Agentenbasierte Modellierung

Thomas Lux

Lehrstuhl für Geld, Währung und Internationale Finanzmärkte

Universität Kiel &

Bank of Spain Chair in Computational Economics,

University Jaume I, Castellón

IMK-Workshop „Pluralismus in der Ökonomie“,
IG-Metall-Bildungsstätte Berlin, 8. – 10. 8.2014

Part of this research has received funding from the European Union’s Seventh Framework Programme under grant agreement no. 612955

Plan des Workshops

- Was passiert an Finanzmärkten? Die Daten und ihre zentralen Eigenschaften
- Erklärung des Marktgeschehens durch agentenbasierte Modelle
- *Technicalities*: ABMs als stochastische Systeme
- Eine empirische Anwendung: Analyse von *animal spirits*
- Der Interbankenmarkt: Stilisierte Fakten und ABMs

**If we restrict ourselves to models which can be solved analytically,
we will be modeling for our mutual entertainment, not to maximize explanatory or predictive power.“**

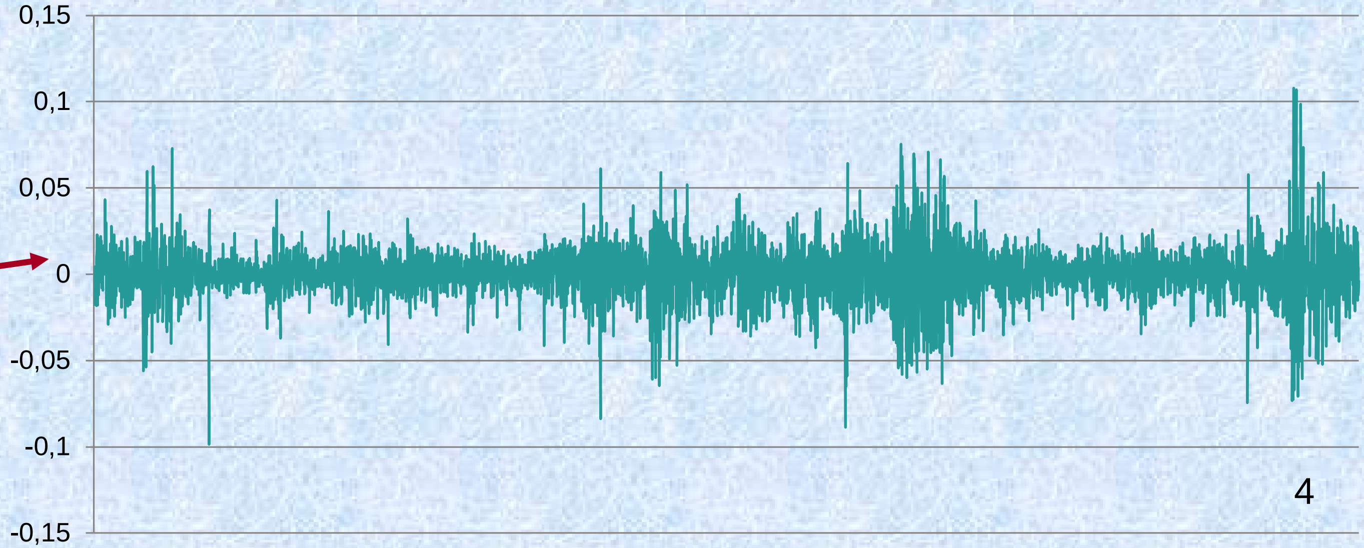
HARRY M. MARKOWITZ, Nobel Laureate in Economics

“.. Macro activity is essentially the result of the interactions between agents..”
(Economist James Ramsey, 1996)

Preliminaries
1. A few time series…

DAX
Index

Returns:
relative daily
changes

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daily log returns
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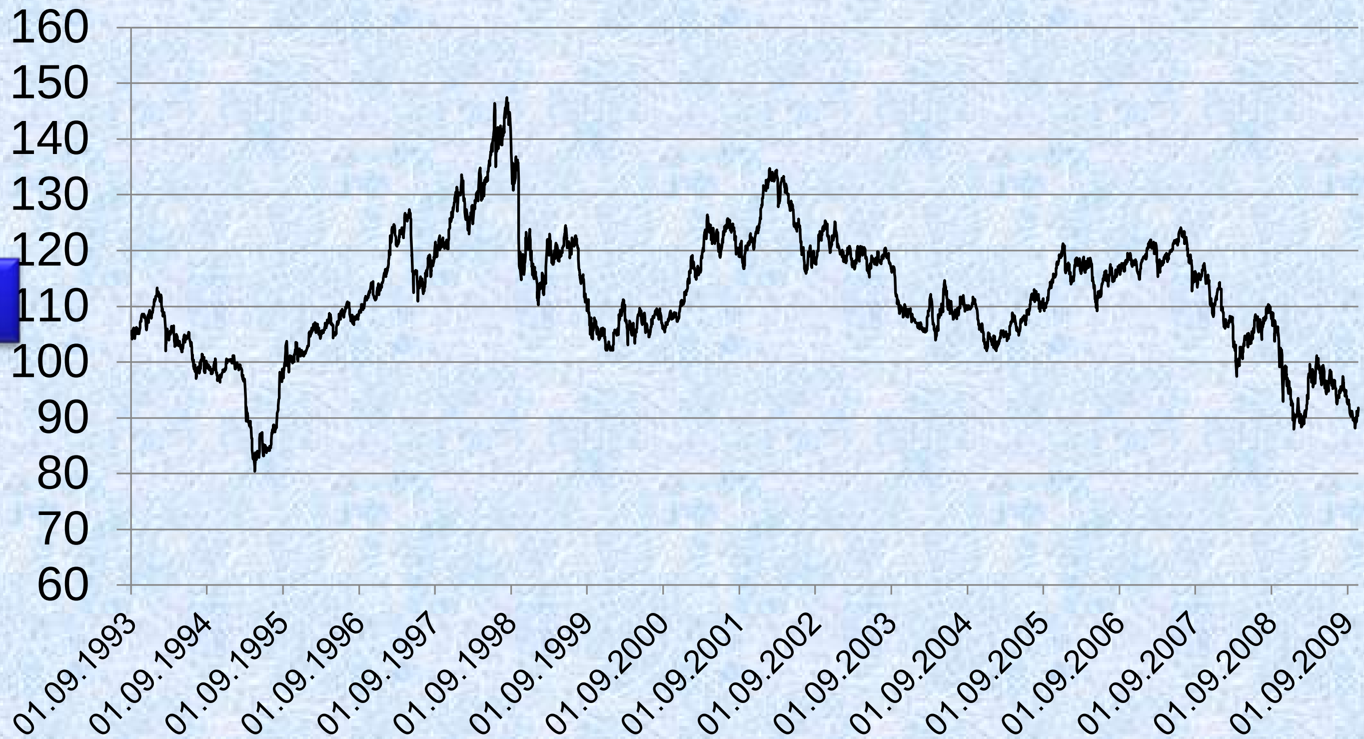
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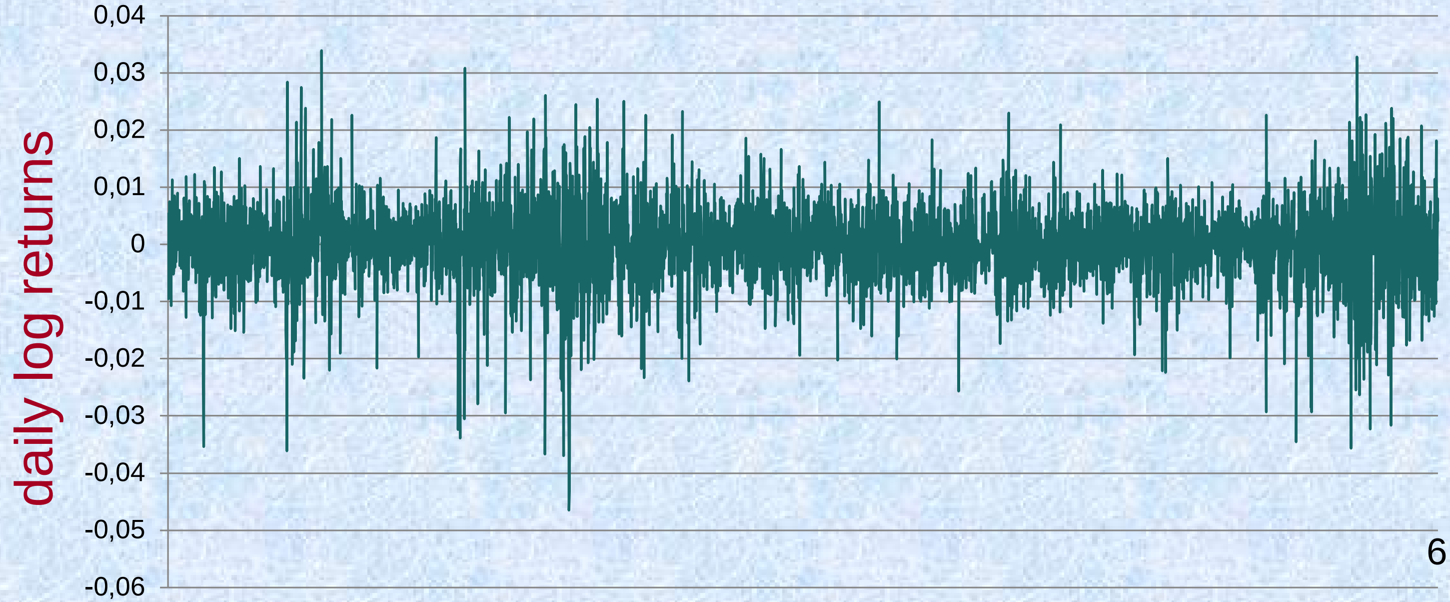
daily log returns
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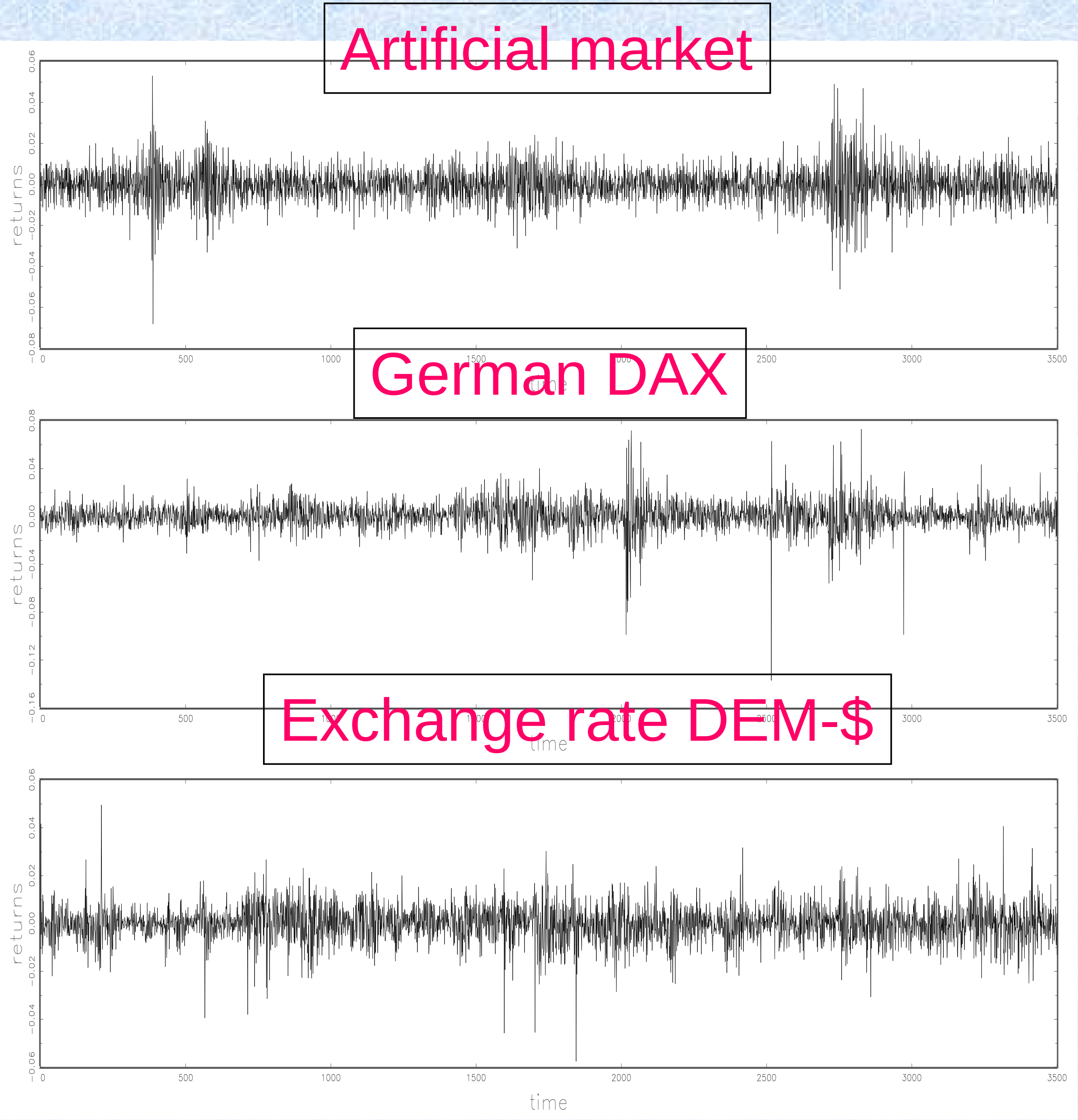
-0,06

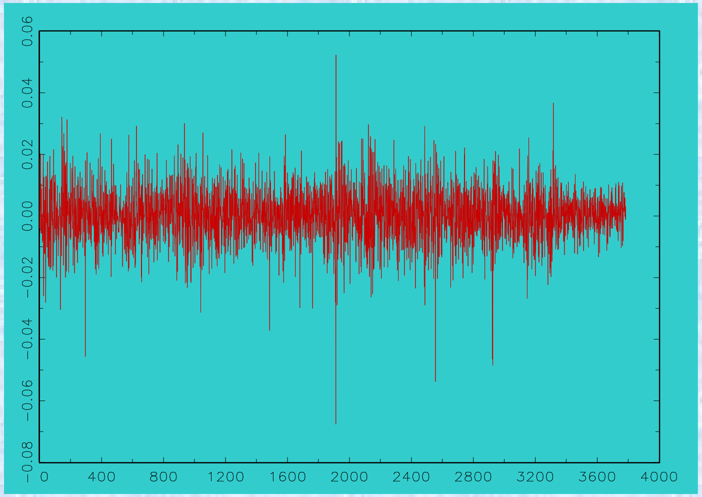


returns =

$\ln(p_t) - \ln(p_{t-1})$

Erstaunliche Beobachtung: Renditen sehen immer ähnlich aus

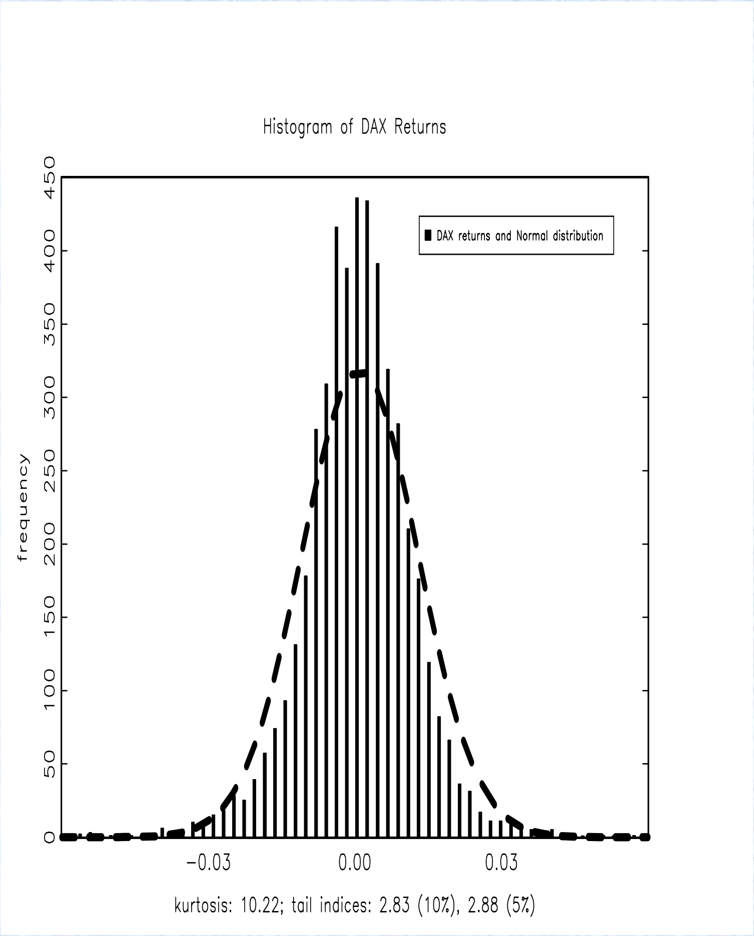
- keine zeitlichen Muster
- Mittelwert ~ 0
- ziemlich symmetrisch um Null
- wirken wie Zufallsprozess

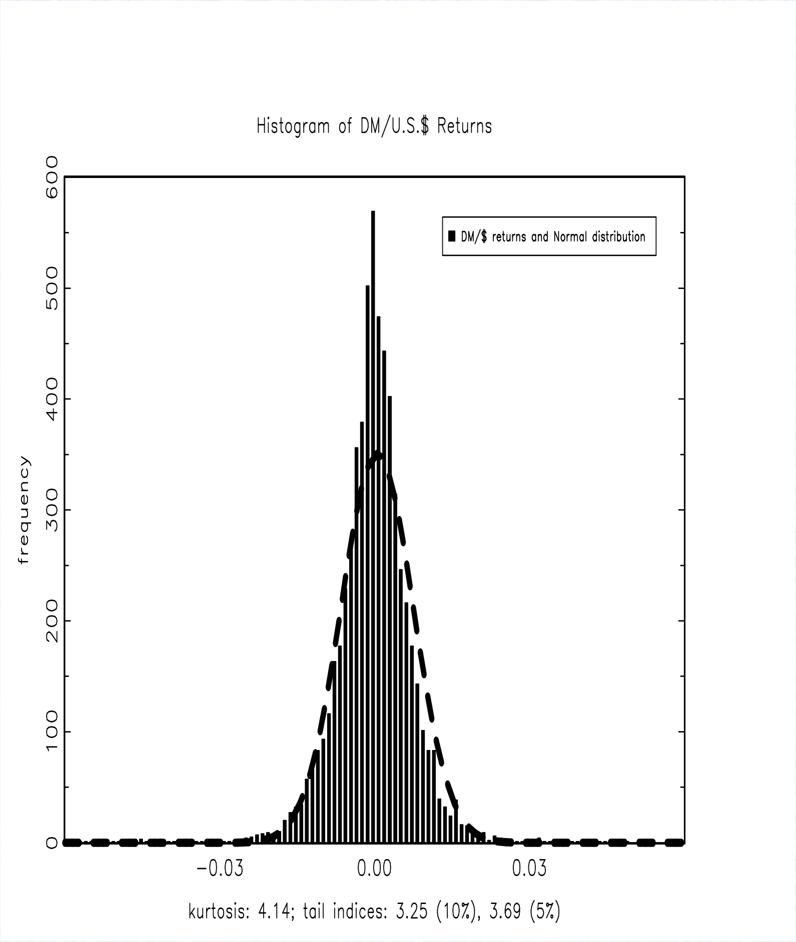


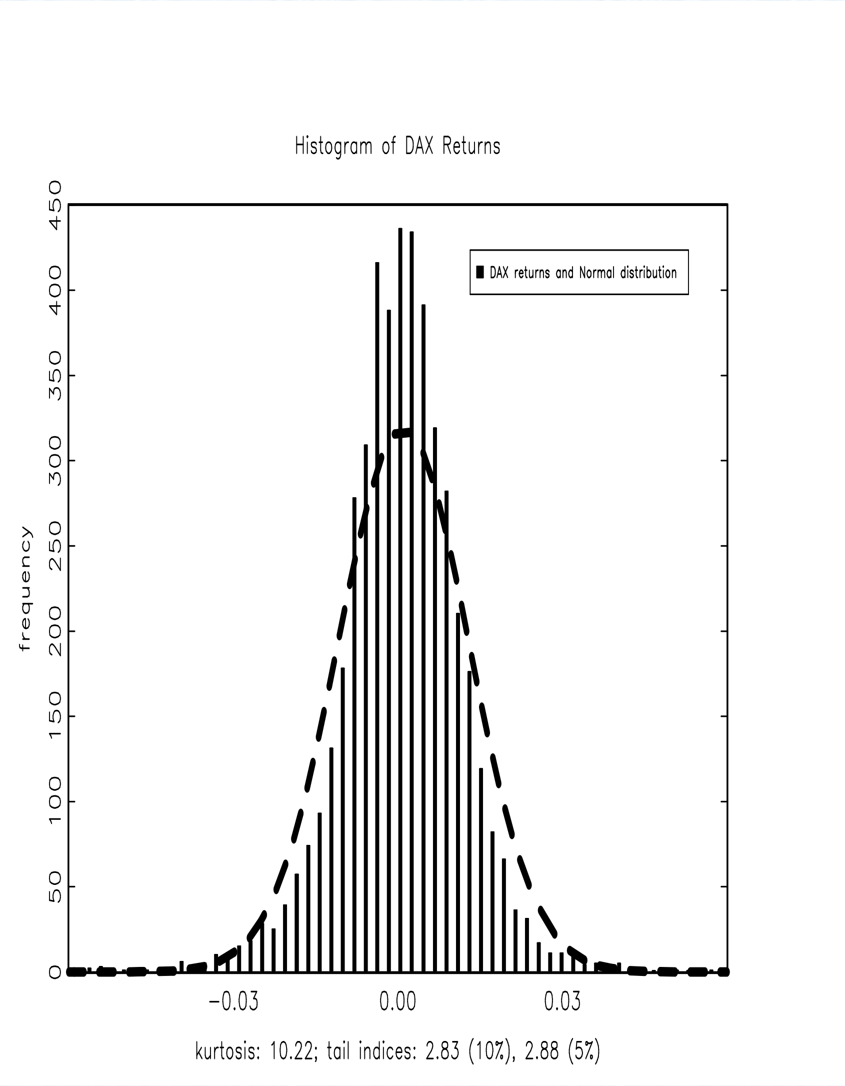
- Konsequenzen: Gewinne/Verluste sind nicht vorhersagbar, Handel ist wie Teilnahme an einem Glücksspiel

- Achtung: auf der Ebene täglicher Daten ist ein jährlicher Zuwachs selbst von 10% kaum zu erkennen: 0.1 / 250 = 0.0004 (tägl. Mittelwert für 10% Jahresrendite)

