^* = 1$, we have the equilibrium value of the investment share given by

$$\left(\frac{I_t}{Y_t}\right)^* = v(g_I + \delta)$$

Thus, according to the Cambridge growth model, a higher (lower) rate of growth of investment implies higher (lower) equilibrium growth rates of capacity output and output. Moreover, with a constant capacity utilization rate, this result is possible because the equilibrium level of the investment share is positively related to the growth rate of investment and, therefore to the equilibrium growth rates of output and capacity output. This last result is shared by the supermultiplier growth model although the process by which it is achieved is different from the process featured in the Cambridge growth model.

It is important to note, however, that the growth determining role of investment in the Cambridge model actually follows from its effect on productive capacity and *not* from its influence on aggregate demand. As we saw above, in the Cambridge growth model, the level of existing capacity output determines the level of aggregate demand. Thus, a higher (lower) growth rate of investment only causes a higher (lower) rate of output growth because it raises (reduces) the level of the investment share of output by (increasing) reducing the share of induced consumption in both actual and capacity output. This higher share of investment in capacity increases the pace of capital accumulation and the growth of capacity (or potential) output. Therefore, the Cambridge model displays, in fact, a supply (capacity) constrained pattern of economic growth, and hence, it is *not* a demand-led growth model.

Finally, we can substitute the expression for the equilibrium level of the investment share in the equation relating the investment share and the saving ratio. Doing so we can obtain the following result

$$(1 - \omega^*) = \frac{v(g_I + \delta)}{s_\pi} \qquad (18).$$

The last equation shows us the determinants of the required level of income distribution in the Cambridge model. In particular, we can see that, given v, δ and s_π, a higher (lower) growth rate yields a higher (lower) profit share of output. Thus, according to the Cambridge growth model and in contrast with the supermultiplier model, there exists a theoretically necessary relationship between income distribution and economic growth, the profit share of output (the wage share) is positively (negatively) related to the rate of economic growth. In the Cambridge model, such relationship is necessary for obtaining a growth path characterized by the equilibrium between aggregate demand and aggregate output with a constant degree of capacity utilization that is equal to its normal value. Therefore, the closure provided by the Cambridge model requires the endogenous determination of an appropriate level of income distribution, which must

[17] The adjustment process between the rate of capital accumulation and the expansion rate of investment can also be explained as follows. As we saw before, $\dot{g}_{Kt} = (g_{Kt} + \delta)(g_{It} - g_{Kt})$, so, since $g_{It} = g_I$, if initially $-\delta < g_{K0} \neq g_I$, then the capital accumulation rate would converge to the investment growth rate because from the last differential equation we can verify that $\dot{g}_{Kt} \gtrless 0$ according to $g_{It} \gtrless g_{Kt}$.

be compatible with various combinations of the values of the model's exogenous variables and parameters.[18]

2.2. Neo-Kaleckian growth models

We shall now compare the supermultiplier and the neo-Kaleckian growth models.[19] Like in the supermultiplier model, in the neo-Kaleckian growth model the level of aggregate demand determines the equilibrium level of aggregate output. Also, as occurs with the supermultiplier model, income distribution is exogenously determined and, therefore, cannot be part of the adjustment mechanism that allows the existence of the equilibrium path as in the Cambridge model. However, in contrast to the supermultiplier model, in the neo-Kaleckian models aggregate investment does not follow the capital stock adjustment principle. For the sake of simplicity, we will first present the model considering that aggregate investment is autonomous and grows at an exogenously determined growth rate $g_I > 0$.[20] Also, differently to the supermultiplier model, there is no autonomous component in aggregate consumption in the neo-Kaleckian models. In fact, in our representation of the basic neo-Kaleckian model we utilize the same specification for the consumption function used in the Cambridge model above. The only, but important, difference being that in the neo-Kaleckian model the wage share is exogenously determined and, accordingly, the marginal propensity to consume (equal to the average propensity to consume) is also an exogenous variable.

