A Keynesian dynamic stochastic labor market disequilibrium model for business cycle analysis.

Christian Schoder∗

preliminary and incomplete draft – please do not quote

October 22, 2014

Abstract

A micro-founded Dynamic Stochastic Labor Market Disequilibrium model for Keynesian business cycle analysis is proposed. Investment, price setting and capital utilization decisions are consistent with a profit-maximization objective of the firm. The consumption function is derived from a precautionary saving motive of the household as well as from an inner conflict of individuals between instantaneous and inter-temporal utility maximization. A collective Nash bargaining process determines wage inflation with the labor market affecting the relative bargaining power. The predicted responses to policy shocks are contrasted to those of a neoclassical Dynamic Stochastic General Equilibrium model and a traditional Post-Keynesian model. While sharing the type of micro-foundations of the former, the model proposed shares the transmission mechanisms of the latter. It is therefore well suited for Keynesian business cycle analysis.

Keywords: Post-Keynesian economics, micro-foundations, dynamic stochastic labor-market disequilibrium model, dynamic stochastic general equilibrium model

JEL Classification: B41, E12, J52

∗Vienna University of Economics and Business, Welthandelsplatz 1, Building D4, 1020 Vienna. Email: christian.schoder@wu.ac.at
1 Introduction

To study the business cycle, much of the Keynesian literature employs aggregative models in the Cambridge tradition of Kalecki (1971), Robinson (1956, 1962) and Kaldor (1982) which we shall refer to as Traditional Post-Keynesian (TPK). Models in this vein feature a rich set of economic dynamics and perceive fluctuations as a demand-side phenomenon generated by the interaction of distribution, financial leverage and aggregate demand (cf. Taylor 1985, 2012, Flaschel 2009 and Schoder 2014c). The core feature of this model class is the principle of effective demand according to which output is determined by aggregate demand. It implies that the labor market exhibits Keynesian unemployment resulting from a lack of aggregate demand typically allowing for labor market conditions feeding back into the determination of wage growth (cf. Hein and Stockhammer 2010, Taylor 2004). As a core weakness, this model class has been criticized from an economic theory perspective for the lack of explicit micro-foundations. Behavioral relations are anchored in stylized and highly contested empirical observations such as the Keynesian consumption function, rather than being derived from goal-oriented behavior of economic agents. This shortcoming has contributed immensely to the marginalization of Keynesian macroeconomic theory since the critique by Lucas (1976) and has been acknowledged by many Keynesian authors including Farmer and Foley (2009), Skott (1989a, 2012a), Tavani (forthcoming) and Murota and Ono (2010) seeking to provide explicit micro-foundations for selected Keynesian behavioral relations.

Neoclassical Dynamic Stochastic General Equilibrium (DSGE) models, on the other hand, which have become the mainstream in business cycle analysis do not allow for Keynesian unemployment simply due to the assumption of a general equilibrium (cf. Clarida et al. 1999, Woodford 2003 and Smets and Wouters 2003). The economy is supply-side determined since not necessarily market-clearing but structural short and long-run equilibria on the factor markets combined with an aggregate production function imply a unique level of output and employment.

The aim of the present paper is to combine the advantages of both approaches to business cycle analysis. We present a model which we refer to as Dynamic Stochastic Labor Market Disequilibrium (DSLMD) model. It provides one possible consistent set of micro-foundations for the traditional Keynesian aggregative approach. With DSGE models it shares anchoring all economic decisions in goal-oriented economic behavior. With TPK models it shares the principle of effective demand as well as the possibility of endogenous cycles and persistent responses to temporary shocks. Using any model which is Keynesian by theory (effective demand) and mainstream by methodology (micro-foundation) may contribute considerably to the return of Keynesian ideas to orthodox macroeconomics. As the present contribution will argue, the core features of a Keynesian economy

---

1 In the long run, unemployment may become structural, i.e. independent of aggregate demand, in some model variants (Skott 1989b, Dumenil and Lévy 1999, Shaikh 2009). In Lavoie (1995), Dutt (1997), and Schoder (2012) hysteresis effects allow unemployment to be demand-driven even in the long run. This, however, implies a non-stationary rate of capacity utilization which contradicts empirical observation (Skott 2012b, Schoder 2014a). Schoder (2014c) shows that a demand-driven unemployment rate can be reconciled with a stationary rate of capacity utilization by assuming an endogenous capital productivity.

2 For well-known early critiques of the Keynesian consumption function, for instance, see Duesenberry (1949), Friedman (1957) and Modigliani and Brumberg (1954).

3 Unemployment, if it exists at all, does not follow from a lack of aggregate demand but from labor market imperfections such as search and matching frictions (Mortensen and Pissarides 1994, Gertler et al. 2008) or disequilibrium wages due to efficiency considerations arising from asymmetric information between employers and employees (Shapiro and Stiglitz 1984). Hence, unemployment is structural. In the steady state, monetary and fiscal policy is neutral to unemployment. Out of steady-state, these policies affect unemployment only through their impact on the labor market imperfections. Without market imperfections which go beyond wage rigidities, no unemployment exists.
can be traced back to a type of micro-foundation consistent with conventional macroeconomics if crucial assumptions of the latter regarding the nature of the labor market and consumption choice are adapted. When presenting the model, we argue that the core behavioral equations anchored in goal-oriented economic behavior are consistent with those of much of the traditional Keynesian literature on consumption and investment as well as price and wage setting.

Note that the core assumption in the DSGE literature which makes the business cycle supply-driven is that the nominal wage adjusts in order to eliminate unemployment completely or reduce it to a structural level at any point in time. In the DSLMD model outlined here, this assumption is dropped and replaced by the assumption that the rate of wage inflation is determined by a collective bargaining process between firms’ and workers’ representatives. Moving from a decentralized to a centralized wage setting mechanism is the crucial step for the transformation of a DSGE into a DSLMD model. Once the assumption of labor-market clearing is replaced by a collective wage bargaining process, the model, in particular consumption, becomes indeterminate. This is because steady-state consumption in DSGE models is the residual predetermined output not invested or consumed by the government. To solve the problem of indeterminacy, we derive a Keynesian type of consumption function relating aggregate consumption to aggregate income. In particular, we follow Carroll (1997), Carroll and Jeanne (2009), Carroll and Toche (2009) and introduce an uninsurable risk of permanent income loss to the household’s problem which induces the household to accumulate precautionary savings to an extent which depends on current income. In turn, this implies consumption to be an increasing function in current income.

A key implication of the precautionary saving motive in the consumer problem is a positive response of consumption to the interest rate. To reverse this relationship, we follow Schoder (2014d) and additionally model the consumption and saving decisions of the household as the outcome of a conflict between two inner selves inhabiting the individual: the doer and the planner. This innovation follows loosely and formalizes Shefrin and Thaler’s (1988) behavioral life-cycle hypothesis. The doer chooses consumption so as to maximize instantaneous utility without caring about constraints. The planner, however, concerned about life-time utility and taking into account the budget constraint can enforce willpower which infuses bad conscience and reduces the doer’s utility proportional to consumption. A sensitivity parameter captures how effective the planner’s willpower is in disciplining the doer. The crucial property of this sensitivity parameter is that it may increase with the interest rate.

Using calibrations based on estimations by Schoder (2014d,b), the DSLMD, TPK and DSGE models are compared with respect to their short and long-run responses to policy shocks. The core insight is that Keynesian macroeconomics is highly consistent with orthodox micro-foundations. In particular, we find the following: (a) The Taylor principle, which has been used excessively by conservative macroeconomists (cf. Taylor 1993) to urge the monetary authorities to respond aggressively to changes in the inflation rate in order to ensure stability and which has been strongly objected to by Keynesian macroeconomists (cf. Arestis 2009), is a condition for the existence of a unique solution only for the DSGE but not for the DSLMD or TPK model. (b) Despite the different

---

4Steady-state output is determined by factor market equilibria, which are independent of aggregate spending, and a production function.(cf. Smets and Wouters 2003). Note that the Euler equation for consumption obtained from the household’s optimization problem only determines optimal consumption smoothing but not the level of optimal consumption. Without labor-market clearing, i.e. at the presence of unemployed labor going beyond a structural level, the resource constraint—demand cannot exceed what is produced—becomes a goods market equilibrium condition—aggregate demand generates an aggregate income which, in turn, generates this very aggregate demand—as known from the textbook IS model. Since output now depends on aggregate spending, consumption is indeterminate.
natures of the DSGE model on the one hand and the DSLMD and TPK models on the other, the responses to shocks, especially to temporary ones, are rather similar. This is mainly because of the labor market feedback in the Keynesian models according to which decreasing unemployment causes wage inflation to accelerate. If this feedback mechanism is weak, the response of quantities to shocks is more pronounced in the Keynesian models than in the DSGE model. (c) The multiplier effects of permanent government spending shocks are more pronounced in the DSLMD and TPK models than in the DSGE model especially when the feedback of the labor market to wage formation is weak. (d) Because of the assumption of a propensity to save independent of the source of income, a rise in wage inflation causes a contraction in the DSLMD and TPK models. (e) This allows for Goodwin type of cycles if the wage formation responds sluggishly to changes in the labor market conditions.

The remainder of the paper proceeds as follows: In the next section, the behavioral relations implied by goal-oriented economic behavior are discussed and compared to the behavioral rules posed by TPK models. The third section deals with expectation formation and model solution. In the fourth section, the macroeconomic responses to policy shocks are studied for the model variants considered. The emphasis lies on comparing the transmission mechanisms in the different models. The last section concludes.

2 Motivating economic decisions by goal-oriented behavior

Apart from three decisive modifications, i.e. replacing the assumption of labor market clearing by a collective wage bargaining process, introducing an uninsurable risk of permanent income loss to the household’s problem and modeling the household’s consumption and saving choices as results of an inner conflict, the building blocks of the model presented here are reminiscent of a DSGE model with firm-specific capital as proposed by Woodford (2005) and Sveen and Weinke (2007, 2009). All model equations are derived explicitly in Appendix B. Details on the household and firm behavior can also be found in Schoder (2014d). In this section, we focus on the economic content with a limited use mathematics.\(^5\)

The economy considered comprises households, a final good firm, intermediate good firms, a fiscal authority, a monetary authority as well as firms’ and workers’ representatives. The capital stock is owned by the household, but managed by the firm. Hence, decisions are made by the firm but profits are completely distributed to the households. Capital is firm-specific and cannot be simply moved to another firm. Hence, there is no spot market for capital services. The population and labor embodied productivity grow at constant rates.

2.1 The household sector


**Risk of income loss and inner conflict.** Individuals are born into generations which grow in size by a constant rate. Each household is born as part of the labor force and supplies labor

\(^5\)The Dynare codes for the three model variants used for computing the steady-states and the impulse-response functions discussed below can be obtained from the author upon request.
hours which will be (partly) employed by the firm. Each period, the household may drop out of the labor force with a known probability loosing all sources of income which poses and uninsurable risk. Once the household is inactive, i.e. has left the labor force, it cannot return. However, it faces the risk of death with a given probability. Hence, the household accumulates precautionary savings in order to insure against the risk of permanent income loss.

The active household consists of an individual inhabited by two souls: the *doer* and the *planner*. The doer seeks to maximize instantaneous utility by desiring to consume as much as possible without concern about financial or resource constraints. Yet, the planner can enforce willpower which infuses bad conscience to the doers utility function and aims at disciplining the doer.

The doer chooses consumption so as to maximize instantaneous utility which we assume to be a function in consumption as well as several variables taken as given by the doer. These are the willpower enforced by the planner, a sensitivity measure by which a given level of willpower generates disutility, and the labor supply of the household. We assume that consumption, willpower and the sensitivity measure affect utility jointly. This means that the utility reducing bad conscience infused by the planner not only depends on the willpower enforced and the sensitivity measure by which it is effective in disciplining the doer but also on the level of consumption. The higher the level of consumption, the higher the bad conscience of the doer, for a given willpower and sensitivity parameter. What level of consumption does the doer choose? Overall, at low levels of consumption a rise in consumption will increase instantaneous utility for any given level of effective willpower (which is the willpower weighted by the sensitivity measure). Yet, as consumption increases, marginal utility decreases. At some consumption level, the additional bad conscience starts exceeding the direct utility of consumption. Then a rise in consumption will decrease utility. The doer chooses consumption associated with the maximum overall utility.