Verteilung von Renditen versus Normalverteilung

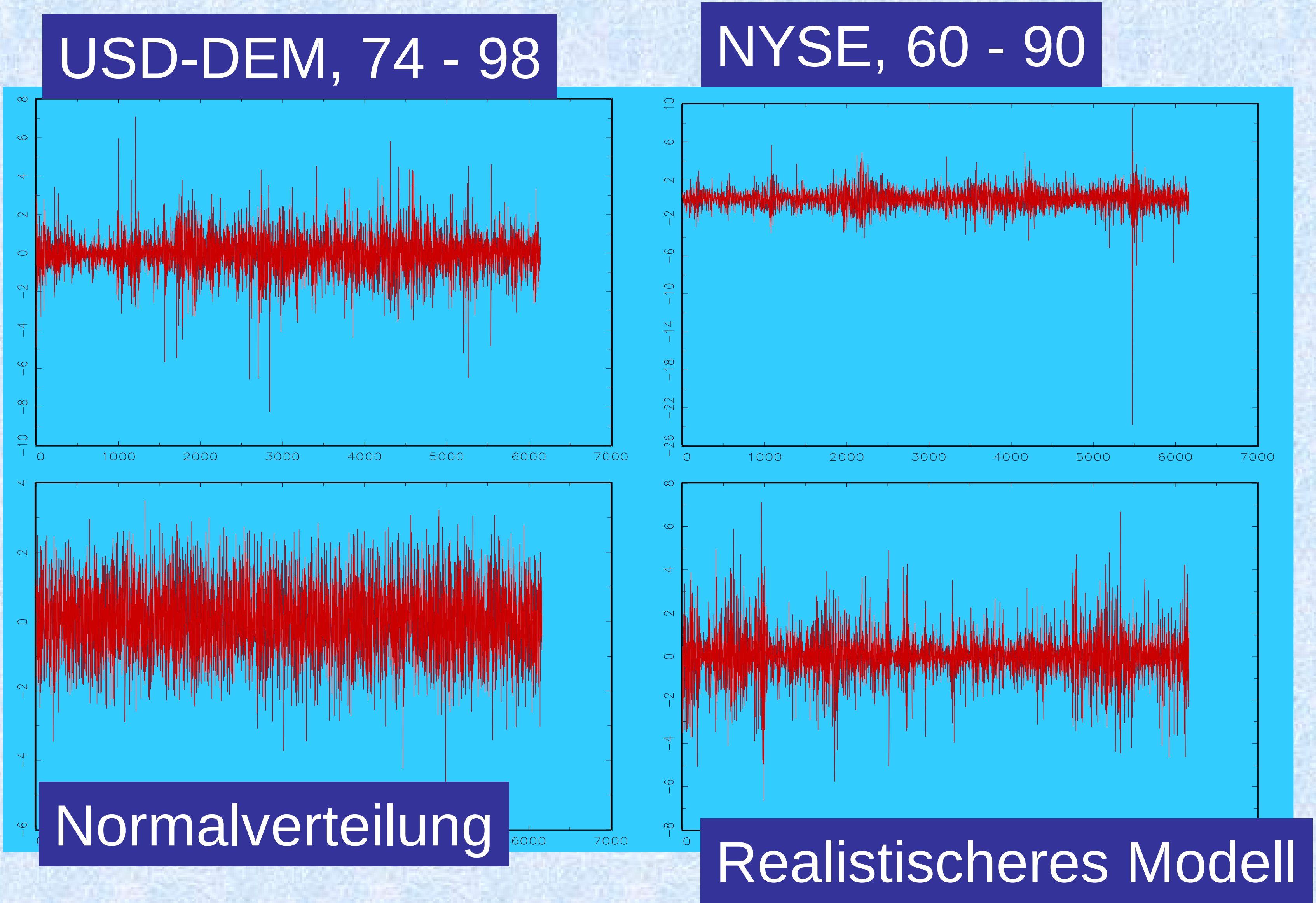






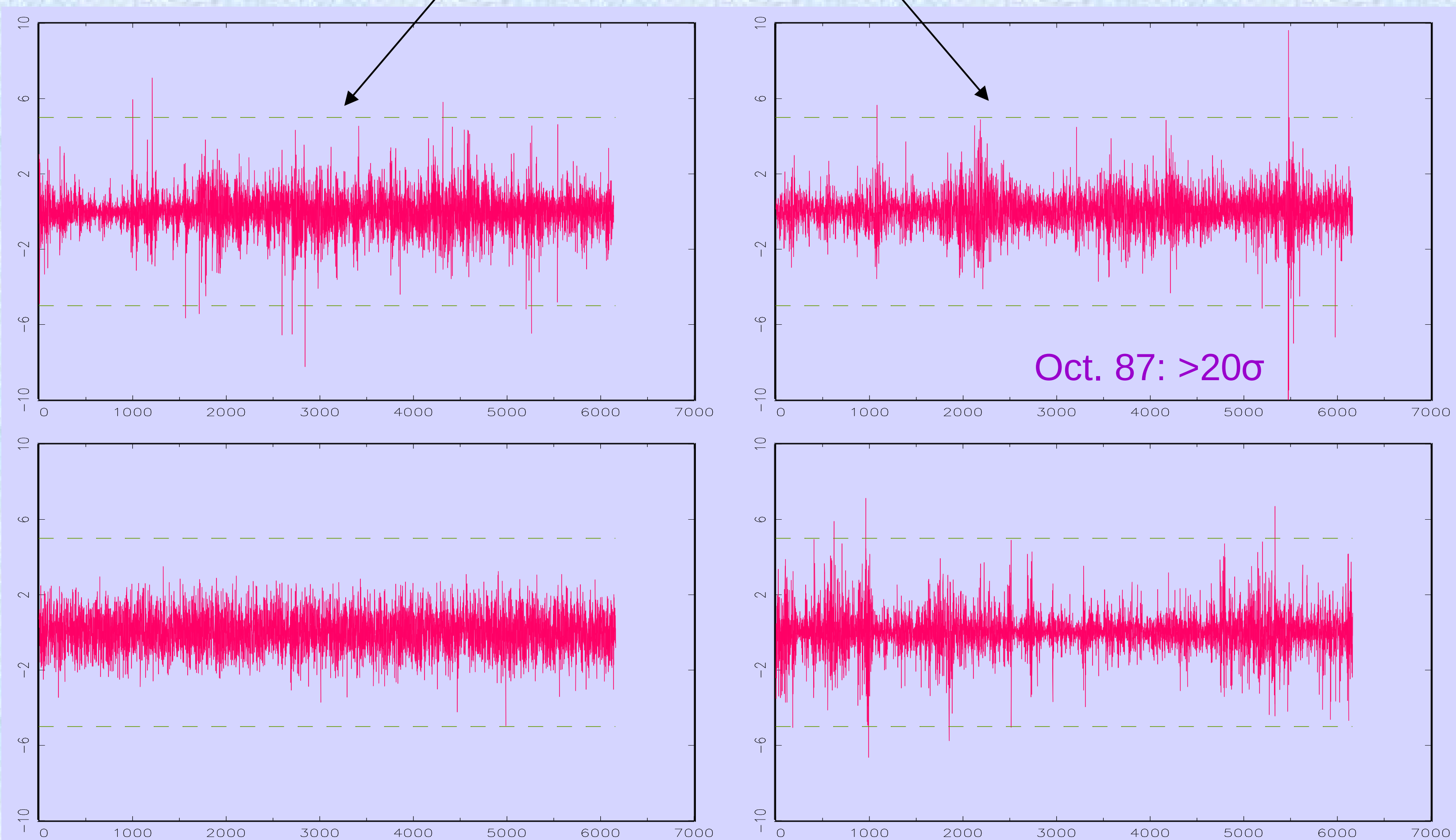
- symmetrisch
- glockenförmig
- … aber nicht Normal
- sondern
  - zu viele sehr kleine R
  - zu wenige mittelgroße R
  - zu viele sehr große R

Der Unterschied

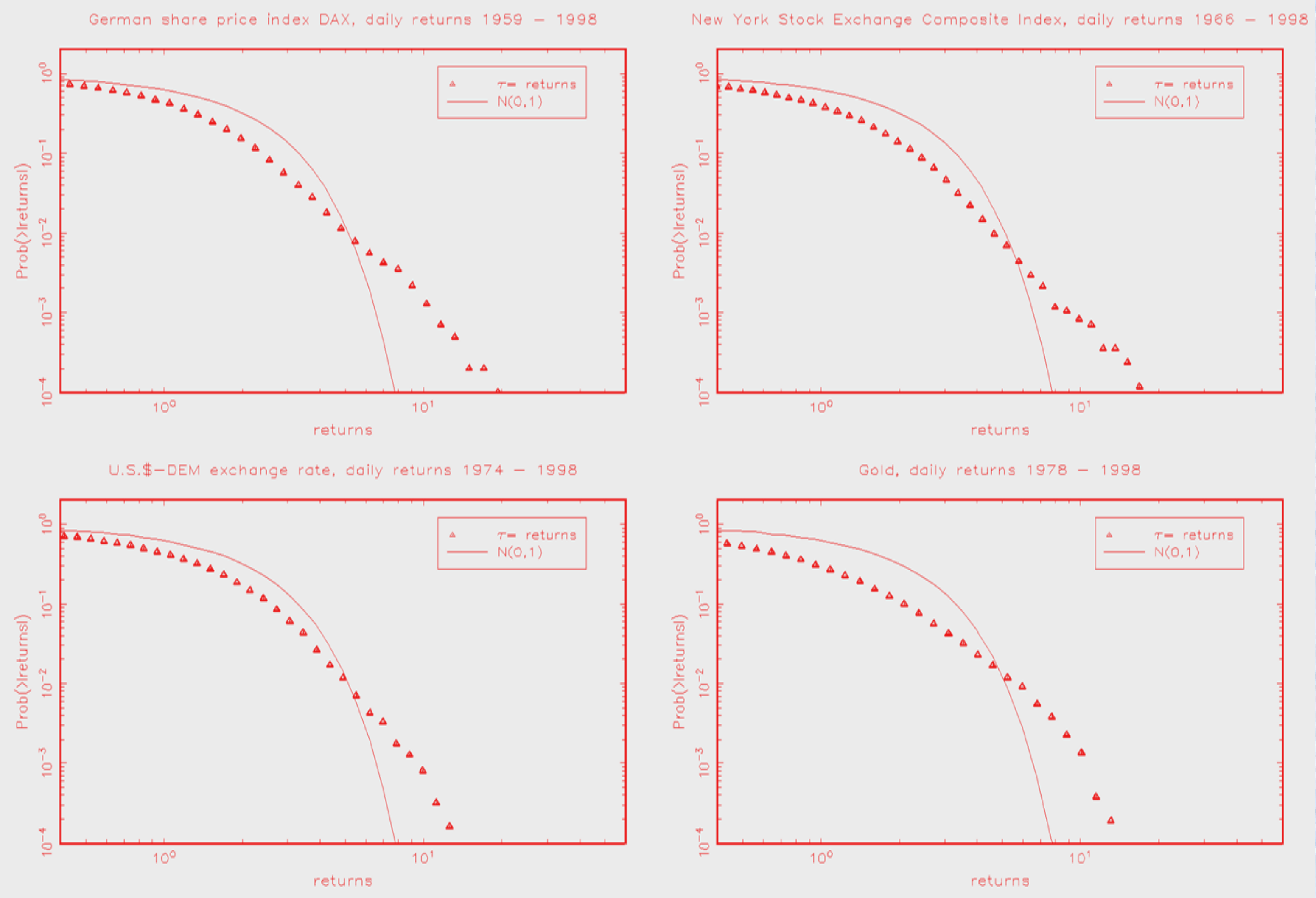


The Wild Things: Vergleich standardisierter Renditen

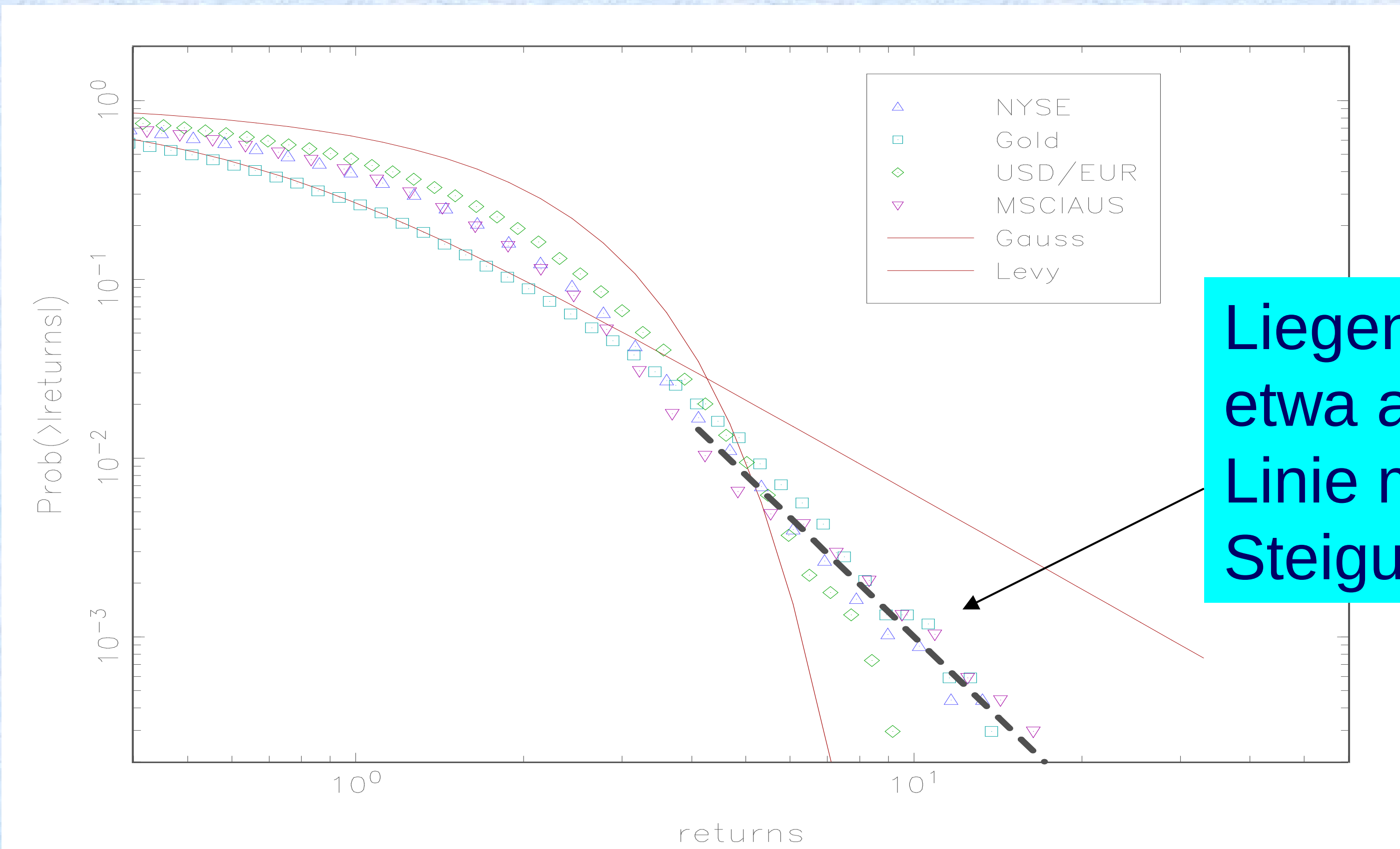
mehr als 5 mal ‚normale‘ Fluktuation



Die Häufigkeit großer Ausschläge werden in logarithmischer Darstellung besser sichtbar



Die Gesetzmäßigkeit großer Ausschläge



Liegen alle etwa auf einer Linie mit Steigung -3

Ein universelles *Power Law* der Renditen

Log(Häufigkeit) = c – α*

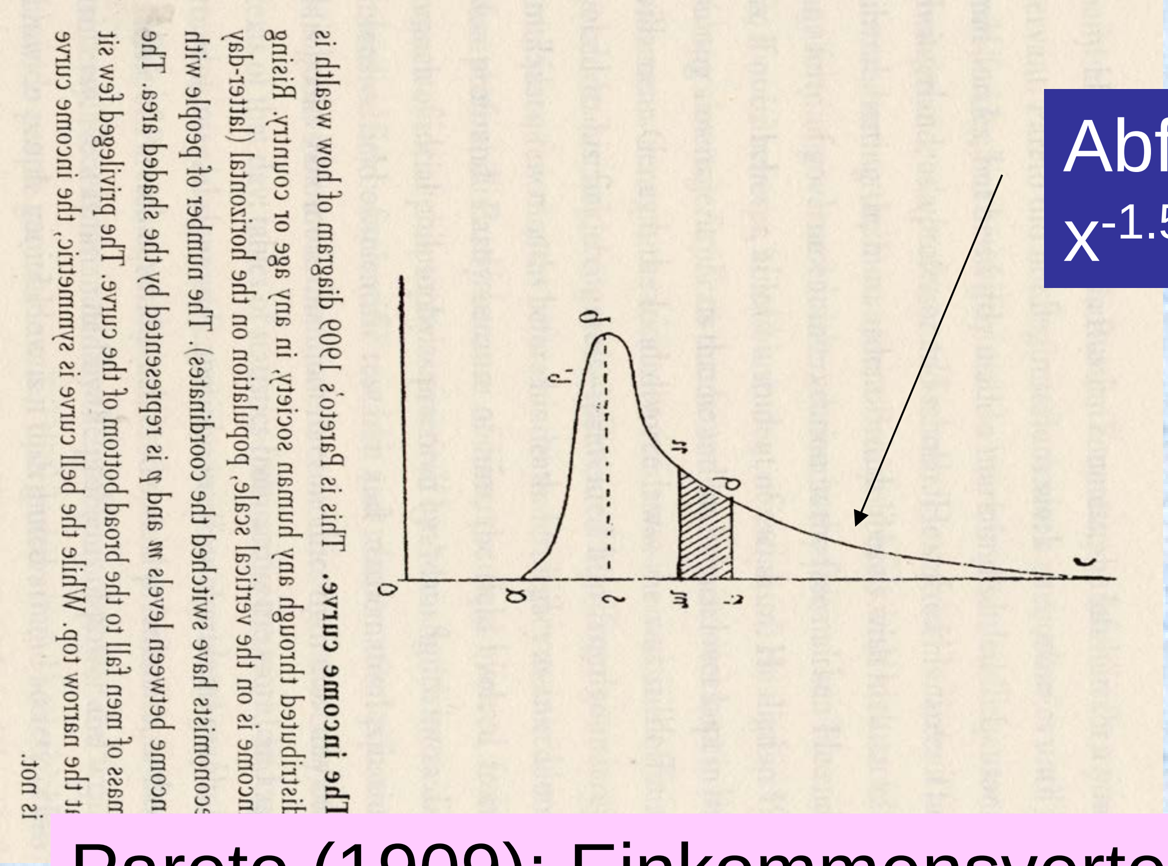
Log(Schwellenwert)

oder

Häufigkeit = Konst. * $x^{-\alpha}$

Power law,
Pareto-Gesetz

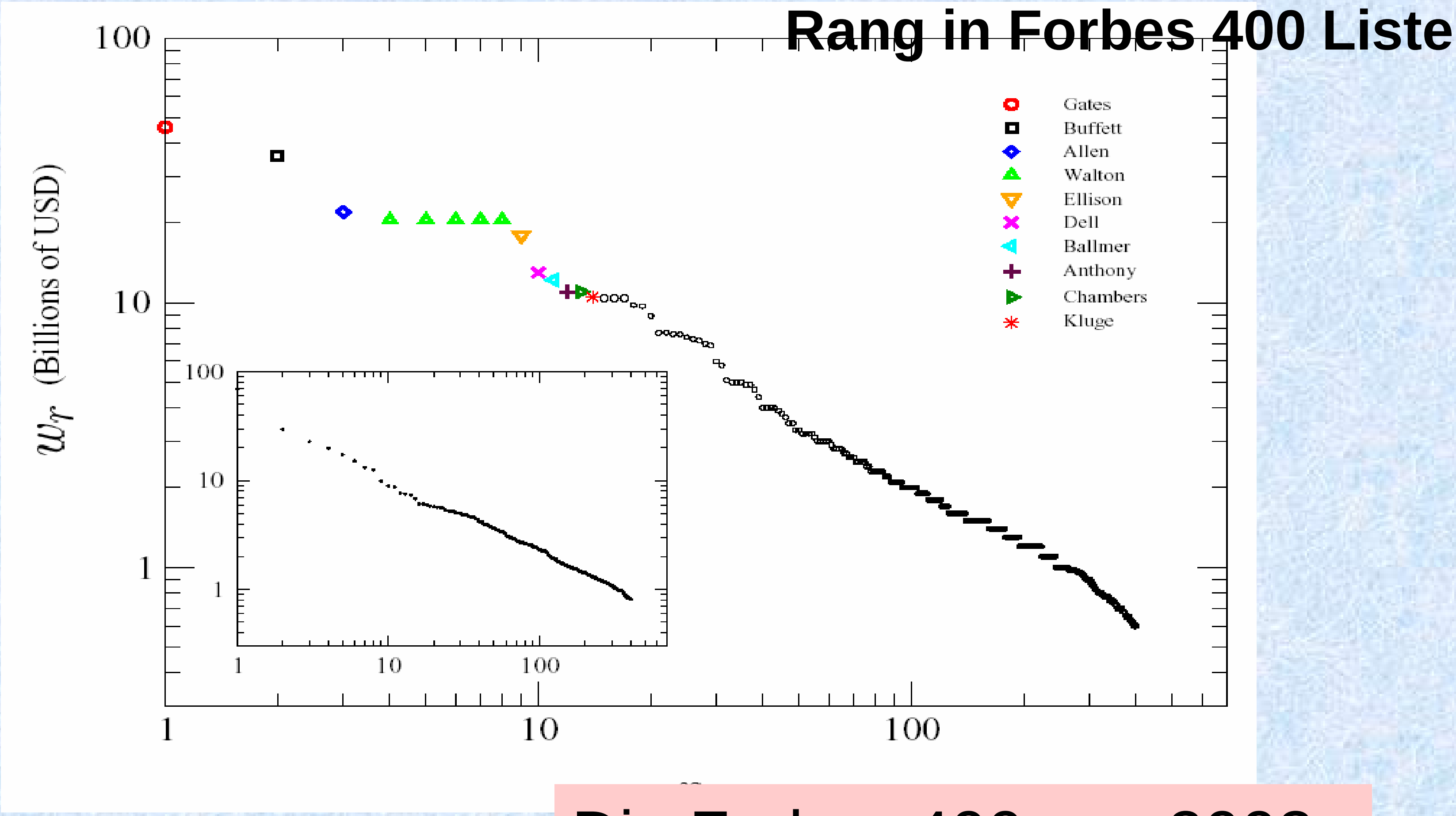
Ähnliche Gesetzmäßigkeiten auch in anderen Bereichen



