

# Agentenbasierte Modellierung

## II. Interbankenmarkt

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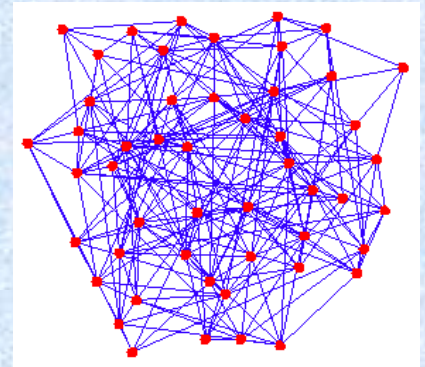
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## **Evidence for Success of Agent-Based Modelling?**

### **Areas outside Economics**

- ABMs are the dominating new paradigm for modelling traffic flows
- ABMs are standard tool in epidemics research (with good coincidence of results between agent-based micro models and phenomenological macro models)
- Apparently, great interest in the U.S. Military and Homeland Security department...  
(modelling of combat, dispersed terrorist activity, impact of attacks on population etc.)
- Many central banks have now started to implement ABMs within network models of interbank connections

# Research on Networks in Economics and Beyond



- Economic literature: network research mostly of game-theoretic nature
- Social sciences: mainly empirical work on determinants of network formation

(see *Journal Social Networks* etc.)

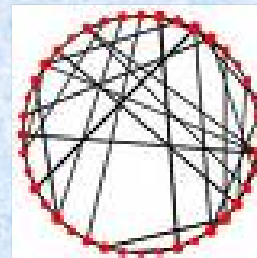
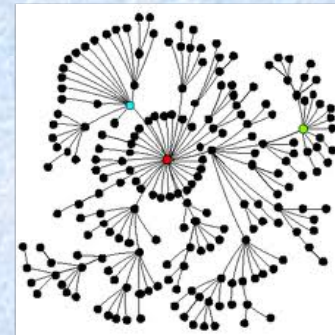
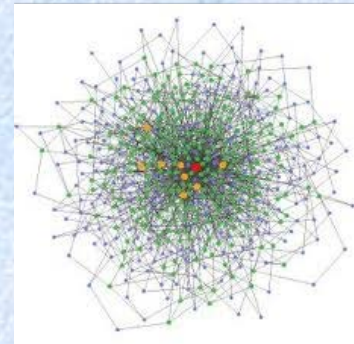
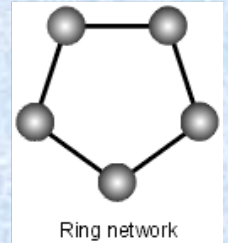
- Natural sciences: typology of networks and their characteristics (small world, random, scale-free..)

(cf., Barabási, *Linked: The New Science of Networks*, 2002)

- Recently intensifying research on interbank linkages and systemic risk

# A Short Network Typology

- Of course, there can be a variety of simple forms, but the more often encountered are:
- Random networks: fixed prob. of connections between nodes (Erdős & Rényi)
- Scale-free network: degree distribution is power law  $P(k) \approx k^{-\mu}$
- Small-world networks: distance  $L \approx \log(N)$   
(three degrees of separation)
- Core-periphery networks





# The importance of the *systemic* aspect

- Failure of individual bank poses risk to further financial institutions and even the economy as a whole (domino effects, systemic risk)
- -> banking system as network where each node represents a bank and each link represents a lending relationship
- knowledge is required about the structure of the interbank market (mapping the financial system)
- ... and on the role of particular institutions (systemic importance)
- before we can proceed to simulations, we need information about the structure of the interbank network

# Existing Literature

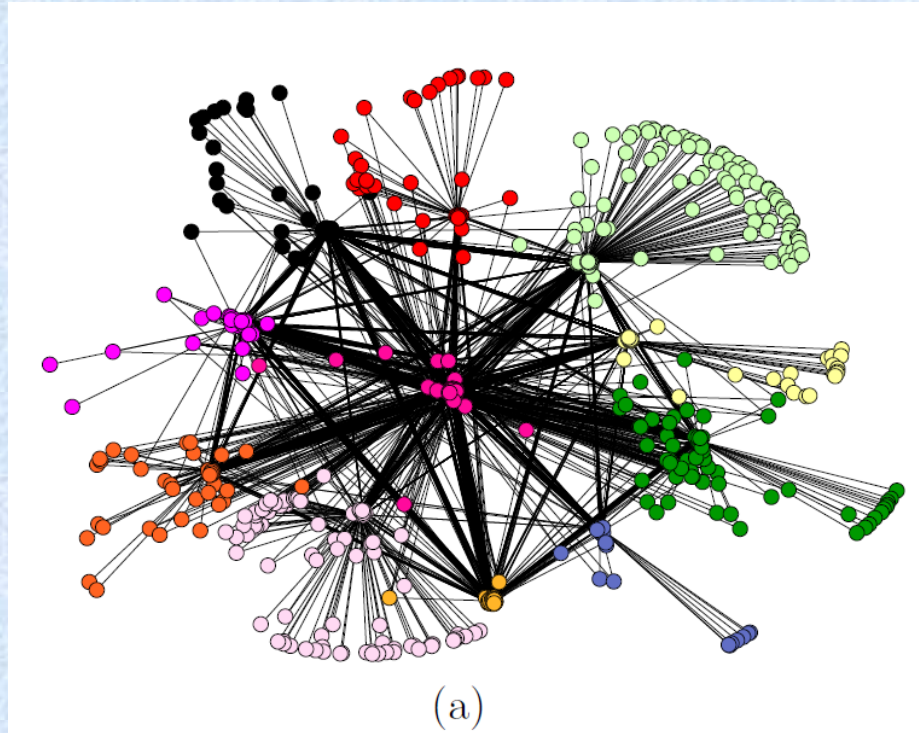
- Starting a few years before the crisis:  
Network studies of interbank data in the style of natural sciences
  - Germany: Upper and Worms (2004)
  - Austria: Boss et al. (2004)
  - US Fedwire network: Soramäki et al. (2006), May et al. (2008)
  - Italian e-MID data: de Masi et al. (2006), Iori et al. (2008), etc.
- Typical questions: Is network scale-free or random, descriptive statistics of connectivity

# Terminology

- (Interbank) Network:  $N$  nodes (banks), connected by  $M$  directed edges (credit flows)
- $D_{\{N \times N\}}$ : Matrix of interbank claims. Element  $d_{ij}$  is the total value of credit extended from  $i$  to  $j$  within a certain period
- $A_{\{N \times N\}}$ : Adjacency matrix. Element  $a_{ij} = 1$  if  $a_{ij} > 0$ .
- In- and out-degree: number of incoming/outgoing links per bank  $i$



## Example: The banking network of Austria

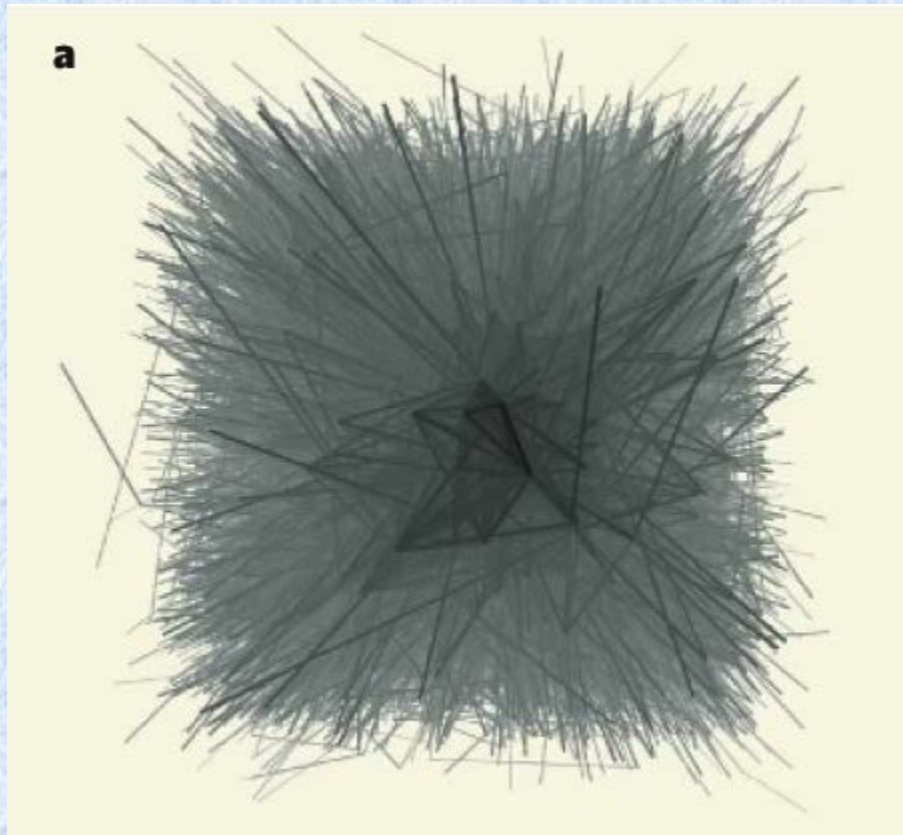


*The banking network of Austria* (a). Clusters are grouped (colored) according to regional and sectorial organization: R-sector with its federal state sub-structure: RB yellow, RSt orange, light orange RK, gray RV, dark green RT, black RN, light green RO, light yellow RS. VB-sector: dark gray, S-sector: orange-brown, other: pink.

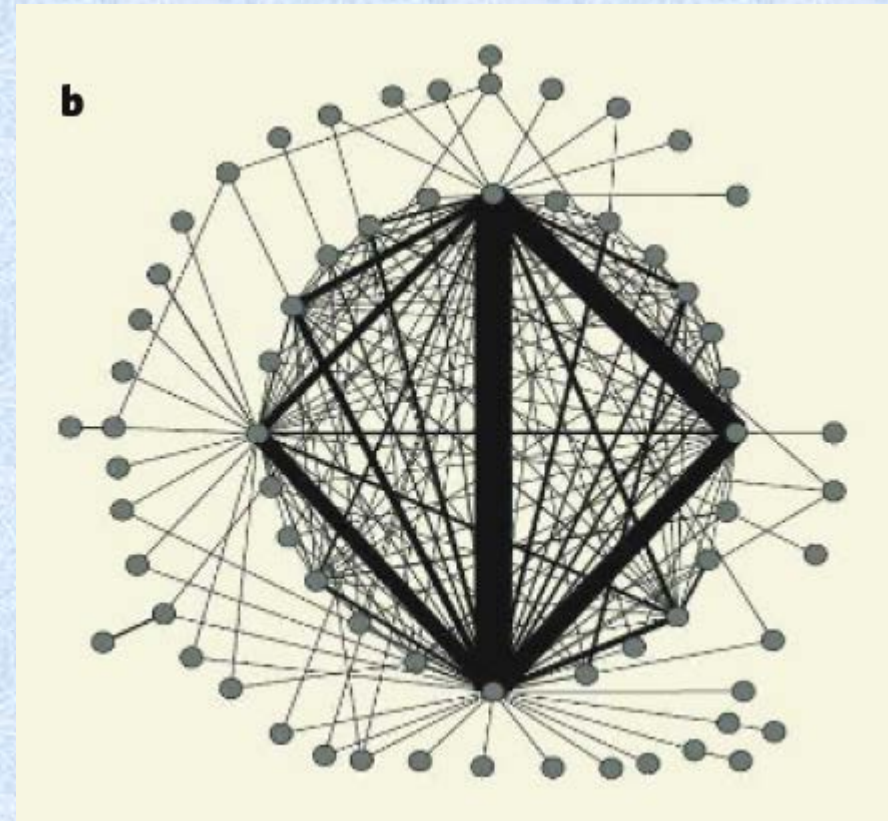
From: Boss *et al.*, *Quantitative Finance* 4, 2004



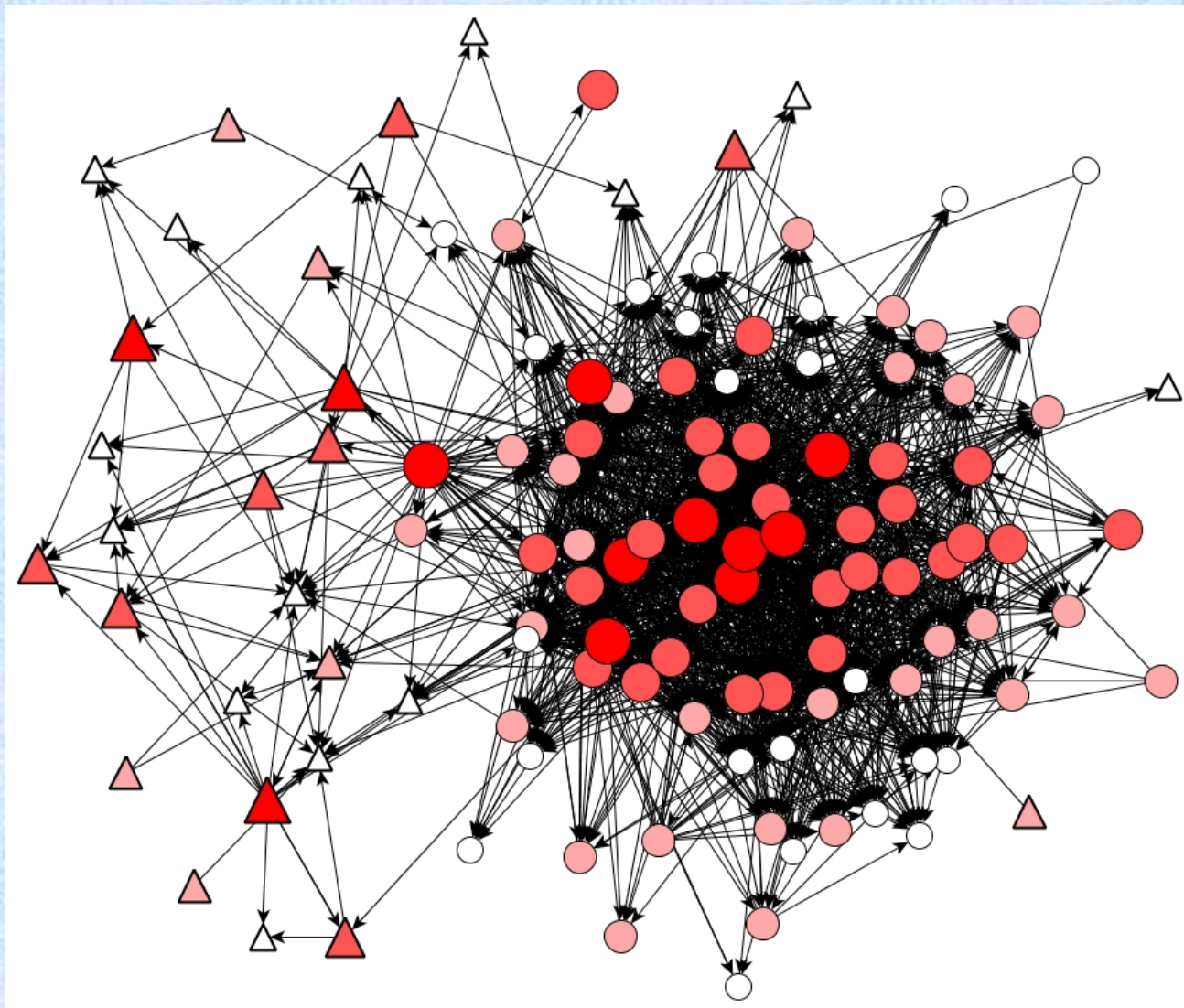
# The Fedwire interbank payment network



The entire system



The core: 66 banks  
with 75% of daily  
value of transfers



Snapshot of  
the e-MID  
network at  
2010/4

Triangles:  
foreign banks  
(20)

Dots: Italian  
banks (89)

Size and  
brightness  
indicate size as  
lender



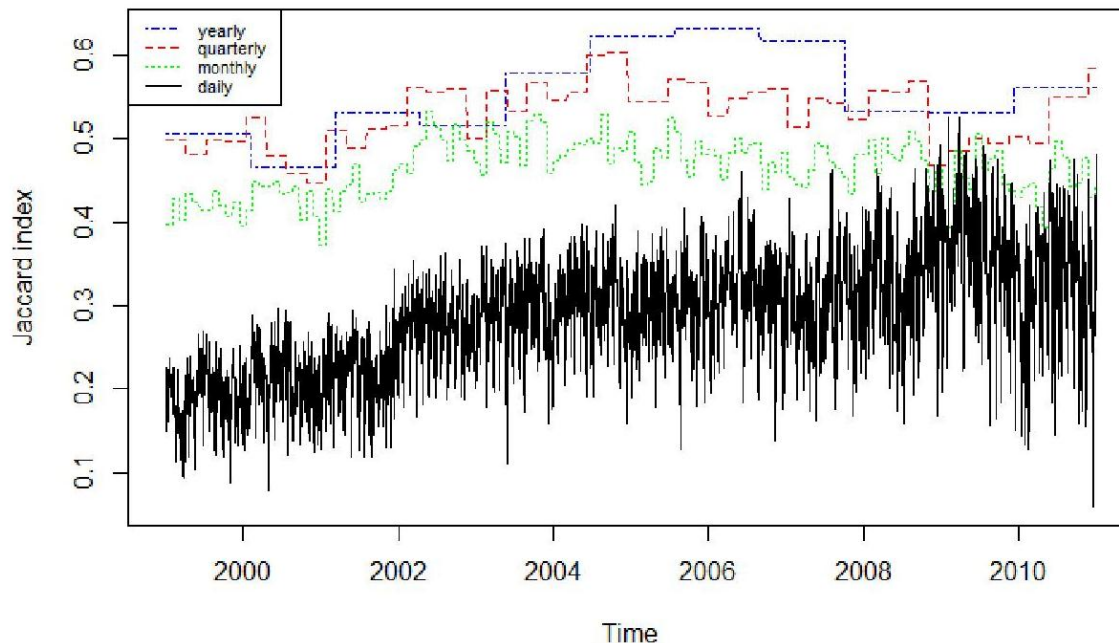
## Insights from recent literature

(various papers with K. Finger and D. Fricke on e-MID)

- *What format of data to use?* Interbank relations aggregated over longer horizons (from monthly) provide for much more stable statistics than high-frequency data
- *Disassortative mixing*: high-degree nodes are more likely to have associations with low-degree nodes
- A *core-periphery* structure provides a better fit than alternative network models
- *Distribution of links* does not correspond to the scale-free ideal
- Standard network formation models cannot reproduce the entirety of statistical findings

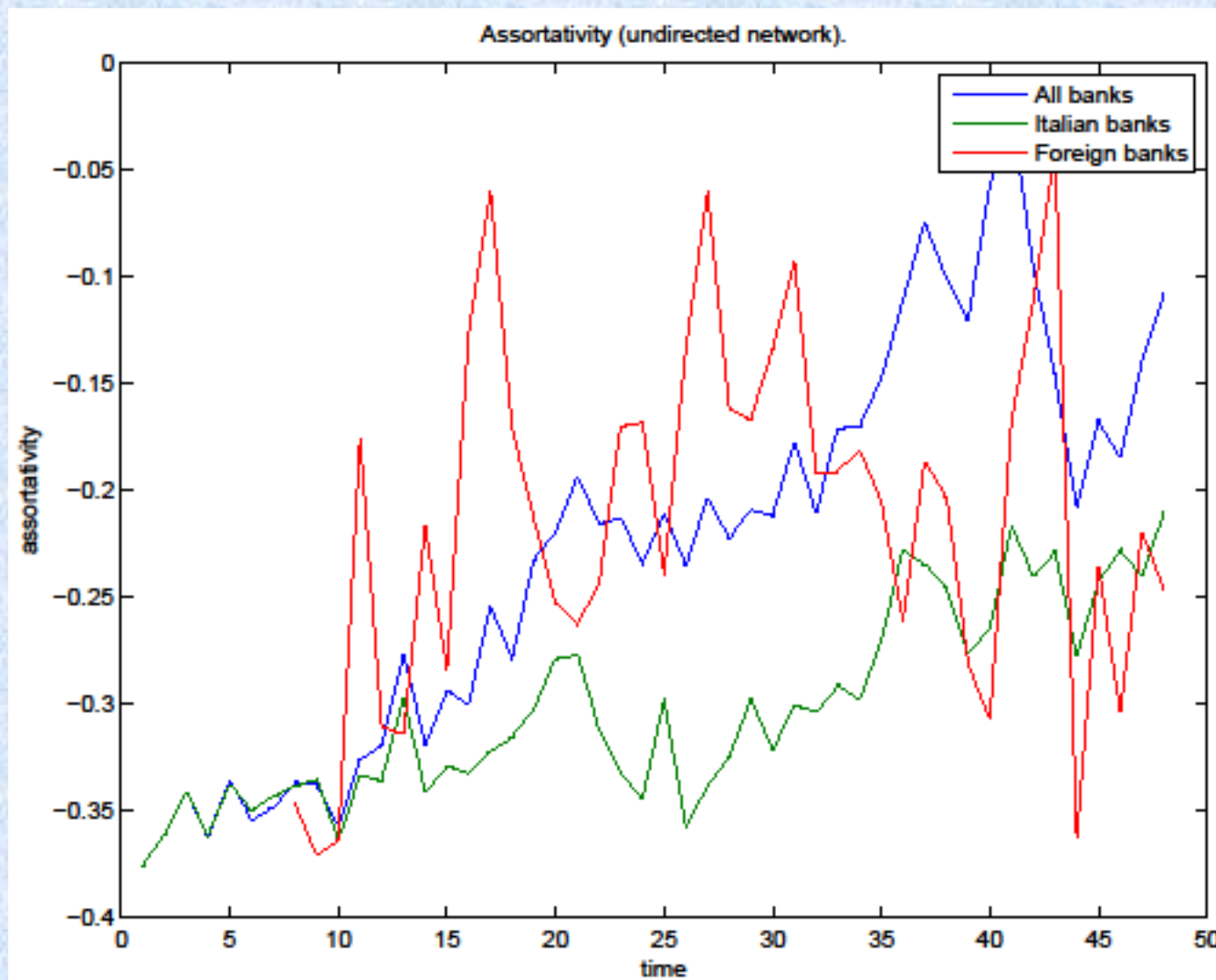


*Time horizon:* daily networks behave very erratically, they are probably best considered as incomplete sampling from an underlying dormant network, of which only few links are activated, more stability for monthly, quarterly networks

