

Greenhouse Gas and Cyclical Growth: A Medium-run Keynesian, Long-run Ricardian Simulation Model

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Key Features I

Output and capital accumulation are demand-driven in medium run – the Keynesian aspect of the model.

Higher capital per capita increases output which in turn increases the speed of CO₂ accumulation.

Higher atmospheric CO₂ concentration reduces output and growth of capital per capita.

Hence we have a variation on “typical” predator-prey dynamics – CO₂ is the predator and capital per capita the prey. Numerical simulations suggest an upswing in capital per capita for around eight decades, followed by a crash *of output and capital only*.

Key Features II

Contrary to familiar fox-and-rabbit models, the decay rate of CO₂ in the atmosphere is *very* slow (the “fox” is almost immortal). *Concentration remains high*, blocking any chance of economic recovery.

We follow the usual growth theory convention of setting up a model that converges to a steady state.

In practice the system *must* converge to a *stationary* state with constant capital stock, CO₂ concentration, etc. Otherwise, CO₂ accumulation will overwhelm the economy – the Ricardian aspect.

Key Features III

Investment in mitigation of CO₂ accumulation can offset the crash, and lead to a non-dismal stationary state. In numerical simulations the share of output required is on the order of total world “defense” spending.

Begin with a slightly formal review of how the model hangs together in numerical simulations, then the dynamics and steady state, and finally the cyclical convergence mentioned above.

Model Architecture I

3 dynamic variables: CO₂ concentration in ppmv (G), capital stock per capita (κ), labor productivity (ξ).

Increase in G (or \dot{G}) is proportional to output (X) with factor of proportionality reduced by outlay on mitigation (m) as share of X .

Increase in κ (or $\dot{\kappa}$) driven by investment/capital ($g = I/K$) less depreciation (rate δ) and population growth rate (n)

Productivity growth rate ($\hat{\xi}$) driven by growth rate of energy intensity or the energy/labor ratio ($e = E/L$)

Model Architecture II

Ricardian long run: $\dot{G} = \dot{\kappa} = \dot{\xi} = \dot{e} = n = 0$. So G , κ , X , capital stock (K), employment (L) and population (N) are all constant.

Keynesian medium run: X and L determined by effective demand driven by g and m

Medium run “capital utilization” $u = X/K$ *increases* with profit share π (“profit-led”) via an increase in investment demand $g(\pi)$ so $u = u(\pi)$.

Model Architecture III

Assume that π decreases with $\lambda = L/N$ in a “profit squeeze” à la Marx/Kalecki/Goodwin – this response assures medium-run stability. In one variant π is squeezed by G as CO₂ concentration drives up costs.

In another variant, π decreases just with λ but higher CO₂ concentration raises the depreciation rate (“capital destruction”)

Key identity $\lambda = \kappa u / \xi$ means that higher κ increases λ , and reduces π and g . Lower g means that growth of κ slows, i.e. $d\dot{\kappa}/d\kappa < 0$ so growth in capital per capita is (locally) dynamically stable.

Model Architecture IV

An alternative closure would make demand “wage-led,” in which case higher G would “naturally” reduce labor share or increase π .

For medium-run stability there would have to be a high employment “wage squeeze.” Two problems:

At least at business cycle frequencies a wage squeeze or $\partial\pi/\partial\lambda > 0$ is counter-factual

A higher $g(\pi)$ in response to $\partial\pi/\partial\lambda > 0$ and $\partial\lambda/\partial\kappa > 0$ would make $d\dot{k}/d\kappa > 0$ and growth would be dynamically unstable.

Dynamics I -- CO₂ accumulation

Following the “Kaya identity” from climate science, CO₂ accumulation equation is

$$\dot{G} = \chi E - \mu(m)X - \omega G = [(\chi/\varepsilon) - \mu(m)]X - \omega G$$

with $\varepsilon = X/E$ as energy productivity. Higher mitigation m reduces factor of proportionality of \dot{G} wrt X . Dissipation parameter ω for G is very small.

There is a steady state at

$$G = \omega^{-1}[(\chi e/\xi) - \mu(m)]uN\kappa$$

Note that steady state G is proportional to steady state u , κ , and N (a Malthusian touch – numbers below).

Dynamics II– Growth of capital stock per capita

Capital stock basically scales the system (there is no aggregate production or cost function although the identity $\lambda = \kappa u / \xi$ and $\partial \pi / \partial \lambda < 0$ do apply).

Capital per capita accumulates with real investment as $\dot{\kappa} = \kappa(g - \delta - n)$. (Recall stability discussion above.)

With $n = 0$ there is a steady state at $g = \delta$.

Higher m shifts up medium run g and steady state κ .

Steady state with profit squeeze from both high employment and CO₂ concentration I

In a steady state $\delta = g \Rightarrow \pi$ from investment demand. Then $\pi \Rightarrow u$ from *demand side* macro balance. In medium run we have $\pi = F(\lambda, G) = F(\kappa u / \xi, G)$ with negative partials from profit squeeze. Hence G and κ must trade off in steady state to hold π constant.

[Contrast, say, Solow model where $\delta = g = sf(\kappa, G)$ so $\delta \Rightarrow \kappa$ from *supply side*. Also $\partial f / \partial G < 0$.]