The willpower sensitivity is assumed to depend on the real interest rate. Given the level of willpower enforced by the planner, a rise in the interest rate can be expected to increase the doer’s bad conscience of consuming, which can be captured by a rising sensitivity measure.

**The active household’s problem.** The other inhabitant of the individual, i.e. the planner, knows the doer’s consumption choice for any given level of willpower and sensitivity. Internalizing the doer’s solution, the planner then chooses inter-temporal paths for willpower and labor supply in order to maximize discounted expected life-time utility. In doing so, the planner faces two constraints which he or she has to take into account: first, the behavior of the doer as discussed above; second, the inter-temporal budget constraint. Real wealth tomorrow is the part saved out of today’s wealth and household income plus the interest on it.

Note that, due to the inter-temporal nature of the planner’s problem, he or she has to form *i*-periods-ahead expectations $E_t x_{t+i}$ about future realizations of any relevant variable $x$ in period $t$ for $i \to \infty$. At this stage, we do not need to specify how these expectations are formed. The only requirement is that expectations formed in different periods are the same if the information set is

---

6Hence each household will be affected by unemployment to the same extent.

7For simplicity assume that the inactive household is dominated by the planner who fully controls the doer without cost.

8For details, see Schoder (2014d).

9As discussed in more detail in Carroll and Jeanne (2009) and Schoder (2014d), we assume a non-distortionary transfer from non-newborn households to newborn households which ensures real wealth to be equal across households and facilitates aggregation.
the same. Hence, expectations could be naive, adaptive, based on statistical forecasting or learning, or rational in order to solve the household’s problem.

The planner of the active household will choose the level of consumption such that the current period’s marginal utility of consumption equals the discounted expected marginal utility of the next period. Yet, this expected marginal utility includes the risk of dropping out of the labor force. Because of the inter-temporal link of consumption today and tomorrow and because of the fact that the active household may become inactive tomorrow, it thinks today about tomorrows consumption choice for the potential case of an income loss. In this case, consumption would be chosen according to considerations of the inactive household. Hence, the active household’s consumption choice today is affected by the inactive household’s consumption choice which may become relevant tomorrow.

The inactive household’s problem. We follow Carroll (1997), Carroll and Jeanne (2009), Carroll and Toche (2009) and assume that newly inactive households obtain a consumption path from a type of Blanchard (1985) insurance company. Once they drop out of the labor force, they hand all real wealth over to a perfectly competitive insurance company and receive annuities in return as long as they are alive. This assumption facilitates aggregation. Because of the law of large numbers, death is a microeconomic but not a macroeconomic risk. Hence, there is no accidental bequests. Recall that inactive households are not subject to an inner conflict between the doer and the planner. Then, it can be shown, that the solution of the inactive household’s problem implies consumption to be proportional to real wealth. Since we assume that the active household internalizes this solution of the inactive household’s problem, the former’s expected marginal utility of consumption discussed above depends on the level of previously accumulated wealth. This is the crucial property of the active household’s solution and gives rise to a Keynesian type of consumption function in the steady state.

The consumption function. As argued above, the solution for the inactive household’s problem implies

$$\tilde{C}_i^t = \kappa \tilde{B}_i^t$$  \hspace{1cm} (1)

where $\tilde{C}_i^t$ and $\tilde{B}_i^t$ denote for the inactive household detrended aggregate consumption and detrended real wealth in time $t$ and where $\kappa > 0$ is a constant implied by the household’s problem. To derive the aggregate budget constraint for the inactive households, note that they do not receive any income and that tomorrow’s aggregate wealth of inactive households is also contributed to households which will become inactive tomorrow. Hence, we get

$$E_t \hat{B}_{t+1}^i = E_t \hat{R}_{t+1} \frac{1}{\Gamma} (-\tilde{C}_i^t + \tilde{B}_i^t) + U E_t \tilde{B}_{t+1}^a$$  \hspace{1cm} (2)

where $\hat{R}_{t+1}$ is the real interest factor determined by the nominal interest rate and expected price inflation and $\Gamma$ is the deterministic growth factor of the economy. $E_t$ is the expectations operator. To obtain the aggregate consumption Euler equation for the active households, we combine the solutions for both household types to get

$$\frac{1}{\theta_t} (\tilde{C}_a^t)^{-\rho} = E_t \beta \hat{R}_{t+1} (1 + \gamma)^{-\rho} \left( (1 - U) \frac{1}{\theta_{t+1}} (\tilde{C}_{t+1}^a)^{-\rho} + U (\kappa \tilde{B}_{t+1}^a)^{-\rho} \right)$$  \hspace{1cm} (3)
where $\theta$, $\rho$ and $\gamma$ are the willpower sensitivity parameter, the parameter of relative risk aversion in the utility function and the growth rate of labor embodied productivity, respectively. Eq. (3) states that, in the aggregate, the current period’s marginal utility of consumption equals the discounted expected marginal utility of the next period. The crucial difference to conventional consumption Euler equations is that the expected marginal costs depends on real wealth. Why is that? Note that, because of (1), $\kappa \tilde{B}_{t+1}$ in (3) is the consumption of a newly inactive household. This is exactly the consumption level the active household expects for the next period with probability $U$, i.e. when dropping out of the labor force. Note that (3) collapses to the standard consumption Euler equation if there is no risk of permanent income loss, i.e. $U = 0$. Then consumption would be independent of wealth.

Let us now derive the active household’s aggregate budget constraint. Note that the overall wealth saved today for tomorrow by active households will be divided in tomorrow’s wealth of active households and tomorrow’s wealth of newly inactive households. The aggregate budget constraint therefore is
\[
E_t \tilde{B}_{t+1}^a = E_t R_{t+1} \left( \frac{1}{1 + U} (\tilde{Z}_t - \tilde{C}_t^a + \tilde{B}_t^a) \right)
\]
where $\tilde{Z}_t$ is the active household’s net income.

Let us consider the economy at the steady state, i.e. when $x_t = E_t x_{t+1} = x$ for any variable $x_t$. In this case, the specification of the willpower sensitivity parameter implies $\theta = 1$ (see Appendix B). Then, the two eqs. (3) and (4) feature three variables, i.e. $C^a$, $B^a$, $Z$. Hence, conditional on income, we can compute equilibrium consumption and wealth. Seen from a different angle, we can divide both eqs. (3) and (4) by $Z^{-\rho}$ and $Z$, respectively. Then we have two equations in the consumption-income ratio and the wealth-income ratio for which the existence of a unique solution can be shown under certain parameter constellations. As we can see, introducing the risk of permanent income loss to the conventional consumer problem implies the existence of an equilibrium consumption-income ratio. With rising income, consumption will increase by a fixed proportion which is very similar to a Keynesian consumption function. The core difference is that in our model, the marginal propensity to consume is endogenous and based on explicit micro-foundations. In particular, it depends on the nominal interest rate, the rate of inflation and the willpower sensitivity parameter out of the steady state.

Note that in the conventional case of $U = 0$ no unique solution for the consumption-income ratio and the wealth-income ratio exists. Even for a given income consumption would not be determined by the household’s problem.

In TPK models, a behavioral relationship between consumption and income is typically assumed based on stylized empirical observations (Lavoie 1992). A common stock-flow-consistent specification relates consumption to disposable income and wealth such as $\tilde{C}_t = c_z \tilde{Z}_t + c_b \tilde{B}_t$ with the budget constraint $\tilde{B}_{t+1} = \tilde{R}_{t+1} (\tilde{Z}_t - \tilde{C}_t + \tilde{B}_t)$ (cf. Godley and Lavoie 2012). How do the DSLMD and TPK consumption theories differ? Note that the TPK budget constraint is the same as the DSLMD budget constraint aggregated over the two household types. Normalizing the TPK consumption function by income at the steady state leads to $\tilde{C}/\tilde{Z} = c_z + c_b \tilde{B}/\tilde{Z}$. Normalizing the budget constraint by income and substituting into the TPK consumption function yields $\tilde{C}/\tilde{Z} = c_z + c_b \tilde{R}/(1 - \tilde{R})(1 - \tilde{C}/\tilde{Z})$. We can now see that for a given $\tilde{R}$ and a given $c_b$ there exists a

\[10\] An important feature of PK consumption functions is that the propensity to save out of wage income is higher than out of profit income. For the sake of comparability, we do not follow this distinction here.
c_z such that the consumption-income ratio of the TPK model equals the one of the DSLMD model. Hence, at the steady state the DSLMD and TPK consumption theories are equivalent. What about the dynamics out of steady-state? In the TPK model, these are fully captured by the constant propensities to consume out of income and wealth. In the DSLMD model, these propensities are endogenous and will be revisited in Section 4.

**Labor supply.** The active household’s problem obviously implies a positive relationship between labor supply and the real wage. This, however, is not a crucial property of the labor market for the DSGE or DSLMD models to work. As long as the supply curve’s slope is higher than the demand curve’s slope, the nominal wage will always adjust to equilibrate supply and demand in the DSGE framework. In the DSLMD framework, as discussed below, the real wage will turn out to decrease with the unemployment rate through collective wage bargaining. Stability then requires that the unemployment rate increases with the real wage. This holds for any labor supply curve as long as it has a higher slope than the demand curve. In the TPK model, labor supply is assumed to be constant. Note that stability then requires labor demand to fall in the real wage.

### 2.2 The firm sector

The firm sector is similar to DSGE models assuming firm-specific capital as outlined in Woodford (2005) and Sveen and Weinke (2007, 2009). Nevertheless, as we will argue below, it is highly consistent with Keynesian theories of the firm (cf. Taylor 2004).

There are two types of firms: a perfectly competitive firm aggregating intermediate goods into a final good used for consumption and investment, and a continuum of monopolistically competitive firms producing a differentiated good using capital and labor input. This distinction is used in order to reconcile in a simple way market power in production (due to heterogenous intermediate goods) and having one single consumption and investment good (due to a homogenous final good). We further assume capital to be firm specific. It cannot be transferred from one firm to the other and, hence, there is no capital market. Labor is rented from households.

**Final good firm.** A representative final good firm bundles a continuum of differentiated intermediate goods into a final good and sells it on a perfectly competitive market. Taken as given the price of the intermediate good, the elasticity of substitution of inputs given by its technology as well as the overall demand for the final good, its demand for the intermediate good can simply be obtained from cost minimization considerations. The result of this problem is an inverse relationship between the demand for an input and its price for a given output of final goods. This demand schedule will be assumed to be part of the information set of the intermediate good firm. It will turn out to be important when choosing the optimal price.

**The intermediate good firm’s problem.** There is a continuum of intermediate good firms each producing a differentiated good according to a constant-returns-to-scale Cobb-Douglas production function in capital and labor with labor embodied productivity growing at a deterministic rate.\(^{11}\) Intermediate goods are sold on a monopolistically competitive market. Facing a quadratic

---

\(^{11}\)Note that the choice of a Cobb-Douglas production function is not crucial for neither the DSGE nor DSLMD model. Any production function with increasing marginal costs in the short run, i.e. at a given capital stock, and constant marginal costs in the long run, i.e. at a fully adjusted capital stock, may be chosen. The production
adjustment cost, the firm purchases investment goods to accumulate capital. These adjustment costs affect deprecation. We assume Rotemberg (1982) price setting. Price setting is subject to quadratic adjustment costs which are assumed to be transferred to the households as profit income.\footnote{We assume price rigidities in the vein of Rotemberg (1982) instead of Calvo (1983), since the former implies that an acceleration of wage inflation increases real marginal costs, and hence the real wage, while the latter does not.} Taking as given output, the overall price level, its capital stock which is predetermined, and the nominal wage as well as the law of motion of capital, the production function and the demand function for intermediate goods, the firm chooses an inter-temporal path of prices, labor demand, and investment to maximize the discounted sum of expected future cash-flows. The cash-flow is sales minus costs where costs include wages, investment, interest payments and price adjustment costs.\footnote{Note that we implicitly assume sufficiently large costs of market entry required to keep the number of firms constant despite positive profits in the steady state.} The cash-flow minus deprecation are the profits distributed to the households.