Abfall wie: $x^{-1.5}$

Pareto (1909): Einkommensverteilung

... gilt auch heute noch



Die Forbes 400 von 2003 ,
Steigung = 1.7

Verteilung von Firmengrößen

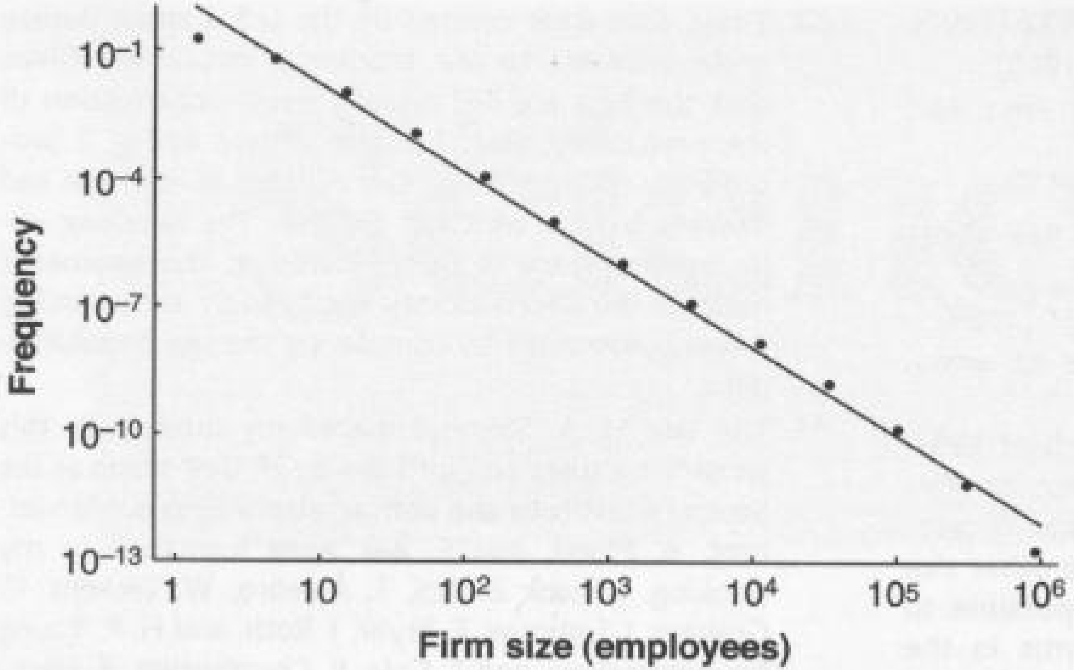
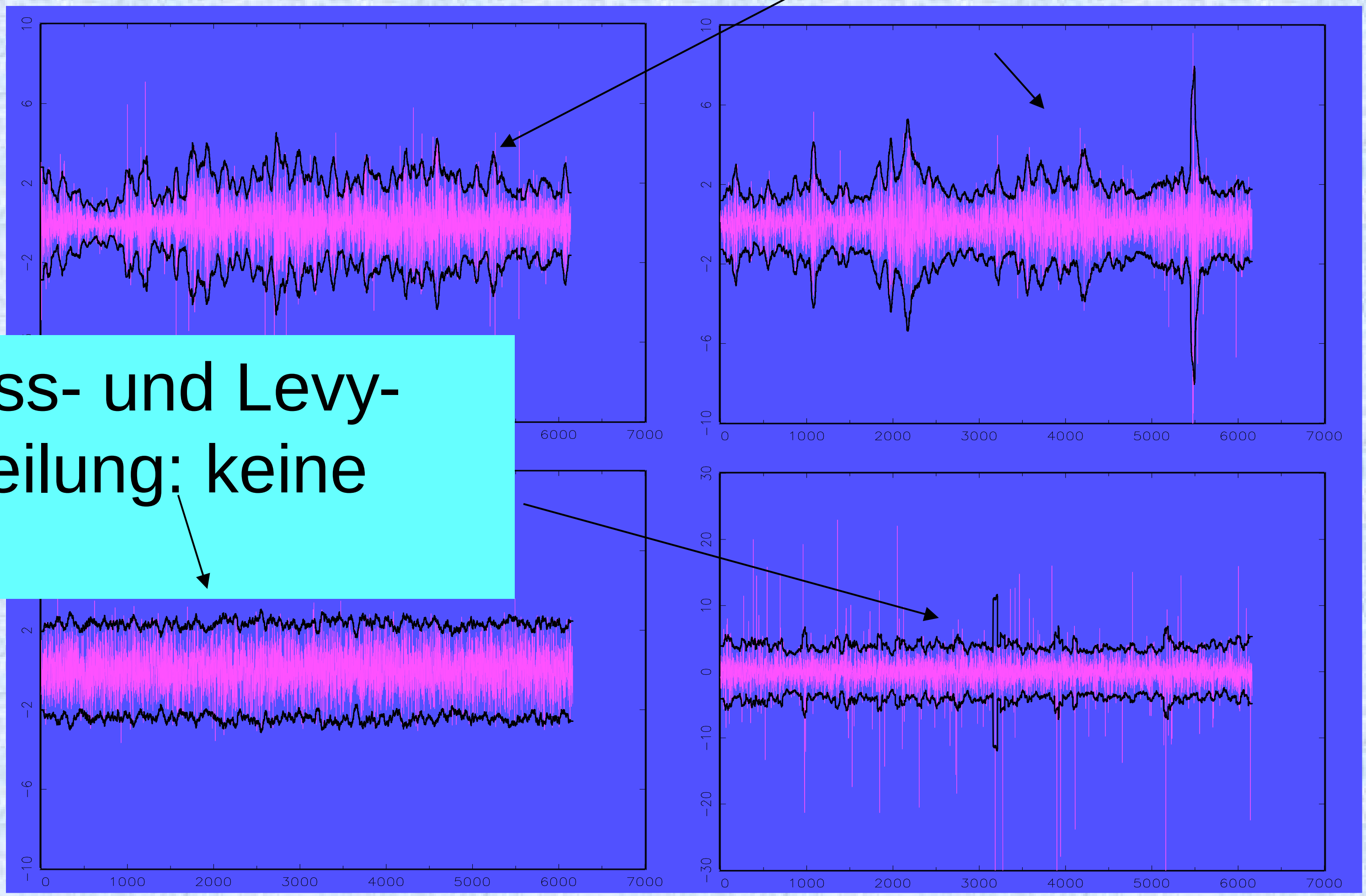


Fig. 1. Histogram of U.S. firm sizes, by employees. Data are for 1997 from the U.S. Census Bureau, tabulated in bins having width increasing in powers of three (*30*). The solid line is the OLS regression line through the data, and it has a slope of 2.059 (SE = 0.054; adjusted $R^2 = 0.992$), meaning that $\alpha = 1.059$; maximum likelihood and nonparametric methods yield similar results. The data are slightly concave to the origin in log-log coordinates, reflecting finite size cutoffs at the limits of very small and very large firms.

Source: Robert L. Axtell, Zipf Distribution of U.S. Firm Sizes, *Science* 293,
7 September 2001, pp. 1818-1820

Noch mehr Pow
Volatilitätsc

Irreguläre Zyklen im Ausmaß der Schwankungen



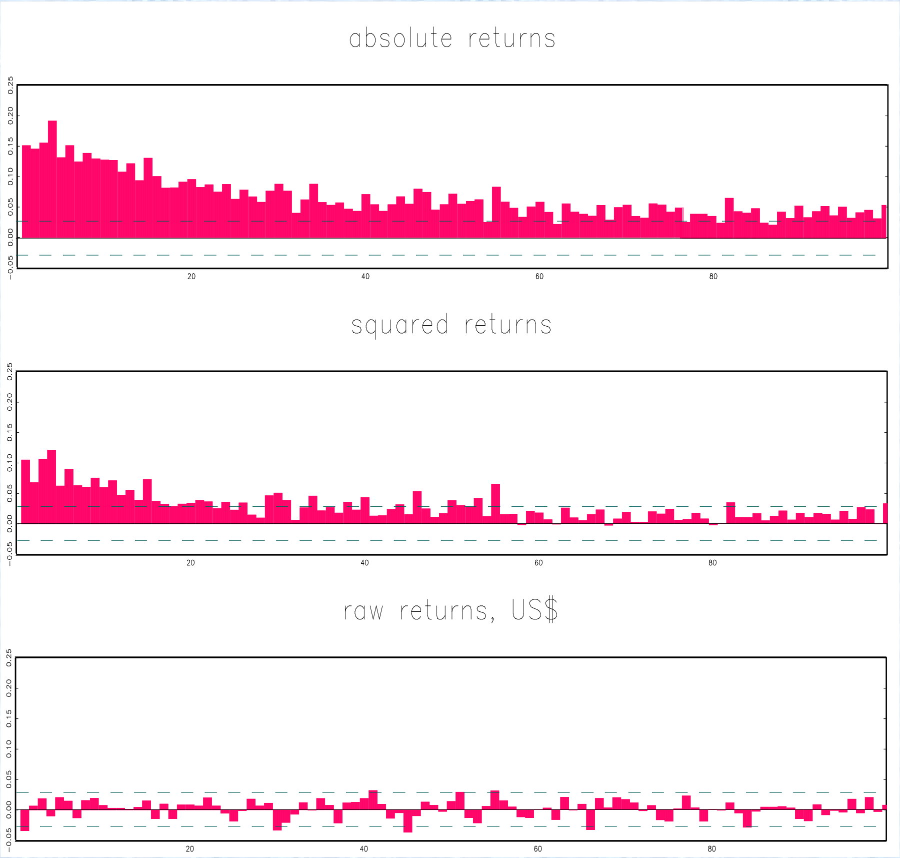
Gauss- und Levy-Verteilung: keine VC

Volatilitäts-Clustering

- Ausmaß der Schwankungen schwankt selbst über die Zeit (Risiko ist zeitabhängig!)
- Märkte wechseln zwischen ruhigen und stürmischen Phasen (market timing!)
- Ist der Markt gegenwärtig eher volatil, so bleibt er dies wahrscheinlich auch in der nahen Zukunft
- Ist der Markt gegenwärtig eher ruhig, so bleibt er dies wahrscheinlich auch in der nahen Zukunft... (Vorhersagbarkeit!)

Volatilität: ebenfalls power law-Verhalten

- Zeitliche Abhängigkeit von Maßen der Volatilität: ret^2, abs(ret)
- Hyperbolischer Abfall der ACF-Funktion



Verallgemeinerung: Multi-Scaling und Multi-Fraktalität

- Vassilicos, Demos, Tata, 1993, Evidence für Multi-Fraktalität
- Ghasghaie et al., *Nature*, 1996, Turbulente Kaskaden auf Finanzmärkten?
- Mandelbrot, Calvet and Fisher, 1997, -> Calvet and Fisher, 2002, Lux 2008:

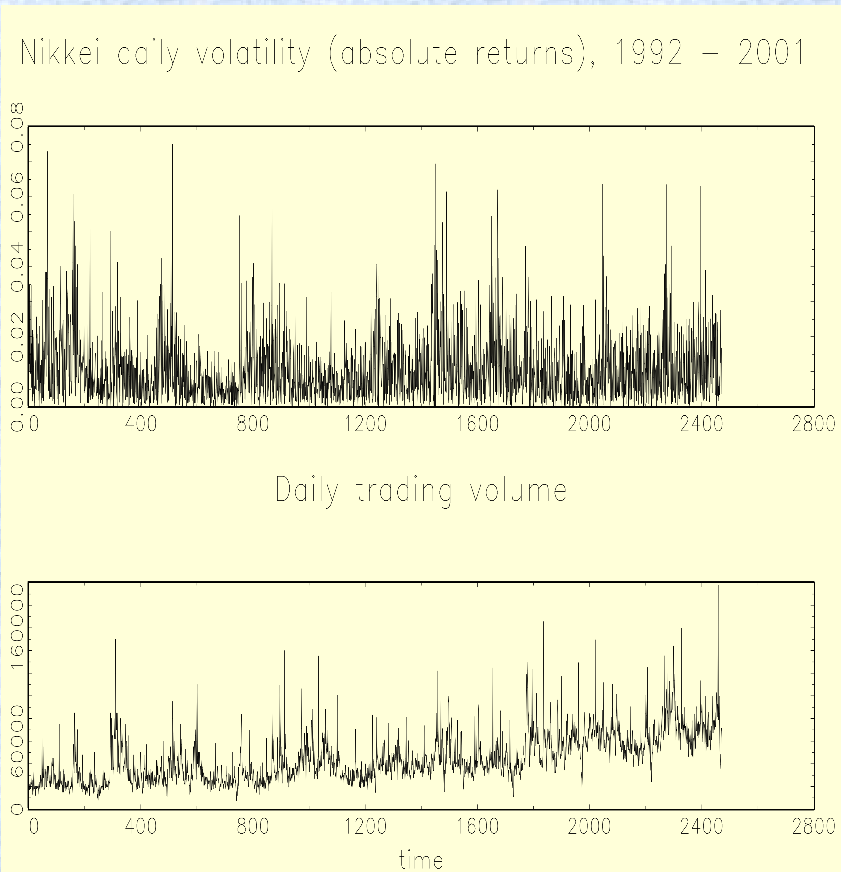
Multifraktales Renditemodell

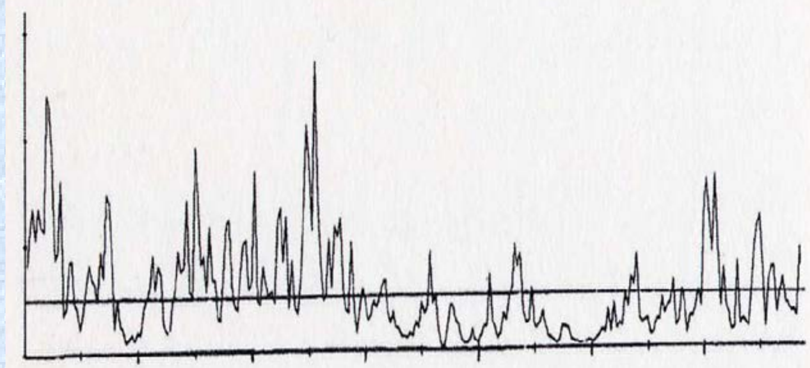
$$E\left[\left|r_t^q r_{t-\Delta t}^q\right|\right] \sim \Delta t^{-\gamma(q)}$$

Multi-Fraktalität:

Nichtlineares Skalierungsverhalten

Weitere Beobachtungen: Handelsvolumen verhält sich ähnlich wie „Volatilität“ ... und beides ähnelt turbulenten Strömungen





A turbulent wind: In the atmosphere. A chart of the original multifractal simulation (Mandelbrot 1972) of changing wind speed as it bursts into and out of gusty, turbulent flow. Notice how the peaks and troughs cluster together.

Zwischenfazit: Die ‘Stilisierten Fakten’

- Stochastizität von Kursänderungen

Kurse verhalten sich wie Random Walk (Martingal): $s_t = s_{t-1} + \varepsilon_t$

- Dicke Enden der Renditeverteilung

Wahrsch. großer Ausschläge: Prob(ret. > x) $\sim$ $x^{-\alpha}$ mit $\alpha \in [2,5]$

- Volatilitätscluster

Autokorrelation in Volatilitätsmaßen, z.B.. ret^2, abs(ret) etc.

sogar Langzeitabhängigkeit: $E[|r_t \, r_{t-\Delta t}|] \sim \Delta t^{-\gamma} = \Delta t^{2d-1}$, $d \approx 0.35$

power laws

bzw. ***scaling laws***

Unabhängig von Ort und Zeit!

Universelles prä-asymptotisches Verhalten: langsame Konvergenz zur Normalverteilung

‘Traditioneller’ Ansatz in der Ökonomie

- Extrem reduktionistisch: repräsentativer Akteur (RA)

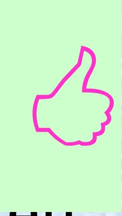
  (ein Agent als repräsentativer Anleger – oder repräsentativer Akteur der ganzen Volkswirtschaft)

- Mikrofundierung im Sinn eines vollständigen Maximierungsproblems des RA über einen unendlichen Zeithorizont unter *rationalen Erwartungen*

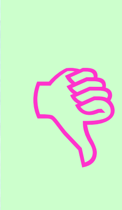
- Keine Interaktion, Mikro = Makro, keine Emergenz, kein Zugang zu stilisierten Fakten

In der Finanzökonomik: Theorie effizienter Märkte

- Aufgrund der Rationalität der Marktteilnehmer: Marktkurs reflektiert alle relevanten Infomationen über zukünftige Preise und Dividenden
- Kurse reagieren augenblicklich und *korrekt* auf neue Informationen
- -> Kursbewegungen müssen zufällig (stochastisch) erscheinen



- Erklärt aber nicht die anderen stilisierten Fakten



Stimmt das denn: sind Märkte effizient?

- Obwohl wir neue Infomationen nicht vorhersagen können, sollten wir sie ex post identifizieren können
- Große Ausschläge sollten erklärbar sein
- Sind sie aber häufig nicht

##	Date	Percent Change	New York Times Explanation[3]
1	Oct. 19, 1987	-20.47	Worry over dollar decline and trade deficit, fear of US not supporting dollar.
2	Oct. 21, 1987	9.1	Interest rates continue to fall; deficit talks in Washington; bargain hunting.
3	Oct. 26, 1987	-8.28	Fear of budget deficits; margin calls; reaction to falling foreign stocks.
4	Sep. 3, 1946	-6.73	"No basic reason for the assault on prices."
5	May 28, 1962	-6.68	Kennedy forces rollback of steel price hike.
6	Sep. 26, 1955	-6.62	Eisenhower suffers heart attack.
7	Jun. 26, 1950	-5.38	Outbreak of Korean War.
8	Oct. 20, 1987	5.33	Investors looking for "quality stocks".
9	Sep. 9, 1946	-5.24	Labor unrest in maritime and trucking industries.
10	Oct. 16, 1987	-5.16	Fear of trade deficit; fear of higher interest rates; tension with Iran.
11	May 27, 1970	5.02	Rumors of change in economic policy. "The stock surge happened for no fundamental reason."

Table 1: Eleven largest Post-War Movements in S&P index and Their "Causes". Source: Cutler et al. (1989).

Cf. Cutler, Poterba, Summers, Journal of Portfolio Management, 1989

Erklärungen der Stilisierten Fakten durch Modelle mikroökonomischer Interaktion

“Statistical physicists have determined that physical systems which consist of a ***large number of interacting particles obey universal laws*** that are independent of the microscopic details. This progress was mainly due to the development of scaling theory. Since economic systems also consist of a large number of interacting units, it is plausible that scaling theory can be applied to economics“

aus: Stanley, H. *et al*. Can Statistical Physics Contribute to the Science of Economics, in: *Fractals* 4 (1996)

interacting units -> market participants

scaling laws -> stylized facts: volatility clustering, fat tails , independent of the microscopic details!

Auch in der Ökonomie: Neue Perspektiven

- “Statistical laws apply in physics and social sciences”

  (Physicist Majorana,1942, and a number of other independent forerunners: Weidlich 1983, Farjoun and Machover, 1983,…)

- “.. Macro activity is essentially the result of the interactions between agents..”

  (Economist Ramsey, 1996)

- “...there is no plausible justification that for the assumption that the aggregate of individuals acts itself like an individual maximizer”

  (Economist Kirman, 1992)

Stilisierte Fakten als Emergente Eigenschaften eines Multi-Agent Systems

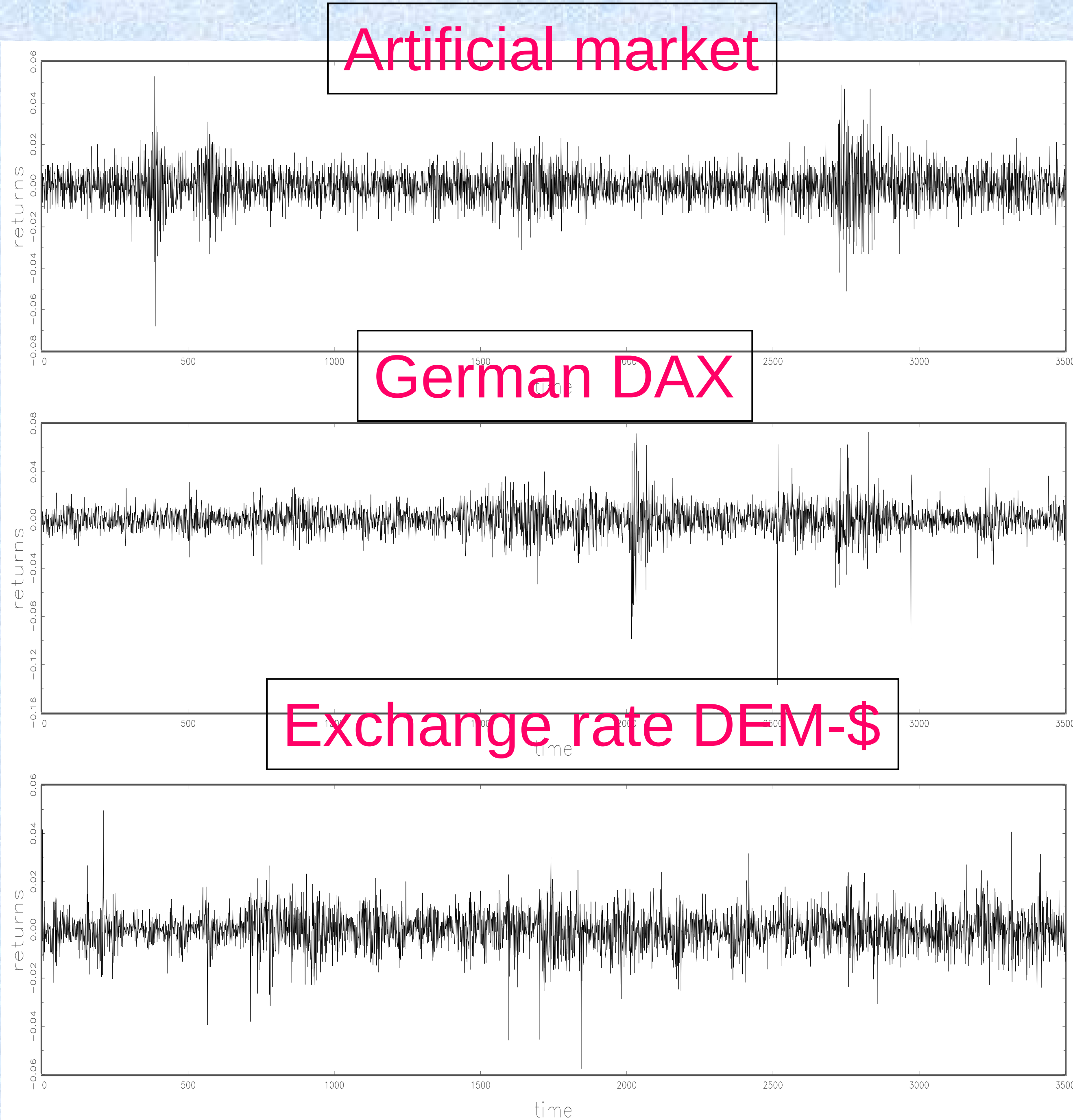
Effiziente Märke vs. Interagierende Agenten

EMH: Preisänderungen sind unmittelbare und unverfälschte Reaktion auf neue Informationen

-> statistische Eigenschaften der Renditen sind *reine Reflektion* der Eigenschaften des *news arrival process*

Interacting Agent Hypothesis: Preisbildung resultiert endogen aus dem Zusammenspiel vieler Investoren mit unterschiedlichen Informationen, Strategien etc.