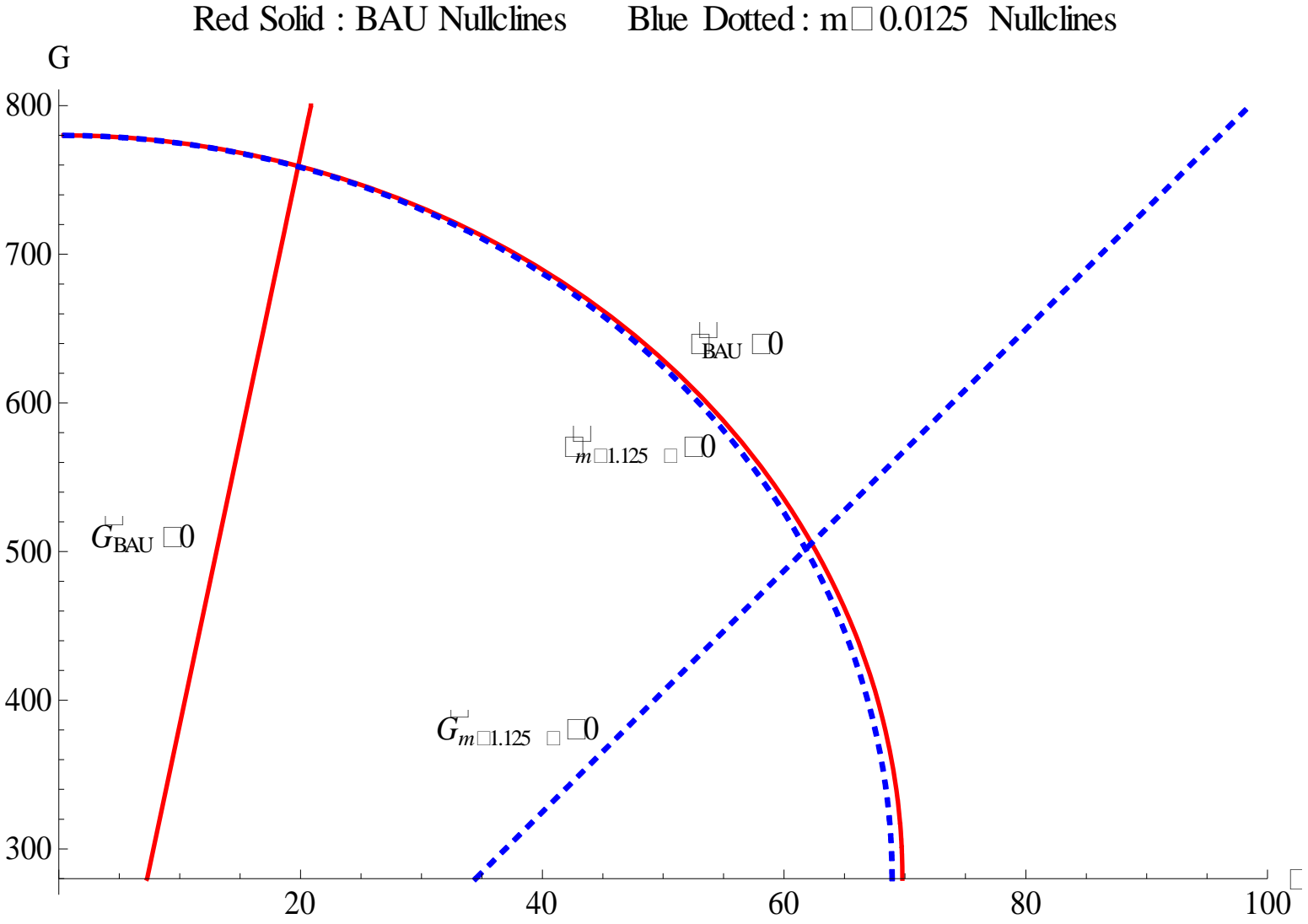
Steady state with profit squeeze from both high employment and CO₂ concentration II

Slope of the nullcline for G is sensitive to m so mitigation can support a non-dismal steady state.

But with no mitigation, $\kappa < 20$ and $G = 759$ in “business as usual”(BAU) dismal steady state (Initial values are $\kappa = 28.57$ and $G = 400$.)

Mitigated steady state $G = 486$ might correspond to 1°C global warming; BAU steady state with $G = 759$ to 3 or 4°C.

Nullclines for per capita capital stock (κ) and CO₂ concentration (G) when the profit share decreases with both G and κ



