Growth, Income Distribution and Autonomous Public Expenditures

Olivier Allain

Université Paris Descartes, Sorbonne Paris Cité, & Centre d'Economie de la Sorbonne
Mail address: olivier.allain@parisdescartes.fr

16th Conference of the Research Network Macroeconomics and Macroeconomic Policies (FMM) – *The State of Economics after the Crisis* – 25 - 27 October 2012, Berlin

Fourth version

Abstract. We present a Kaleckian model which is enriched by introducing autonomous public expenditures growing at an exogenous rate. We show that the usual properties are not affected in the short run: an increase in the profit share thus causes a decrease in the rate of capital accumulation (growth is wage-led). But long run properties are strongly affected: public expenditures play a role of automatic stabilizer such that accumulation rate converges toward the growth rate of public expenditures. The effect of a change in income distribution on the growth rate is then only transient. However, the impacts on the variables in level (output, capital stock, labor…) remain permanent. We also show that this theoretical framework can provide a solution (depending on the parameters) to the 'second' Harrod knife-edge problem. In this case, Kaleckian outcomes are consistent with the convergence of the current utilization rate toward the 'normal' one, a result which has not been achieved in the existing literature.

Key words: Kaleckian models, Utilization rate, Harrod instability, Income distribution, Automatic stabilizers.

JEL codes: E12, E2, E25, E62

1. Introduction

The aim of this paper is to contribute to the intense debates about the long run properties of income distribution and growth models. These models rest on three fundamental hypotheses. Firstly, they are demand restricted, taking up the Keynesian effective demand principle. Secondly, national income distribution is supposed to affect economic activity since the propensity to save out of wages is lower than the propensity to save out of profits. Thirdly, investment is assumed to be partly exogenous and partly endogenous, depending on capacity utilization as well as on profitability.

While there can be a consensus on the short run properties of these models (the paradox of thrift occurs and economic growth can be wage-led), oppositions can be drawn in the long run between three strands of thought. In Kaleckian models[1] the short run properties persist in the long run.

ose which keep the short run properties and those for which some short run properties are reversed (mainly growth becoming profit-led). The former are known as while the latter may be Cambridgian or Marxian[2]. The key point behind this opposition is the pattern of the utilization rate of capacity. For Cambridgian or Marxian economists, this rate must converge in the long run toward the normal rate. For the Kaleckian economists on the contrary, the utilization rate remains endogenous: there is then a remaining gap between the current and normal utilization rates, except if the latter converges toward the former.

In summary, either long run growth is wage-led but the utilization rate can't converge toward the normal one, or the utilization rate converges toward the normal one but growth must be profit-led.

The debates have recently taken a new turn with the use of empirical arguments. In short, utilization rate data often show a greater stability than it is expected after reading Kaleckian models[3]. Of course, these empirical arguments are open to the criticisms. But they already give rise to the question whether it is possible to formally combine Kaleckian results in the long run with a rate of capacity utilization converging toward the normal one. The present paper is a step in this direction.

We present a Kaleckian model which is just amended in order to introduce, near the autonomous component related to investment, another autonomous demand component characterized by its own growth rate. Such amendment has not really been taken into account in the literature: for instance, government expenditures or public deficit are assumed to be proportional with the capital stock and then to grow at the same rate[4]; and when exports are introduced, they are partly autonomous, but the results of the models don't fully take into account the consequences of this exogeneity[5].

Of course, the model conclusions may differ depending on the nature of the autonomous demand component that is taken into account and the source of its financing. In this paper, we introduce autonomous public expenditures growing at an exogenous rate. Because of the complexity of the issue of debt interests, we assume for sake of simplicity that government

[1] See Allain [2009], Blecker [2002], Dutt [2011], Hein *et al*. [2011, 2012], or Sawyer [2012] for recent contributions and surveys.

[2] The distinction between these currents of thought is beyond the scope of this article (see for instance Lavoie [2012]). What they have in common (from the point of view of the present paper) is that the current utilization rate converges toward the normal one in the long run. Some of these models are briefly presented further in the text.

[3] See Allain and Canry [2008], Nikiforos [2011], or Skott [2010, 2012]. See also Dallery [2007] who questions the stability of Kaleckian models through a simulation approach.

[4] See Blecker [2002], Commendatore *et al*. [2005], Sawyer [2012] or also You and Dutt [1996]. Lavoie [2000] assuming exogenous public expenditures is an exception but he himself limits its analysis to the short run whereas we address the long run issues. See also Chatelain [2010].

[5] See Blecker [1998, 2002, 2011]. His 1998's model in particular is explicitly built on the contradictory assumptions that exports and national income growth rates may differ from each other, but that the share of exports in national income remains unchanged. That is the kind of contradiction we tackle in this paper. However, open economies address specific issues which will be analyzed in further researches.

endogenously adjusts the tax rate in order to preserve the budget balance[6]. That is to say that emphasis is put on the impact of the autonomous demand component rather than on fiscal policy issues.