**Optimal price setting.** The solutions of the intermediate firm’s problem are derived in Appendix B. Here, we want to briefly characterize the choices and how they are related to the Keynesian literature. What is the optimal price? Without price adjustment costs, the firm would set the price with a mark-up on nominal marginal costs. The mark-up is determined by the elasticity by which the final good firm can substitute intermediate goods to produce a given amount of final goods. Obviously, if the elasticity is high (low), the mark-up will be low (high). With price adjustment costs, prices will be set lower than without. Hence, the imputed mark-up over marginal costs will also decrease. Price adjustment costs ensure that faster wage inflation leads to an under-proportional increase of price inflation and, hence, to a larger real wage. The vehicle of distinguishing between a perfectly competitive final good firm and a monopolistically competitive intermediate good firm allows us to introduce mark-up pricing over marginal costs. The final good firm’s elasticity of substitution between differentiated intermediate goods as inputs is the source of the monopoly power of the intermediate good firms and, hence, the mark-up. This is highly consistent with Kalecki’s (1971) *degree of monopoly* which is typically referred to the Keynesian literature to justify the price mark-up over wage costs.

It is remarkable to note that Kaleckian mark-up pricing can be obtained from our framework by merely assuming that no substitution between capital and labor is possible. Then, marginal costs are proportional to wages and, assuming no price adjustment costs, the Kaleckian mark-up is equal to the imputed mark-up implied by the DSLMD model. Two more implications are worth to note: First, introducing price adjustment costs in a conventional firm’s profit maximization problem is a viable micro-foundation to obtain a cyclical mark-up which, in traditional Keynesian models of the Kaleckian type, is typically simply assumed (cf. Lavoie 1992). With stronger wage inflation which will be shown to move pro-cyclically due to labor market tightening, the imputed mark-up of prices over wages decreases. Second, the substitutability as implied by the Cobb-Douglas production function is another source of variation of the imputed mark-up over wages absent in the TPK model.

The cash-flow minus deprecation are the profits distributed to the households.
Optimal labor demand. Since the Cobb-Douglas production function allows for input substitution, the labor demand is not proportionally linked to output as assumed in TPK models. The optimizing firm will choose labor input such that the marginal revenue product of labor is equal to the marginal cost of labor. The latter is simply the real wage. The former is the marginal product of an additional unit of labor weighted by the marginal cost of a unit of output. To get a better understanding of this result, assume that the real wage is lower than the marginal revenue product of labor. Then, a one-unit rise in labor input would increase output by the marginal product of labor. For a given marginal cost of output, this translates into an increase in costs which exceeds the real wage. Hence, an expansion of labor demand is beneficiary which lowers the marginal product of labor. Note that the marginal product of labor is constant if a production function is assumed which does not allow input substitution which is typically implied by TPK models. Then the relationship between marginal costs of output and the real wage is proportional. Moreover, labor demand is directly linked to the production function in this case.\footnote{Note further that the firm’s marginal product of labor depends on its previously chosen capital stock. Hence, the pricing decision is not independent from the capital accumulation decision since capital is not purchased on a spot market.}

Investment and Tobin’s $q$. The firm chooses the path of the capital stock taking into consideration the law of motion of capital as well as capital adjustment costs and taking demand as given. The firm’s problem is basically the well-known Tobin’s (1969) $q$-theory of investment. The firm has to consider two questions.

First, in the short-run, what is the optimal response of investment to shocks such as a change in expected sales? The answer lies in the nature of the adjustment costs. Since these are assumed to be quadratic, a sharp adjustment in investment might cause a strong depreciation of the capital stock and, hence, increase costs. A slow adjustment might cause a temporary output-capital ratio which is too high to be cost efficient. As is well known from Tobin’s investment theory, the relevant signal for the firm comes from $q_t$ which is the Lagrangian multiplier for the law of motion constraint in the firm’s optimization problem. It measures how much profits the firm would gain by having one more unit of capital installed in the next period. It is the marginal value of an additional unit of capital taking into account capital adjustment costs. The optimality condition for investment then states that, in any period, investment should be chosen such that the marginal loss measured in terms of profits due to an one-unit increase in investment is equal to the marginal gain in terms of profits due to an extra unit of capital in the next period implied by a one-unit increase of investment today, i.e. $q_t$. Hence, for a given $q_t$, the optimality condition for investment tells us what the optimal investment will be in period $t$. If $q_t = 1$, then the marginal adjustment costs have to be zero which will only be the case when investment only covers depreciation and the capital stock does not change.

Second, in the long-run, what is the optimal capital stock for a given level of output? The answer is implied by the production function. It is the capital stock which minimizes the costs of production, i.e. maximizes profits. In the short-run, the optimality condition for the capital stock requires that the capital stock has to be chosen such that the negative marginal cost of capital, i.e. the marginal revenue product of capital is equal to the opportunity cost of a unit of capital which is the real interest minus capital gains.\footnote{Note that in our model the marginal return on capital is not measured by the firm’s marginal revenue product of capital due to the absence of a rental market for capital services. Rather, it captures the reduction of nominal labor costs that can be afforded after a one-unit increase in the capital stock in order to produce a given level of output.} Given the decision on investment today, which is implied
by the optimality condition for investment and today’s $q_t$ and which implies the capital stock for tomorrow, the optimality condition for capital determines tomorrow’s $q_{t+1}$.

The dynamics are illustrated in Figure 1. The optimality condition for capital implies for the $k_t$-nullcline on which $k_{t+1} - k_t = 0$ and, hence, $i_t = 0$ that $q_t = 1$, where $k_t$ and $i_t$ denote the capital stock and investment. In times of $q_t > 1$, it is worthwhile to expand the capital stock since the marginal gain in terms of profits from an extra unit of capital exceeds the marginal loss from an extra unit of capital. For points below the curve the reverse holds. The $q_t$-nullcline implies an inverse relationship between $q_t$ and $k_t$. For an initial value of output, the steady state capital stock is $k_0^*$. Suppose output increases shifting the $q$-nullcline to the north. $q$ jumps upwards immediately causing an expansion of the capital stock converging to a new steady state, $k_1^*$.

**Steindl meets Tobin.** Even though the underlying investment theory is based on Tobin’s $q$, it implies a relation between the rate of capital accumulation and the gap between the current rate of capacity utilization, $v_t$, and the so-called *normal* rate of capacity utilization, $v_t^*$, which is a popular behavioral rule for investment in the Keynesian literature (cf. Steindl 1952). The rate of capacity utilization is the ratio between output $y_t$ and full-capacity output $y_{c;t}$. To derive the investment function from Tobin’s $q$ theory, first the notion of full-capacity output has to be motivated in the context of our firms optimization problem.

Note that the Fed provides data on the rate of capacity utilization as well as its components, production output and full-capacity output. The questionnaire asks: “Full Production Capability - The maximum level of production that this establishment could reasonably expect to attain under normal and realistic operating conditions fully utilizing the machinery and equipment in place. In estimating market value at full production capability, consider the following […] Assume only the machinery and equipment in place and ready to operate will be utilized. Do not include facilities or equipment that would require extensive reconditioning before they can be made operable. […] Assume number of shifts, hours of plant operations, and overtime pay that can be sustained under normal conditions and a realistic work schedule. […]”

What does this imply for the firms considered here? The maximum production level that can be sustained under normal conditions may be interpreted as the maximum output which still allows profits to be positive for a given capital stock, real wage and interest rate. Hence, we shall define
Figure 2: Marginal revenues, marginal costs and average costs after a demand shock in the short run (upper panel) and long run (lower panel)

full-capacity output, $y_{c,t}$, as the level of output at which average costs equal marginal revenues for a given capital stock, real wage and interest rate and steady-state full-capacity output, $y^*_{c,t}$, as full-capacity output with the capital stock, real wage and interest rate at the steady state.

This interpretation is illustrated in Figure 2 depicting marginal revenues, marginal costs and
average costs with respect to output. We start at the steady state in time 0. Optimal price setting of the firm implies that the real wage $\omega_0$ will be such that the marginal costs are equal to marginal revenues at the level of output $y_0$. With a given real wage $\omega_0$ and capital stock $k_0$, full-capacity output, $y_{c,0}$, is where average costs are equal to marginal revenues. The rate of capacity utilization is equal to the normal rate, i.e. $v_0^* = y_0/y_{c,0}$, since the capital stock is fully adjusted. Suppose there is a permanent demand shock with output increasing to $y_1$ in time 1. With a given capital stock, price setting implies the real wage to fall to $\omega_1$ such that marginal costs cut marginal revenues at the new level of output. Average costs decrease slightly because of lower real wages for any level of output given the initial capital stock. Hence, full-capacity output increases slightly to $y_{c,1}$. Overall, the rate of capacity utilization goes up to $v_1 = y_1/y_{c,1}$. The corresponding normal utilization rate is the utilization rate after full adjustment of the capital stock, real wages and the interest rate which is achieved in time 2, i.e. $v_1^* = y_2/y_{c,2}$ corresponding to the new steady state. A rising capital stock will induce pricing to increase the real wage such that the marginal cost curve remains unchanged from time 1 to 2. The average cost curve however will now cut the marginal revenue curve at a higher level of output due to a higher real wage and a higher capital stock. Overall, the adjustment of the capital stock between time 1 and 2 will be associated with the utilization rate exceeding the normal utilization rate.

Let $v_t \equiv y_t/y_{c,t}$ and $v_t^* \equiv y_t^*/y_{c,t}^*$ where the asterisk indicates steady-state values. The previous considerations imply that a capacity utilization rate exceeding the steady-state capacity utilization rate will be associated with a positive rate of investment. Hence, the firm’s investment behavior can be approximated by

$$\frac{i_t}{k_t} = \delta + f(v_t - v_t^*)$$

with $f_\nu(\cdot) > 0$ and $f(\cdot) = 0$ for $v_t = v_t^*$. Note that the real wage (and, therefore, for a given level of productivity also the wage share) as well as the interest rate are captured in the utilization differential through their effect on capacity output. Hence, income distribution as well as monetary policy affects investment through changes in the rate of capacity utilization. Explicitly including distribution in the investment function as has become the convention in much of the Keynesian literature since Bhaduri and Marglin (1990) is not required. Note further that the interpretation of the normal rate of utilization is based on cost and profit considerations which differs from the interpretation put forward in parts of the TPK literature which emphasizes the role of idle steady-state capacity as a means to deter market entry of potential rival firms.\(^\text{16}\)

To summarize, the remarkable implication of these considerations is that all relevant information of the firm’s investment behavior captured by Tobin’s $q$ is also contained by the utilization differential as long as capacity output is properly defined. Hence, the steady-state rates of investment in the DSLMD and TPK model are the same. Yet, the dynamics out of steady-state differ since, in the DSLMD model, $f(v_t - v_t^*)$ is represented by a highly non-linear and dynamic term in Tobin’s $q$ whereas the TPK specification of investment will crudely approximate $f(v_t - v_t^*)$ by a linear function.

\(^{16}\)Interpreted in the context of the current model, the results obtained by Schoder (2012) for an analysis for US industries suggest that the steady-state rate increases after a positive output shock. That means that the steady-state capacity output increases less-than-proportional to the initial increase in demand.
2.3 Accounting, definitions and policy.

Apart from the economic behavior outlined above, the model is characterized by accounting relations and policy rules which we briefly summarize here. The crucial accounting relation is the macroeconomic balance condition stating that real expenditures are the sum of consumption, investment and government expenditures.

Fiscal policy involves government consumption and lump-sum taxes. We suppose a balanced budget at all times. Government expenditures are assumed to follow an auto-regressive process. Hence a parameter indicates the persistence of a government shock.

The monetary authority is assumed to set the interest rate according to the Taylor rule with interest smoothing. This means that the interest rate is a weighted average of the previous interest rate and the newly desired interest rate which follows from the deviation of the inflation rate from target. Note that the monetary policy rule features a secular interest prevailing at the steady-state. For a given target inflation rate, this secular rate is the interest rate which implies the inflation rate to meet the target in the steady state. Its size and interpretation will depend on the closure of the model, as discussed next. Note that credit is endogenous and determined by the investment decisions of the firms. The central bank accommodates any level of credit demand.

2.4 Model closures

Here, we consider two distinct closures of the model: the conventional Dynamic Stochastic General Equilibrium (DSGE) closure assuming labor market clearing and a new Dynamic Stochastic Labor Market Disequilibrium (DSLMD) closure which models the rate of nominal wage inflation as the outcome of a collective Nash bargaining process between workers' and firms' representatives. Let us discuss these closures in more detail.

**DSGE: labor market clearing.** Assuming labor market clearing at all times closes the above model and imposes a general equilibrium. Setting additionally the probability of permanent income loss to zero, $U = 0$, one obtains a standard medium-scale DSGE model. In that case, overall consumption will simply be consumption of the active households. Then, the consumer problem discussed above characterizing the optimal consumption choice collapse to the standard Euler equation.