Steady state with capital destruction

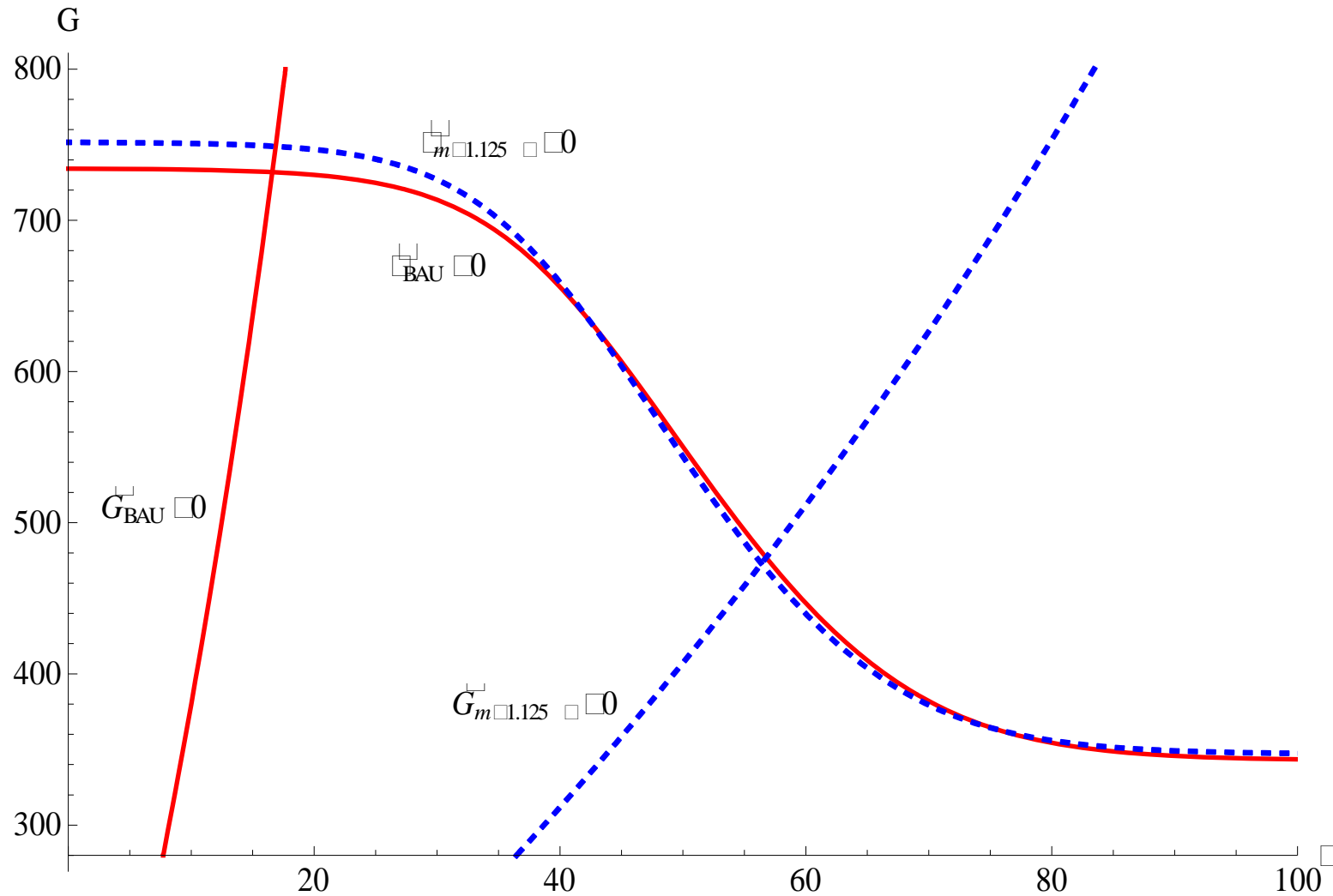
In the second medium run variant, there is no direct adverse effect of G on π , but higher CO₂ concentration raises the depreciation rate δ – there is destruction of capital stock.

Now in steady state, higher G and δ must lead to higher π (investment is profit-led). But a higher π must be associated with a lower κ via lower λ .

Again we get a trade-off between G and κ . Mitigation can again support a high level steady state. No mitigation leads to low level stagnation.

Nullclines for capital stock per capita (κ) and CO₂ concentration (G) when higher G increases capital depreciation.

Red Solid : BAU Nullclines , Blue Dashed: $m=0.0125$ Nullclines



Levels of Key Variables in Steady States

Profit share decreases with both κ and G			
	Initial value	BAU	Mitigated
G	400	759.4	486.2
κ	28.6	19.8	63.0
X/N	8.6	5.6	18.3
λ	0.429	0.153	0.5
Higher G increases depreciation rate			
G	400	698.6	464.7
κ	28.6	20.3	57.3
X/N	8.6	6.6	17.2
λ	0.429	0.181	0.468

Steady state responses when the profit share decreases with both G and κ

Lots of numbers in next slides – note elasticities with respect to N , ξ , and m .

Higher steady state population strongly reduces κ and X/N under BAU; relatively weak impact on G .
Magnitudes reverse in mitigated solution.

Higher labor productivity (which also raises energy productivity) increases κ , G , and X/N , more strongly in mitigated solution.

Higher m has generally beneficial effects.

Derivatives of κ , G , u and λ , wrt select magnitudes at steady state when the profit share decreases with both G and κ

Responses	BAU	MITIGATED
	sw	Sw
d_{ss}	27.5363	62.0069
$d_{G,ss}$	□28.4175	□250.939
$d_{u,ss}$	□0.438389	□0.475731
$d_{\lambda,ss}$	□0.023337	□0.323615
$d_{N,ss}$	□0.859272	□11.8942
	s □	s □
d_{ss}	10.0479	21.8808
$d_{G,ss}$	□10.3695	□88.5503
$d_{u,ss}$	□0.180331	□0.166749
$d_{\lambda,ss}$	□0.0194971	□0.112268
$d_{N,ss}$	□0.717017	□4.12638
	N	N
d_{ss}	□1.87739	□2.11765
$d_{G,ss}$	3.98185	33.9201
$d_{u,ss}$	□0.00148978	□0.0193239
$d_{\lambda,ss}$	□0.0153296	□0.0498982
$d_{N,ss}$	□0.538804	□0.616721
	□	□
d_{ss}	0.0282638	1.13426
$d_{G,ss}$	1.08369	9.20904
$d_{u,ss}$	□0.000405454	□0.00524628
$d_{\lambda,ss}$	□0.00417233	□0.013581
$d_{N,ss}$	0.0081116	0.330331