Our model leads to two important and original outcomes. The first one is that the rate of growth of public expenditures plays a stabilizer role on the rate of accumulation. The mechanism relates to the share of public expenditures in aggregate demand: a rise in the profit share results in a decrease of both utilization and growth rates in the short run; but it also results in an increase in the public expenditures' share which exerts a pressure both on the rate of utilization and on the propensity to save; then a turnaround of the utilization and growth rates. Eventually, it is the accumulation rate which adjusts to the rate of growth of public expenditures. However, although the impact on growth vanishes in the long run, their occurrence in the short run causes permanent effects on the variables in level (output, capital stock, labor, *etc*.). A rise in the profit share induces negative effects on economic activity, as in the canonical Kaleckian model, even if the long run rate of growth of capacity is not affected.

The second result is that our model can provide a solution to the 'second' Harrod knife-edge problem. We assume entrepreneurs react to the gap between the current and normal rates of capacity utilization by adjusting their rate of accumulation. As it is well known, such behavior in a demand constraint model usually generates instability (the 'second' Harrod problem). On the contrary, it can be shown in our framework (and depending on the parameters) that this entrepreneurs' behavior could be necessary in order for the current utilization rate to converge toward the normal rate. The intuition behind is that instability is more than offset by the stability resulting from the first mechanism. As a consequence, Kaleckian outcomes can be consistent with the convergence of the current utilization rate toward the 'normal' one, a result which has not been achieved in the existing literature.

Section 2 is devoted to the canonical Kaleckian model with endogenous public expenditures and a balanced public budget constraint hypothesis. It includes a discussion about the long run equilibrium with a special focus on Harrod knife-edge instability problem. The model with autonomous public expenditures is presented in section 3 where a distinction is made between short run (goods market equilibrium), medium run (stability of the share of public expenditures), and long run (convergence of the current rate of capacity utilization toward the normal one). A brief comparison between this model and Serrano's Sraffian supermultiplier (Serrano [1995a, 1995b]) is proposed in Section 4.

2. The canonical Kaleckian model with endogenous public expenditures and a balanced public budget constraint hypothesis

2.1. *Model resolution and economic interpretation*

Let us suppose a closed economy whose aggregate production function is given by:

$$Y_t = qL_t = u_t K_t \ ,$$

where Y_t, L_t and K_t correspond to production volume, labor input and capital stock; q is the fixed labor productivity coefficient, and u_t the current utilization rate of capacity. Aggregate demand (Y_t^d) is given by:

[6] You and Dutt [1996] take the debt dynamics into account but in a model where public expenditures are endogenous.

$$Y_t^d = C_t + I_t + G_t,$$

where C_t, I_t and G_t represent consumption, investment and public expenditures. Public expenditures are assumed to be related to consumption rather than accumulation. They are represented by:

$$G_t = \beta Y_t,$$

Tax revenue results from an income tax whose rate (τ) is supposed to be the same for wages and profits:

$$T_t = \tau Y_t.$$

Of course, under a balanced public budget constraint hypothesis, we have $\beta = \tau$[7]. In addition, workers are supposed to consume their whole wages. Consumption and savings are then given by:

$$C_t = (1-\tau)(1-s_\pi \pi)Y_t,$$

$$S_t = (1-\tau)s_\pi \pi Y_t,$$

where π is the profit share and s_π the propensity to save out of profits. Finally, noting u_n the 'normal' rate of capacity utilization from the entrepreneurs' point of view, the investment function can be written:

$$I_t = [\gamma + \gamma_u(u_t - u_n)]K_t,$$

where γ_u corresponds to the sensibility of the rate of capital accumulation ($g_{Kt} = I_t/K_t$) to the gap between the current and normal utilization rates. Of course, $u_t = u_n$, results in $g_{Kt} = \gamma$. As a consequence, the γ parameter can be understood as the average firms' expectation of the secular rate of growth (subject to animal spirits)[8]. Let us rewrite this function as:

$$I_t = \gamma_u Y_t + (\gamma - \gamma_u u_n)K_t.$$

It clearly appears that all the aggregate demand components are endogenous, but the fraction of capital accumulation which relates on the existing capital stock $(\gamma - \gamma_u u_n)$.

Substituting C_t, I_t and G_t in the aggregate demand function and solving gives the goods market equilibrium utilization rate[9]:

$$u_t^* = \frac{\gamma - \gamma_u u_n}{(1-\tau)s_\pi \pi - \gamma_u}.$$

The equilibrium rate of accumulation is then:

$$g_{Kt}^* = \gamma + \gamma_u(u_t^* - u_n),$$

and the after-tax rate of profit:

$$r_t^* = (1-\tau)\pi u_t^*.$$

These are the main results of the canonical Kaleckian model. The main comparative static results are reported in Table 1. Every column give the qualitative impact that a change in a

[7] See You and Dutt [1996] for a model which takes public deficit and debt into account.

[8] For sake of simplicity and without loss of generality, profitability is not included in the investment function.

[9] The Keynesian stability condition is supposed to hold, that is $(1-\tau)s_\pi \pi - \gamma_u > 0$. As a consequence, u_t^* numerator must also be positive ($\gamma - \gamma_u u_n > 0$) for the rate of capacity utilization to be positive.

given parameter (in columns) has on the short run equilibrium value of the endogenous variables u_t^*, g_t^* and r_t^* (in rows).