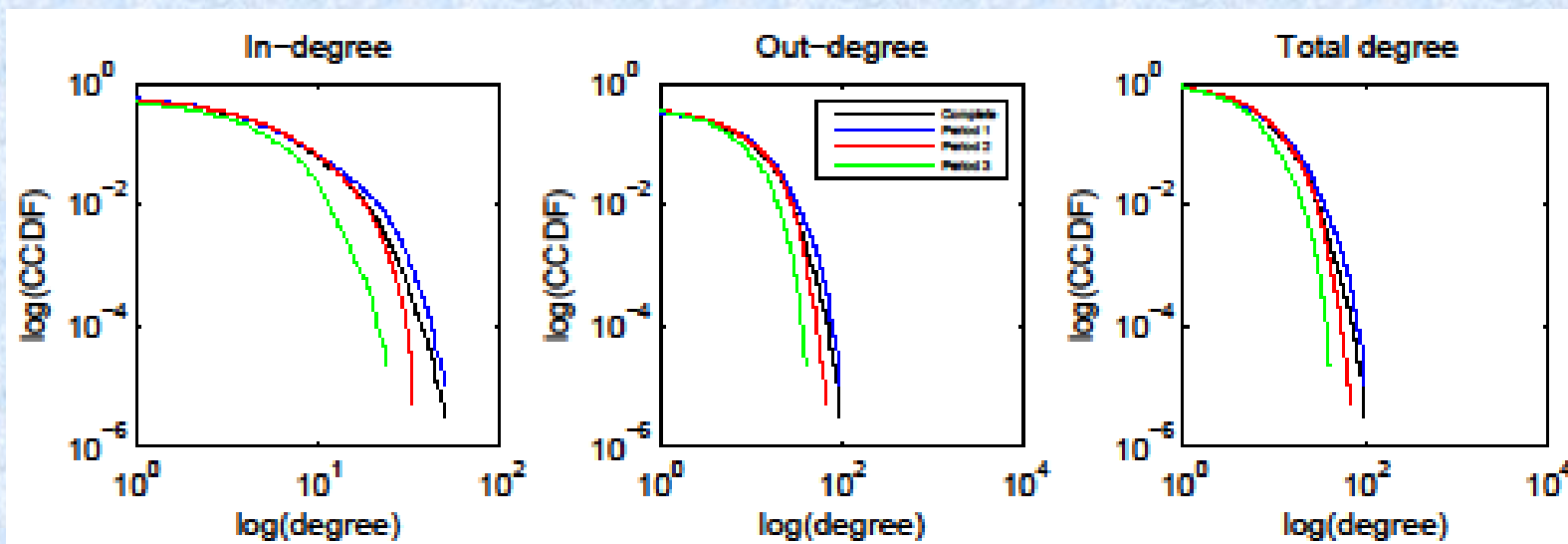
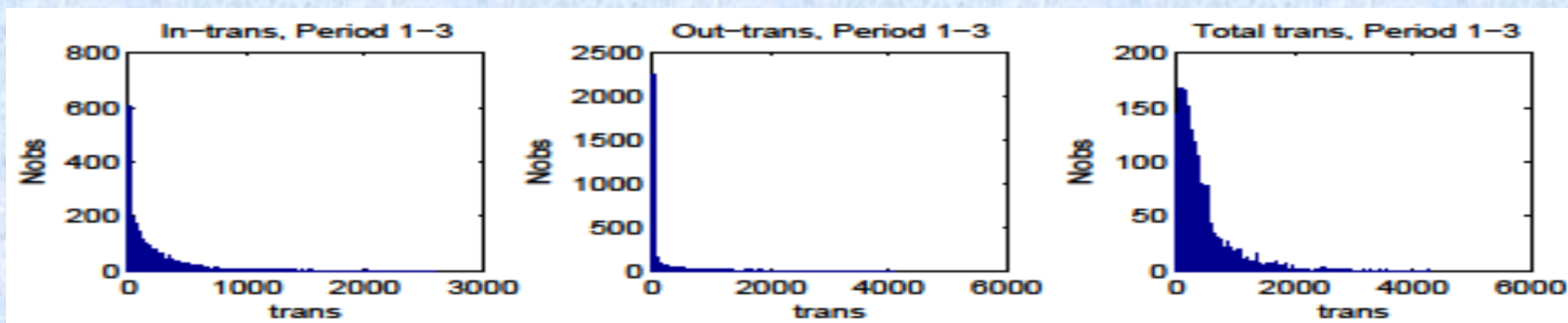


Jaccard Index for daily (black), monthly (green), quarterly (red) and yearly (blue) networks.

$$\text{Jaccard index: } J = \frac{M_{11}}{M_{01} + M_{10} + M_{11}},$$



An important observation: Negative *assortativity* ,  
i.e. negative correlation between degrees of trading partners,  
small banks trade more with large banks and *vice versa*

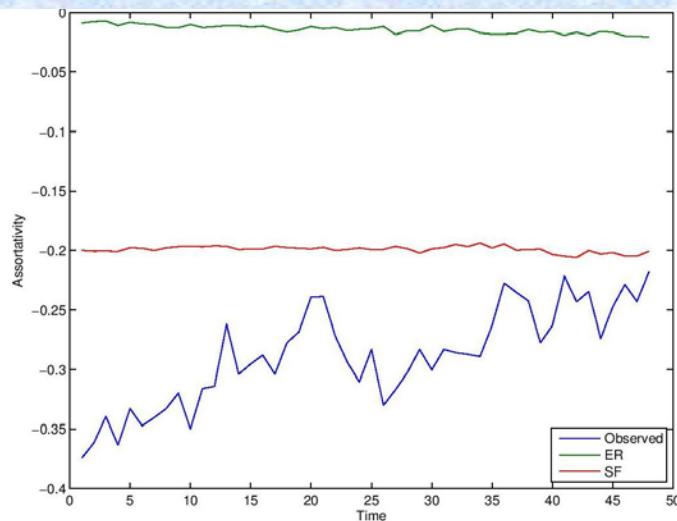


***Degree distributions:*** exponential rather than power-law decline of cdf, best fits by negative Binomial, Weibull, Gamma, Exponential distributions, same for no. of transactions, volume

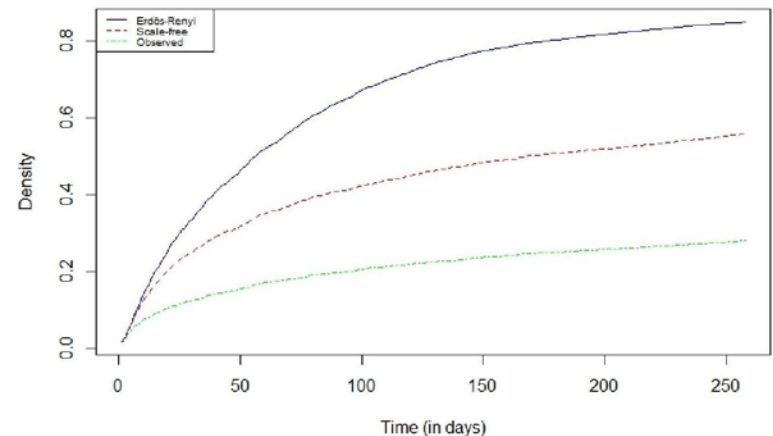


# The ensemble of stylized facts cannot be reproduced by standard network models

- Assortativity higher than in benchmark random and SF networks
- Density saturates over time (evidence for long-term links)



Assortativity coefficient  $r$  of observed (blue) and random networks for ER (green), and SF networks (red) for the undirected version of the network over time. The densities in the SF and random networks match the observed ones for the network of Italian banks. A scaling exponent of 2.3 was used for the in- and out-degrees in the SF networks. Results for random and SF



Data for 1999. Density for the aggregated Erdős-Rényi (blue), Scale-Free (red) with  $\alpha = 2.3$ , and observed networks (green). Aggregation period in days. Note: we do not plot standard deviations, since these are negligible.

# Core-Periphery Network Analysis

**Simplest case: Discrete model:**  $c_i$  = core membership of agent  $i$ ,  $c_i = 0$  or  $1$

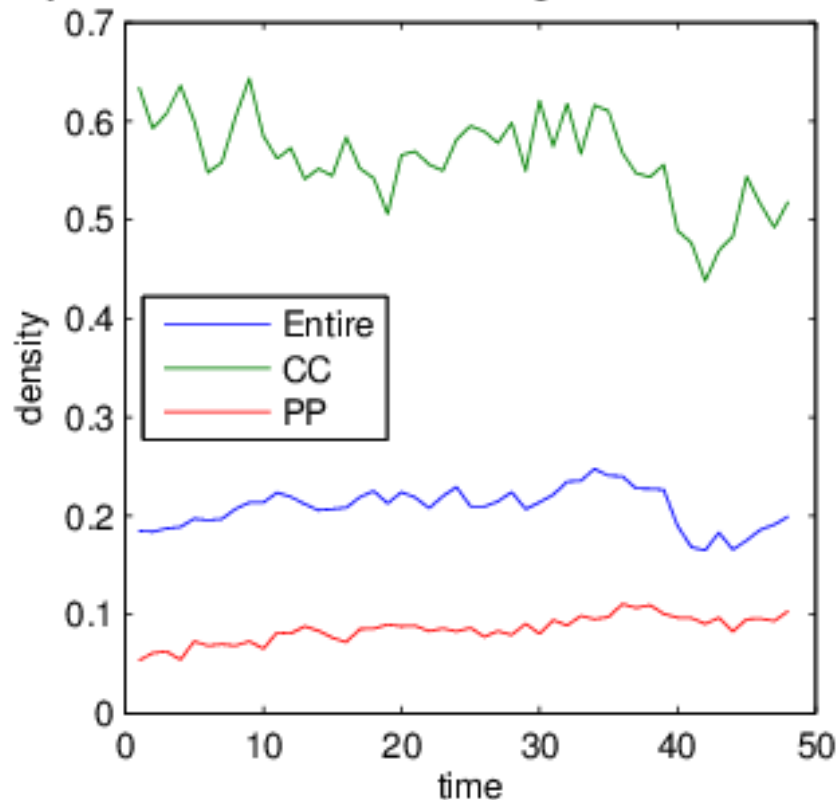
In order to determine the core, we match the empirical distribution of links to an idealized pattern (cf. Borgetti and Everett, *Soc. Networks* 1999)

$$P = \left( \begin{array}{cccc|cccc} 1 & 1 & 1 & \dots & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & \dots & 0 & 1 & 0 & 0 \\ \dots & \dots & \dots & \dots & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right)$$

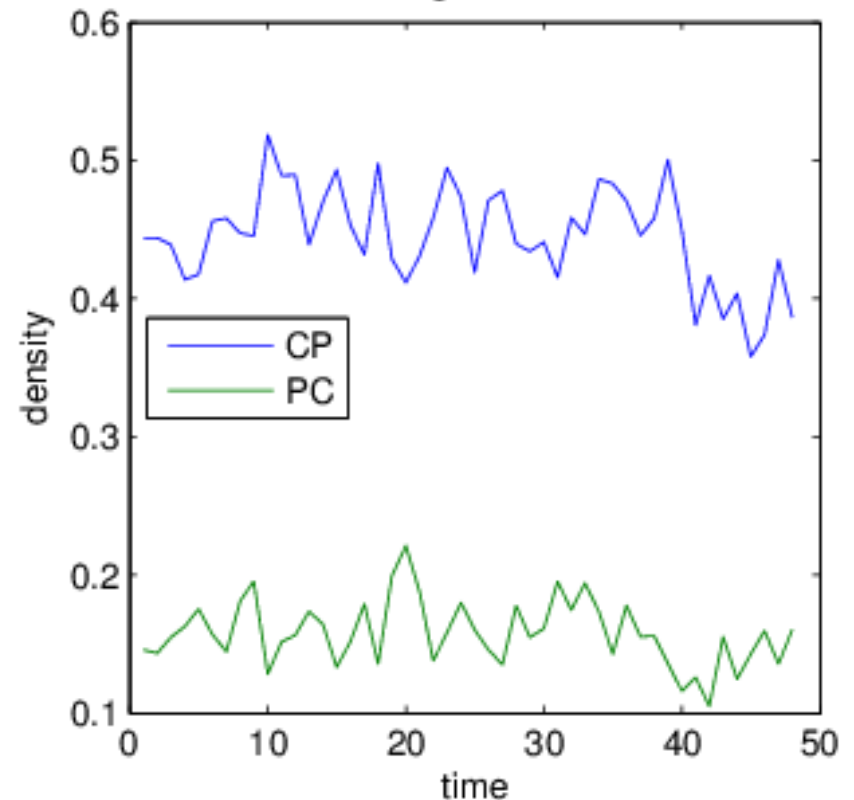
Idealized pattern matrix:

$$P_I = \begin{pmatrix} CC & CP \\ PC & PP \end{pmatrix} = \begin{pmatrix} 1 & CP \\ PC & 0 \end{pmatrix}$$

Density in the entire network, and diagonal blocks. Discrete model.



Off-diagonal blocks.

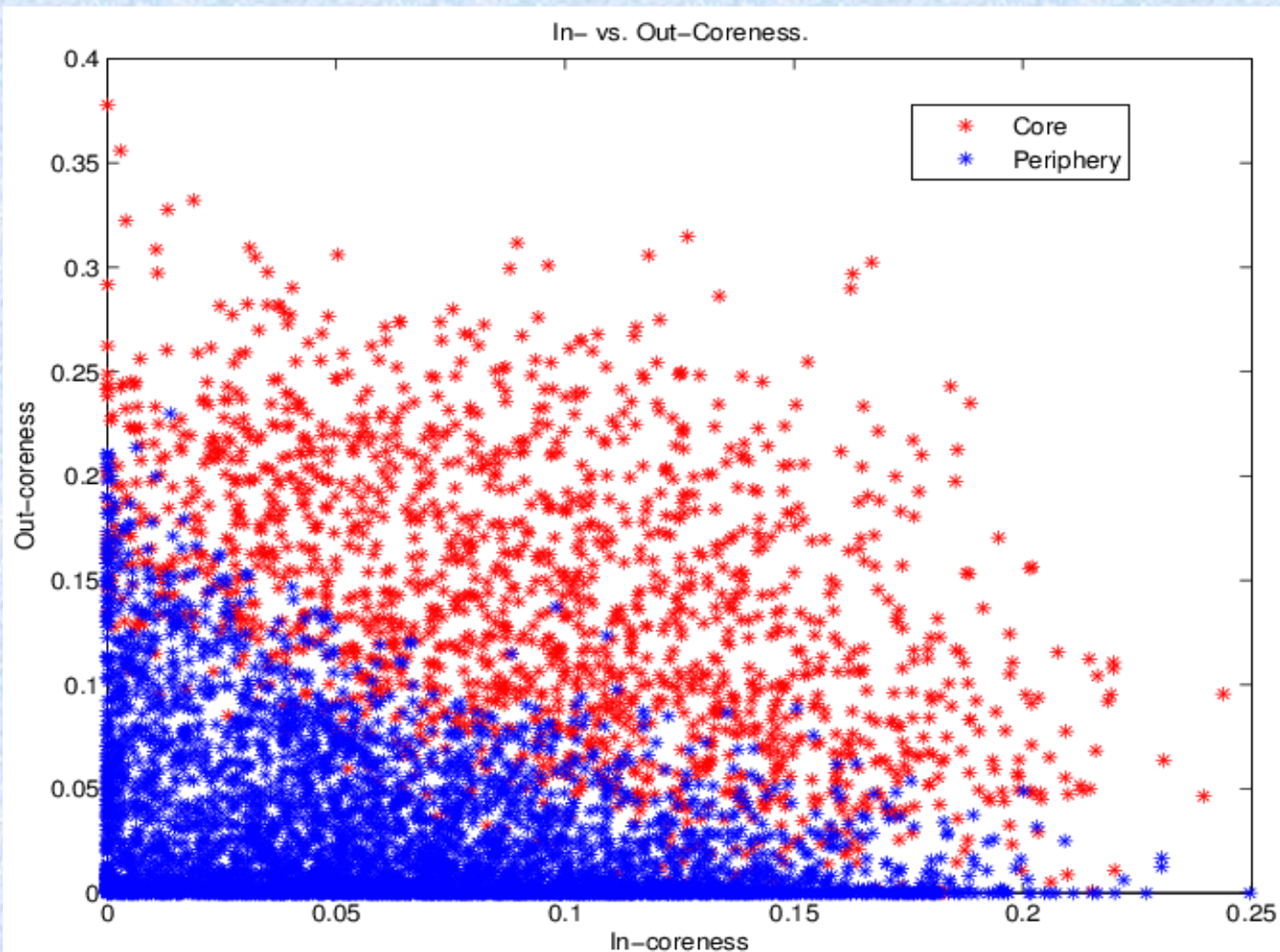


## Network density:

- Is also very stable over time as a whole as well as in the subsets of the network,
- Points to relevant asymmetry (CP vs PC)



# Added value of an asymmetric continuous CP model: In-coreness vs out-coreness



->  $u$  and  $v$  are virtually uncorrelated (correlations are  $\sim -0.07$ )

# What happened to the network during the financial crisis?

From CP analysis:

- Overall activity dropped significantly at quarter 37/38 (Lehman's default)
- Less trading within the core banks' block (CC) as well as in PP
- Core banks lend less to periphery (CP) while PC remained relatively stable
- Jaccard indices showed no significant change, i.e. much of the network structure remained intact

-> major disruption in CC which was partially compensated by PC, C banks started to hoard liquidity

# Another Look at Dynamics: “Friendship” between banks? (Finger and Lux, 2014)

- Application of the *stochastic actor-based model* from sociology: the major determinant of link formation is size and previous intensity of connections,
- Interest rates relatively unimportant