Responses	BAU	MITIGATED
	e	e
d_{ss}	0.173074	6.94567
$d_{G,ss}$	6.63598	56.3917
$d_{u,ss}$	□0.00248281	□0.0321257
$d_{\lambda,ss}$	□0.0255493	□0.0831638
$d_{N,ss}$	0.0496715	2.02278
	□	□
d_{ss}	□66.8586	□365.036
$d_{G,ss}$	141.804	5847.09
$d_{u,ss}$	0.95911	1.68839
$d_{\lambda,ss}$	□0.00012202	□0.000685853
$d_{N,ss}$	□19.1882	□106.309
	m	m
d_{ss}	1199.28	6181.23
$d_{G,ss}$	□2584.55	□100.014.
$d_{u,ss}$	□17.2041	□28.5899
$d_{\lambda,ss}$	0.00218875	0.0116137
$d_{N,ss}$	356.553	1840.61
	□	□
d_{ss}	□124.275	□1307.77
$d_{G,ss}$	□947.702	□8049.28
$d_{u,ss}$	1.78277	6.0488
$d_{\lambda,ss}$	□0.000226809	□0.00245712
$d_{N,ss}$	□35.6666	□380.861

Elasticities of κ , G , u and λ , wrt select magnitudes at steady state when the profit share decreases with both G and κ

Elasticity	BAU	MITIGATED
$\frac{\partial \ln \kappa}{\partial \ln sw}$	0.296497	0.210116
$\frac{\partial \ln G}{\partial \ln sw}$	0.00798269	0.110093
$\frac{\partial \ln u}{\partial \ln sw}$	0.328973	0.34845
$\frac{\partial \ln \lambda}{\partial \ln sw}$	0.0324758	0.138333
$\frac{\partial \ln \lambda}{\partial \ln \kappa}$	0.0325534	0.138416
$\frac{\partial \ln \kappa}{\partial \ln s}$		
$\frac{\partial \ln G}{\partial \ln s}$	0.142003	0.0973169
$\frac{\partial \ln u}{\partial \ln s}$	0.00382319	0.0509905
$\frac{\partial \ln \lambda}{\partial \ln s}$	0.177615	0.160306
$\frac{\partial \ln \lambda}{\partial \ln \kappa}$	0.0356115	0.0629886
$\frac{\partial \ln \lambda}{\partial \ln \lambda}$	0.0356535	0.0630267
$\frac{\partial \ln \kappa}{\partial \ln N}$		
$\frac{\partial \ln G}{\partial \ln N}$	0.947583	0.336373
$\frac{\partial \ln u}{\partial \ln N}$	0.0524319	0.697588
$\frac{\partial \ln \lambda}{\partial \ln N}$	0.0524048	0.66347
$\frac{\partial \ln \lambda}{\partial \ln \kappa}$	0.999986	0.999842
$\frac{\partial \ln \lambda}{\partial \ln \lambda}$	0.956852	0.336423
$\frac{\partial \ln \kappa}{\partial \ln e}$		
$\frac{\partial \ln G}{\partial \ln e}$	0.0524138	0.661962
$\frac{\partial \ln u}{\partial \ln e}$	0.0524285	0.695837
$\frac{\partial \ln \lambda}{\partial \ln e}$	0.0524014	0.661805
$\frac{\partial \ln \lambda}{\partial \ln \kappa}$	0.999986	0.999843
$\frac{\partial \ln \lambda}{\partial \ln \lambda}$	0.0529265	0.662061