Table 1. Canonical model impact effects

	γ	s_π	π	τ
u_t^*	+	– (paradox of thrift)	– (stagnationism)	+
g_t^*	+ (animal spirits)	–	– (wage-led growth)	+
r_t^*	+	–	– (paradox of costs)	+

More optimistic entrepreneurs' expectations (animal spirits) increase both activity and growth. A rise in the capitalists' propensity to save weights consumption down, and then results in a cut in the rate of utilization (paradox of thrift). For its part, a rise in profit share causes a decline of activity and growth (stagnationist regime and wage-led economics growth) because it increases savings and reduces consumption. In addition, the Kaleckian model is such that a rise in π induces a proportionally higher cut in the utilization rate and, consequently, a decrease in the after-tax rate of profit. As a consequence, the rise in π is detrimental to capitalists as it is to workers (paradox of costs). Eventually, in accordance with the Haavelmo theorem, a balanced growth of public expenses has a positive impact on economic activity and growth.

2.2. *The rate of capacity utilization in the long-run*

As the equilibrium utilization rate u_t^* only depends on exogenous parameters, there is no guarantee about the equality u_t^* and u_n. How could the current rate go back to its normal value? Intuitively, it can be expected that entrepreneurs adjust their expected secular rate of growth (γ becomes γ_t). Starting from the accumulation function and assuming $u_t = u_n$, it comes that:

$$g_{Kt|u_n} = \gamma_t,$$

so that:

$$g_{Kt}^* - g_{Kt|u_n} = \gamma_u(u_t^* - u_n).$$

The adjustment function then should be:

$$\dot{\gamma}_t = \psi\gamma_u(u_t^* - u_n),$$

with $\psi > 0$. But, as it is well known, such behavior makes worse the situation because $\partial\dot{\gamma}_t/\partial\gamma_t > 0$: a fall in u_t^* results in a decrease of γ_t which induces another fall in u_t^*, *etc*. It is the Harrod knife-edge problem. Literature proposes other mechanisms that are here briefly surveyed, focusing on the long run responses to a rise in the profit share[10].

In the early Cambridge models (Robinson [1956, 1962], Kaldor [1955-56, 1957]), the convergence between the current and normal rates of capacity utilization results from a price mechanism: entrepreneurs react to a fall in u_t^* by decreasing goods prices via a cut in their profit margins. That allows an increase of aggregate demand and the restoration of u_n. In this framework, utilization rate and profit share go hand in hand, as in a profit-led model, albeit income distribution is endogenous in the long run (while it is exogenous in the short run).

[10] See Hein *et al*. [2011].

Another solution had been proposed by Duménil and Lévy [1999] who introduce the interest rate in the accumulation function and assume central banks have a goal in term of economic activity: monetary authorities react to a fall in u_t^* by diminishing the interest rate because there is no risk of inflation. This policy boosts investment, there is a rise in aggregate demand and the utilization rate is brought back to its normal value. Eventually, the combination of a higher π, $u_t^* = u_n$, and a lower interest rate transforms the wage-led short run model in a profit-led long run model.

According to Skott [2010, 2012], the model may suffer from locally Harrodian instability. However, the entrepreneurs' behavior is finally supposed to restore stability: the fall in u_t^* leads to an increase of unemployment. When unemployment is sufficiently high, entrepreneurs start accumulating capital again because they can easily hire workers to work on new equipment (formally, there is a rise in γ). In the opposite, an increase in u_t^* makes it difficult for firms to hire workers, then a decline in accumulation (a fall in γ). This mechanism gives rise to a limit cycle around a steady growth path which rests on labor supply growth.

For its part, Shaikh [2007] makes a distinction between retained earnings and households' savings. He also assumes that the retention ratio of firms (that is the share of retained profits) positively depends on the gap between the current and normal utilization rates. In this framework, a fall in u_t^* leads to a decrease of the retention ratio, and then to a decline of the global propensity to save. The rise of aggregate demand brought back u_t^* to its normal value. Eventually, as for Duménil and Lévy, the wage-led short run model becomes a profit-led long run model.

In another article, Shaikh [2009] focuses on the accumulation function where the γ parameter is replaced by the rate of growth of expected demand which is assumed to be perfectly foresight by entrepreneurs (that is with zero-mean errors). Perfect foresights combined with an accumulation process in order to come back to the normal rate of capacity utilization provide a stable adjustment around the Harrod warranted path. As a result, paradoxes of thrift and costs occur in the short run but they reverse in the long run.

At this stage, it is interesting to come back to the Kaleckian authors to precise their position according to the long run rate of capacity utilization[11]. Three main directions have been proposed. The first one is to question either the uniqueness of the normal utilization rate (preferring the idea of a corridor of stability), or the pertinence of long run analysis (preferring medium run or provisional equilibria). A second approach consists in assuming firms have multiple targets the realization of which may be mutually exclusive. As a result, entrepreneurs have to accept a lasting gap between u_t^* and u_n (Dallery and van Treeck [2011]). The third direction is to reverse the way of convergence: Lavoie [1996, p. 127] for instance assumes entrepreneurs have an adaptative behavior, setting the normal rate of capacity utilization according to past conventions and recent experience.

In summary, either long run growth is wage-led but the current utilization rate can't converge toward the normal one, or the utilization rate converges toward the normal one but growth must be profit-led[12]. In latter models, some short run outcomes (growth is wage-led) are reversed in the long run (growth becomes profit-led); and what appears to be a good policy (wages expansion) turn out to be a bad one.