With these hypotheses, aggregate demand is given by the following expression

$$D_t = \big(\omega + c_\pi(1-\omega)\big)Y_t + I_t$$

and in equilibrium between aggregate demand and output we have

$$Y_t^* = \left(\frac{1}{s}\right)I_t = \left(\frac{1}{s_\pi(1-\omega)}\right)I_t \qquad (19).$$

According to the last equation aggregate investment is the main determinant of the equilibrium level of output. Further, given income distribution (i.e. the wage share), the value of the multiplier $1/s = 1/\big(s_\pi(1-\omega)\big)$ is constant. Thus, in the neo-Kaleckian model, as can be verified from the last equation, the pace investment expansion determines the equilibrium output growth rate of the economy for a given level of income distribution. That is, we have

$$g^* = g_I$$

So like the supermultiplier growth model, the neo-Kaleckian model produces a demand-led growth pattern. But while in the supermultiplier model we have a consumption-led growth pattern, the neo-Kaleckian model generates an investment-led growth pattern.

From equation (19) we can also verify that

$$S_t^* = sY_t^* = s_\pi(1-\omega)Y_t^* = I_t$$

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[18] Contrast equation (18) with equation (12) that represents the closure provided by the supermultiplier growth model.

[19] These models descend from Kalecki (1971) and Steindl (1952 and 1979) original contributions. The modern neo-Kaleckian model was presented originally by Dutt (1984) and Rowthorn (1981). See also Dutt (1990) and Lavoie (2014) for a formalization and comparison of the neo-Kaleckian model with alternative growth models. For a detailed survey of the literature, see Blecker (2002).

[20] Note that our presentation of the neo-Kaleckian model has some minor differences in relation to the usual presentation of these models. The main difference is that we specify the investment function in terms of the determinants of the growth rate of investment, while the usual neo-Kaleckian specification is in terms of the determinants of the desired rate of capital accumulation. Observe that this difference does not affect the equilibrium values of the model's endogenous variables because, since by definition $g_{It} = g_{Kt} + \dot{g}_K/(g_{Kt} + \delta)$ and since, in equilibrium, we have $\dot{g}_K = 0$, then in equilibrium we obtain $g_I^* = g_K^*$. Moreover, as we shall see shortly, our specification of the investment function does not affect also the equilibrium stability condition. Therefore, we claim that nothing essential is altered by our particular specification of the investment function of the neo-Kaleckian model.

and

$$\frac{I_t}{Y_t^*} = \frac{S_t^*}{Y_t^*} = s = s_\pi(1-\omega)$$

Thus, according to the first of the two equations above, the level aggregate saving adjusts to the level of aggregate investment through the variation of the level of aggregate output, the only endogenous variable in the equation. Note however that, as we can verify from the second equation above, given s_π and ω, the saving ratio, equal to the marginal propensity to save s, is exogenously determined and thus it determines the investment share of output in the neo-Kaleckian model. This feature of the model contrasts with the related result obtained from the Cambridge and supermultiplier growth models. Indeed, as we pointed out before, in these latter models the investment share of output determines the saving ratio. In the Cambridge model this result follows from changes in income distribution and in the marginal propensity to save, while in the supermultiplier growth model the same result follows from the existence of an autonomous component in aggregate consumption which makes the saving ratio endogenous even though income distribution and the marginal propensity to save are given exogenously. In contrast, the neo-Kaleckian model assumes that income distribution (and thus the marginal propensity to save) is exogenously determined and that there is no autonomous consumption component, which implies, in combination with the other assumptions of the model, the exogeneity of the saving ratio.

Now, since the saving ratio is an exogenous variable and it determines the investment share of output, then the latter variable cannot be changed according to the requirements of the pace of economic growth. So, in contrast with the supermultiplier growth model, according to the neo-Kaleckian growth model, given income distribution, a change in the investment and output growth rates does not have any effect on the equilibrium value of the investment share of output. More importantly, from equation (2) we can verify that in the neo-Kaleckian model the rate of capital accumulation can only be reconciled with the output/demand growth rate if the degree of capacity utilization is properly adjusted. Indeed, since in the neo-Kaleckian model the saving ratio determines the investment share of output, then, according to equation (2), the rate of capital accumulation is given by

$$g_{Kt} = \left(\frac{s_\pi(1-\omega)}{v}\right)u_t - \delta$$

On the other hand, we saw that in the neo-Kaleckian model the growth rate of investment determines the equilibrium growth rate of output, so we have that $g^* = g_I$. Thus, using these results in equation (3), we obtain the following differential equation for the dynamic adjustment of the degree of capacity utilization

$$\dot{u} = u_t\left(g_I - \left(\frac{s_\pi(1-\omega)}{v}\right)u_t + \delta\right) \qquad (20)$$