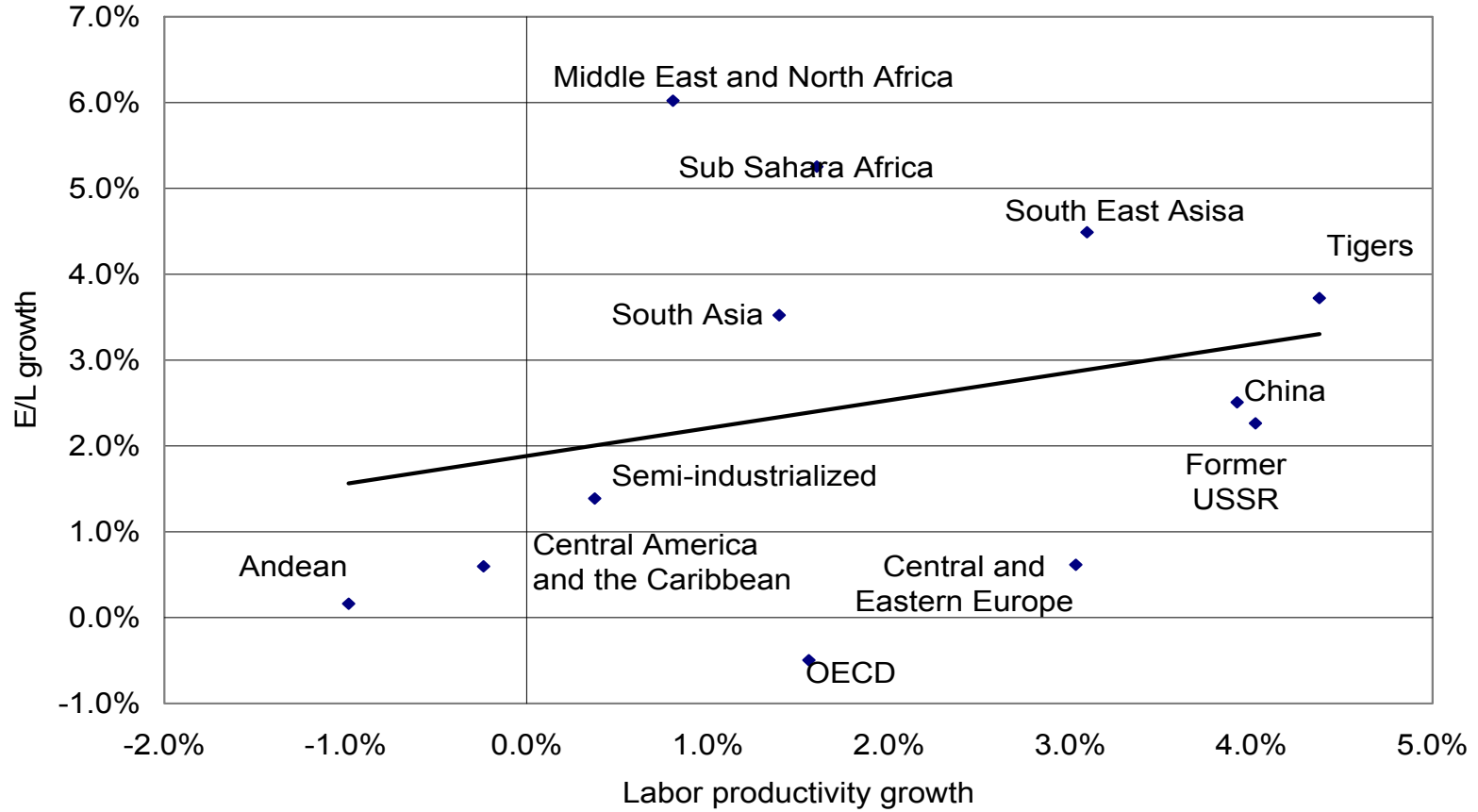
Elasticity	BAU	MITIGATED
$\frac{\partial \ln \kappa}{\partial \ln e}$	0.0524138	0.661962
$\frac{\partial \ln G}{\partial \ln e}$	0.0524285	0.695837
$\frac{\partial \ln u}{\partial \ln e}$	0.0524014	0.661805
$\frac{\partial \ln \lambda}{\partial \ln e}$	0.999986	0.999843
$\frac{\partial \ln \lambda}{\partial \ln \kappa}$	0.0529265	0.662061
$\frac{\partial \ln \kappa}{\partial \ln m}$		
$\frac{\partial \ln G}{\partial \ln m}$	0.947583	1.62817
$\frac{\partial \ln u}{\partial \ln m}$	0.0524319	3.37659
$\frac{\partial \ln \lambda}{\partial \ln m}$	0.947359	1.62779
$\frac{\partial \ln \lambda}{\partial \ln \kappa}$	0.000223508	0.0003859
$\frac{\partial \ln \lambda}{\partial \ln \lambda}$	0.956852	1.62842
$\frac{\partial \ln \kappa}{\partial \ln \lambda}$		
$\frac{\partial \ln G}{\partial \ln \lambda}$	0.	1.22731
$\frac{\partial \ln u}{\partial \ln \lambda}$	0.	2.57106
$\frac{\partial \ln \lambda}{\partial \ln \lambda}$	0.	1.22701
$\frac{\partial \ln \lambda}{\partial \ln \kappa}$	0.	0.000290889
$\frac{\partial \ln \lambda}{\partial \ln \lambda}$	0.	1.25507
$\frac{\partial \ln \kappa}{\partial \ln \lambda}$		
$\frac{\partial \ln G}{\partial \ln \lambda}$	0.326176	1.0802
$\frac{\partial \ln u}{\partial \ln \lambda}$	0.0648913	0.860799
$\frac{\partial \ln \lambda}{\partial \ln \lambda}$	0.326099	1.07994
$\frac{\partial \ln \lambda}{\partial \ln \kappa}$	0.0000769356	0.000256021
$\frac{\partial \ln \lambda}{\partial \ln \lambda}$	0.329366	1.08036