[11] See Hein *et al*. [2012].

[12] The only exception is Skott [2010, 2012] whose model exhibits a cycle around a steady growth path.

3. The introduction of autonomous public expenditures

The introduction of autonomous public expenditures makes it necessary to distinguish between three equilibria. *Short run* corresponds to the goods market equilibrium. But the share of public expenditures in aggregate demand (or in capital stock) is not necessarily stable in the short run; it will be shown that this share converges toward a position of rest which we call the *medium run* equilibrium. However, the current and normal rates of capacity utilization may differ at this stage. We then analyze the impact of entrepreneurs' behavior adjusting their accumulation rate in order to fill the gap between the two utilization rates. We have shown in the previous section that such a behavior leads to instability (the Harrod knife-edge problem). We will show here that, depending on the parameters, this behavior can induce the convergence toward the normal rate of capacity utilization; that is our *long run* equilibrium. These three equilibria are analyzed in three distinct sub-sections. A fourth sub-section is devoted to a comparison between our analysis with the Sraffian supermultiplier.

3.1. The short run equilibrium

In the previous section, public expenditures were assumed to be completely endogenous which a strong hypothesis is. Indeed, these expenditures can be expected to be partly autonomous. But the mix of autonomous and endogenous components raises formal difficulties about deficit and debt issues. We consequently assume completely autonomous public expenditures, growing at an exogenous rate α, that is:

$$G_t = G_0 e^{\alpha t}.$$

Tax revenue is specified as before, but now the tax rate must be endogenous in order to balance the public budget:

$$\tau_t = \frac{\lambda_{Gt}}{u_t},$$

where $\lambda_{Gt} = \frac{G_0 e^{\alpha t}}{K_t}$.

Taking these amendments into account, savings and accumulation respectively become:

$$g^s = \frac{S_t}{K_t} = (u_t - \lambda_{Gt}) s_\pi \pi,$$

$$g^i = \frac{I_t}{K_t} = \gamma + \gamma_u (u_t - u_n),$$

where the after-tax rate of profit is now $r_t = (u_t - \lambda_{Gt})\pi$. Let us emphasize that the average firms' expected secular rate of growth (γ) may differ from the rate of growth of public expenditures (α). The *short run* goods market equilibrium is then given by[13]:

$$u_t^* = \Phi(\gamma - \gamma_u u_n + s_\pi \pi \lambda_{Gt}),$$

where $\Phi = (s_\pi \pi - \gamma_u)^{-1}$ is the Keynesian multiplier. We also have:

$$g_{Kt}^* = \gamma + \gamma_u (u_t^* - u_n).$$

[13] The Keynesian stability condition is still supposed to hold, that is $s_\pi \pi - \gamma_u > 0$. As a consequence, the numerator must also be positive ($\gamma - \gamma_u u_n + s_\pi \pi \lambda_{Gt} > 0$) for the rate of capacity utilization to be positive. Note that this constraint is less restrictive than in the previous formulation because the expression $\gamma - \gamma_u u_n$ can now be negative.

The comparative static results are reported in Table 2. Note that the sign of the derivatives with regard to s_π and π is that of $\lambda_{Gt} - u_t^*$.[14] We suppose here that $\lambda_{Gt} - u_t^* < 0$, which is consistent with the results of the following sub-section. In summary, the short run equilibrium presents the same results than the canonical Kaleckian model.

Table 2. Short run impact effects

	γ	s_π	π	α
u_t^*	+	–	–	0
g_t^*	+	–	–	0
τ_t^*	–	+	+	0
r_t^*	+	–	–	0
		Assuming $\lambda_{Gt} - u_t^* < 0$		

3.2. The medium run equilibrium

The crucial point of the model is that the short run equilibrium (u_t^*) is not only composed by exogenous parameters. It now includes the variable λ_{Gt} which varies as soon as the accumulation rate differs from the rate of growth of public expenditures (α). It is then necessary to make a distinction between u_t^* which only relates to the goods market equilibrium and u_t^{**} which also relates to a position of rest ($\dot{\lambda}_{Gt} = 0$) where[15]:

$$\dot{\lambda}_{Gt} = \lambda_{Gt}(\alpha - g_{Kt}^*).$$

The *medium run* equilibrium $(u_t^{**}, \lambda_{Gt}^{**})$ is then the solution of the system:

$$\begin{cases} u_t^* = \Phi(\gamma - \gamma_u u_n + s_\pi \pi \lambda_{Gt}) \\ \lambda_{Gt}(\alpha - g_{Kt}^*) = 0 \end{cases}$$

The second equation has two solutions, one of which being $\lambda_{Gt}^{\sim} = 0$ inducing $u_t^{\sim} = \Phi(\gamma - \gamma_u u_n)$. This solution brings back the analysis to the canonical Kaleckian model which has here little interest.