The last equation shows that if the investment growth rate is higher (lower) than the rate of capital accumulation, then the degree of capacity utilization increases (declines) and this raises (reduces) the pace of capital accumulation. As a result, the capital accumulation rate converges to the investment growth rate through changes in the degree of capacity utilization. Therefore, in the equilibrium path of the neo-Kaleckian model we have

$$g_K^* = g_I = \left(\frac{s_\pi(1-\omega)}{v}\right)u^* - \delta$$

Now, solving the last equation for the equilibrium degree of capacity utilization we obtain

$$u^* = \frac{v(g_I + \delta)}{s_\pi(1-\omega)} \qquad (21)$$

Equation (21) shows the determinants of the required degree of capacity utilization u^* in the simplified version of the neo-Kaleckian model here presented. This latter rate is the one that reconciles the rate of capital accumulation with the pace of economic growth and, therefore, allows the existence of an equilibrium growth path in the model. Observe that, in its role as an adjusting variable, the equilibrium degree of capacity utilization has to able to assume any value between zero and one, no matter how implausible this may be. Therefore the neo-Kaleckian model is not compatible with the related notions of planned spare capacity and normal (or desired) capacity utilization rate. Indeed, if we suppose the existence of a normal degree of capacity utilization, the closure provided by the model implies that it would be possible to have large and persistent deviations of the equilibrium degree of capacity utilization from its normal level and also that such divergence would not have any repercussion on capitalist investment decisions.[21]It is important to be remarked that the required long run endogeneity of the equilibrium degree of capacity utilization does not depend on the particular specification for the investment function adopted here, being in fact valid for all the usual specifications of the investment function in neo-Kaleckian models.[22] Actually, the necessity concerning the variability of the equilibrium degree of capacity utilization follows from the specification of the consumption function and *not* from the particular formulation of the investment function adopted in the model. As we argued above, it is the rigidity of the investment share of output implied by the exogeneity of the saving ratio that leads to the requirement of the long run variability of the equilibrium capacity utilization rate.

Further, note that admitting that the capital stock adjustment principle regulates investment decisions in the context of a neo-Kaleckian model only leads to an instability process of the Harrodian type.[23] Indeed, suppose, following Skott (2008), that in trying to adjust the actual degree of capacity utilization to its normal level, capitalist firms change the investment growth rate according to $\dot{g}_I = \eta(u_t - \mu), \eta > 0$. Thus, in the equilibrium path of this particular model (i.e. with $\dot{g}_I = \dot{u} = 0$), we would have $u^* = \mu$ and $g_I^* = (s_\pi(1-\omega)/v)\mu - \delta$. Now, if initially we have $u_0 = \mu$ and $g_{I0} \gtrless g_I^*$, then, according to equation (20), we would have $\dot{u} \gtrless 0$, which implies that thereafter we would have $u_t \gtrless \mu$ and, hence, $\dot{g}_I \gtrless 0$ and $g_{It} \gtrless g_I^*$. So the equilibrium rate of growth would be unstable. Thus, according to the neo-Kaleckian growth model, we would have a dilemma: either, on the one hand, we assume way the possibility of an adjustment of the actual degree of capacity utilization towards its normal level and admit the possibility of obtaining an equilibrium path with an implausibly high or low equilibrium degree of capacity utilization or, on the other, we allow an adjustment of the actual to the normal degree of capacity utilization and obtain an unstable growth trajectory as we just saw. Observe, however, that the dilemma exists only if we restrict ourselves to the set of assumptions of the neo-Kaleckian model. In fact, once we admit the existence of an autonomous component in aggregate consumption the saving ratio becomes an endogenous variable and the investment share of output can change allowing the adjustment of the actual degree of capacity utilization to its normal level, as we have in the supermultiplier growth model.[24]

Finally, we shall analyze the role of income distribution in the neo-Kaleckian growth model. Thus, in the very simple version of the model presented here there is no relationship between the pace of economic growth and the level of functional income distribution. Since the investment growth rate is supposed to be an exogenous variable in the model, the equilibrium rate of output growth does not affect

[21]Compare equation (21) with the theoretical closure associated with the supermultiplier growth model as represented by equation (12) above.