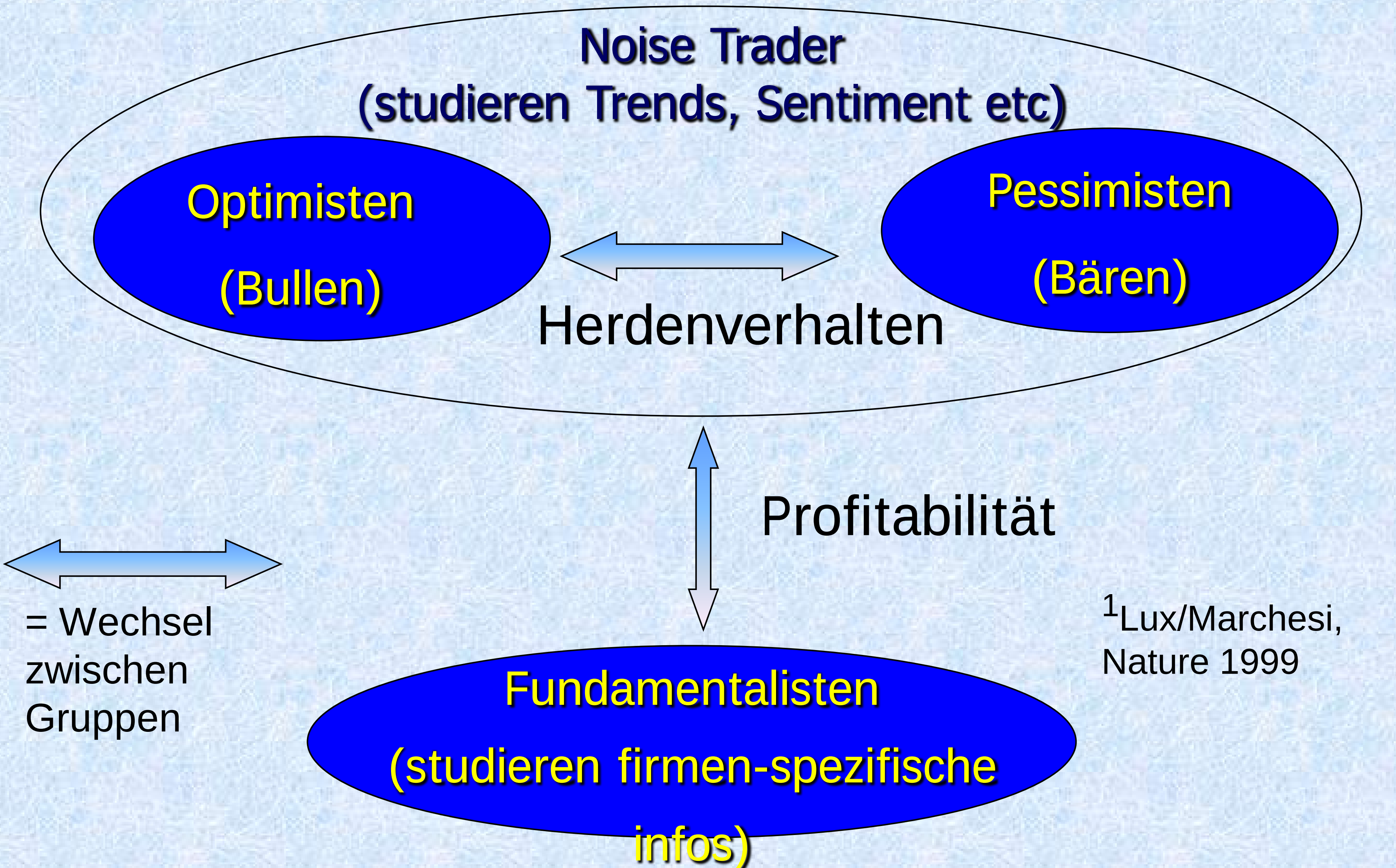
Marktinteraktion generiert das typische Erscheinungsbild der Kursdynamik



returns =

$\ln(p_t) - \ln(p_{t-1})$

Ein Markt mit interagierenden heterogenen Investoren[1]



[1]Lux/Marchesi, Nature 1999

Formalisierung:

- Übertrittswahrscheinlichkeiten für Wechsel zwischen Gruppen (à la Partikel-Dynamik)

- Wahrsch. sind zeitvariabel und endogen, enthalten psychologische wie ökonomische Komponenten

Beispiel: Wechsel zw. Bulls (+) und Bears(-)

$\pi_{+-} = v_1 \exp(U_1)$ and $\pi_{-+} = v_1 \exp(-U_1)$,

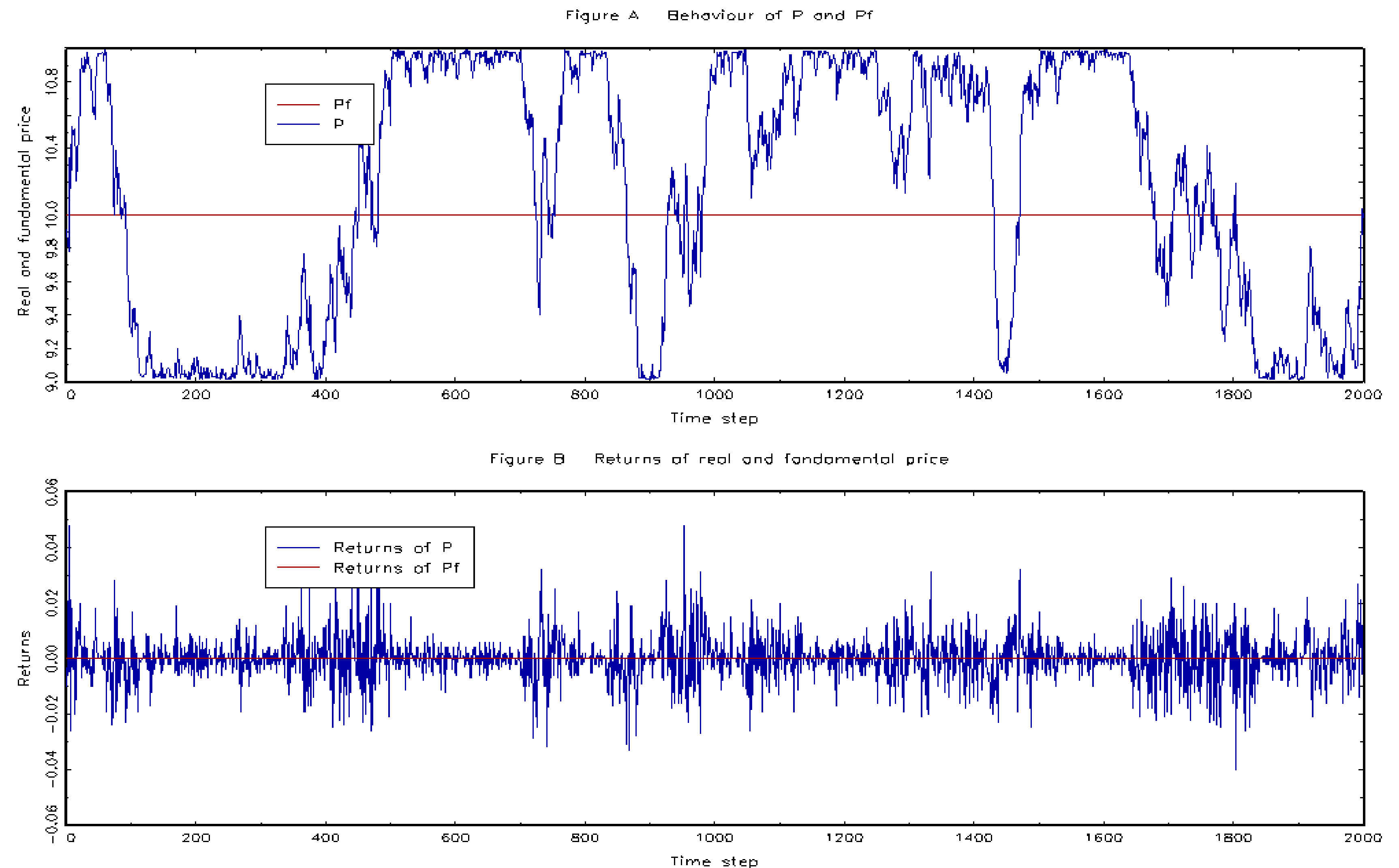
with: $$U_1 = \alpha_1 x + (\alpha_2 / v_1) \frac{p'(t)}{p}$$

x: Majorität (flows)

p'(t): Preistrend (charts)

Einfachstes Modell: nur Herdenverhalten unter NTs, kein Wechsel zwischen NTs und Fundis,

Bi-stabile Verteilung führt zu Volatilitätsclustering



Modell mit drei Gruppen

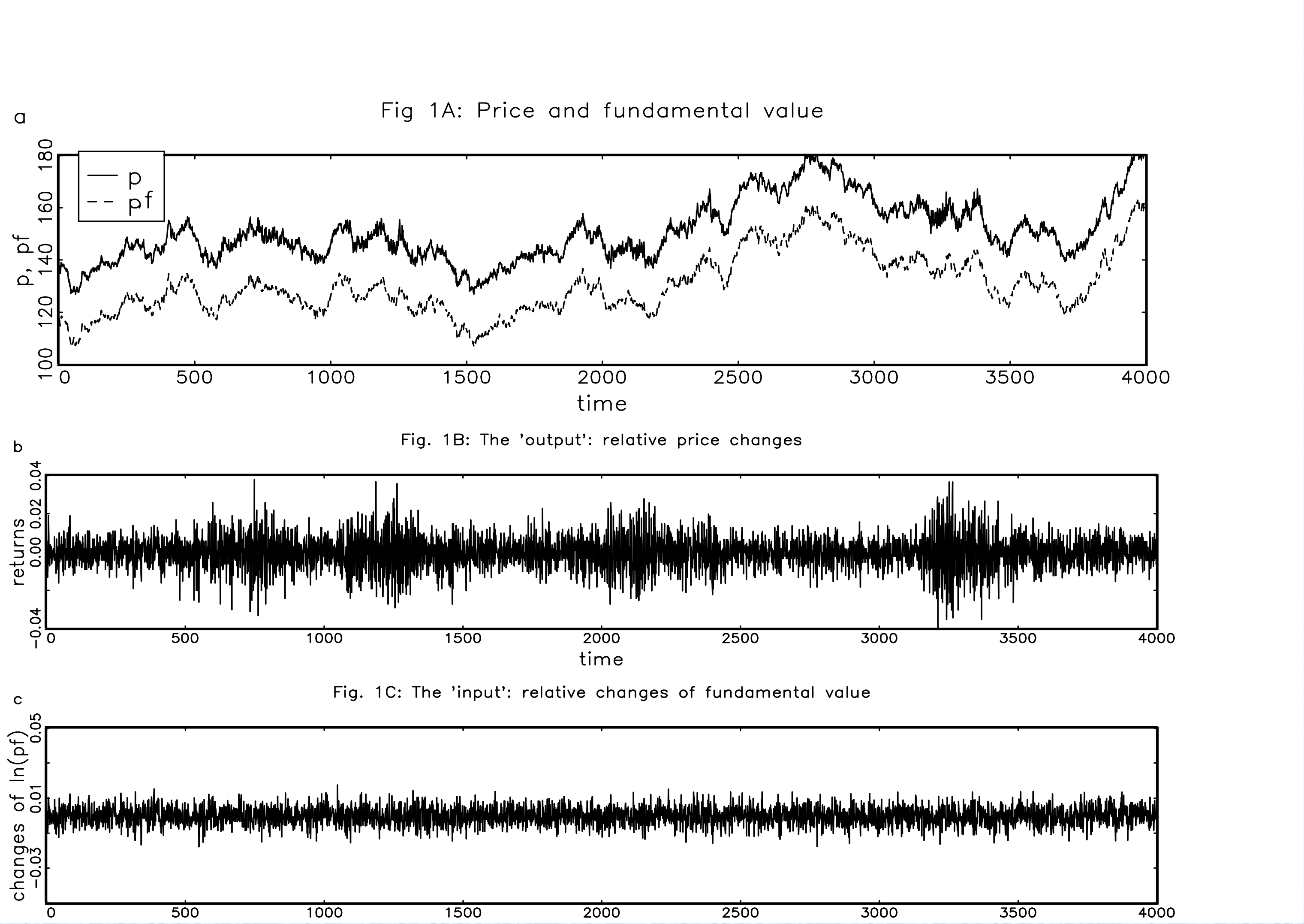
- Händler wechseln nun zwischen fundamentalistischer und Noise Trader-Strategie

- Übertritte werden getrieben von der (kurzfristigen) Profitabilität der Strategien (etwas mehr an Ökonomie)

- Es werden exogene Infomationen über zukünftige Dividenden in den Markt gegeben, die einem Wiener Prozeß folgen:

$$\ln(p_{f,t}) = \ln(p_{f,t-1}) + \varepsilon_t \Delta t \quad \text{mit } \varepsilon_t \sim N(0, \sigma_\varepsilon)$$

-> ***news arrival process ohne realistische Eigenschaften***



Typical snapshot from a simulation run. The upper panel depicts the market price p (solid line) and the fundamental value p_f (dotted line). The latter series has been shifted vertically for better visibility. The middle and bottom panel show returns and log changes of the fundamental value, respectively.

Ergebnis: “On-Off Intermittency”

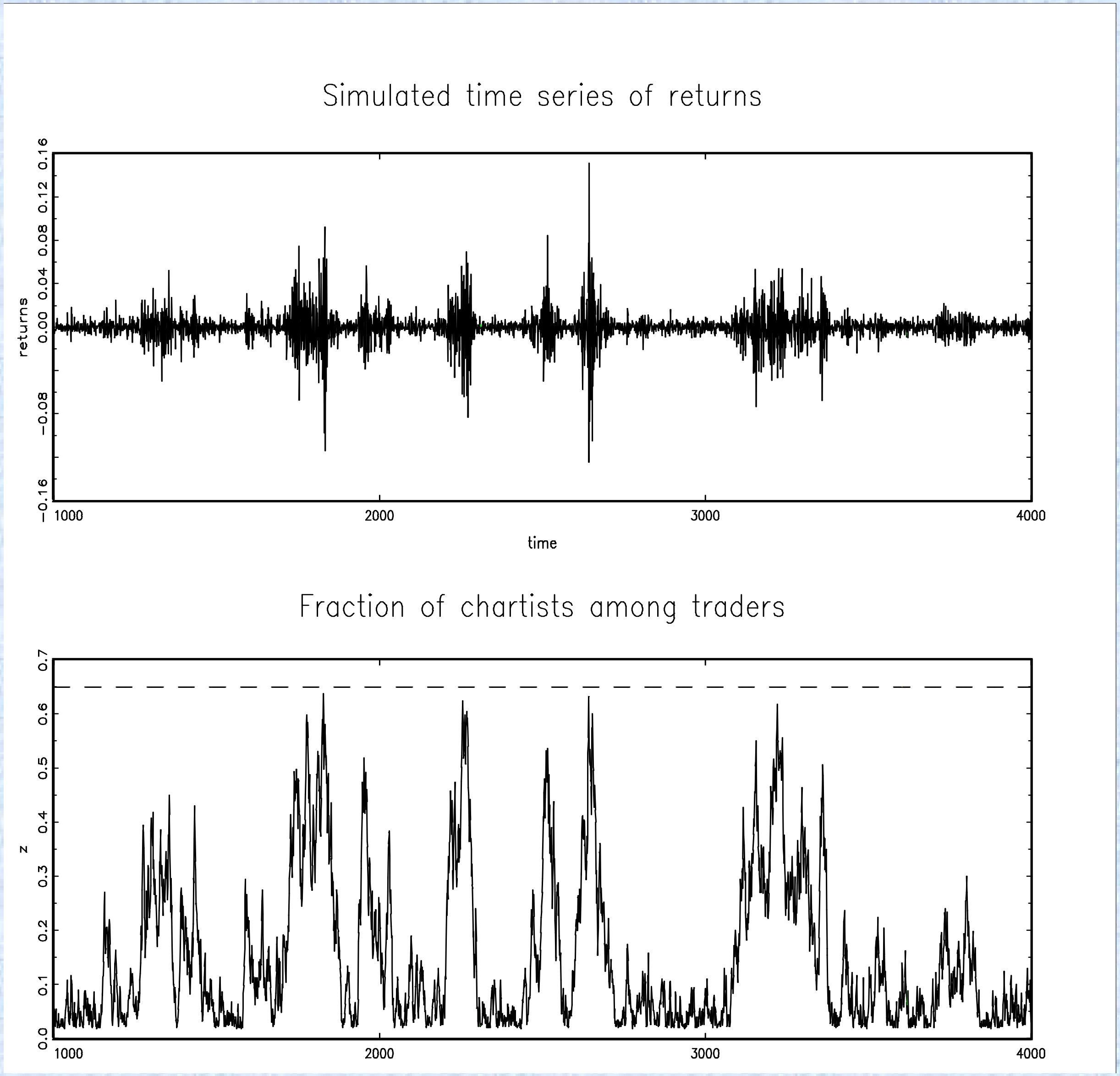
- das System weist ein eindeutiges Gleichgewicht auf, das abhängig von der Zusammensetzung der Händlerpopulation stabil oder instabil sein kann

- die Strategiewahl der Händler ist aber um das Gleichgewicht unbestimmt, so dass das System erratisch zwischen stabilen und instabilen Phasen wechselt

- plötzliche (temporäre) Destabilisierung durch zufällige Verhaltensänderungen oder exogene Schocks

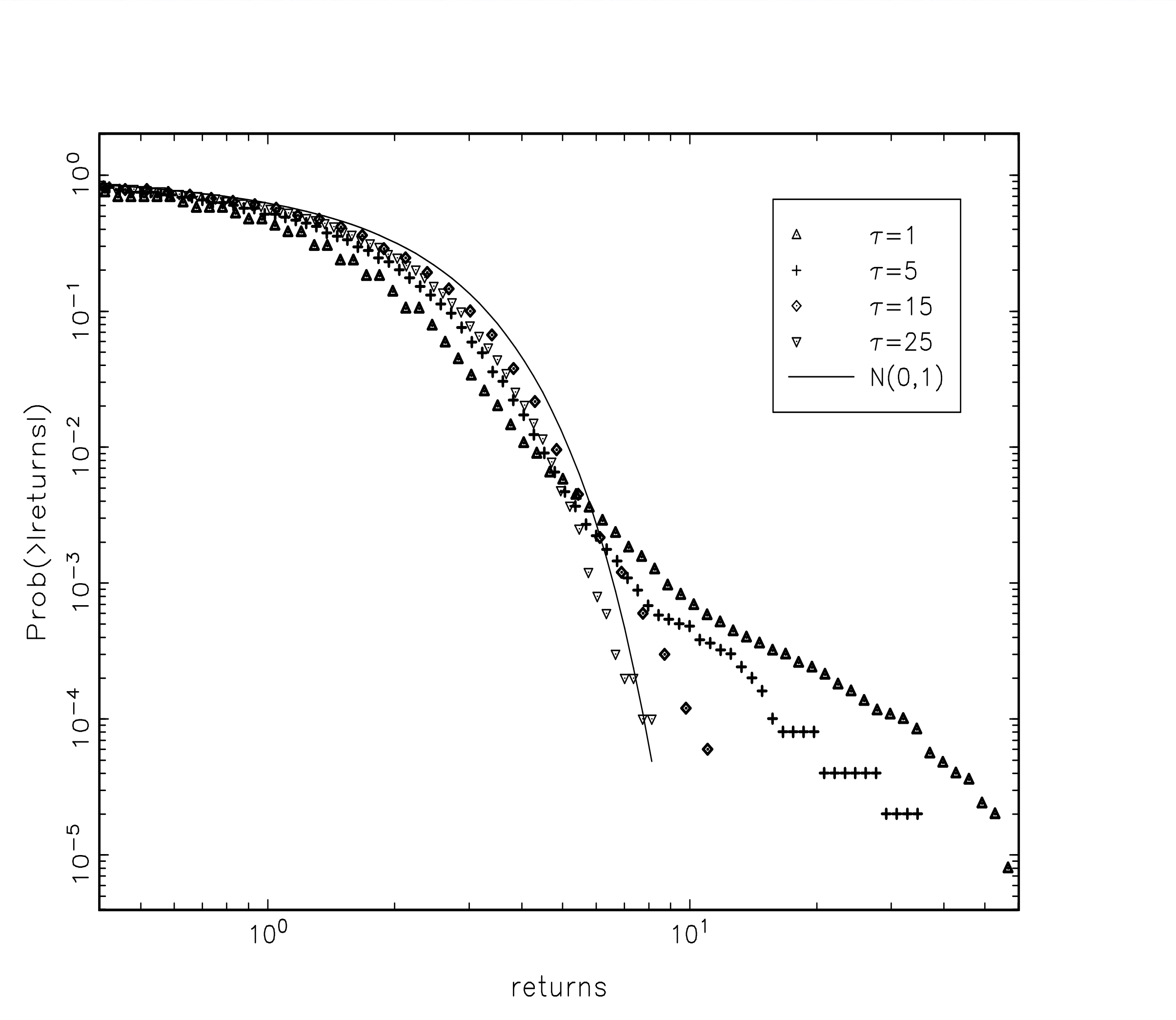
- Unkoordinierte Aktivitäten der Händler generieren realistische Zeitreihen)mit fat tails, volatility clustering)

- Resultierende Zeitreihen sind auch mit Informationseffizienz vereinbar (d.h., EMH wird nicht abgelehnt)



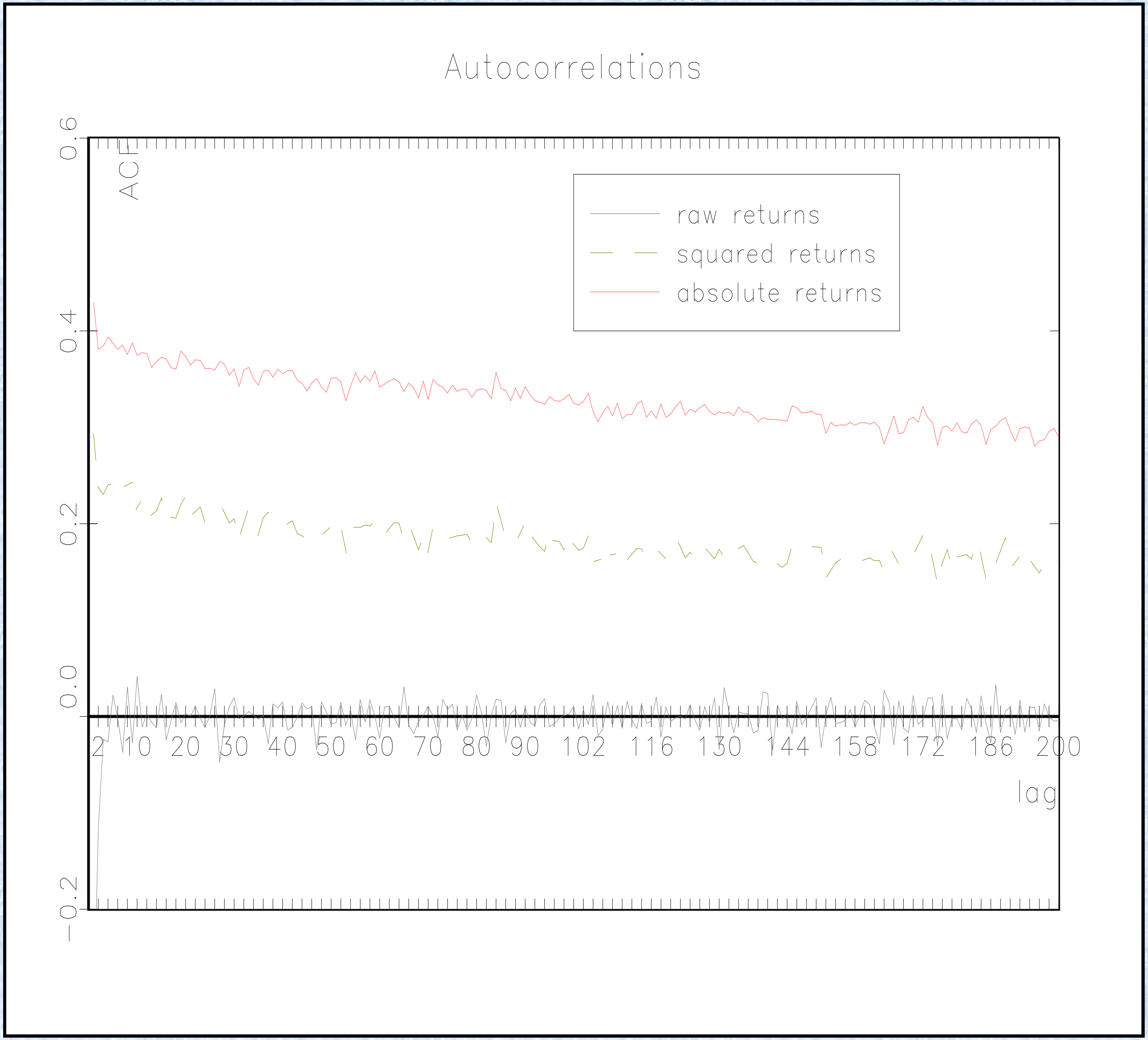
Example of the Dynamics: returns and simultaneous development of the fraction of chartists. The broken line indicates the critical value at which a loss of stability occurs.