On the other hand, solving the system for a positive λ_{Gt} results in:

$$u_t^{**} = \frac{\alpha - \gamma}{\gamma_u} + u_n,$$

and:

$$\lambda_{Gt}^{**} = u_t^{**} - \frac{\alpha}{s_\pi \pi}.$$

The signs of λ_{Gt}^{**} and u_t^{**} being a matter of parameters, we thereafter assume that the equilibrium has significant positive values from an economic point of view. In addition, the resulting accumulation, tax and after-tax profit rates are given by:

$$g_{Kt}^{**} = \alpha,$$

$$\tau_t^{**} = 1 - \frac{\alpha}{s_\pi \pi u_t^{**}},$$

$$r_t^{**} = \frac{\alpha}{s_\pi}.$$

[14] Actually, $du_t^*/ds_\pi = (\lambda_{Gt} - u_t^*)\pi/(s_\pi\pi - \gamma_u)$ and $du_t^*/d\pi = (\lambda_{Gt} - u_t^*)s_\pi/(s_\pi\pi - \gamma_u)$.

[15] Let us remind that the rate of capacity utilization is assumed to adjust instantaneously at every period in order to preserve the goods market equilibrium.

The stability conditions depend on the first and second derivatives of the public expenditures' share:

$$\frac{d\dot{\lambda}_{Gt}}{d\lambda_{Gt}} = \alpha - \Phi s_\pi \pi(\gamma - \gamma_u u_n) - 2\Phi\gamma_u s_\pi \pi \lambda_{Gt},$$

$$\frac{d^2\dot{\lambda}_{Gt}}{d\lambda_{Gt}^2} = -2\Phi\gamma_u s_\pi \pi < 0.$$

The second derivative being negative, the function $\dot{\lambda}_{Gt}(\lambda_{Gt})$ is an inverted u-shaped relationship with two roots ($\lambda_{Gt}^{\sim} = 0$ and λ_{Gt}^{**}). The condition for λ_{Gt}^{**} to be both positive and stable is that the first derivative is positive for $\lambda_{Gt} = 0$:

$$\left.\frac{\partial\dot{\lambda}_{Gt}}{\partial\lambda_{Gt}}\right|_{\lambda_{Gt}=0} = \alpha - \Phi s_\pi \pi(\gamma - \gamma_u u_n).$$

Assuming this condition holds[16], economics monotonically converges toward its stable long run equilibrium $(u_t^{**}, \lambda_{Gt}^{**})$. An important property of the model is that $\lambda_{Gt}^{**} < u_t^{**}$, which confirms the short run outcomes in Table 2.

The comparative static results in Table 3 deserve attention because some of them seem at odds with Kaleckian results. Actually, they are not. We will argue that they might be a faithful extension to the short run Kaleckian model.

Table 3. Medium run impact effects

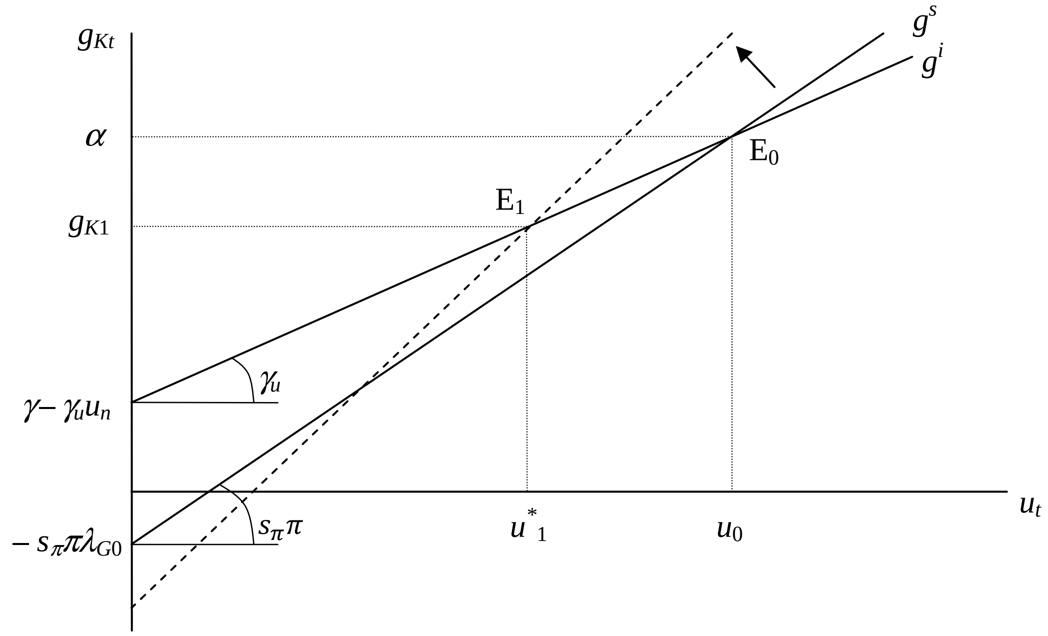
	γ	s_π	π	α
λ_{Gt}^{**}	–	+	+	+
u_t^{**}	–	0	0	+
g_t^{**}	0	0	0	+
τ_t^{**}	–	+	+	–
r_t^{**}	0	–	0	+

The next figures help to explain the underlying mechanisms. The goods market equilibria correspond to the intersections between the two straight lines representing capital accumulation (g^i) and savings (g^s). The initial short run equilibrium (E_0) is assumed to be a position of rest ($g_{Kt} = \alpha$). Let us suppose an increase in the profit share (π). This change will have distinct effects in short and medium run.

In the short run (Figure 1), the rise in π leads to a counterclockwise rotation and a downward shift of the intercept of g^s. The equilibrium moves to E_1. As in the canonical model there is a decrease of u and g_K.