Note that the DSGE closure implies a steady-state rate of interest which follows from the consumption Euler equation. It is the interest rate which makes the household indifferent of moving a marginal unit of consumption over time and has been referred to as the natural rate of interest.

The assumption of labor market clearing lends a neoclassical character to the model. Nominal wage setting ensures that labor supply equals labor demand (even if wage setting rigidities were introduced). Inputs to production are fully employed in equilibrium and output is then determined by the production function. The macroeconomic balance condition now has the interpretation of a resource constraint with total output feeding the demand components. Output is supply-driven even in the short run: Along the adjustment to the steady state, e.g. after a fiscal shock, an increase in production can only be achieved since households are willing to provide more labor because of higher real wages. Since the government consumes more output, private consumption will be crowded out.

Moreover, it is worth but not surprising to note that in the DSGE model the household’s problem does not determine the level of consumption, but inter-temporal consumption smoothing
depending on the expected interest and inflation rates. The level of consumption is determined by the resource constrained: every output not invested and consumed by the government will be consumed by the households.

**DSLMD and TPK: collective wage bargaining.** As an alternative to the assumption that the nominal wage adjusts to clear the labor market, we assume here that the rate of wage inflation is subject to a bargaining process between a workers’ and a firms’ representative. The respective return functions are crucial for the bargaining game. We take the steady-state real wage, $\bar{\omega}(\pi^w)$, as the worker’s return and the steady-state profit rate, $r(\pi^w)$, as the firm’s return. The former can be shown to increase and the latter to decrease in the rate of wage inflation. Hence, we suggest that the bargaining parties are concerned with the long-run implications of the bargaining. Nevertheless, the bargaining game is affected by the short run by assuming that the last period’s state of the labor market determines the relative bargaining power.

We consider a Nash solution to the bargaining problem which is the rate of wage inflation $\pi^w_t$ solving the joint maximization problem

$$\max_{\pi^w_t} [\bar{\omega}(\pi^w_t)]^{v_t} [r(\pi^w_t)]^{1-v_t}$$

with

$$v_t = (1 - u_{t-1}) \nu \mu_t$$

where $\nu$ is a scaling parameter and $\mu_t$ is an auto-regressive shock to the bargaining power. The first order optimality condition of this problem characterizes the evolution of the desired rate of wage inflation, $\pi^w_t$, i.e.

$$1 = (1 - 1/v_t) \frac{\bar{\omega}(\pi^w_t)}{r(\pi^w_t)} \frac{r'(\pi^w_t)}{\bar{\omega}'(\pi^w_t)}$$

The evolution of the rate of wage inflation is then assumed to be

$$\pi^w_t = \rho w \pi^w_{t-1} + (1 - \rho w) \pi^w_t.$$  

In the DSGE model, the long-term interest rate is such that households have the same detrended consumption every period for a given inflation target and is therefore referred to as the natural rate of interest. In the DSLMD model, there is no natural rate of interest. Rather, the steady-state interest rate is implied by the monetary policy rule. The only condition is that the steady-state interest rate is between the inflation target and the nominal growth rate of the economy. The former ensures that the real interest rate is positive in the long run; the latter that savings do not grow faster than the economy in the long run.

Without the assumption of labor market clearing and with consumption depending on current income through precautionary savings motives, the DSLMD model has a Keynesian character. Since labor is not fully employed and involuntary unemployment persists, the macroeconomic balance condition cannot be interpreted as a resource constraint. Rather, it is a goods market equilibrium condition stating that aggregate output needs to equal aggregate spending. Business fluctuations are demand-driven. A demand shock affects output without requiring households to provide more resources, i.e. labor, since unemployed labor can be employed. An accelerator effect is predicted
since consumption and investment move in the same direction of the demand shock. Labor market conditions then change, affect the bargaining process over wages and move the economy back to the steady state.

Note that the DSLMD model is indeterminate for \( U = 0 \), i.e. in the absence of the risk of permanent income loss. In this case, the household’s consumption Euler equation characterizes the optimal evolution of consumption over time for a given interest and inflation rate, as in the DSGE model, but not the level of consumption in the steady state. Since output is determined by aggregate spending decisions and, hence, a resource constraint is absent, which consumption can be obtained from in the DSGE model, consumption is not determinate in the steady state of the DSLMD model.

2.5 How to solve the models?

Both the DSGE and the DSLMD model are highly non-linear. To study their characteristics, the model dynamics need to be approximated in the neighborhood of the steady state. Approximation is done by computing the steady state followed by a first-order Taylor expansion around the steady state. The resulting log-linearized first-order dynamical system is then solved assuming rational expectations (DSGE and DSLMD model) and static expectations (TPK model).

Expectations. The result is a log-linearized dynamical model which we would like to express in auto-regressive form. To do so, let us collect all log-linearized state or predetermined variables in a \((n \times 1)\) vector \( X_{1,t} \). The realizations of these variables are known before the stochastic elements of the model have been realized. To be precise, a predetermined variable is a function of only variables being part of the full information set in time \( t \), hence \( E_{t}X_{1,t+1} = X_{1,t+1} \). For instance, the capital stock or wealth in period \( t+1 \) are known already in period \( t \) and are independent of economic choices or shocks in period \( t+1 \). We also collect all log-linearized jump or forward-looking variables in a \((m \times 1)\) vector \( X_{2,t} \). This vector includes all variables whose realizations are known only once the stochastic shocks have been realized. Hence, a forward-looking variable can be a function of any variable in the information set in \( t+1 \). These variables include e.g. consumption, Tobin’s \( q \), inflation, etc. Finally, we collect all exogenous variables in a \((k \times 1)\) vector \( V_{t} \). The log-linearized model can then be compactly represented as

\[
A \begin{bmatrix} X_{1,t+1} \\ E_{t}X_{2,t+1} \end{bmatrix} = B \begin{bmatrix} X_{1,t} \\ X_{2,t} \end{bmatrix} + CV_{t}
\]

where the \((n+m) \times (n+m)\) matrices \( A \) and \( B \) as well as the \((n+m) \times k\) matrix \( C \) collect the model parameters. Note that the expectation operator is not required for \( X_{1,t+1} \) since these variables are known in \( t \). Note further that so far we have not assumed rational expectations. Each line of the system of equations represented above corresponds basically to a model equation. A solution of this system of equation which includes forward-looking variables is a characterization of the model variables in only predetermined variables since the forward-looking values are not known. One simple way to solve the model is to assume rational expectations, i.e. \( E_{t}X_{2,t+1} = E_{t}(X_{2,t+1}|\Omega_{t}) \) where \( \Omega_{t} \) is the information set in time \( t \) which includes at least all past and current values of \( X_{1}, X_{2} \) and \( V \). In this case, i.e. when we assume that each economic agent knows the entire model, we can pre-multiply both sides by \( A^{-1} \) to obtain

\[
\begin{bmatrix} X_{1,t+1} \\ E_{t}X_{2,t+1} \end{bmatrix} = F \begin{bmatrix} X_{1,t} \\ X_{2,t} \end{bmatrix} + HV_{t}
\]  (8)
where $F = A^{-1}B$ and $H = A^{-1}C$. Note that one could also assume different ways of expectation formation in this type of models such as adaptive expectations (cf. Sidrauski 1967). In the TPK model variant, expectations are assumed to be backward-looking. In particular, we assume $E_t X_{2,t+1} = X_{2,t}$. In this case, (8) can be solved simply. In the DSLMD and DSGE model, however, we assume rational expectations. We want to show that Keynesian results can be produced even with these forward-looking expectations.

A solution $(X_{1,t}, X_{2,t})$ of the model is a sequence of functions of variables in the information set $\Omega_t$ which is consistent with (8). Blanchard and Kahn (1980) shows how to derive such a solution which shall be skipped here. Note that a unique solution only exists if the number of eigenvalues of $F$ outside the unit cycle is equal to the number of forward-looking variables, an assumption which holds in both of our models with the calibration chosen.

**Determinacy and the Taylor principle.** If the number of unstable eigenvalues of $F$ is lower then the number of forward-looking variables, there exists an infinite number of solutions and an indeterminacy problem arises. If it is larger, then there is no stable solution. A core implication of any DSGE model is the so-called *Taylor principle*. It says that the monetary authority needs to respond aggressively enough to deviations of the inflation rate from the target in order to achieve determinacy of the model. Only in this case, i.e. when a rise in the inflation rate provokes a rise in the interest rate such that the real interest rate rises, will $F$ exhibit enough unstable eigenvalues for the solution to be unique. Hence the response of the monetary policy instrument to a 1%-point increase in inflation typically exceeds 1 in DSGE models. Note that such an aggressive monetary policy response to inflation is not required in the DSLMD framework. In contrary, with the response parameter to inflation exceeding 1, the model would exhibit more unstable eigenvalues than forward-looking variables and no stable solution would exist. This is a remarkable result since the Taylor principle collapses with introducing a precautionary saving motive to the household’s problem (cf. also Schoder 2014d).

## 3 Impulse-response analysis

This section contrasts the proposed DSLMD model with both the TPK model and the conventional DSGE model. We first discuss the calibration of the models and subsequently study the model predictions of the macroeconomic effects of a variety of shocks. We consider three shocks: a fiscal policy shock, a monetary policy shock and a shock to the wage bargaining power. Note that despite a similar calibration, the three models imply different shares of consumption, investment and government spending in aggregate demand. Since the output response to any shock depends on the relative demand shares, the former are not comparable across models. In order to achieve the comparability of output responses, we report for the DSLMD and the TPK models the percentage deviation of output from the steady state of the DSGE model.

**Calibration.** A list of variables and parameters as well as a description can be found in Appendix A. The equations characterizing the three models considered here are discussed in Appendix B. In order to simulate the responses to macroeconomic shocks, the models need to be calibrated. Details are spilled out in Appendix C. To facilitate comparing the three models, we choose parameter values

---

17 The Dynare codes can be obtained from the author upon request.
which are as similar as possible across models. For the DSGE and DSLMD models, calibration is based on Schoder (2014d,b). A few parameters are calibrated according to conventions in the literature (cf. Smets and Wouters 2003, Carroll and Jeanne 2009). A few parameters are set in order to match averages in the data on the Euro Area between 1970 to 2009, in particular, the average growth rates of per-capita income and of the labor force. The inflation target is chosen to be consistent with a 2% annual inflation target. Given the inflation target, we set the interest rates in the DSGE models such that the steady state to exist. In the DSLMD model, the scaling parameter for labor supply has been calibrated such that the steady-state unemployment rate is $u = 0.075$. In the TPK model, labor supply has been set accordingly. The remaining parameters of the DSGE and DSLMD models are based on the estimates by Schoder (2014d,b) using Bayesian Maximum Likelihood for the Euro Area. Note that even though wage formation is highly persistent in reality, we assume in the baseline specification that the rate of wage inflation in the DSLMD model is determined by the state of the labor market only and not by its previous value. We have chosen this calibration in order to have the DSGE model as comparable as possible which does not feature wage rigidities either in the variant used. Where applicable the TPK model uses the same parameter values than the DSLMD model. The DSLMD and the TPK model share the same steady-state. Only the dynamics around the steady-state differ.

Budget-neutral fiscal policy shock in the baseline models. The impulse-response functions (IRFs) for a permanent, i.e. $\rho_G = 1$, budget-neutral government spending shock are plotted in Figure 3. We assume for the sake of simplicity that lump-sum taxes increase by the same amount as government expenditures in order to keep the public budget balanced. Let us first collect some remarkable observations with the underlying mechanism becoming clear below: First, the impact multiplier on output is larger for the DSLMD and TPK models than for the DSGE model. Second, the adjustment, especially of quantities, is much slower for the latter then for the former. Third, the long-run impact on output and its components is lowest for the TPK model. Fourth, output and consumption overshoot in the Keynesian models while adjustment is more gradual in the neoclassical model. To understand these observations let us study the transmission channels of a permanent budget-neutral fiscal expansion in each model.

In the DSGE model, the rise in government spending financed by a contraction of the households’ budget is expansionary. The positive short and long-run output multipliers ranging from 0.3 to 0.4 are due to the crowding-out of consumption. With a lower level of consumption, its marginal utility increases. Hence, households are willing to supply more labor. This affects output through the supply side. A rising demand induces firms to raise investment slowly due to capital adjustment costs. Since households in the DSGE model are concerned about life-time income and not about current income, excessive consumption smoothing implies a very smooth adjustment of quantities. Note that the nominal wage adjusts immediately to clear the labor market. A slowly but persistently rising output increases marginal costs which, given the mark-up, raise inflation above the monetary authority’s target. Despite low rates of price inflation the Taylor principle requires a strong response of the interest rate which increases and slowly returns back to the steady state. To summarize, real and nominal adjustment in the DSGE model after a permanent shock is immediate and, hence, does not generate much variation in the data.