Power laws im Modell

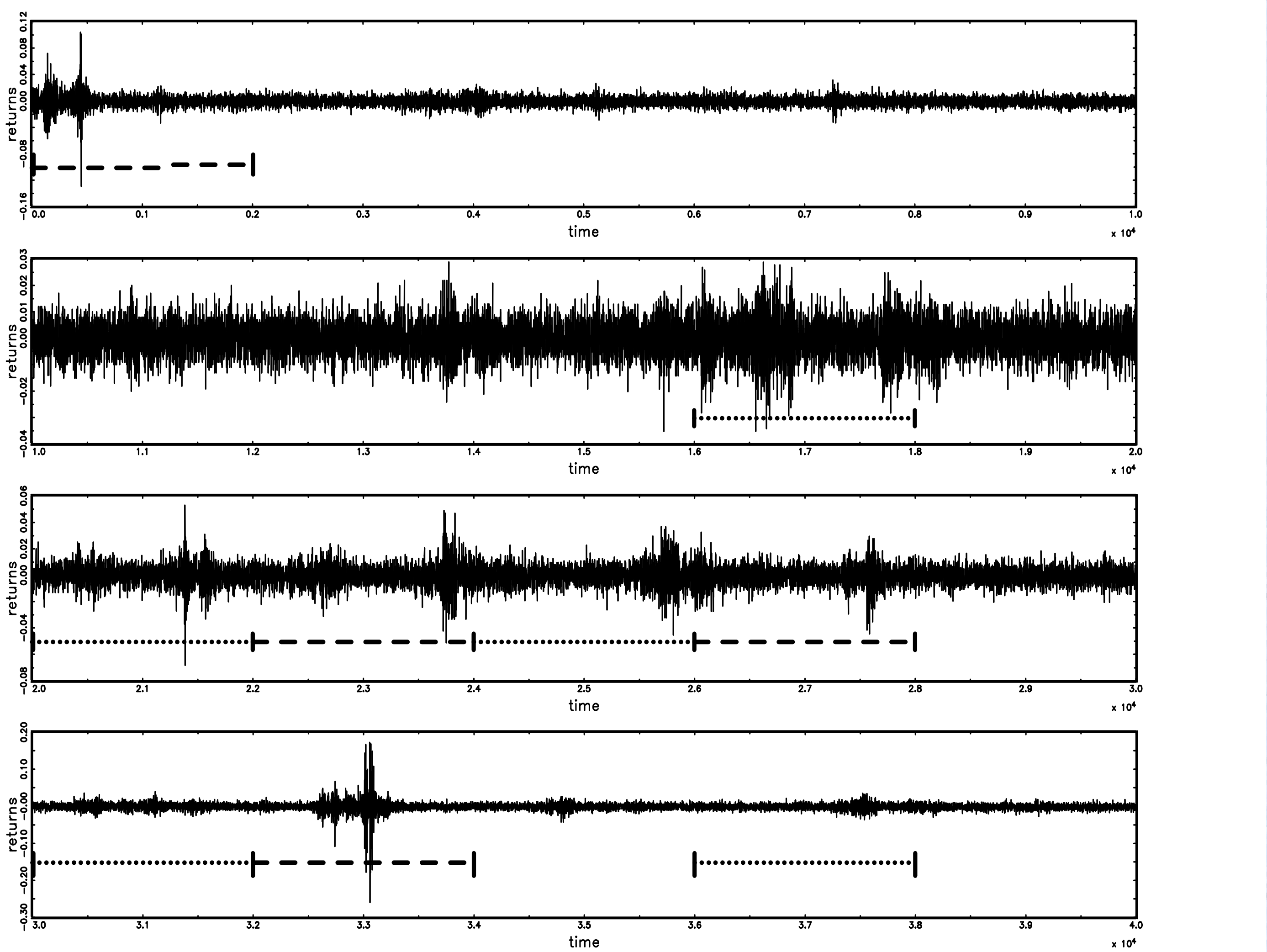


Loglog plot of the cumulative distribution of returns at different levels of time aggregation. For comparison, the solid line gives the cumulative distribution of the standard Normal.

Estimation of the exponent α gives 2.64 ± 0.077 at unit time steps ($\tau = 1$).

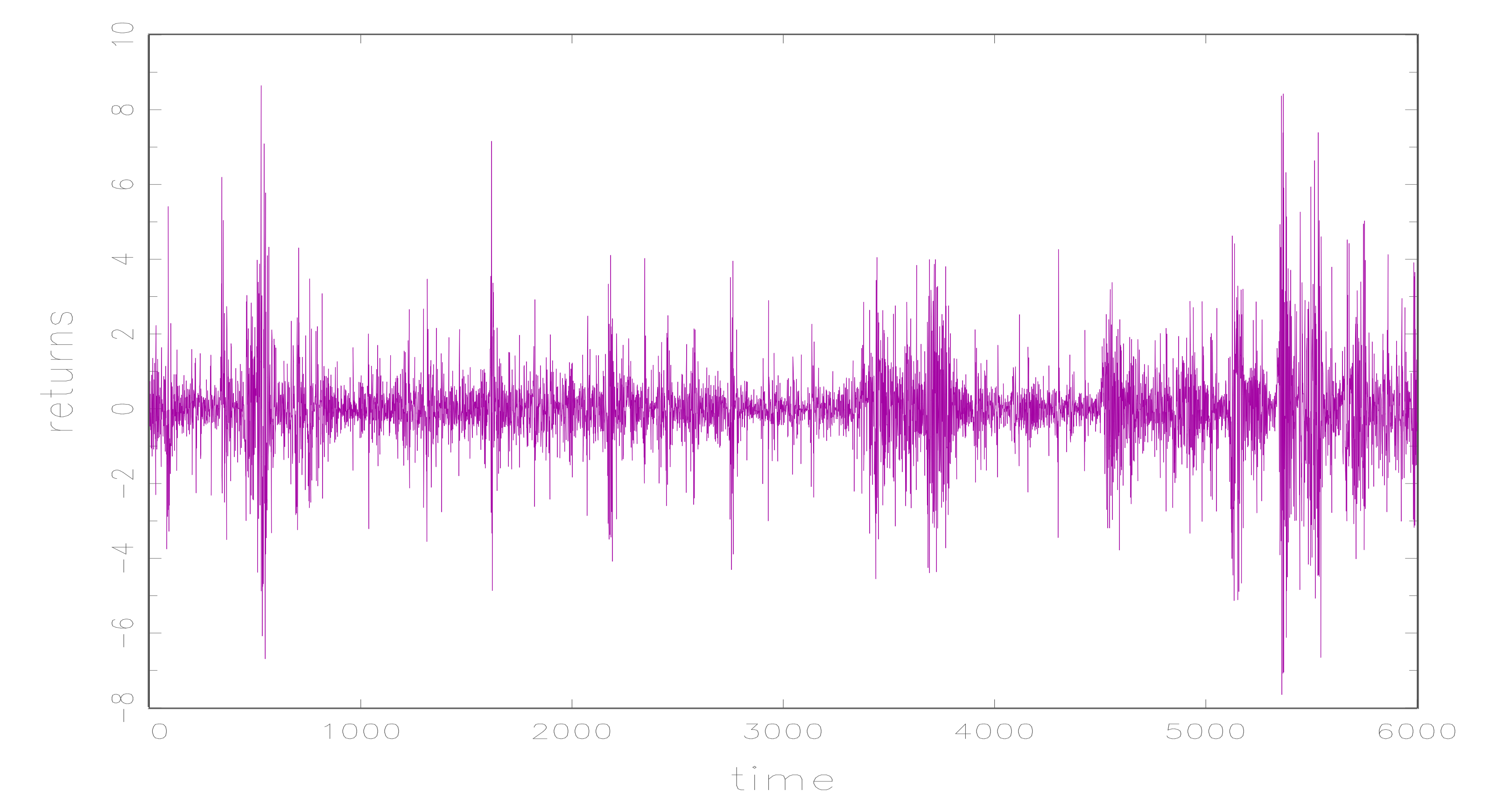

Autocorrelations
ACF
0.6
0.4
0.2
0.0
−0.2
raw returns
squared returns
absolute returns
2 10 20 30 40 50 60 70 80 90 102 116 130 144 158 172 186 200
lag

Zwischen Stochastik und deterministischer Dynamik: Struktur aus dem Nichts



Broken and dotted lines mark subperiods with clear rejection from the BDS test (----) and ambiguous results (......), respectively.

Noch (viel) komplexer: Händler mit künstlicher Intelligenz (hier genetische Algorithmen)



Renditen des GA-Modells: ähnliche Mechanismen, ähnliche Ergebnisse

Komplexere Modellstruktur

- Händler entscheiden über die Zusammensetzung ihres Portfolios (heimische vs. ausländische Wertpapiere)
- Ihre Spar- und Portfolioentscheidungen werden mittels genetischer Algorithmen modelliert. Sie *lernen*, ihren Nutzen zu maximieren
- Wiederum: ruhige und volatile Marktphasen, Destabilisierung durch Verhaltensänderungen
- Wechsel zwischen sehr homogenem und heterogenerem Marktverhalten

Now, we get more technical on modelling interaction, opinion formation and animal spirits

Agent-based models with social interactions inspired by statistical physics: a time-honoured legacy

- Föllmer (1974): “Random economies with interacting agents”

- Weidlich and Haag (1983): “Quantitative Sociology”

- Schelling: “Micromotives and Macrobehavior”

- Aoki (1996, 2002), Aoki and Yoshikawa (2007): “Macroeconomic Aggregates and Mean Field Dynamics”

- Brock and Durlauf (2001): Discrete choice with social interactions

Available models are successful in explaining empirical regularities, but have not been really implemented empirically

Missing is:

- estimation of the underlying parameters,
- comparison of models,
- goodness of fit.

- In the following, an agent-based model for opinion formation via social interaction will be introduced (the *Weidlich* model)

- We develop a general rigorous methodology for parameter estimation on the base of aggregate data (i.e. survey data) from a microscopic model

- Illustration: estimation of parameters of *Weidlich* model for economic survey data (ZEW survey)

Background on Survey Data

- Survey data are mostly used to forecast key economic quantities (GDP, IP, stock prices), but are seldomly treated as endogenous variables

- What drives expectations expressed in surveys? Rational expectations vs. animal spirits

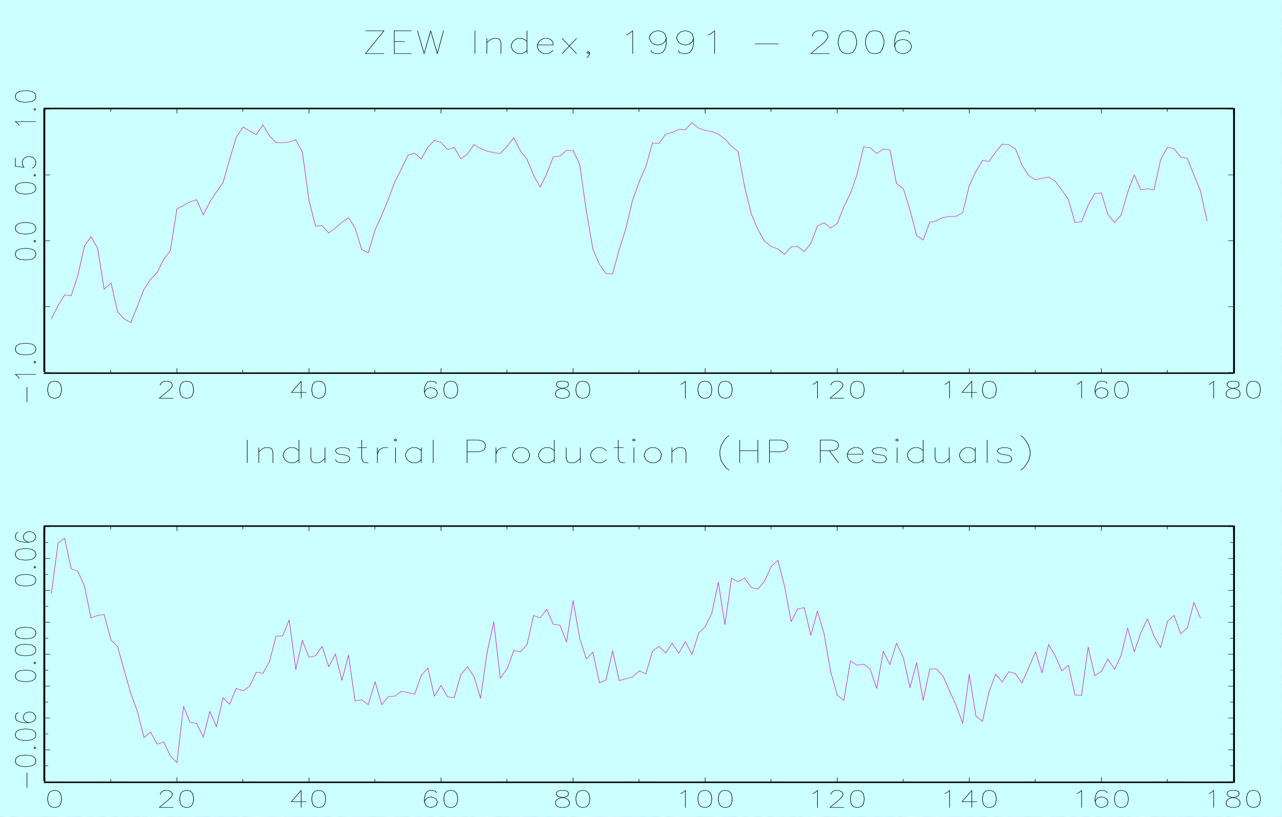
- RE tests typically negative, recent revival of interest in animal spirits (Akerlof and Shiller)

- Research question: Can we identify animal spirits at work? Can we identify the influence of social interaction on opinion formation?

Application I: Macro Sentiment

ZEW Index of Economic Sentiment, 1991 – 2006,

Monthly data, index = #positive - # negative, ca. 350 respondents



A Canonical Interaction Model à la Weidlich

- Two opinions, strategies etc: + and –
- A fixed number of agents: 2N
- Agents switch between groups according to some transition probabilities $w_\downarrow$ and $w_\uparrow$

v: frequency of switches,

U: function that governs switches

α_0, α_1: parameters

$$w_\uparrow = v * \exp(U)$$

$$w_\downarrow = v * \exp(-U)$$

$$U = \alpha_0 + \alpha_1 x$$

$$x = \frac{n_+ - n_-}{2N}$$

Sentiment index

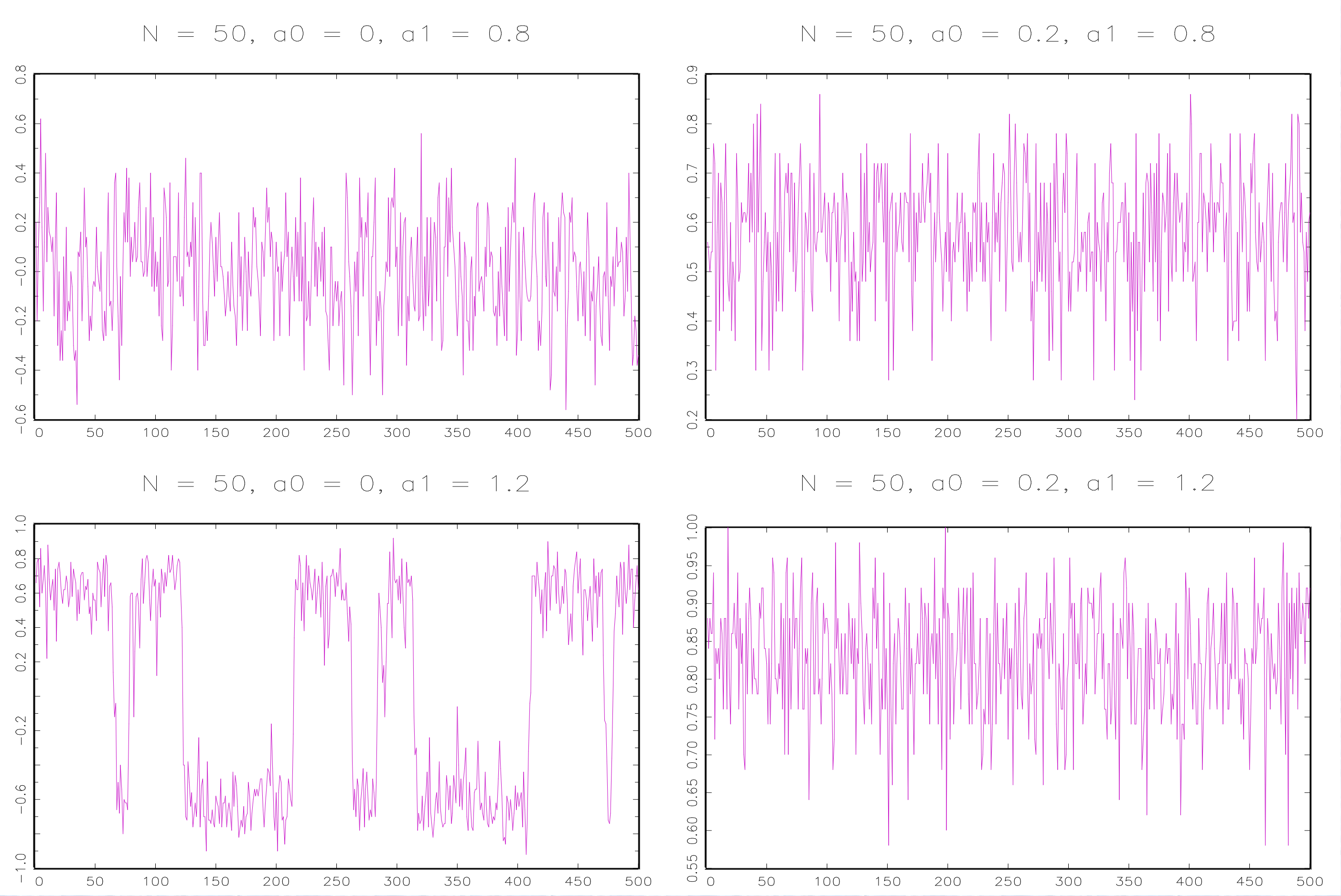
Remarks

- The model is designed as a continuous-time framework, i.e. $w_{\downarrow}$ and $w_{\uparrow}$ are Poisson rates (jump Markov process)
- The "canonical" model allows for interaction (via α_1) and a bias towards one opinion (via α_0), but could easily be extended by including arbitrary exogenous variables in U
- The framework corresponds closely to that of "discrete choice with social interactions", it formalizes non-equilibrium dynamics, while DCSI only considers RE equilibria

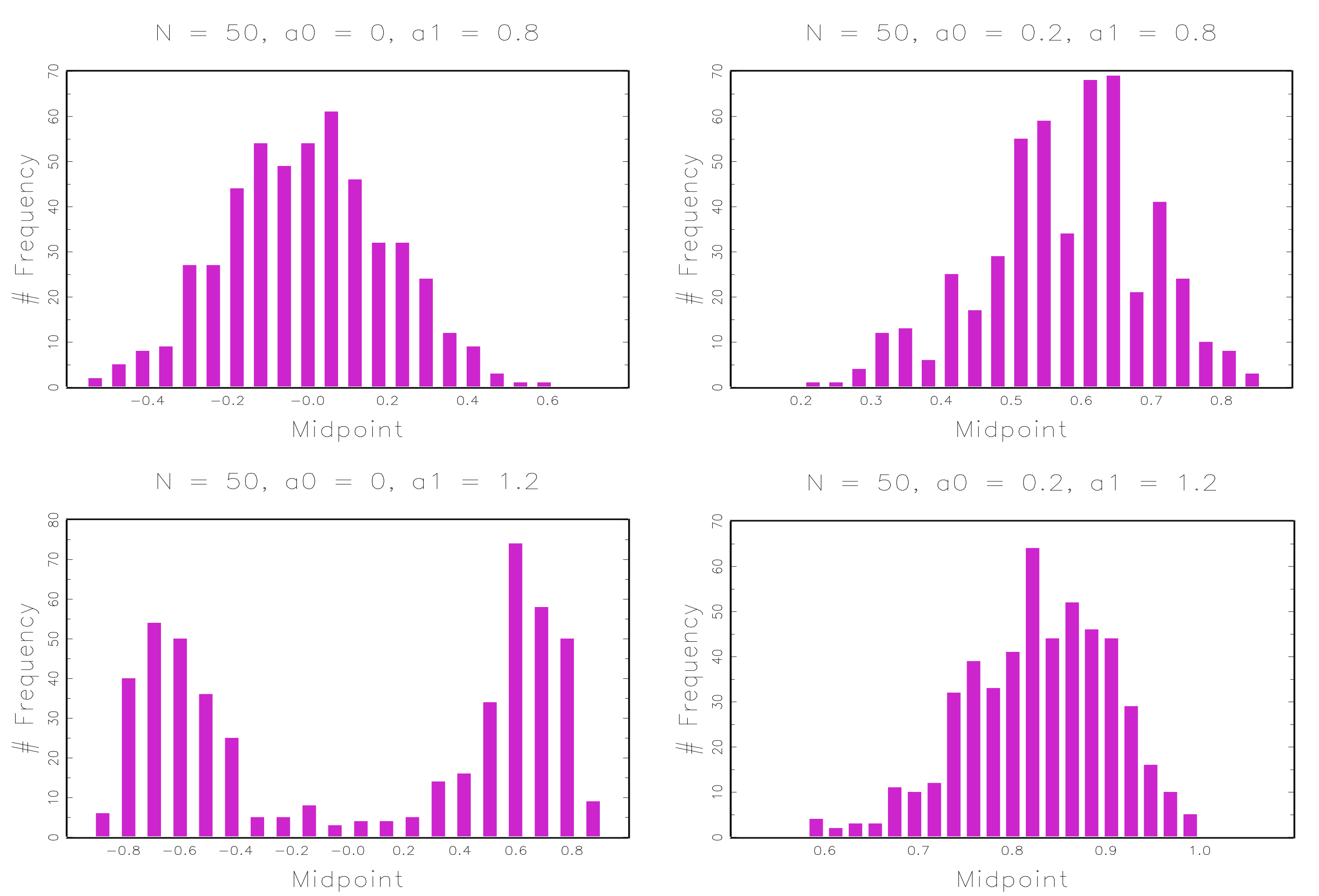
Theoretical Results

- For $\alpha_1 \leq 1$: uni-modal stationary distribution with maximum $x^* =, >,< 0$ (for $\alpha_0 =,>,< 0$)
- For $\alpha_1 > 1$ and α_0 not too large: bi-modality (symmetric around 0 if $\alpha_0 = 0$, asymmetric otherwise)
- If $|\alpha_0|$ gets too large: return to uni-modality (with maximum $x^* >,< 0$ for $\alpha_0 >,< 0$)

Some simulations



Thomas Lux
Department of Economics
University of Kiel


N = 50, a0 = 0, a1 = 0.8
N = 50, a0 = 0.2, a1 = 0.8
N = 50, a0 = 0, a1 = 1.2
N = 50, a0 = 0.2, a1 = 1.2
# Frequency
Midpoint

How to Arrive at Analytical Results?