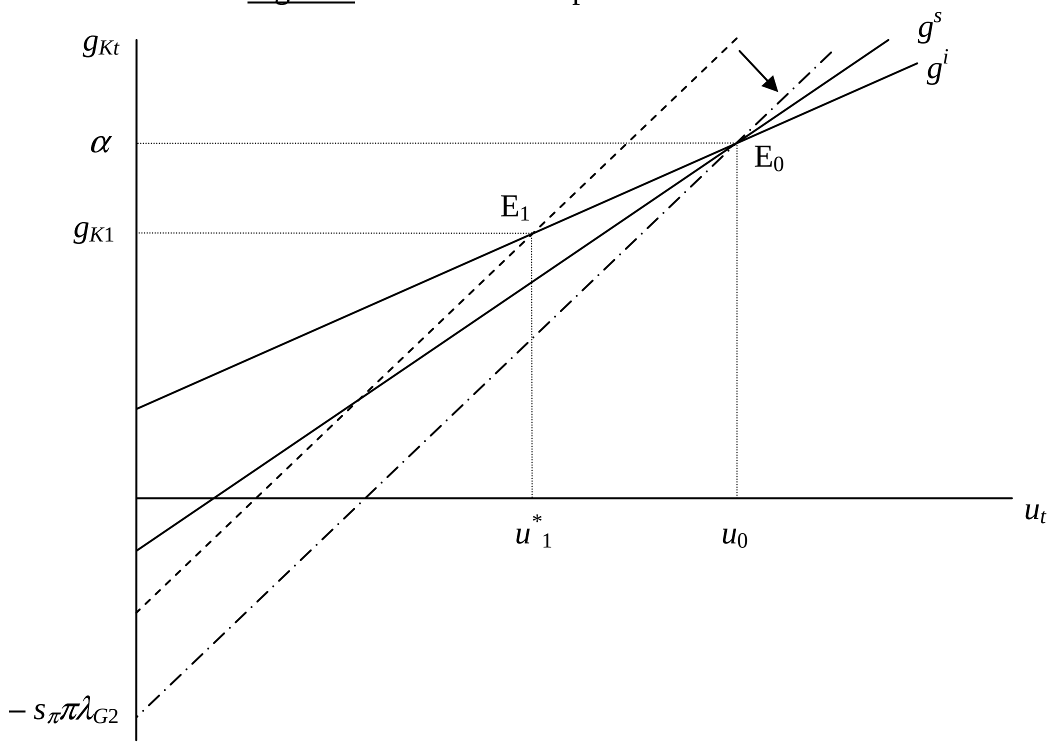
[16] It is not a too restrictive assumption since γ can be negative.

Figure 1. Short run impact of a rise in π



But this solution is not stable because $g_{K1} < \alpha$. There is then a rise in the public expenditures share (λ_G) which supports aggregate demand and reduces savings. More precisely, a portion of profits which was intended for capitalists' saving is now redirected toward public expenditures via the increase of the endogenous tax rate: g^s shifts downward on Figure 2, until the economy finds once again a position of rest in E_0.

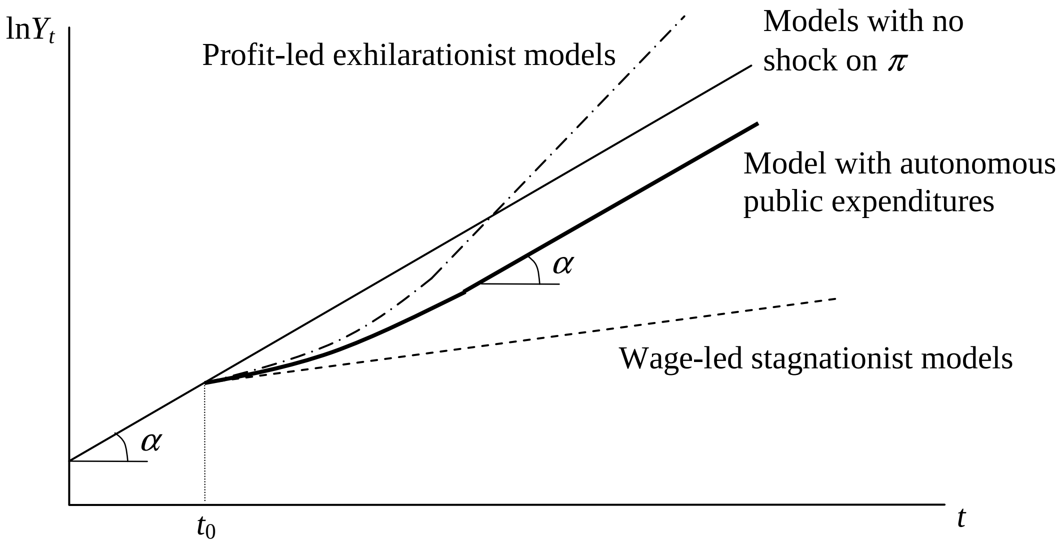
Figure 2. Medium run impact of a rise in π



Eventually, the main result is that the negative effects on capacity utilization and economic growth vanish in the medium run. Does it mean that the rise in the profit share has no impact in the medium run? Actually, the answer is no. Let us look at Figure 3 to be convinced. The normal straight line represents the temporal evolution of output assuming there is no shock on

π. The bold straight line represents the evolution of capital stock assuming an increase in π at time t_0. In the medium run, the economic growth rate is brought back to α. But this rate is temporary lower than α. The output level thus remains permanently lower than it would have been with an unchanging profit share. In this sense, the medium run analysis doesn't contradict the short run one. The model is thus consistent with other Kaleckian models for which, in addition, the growth rate is durably affected (dashed line). On the opposite, the profit-led model (mixed line) leads to positive medium run effects on the output level despite temporary short run negative effects.