In the TPK model, the rise in budget-neutral government spending immediately transfers funds from household’s which save part of their income to the government which spends all of it. Further, a jump in capacity utilization causes an increase in investment and hence output and consumption
through the accelerator effect. Then, two contradicting mechanisms set in which in sum cause output and consumption to decrease while investment still increases. The negative dominating effect originates in the wealth effect in the consumption function. Despite lower disposable income households still have large savings at the beginning. However, after the tax shock, their consumption-income rate exceeds one, i.e. they dissave and use up wealth for consumption until a new and lower wealth-income ratio has been reached. During the time of adjustment consumption decreases with decreasing wealth for a given propensity to consume out of wealth. The second mechanism insufficient to compensate for the erosion of disposable income is the rise in investment triggered by a rise in the rate of capacity utilization. Note that capacity utilization still increases after the shock even though output decreases. The reason for this can be found in full-capacity output which increases with the capital stock but decreases with the real wage and the interest rate. All of these variables increase: the capital stock through investment; the interest rate through the response of the monetary authority to faster price inflation which, in turn, can be explained by faster wage inflation due to labor market tightening; and the real wage also due to faster wage inflation and the presence of price adjustment costs increasing with price inflation. The investment boom is over in the TPK model, once the labor market returns to the steady state causing the interest rate and the real wage to decrease and, hence, capacity output to rise and capacity utilization to fall. In the long run, these mechanism lead to a new equilibrium of the TPK model which features a level of output, consumption and investment lower than implied by the other models.

The mechanism at work in the DSLMD model are very similar to the TPK model as can be seen by the IRFs. The crucial difference is that behavioral relations are endogenous in the former. For instance, while the marginal propensities to consume out of income and wealth are invariant to the government shock in the TPK model, they are fully endogenous in the DSLMD model and depend on the interest rate and the inflation rate. Further, while the response of investment to a utilization gap is exogenous in the TPK model, it is endogenous in the DSLMD and DSGE models and represented by Tobin’s $q$. Similar to the TPK model, the budget-neutral fiscal expansion implies a large impact effect on output and consumption which investment adjusting slowly due to capital adjustment costs. Again the impact effect is due to the transfer of partly saved funds to completely spent funds. Consumption, then, declines due to the decrease in the consumption-income ratio. This is because at a higher income active households increase saving over-proportionally in order to accumulate wealth to reach a higher long-run wealth-income ratio. Additionally the drop in the real interest rates causes the willpower sensitivity parameter to decrease. Hence, the planner has a harder time to discipline the doer and consumption goes up. The positive long-run multiplier effect on investment can be explained as follows: The higher level of output requires a higher level of capital for production. For a given depreciation rate, break-even investment will be higher at a higher capital stock.

Finally note that wages are perfectly flexible in all model variants. In the DSGE model, nominal wages adjust immediately to clear the labor market. In the DSLMD and TPK models, collective wage bargaining fully incorporates labor market conditions with a lag of one period. Below we will study how the DSLMD and TPK models respond to shocks when wage inflation is independent of the state of the labor market as well as when the adjustment of wage inflation exhibits inertia.

**Monetary policy shock in the baseline models.** The effects of a temporary contractionary monetary policy shock are plotted in Figure 4 for the baseline DSLMD, TPK and DSGE models. In the TPK model used here, contractionary monetary policy has a positive impact effect on demand
and its components. Investment increases due to the raise in capacity utilization which results from a drop in capacity output due to higher interest rates. Through the accelerator effect, this feeds into higher consumption and output. We shall not put too much emphasis on this result since TPK models analyzing monetary policy effects typically capture negative interest rate effects in the investment function.

Let us focus instead on comparing the DSLMD and DSGE models. The effectiveness of monetary policy is higher in the former than in the latter as can be seen by the responses of output, consumption and investment. Excessive consumption smoothing as observed in the DSGE model is prevented in the DSLMD model by the inner conflict between the doer and the planner. A higher real interest rate increases the bad conscience experienced by the doer for a given level of willpower and consumption. This induces the doer to reduce consumption. Through an accelerator effect on investment, output is therefore very sensitive towards changes in the interest rate. Note however that this result depends strongly on $\phi_\theta$. Without an inner conflict, i.e. with the sensitivity parameter $\theta_t$ being independent of the real interest rate, the effect of monetary policy on output

Figure 3: Responses to a 1% permanent budget-netural government spending shock in the DSLMD (blue-solid), PK (green-dashed) and DSGE (red-dotted) model as deviations from the steady state.
is much smaller. This is because, in this case, consumption responds positively to a rise in the interest rate, which at first sight is a puzzling result. It is due to the precautionary saving motive of the household. A rise in the interest rate causes the equilibrium wealth-income ratio to decrease. Moreover, the lower level of wealth can be reached with lower foregone consumption, i.e. saving, due to higher interest rates. Hence, the consumption-income ratio goes up with rising interest rates as can be seen in the Figure.

Another difference between the DSGE and DSLMD responses is that the immediate negative impact on output and the demand components is succeeded by an overshooting after the recovery in the latter. This is due to the falling wage inflation rate which also overshoots during the recovery due to tightening labor markets. Note that a rising real wage reduces output due to higher marginal costs and lower investment.

Bargaining power shock in the baseline models. A core question in Keynesian economic analysis is how income distribution affects economic activity—a debate known as the profit-led...
vs. wage-led dichotomy. Much of this literature suggests that income distribution is an exogenous variable or policy instrument.\textsuperscript{18} Typically, the effect of an exogenous shift in the income distribution is the subject of analysis. Within our dynamic framework the dichotomy of wage-led/profit-led demand is not straightforward. With endogenous distribution, it is not evident what the appropriate policy variable to be shocked should be. Here, we consider a permanent increase in the workers’ collective wage bargaining power $\mu_t$ to an extent which corresponds to an increase of the bargaining power of 1 percentage point. Since the rate of nominal wage inflation as such is not a policy variable but endogenous, this exercise is the micro-founded equivalent of the discussion of profit-led vs. wage-led demand regimes.

The responses to a permanent bargaining power shock are plotted in Figure 5 for the baseline specifications of the DSLMD and TPK models. Since the propensity to save out of wage income is not lower than out of profit income in our model and since wages are a cost of production, one may suspect our model to be profit-led. While aggregate demand and its components decrease in the long run, the rise in the bargaining power is slightly expansionary in the short run. What is the transmission mechanism? In both models, the rise in the wage bargaining power accelerates wage inflation immediately for a given unemployment rate. Then the two stories take different paths. In the DSLMD model, the fall in the real interest rate causes an expansion of consumption through the inner-conflict of the individual. By the extent the real interest rate recovers consumption decreases even though the consumption-income ratio goes up. The rise in the real interest rate is triggered by the fall in the rate of wage inflation which results from deteriorating conditions on the labor market.

In the TPK model, the decline in output and consumption is slowed down by a temporary investment boom. It originates in the drop of capacity output which, in turn, results from faster wage inflation causing real wages and interest rates to go up.

Both models predict that in the long run the rise in the bargaining power did not affect the real wage or the rate of wage inflation. Yet, unemployment has increased and demand decreased. Note that this result has to be interpreted with caution since many simplifying assumptions have been made in order to keep the core model mechanisms tractable. Relaxing the assumption of household homogeneity by considering a rich and a poor household with different propensities to save may well turn our economy wage-led.

The role of labor-market feedback. In many static TPK models, distribution is assumed to be exogenous (cf. Bhaduri and Marglin 1990, Stockhammer 2006, Hein 2007). This corresponds to the case of no labor-market feedback to the wage formation process in our DSLMD and TPK framework. Figure 6 plots the IRFs for a 1% permanent and budget-neutral increase in government spending for the DSLMD and TPK models with baseline specifications as well as a constant nominal wage inflation set at the steady-state level and independent of labor market conditions. Comparing for each model the IRFs of the two scenarios illustrates the crucial role which the feedback of labor-market conditions on wage formation plays. Overall, a permanent budget-neutral increase in government spending of 1% causes output to increase permanently by 0.6% in both models. This is considerably higher than in the case of labor-market feedback to wage formation. Note that the rate of unemployment drops accordingly. The immediate reason for the permanent expansionary effect is that consumption is not crowded out. This, in turn, is because of the evolution of the real

\textsuperscript{18}See, for instance, Naastepad and Storm (2007), Stockhammer et al. (2009), Hein and Vogel (2008), Stockhammer and Stehrer (2011) and Onaran and Galanis (2012).
wage. With labor-market feedback, raising real wages cut into profits and reduce economic activity due to the profit-led character of our economy. Without, the real wage decreases in the DSLMD and stays constant in the TPK model.

The role of the labor market for cyclicality. Empirically, there seems to exist strong evidence for Goodwin-type of cycles with the wage share following utilization which has been observed for the US by Barbosa-Filho and Taylor (2006) and Zipperer and Skott (2010) and for European economies by Flaschel (2009). This is because high rates of utilization implying low unemployment and strong trade unions tend to cause profit margins to go up rather than down (cf. Steindl 1979, Kurz 1994). So far, our Keynesian models have not been specified to generate such a cyclical adjustment to shocks. Here, we want to argue that our micro-founded DSLMD model is able to do so. Figure 7 plots the IRFs for a 1% temporary and budget-neutral increase in government spending for the DSLMD model with baseline specification ($\rho_w = 0$) and a highly persistent wage setting ($\rho_w = 0.95$). Introducing persistence in the wage formation process by raising the auto-regressive
part of the wage inflation, causes the economy to readjust to the steady state in cycles. In this case, expansionary fiscal policy immediately reduces unemployment which, now, does not immediately trigger an upward adjustment of the wage inflation rate. Rather, it increases only gradually. Price inflation rises causing the real wage to decrease contributing to the boom phase. Wage inflation accelerates eventually raising the real wage and cutting into consumption and investment. Output starts to fall with real wages rising further and causing output to undershoot at the trough. These profit-squeeze dynamics are equivalent to the findings of Taylor (2012) and Schoder (2014c) using aggregative Keynesian models.

4 Concluding remarks

In the present paper we have sketched a Dynamic Stochastic Labor-Market Disequilibrium (DSLMD) model with involuntary unemployment due to insufficient aggregate demand. The model
features the Keynesian principle of effective demand according to which output is determined by aggregate spending decisions, investment creates the corresponding savings through an adjustment of aggregate income and unemployment results from a lack of aggregate demand. Additionally, the behavioral relations have been derived from inter-temporal optimization problems of the economic agents as standard in the modern macroeconomic literature.

We have started from a medium-scale Dynamic Stochastic General Equilibrium (DSGE) model with firm-specific capital and dropped the assumption of labor-market clearing. Instead, the nominal wage has been taken as a policy variable subject to collective bargaining. Then, the resource constraint turns into a traditional goods market equilibrium condition as known from the textbook IS-LM model. This implies, however, that the model becomes indeterminate since, in DSGE models, consumption is obtained from the resource constraint. To solve that problem, the household’s problem is changed by introducing an uninsurable risk which causes households to accumulate precautionary savings which turn out to be proportional to income. This implies a Keynes-type of
consumption function which makes the model determinate. Additionally, the individual’s choice of consumption and saving is modeled as an inner conflict between two selves. This implies an inverse relationship between consumption and the interest rate.

The DSLMD model has then been compared to DSGE and Traditionan Post-Keynesian (TPK) models regarding the predictions of responses to fiscal policy and monetary policy shocks as well as to shocks to the bargaining relationship between workers and firms. We obtain the following results: (a) The Taylor principle is a condition for the existence of a unique solution only for the DSGE but not for the DSLMD or TPK model. (b) Despite the different natures of the DSGE model on the one hand and the DSLMD and TPK models on the other, the responses to shocks, especially to temporary ones, are rather similar. This is mainly because of the labor market feedback in the Keynesian models according to which decreasing unemployment causes wage inflation to accelerate. If this feedback mechanism is weak, the response of quantities to shocks is more pronounced in the Keynesian models than in the DSGE model. (c) The multiplier effects of permanent government spending shocks are more pronounced in the DSLMD and TPK models than in the DSGE model especially when the feedback of the labor market to wage formation is weak. (d) Because of the assumption of a propensity to save independent of the source of income, a rise in wage inflation causes a contraction in the DSLMD and TPK models. (e) This allows for Goodwin type of cycles if the wage formation responds sluggishly to changes in the labor market conditions.