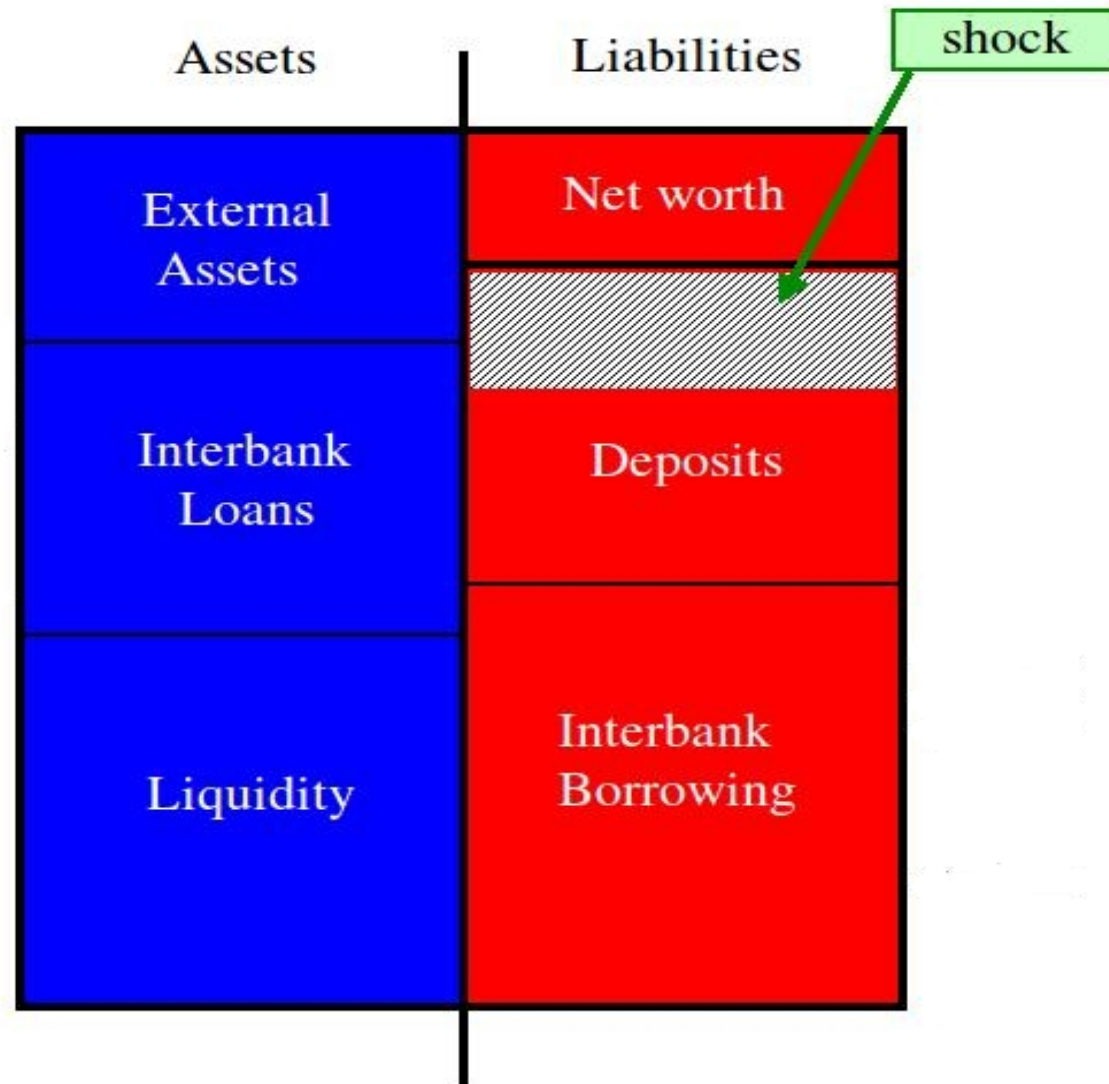
effect	category	mean	s.e.	p-value	significance
rate		36.055			
size	rate	.1566	(.00084)	.001***	[32 -4]
density (outdegree)	structural	-1.2010	(.0647)	.001***	[-36 -]
reciprocity	structural	.6797	(.0508)	.001***	[23 -13]
reciprocity persistence	structural	-.7371	(.0957)	.001***	[-8 28]
transitive triplets	structural	-.0029	(.0010)	.006***	[-9 27]
3-cycles	structural	-.0236	(.0026)	.001***	[-34 2]
indegree pop. (sqrt)	structural	.3580	(.0061)	.001***	[36 -]
outdegree act. (sqrt)	structural	.2157	(.0056)	.001***	[36 -]
past trades	dyadic cov.	.1946	(.0086)	.001***	[36 -]
past rates	dyadic cov.	.4087	(.3959)	.309	[9 7 20]
size popularity	actor cov.	.0065	(.0078)	.410	[8 6 22]
size activity	actor cov.	-.0437	(.0115)	.001***	[4 21 11]
size similarity	actor cov.	.0536	(.0170)	.003***	[6 - 30]
core popularity	actor cov.	.2174	(.0239)	.001***	[27 - 9]
core activity	actor cov.	.2497	(.0315)	.001***	[27 1 8]
core similarity	actor cov.	-.0062	(.0129)	.632	[3 6 27]
lending rate pop.	actor cov.	-.0478	(.0754)	.530	[2 4 30]
lending rate act.	actor cov.	-.2855	(.1831)	.128	[5 9 22]
lending rate sim.	actor cov.	-.1581	(.0618)	.015**	[3 11 22]
borrowing rate pop.	actor cov.	-.3885	(.2820)	.177	[8 13 15]
borrowing rate act.	actor cov.	.0991	(.1179)	.407	[5 3 28]
borrowing rate sim.	actor cov.	-.1146	(.0547)	.044**	[3 7 26]



# I. Towards a More Realistic Simulation Framework

- The small literature on interbank simulation models falls into two categories:
  - ✓ Counterfactual simulations: disaggregation from macro data
  - ✓ Theoretical models using one of the well-known classes of networks for link formation, e.g. random network etc (Nier et al, JEDC 2007, theoretical approach: May and Arinaminpathy, 2010)
- We follow the second approach but attempt to integrate certain stylized facts of the interbank market: *realistic degree distributions, disassortative mixing, and proximity to CP structure*

## The basic framework: Banks' balance sheet structure



## A few book keeping conventions

- Bank's assets consist of interbank loans and external assets:

$$A_i = l_i + e_i$$

- Liabilities consist of inter-bank borrowing, deposits and net worth

$$I_i = b_i + d_i + \eta_i$$

- Solvency requires  $\eta_i > 0$
- Following Nier et al., we impose the following structural restrictions:

$$e_i = \theta A_i; l_i = (1 - \theta) A_i$$

## Adding realistic features

- Bank's size is drawn from a Pareto law

$$p(A_i) \sim A_i^{-\tau} \text{ within } [a, b], \text{ and } \tau = 2$$

- Given the size of two nodes, we decide about connections according to one of the three probability functions

$$P_1(A_i, A_j) = \left(\frac{A_i}{A_{\max}}\right)^\alpha \left(\frac{A_j}{A_{\max}}\right)^\beta,$$

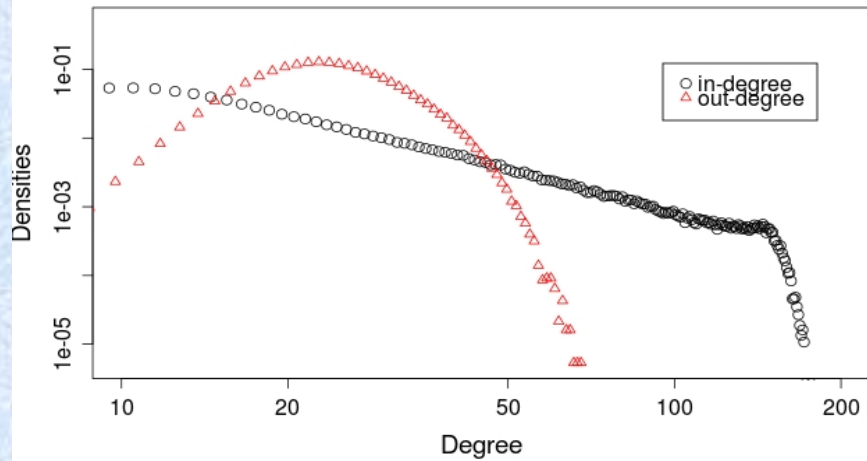
$$P_2(A_i, A_j) = c(A_i + A_j),$$

$$P_3(A_i, A_j) = \theta(A_i + A_j - z).$$

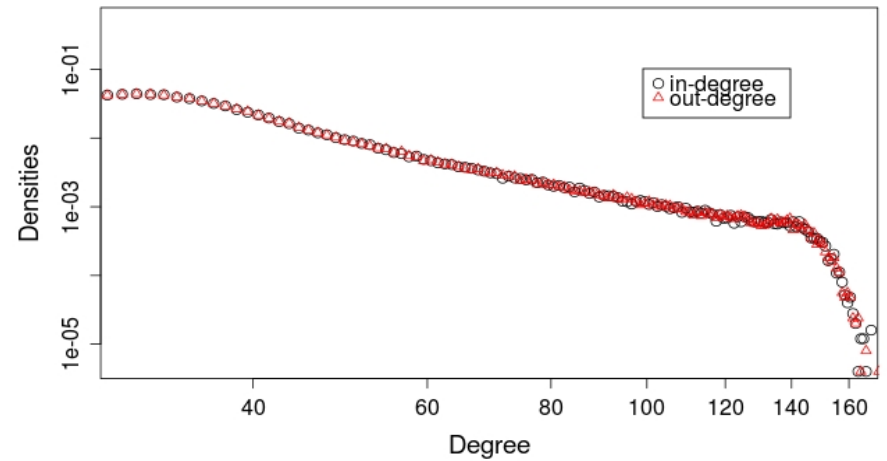
leading to disassortative mixing and power-law degree distribution.



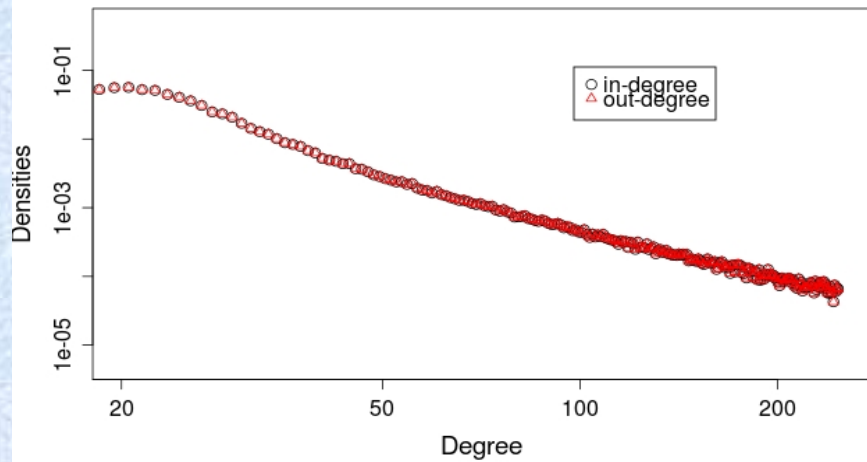
**Degree distribution (P1)**



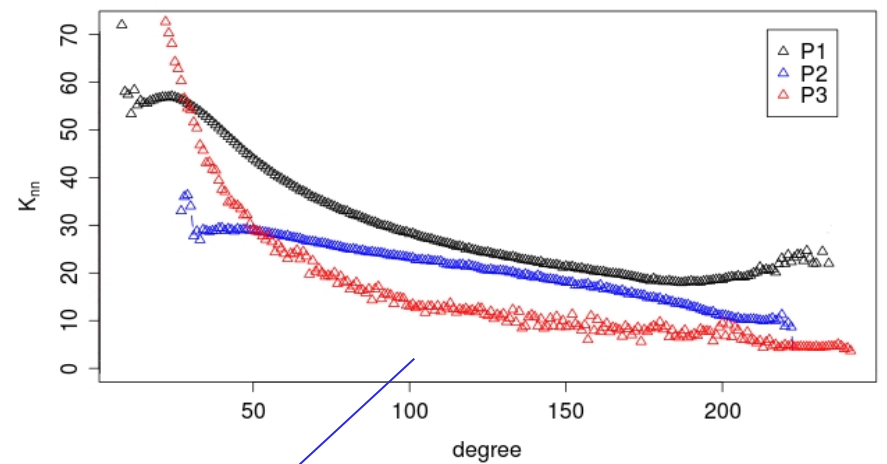
**Degree distribution (P2)**



**Degree distribution (P3)**



**Disassortative behavior**



Mean neighbor degree

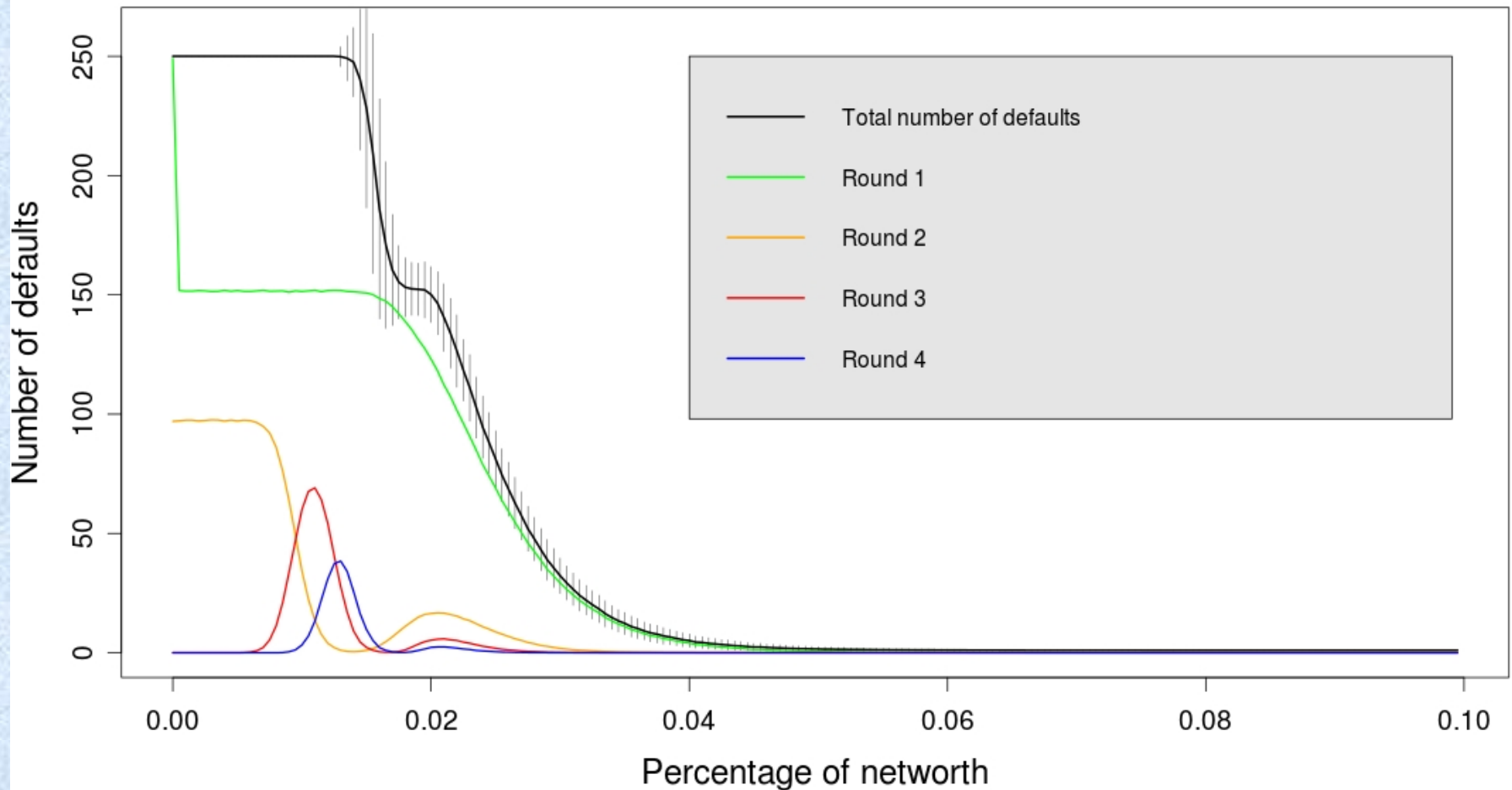
## Closing the system

- Loans are determined by: 
$$l_{ij} = \frac{l_i a_{ij}}{\sum_j a_{ij}}, a_{ij} \in \{0,1\}$$
- Deposits are adjusted to equate assets and liabilities:

$$d_i = e_i + l_i - \eta_i - b_i$$

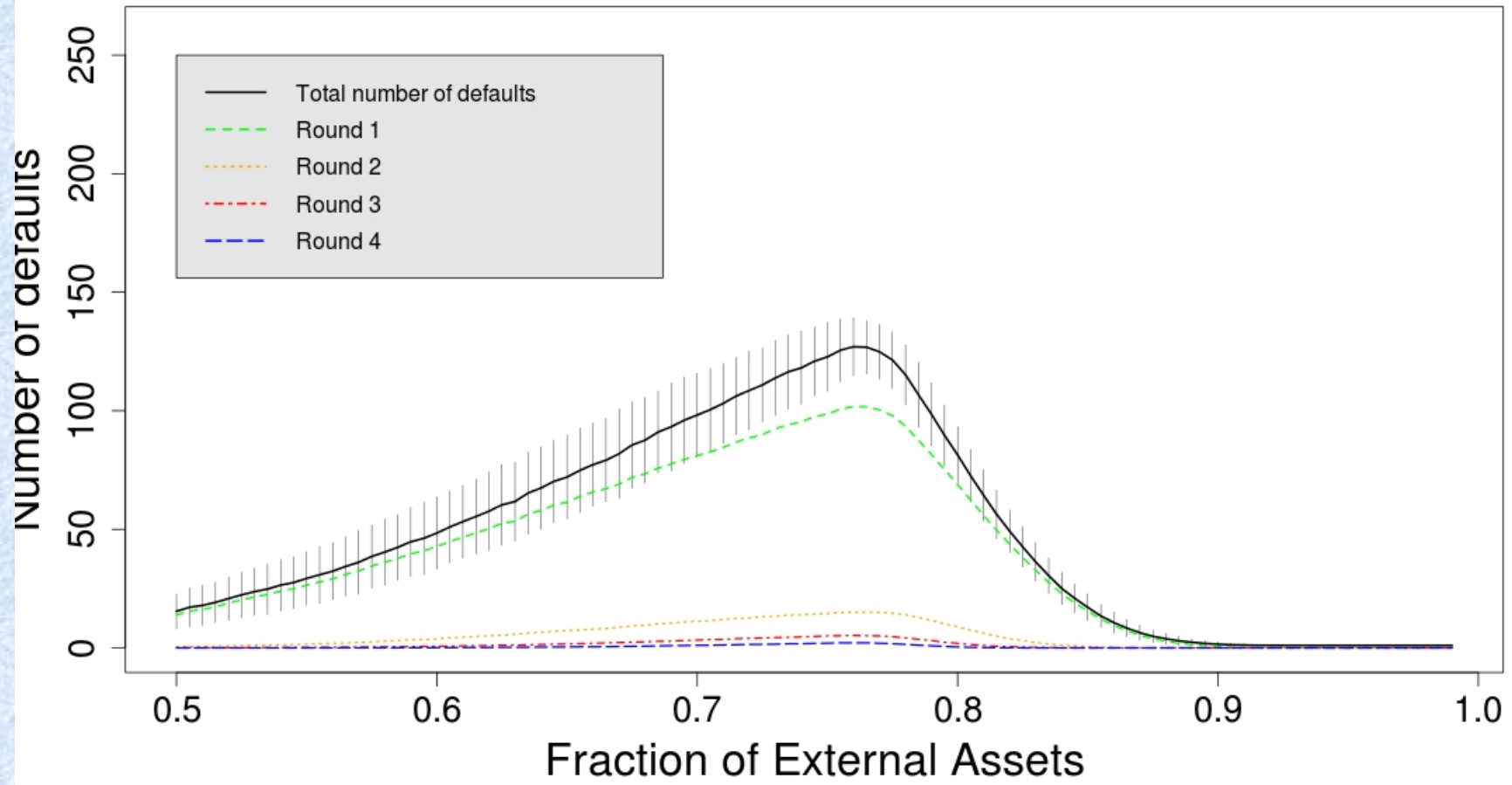
- Level of net worth  $\eta_i$  will be varied in experiments
- In case of insolvency or defaults: after effects via interbank lending, losses absorbed first by net worth, then by creditor banks