[22]For instance, the result under discussion is valid for the investment function given by equation (22) (below in the text). Indeed, the endogenous character of the equilibrium degree of capacity utilization and its implications are maintained as can be verified from equation (23) (below in the text) which is the equation that shows the determinants of the equilibrium degree of capacity utilization corresponding to the investment function represented by equation (22). The same point is valid for other investment functions that frequently appear in the neo-Keleckian growth literature and, in particular, it is valid in the case of the investment function given by $g_{It} = \alpha + \beta(u_t - \mu)$ with $\alpha, \beta > 0$ and μ exogenous, Note that in the latter investment function the normal degree of capacity utilization appears as an argument. Nonetheless, the endogenous character of the equilibrium degree of capacity utilization is also maintained in this case and the corresponding value of the equilibrium rate is given by $u^* = \big(v(\alpha + \delta - \beta\mu)\big)/(s_\pi(1-\omega) - \beta v)$.

[23]In this connection, see Hein, Lavoie and van Treeck (2012) for a survey on Harrodian instability and the tendency for the normal capacity utilization rate in neo-Kaleckian growth models.

[24]See Allain (2014) for a growth model that comes from the Kaleckian tradition pointing in the direction of a closure similar to the one provided by the supermultiplier growth model presented in this paper.

and is not affected by the level of the wage share of output. Nevertheless, a change in the wage share has a level effect over the equilibrium value of output according to the simple neo-Kaleckian model under analysis. In fact, an increase (decrease) in the wage share, raises (reduces) the value of the multiplier $1/\left(s_\pi(1-\omega)\right)$ and, through it, such a change has a positive (negative) level effect on equilibrium output. These two latter results are shared with the supermultiplier growth model. On the other hand, in contrast with the latter model, from equation (21) we can see that, in the neo-Kaleckian model, an increase (decrease) in the wage share leads to an increase (a reduction) in the equilibrium degree of capacity utilization. A model that presents this type of result is classified in the neo-Kaleckian literature as a "stagnationist" or "wage led aggregate demand" model.[25]

We must say, however, that the independence between the pace of economic growth and income distribution in the simple version of the neo-Kaleckian model presented above is a direct consequence of the specific investment function adopted, which, as we saw, assumes the rate of investment growth to be completely exogenous. Indeed, if we consider the more usual formulations of the investment function in the neo-Kaleckian models, then we can obtain a causal relationship running from income distribution to the pace of economic expansion. So let us consider, for instance, a linear version of the investment function suggested by Marglin & Bhaduri (1990) and Bhaduri & Marglin (1990)

$$g_{It} = \alpha + \beta u_t + \rho(1-\omega) \tag{22}$$

where $\alpha > 0$ is an autonomous component of the investment function, $\beta > 0$ is a parameter measuring the sensitivity of the growth rate of investment to the capacity utilization rate, and $\rho > 0$ is a parameter measuring the sensitivity of the investment growth rate with respect to the profit share (i.e. $(1-\omega)$). The introduction of an induced component βu_t in the investment function turns the investment growth rate into an endogenous variable of the model that positively depends on the actual degree of capacity utilization. With such specification for the investment function, the equilibrium value of the degree of capacity utilization is given by

$$u^* = \frac{v\left(\alpha + \delta + \rho(1-\omega)\right)}{s_\pi(1-\omega) - \beta v} \tag{23}$$

From the equation above we can see that an equilibrium with a positive value for the degree of capacity utilization requires that $\beta < s_\pi(1-\omega)/v$.[26] Also, it can be shown that, *ceteris paribus*, an increase (decrease) in the wage share raises (reduces) the equilibrium degree of capacity utilization.[27] Thus, the model still is classified as "stagnationist" or "wage led aggregate demand". Now, substituting this last result in the investment function, we obtain the equilibrium level of the investment growth rate as follows

$$g_I^* = \alpha + \beta\left(\frac{v\left(\alpha + \delta + \rho(1-\omega)\right)}{s_\pi(1-\omega) - \beta v}\right) + \rho(1-\omega)$$

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[25]See Blecker (2002) for discussion of the neo-Kaleckian models based on this type of classification.