The present paper has shown that a fundamentally Keynesian economy can be characterized by a set of micro-foundations consistent with mainstream methodology. We do not claim that the proposed micro-foundation is the only viable one. In particular, the formation of expectations should be based on a more realistic footing in future research. Yet, explicitly anchoring behavioral relations in goal-oriented considerations of the economic agents can no longer be seen as an obstacle for Keynesian analysis. Quite the contrary, modeling economic behavior rather than postulating it comes with considerable benefits for Keynesian macroeconomics: First, the methodological inconsistency arising from using a modeling approach to explain macro-phenomena but a verbal approach to explain micro-phenomena can be overcome by consistently modeling economic behavior as well as its interaction. Second, the parameters characterizing behavioral rules in TPK models are highly endogenous to policy which has been known since Lucas (1976) but vehemently played down by the TPK mainstream. Neglecting Lucas’ critique may well have been fueled by the fear among Keynesians that the mainstream rational-expectations, general-equilibrium solution is a necessary implication of his objection. As has been argued in the present paper, however, this fear is ill-founded. The framework proposed demonstrates one possible way of how to address Lucas’ critique within the framework of Keynesian economic analysis.

References


# A Description of variables and parameters

<table>
<thead>
<tr>
<th>Variables</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y_t$</td>
<td>Real aggregate output</td>
</tr>
<tr>
<td>$Z_t$</td>
<td>Real aggregate household income net of taxes, depreciation and adjustment costs</td>
</tr>
<tr>
<td>$C_t$</td>
<td>Real aggregate consumption</td>
</tr>
<tr>
<td>$I_t$</td>
<td>Real aggregate investment</td>
</tr>
<tr>
<td>$G_t$</td>
<td>Real government spending</td>
</tr>
<tr>
<td>$K_t$</td>
<td>Real aggregate capital stock</td>
</tr>
<tr>
<td>$T_t$</td>
<td>Real aggregate lump-sum taxes</td>
</tr>
<tr>
<td>$C^a_t$</td>
<td>Real aggregate consumption of active households</td>
</tr>
<tr>
<td>$C^i_t$</td>
<td>Real aggregate consumption of inactive households</td>
</tr>
<tr>
<td>$B^a_t$</td>
<td>Real aggregate wealth of active households</td>
</tr>
<tr>
<td>$B^i_t$</td>
<td>Real aggregate wealth of inactive households</td>
</tr>
<tr>
<td>$Y_{c,t}$</td>
<td>Capacity output</td>
</tr>
<tr>
<td>$E_t$</td>
<td>Labor embodied productivity</td>
</tr>
<tr>
<td>$L_t$</td>
<td>Labor demand</td>
</tr>
<tr>
<td>$N_t$</td>
<td>Aggregate labor supply</td>
</tr>
<tr>
<td>$W_t$</td>
<td>Nominal wage per unit of labor</td>
</tr>
<tr>
<td>$P_t$</td>
<td>Price level in the economy</td>
</tr>
<tr>
<td>$p_t$</td>
<td>Price level of the individual firm</td>
</tr>
<tr>
<td>$\theta_t$</td>
<td>Willpower sensitivity parameter</td>
</tr>
<tr>
<td>$x_t$</td>
<td>Willpower</td>
</tr>
<tr>
<td>$i_t$</td>
<td>Nominal interest rate</td>
</tr>
<tr>
<td>$\pi_t$</td>
<td>Rate of price inflation</td>
</tr>
<tr>
<td>$\pi_t^*$</td>
<td>Rate of nominal wage inflation net of labor embodied productivity growth</td>
</tr>
<tr>
<td>$\pi_t^{**}$</td>
<td>Bargaining agreement on the rate of nominal wage inflation net of labor embodied productivity growth</td>
</tr>
<tr>
<td>$s_t$</td>
<td>Scaling variable</td>
</tr>
<tr>
<td>$\tau_t$</td>
<td>Non-distortionary transfer between households</td>
</tr>
<tr>
<td>$\epsilon_t$</td>
<td>Number of employed households</td>
</tr>
<tr>
<td>$\omega_t$</td>
<td>Real wage per unit of labor</td>
</tr>
<tr>
<td>$\rho_{t-1,t}$</td>
<td>Stochastic discount factor</td>
</tr>
<tr>
<td>$v_t$</td>
<td>Rate of capacity utilization</td>
</tr>
<tr>
<td>$\phi_t$</td>
<td>Real marginal costs</td>
</tr>
<tr>
<td>$u_t$</td>
<td>Unemployment rate</td>
</tr>
<tr>
<td>$r_t$</td>
<td>Profit rate</td>
</tr>
<tr>
<td>$\mu_t$</td>
<td>Autoregressive shock to the workers’ relative bargaining power</td>
</tr>
<tr>
<td>$v_t$</td>
<td>Workers’ relative bargaining power</td>
</tr>
<tr>
<td>$\epsilon_{G,t}$</td>
<td>Innovation to real government spending</td>
</tr>
<tr>
<td>$\epsilon_{i,t}$</td>
<td>Innovation to the interest rate</td>
</tr>
<tr>
<td>$\epsilon_{\mu,t}$</td>
<td>Innovation to the workers’ relative bargaining power</td>
</tr>
</tbody>
</table>
Parameters

- \( \phi_p \): Elasticity of the willpower sensitivity parameter w.r.t. deviations of the interest rate from the steady state
- \( \beta \): Household’s discount rate
- \( \rho \): Inverse of the elasticity of intertemporal substitution of consumption
- \( \psi \): Scaling parameter of labor supply
- \( \eta \): Inverse of the Frisch labor supply elasticity
- \( \gamma \): Growth rate of labor-embodied productivity
- \( \kappa \): Growth rate of population
- \( \Gamma \): Growth factor of labor-embodied productivity and population
- \( D \): Probability of death
- \( U \): Probability of dropping out of the labor force
- \( \kappa \): Steady-state consumption-wealth ratio of inactive households
- \( c_z \): Marginal propensity to consume out of income
- \( c_b \): Marginal propensity to consume out of wealth
- \( \pi^* \): Target inflation rate of monetary authority
- \( \delta \): Steady-state rate of capital depreciation
- \( \epsilon_p \): Inverse of the elasticity of substitution minus one
- \( \lambda \): Steady-state debt-capital ratio
- \( \tau_p \): Price adjustment cost scaling parameter
- \( \tau_i \): Investment adjustment cost scaling parameter
- \( \alpha \): Output elasticity of capital
- \( \phi_i \): Utilization effect on investment
- \( i \): Long-run Interest rate target of the monetary authority
- \( \phi_x \): Elasticity of the interest rate w.r.t. inflation
- \( \phi_y \): Elasticity of the interest rate w.r.t. output
- \( \nu \): Bargaining power scaling parameter
- \( \rho_G \): Persistence of a government spending shock
- \( \rho_i \): Persistence of an interest rate shock
- \( \rho_w \): Persistence of a workers’ relative wage bargaining power shock
B Model appendix

This appendix derives all aggregated model equations characterizing our DSLMD and DSGE economy. The corresponding model equations for the TPK model will be stated along the way. Let $\Gamma \equiv (1 + n)(1 + \gamma)$ denote the deterministic growth factor of the economy. Then, we use the following notation: $\dot{X}_i \equiv \frac{X_i}{\Delta t}$. An exception is wealth: $\tilde{B}_t \equiv \frac{B_t}{P_t}$. Note that a description of the variables can be found in Appendix A.

B.1 Households

Willpower sensitivity. The willpower sensitivity, $\theta$, is specified as

$$\theta_t = \left( \frac{1 + \frac{\rho_t + \pi_t}{\kappa}}{1 + \frac{\rho_t + \pi_t}{\kappa} - 1} \right) \phi_t. \tag{B.1}$$

Inactive households. Let us now derive the inactive household’s first order conditions (FOCs) and budget constraint. Newly inactive households obtain a consumption path from an insurance company. The problem reads

$$\max_{c_{t+1}^i, b_{t+1}^i, P_{t+1}^i} \sum_{j=0}^{\infty} \beta^{1+j} (1 - D)^{(c_{t+1}^i + 1)\rho_j} - \frac{1}{1 - \rho} c_{t+1}^i + b_{t+1}^i = \frac{1 + \frac{\rho}{\kappa}}{1 - D} (-c_{t+1}^i + b_{t+1}^i).$$

The Lagrangian characterizing this problem is

$$L = E \sum_{j=0}^{\infty} \beta^{1+j} (1 - D)^{(c_{t+1}^i + 1)\rho_j} + \lambda_{t+1} \left( \frac{1 + \frac{\rho}{\kappa}}{1 - D} (-c_{t+1}^i + b_{t+1}^i) - b_{t+1}^i \right).$$

The FOC w.r.t. consumption is

$$c_{t+1}^i - \rho = \lambda_{t+1} \frac{1 + \frac{\rho}{\kappa}}{1 - D} \lambda_{t+1}.$$

and w.r.t. wealth is

$$\lambda_t = \beta (1 - D) \frac{1 + \frac{\rho}{\kappa}}{1 - D} \lambda_{t+1}.$$

Combining the two FOCs leads to

$$c_{t+1}^i - \rho = \beta c_{t+1}^i \left( \frac{1 + \frac{\rho}{\kappa}}{1 - D} \right) \left( \frac{1 + \frac{\rho}{\kappa}}{1 - D} \right)$$

$$c_{t+1}^i = \beta \frac{1 + \frac{\rho}{\kappa}}{1 - D} c_{t+1}^i$$

$$c_{t+1}^i = \left( \frac{1 + \frac{\rho}{\kappa}}{1 - D} \right) c_{t+1}^i$$

The budget constraint can be rearranged and iterated forward and, then, the previous result can be used to get

$$\frac{b_t^i}{P_t} = \left( \frac{1 + \frac{\rho}{\kappa}}{1 - D} \right) \left( \frac{1 + \frac{\rho}{\kappa}}{1 - D} \right) + c_t^i$$

$$= \frac{1 + \frac{\rho}{\kappa}}{1 - D} \left( \frac{1 + \frac{\rho}{\kappa}}{1 - D} \right) + c_t^i$$

$$= \sum_{n=0}^{\infty} \left( \frac{1 + \frac{\rho}{\kappa}}{1 - D} \right)^n \beta (1 - D)^n c_{t+n}^i$$

$$= \sum_{n=0}^{\infty} \left( \frac{1 + \frac{\rho}{\kappa}}{1 - D} \right)^n \beta (1 - D)^n \left( \frac{1 + \frac{\rho}{\kappa}}{1 - D} \right) c_{t+n}^i$$

$$= \frac{1 + \frac{\rho}{\kappa}}{1 - D} c_t^i$$

$$\tilde{C}_t^i = \kappa \tilde{B}_t^i \tag{B.2}$$
The Lagrangian characterizing this problem is that wealth is distributed equally across each active household at any point in time which facilitates aggregation. The FOCs w.r.t. consumption and wealth are

\[ \tau_s \] and \[ \tau \] inactive tomorrow. Hence, we get

Active households. The active household’s problem reads

\[ \beta^t \left( \frac{c^t}{\theta_t} - \lambda_t (1 + i_{t-1}) \right) = 0 \]

\[ \lambda_t (1 + i_{t-1}) = \frac{(c^t)^{-\rho}}{\theta_t} \]

and

\[ -\beta \lambda_t \frac{1}{P_t} + (1 - U) \beta^{t+1} \lambda_{t+1} (1 + i_t) \frac{1}{P_{t+1}} + U \beta^{t+1} \lambda_{t+1} (1 + i_t) \frac{1}{P_{t+1}} = 0 \]

\[ \lambda_t = \beta E_t \lambda_{t+1} \frac{1 + i_t}{1 + \pi_{t+1}} \]

respectively. Now, substitute out \( \lambda_t \) from the second FOC and note that there are two expected states in the next period. Note that we have assumed a transfer between newborn and non-newborn households which ensures that each household has the same wealth which will facilitate aggregation. Let \( \xi_t = \frac{(1+n)^{i_{t+1}}}{n} \) be number of employed households growing at factor \( 1 + n \). Note that the deterministic growth factor is \( \Gamma = (1 + n)(1 + \gamma) \). The aggregate detrended Euler equation for consumption can be obtained as

\[ \frac{1}{1 + i_{t-1}} \left( \frac{(c^t)^{-\rho}}{\theta_t} \right) = \beta \frac{1 + i_t}{1 + \pi_{t+1}} \left( (1 - U) \frac{1}{1 + i_t} \left( \frac{(c^t_{t+1})^{-\rho}}{\theta_{t+1}} \right) + U \frac{1}{1 + \pi_{t+1}} (C^t_{t+1})^{-\rho} \right) \]

\[ \frac{(c^t)^{-\rho}}{\theta_t} = \beta \frac{1 + i_{t-1}}{1 + \pi_{t+1}} \left( (1 - U) \left( \frac{(c^t_{t+1})^{-\rho}}{\theta_{t+1}} \right) + U (C^t_{t+1})^{-\rho} \right) \]

\[ \frac{C^t}{\theta_t} = \beta \frac{1 + i_{t-1}}{1 + \pi_{t+1}} \left( (1 - U) \left( \frac{C^t_{t+1}}{1 + \pi_{t+1}} \right) + U (C^t_{t+1})^{-\rho} \right) \]

\[ \frac{(c^t)^{-\rho}}{\theta_t} = \beta \frac{1 + i_{t-1}}{1 + \pi_{t+1}} \left( (1 - U) \left( \frac{(c^t_{t+1})^{-\rho}}{\theta_{t+1}} \right) + U (C^t_{t+1})^{-\rho} \right) \]

Let us derive the active household’s aggregate budget constraint. As stated in the budget constraint above, we assume a transfer which ensures that every active household has the same wealth-income ratio which facilitates aggregation.