# Simulations: We wipe out the largest bank and study the repercussions in the system

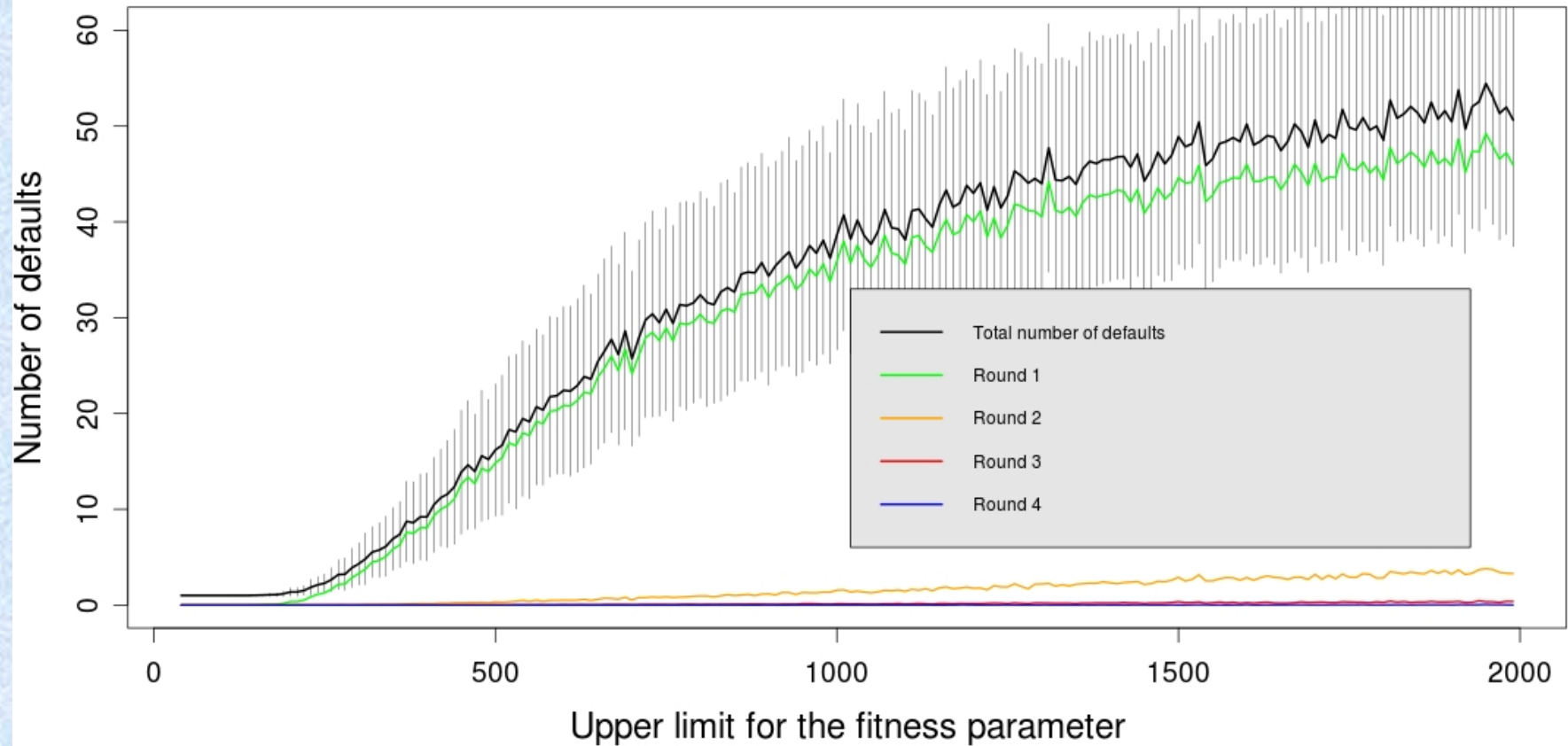




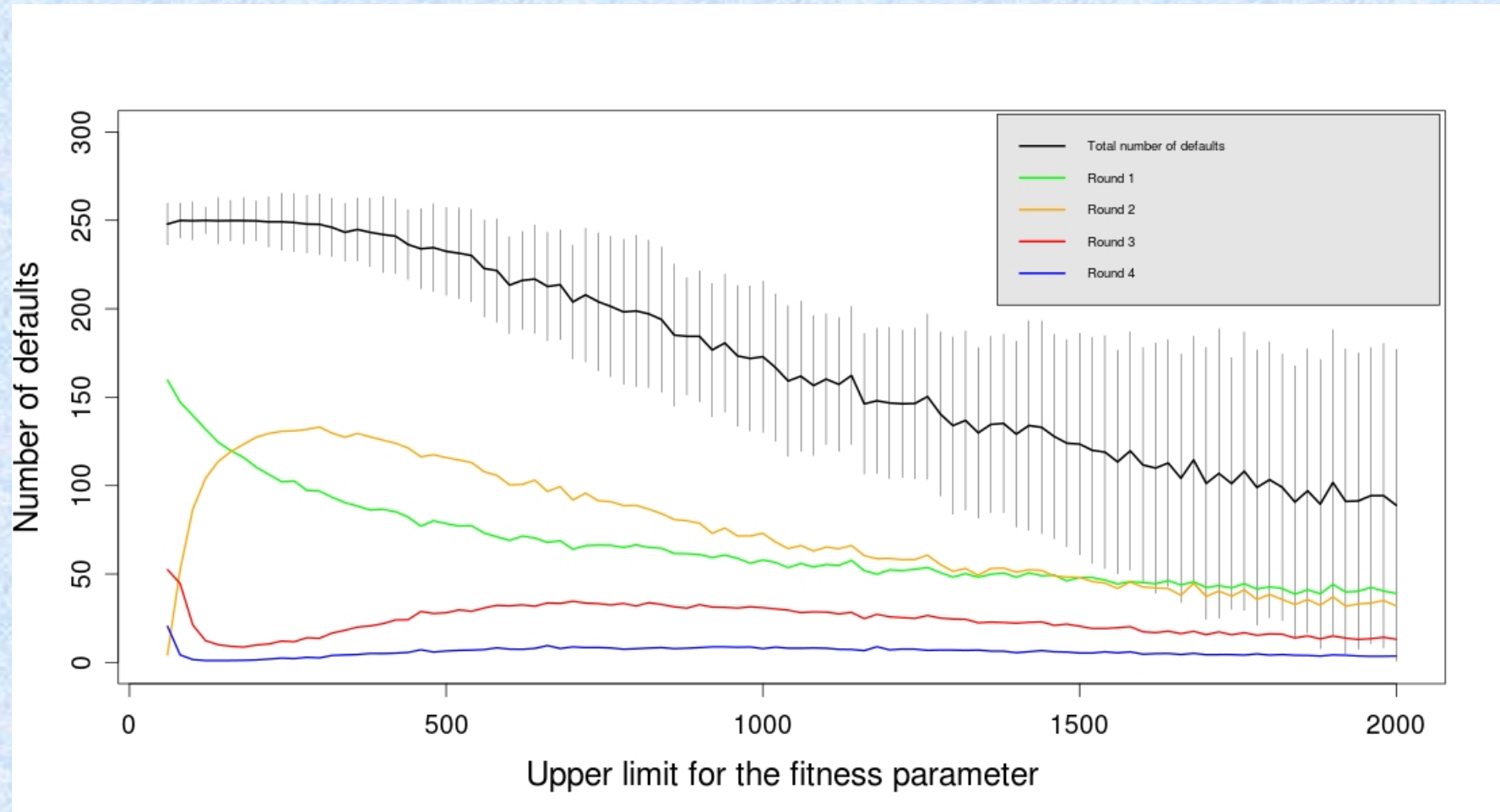
# Interbank exposure: non-monotonic influence



# Absolute size of largest bank: Variation of $b$ and number of defaults ( $\eta = 0.1$ , $\theta = 0.8$ )

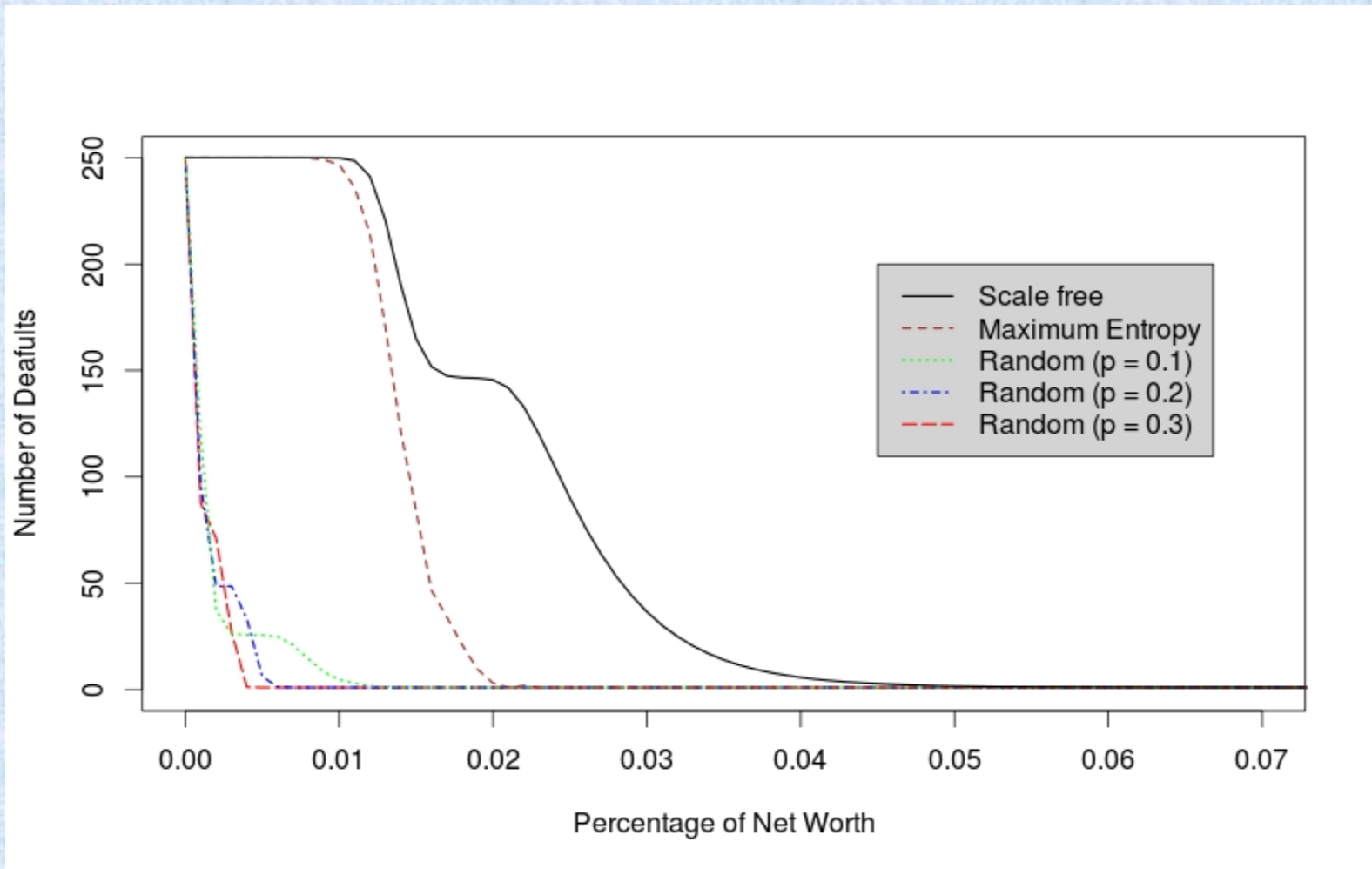


# Absolute size of largest bank: Variation of $b$ and number of defaults ( $\eta = 0.01$ , $\theta = 0.8$ )





# Comparison of number of defaults for scale-free, random and max entropy networks ( $\eta = 0.01$ , $\theta = 0.8$ )

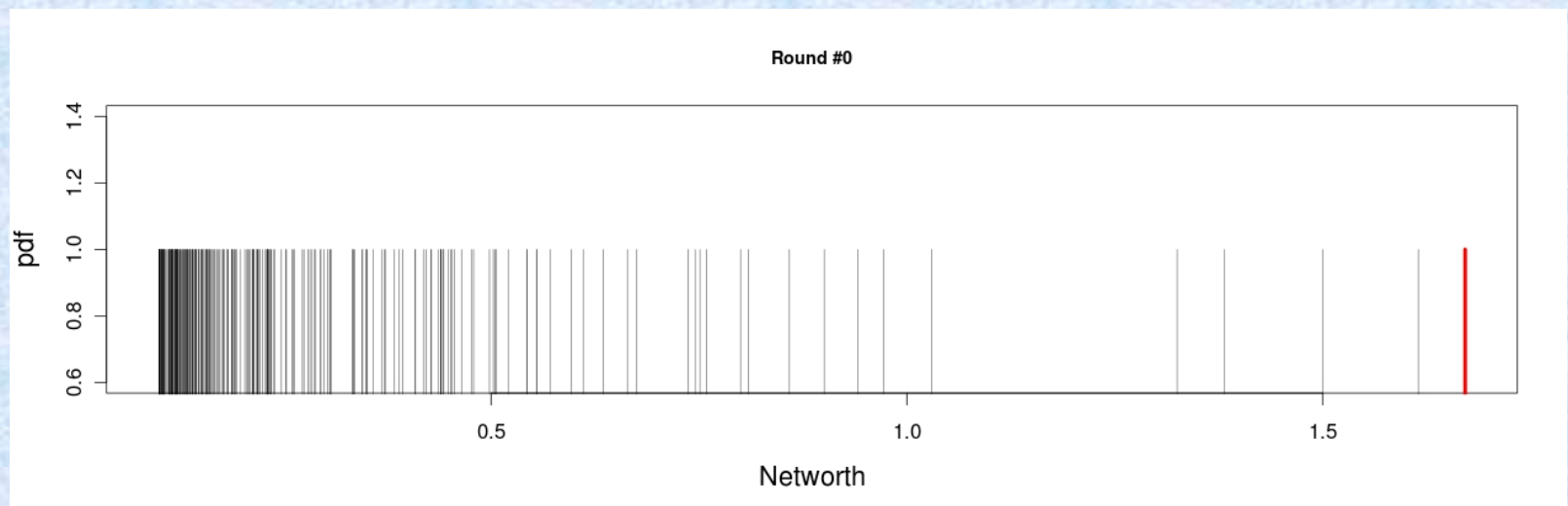


# A Theoretical Approach: Investigating the Dynamics of the System via Mean-Field Approximations

- Consider the state function of extant levels of net worth:

$$\Phi(\vec{\eta}) = \Phi(\eta_1, \eta_2, \dots, \eta_n)$$

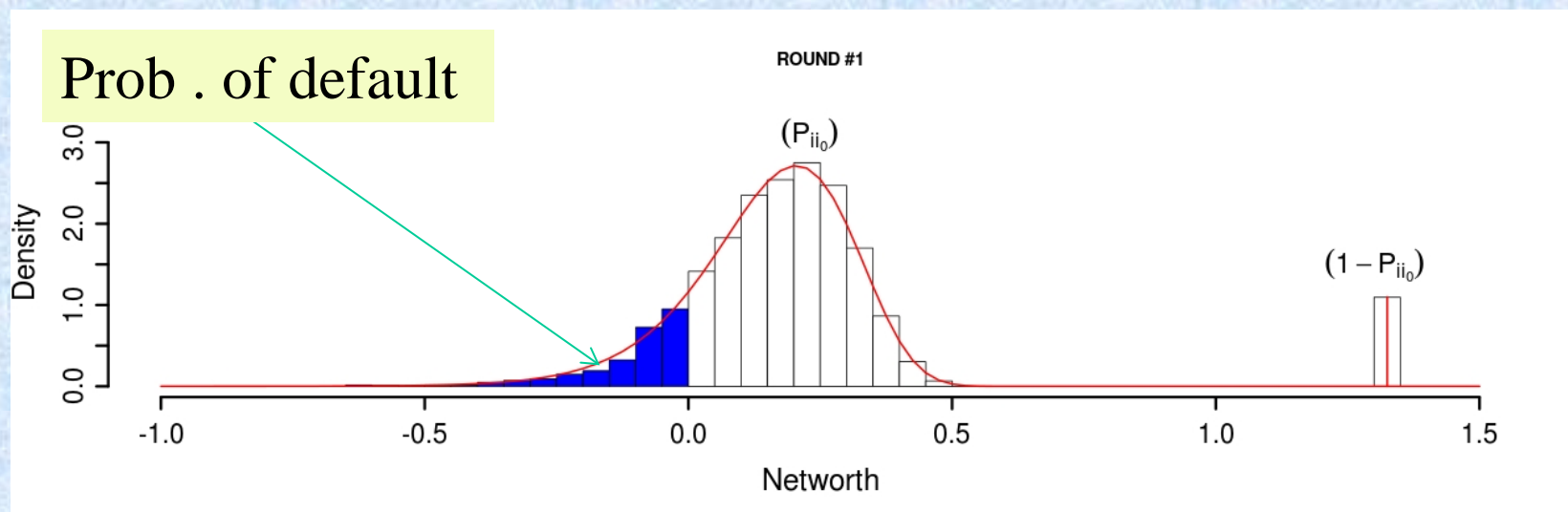
- At “time”  $t = 0$ , we record the known initial conditions:



Knowing the initial distribution, but only the probabilistic laws for the connections, one can track the *expected* propagation of shocks

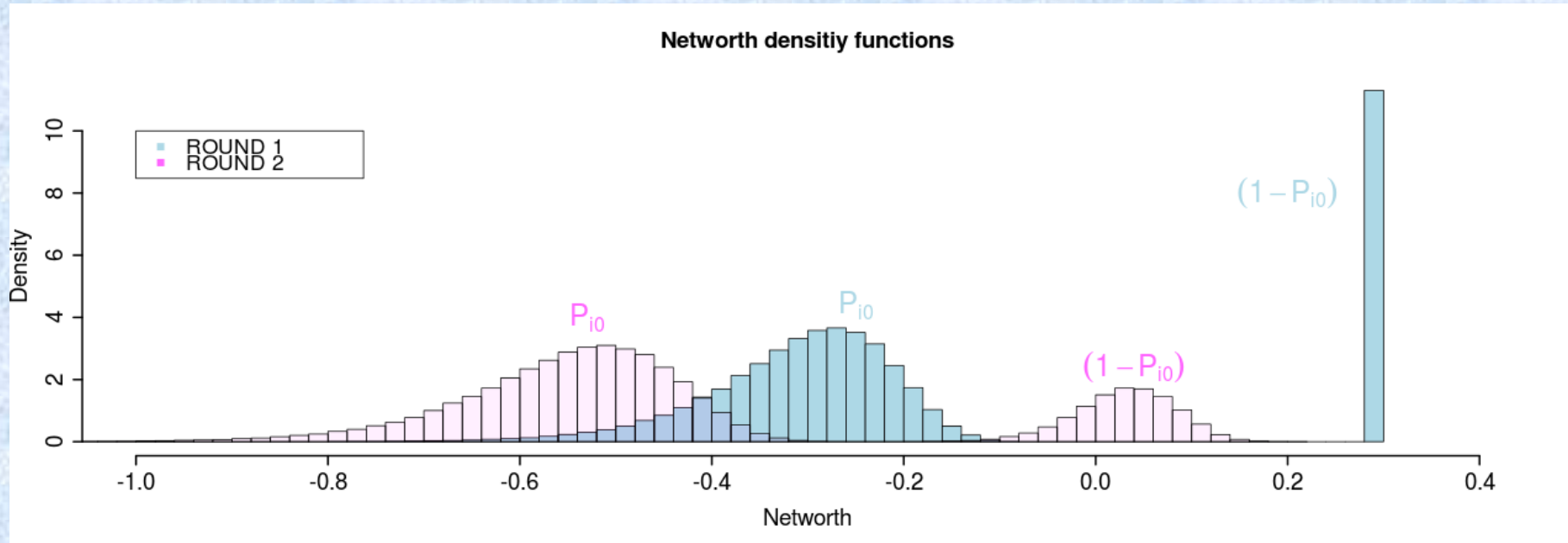
Marginal distribution of  $i$ 's net worth immediately after the default of bank  $i_0$ :

- bank  $i$  remains unaffected with prob.  $1 - p_{ii_0}$
- it is affected with prob.  $p_{ii_0}$ , and the distribution then depends on the stochastic realization of its loan to  $i_0$



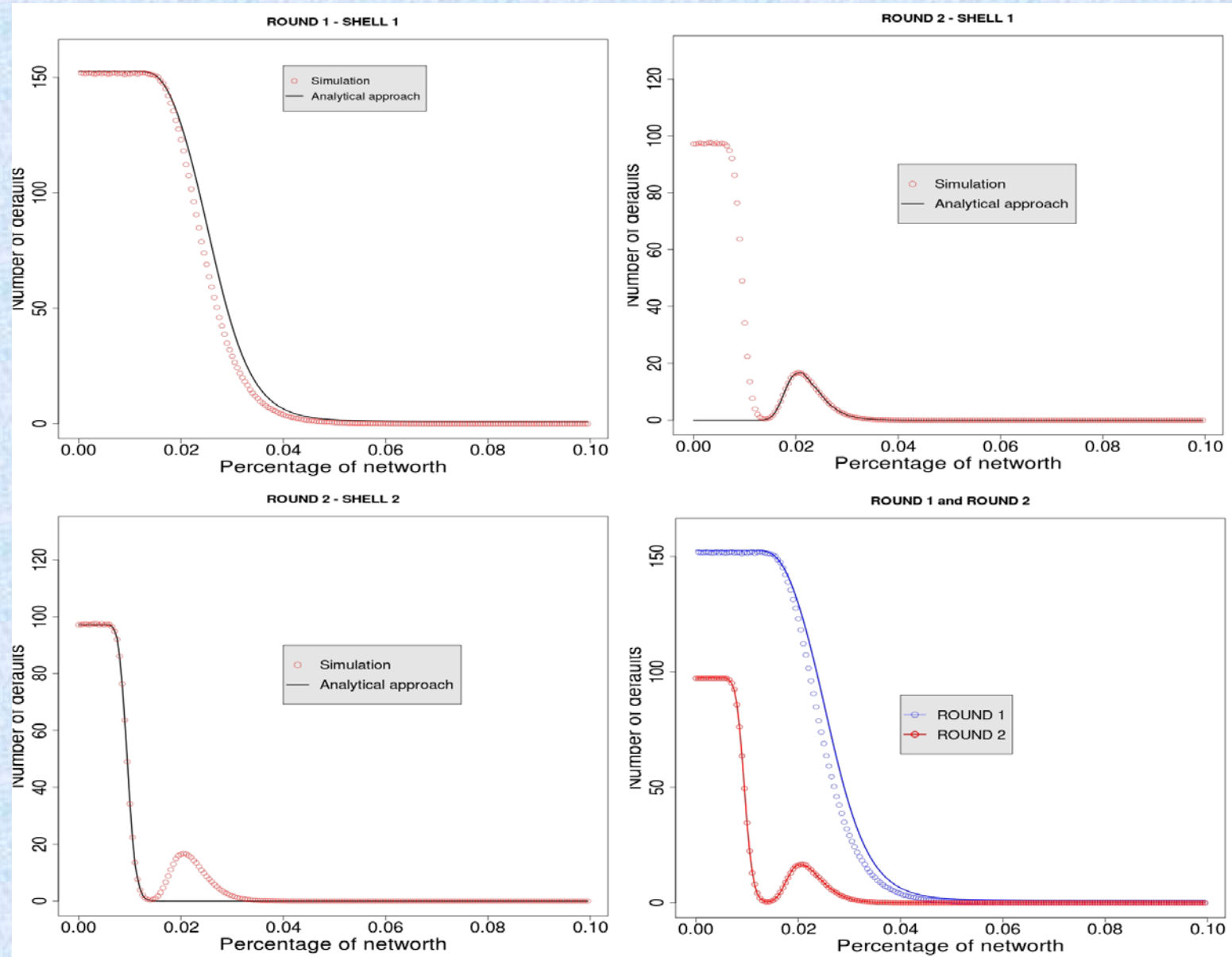


## Second round: further connections are taken into account



With known stochastic rules for the connections and loan sizes, we can track these distributions (for the whole ensemble of banks) via numerical approximations to the joint density of the state function.