Dynamics III – Productivity growth

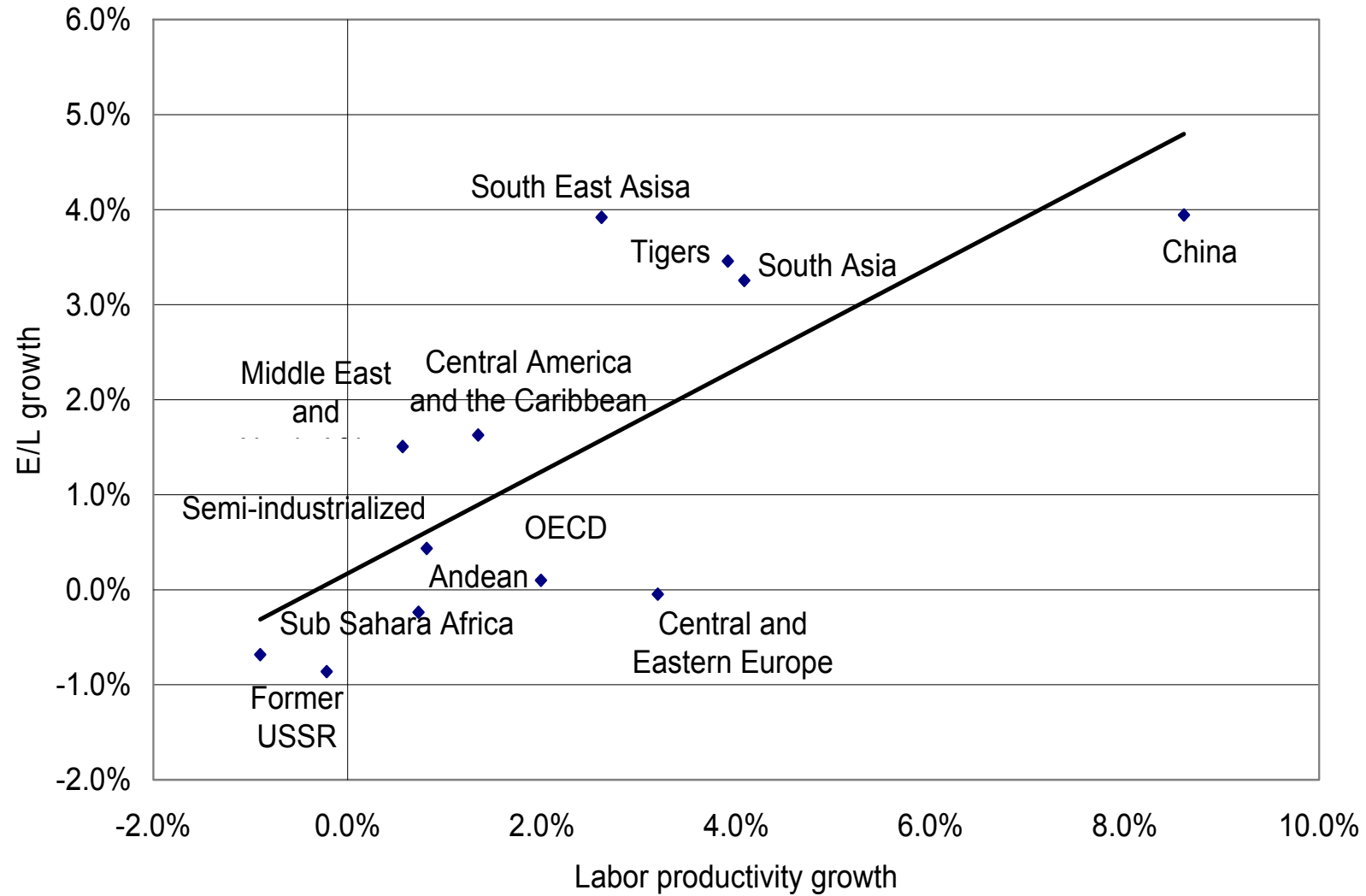
These steady state results presuppose constant levels of population (initial level = 7 billion, final = 10), energy intensity $e = E/L$ (initial = 4 kilowatts per employed worker, final = 6) and labor productivity $\xi = X/L$ (initial = \$20,000/worker, final = \$35,000)

See next slides for empirical relationships between growth rates of e and ξ .

Growth of energy to labor ratio and labor productivity: 1970-1990



Growth of energy to labor ratio and labor productivity: 1990-2004



Dynamics IV – Productivity growth

So an ostinato theme in Ecological Economics is that labor productivity is closely tied to energy intensity – (nearly) true historically.

Hence assume that producers choose a growth rate of energy intensity that converges to steady state level, and labor productivity growth is determined as

$$\dot{\xi} = \xi T \hat{e}$$

with $T = 1.5$.

Energy productivity for use in \dot{G} equation is set by the identity $\varepsilon = \xi / e$.

Transient paths to steady state – Business as usual (BAU) dynamics I

We set up simulations to track model dynamics toward a steady state. Growth trajectories are affected by assumed rates of increase of population, labor productivity, and energy intensity (logistic curves between initial and final levels).

First look at BAU growth when there is an adverse effect of CO₂ concentration on profitability (similar results when higher CO₂ increases depreciation rate).