Figure 3. Short and medium run impacts of a rise in π



Of course, the rise in the profit share causes an increase in the before-tax rate of profit (πu_t^*). But it is entirely offset by the rise in tax rate (resulting from the increase in λ_G), therefore an after-tax rate of profit which is brought back to its initial value.

Finally, public expenditures play a role of 'growth automatic stabilizer' although there is no public deficit (whereas automatic stabilizers usually rest on public deficit variations depending on tax income being more endogenous than expenditures). Stabilization here comes from tax rate adjustments which makes it possible to transfer some income from capitalists' saving to public expenditures, or conversely. Moreover, stabilization doesn't apply to the production level which is permanently affected by exogenous shocks; stabilization only applies to the rate of capital accumulation which is brought back toward α in the long run. Note that a higher α has a positive impact on every endogenous variable, even on the profit rate thanks to public expenditures supporting activity.

3.3. *The long run equilibrium*

In the medium run, the rate of capacity utilization u_t^{**} may differ from the normal rate u_n. The last step consists in introducing the above 'Harrodian mechanism' assuming firms adjust their accumulation rate (via γ_t) depending on the gap between the current and normal utilization rates (see above):

$$\dot{\gamma}_t = \psi \gamma_u (u_t^* - u_n).$$

The *long run* equilibrium is therefore the solution of the system:

$$\begin{cases} u_t^{**} = \frac{\alpha-\gamma_t}{\gamma_u} + u_n \\ \lambda_{Gt}^{**} = u_t^{**} - \frac{\alpha}{s_\pi \pi} \\ \quad u_t^{**} = u_n \end{cases}$$

It results that:

$$u_t^{***} = u_n, \qquad \lambda_{Gt}^{***} = u_n - \frac{\alpha}{s_\pi \pi},$$

$$\gamma_t^{***} = \alpha, \qquad g_{Kt}^{***} = \alpha,$$

$$\tau_t^{***} = 1 - \frac{\alpha}{s_\pi \pi u_n} \qquad r_{Kt}^{***} = \frac{\alpha}{s_\pi}.$$

The local stability conditions of this equilibrium depend on dynamics of both γ_t and λ_{Gt}. The former is based on the two partial derivatives:

$$\frac{\partial \dot{\gamma}_t}{\partial \lambda_{Gt}} = \psi \Phi \gamma_u s_\pi \pi,$$

$$\frac{\partial \dot{\gamma}_t}{\partial \gamma_t} = \psi \Phi \gamma_u.$$

In the same way (but after linearization), the λ_{Gt} dynamics is given by:

$$\frac{d\dot{\lambda}_{Gt}}{d\lambda_{Gt}} = -\gamma_u \Phi (s_\pi \pi u_n - \alpha),$$

$$\frac{\partial \dot{\lambda}_{Gt}}{\partial \gamma_t} = -\Phi (s_\pi \pi u_n - \alpha).$$

The stability can be analyzed by means of the Jacobian matrix which is:

$$\begin{pmatrix} \psi \Phi \gamma_u & \psi \Phi \gamma_u s_\pi \pi \\ -\Phi (s_\pi \pi u_n - \alpha) & -\gamma_u \Phi (s_\pi \pi u_n - \alpha) \end{pmatrix}$$

For the equilibrium to be stable, the matrix determinant must be positive whereas the trace must be negative. The determinant is:

$$DET = \psi \Phi \gamma_u (s_\pi \pi u_n - \alpha).$$

The sign of *DET* depends on the sign of the term in parentheses which is of course a matter of parameters. It can be positive as well as negative under reasonable economic assumptions. A negative determinant indicates a *saddle point*. We thereafter assume that *DET* > 0. Let us also underline that *DET* sign doesn't depends on the value of ψ.

The trace is given by:

$$TR = -\gamma_u \Phi (s_\pi \pi u_n - \alpha - \psi).$$

It can be deduced that:

$$TR < 0 \Leftrightarrow \psi^{\sim} < s_\pi \pi u_n - \alpha$$

In summary, assuming a positive determinant, the necessary condition for the system to converge toward its long run solution is that $\psi < \psi^{\sim}$. In addition, the system trajectory depends on the discriminant of its eigenvalues, that is:

$$\Delta = TR^2 - 4.DET.$$

It can be shown that:

$$\Delta = 0 \Leftrightarrow \psi^2 - 2(1+\rho)\Omega\psi + \Omega^2 = 0$$

where $\Omega = s_\pi \pi u_n - \alpha$ and $\rho = \frac{2}{\Phi\gamma_u}$. The roots of this quadratic function rest on the value of the ψ parameter:

$$\psi_1 = \Omega\left[(1+\rho) - \sqrt{\rho(2+\rho)}\right],$$

$$\psi_2 = \Omega\left[(1+\rho) + \sqrt{\rho(2+\rho)}\right].$$

The terms in brackets being respectively lower and higher than unity, it comes that $\psi_1 < \psi^{\sim} < \psi_2$. Assuming *DET* > 0, the results can then be summarized as follows (see the phase diagram[17] in Figure 4):

a. $\psi < \psi_1 \Rightarrow \Delta > 0$ and *TR* < 0: λ_{Gt} and γ_t converge monotonically toward their long run equilibrium (*stable node*).

b. $\psi_1 < \psi < \psi^{\sim} \Rightarrow$ *TR*, $\Delta < 0$: λ_{Gt} and γ_t converge via damped oscillations (*stable focus*).