# Comparison between simulations and 'theory'



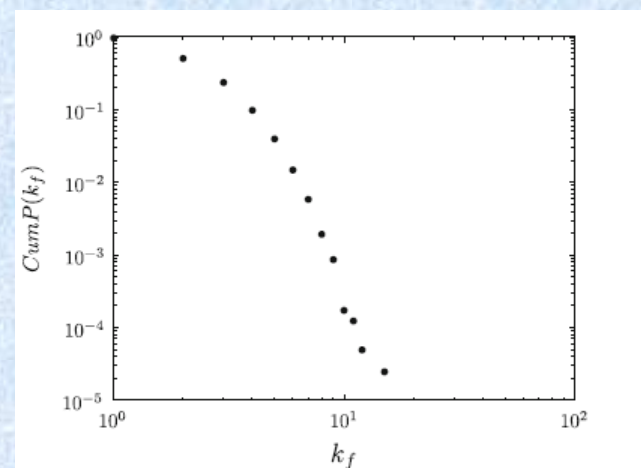
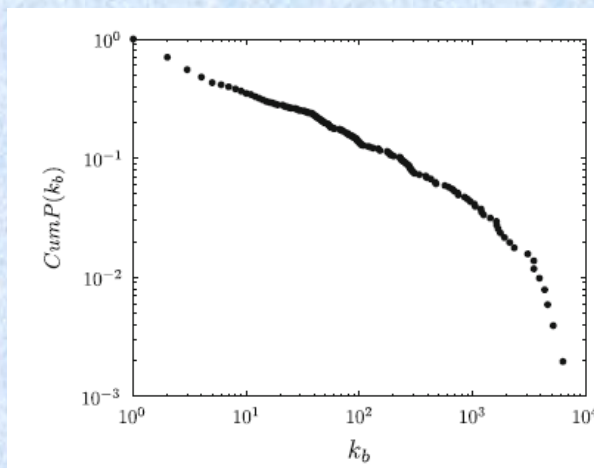
# Adding Other Channels of Contagion

- Funding risk (Halaj and Kok, 2013)
- Portfolio overlaps and valuation effects (Huang et al., 2012, Montagna and Kok, 2013)
- Joint exposure via derivatives
- Joint exposures via loans to same counterparty

New Features: Bipartite or tripartite network structures

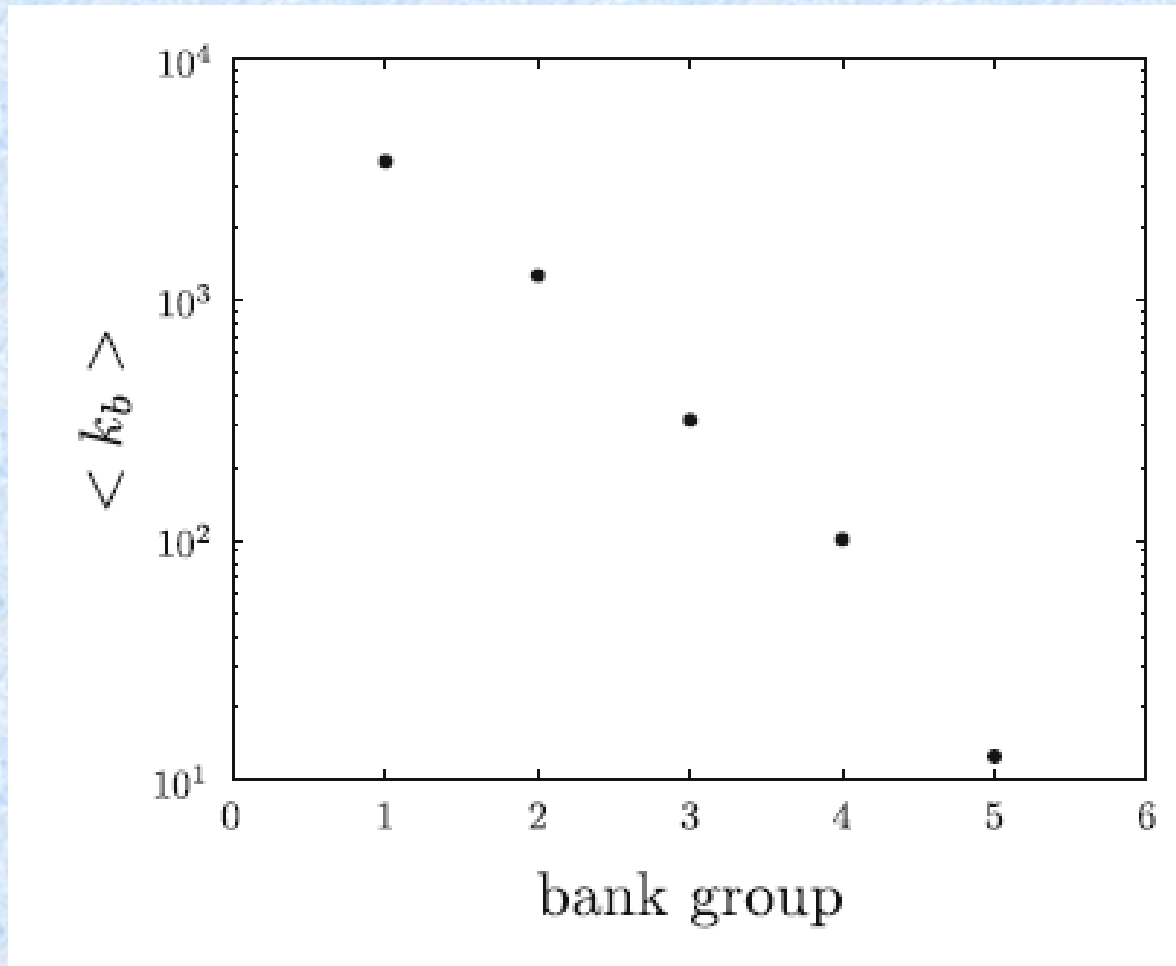
# What do we know about the firm-bank credit network?

- Banks typically have more links and a broader link distribution than firms
- From Italian data: mean degree of firms = 1.8, for banks = 149, maxima are 15 and 6699, respectively
- While not monotonic, there is a tendency of the no. of links to increase with size for both banks and firms





Large banks have more credit links



# Modelling the Firm-Bank Network

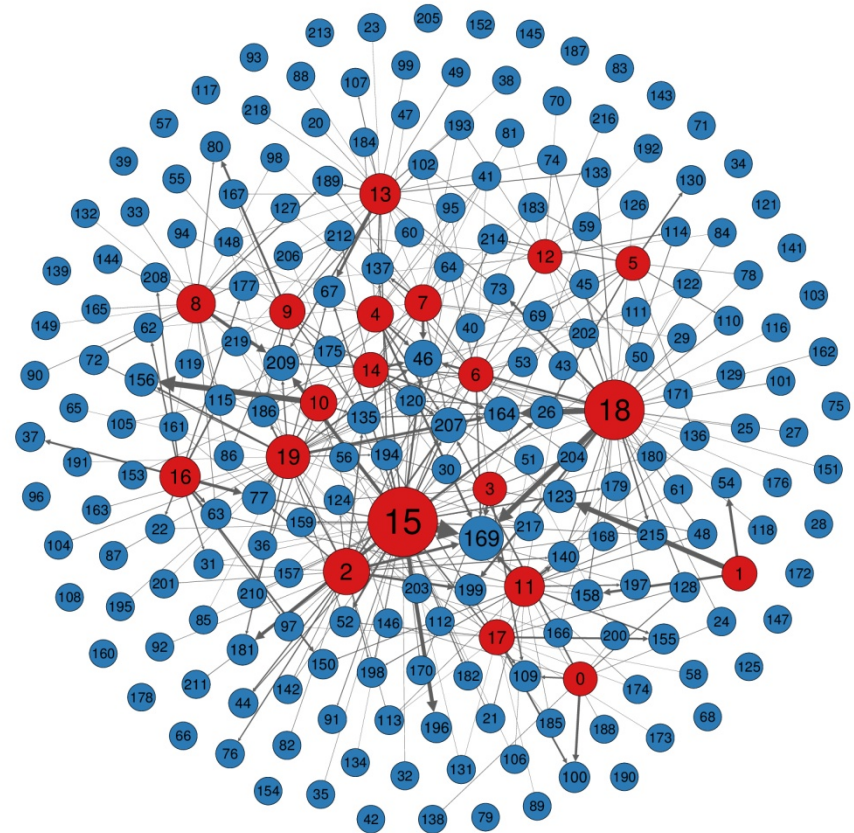
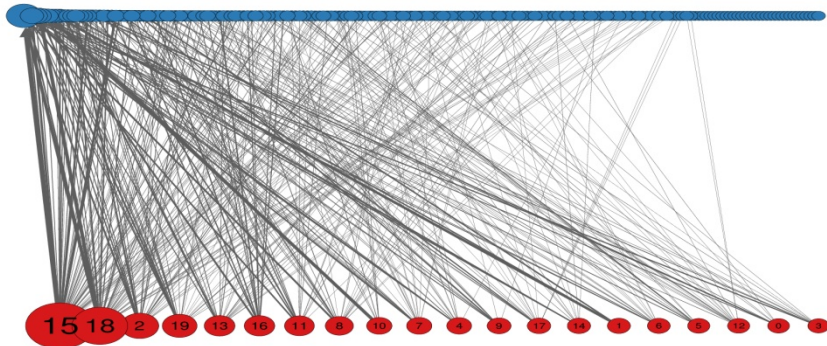
- Honoring Zipf's law, we consider a fat-tailed size distribution for both banks and firms (or their loans)
- To capture size dependence and heterogeneity, we draw the number of links per bank and firm from Poisson distributions with size-dependent parameter

$$\lambda_{i,(j)} = \overline{\lambda_{(j)}} A_{i,(j)}, j \in \{b, f\}$$

- Normalization so that averages for banks and firms are in harmony with empirical findings
- Links are then matched randomly until either the aggregate links of banks or firms are exhausted

$$\overline{\lambda}_f = 2, \overline{\lambda}_b = 20$$

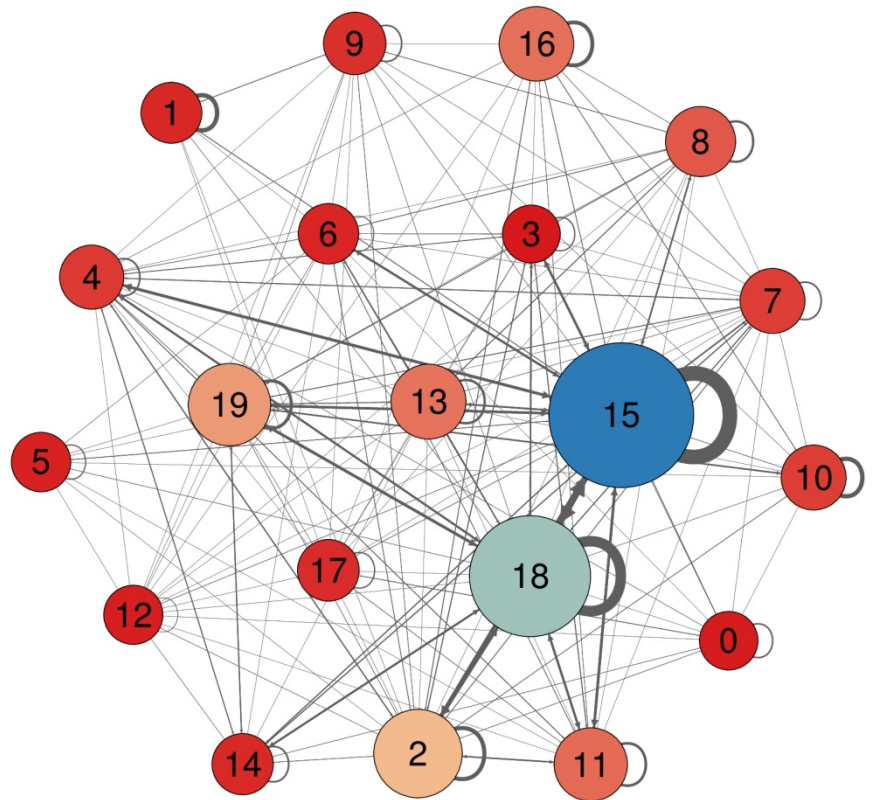
A bipartite network  
of firm and bank  
connections,  
 $N_b = 20$ ,  $N_f = 200$





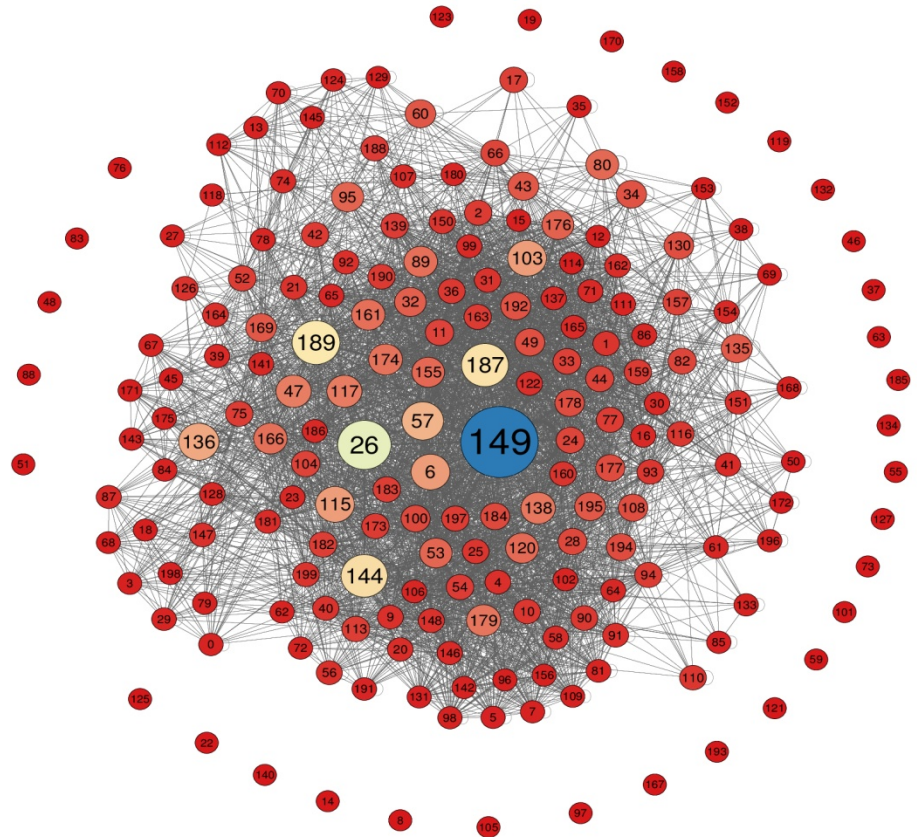
The resulting connections between *banks* via joint exposures, given by  $M M^T$

$M$ : incidence matrix of dimension  $N_b \times N_f$



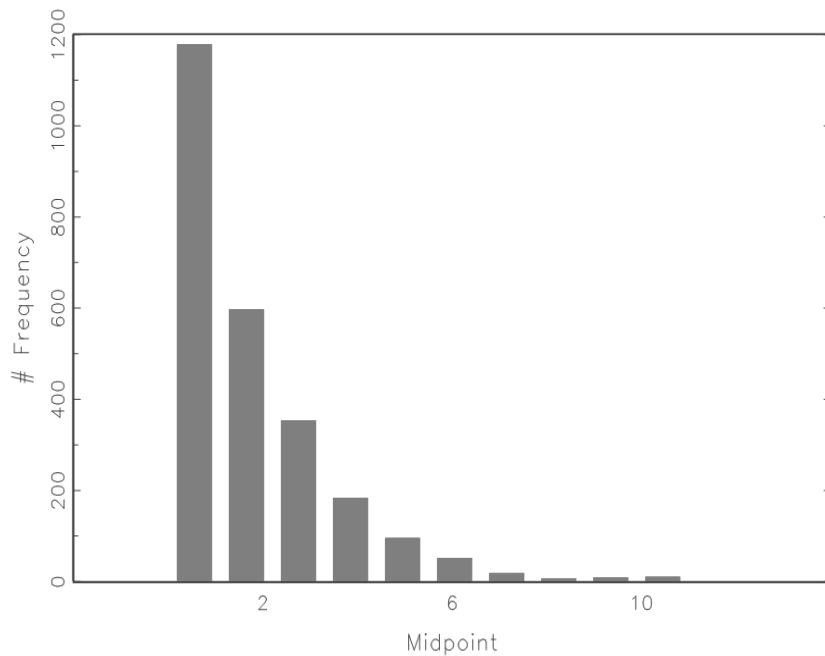


The resulting  
connections between  
*firms* via joint  
exposures, given by  
 $M^{TM}$

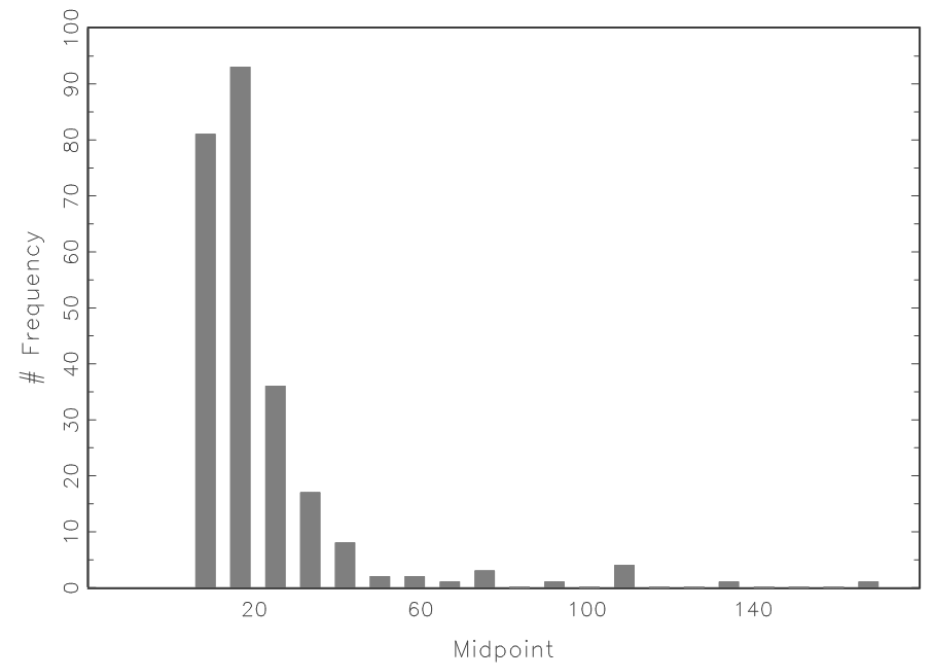


# Degree distributions of firms and banks

Distribution of Links of Firms, #firms = 2500



Distribution of Links of Banks, #banks = 250



## **Application: We now consider as external shocks the failure of a specific company**

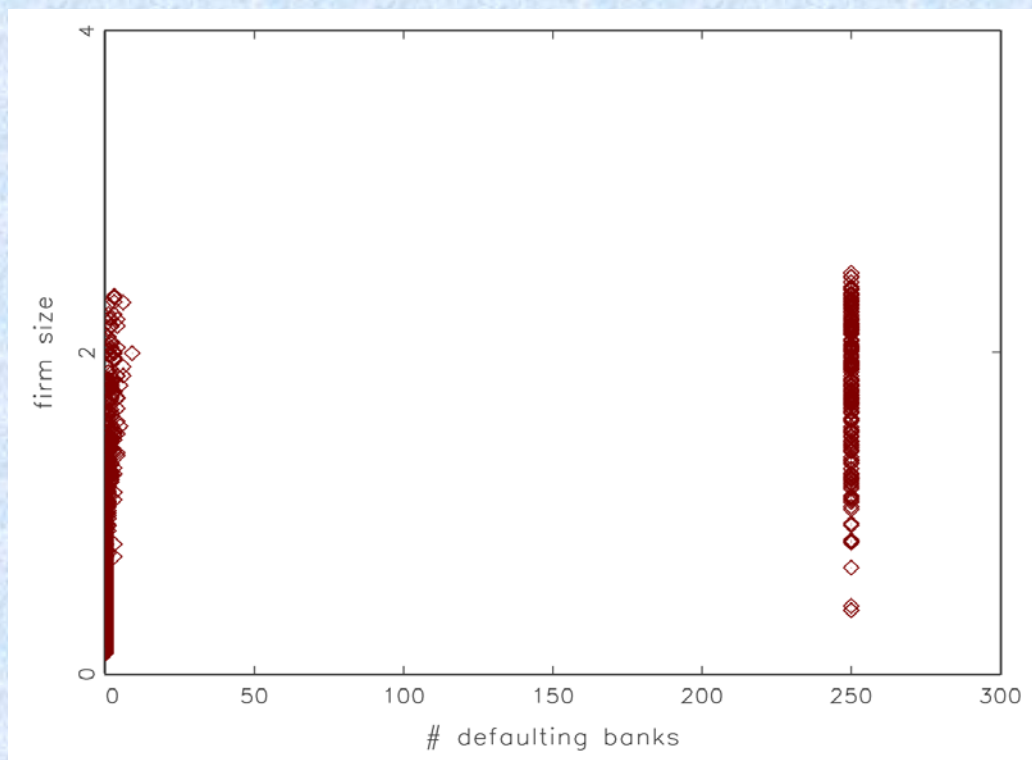
- $N_b = 250$ ,  $N_f = 10,000$ ,  $\lambda_f = 2$ ,  $\lambda_b = 80$ ;  $\alpha = 1.2$  (Pareto index)
- External assets: 50 % of balance sheet
- Equity ratio: 3 %