- Our system is quite complex: 2N coupled Markov jump processes with state-dependent, non-linear transition rates
- Solution via ***Master equation***: full characterization of time development of pdf: can be integrated numerically, but is too computation intensive with large population
- More practical: ***Fokker-Planck equation*** as approximation to transient density

How to Arrive at Analytical Results?

x in steps of 1/N

Different formalisms for jump Markov models of interactions:

- Master equation: full characterization of time development of pdf: can be integrated numerically, but is too computation intensive with large population
- Fokker-Planck equation:

$$\frac{dP(x,t)}{dt} = w_{\uparrow}P(x-\frac{1}{N},t) + w_{\downarrow}P(x+\frac{1}{N},t) - (w_{\uparrow}+w_{\downarrow})P(x,t)$$

$$\frac{\partial p_x}{\partial t} = \frac{\partial}{\partial x}\mu(x)p_x + \frac{\partial^2}{\partial x^2}g(x)p_x$$

drift:-μ

Diffusion: 2*g

Fokker-Planck-Equation:

expanding the step-operators for x in Taylor series up to the second order and neglecting the terms $o(\Delta x^2)$, we end up with the following FPE:

$$\frac{\partial p_x}{\partial t} = \frac{\partial}{\partial x}\mu(x)p_x + \frac{\partial^2}{\partial x^2}g(x)p_z$$

$$-\mu(x) = \frac{n_-}{2N}w_\uparrow(x) - \frac{n_+}{2N}w_\downarrow(x)$$

$$g(x) = \frac{1}{2N^2}\left(\frac{n_-}{2N}w_\uparrow(x) + \frac{n_+}{2N}w_\downarrow(x)\right)$$

NB: Derivation of Macroscopic Laws of Motion from the Microdynamics

The expected value of x at time t is given by:

$$\bar{x}_t = \sum_{x=-1}^{1} xP(x;t)$$

and its change in time is obtained from the Master equation:

$$\frac{d\bar{x}_t}{dt} = \sum_{x=-1}^{1} x \frac{dP(x;t)}{dt}$$

In general:

$$\frac{d\bar{x}_t}{dt} = \sum_{x} x \sum_{x'} (w(x|x' ,t)P(x' ,t) - w(x'|x,t)P(x;t))$$

With a few simple manipulations we get:

$$\frac{d\overline{x}_t}{dt} = \sum_x \sum_{x'} x w_{xx'} P(x';t) - \sum_x \sum_{x'} x w_{x'x} P(x;t)$$

$$= \sum_x \sum_{x'} x' w_{x'x} P(x;t)) - \sum_x \sum_{x'} x w_{x'x} P(x;t)$$

$$= \sum_x \sum_{x'} (x' - x) w_{x'x} P(x,t)$$

$$= \sum_x a_{x,1} P(x;t) = \overline{a_{x,1}}$$

$= a_{x,1}$

$$with\ w_{x'x} = w(x' | x,t)$$

$a_{x,1}$ is the so-called *first jump moment* which, in our framework, coincides with the drift term of the Fokker Planck equation:

$$a_{x,1} = \sum_{x'} (x' - x) w_{x'x} = \frac{1}{N} w_{\uparrow}(x) + \left(-\frac{1}{N} \right) w_{\downarrow}(x)$$

$$= \frac{1}{N} (w_{\uparrow}(x) - w_{\downarrow}(x))$$

The change in time of the expected value of x is, therefore, given by the expectation of $a_{x,1}$, i.e., the average jump of x weighted by the probability of x at time t, $P(x;t)$:

$$\frac{d\bar{x}_t}{dt} = \sum_{x} a_{x,1} P(x;t) = \overline{a_{x,1}}$$

Approximation of mean-value equation:

First-order approximation of the mean-value dynamics yields:

$$\frac{d\overline{x}_t}{dt} = a_{x,1}(\,\overline{x}\,)$$

which is exact if the first jump moment is linear in x.

A *second-order approximation* adds a correction term depending on the fluctuations:

$$\frac{d\overline{x}_t}{dt} = E[a_{x,1}(\overline{x}) + \underbrace{(x-\overline{x})}_{=0}\, a'_{x,1}(\overline{x}) + \frac{1}{2}\underbrace{(x-\overline{x})^2}_{=\sigma_x^2}\, a''_{x,1}(\overline{x}) + \ldots]$$

$$\approx \underbrace{a_{x,1}(\overline{x})}_{\text{pure mean value dynamics}} + \underbrace{\frac{1}{2}\sigma_x^2 a''_{x,1}(\overline{x})}_{\text{second-order correction}}$$

Implementation: The population transition rates are:

$$w_{\uparrow}(x) = N(1-x)ve^{\alpha_0+\alpha_1 x} = n_{-}ve^{\alpha_0+\alpha_1 x}$$

$$w_{\downarrow}(x) = N(1+x)ve^{-\alpha_0-\alpha_1 x} = \mathrm{n}_{+}ve^{-\alpha_0-\alpha_1 x}$$

The first jump moment is:

$$\begin{aligned} a_{x,1} &= \sum_{x'}(x'-x)w_{x'x} \\ &= \frac{1}{N}[N(1-x)ve^{\alpha_0+\alpha_1 x} - N(1+x)ve^{-\alpha_0-\alpha_1 x}] \end{aligned}$$

Using the hyperbolic trigonometric functions, the first-order approximation to the mean-value dynamics becomes:

$$\frac{d\bar{x}_t}{dt} = 2v(Tanh(\alpha_0 + \alpha_1\bar{x}) - \bar{x})Cosh(\alpha_0 + \alpha_1\bar{x})$$

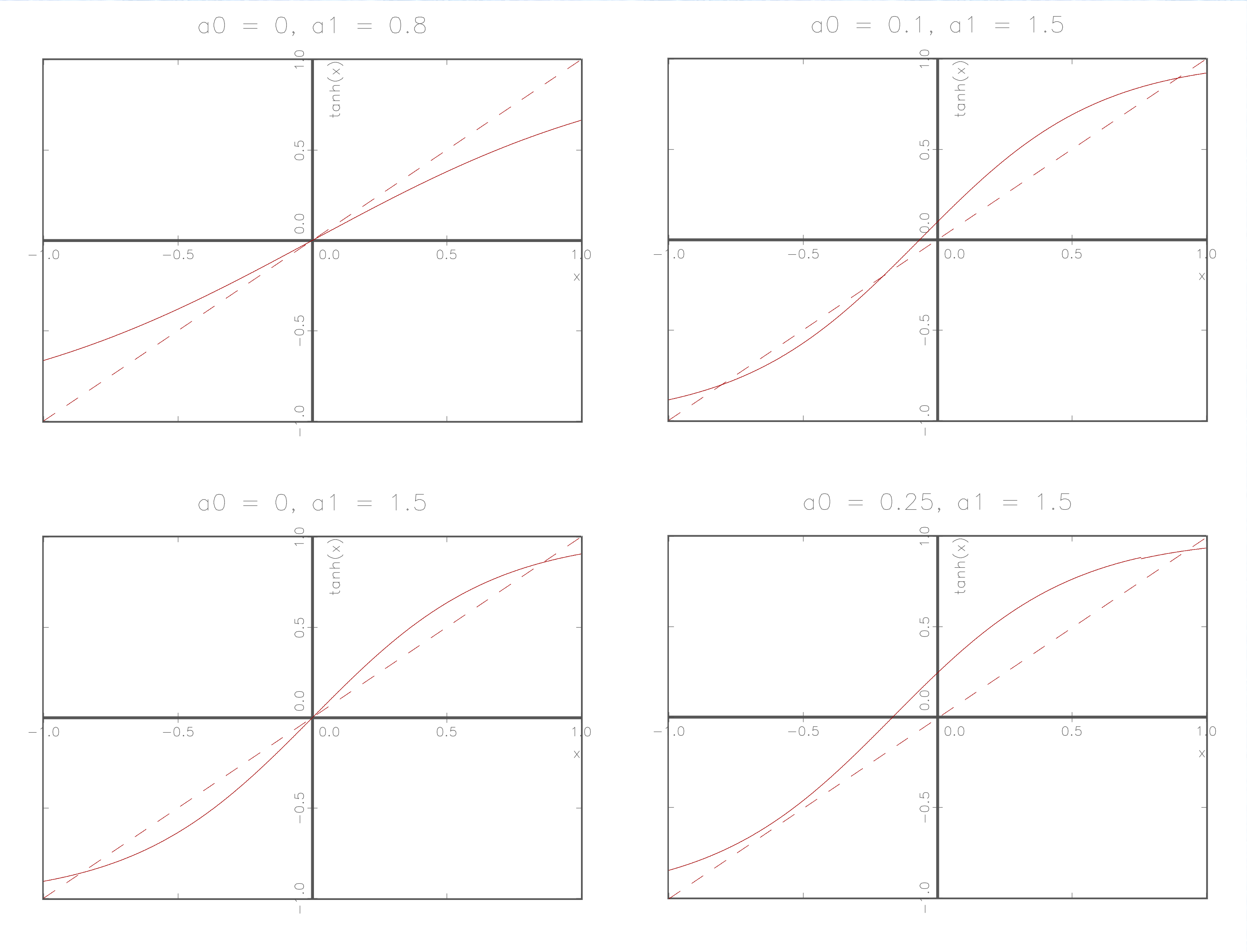
Stationary states are obtained from:

$$\frac{d\bar{x}_t}{dt} = 0 \Rightarrow \bar{x}^* = Tanh(\alpha_0 + \alpha_1\bar{x}^*)$$

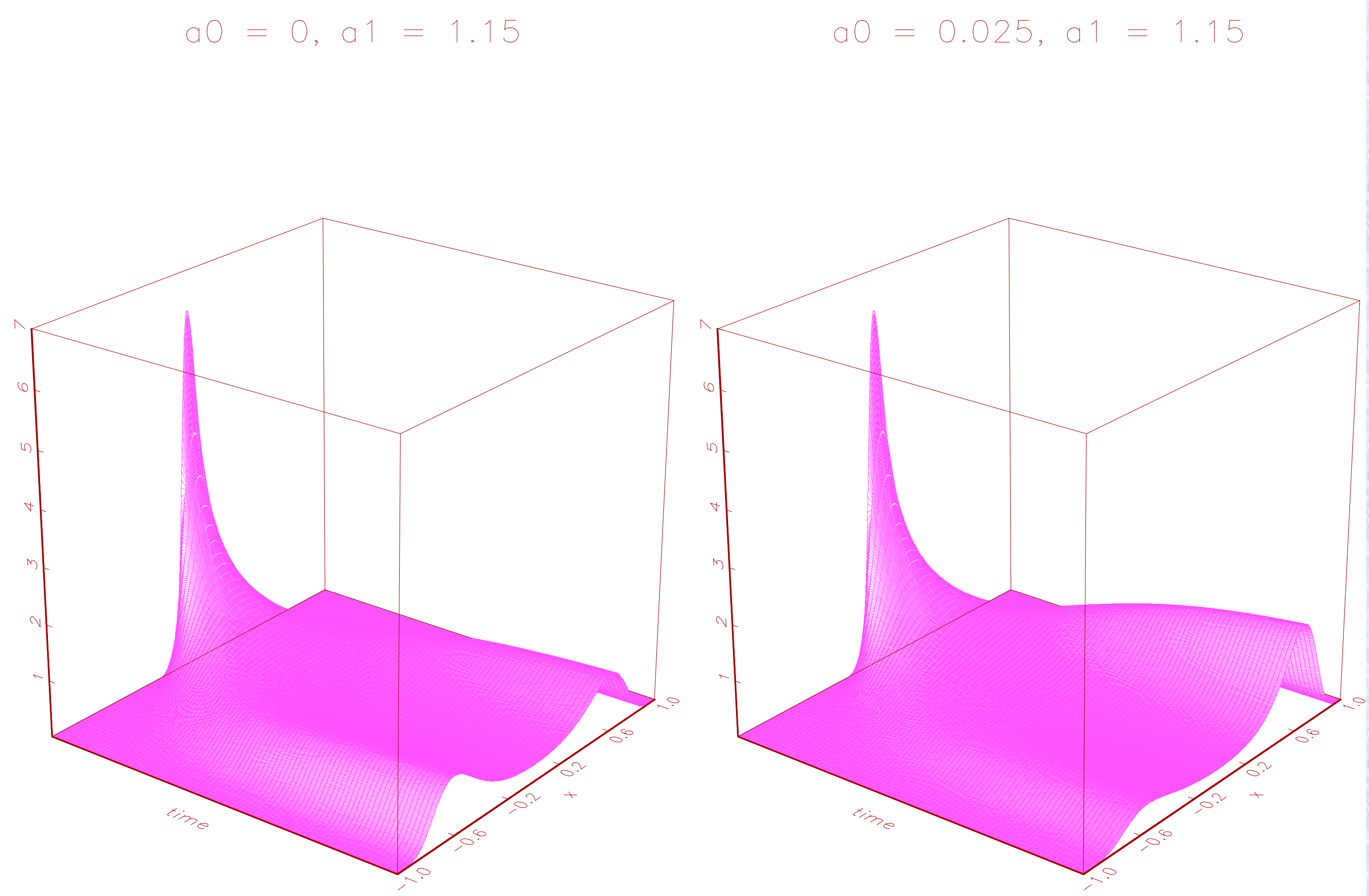
Features:

• unique stable steady state x* for α_0 small, $\alpha_1 \leq 1$,

• multiple steady states $x_\pm^* \neq 0$ for α_0 small, $\alpha_1 > 1$, the formerly unique steady state x* becomes unstable.



Various cases of the stochastic herding model



Examples of Transient Densities from Fokker-Planck Equation

Second-order approximation:

$$\frac{d\overline{x}_t}{dt} = 2v(Sinh(\alpha_0 + \alpha_1\overline{x}) - \overline{x}Cosh(\alpha_0 + \alpha_1\overline{x})) \ +$$

$$\underbrace{v((\alpha_1{}^2 - 2\alpha_1)Sinh(\alpha_0 + \alpha_1\overline{x}) - \overline{x}\alpha_1 Cosh(\alpha_0 + \alpha_1\overline{x}))}_{=\frac{1}{2}a''_{x,1}(\overline{x})}\sigma_x^2$$

To implement the correction term, we need information about the time evolution of the variance…

Master equation for opinion index *x*:

$$\frac{dP(x;t)}{dt} = w_{\downarrow}\left(x+\frac{1}{N}\right)P\left(x+\frac{1}{N};t\right) + w_{\uparrow}\left(x-\frac{1}{N}\right)P\left(x-\frac{1}{N};t\right)$$

$$-\left(w_{\uparrow}(x)+w_{\downarrow}(x)\right)P(x;t)$$

The Fokker-Planck Equation as a Taylor Series Expansion of the Master Equation

Since for large N, x is close to a continuous quantity, we can perform a Taylor expansion with respect to Δx. Rearranging gives:

$$\frac{dP(x;t)}{dt} = w_{\uparrow}\left(x-\frac{1}{N}\right)P\left(x-\frac{1}{N};t\right) - w_{\uparrow}(x)P(x;t)$$

$$+ w_{\downarrow}\left(x+\frac{1}{N}\right)P\left(x+\frac{1}{N};t\right) - w_{\downarrow}(x)P(x;t)$$

A second-order approximation of the first and second group of components on the right-hand side around x yields:

interpretation as quasi-continuous function of x

Estimation: for a time series of discrete observations X_s of our canonical process, the likelihood function reads

$$-\log f_0(X_0 \mid \theta) - \sum_{s=0}^{T-1} \log f(X_{s+1} \mid X_s, \theta)$$

with discrete observations X_s, the Master or FP equations are the exact or approximate laws of motion for the transient density and allow to evaluate log $f(X_{s+1}|X_s,\theta)$ and , therefore, to estimate the parameter vector θ ($\theta = (v, \alpha_0,\alpha_1)$')!

Kramers-Moyal expansion from statistical physics

$$\frac{\partial P(x;t)}{\partial t} = \frac{\partial}{\partial x}[w_{\uparrow}(x)P(x;t)]\left(-\frac{1}{N}\right) + \frac{1}{2}\frac{\partial}{\partial x^2}[w_{\uparrow}(x)P(x;t)]\left(-\frac{1}{N}\right)^2$$

$$+\frac{\partial}{\partial x}[w_{\downarrow}(x)P(x;t)]\frac{1}{N} + \frac{1}{2}\frac{\partial}{\partial x^2}[w_{\downarrow}(x)P(x;t)]\left(\frac{1}{N}\right)^2$$

$$\Rightarrow \frac{\partial P(x;t)}{\partial t} = -\frac{\partial}{\partial x}[(w_{\uparrow}(x)-w_{\downarrow}(x))P(x;t)]\frac{1}{N}$$

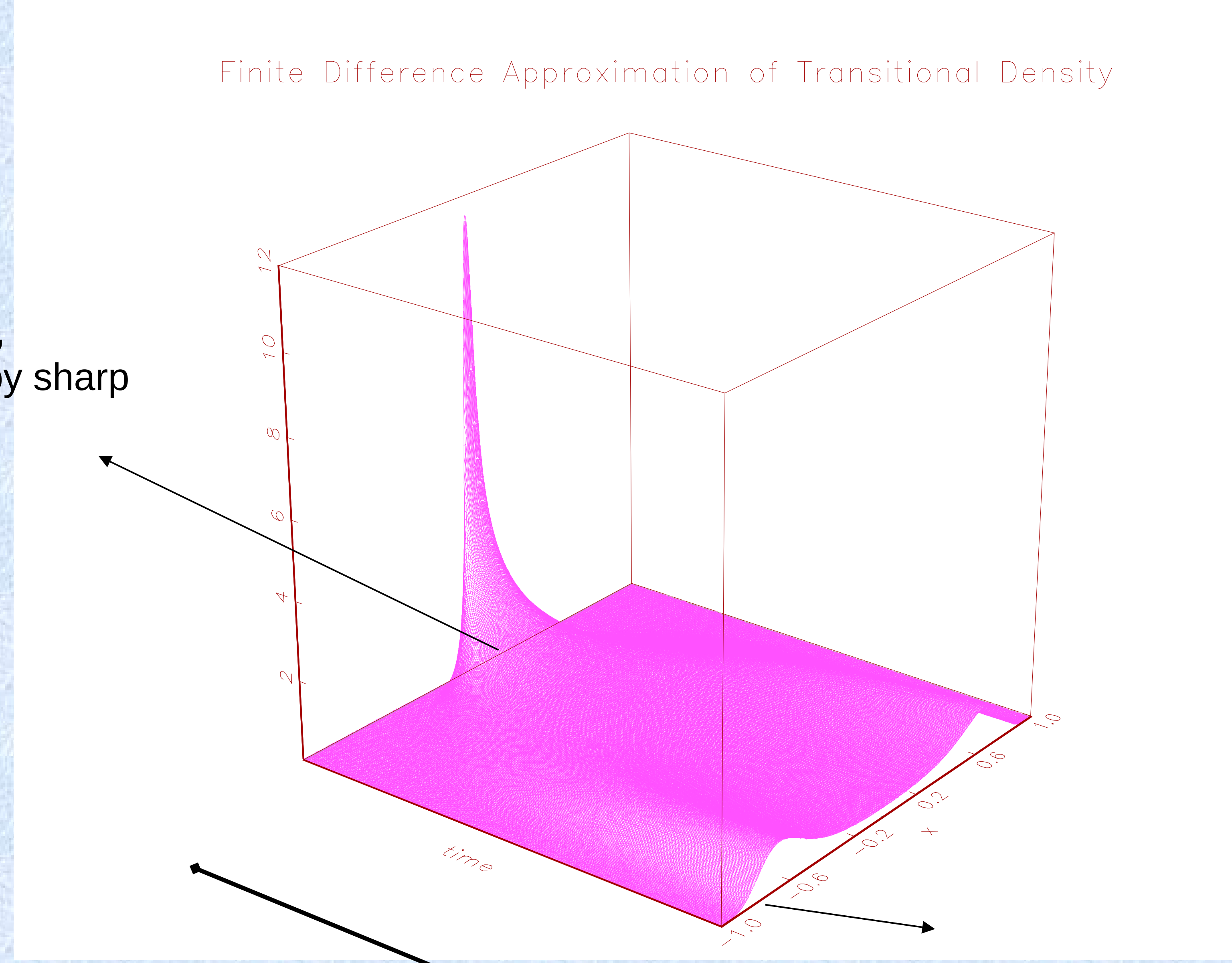
$$+\frac{1}{2}\frac{\partial}{\partial x^2}[(w_{\uparrow}(x)+w_{\downarrow}(x))P(x;t)]\frac{1}{N^2}$$

Fokker-Planck equation

Implementation

- Usually no analytical solution for transient pdfs from Master or FP equations
- Numerical solution of Master equation too computation intensive if there are many states x (i.e., particularly with large N)
- Numerical solutions of FP equation is less computation intensive, various methods available for discretization of stochastic differential equations

Observation X_s, approximated by sharp Normal distr.