Transient paths to steady state – BAU dynamics II

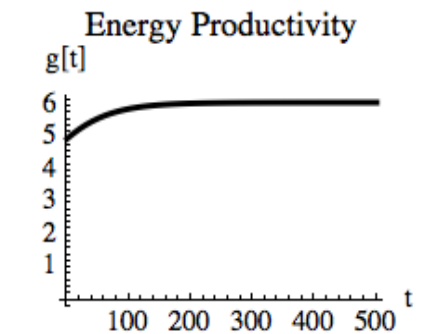
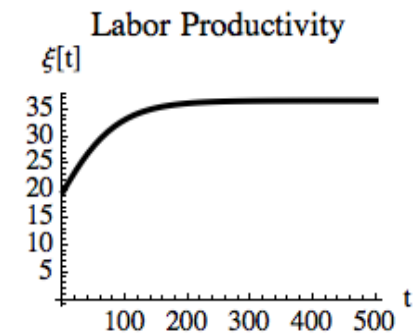
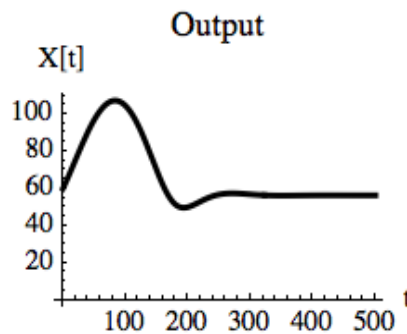
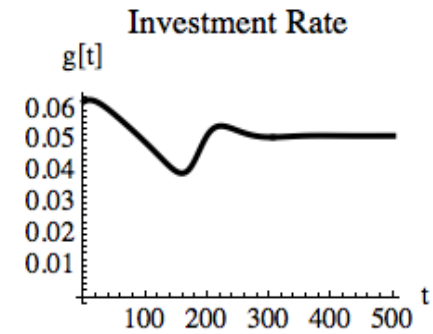
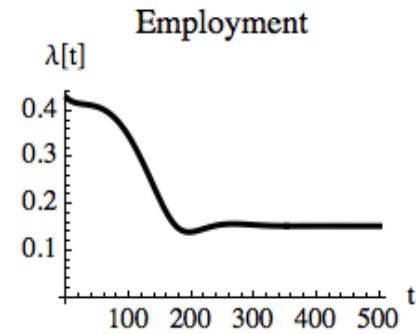
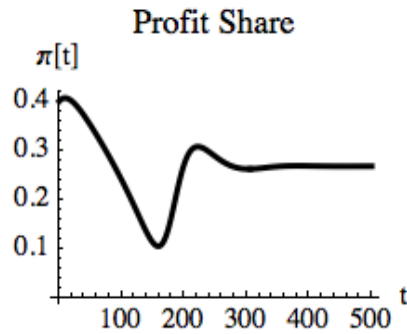
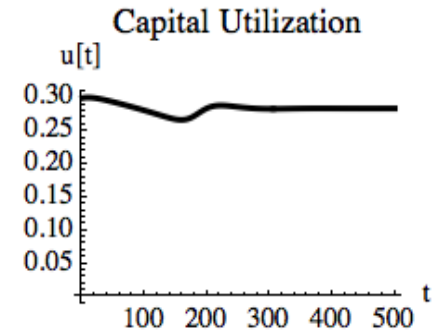
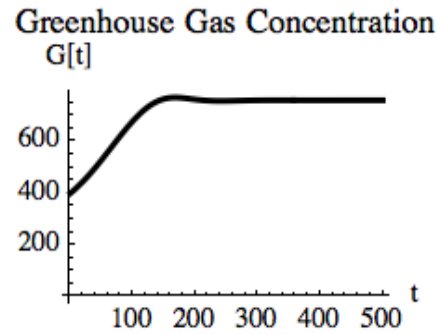
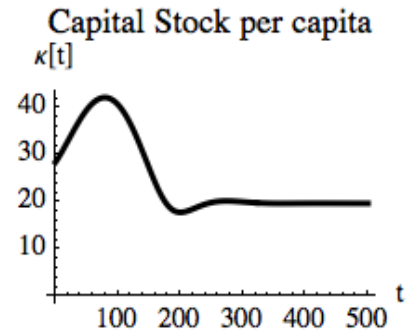
Cyclical growth with crashes in capital per capita and output after around 8 decades.

CO₂ concentration *stabilizes* at well over 700 ppmv, so an atmospheric temperature increase of 3-4° Celsius. Output cannot recover.

Output stabilizes near its initial level of \$60 trillion so output per capita falls by around 35% at a final population level of 10 billion.

BAU simulation when the profit share decreases with both κ and G

Variant One: BAU Scenario
 — BAU, m=0



Transient paths to steady state – Climate mitigation dynamics I

Now look at growth with mitigation at initial cost of \$160 per metric ton of carbon, or \$44 per ton of CO₂ (mid-range of current estimates).

With mitigation outlay of 1.25% of world output (\$60 trillion initially) CO₂ concentration can be stabilized. This outlay is around one-half of current level of defense spending.

Transient paths to steady state – Climate mitigation dynamics II

The macro economy basically follows the growth path to a stationary state that would be observed in the absence of global warming.

BAU and 1.25% mitigation scenarios broadly correspond to the highest and lowest damage paths in the IPCC 2013

“Front-loading” mitigation leads to more favorable results ($G \approx 400$) – a “climate policy ramp” would be harmful.