We consider:

- Initial default: any one of the  $N_f$  firms
- Knock-on effects (I) through interbank contagion (as before)
- Knock-on effects (II) through lack of funding for firms

# Cumulative Defaults vs. Size of Initial Disturbance

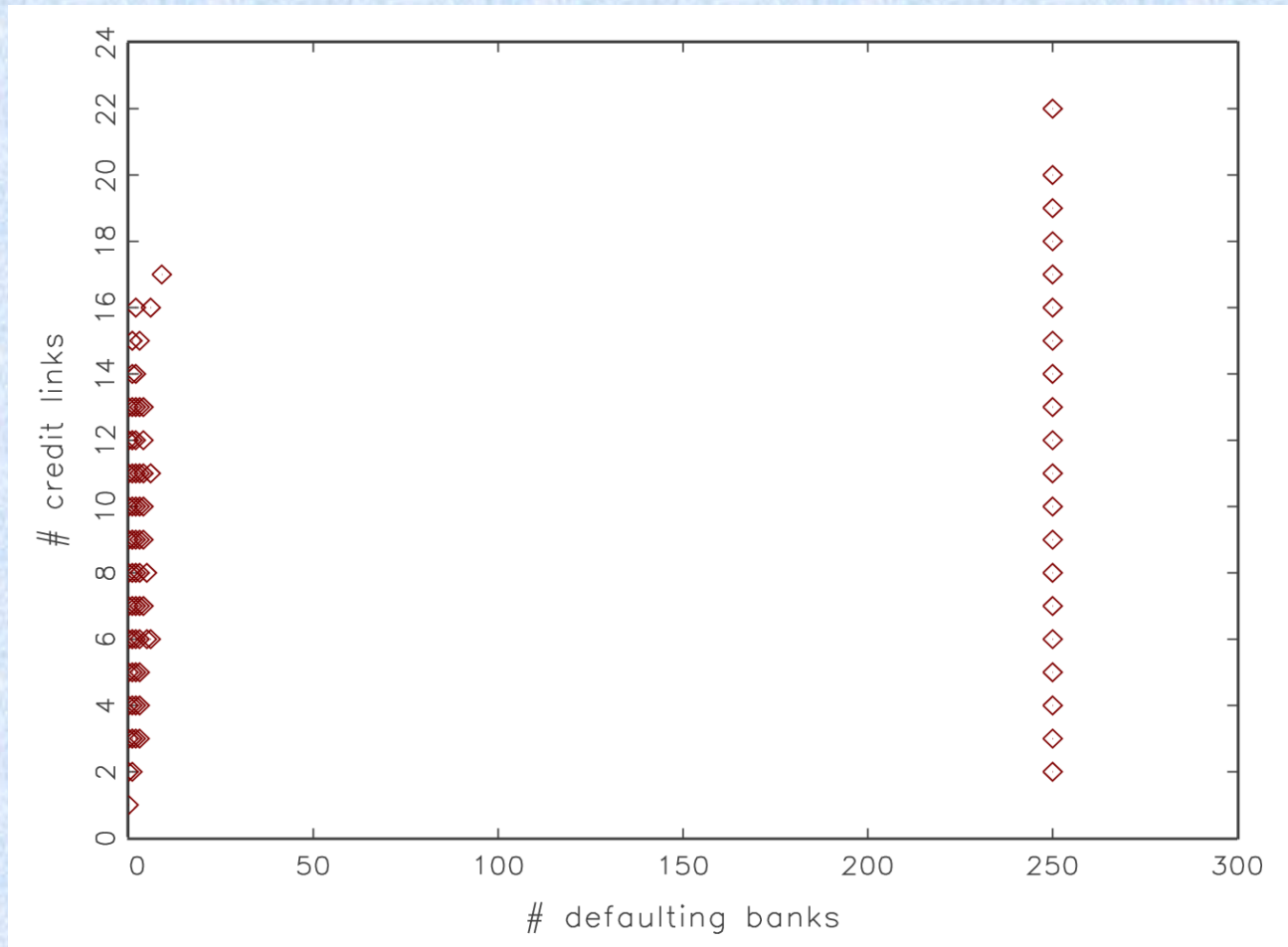
- Huge heterogeneity of no. of defaults
- almost uncorrelated to size of firm, but dependent on exact position in the network



All firms have at least one connection to a bank



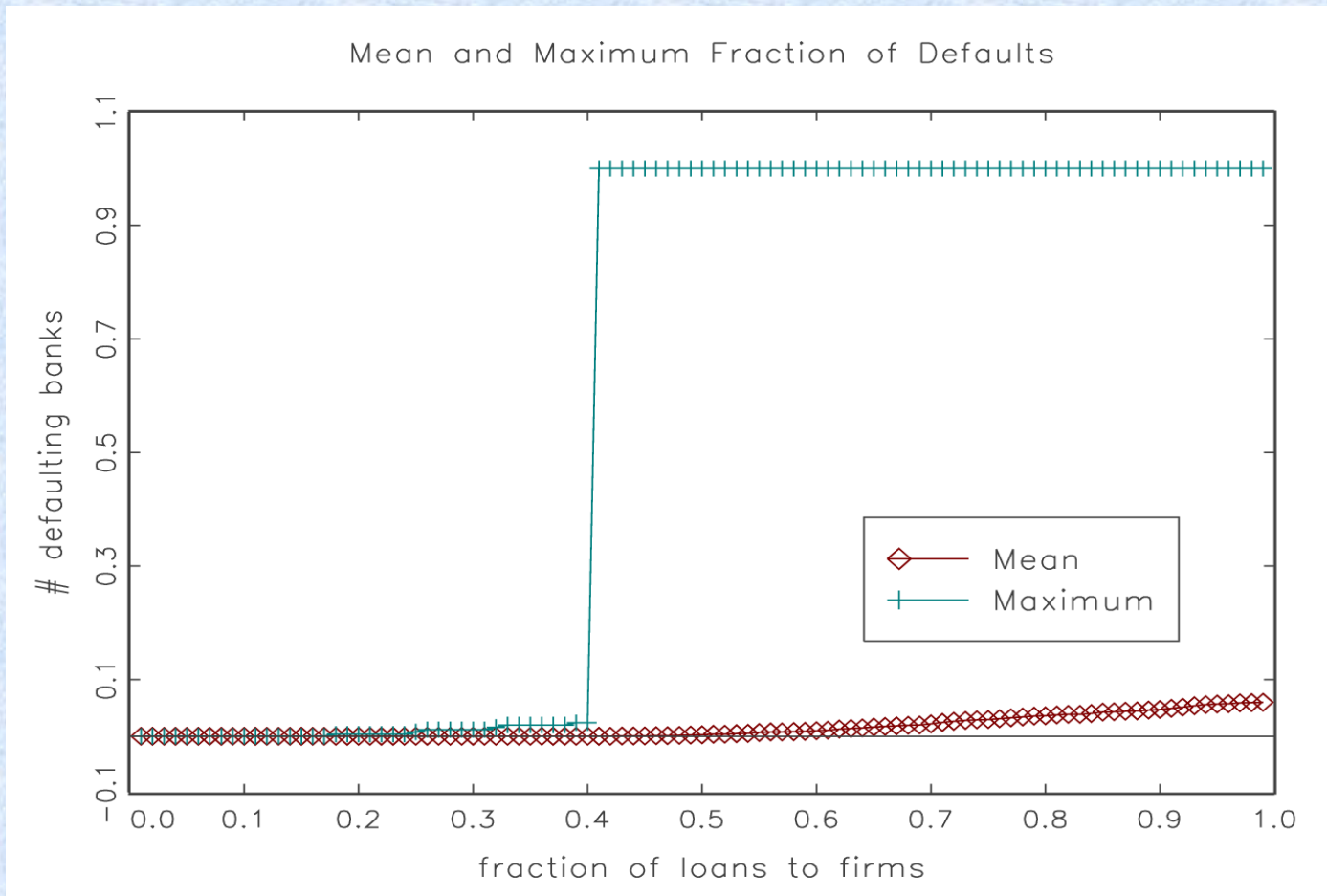
**also independent of no. of links**



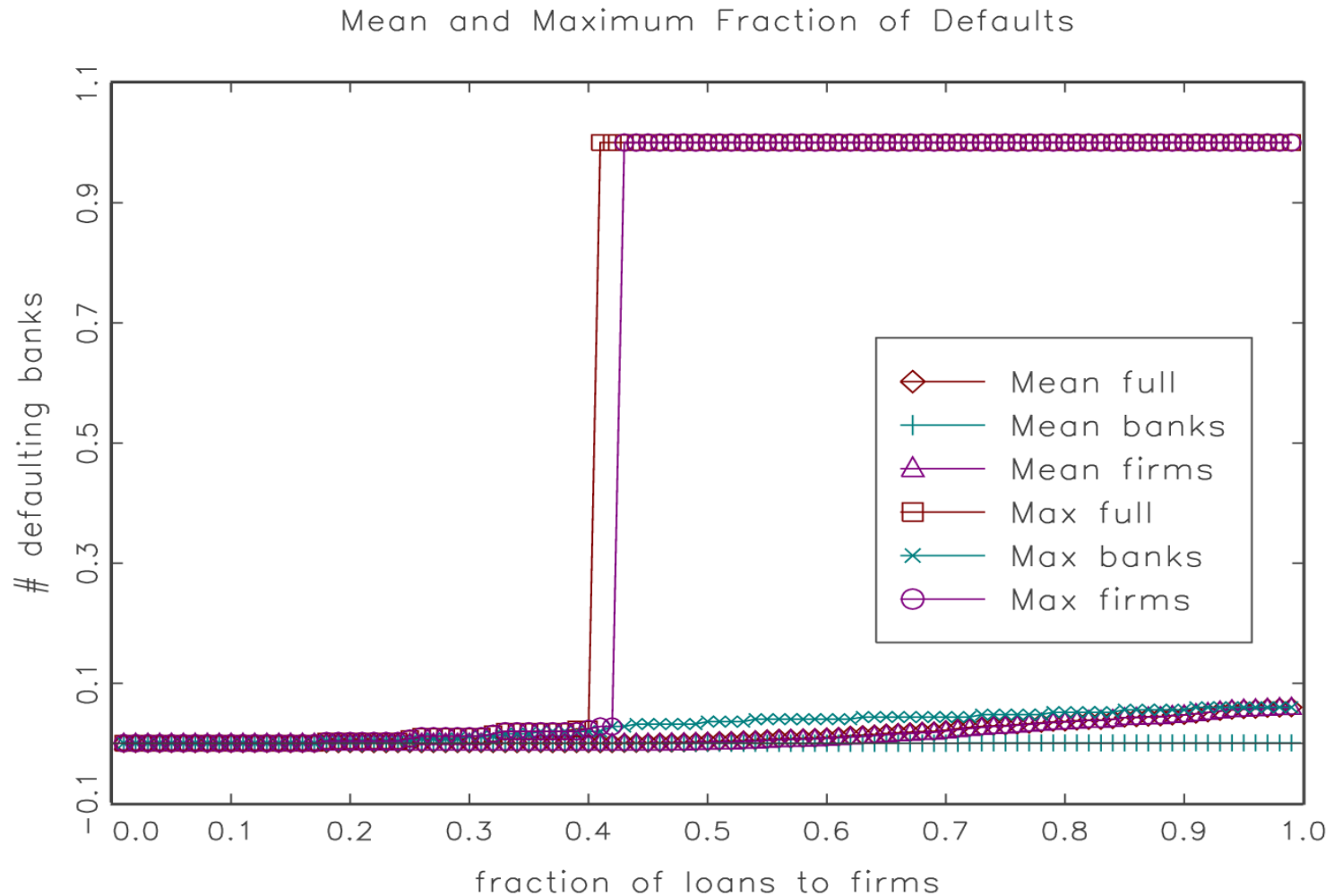
Probit model shows significant coefficients for size and degree, but forecasting is dismal.

# Similar: Role of External Assets vs. Interbank Exposures

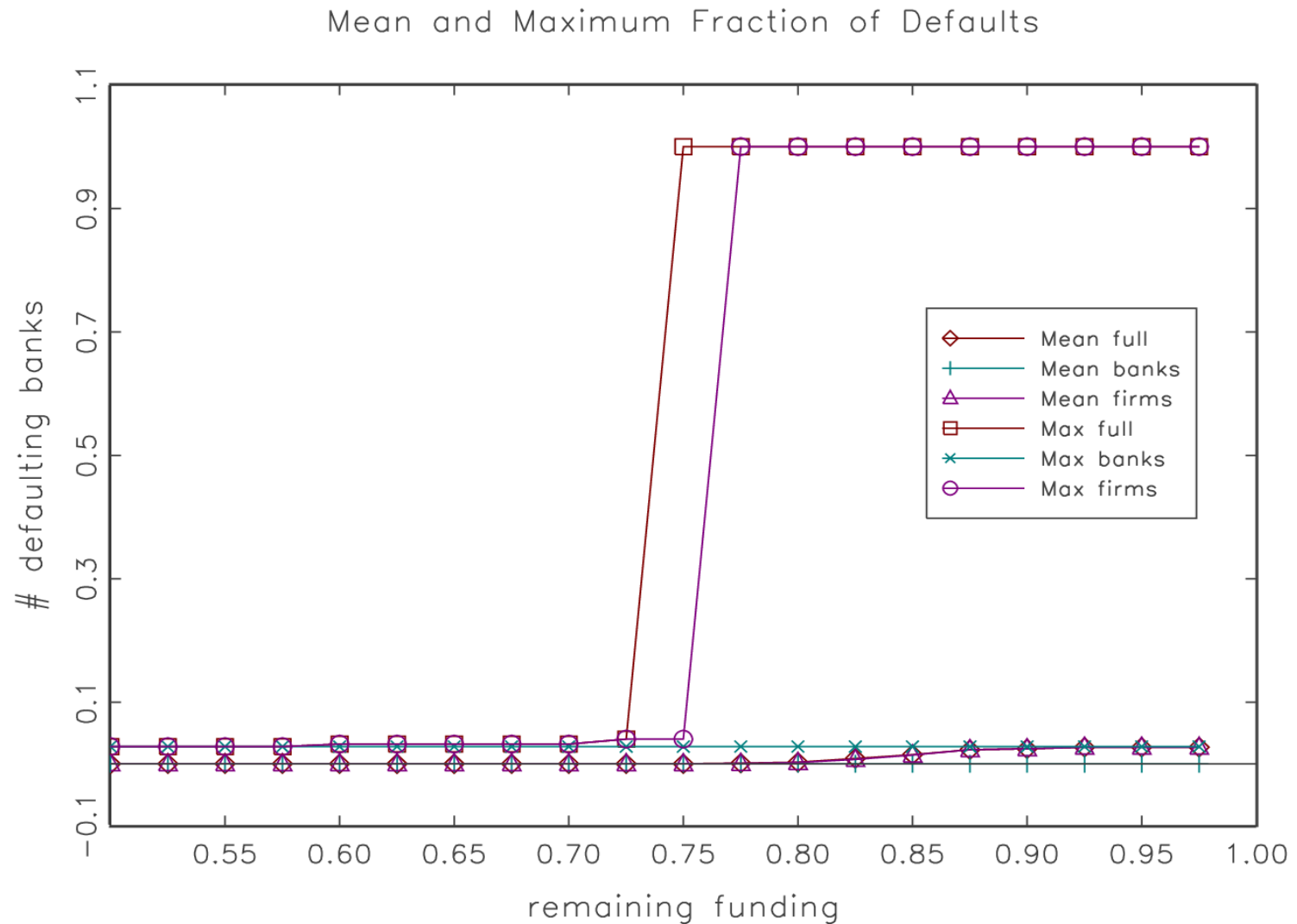
Mean and maximum no. of defaults over all sources of shocks (i.e., firms)



# Firm-Bank vs Bank-Bank Channel of Contagion

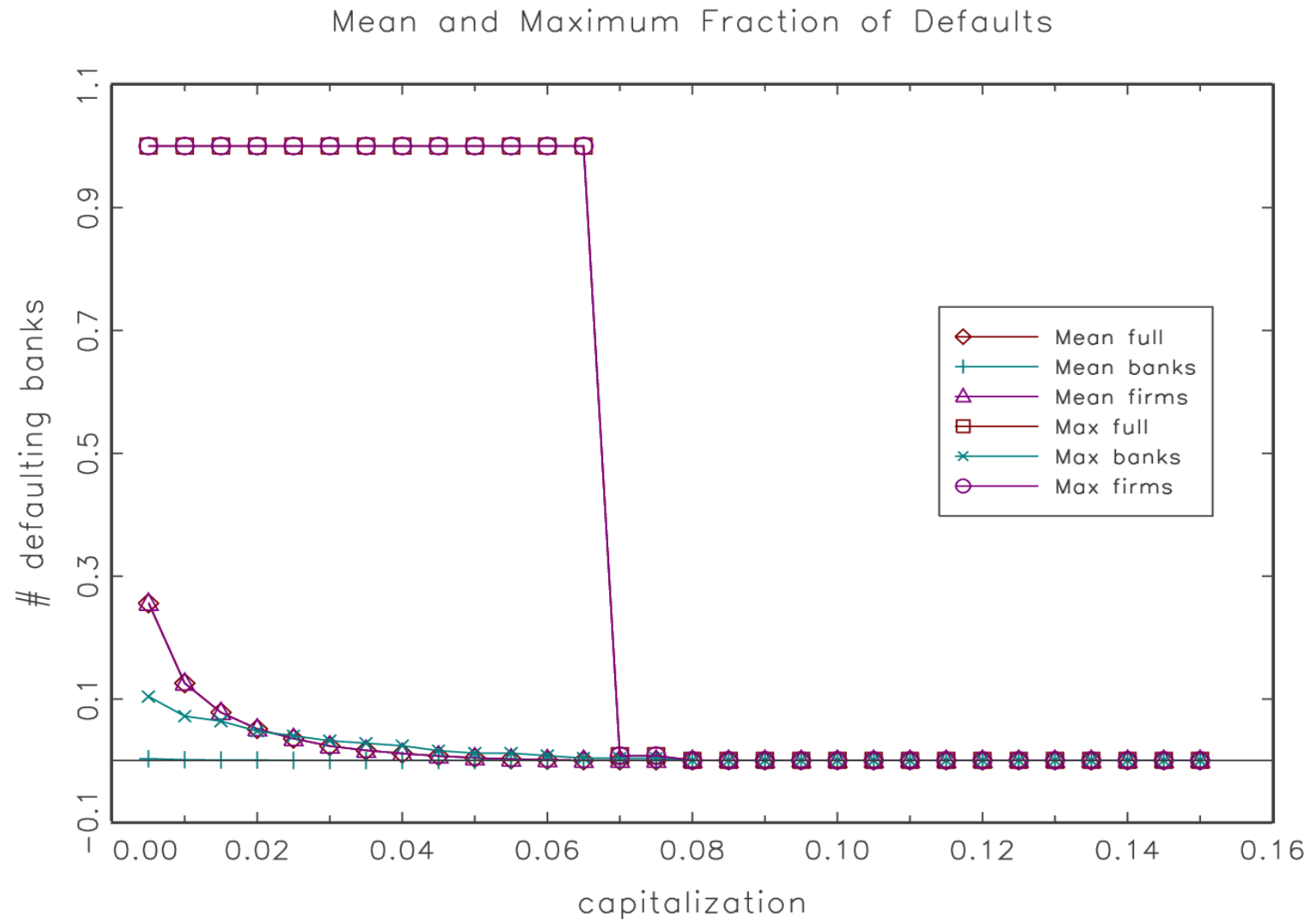


# Remaining Funding for Survival of Firm

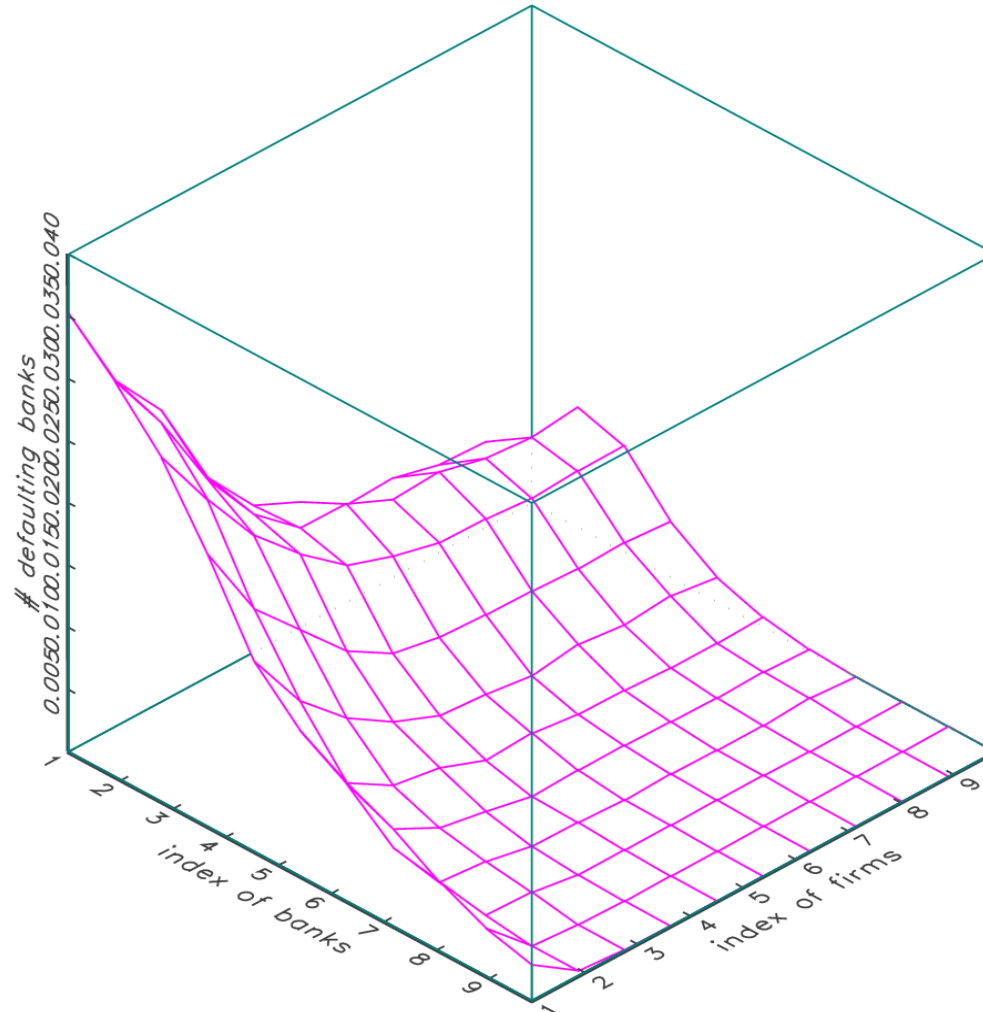




# Role of Capitalization



# Heterogeneity of Size: Different Pareto Indices for Banks and Firms



# Theoretical Analysis: The Reason for the Danger of System-Wide Breakdowns

- Degree distributions of banks and firms are Poisson-Pareto mixtures:

$$P_I(k) = \int_{\underline{\lambda}_1}^{\infty} \frac{e^{-\lambda_1} \lambda_1^k}{k!} \alpha \frac{\lambda_1^\alpha}{\underline{\lambda}_1^{\alpha+1}} d\lambda_1$$

- With probability generating functions:

$$f_o(s) = E[s^k] = \sum_{k=0}^{\infty} p_k s^k = \int_{\underline{\lambda}_1}^{\infty} e^{\lambda_1(s-1)} \alpha \frac{\lambda_1^\alpha}{\underline{\lambda}_1^{\alpha+1}} d\lambda_1$$

- $f_1(s)$ : probability distr. of edges reached from an arbitrary chosen node

- Connecting both parts of the bipartite network, the degree distribution of the banks to which the firms from a randomly drawn link are connected, is given by

$$G_I(s) = g_1(f_1(s))$$

- It is known that a giant connected component emerges if

$$G_I'(1) = g_1'(f_1(1)) * f_1'(1) = 1$$

- Leads to

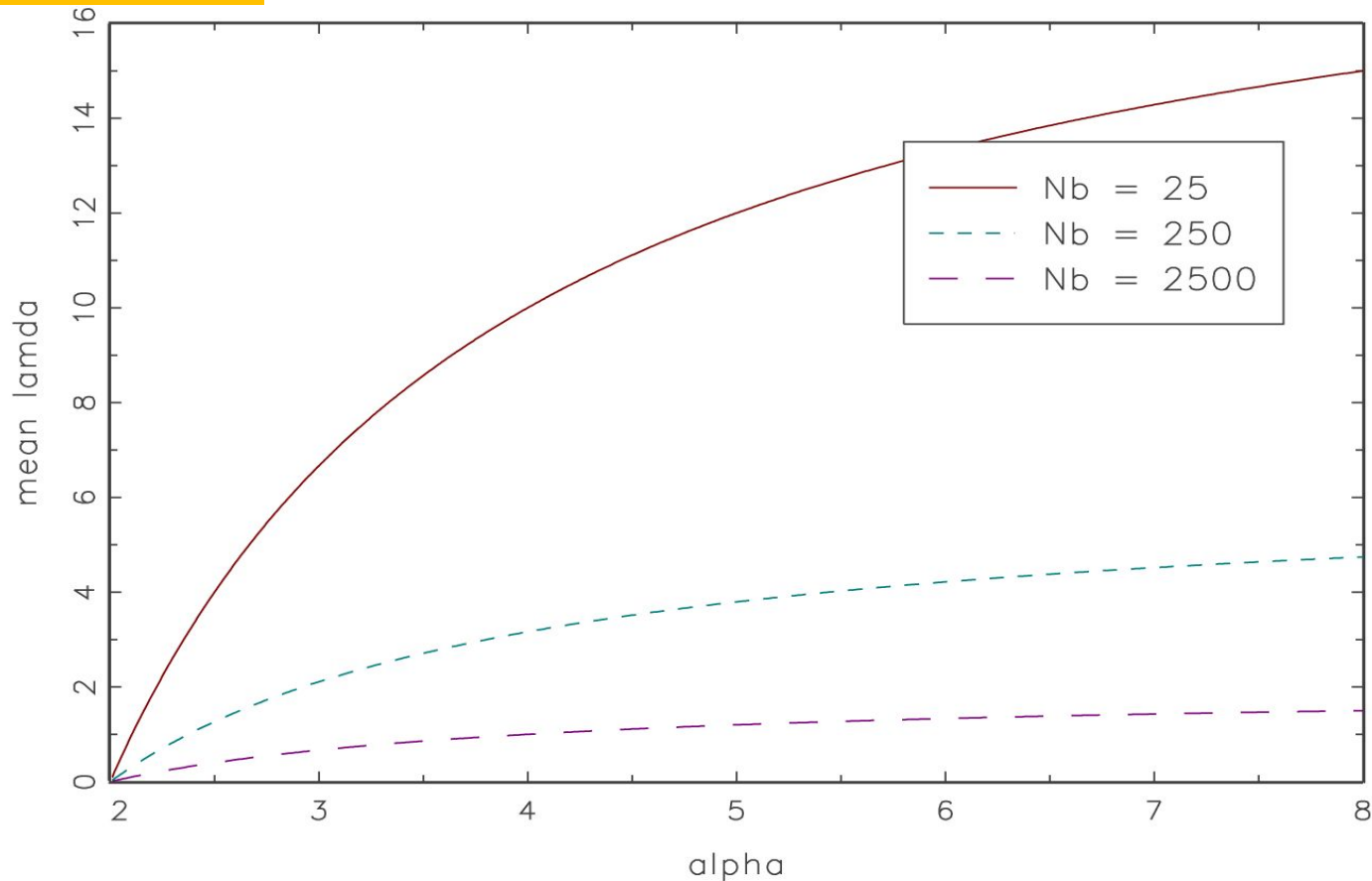
$$\lambda_b = \frac{\alpha - 2}{\alpha} \sqrt{\frac{N_f}{N_b}}$$



# Condition for Giant Component

$$\lambda_b = \frac{\alpha - 2}{\alpha} \sqrt{\frac{N_f}{N_b}}$$

Bifurcation to Giant Clustered Component



$$N_f = 10000$$

# Extension: Towards A Dynamic Model of the Interbank Market

Networks constructed from overnight credit contracts display some stylized facts:

- Dissassortativity
- Proximity to core-periphery structure
- Pronounced asymmetry between all statistics for in-  
*versus* out-activity
- Strong persistence (Jaccard index)
- Large banks mostly have high centrality
- Large banks are mostly net lenders

# A very simple dynamic model

- Ensemble of banks with *power-law distribution* of balance sheet size
- Simple balance sheet structures
- Banks are facing liquidity shocks that are mean-reverting and have mean zero
- Liquidity is reallocated in the system through borrower-initiated trades in interbank market
- Banks decide about potential lender via a trust function depending on past experience

## Dynamic evolution

- Banks are hit in every period by liquidity shocks:

$$shock_{i,t} = \beta(\bar{d}_i - d_{i,t}) + \sigma_i \varepsilon_{i,t}$$

- ...mean-reverting to bank-specific mean, with bank-specific size of random shock
- If  $shock < 0$ : bank asks for credit at other banks choosing creditor according to a „trust“ function
- If  $shock > 0$ : credit is paid back; if no credit remaining, liquidity increases



## Dynamic evolution ...ct'd

Trust function: Matrix  $T_{ij}$  with entries in range (0,1)

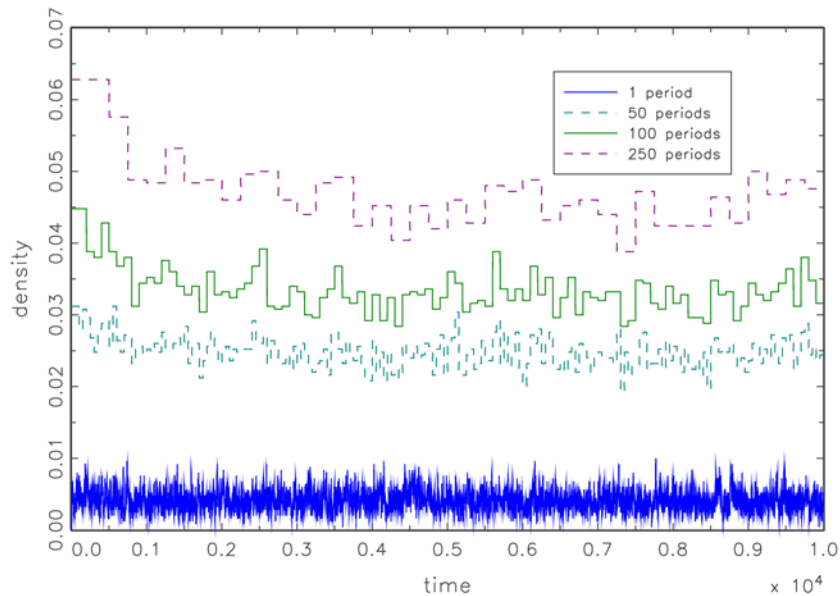
- Banks contact partners with highest trust value
- If credit is granted: trust increases; it decreases if credit is declined (reinforcement learning)
- In this minimal model: credit is declined only if liquidity of creditor bank is below a certain lower threshold:

$$m_{i,t} > \overline{m_i} \equiv \mathcal{A}_i$$

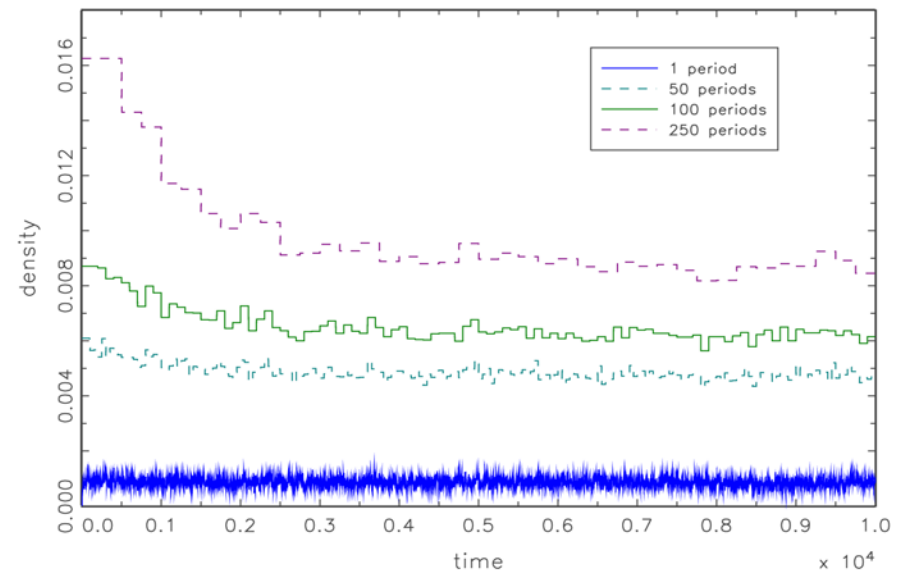
- Note: nothing is happening in this simple model to external assets and net worth of banks!

# Results: The system converges to a statistical equilibrium

Temporal Evolution of Density, # = 50 Banks

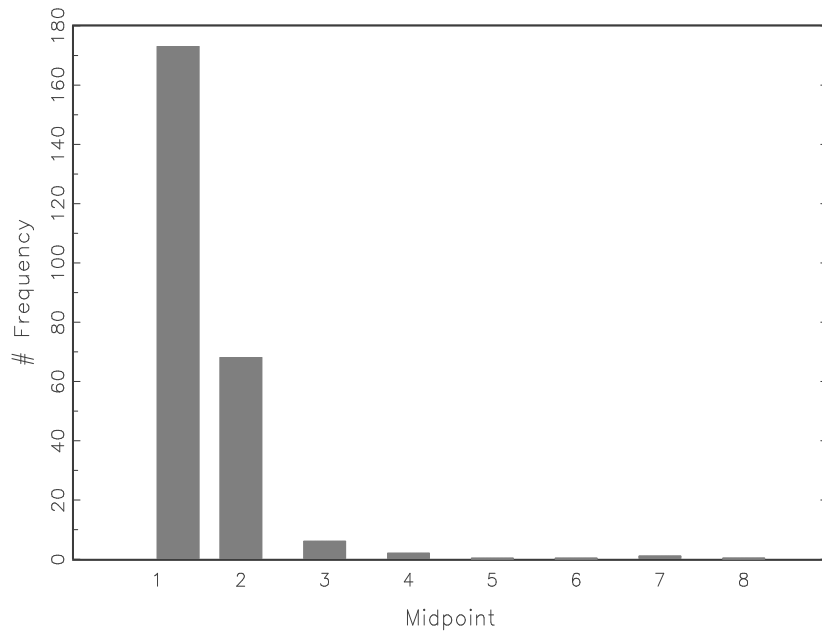


Temporal Evolution of Density, # = 250 Banks

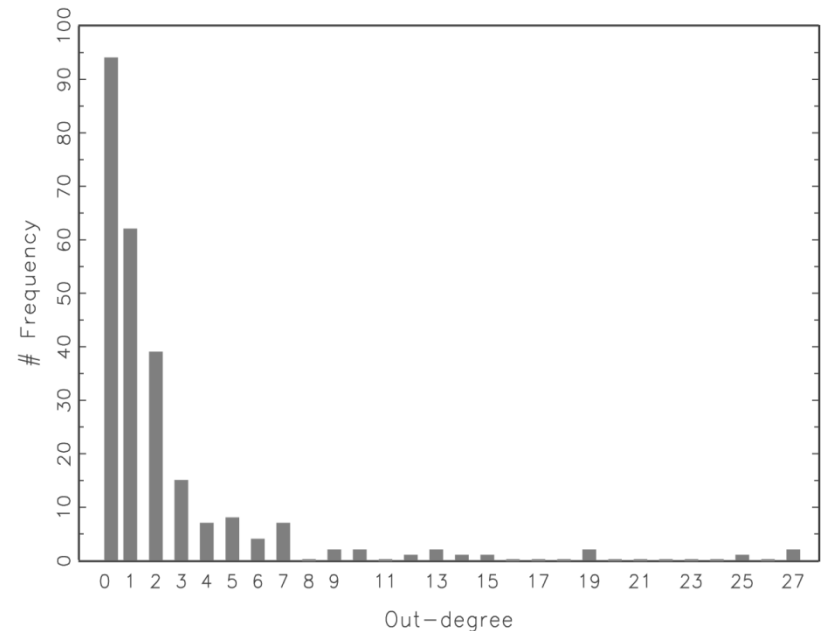


Underlying parameters are:  $a = 5$ ,  $b = 100$ ,  $\alpha = 2$  (size distribution),  $N = 50$  and  $250$ ,  $\theta = 0.8$ ,  $\gamma = 0.08$ ,  $\beta = 0.5$ ,  $\sigma_i \sim 0.025 A_i$ ,  $\vartheta = 0.04$

Distribution of In-Degrees, #banks = 250

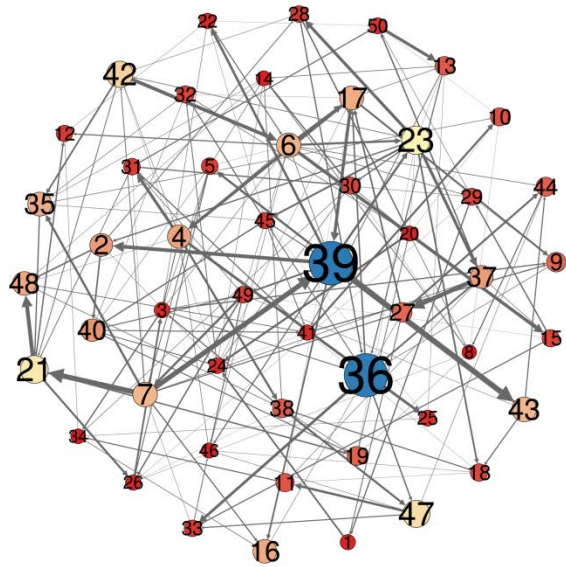


Distribution of Out-Degrees, #Banks = 250

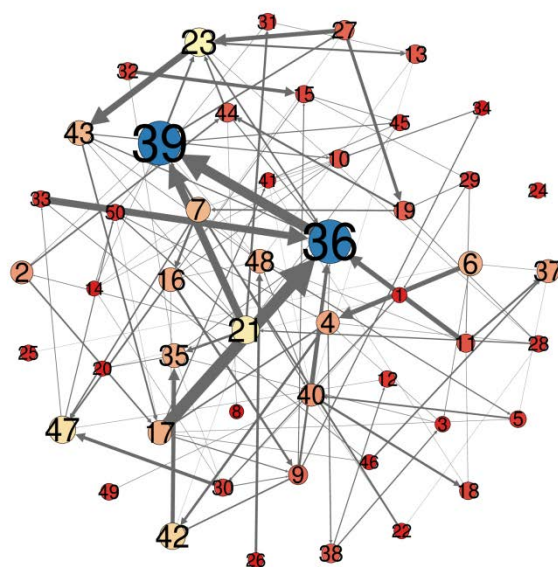


Typical asymmetry: many banks have zero out-links, but some have relatively many,  
distribution of in-links much more narrow