34
After the transfer the individual wealth is \((1 - \tau)\frac{w^*}{P_t}\) for both non-newborn and newborn households. The aggregate wealth is still the same. How to aggregate the budget constraint of the active households? Note that the overall wealth saved today for tomorrow by active household will be divided in tomorrow’s wealth of active households and tomorrows wealth of newly inactive households. Recall further that the aggregation of the individual wealth, \((1 - \tau)\frac{w^*}{P_t}\), is \(\frac{w^*}{P_t}\). The aggregate budget constraint normalized by aggregate households net income can be obtained by

\[
\frac{b^{a}_{t+1}}{P_{t+1}} = (1 + i_{t-1})\left( \frac{wa}{P_t}n_t + \pi^a_t - t_t - c^a_t + (1 - \tau)\frac{b^a_t}{P_t} \right)
\]

\[
\frac{b^{n}_{t+1}}{P_{t+1}} = (1 + \pi^a_{t+1}) = (1 + i_{t-1})(z_t - c^a_t + (1 - \tau)\frac{b^a_t}{P_t})
\]

\[
\frac{b^{n}_{t+1}}{P_{t+1}}(1 + \pi^a_{t+1}) \Xi_t = (1 + i_{t-1})(z_t \Xi_t - c^a_t \Xi_t + (1 - \tau)\frac{b^a_i}{P_i} \Xi_t)
\]

\[
\left( \frac{B^a_{t+1}}{P_{t+1}} + \frac{B^n_{t+1}}{P_{t+1}} \right)(1 + \pi^a_{t+1}) = (1 + i_{t-1})(Z_t - C^a_t + \frac{B^n_t}{P_t})
\]

\[
\bar{B}_{t+1} = \frac{1 + i_{t-1}}{1 + \psi_B} \frac{1}{\psi} \left( \frac{\bar{Z}_t}{\psi} - \bar{C}_t + \hat{B}_t \right)
\]

where

\[
\bar{Z}_t = \bar{Y}_t - \left( \bar{I_t} + \frac{\tau}{2} (\bar{I}_{t-1} - \bar{I}_t) \right) \bar{I}_t + \bar{\delta}_t - \bar{T}_t
\]

is the active households' detrended aggregate real net income. Consumption over effective labor input can simply be obtained by

\[
\bar{C}_t = \bar{C}_1 + \bar{C}_i.
\]

The FOC of the active household w.r.t. labor supply is

\[
\beta_t \left( - s^1 - \psi n^a_t + \lambda_t (1 + i_{t-1}) \frac{w_t}{P_t} \right) = 0
\]

\[
s^1 - \psi n^a_t = \lambda_t (1 + i_{t-1}) \frac{w_t}{P_t}
\]

Note that \(\lambda_t\) is not stationary and growing at rate \(\gamma\) in the steady state. Yet, \(s_t\) is growing at the same rate compensating the growth in \(\lambda_t\). Substituting out \(\lambda_t\) from the third FOC, and noting that \(N_t = n_t \Xi_t\), \(\tilde{N}_t = \frac{N_t}{\Xi_t}\), \(\tilde{w}_t = \frac{w_t}{(1 + \gamma)\Xi_t}\), and \(C^a_t = \tilde{c}_t \Xi_t\) and recalling that \(s_t = (1 + \gamma)^t\) yields

\[
s^1 - \psi n^a_t = \left( \frac{c^a_t}{\theta_t} \right)^{\psi n^a_t} \frac{w_t}{P_t}
\]

\[
s^1 - \psi \left( \frac{N_t}{\Xi_t} \right)^{\psi} = (\frac{C^a_t}{\theta_t})^{\psi} \frac{w_t}{P_t}
\]

\[
s^1 - \psi \tilde{N}^a_t = \left( \frac{C^a_t (1 + \gamma)^t}{\theta_t} \right)^{\psi} \frac{w_t (1 + \gamma)^t}{P_t}
\]

\[
s^1 - \psi \tilde{N}^a_t = \left( \frac{C^a_t (1 + \gamma)^t}{\theta_t} \right)^{\psi} \tilde{w}_t
\]

\[
\psi \tilde{N}^a_t = \left( \frac{C^a_t}{\theta_t} \right)^{\psi} \tilde{w}_t
\]

**Households in the TPK model.** In the TPK model, (B.1)-(B.6) and (B.8) are replaced by a consumption function, the law of motion of wealth and a constant labor supply which are specified as

\[
\bar{C}_t = c_0 \bar{Z}_t + c_0 \bar{B}_t,
\]

\[
\bar{B}_{t+1} = \frac{1 + i_{t-1}}{1 + \psi_B} \frac{1}{\psi} (\bar{Z}_t - \bar{C}_t + \bar{B}_t)
\]

and

\[
\bar{N}_t = \bar{N},
\]

respectively.
B.2 Firms

**Final good firms.** Taken as given price $p_{t,t}$, its demand for the intermediate good $y_{t,t}$ can be obtained from the following cost minimization problem:

$$
\min_{y_{t,t}} \int_{0}^{1} p_{t,t} y_{t,t} \, dt \\
\text{s.t. } Y_{t} = \left( \int_{0}^{1} \frac{1}{y_{t,t}} \, dt \right)^{1+\epsilon_{p}},
$$

where $1 + 1/\epsilon_{p} > 1$ is the elasticity of substitution. Noting that the Lagrangian multiplier of the constraint is equal to the aggregate price index, $P_{t}$, one can show the FOC to read

$$
y_{t,t} = \left( \frac{p_{t,t}}{P_{t}} \right)^{-(1+1/\epsilon_{p})} Y_{t}.
$$

**Intermediate good firms.** Taking as given sales, the price level, the capital stock, and the wage rate well as the law of motion of capital, the production function and the demand function for intermediate goods, the firm chooses $\{p_{t+t}, l_{t+t}, i_{t+t}, k_{t+t+1}\}$ to maximize inter-temporal profits. The optimization problem reads

$$
\max_{p_{t+t}, l_{t+t}, i_{t+t}, k_{t+t+1}} \mathbb{E}_{t} \sum_{j=0}^{\infty} \frac{P_{t}}{P_{t+j}} m_{t,t+j} \left( p_{t+t} y_{t+t} - W_{t+t+l_{t+t}} - P_{t+t} i_{t+t} - \lambda P_{t+t} i_{t+t-1} k_{t+t} - \right) \\
\text{s.t. } k_{t+t+1} = \left( 1 - \frac{\pi_{t+t}}{2} \left( \frac{i_{t+t}}{t_{t+t-1}} - \Gamma \right)^{2} + \delta k_{t+t} \right) \\
y_{t+t} = \left( p_{t+t} \right)^{1-\alpha} \left( \frac{P_{t+t}}{P_{t+t,j}} \right) \\
y_{t+t} = \left( p_{t+t} \right)^{-(1+1/\epsilon_{p})} Y_{t+t}
$$

where

$$
m_{t,t+j} \equiv \beta \mathbb{E}_{t} u_{j} \left( c_{t+t+1}, c_{t+t-1}, \theta_{t+t}, m_{t+j} \right) = \mathbb{E}_{t} \prod_{s=1}^{j} \frac{1 + \pi_{t+s}^{p}}{1 + \pi_{t+s-1}^{p}} \frac{1 + i_{t+s-1}}{1 + u_{t+s-1}}
$$

is the stochastic discount factor which expresses the value of a unit real profit in time $t + j$ in terms of the value of a unit real profit in time $t$. The Lagrangian of the intermediate good firm choosing $p_{t+t}, k_{t+t+1}, i_{t+t}, l_{t+t}, u_{t+t}$ is after substituting $y_{t+t} = \left( \frac{p_{t+t}}{P_{t+t}} \right)^{-(1+1/\epsilon_{p})} Y_{t+t}$ into the objective function

$$
L = \mathbb{E}_{t} \sum_{j=0}^{\infty} \frac{P_{t+j}}{P_{t+t}} m_{t,t+j} \left( p_{t+t} \left( \frac{P_{t+j}}{P_{t+t}} \right)^{-(1+1/\epsilon_{p})} Y_{t+t} - W_{t+t+l_{t+t}} - P_{t+t} i_{t+t} - \right) \\
- \frac{1}{\epsilon_{p}^{2}} \left( \frac{p_{t+j}}{P_{t+t}} - 1 \right)^{2} P_{t+j} Y_{t+t} - \lambda P_{t+t} i_{t+t-1} k_{t+t} + \\
+ P_{t+t} \left( \left( 1 + \gamma \right)^{t+t} l_{t+t} \right)^{1-\alpha} \left( \frac{p_{t+t}}{P_{t+t}} \right)^{-(1+1/\epsilon_{p})} Y_{t+t} + \\
+ P_{t+t} q_{t+t} \left( 1 - \frac{\pi_{t+t}}{2} \left( \frac{i_{t+t}}{t_{t+t-1}} - \Gamma \right)^{2} + \delta k_{t+t} - k_{t+t+1} \right)
$$