[26] The inequality $\beta < s_\pi(1-\omega)/v$ is also necessary for the stability of the equilibrium of the model. To see this, note that, in the case of the present version of the model, the equation for the dynamic adjustment of the degree of capacity utilization (equation (20)) would be given by $\dot{u} = u_t(\alpha + \beta u_t + \rho(1-\omega) - (s_\pi(1-\omega)/v)u_t + \delta) = u_t\left(\alpha + \rho(1-\omega) - \left((s_\pi(1-\omega)/v) - \beta\right)u_t + \delta\right)$. Observe that now a change in the capacity utilization rate affects both the investment growth rate and pace of capital accumulation in the same direction. Thus, if initially the investment growth rate were higher (lower) than the capital accumulation rate, then the capacity utilization rate would increase (decline). This latter change would produce the adjustment between investment growth and capital accumulation rates only if the impact of a change in the capacity utilization rate over the capital accumulation rate is greater than the impact of such a change on the pace of investment growth. That is, only if $\partial g_{Kt}/\partial u_t = s_\pi(1-\omega)/v > \beta = \partial g_{It}/\partial u_t$, which is the inequality mentioned above.

[27] Taking the partial derivative of u^* with respect to ω we obtain that $\partial u^*/\partial\omega = (v[\rho\beta v + s_\pi(\alpha+\delta)])/(s_\pi(1-\omega) - \beta v)^2 > 0$, which justifies the "stagnationist" or "wage led aggregate demand" classification attributed to the model. However, if a nonlinear specification of the positive effect of the share of profit on the investment function was adopted then it would be possible, according to neo-Kaleckian literature, to obtain a negative relationship between $u^*$and ω. In this case, following the suggestion of Marglin and Bhaduri (1990), the model would be classified as "exhilarationist" (or "profit led aggregate demand"). See Blecker (2002) for a detailed discussion of these topics. Moreover, according to Lavoie (2014), even retaining the linear specification of the investment function we can obtain the "exhilarationist" result if we admit negative values of the parameter α.

Thus, since u^* is positively related to the level of the wage share, then the effect of a change in the wage share on the investment growth rate can be either positive or negative according to the value of the parameter ρ. As can be seen from the equation above, a higher value of the latter parameter reduces the positive (and indirect) effect of a change of the wage share exerted through the equilibrium degree of capacity utilization and increases the direct contribution of a change in the wage share through the third term on the RHS of the equation above. Hence, for a sufficiently *low* value of ρ the positive effect of a modification in the wage share on the investment growth rate through the capacity utilization rate dominates the direct negative effect related to the term $\rho(1-\omega)$. In this case, according to the neo-Kaleckian literature, the model would produce a wage led growth pattern. On the other hand, for a sufficiently *high* value of ρ we would have the opposite situation and the model would generate a profit led pattern of economic growth.[28] In both cases, a change in income distribution has a permanent growth effect. The existence of a relationship between economic growth and income distribution featured in the last version of the neo-Kaleckian model, also characterizes the Cambridge growth model as we saw. In the latter model there is an inverse relationship between the wage share and the rate of output growth, whereas the last specification of the neo-Kaleckian model admits either, a positive (in the wage led growth case) or a negative (in the profit led growth case) relationship between the two variables. In contrast with these results, the absence of any direct permanent relationship between income distribution and the trend rate rate of economic growth is an important feature of the supermultiplier growth model.