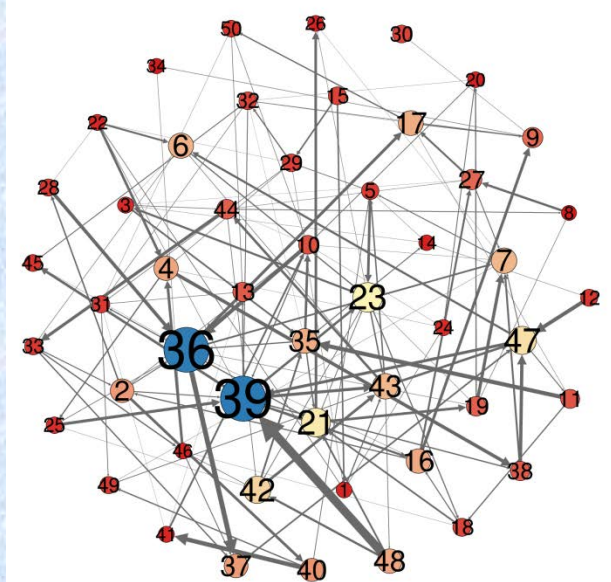
## Development of Network Structure



$t = 100$



$t = 5000$

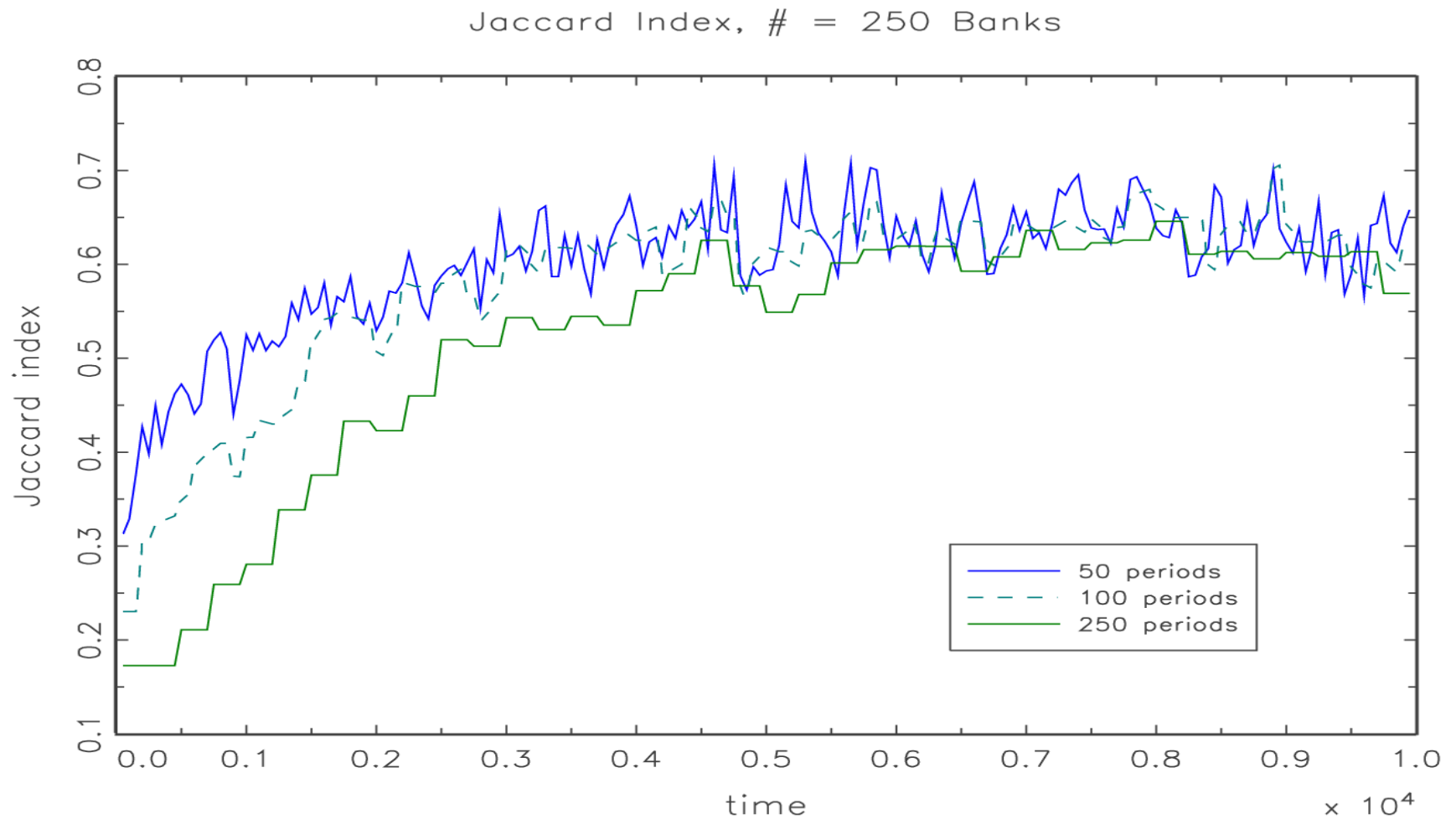


$t = 10000$

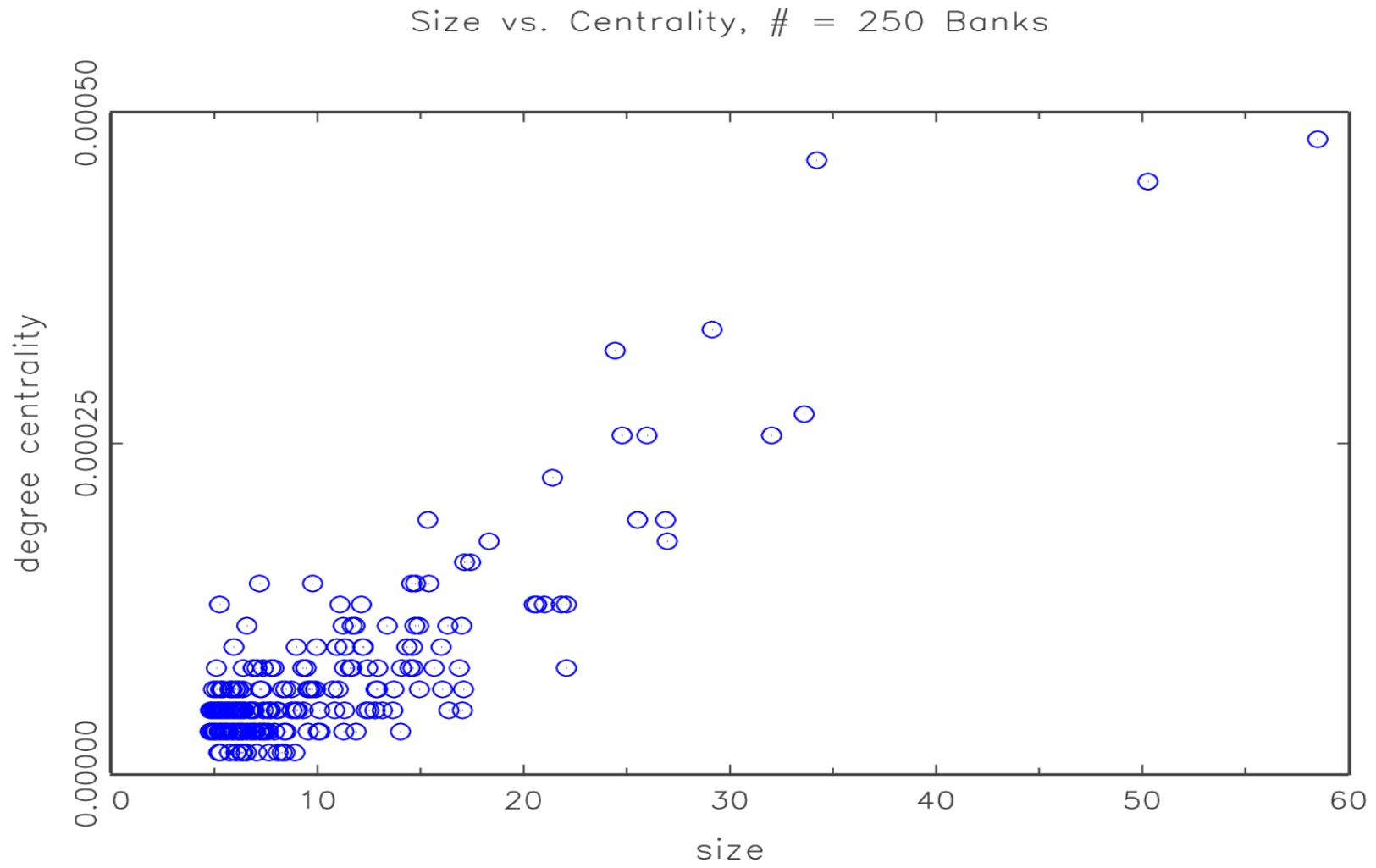
Development of core-periphery structure as documented by Craig/ von Peters, Fricke/Lux and Lelyfeld/in't Veld

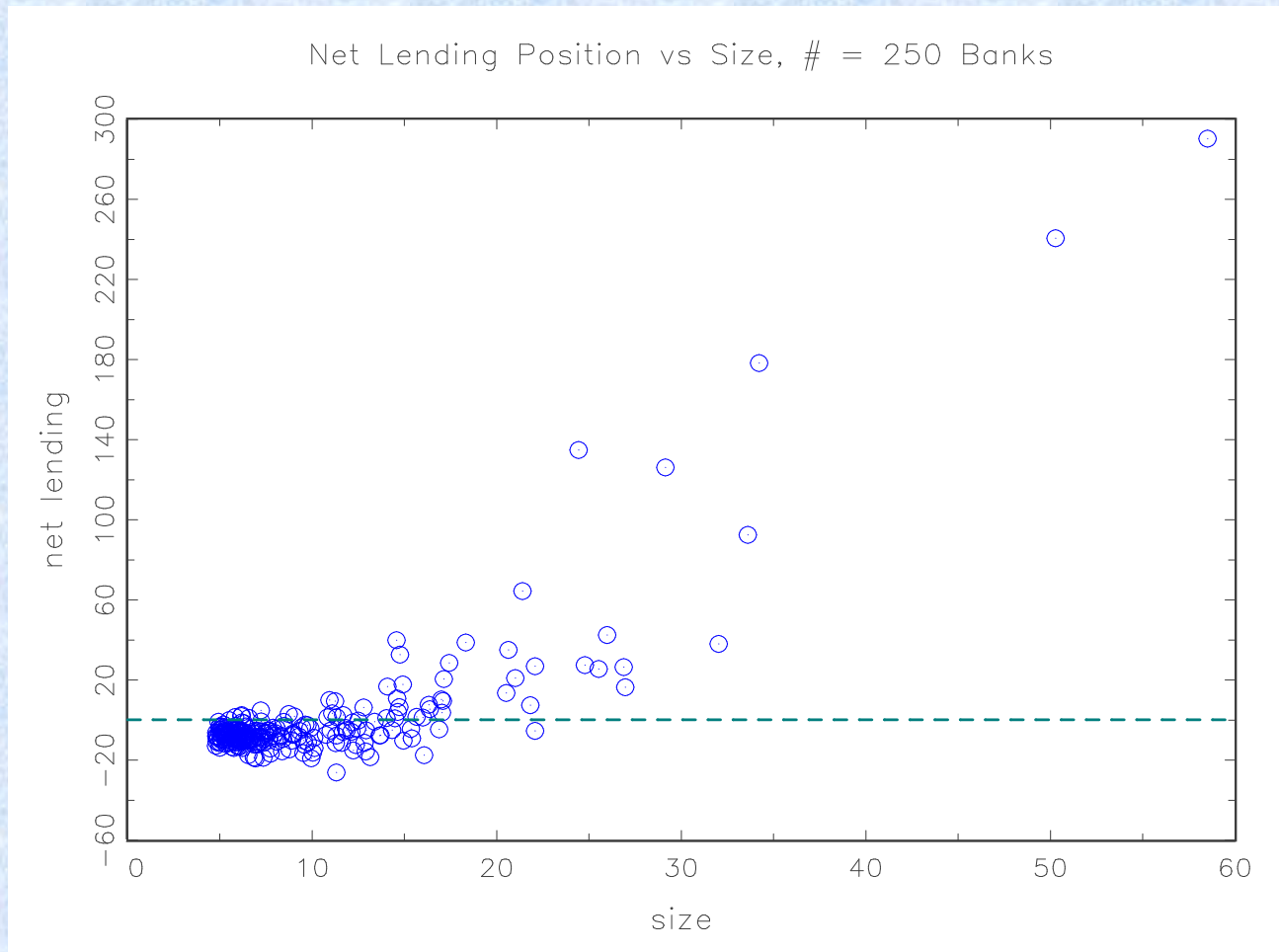


# Assortativity



# Size versus centrality





Larger banks are net lender, smaller banks are net borrowers,  
Despite symmetric shocks!

## Sensitivity analysis: Different size distributions vs homogeneous banking system

alpha	1.2			1.6			uniform		
Aggregation level	50	100	250	50	100	250	50	100	250
Density	0.023 (0.002)	0.028 (0.003)	0.035 (0.004)	0.023 (0.002)	0.029 (0.003)	0.037 (0.004)	0.029 (0.002)	0.041 (0.003)	0.059 (0.003)
Avg. In-degree	1.127 (0.116)	1.396 (0.135)	1.770 (0.192)	1.141 (0.125)	1.427 (0.172)	1.852 (0.225)	1.457 (0.114)	2.036 (0.125)	2.959 (0.141)
Max. In-degree	2.570 (0.714)	3.120 (0.756)	4.060 (1.033)	2.670 (0.697)	3.230 (0.737)	4.280 (1.064)	3.360 (0.523)	4.010 (0.541)	5.250 (0.687)
Max. Out-degree	15.740 (4.775)	17.420 (4.637)	19.060 (4.259)	15.980 (6.307)	17.820 (5.931)	19.560 (5.972)	5.340 (0.879)	6.200 (1.155)	7.620 (1.071)
Jaccard index	0.640 (0.090)	0.665 (0.059)	0.655 (0.043)	0.607 (0.088)	0.643 (0.072)	0.639 (0.050)	0.416 (0.046)	0.497 (0.043)	0.566 (0.030)
Measured at time	250	5,000	10,000	250	5,000	10,000	250	5,000	10,000
Assortativity	-0.096 (0.044)	-0.312 (0.090)	-0.347 (0.097)	-0.079 (0.043)	-0.267 (0.098)	-0.311 (0.105)	-0.040 (0.048)	-0.088 (0.070)	-0.066 (0.081)
Core size	0.178 (0.011)	0.120 (0.013)	0.117 (0.014)	0.179 (0.012)	0.118 (0.014)	0.119 (0.015)	0.185 (0.010)	0.121 (0.004)	0.120 (0.003)
Dep. on lender		0.851 (0.042)			0.840 (0.048)			0.650 (0.024)	
Dep. on borrower		0.196 (0.063)			0.241 (0.078)			0.603 (0.033)	
Correlation size-centrality		0.916 (0.032)			0.909 (0.038)			0.036 (0.143)	
Correlation size-net lending		0.869 (0.083)			0.866 (0.082)			0.030 (0.150)	

Homogeneous system does not match most stylized facts



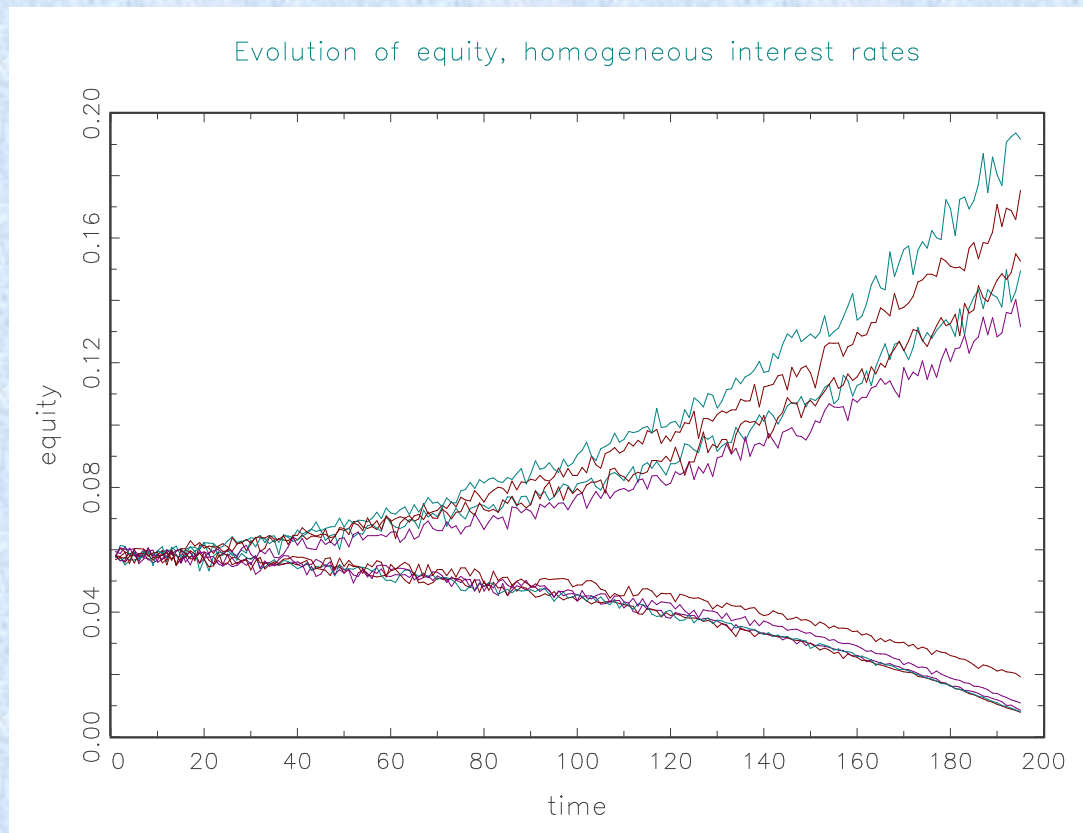
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Avg. In-degree	1.127 (0.116)	1.396 (0.135)	1.770 (0.192)	1.141 (0.125)	1.427 (0.172)	1.852 (0.225)	1.457 (0.114)	2.036 (0.125)	2.959 (0.141)
Max. In-degree	2.570 (0.714)	3.120 (0.756)	4.060 (1.033)	2.670 (0.697)	3.230 (0.737)	4.280 (1.064)	3.360 (0.523)	4.010 (0.541)	5.250 (0.687)
Max. Out-degree	15.740 (4.775)	17.420 (4.637)	19.060 (4.259)	15.980 (6.307)	17.820 (5.931)	19.560 (5.972)	5.340 (0.523)	6.900 (0.541)	7.690 (0.687)
Jaccard index	0.640 (0.090)	0.665 (0.059)	0.655 (0.043)	0.607 (0.088)	0.643 (0.072)	0.639 (0.050)	Empirical: 0.6 (e-MID)		
Measured at time	250	5,000	10,000	250	5,000	10,000			
Assortativity	-0.096 (0.044)	-0.312 (0.090)	-0.347 (0.097)	-0.079 (0.043)	-0.267 (0.098)	-0.311 (0.105)	Empirical: -0.2 (e-MID) -0.5 (Germany)		
Core size	0.178 (0.011)	0.120 (0.013)	0.117 (0.014)	0.179 (0.012)	0.118 (0.014)	0.119 (0.015)			
Dep. on lender		0.851 (0.042)			0.840 (0.048)		Lender: 87 % Borrower: 43 (Germany)		
Dep. on borrower		0.196 (0.063)			0.241 (0.078)				
Correlation size-centrality		0.916 (0.032)			0.909 (0.038)				
Correlation size-net lending		0.869 (0.083)			0.866 (0.082)				

Homogeneous system does not match most stylized facts

# Adding (constant) interest rates

Equity of 5 smallest and 5 largest banks,  $r = 0.015/250$

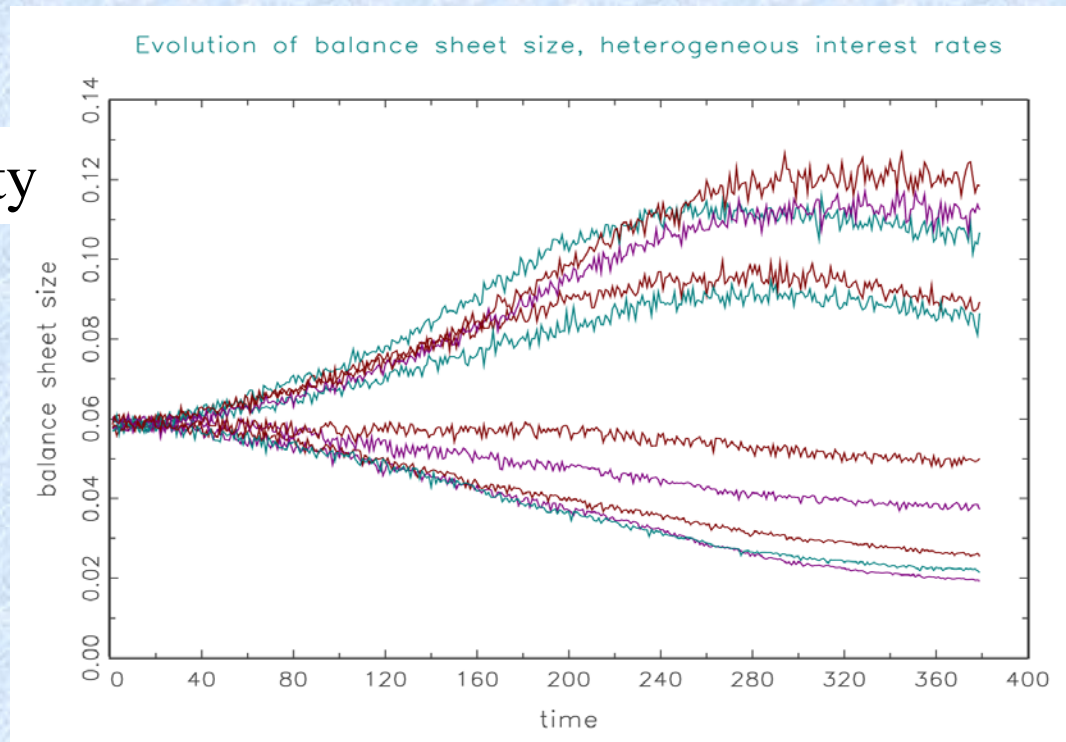


Intermediation benefits the largest banks and is costly to the smaller ones - at zero average liquidity shocks!

# Adding (heterogeneous) interest rates

Equity of 5 smallest and 5 largest banks, same initial conditions

Equity



Relationship-based interest rates compensate for „exploitation“ by intermediators



# Conclusions

- We have extended existing interbank models considering loans to firms and a more realistic structure of the banking sector
- firm-bank links shows a higher potential for cumulative failures than interbank credit
- a dynamic model with reinforcement learning easily replicates the CP structure and other important stylized facts as *emergent phenomena*
- much of the dynamics of a these system is still analytically tractable