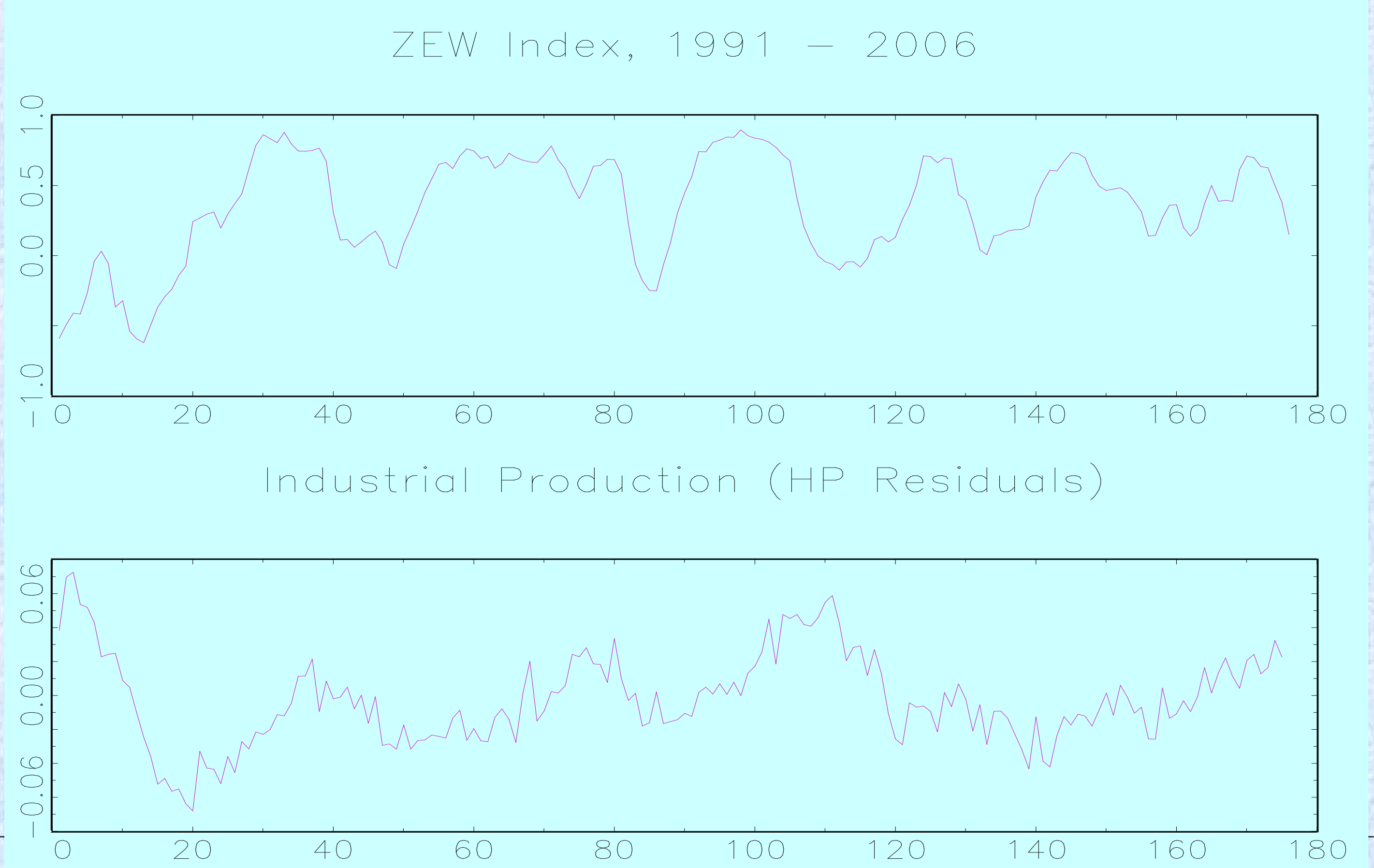
Time interval [s, s+1]

Evaluation of Lkl of observation X_{s+1}

Thomas Lux
Department of Economics
University of Kiel

Empirical Application:
ZEW Index of Economic Sentiment, 1991 – 2006,

Monthly data, index = #positive - # negative, ca. 350 respondents



Extensions of Baseline Model

- introduction of exogenous variables (industrial production, interest rates, unemployment, political variables,…)

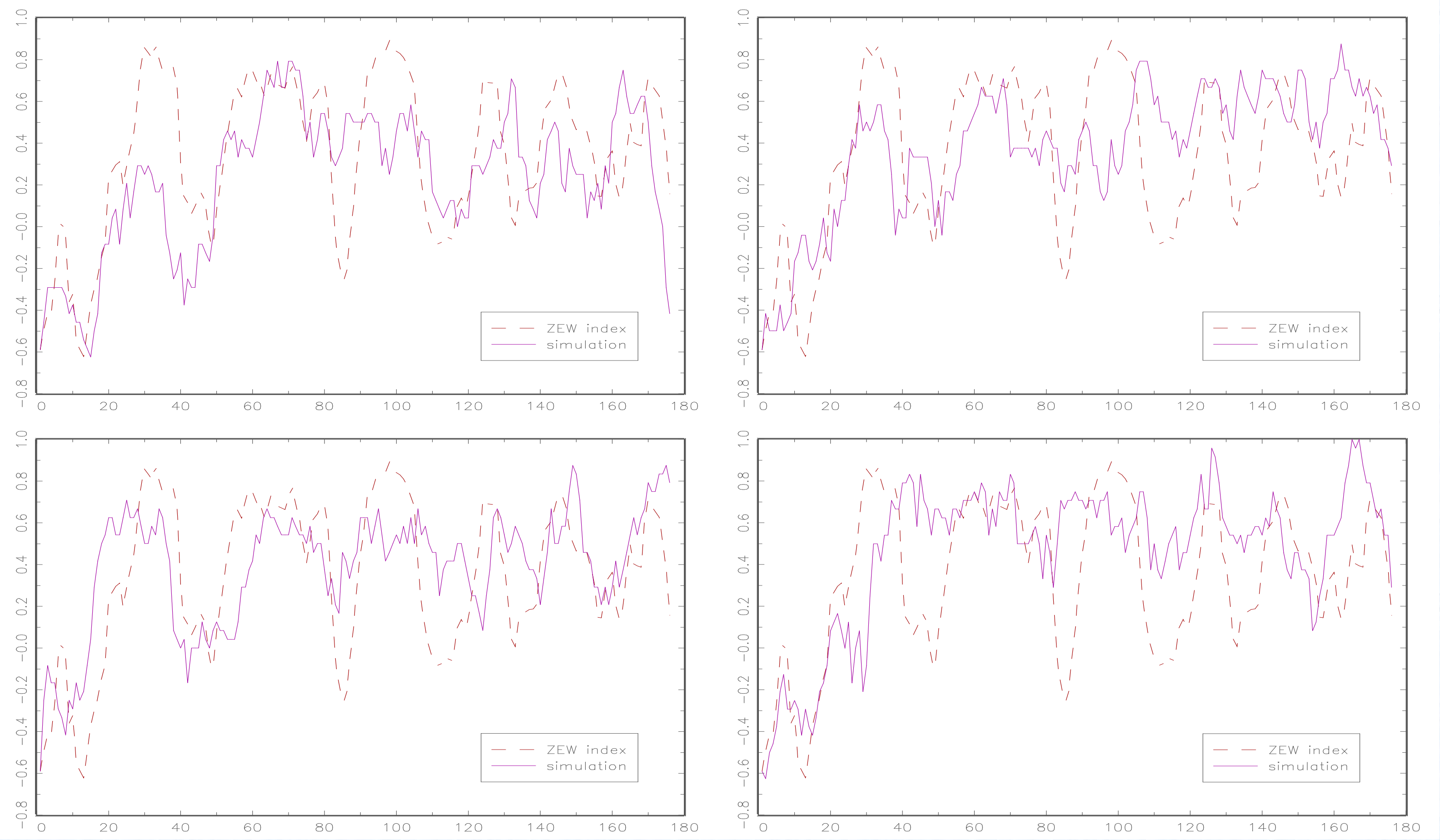
- ‘momentum’ effect

- endogenous N: ‘effective’ number of independent agents

$$U_t = \alpha_0 + \alpha_1 x_t + \alpha_2 IP + \alpha_3 (x_t - x_{t-1})$$

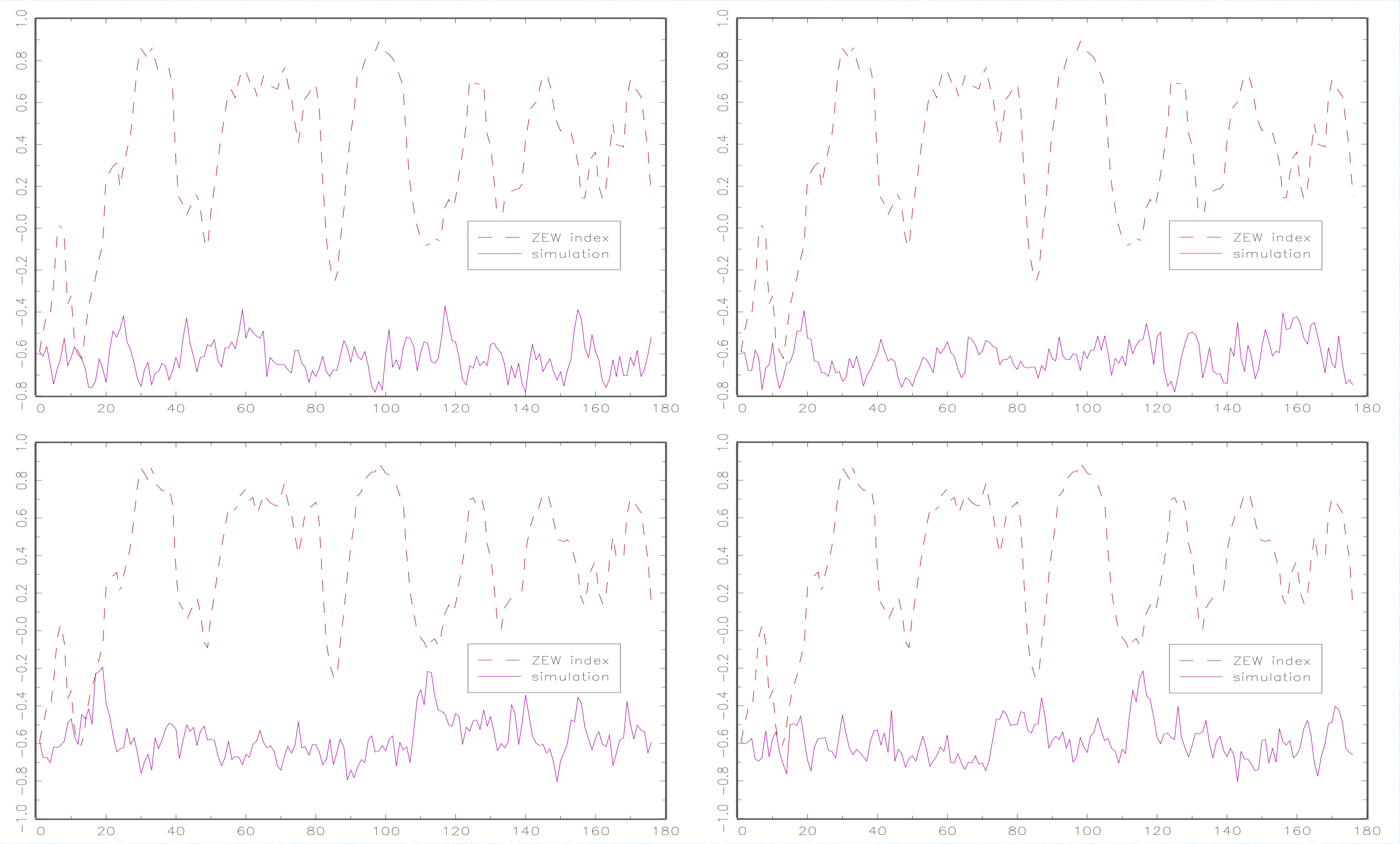
	v	α_0	α_1	α_2	α_3	N	ML	AIC
Model 1 (baseline)	0.78 (0.06)	0.01 (0.01)	1.19 (0.01)			*official: 350/2*	-726.9	1459.8
Model 2 (end. N)	0.15 (0.07)	0.09 (0.06)	0.99 (0.14)			21.21 (9.87)	-655.9	1319.7
Model 3 (feedback from IP)	0.13 (0.06)	0.09 (0.07)	0.93 (0.16)	-4.55 (2.53)		19.23 (8.78)	-650.4	1310.9
Model 4 (moment.)	0.14 (0.05)	0.10 (0.06)	0.91 (0.14)		2.11 (0.76)	27.24 (9.63)	-627.5	1265.1
Model 5 (mom. + IP)	0. 12 (0.05)	0.11 (0.06)	0.86 (0.16)	-2.82 (1.65)	2.23 (0.81)	25.12 (8.95)	-624.9	1261.94

... a few simulations of model V (identical starting value of x, identical influence from IP



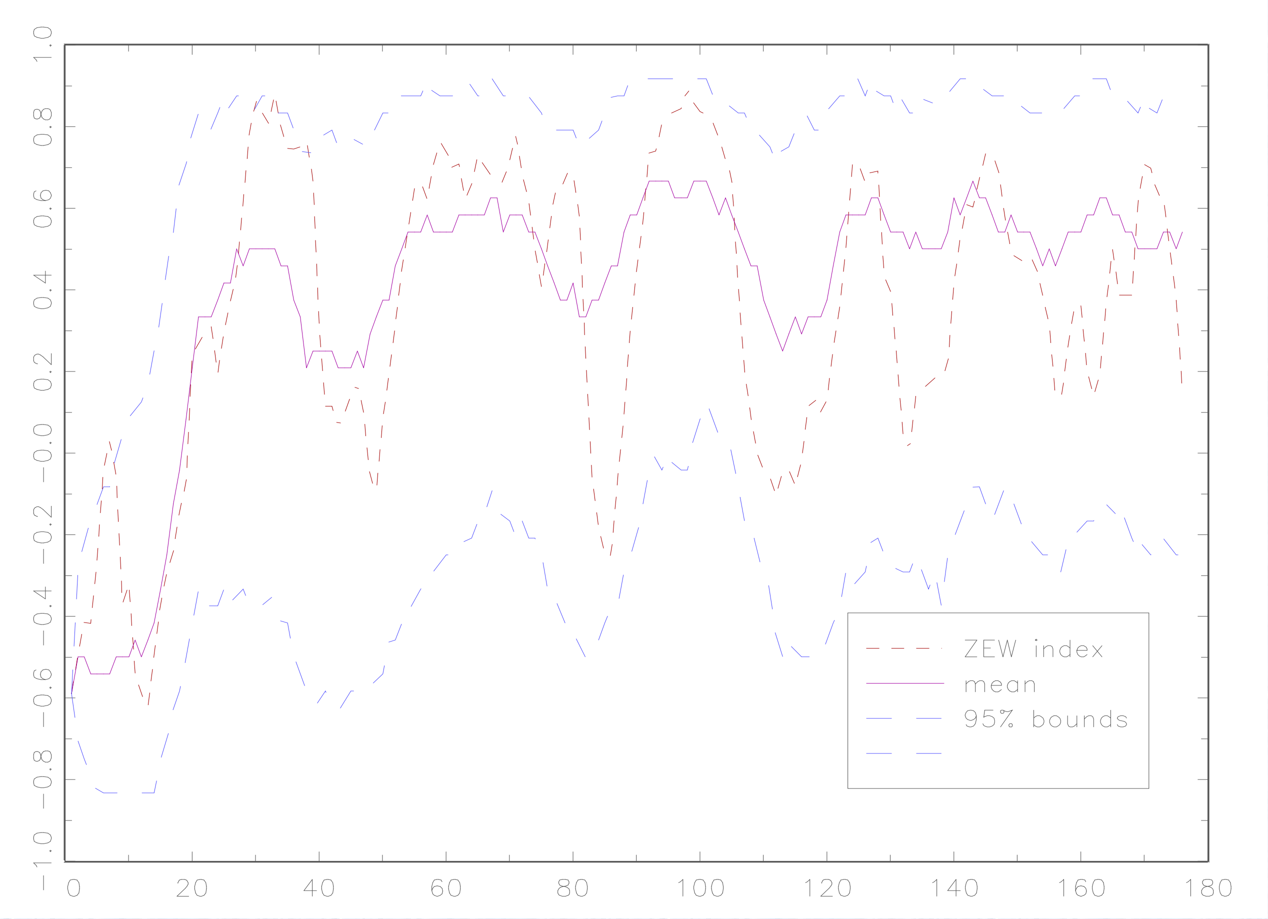
Thomas Lux
Department of Economics
University of Kiel

For comparison: simulations of model I (identical starting value of x) -> no similarity



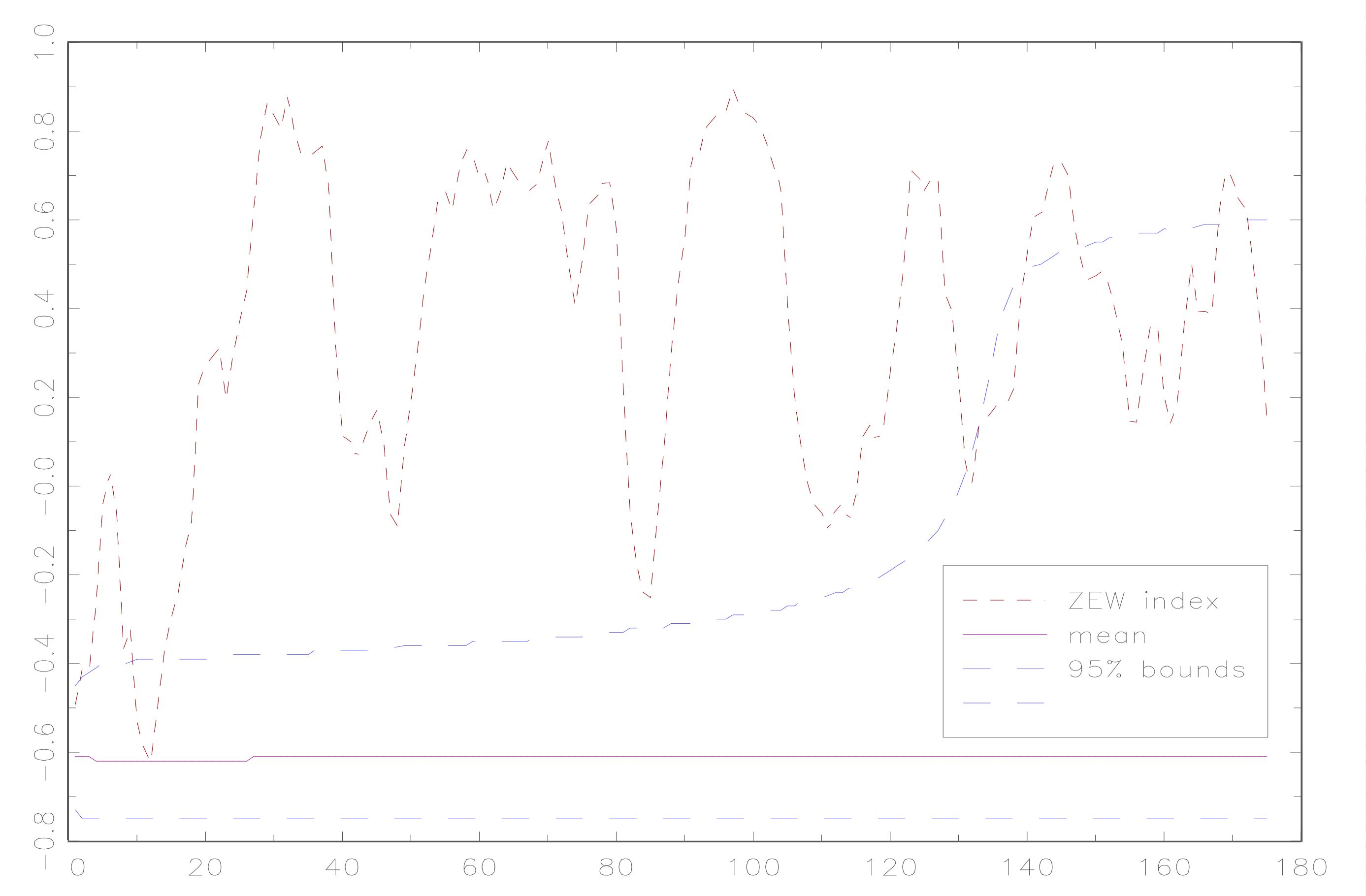
Specification tests: Mean and 95% confidence interval from model 3

(conditional on initial condition and influence form IP)