2.3. General comparison

Table 1 summarizes the principal results obtained from the above comparative analysis. The main conclusion that emerges from this analysis is that the supermultiplier growth model can be considered a heterodox alternative to the Cambridge and neo-Kaleckian growth models in the analysis of the relationship between economic growth, income distribution and effective demand. In this sense, first of all, the supermultiplier growth model shows how is it possible for a heterodox growth model to obtain a tendency towards the normal utilization of productive capacity without relying on the endogenous determination of the level of income distribution as in Cambridge growth model. Thus, the supermultiplier model does not impose the existence of any necessary *a priori*relationship between income distribution and economic growth in the interpretation of economic reality and, therefore, leave open the space for the determination of income distribution from outside the model by political, historical and economic factors not directly and necessarily related to the process of economic expansion. Secondly, the model also shows that the existence of a stable process of demand-led growth does not require the endogenous determination of an equilibrium degree of capacity utilization as in the neo-Kaleckian growth model. Actually, the supermultiplier model shows that the existence of a demand-led pattern of economic growth is fully compatible with the tendency towards the normal utilization of productive capacity. Therefore, it shows that allowing the possible prevalence of arbitrarily high or low rates of capacity utilization and permanent deviations of the actual degree of capacity utilization from its normal level are not necessary requirements for the existence of a demand-led pattern of economic growth.

Table 1 here

3. Final Remarks

In this paper we have shown how the Sraffian Supermultiplier model with its hypotheses of growing non capacity generating autonomous demand and a investment function based on the capital stock adjustment principle provides us with a distinct closure for heterodox growth theory.Using this closure, changes in the propensity to invest can determine the saving ratio by means of changes in the fraction, i.e., the ratio of the average to the marginal propensity to save. We have also shown how the gradual changes in the

[28] For sufficiently low or high value of ρ we mean a value of ρ respectively lower or higher than a critical value $\rho_c = \left(\beta v(\alpha+\beta)\right)/\left((s_\pi(1-\omega)-2\beta v)(1-\omega)\right)$. Therefore, we would have a wage led (profit led) growth pattern as $\rho < \rho_c (\rho > \rho_c)$. That is we have $\partial g_I^*/\partial\omega \gtrless 0$ as $\rho \lessgtr \rho_c$.

marginal propensity to invest that let capacity to adjust to demand allow also the reconciliation of demand led growth, exogenous distribution and a tendency to normal degree of capacity utilization even across steady states. This is in sharp contrast with the Cambridge model that has to resort to an implausible theory of income distribution driven by accumulation in order to retain the idea of a tendency towards normal utilization. It constrast also with the neo-Kaleckian model that maintains the idea of an exogenous determination of income distribution but does so at the cost of the implausible property of the actual degree of utilization being in general always different from the normal degree, even across steady states.[29] In our view (and also of Cesaratto (2015)) the Sraffian Supermultiplier and its closure based on an endogenous saving ratio offer a useful way of combining the classical surplus approach to distribution and the principle of effective demand in a simple model that can be used as a tool to analyse the process of accumulation along the lines suggested by the pioneering work of Garegnani(1962[2015])).

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Table 1

	Cambridge Growth Model	neo-Kaleckian Growth Model	Supermultiplier Growth Model
Output capacity, aggregate output and aggregate demand	*Capacity (potential) output determines the levels of aggregate output and aggregate demand*	*Aggregate demand determines the level of aggregate output*	*Aggregate Demand determines the levels of aggregate output and capacity (potential) output*
Income distribution	*Endogenous*	*Exogenous*	*Exogenous*
Investment share of output and saving ratio	*The investment share of output determines the saving ratio*	*The exogenous saving ratio determines the investment share of output*	*The investment share of output determines the saving ratio*
Pattern of economic growth	*Supply (capacity) constrained growth*	*Demand (investment) led growth*	*Demand (consumption) led growth*
Degree of capacity utilization	*Tends to full capacity utilization rate*	*Tends to an endogenous equilibrium value*	*Tends to the normal degree of capacity utilization*
Investment share of output and the trend rate of economic growth	*There is a theoretically necessary and positive relationship between the two variables*	*No theoretically necessary relationship between the two variables*	*There is a theoretically necessary and positive relationship between the two variables*
Wage share of output (income distribution) and the rate of economic growth	*There is a theoretically necessary and negative relationship between the two variables*	*There is a positive relationship between the two variables in the "wage led growth" case and a negative one in the "profit led growth" case*	*No relationship between the two variables.* *There is a positive wage led output level effect*
Theoretical closure	*Endogenous determination of a required level of income distribution*	*Endogenous determination of a required equilibrium degree of capacity utilization*	*Endogenous determination of a required value of the fraction (i.e. the ratio between the average and marginal propensities to save)*