Transient paths to steady state – Climate mitigation dynamics III

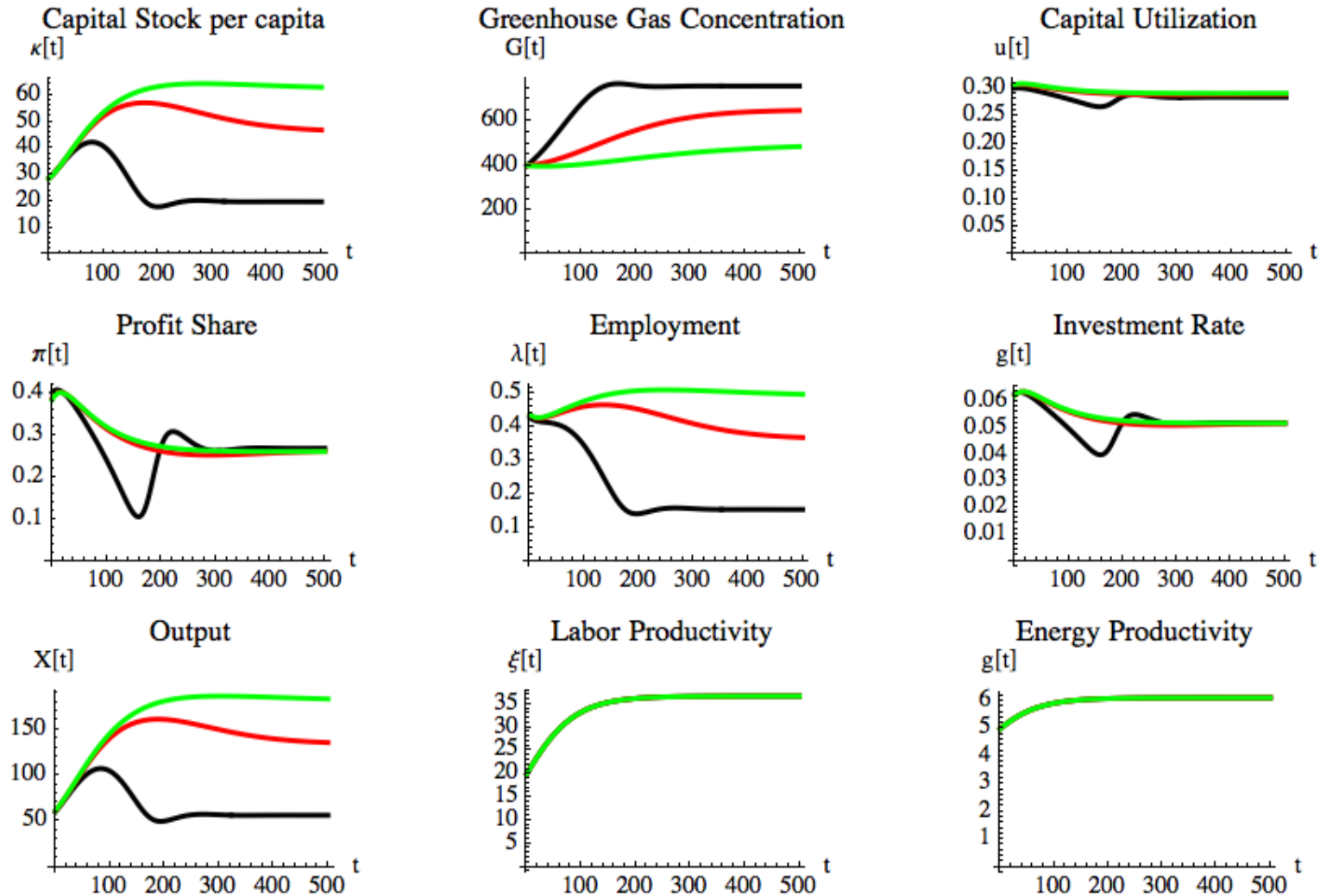
These results are largely driven by convergence dynamics of κ and G to steady state levels.

Same basic pattern appears under variant medium-run adjustments, e.g. higher CO₂ concentration reduces profitability *or* leads to capital destruction via faster depreciation *or* shifts down a neoclassical aggregate production function in a supply-driven full employment Solow growth model.

BAU and mitigation simulations when the profit share decreases with both κ and G

Variant One: BAU and Mitigated Scenarios

— BAU, $m=0$ — $m=0.01$ — $m=0.0125$



Transient paths to steady state – Impacts on labor I

Employment is determined as a “lump of labor,” or $L = X/\xi$. Elasticity of $\lambda = L/N$ w.r.t. ξ is -1 in both BAU and mitigated steady states. It is about -0.8 along transient paths ($\xi \uparrow \Rightarrow X \uparrow$).

BAU steady state λ is 65% below its initial value due to stagnating X and increases over time in ξ and N . λ rises in mitigated solution.

At steady state, $n = \hat{N} = 0$, or gross investment = depreciation. Hence profit rate and saving can fall, or λ can rise slightly.

Transient paths to steady state – Impacts on labor II

Real wage $\omega = (1 - \pi)\xi$.

The profit share stabilizes so that ω (not shown) can rise over time roughly in line with labor productivity.

There are potential positive feedback productivity linkages that we are considering

Kaldor: $\partial \dot{\xi} / \partial g > 0$ so $\xi \uparrow \Rightarrow g \uparrow \Rightarrow \dot{\xi} \uparrow$

Induced innovation: $\partial \dot{\xi} / \partial \omega > 0$ $\xi \uparrow \Rightarrow \omega \uparrow \Rightarrow \dot{\xi} \uparrow$

Kellogg-Sen: $\partial \dot{\xi} / \partial \lambda < 0$ so $\xi \uparrow \Rightarrow \lambda \downarrow \Rightarrow \dot{\xi} \uparrow$

Effects on employment to be explored.

Acknowledgements

Research supported by the Institute for New Economic Thinking (INET) under a grant to the Schwartz Center for Economic Policy Analysis, New School for Social Research. Thanks to Nelson Barbosa, Jonathan Cogliano, Gregor Semieniuk, Rishabh Kumar, Codrina Rada, and Armon Rezai for invaluable contributions.