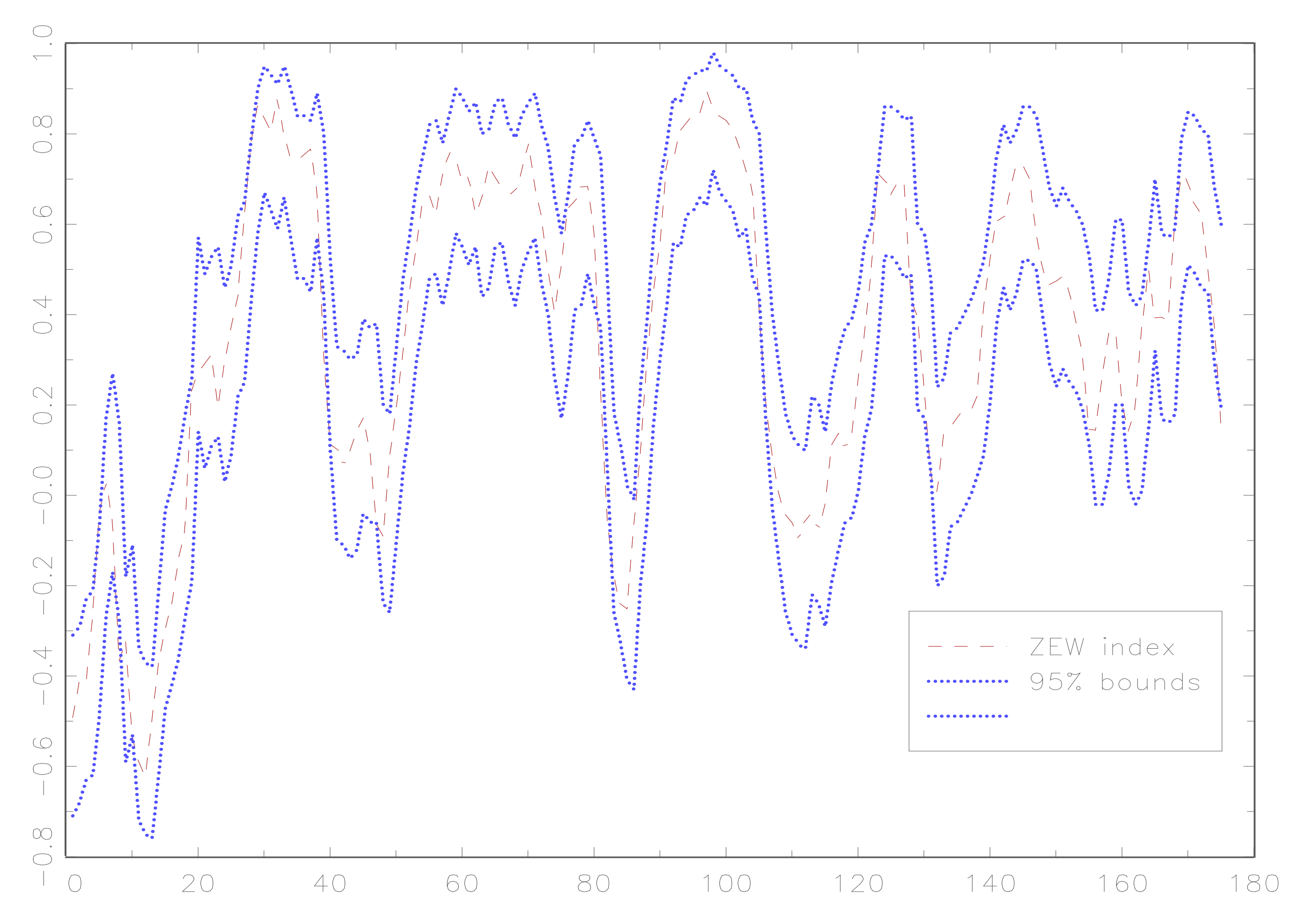
Thomas Lux
Department of Economics
University of Kiel

Mean and 95% confidence interval from model 1



Thomas Lux
Department of Economics
University of Kiel

…are the large shifts of opinion in harmony with the estimated model? 95% confidence interval from period-by-period iterations (model V) (conditional on previous realization and influence form IP)



Thomas Lux
Department of Economics
University of Kiel

Application II: Stock market sentiment, tri-variate set (short/medium run + stock prices)

- data from animusX Investors Sentiment, short and medium run sentiment (one week, 3 months) for German stock market
- categorial data (++,+,0,-,--) expressed as diffusion index
- weekly data since 2004
- online survey, ca 2000 subscribers, ca. 20 – 25 % participation
- incentive: only participants receive results on Sunday evening
- as far as I can see: not used in scientific research so far

Literature on Sentiment

- Stimulated by behavioral finance, noise trader models
- Data on sentiment available for many countries, mostly diffusion indices (# optimistic - # pessimistic)
- Empirical evidence for US: some dynamic interaction between returns (volatility) and sentiment, but no predictive power for near-term returns (Brown and Cliff, 2004)
- In VARs of returns and sentiment: returns are exogenous, sentiment acts as feedback variable (Brown and Cliff, 2004 for US, Kling and Gao, 2008 for Shanghai)

Questions to be pursued:

- Does this efficiency with respect to sentiment also hold for Germany? -> VAR analysis
- Can we come up with a behavioral model of the interaction of sentiment and returns? -> agent-based model

Willkommen

MIND the MAINSTREAM

animusX® - Investors Sentiment zählt zu den führenden Anbietern in Deutschland rund um das Thema Technische Analyse gepaart mit Sentiment und Behavioral Finance.

Mit neuartigen und in Deutschland einmaligen Analysemethoden zur Messung der Anlegerstimmung wird, im Gegensatz zu herkömmlichen Sentimentumfragen, bewusst auf die kausalen Zusammenhänge zwischen der Technischen Analyse und den damit einhergehenden verhaltensorientierten Prozessen der Marktteilnehmer wertgelegt.

animusX® - Investors Sentiment befragt wöchentlich knapp 2.00 Finanzinvestoren, darunter mehr als 295 institutionelle Investoren namhafter Banken, Brokerhäuser, Versicherungen und Vermögensverwalter nach ihrer Einschätzung zu den Kapitalmärkten. Dabei ist die Fragestellung bewusst in der Art gestaltet, dass die Ergebnisse die, aus der Behavioral Finance bekannten, Anomalien der Investoren widerspiegeln sollen.

Sie werden hier demnach keine Standardfragen vorfinden, die Ihnen wohlmöglich aus herkömmlichen Sentimentanalyse bekannt sind...

Probeexemplar

Machen Sie sich hier ein Bild mittels des Probeexemplars über die Ergebnisse und Auswertungen von **animusX®**

Kostenlose Anmeldung

Hier können Sie sich zu **animusX®** anmelden, in den Adminbereich eintreten oder aber Ihre Daten noch einmal anfordern.

Kostenlose Anmeldung

Teilnehmer Login

Userdaten vergessen

Aktuelle Analyse

NEU

animusX®*Analyse*
für die Kalenderwoche 46/2008

DAX: Nach „Fahrplan"

Die Aktienmärkte hatten vergangene Woche, diesund jenseits des Atlantiks, eine Reihe von negativen Wirtschafts- und Konjunkturdaten zu verarbeiten. Exakt an wichtigen Widerständen kam es dann zur Korrektur. Die US-Aktien traf es dabei, im Vergleich zum DAX, besonders hart. Kein netter Willkommensgruß für den 44. Präsident Elected der USA - Barack Obama.

FGBL: Hopp oder Topp!

Über 180 Bp legte der FGBL in der vergangenen Woche zu. Die Zinssenkung seitens der EZB war vermutlich nur ein Grund, aber nicht der Grund. Diese war von den Investoren schließlich mehr als erwartet worden. Was aber treibt die Kurse – haben zu viele zu lange dagegen gehalten?

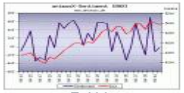
EURUSD: Kurznotiz

Der Euro korrigiert weiterhin seinen ersten Ausbruchsversuch in Richtung 1,38 Dollar. Mit 1,2550 Dollar kommt er an eine wichtige Marke – eine Kreuzunterstützung.

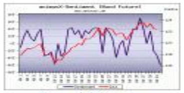
Bei aktiver Beteiligung an den wöchentlichen Umfragen erhalten Sie die Ergebnisse (Grafiken nebst schriftlicher Auswertung) **kostenlos.**

Aktuelles Sentiment

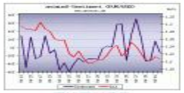
Aktuelles animusX®-Sentiment
(Zum Vergrößern klicken)



DAX



Bund Future



EURUSD

Zugriff auf weitere Sentimentdaten erhalten Sie nach Registrierung und bei aktiver Teilnahme an den wöchentlichen Umfragen.

Referenzen

Helaba Landesbank Hessen-Thüringen | HSBC | HSH NORDBANK | HVB Corporates & Markets | Postbank | DZ BANK Zusammen geht mehr. | SÜDWESTBANK Leistung ist im Süden zu Hause. | NORD/LB

Lesen Sie hier was institutionelle Investoren über **animusX® - Investors Sentiment** denken.

Sentiment from animusX, 2004 - 2008

We use the first 150 data points as in-sample, the remaining 52 as out-of-sample entries throughout

Table 1: Summary Statistics						
Panel A: Full sample (202 observations)						
	Mean	S.D.	Skewness	Kurtosis	ρ_1	*ADF*
S-Sent	0.163	0.376	-0.546	-0.848	0.516	-3.644
M-sent	0.092	0.132	-0.035	-0.363	0.790	-2.295
Returns	0.003	0.022	-0.503	0.488	-0.054	-5.233
Panel B: In-sample (150 observations)						
S-Sent	0.222	0.354	-0.732	-0.464	0.436	-2.352
M-Sent	0.073	0.136	0.204	-0.223	0.777	-2.262
Returns	0.004	0.019	-0.414	0.287	-0.095	-4.017

Notes:

- sentiment is highly persistent, M-Sent more persistent and less volatile
- all variables are stationary

Vector Autoregressive Model

Observations at time t: $y_t = (y_t^1, y_t^2, y_t^3) \equiv (S - Sent, M - Sent, ret.)$

Hypothesized simultaneous system:

$$y_t = \nu + A_1 y_{t-1} + \ldots + A_p y_{t-p} + u_t$$

- “agnostic” statistical approach: can be used to test certain hypotheses (e.g., ‘effficiency’)

- reveals “causal” structures, allows to test for causality

- typical sequence of analysis: specification (how many lags?), estimation, causality, contribution of individual components (error variance decomposition, impulse responses etc.)

Table 3: Coefficient estimates of VAR (2) Model

Panel A: Autoregressive Parameters				
	indep. var.			
dep. var	lag	S-Sent	M-Sent	Ret.
S-Sent	1	0.341	0.813*	2.137
M-Sent	1	-0.028	0.642**	0.306
Ret	1	0.004	0.056**	-0.118
S-Sent	2	0.211	-0.394	-3.025
M-Sent	2	-0.006	0.184*	0.092
Ret	2	0.000	-0.036	-0.206
Panel B: Covariance Matrix				
	indep. var.			
dep. var		S-Sent	M-Sent	Ret.
S-Sent		0.306**	0	0
M-Sent		-0.022**	0.080**	0
Ret		0.017**	-0.001	0.007**

Table 4: Causality Tests			
	sent $\rightarrow$ *ret*	*ret* $\rightarrow$ sent	Instant.causality
Var(1)			
Wald	7.372	1.234	204.208
p-value	0.025	0.540	0.000
Var(2)			
Wald	10.058	3.179	213.073
p-value	0.040	0.528	0.000
Var(5)			
Wald	16.672	15.944	214.100
p-value	0.082	0.101	0.000

Note: this is the opposite of extant literature:

- significant causal effect from sent on returns, but not vice versa!!
- with the animusX data we should be able to predict near-term returns

Extensions/Modifications of Baseline Model

- we use this model both for S-sent and M-sent allowing for cross-influences and dependency on returns

$$U_t = \alpha_0 + \alpha_1 x_t + \alpha_2 y_t + \alpha_3 ret_t$$

- we add a simple diffusion for prices

$$dp_t = (\gamma_1 x_t + \gamma_2 y_t)dt + \sigma_p dZ_2$$

Again: Implementation of Multi-Variate Likelihood Function via Numerical FD Approximations of Fokker-Planck-Equation:

$$\frac{\partial f(z;t)}{\partial t} = -\sum_i \partial_i \left[A_i(z) f(z;t) \right] + \frac{1}{2} \sum_{i,j} \partial_i \partial_j \left[B_{ij}(z) f(z;t) \right]$$

Drift term of variable z_i

Matrix of diffusion terms

Note: we assume that the cross-derivatives in B_{ij} are all equal to zero to avoid some technical complications

1D: S-Sent Table 1: Parameter estimates for uni-variate models

Panel A: Agent-based model of S-Sent (x)		
Param.	Model I	Model II
ν_s	8.851 (2.756)	8.938 (2.741)
α_0	0.008 (0.004)	0.008 (0.004)
α_1 (S-Sent)	1.055 (0.014)	1.055 (0.013)
α_2 (M-Sent)	0.062 (0.025)	0.062 (0.025)
α_3 (ret.)	-0.014 (0.107)	
N	68.452 (14.411)	68.402 (14.376)
LogL	-694.738	-694.740
AIC	1.401.477	1.399.481
BIC	1.399.416	1.399.434

Strong interaction, bi-modality

1D: M-Sent Table 1: Parameter estimates for uni-variate models

Panel B: Agent-based model of M-Sent (y)				
Param.	Model I	Model II	Model III	Model IV
ν_m	0.126 (·)	0.106 (0.118)	0.111 (·)	0.305 (0.034)
β_0	0.069 (·)	0.080 (0.074)	0.073 (·)	0.033 (0.017)
β_1 (M-Sent)	0.046 (·)	-0.118 (1.211)	-0.056 (·)	0.629 (0.096)
β_2 (S-Sent)	-0.011 (·)	-0.013 (0.103)		-0.050 (0.057)
β_3 (ret.)	-0.036 (·)			1.092 (1.034)
M	27.935 (·)	23.553 (25.640)	24.912 (·)	(68)
LogL	-526.058	-526.071	-526.078	-525.511
AIC	1064,116	1062,143	1060,157	1067,022
BIC	1062,056	1062,096	1062,124	1060,975

Moderate interaction, uni-modality (can be captured by OU diffusion)

1D: price

Table 1: Parameter estimates for uni-variate models

Panel D: Diffusion model for prices			
Param.	Model I	Model II	Model III
γ_0	21.270 (10.992)	13.858 (9.616)	
γ_1 (S-Sent)	-33.995 (24.037)		
γ_2 (M-Sent)	165.636 (62.104)	164.872 (62.579)	208.240 (55.262)
σ_p	102.540 (5.940)	103.241 (5.981)	103.952 (6.022)
LogL	-895.322	-896.329	-897,345
AIC	1798,644	1798,658	1798,689
BIC	1800,611	1802,639	1804,683

Significant influence from M-Sent

Note: The models in panels A to c have been estimated via numerical integration of the transitional density, while for the diffusion models in panel D, the exact solution for the transient density could be used. The discretization of the finite difference schems used steps of k = 1/12 and h = 0.01.

Table 2: Parameter Estimates for Bi-Variate Models: S-Sent and M-Sent

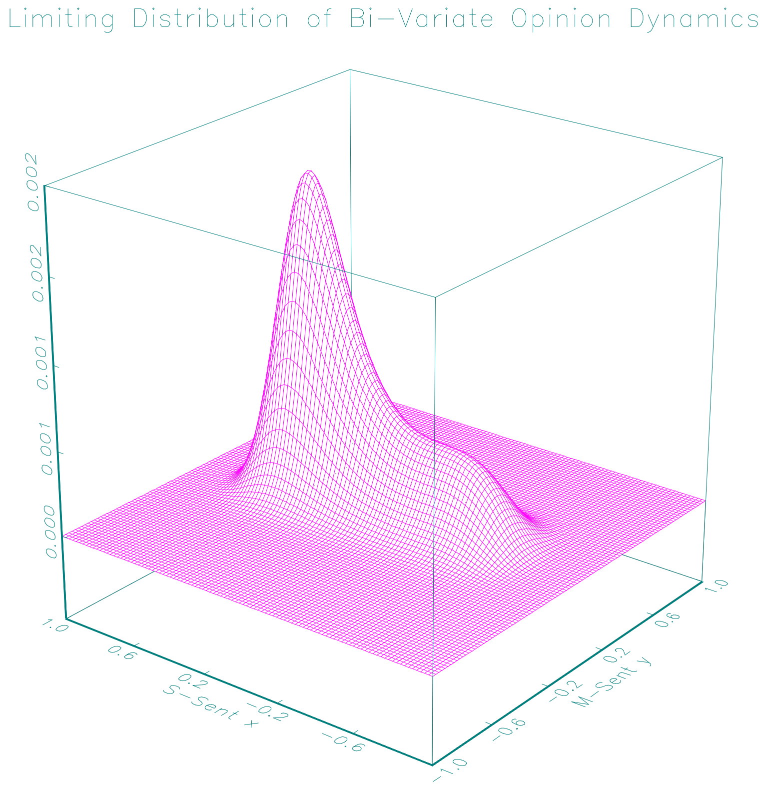
2D: S-Sent + M-Sent

Interaction of S-Sent and M-Sent		
Param.	Model I	Model II
ν_s	9.192 (·)	9.191 (2.838)
α_0	0.010 (·)	0.009 (0.004)
α_1	1.058 (·)	1.058 (0.013)
α_2	0.044 (·)	0.044 (0.025)
N	67.826 (·)	67.809 (14.127)
ν_m	0.295 (·)	0.294 (0.073)
$ß_0$	0.053 (·)	0.053 (0.022)
$ß_1$	0.639 (·)	0.639 (0.127)
$ß_2$	-0.119 (·)	-0.119 (0.056)
M	67.983 (·)	M=N
lkl	-1017.309	-1017.308
AIC	2054,617	2052,616
BIC	2044,5	2044,513

Model I: bi-variate opinion dynamics

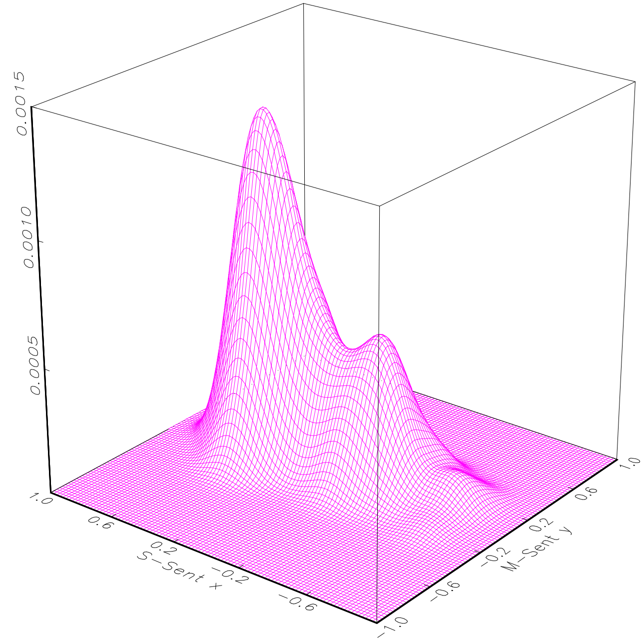
Model II: bi-variate opinion dynamics with identical no. of agents

Note: The models in panels A to C have been estimated via numerical integration of the transitional density using the ADI (alternative direction implicit) algorithm detailed in the Appendix. The discretization of the finite difference schems used steps of k = 1/12 (for time), and h = 0.02 (for S-Sent and M-Sent). In Panels B and C, the discretization of the second space dimension (prices) is chosen in a way to generate the same number of grid points as in the x or y dimension, i.e. Nx = Ny = Np = 100. this amounts to roughly 43 basis points of the DAX index.



Kernel density of in-sample data

Limiting distribution of estimated opinion model



2D: Sent + price

Table 2: Parameter estimates for bi-variate models: S(M)-Sent and Prices

Panel C: S-Sent and prices	
Param.	Model I
ν_s	8.976 (2.819)
α_0	0.013 (0.004)
α_1	1.056 (0.014)
N	64.128 (13.706)
γ_0	-4.844 (9.478)
γ_1	142.540 (27.741)
σ_p	93.014 (6.097)
lkl	-926.593
AIC	1867,186
BIC	1863,111

Panel C: M-Sent and prices			
Param.	Model I	Model II	Model III
ν_m	0.372 (·)	0.295 (0.039)	
$ß_0$	0.023 (·)	0.030 (0.015)	
$ß_1$	0.693 (·)	0.606 (0.100)	
M	80.648 (·)	(68)	
κ			0.240 (0.066)
y_0			0.073 (0.033)
σ_y			0.096 (0.006)
γ_0	7.685 (·)	25.045 (8.639)	19.677 (9.831)
γ_1	7.685 (·)	3.601 (20.804)	74.530 (67.139)
σ_p	7.685 (·)	103.859 (6.280)	103.479 (6.26)
lkl	-768.272	-766.121	-765.406
AIC	1550,544	1544,242	1542,811
BIC	1546,469	1542,181	1540,751

3D

Table 3: Parameter estimates for tri-variate models

Param.	Model I	Model II	Model III	Model IV	Model V
ν_s	9.133 (·)	8.847 (2.703)	8.112 (2.431)	8.427 (2.600)	8.222 (2.443)
α_0	0.008 (·)	0.008 (0.004)	0.009 (0.004)	0.009 (0.004)	0.009 (0.004)
α_1	1.058 (·)	1.057 (0.013)	1.055 (0.014)	1.055 (0.014)	1.056 (0.014)
α_2	0.054 (·)	0.055 (0.026)	0.055 (0.027)	0.057 (0.027)	0.054 (0.027)
N	70.039 (·)	67.950 (14.135)	64.573 (13.666)	64.832 (13.746)	65.378 (13.677)
ν_m	0.385 (·)	0.273 (0.069)	0.267 (0.068)		
β_0	0.043 (·)	0.062 (0.023)	0.060 (0.024)		
β_1	0.758 (·)	0.647 (0.130)	0.627 (0.136)		
β_2	0.117 (·)	-0.165 (0.065)	-0.148 (0.064)		
M	95.842 (·)	M=N	M=N		
κ				0.200 (0.062)	0.206 (0.063)
$\overline{y}$				0.167 (0.056)	0.154 (0.054)
β_1				-0.445 (0.200)	-0.377 (0.184)
σ_y				0.089 (0.006)	0.091 (0.006)
γ_0	-13.595 (·)	-13.631 (10.229)	17.140 (9.703)	-12.994 (10.055)	18.015 (9.682)
γ_1	150.481 (·)	150.819 (28.322)	- -	149.994 (28.018)	- -
γ_2	106.254 (·)	106.271 (60.118)	113.139 (67.360)	99.982 (59.707)	101.224 (67.118)
σ_p	89.381 (·)	89.426 (6.149)	102.330 (6.404)	89.038 (6.099)	102.293 (6.399)
lkl	-1337.176	-1337.022	-1350.211	-1336.736	-1350.003
AIC	2702.353	2700.044	2724.422	2699.472	2724.005
BIC	2684.178	2683.884	2710.277	2683.312	2709.860

Table 3: Parameter estimates for tri-variate models

Models I and II: bi-variate opinion dynamics + price diffusion

Model IV: opinion dynamics for S-Sent
+ OU diffusion for M-Sent
+ price diffusion

Models III and V: restricted models without influence S-Sent -> prices

Price effects are ambiguous

Conclusions

- ☑ evidence for interaction effects in ZEW index ($\alpha_1 \approx 1$) and S-Sent
- ☑ we can identify the determinants of sentiment dynamics and the interaction between different sentiment data
- ☑ in both cases, interaction effects are dominant part of the model
- ☑ we can identify the formation of animal spirits and track their development
- ☑ in multivariate settings: forecasts of ‘hard’ variables become possible