36
The first order conditions are

\[
\frac{\partial L}{\partial p_t} = 0
\]

\[
\frac{1}{\epsilon_p} \dot{y} = -\frac{1}{\epsilon_p} \tau_p \left( \frac{P_t}{P_{t-1}} - 1 \right) \frac{1}{P_{t-1}} Y_t - P_t \varphi_t \frac{1 + \epsilon_p \gamma_t}{\epsilon_p} \gamma_t + \frac{P_t}{P_{t-1}} m_{t+1,t} + \frac{1}{\epsilon_p} \tau_p \left( \frac{P_{t+1}}{P_t} - 1 \right) \frac{P_{t+1}}{P_t^2} P_{t+1} Y_{t+1}
\]

\[
1 = -\tau_p \left( \frac{P_t}{P_{t-1}} - 1 \right) \frac{1}{P_{t-1}} P_t - P_t \varphi_t (1 + \epsilon_p) \frac{1}{P_t} + \frac{P_t}{P_{t-1}} m_{t+1,t} \tau_p \left( \frac{P_{t+1}}{P_t} - 1 \right) \frac{P_{t+1}}{P_t} P_{t+1} Y_{t+1}
\]

\[
1 = (1 + \epsilon_p) \varphi_t - \tau_p \left( \frac{P_t}{P_{t-1}} - 1 \right) \frac{P_t}{P_{t-1}} + m_{t+1,t} \tau_p \left( \frac{P_{t+1}}{P_t} - 1 \right) \frac{P_{t+1}}{P_t} P_{t+1} Y_{t+1}
\]

\[
1 = (1 + \epsilon_p) \varphi_t - \tau_p \left( \frac{P_t}{P_{t-1}} - 1 \right) \frac{P_t}{P_{t-1}} m_{t+1,t} \tau_p \left( \frac{P_{t+1}}{P_t} - 1 \right) \frac{P_{t+1}}{P_t} P_{t+1} Y_{t+1}
\]

\[
1 = (1 + \epsilon_p) \tau_p \left( \frac{P_t}{P_{t-1}} - 1 \right) \frac{P_t}{P_{t-1}} m_{t+1,t} \tau_p \left( \frac{P_{t+1}}{P_t} - 1 \right) \frac{P_{t+1}}{P_t} P_{t+1} Y_{t+1}
\]

\[
1 = (1 + \epsilon_p) \tau_p \left( \frac{P_t}{P_{t-1}} - 1 \right) \frac{P_t}{P_{t-1}} m_{t+1,t} \tau_p \left( \frac{P_{t+1}}{P_t} - 1 \right) \frac{P_{t+1}}{P_t} P_{t+1} Y_{t+1}
\]

\[
1 = (1 + \epsilon_p) \tau_p \left( \frac{P_t}{P_{t-1}} - 1 \right) \frac{P_t}{P_{t-1}} m_{t+1,t} \tau_p \left( \frac{P_{t+1}}{P_t} - 1 \right) \frac{P_{t+1}}{P_t} P_{t+1} Y_{t+1}
\]

where

\[
m_{t-1,t} = \frac{1 + \pi_t^p}{1 + \tau_t-2}
\]

\[
\frac{\partial L}{\partial i_t} = 0
\]

\[
-w_0 + P_t \varphi_t (1 + \gamma) (1 - \alpha)_{(1 - \alpha)_{t-1}^k_t} = 0
\]

\[
-w_1 + P_t \varphi_t (1 + \gamma) (1 - \alpha)_{(1 - \alpha)_{t-1}^k_t} = 0
\]

\[
\varphi_t = \frac{w_t}{P_t} \frac{1}{(1 + \gamma)^{1 - \alpha} (1 - \alpha)} \frac{1}{(1 + \gamma)^{1 - \alpha}} \frac{k_t}{1 - \alpha} \left( \frac{k_t}{1 - \alpha} \right)^{-\alpha}
\]

\[
\varphi_t = \frac{w_t}{P_t} \frac{1}{(1 + \gamma)^{1 - \alpha} (1 - \alpha)} \frac{1}{(1 + \gamma)^{1 - \alpha}} \left( \frac{k_t}{1 - \alpha} \right)^{-\alpha}
\]

\[
\varphi_t = \omega_t \frac{1}{(1 + \gamma)^{1 - \alpha} (1 - \alpha)} \left( \frac{k_t}{1 - \alpha} \right)^{-\alpha}
\]

\[
\frac{\partial L}{\partial i_t} = 0
\]

\[
1 = q_t \left( 1 - \tau_t \left( \frac{I_t}{I_{t-1}} - 1 \right) \right)^2 - \tau_t \left( \frac{I_t}{I_{t-1}} - 1 \right) + E_t m_{t+1,t} \tau_t \left( \frac{I_{t+1}}{I_t} - 1 \right) \left( \frac{I_{t+1}}{I_t} - 1 \right)^2
\]

\[
1 = q_t \left( 1 - \tau_t \left( \frac{I_t}{I_{t-1}} - 1 \right) \right)^2 - \tau_t \left( \frac{I_t}{I_{t-1}} - 1 \right) \left( \frac{I_{t+1}}{I_t} - 1 \right) \left( \frac{I_{t+1}}{I_t} - 1 \right)^2
\]

\[
1 = q_t \left( 1 - \tau_t \left( \frac{I_t}{I_{t-1}} - 1 \right) \right)^2 - \tau_t \left( \frac{I_t}{I_{t-1}} - 1 \right) \left( \frac{I_{t+1}}{I_t} - 1 \right) \left( \frac{I_{t+1}}{I_t} - 1 \right)^2
\]

37
\[ \frac{\partial L}{\partial k_{t+1}} = 0 \]

\[ P_t q_t = \frac{P_t}{P_{t+1}} m_{t,t+1} \left( - \lambda P_{t+1} i_t + P_{t+1} \varphi_t + \left( \left( (1 + \gamma)^t + 1 \right) k_{t+1}^{\alpha} \frac{1}{k_{t+1}^{1-\alpha}} \right) + P_{t+1} q_{t+1} (1 - \delta) \right) \]

\[ q_t = m_{t,t+1} \left( \varphi_{t+1} + \left( \left( (1 + \gamma)^t + 1 \right) k_{t+1}^{\alpha} \frac{1}{k_{t+1}^{1-\alpha}} \right) + q_{t+1} (1 - \delta) - \lambda_t \right) \]

\[ q_t = m_{t,t+1} \left( \varphi_{t+1} \alpha \frac{1}{k_{t+1}^{1-\alpha}} + q_{t+1} (1 - \delta) - \lambda_t \right) \]

(B.16)

The aggregate law of motion of the capital stock normalized by trend growth is

\[ \Gamma \dot{K}_{t+1} = (1 - \delta) \ddot{K}_t + \left( 1 - \frac{\tau_i}{2} \left( \Gamma \frac{\ddot{I}_t}{I_{t-1}} - \Gamma \right) \right) I_t \]

(B.17)

The production function can be aggregated in the following way. Note that all firms set the same price. Hence, \( p_t = P_t \).

\[ y_t = k_t^{\alpha} \left( (1 + \gamma)^t \right) L_t \]

\[ \left( \frac{p_t}{P_t} \right) Y_t = k_t^{\alpha} \left( (1 + \gamma)^t L_t \right) \]

\[ Y_t = \left( \frac{K_t}{(1 + \gamma)^t L_t} \right)^\alpha \left( 1 + \gamma \right)^t L_t \]

\[ \frac{Y_t}{(1 + \gamma)^t L_t} = \left( \frac{K_t}{(1 + \gamma)^t L_t} \right)^\alpha L_t \]

\[ \frac{Y_t}{(1 + \gamma)^t \xi_t} = \left( \frac{K_t}{(1 + \gamma)^t \xi_t} \right)^\alpha L_t \]

\[ \ddot{Y}_t = \ddot{K_t} L_t^{1-\alpha} \]

(B.18)

TPK firms. The TPK firms set prices according to (B.12) but

\[ m_{t-1,t} = m \]

is constant at the steady state. TPK firms produce according to (B.18). Capital evolves according to (B.17) but note that \( \tau_i = 0 \). Even though the production function is Cobb-Douglas, TPK firms do not substitute. Hence, marginal costs are

\[ \varphi_t = \omega_t \left( \frac{1}{1 - \alpha} \left( \frac{\ddot{Y}_t}{K_t} \right)^\frac{\alpha}{\alpha - 1} \right) \]

(B.20)

where the output-capital ratio is fixed at the steady-state level. Then, marginal costs are proportional to wage costs. Instead of (B.15) and (B.16), TPK firms invest according to

\[ \frac{I_t}{K_t} = \Gamma - (1 - \delta) + \phi_t (v_t - v_t^*) \]

(B.21)

B.3 Accounting, definitions and policy.

The following equations hold for all three models. The goods market equilibrium condition reads

\[ \ddot{Y}_t = \ddot{C}_t + \ddot{I}_t + \ddot{G}_t. \]

(B.22)
We define the rate of unemployment as
\[ u_t = 1 - \frac{L_t}{N_t} \]  
(B.23)
The growth rate of the real wage is linked to wage and price inflation according to
\[ \frac{\bar{\omega}_t}{\bar{\omega}_{t-1}} - 1 = \pi_t^w - \pi^r_t. \]  
(B.24)
Fiscal policy involves government consumption and lump-sum taxes. We suppose a balanced budget at all times, i.e.
\[ \check{T}_t = \check{G}_t. \]  
(B.25)
Government expenditures evolve according to
\[ \check{G}_t = \rho \check{G}_{t-1} + (1 - \rho_c) \check{G} + \varepsilon_{c,t}. \]  
(B.26)
The monetary policy rule is assumed to be
\[ i_t = \rho_i i_{t-1} + (1 - \rho_i) \left( i + \phi_r (\pi^p_t - \pi^*) \right) + \varepsilon_{i,t} \]  
(B.27)
The current and desired rates of capacity utilization are
\[ v_t = \frac{\check{Y}_t}{Y_{c,t}} \]  
and
\[ v^*_t = \frac{\check{Y}_t}{\check{Y}_c} \]  
respectively. Capacity output requires marginal costs to be equal to average costs which implies
\[ 1 = \left( \frac{1}{\check{Y}_{c,t}} \right) \frac{\check{W}(\check{Y}_{c,t})}{K_t} + i_t \check{K}_t. \]  
(B.30)

### B.4 Model closures.

The DSGE model is closed by assuming labor market clearing, i.e.
\[ u_t = 0. \]  
(B.31)as well as \( U = 0 \) which implies for the consumption optimality condition
\[ \check{C}^{-\rho}_t = \beta \frac{1 + \check{i}_{t-1}}{1 + \check{\pi}_t} (1 + \gamma)^{-\rho} \check{C}^{-\rho}_{t+1}. \]  
(B.32)
The DSLMD and TPK models are closed by collective wage bargaining. The FOC of the workers’ and firms’ bargaining problem is
\[ 1 = (1 - 1/v_t) \frac{\check{W}(\pi^w_t)}{r' (\pi^w_t)} \frac{\check{W}'(\pi^w_t)}{\check{W}(\pi^w_t)}. \]  
with
\[ v_t = (1 - u_{t-1}) \nu \mu_t \]  
(B.34)and
\[ \ln(\mu_t) = \rho_\mu \ln(\mu_{t-1}) + \varepsilon_{\mu,t}. \]  
(B.35)
The evolution of the rate of wage inflation is assumed to be
\[ \pi_1^w = \rho_w \pi_{1-1}^w + (1 - \rho_w) \pi_{1-1}^w. \]  
(B.36)
The DSLMD model is characterized by eqs. (B.1)-(B.8), (B.12)-(B.18), (B.22)-(B.30) and (B.33)-(B.33), the TPK model by eqs. (B.9)-(B.11), (B.12), (B.17)-(B.30) and (B.33)-(B.33), and finally the DSGE model by eqs. (B.8), (B.12)-(B.18) and (B.22)-(B.32).
## C  Calibration

To facilitate comparing the three models, we choose parameter values which are as similar as possible across models. For the DSGE and DSLMD models, calibration is based on where parameter values are partly taken from the relevant literature and partly estimated for the Euro Area using Bayesian Maximum Likelihood. The model calibrations are as follows. A description of the parameters can be found in Appendix A.

<table>
<thead>
<tr>
<th>General</th>
<th>DSLMD</th>
<th>TPK</th>
<th>DSGE</th>
<th>Firms</th>
<th>DSLMD</th>
<th>TPK</th>
<th>DSGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>n</td>
<td>0.0019</td>
<td>0.0019</td>
<td>0.0019</td>
<td>α</td>
<td>1/3</td>
<td>1/3</td>
<td>1/3</td>
</tr>
<tr>
<td>γ</td>
<td>0.0037</td>
<td>0.0037</td>
<td>0.0037</td>
<td>δ</td>
<td>0.025</td>
<td>0.025</td>
<td>0.025</td>
</tr>
<tr>
<td>u</td>
<td>0.075</td>
<td>0.075</td>
<td>—</td>
<td>τ_p</td>
<td>25</td>
<td>25</td>
<td>25</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>τ_i</td>
<td>15</td>
<td>0</td>
<td>15</td>
</tr>
<tr>
<td>Households</td>
<td></td>
<td></td>
<td></td>
<td>ε_p</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>U</td>
<td>0.025</td>
<td>—</td>
<td>0</td>
<td>λ</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>D</td>
<td>0.005</td>
<td>—</td>
<td>—</td>
<td>φ_i</td>
<td>—</td>
<td>0.05</td>
<td>—</td>
</tr>
<tr>
<td>ρ</td>
<td>2</td>
<td>2</td>
<td>—</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>β</td>
<td>0.998</td>
<td>—</td>
<td>0.998</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>η</td>
<td>1</td>
<td>—</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ψ</td>
<td>898</td>
<td>—</td>
<td>898</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>φθ</td>
<td>0.2</td>
<td>—</td>
<td>—</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ρω</td>
<td>0</td>
<td>0</td>
<td>—</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ecz</td>
<td>—</td>
<td>0.609</td>
<td>—</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>cb</td>
<td>—</td>
<td>0.003</td>
<td>—</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>—</td>
<td>1.382</td>
<td>—</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>