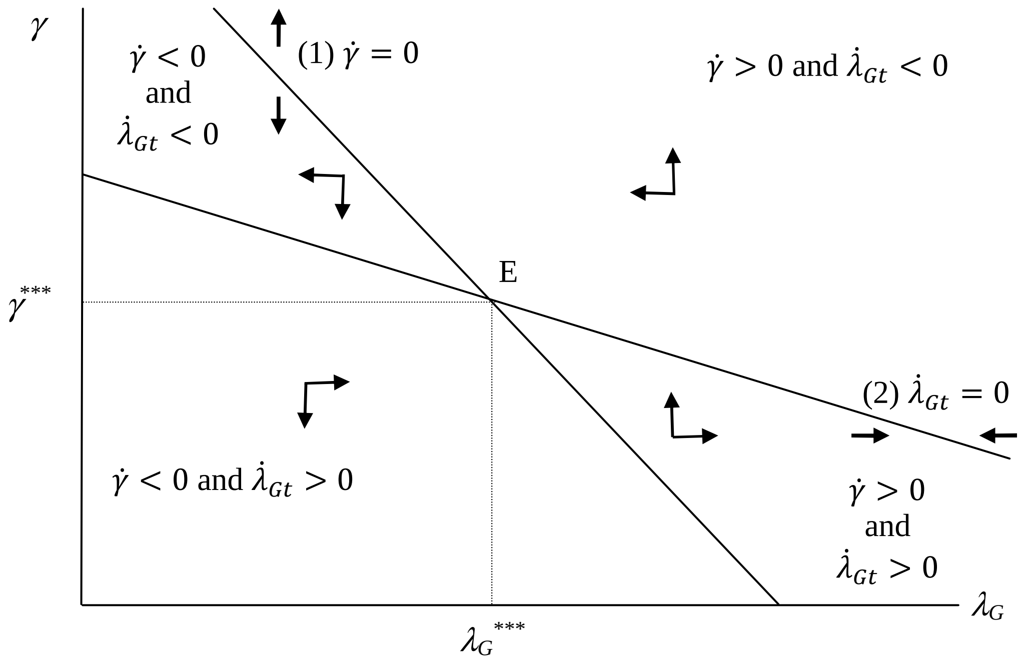
c. $\psi = \psi^{\sim} \Rightarrow$ *TR* = 0 and $\Delta < 0$: oscillations are undamped (equilibrium is *centre*).

d. $\psi^{\sim} < \psi < \psi_2 \Rightarrow$ *TR* > 0 and $\Delta < 0$: the system diverges since λ_{Gt} and γ_t oscillations are unstable (*unstable focus*).

e. $\psi_2 < \psi \Rightarrow$ *TR*, $\Delta > 0$: λ_{Gt} and γ_t monotonically diverges (*unstable node*).

In sum, it is not possible to formulate a univocal conclusion. The best we can say is that there is some room for the system to converge toward its long run equilibrium. For this to happen, the ψ parameter has to be small but positive. In fact, a positive ψ is a necessary condition in order to solve the Harrod knife-edge problem while this condition generates instability in the existing literature.

Figure 4. Dynamics of λ_{Gt} and γ_t (phase diagram).



[17] Line (1) represents $\dot\gamma = 0 \Leftrightarrow \gamma_t = s_\pi\pi(u_n - \lambda_{Gt})$ whereas line (2) represents $\dot\lambda_{Gt} = 0 \Leftrightarrow \gamma_t = \frac{\alpha}{\Phi s_\pi \pi} + \gamma_u u_n - \gamma_u \lambda_{Gt}$. It can be shown that the slope in absolute value is higher for (1) than for (2), and conversely for the intercepts.

In the previous section, it has been shown that a rise in π leads to a permanent cut in both capital stock and output. Configuration (a) leads to the same outcomes. The rise in π causes a decrease in both u_t^* and g_{Kt}^*. As $u_t^* < u_n$, entrepreneurs react in reducing γ_t. That induces another decrease in g_{Kt}^* which slows down the equilibrium restoration (compared with the previous section) resulting from the increase in λ_{Gt}. The convergence being monotonically, capital stock and output are permanently lower than they should have been without an initial change in π. If the equilibrium is a center (configuration c), the accumulation rate g_{Kt}^{***} is also a center. Its decreases are offset by its increases, and conversely. Intuitively, both capital stock and output should oscillate around their initial path. As a consequence, the intermediary configuration (b) should show the same permanent cuts in capital stock and output than in configuration (a): the decreases in g_{Kt}^* being not totally offset by its increases, capital stock and output stabilize on a lower path than the initial one.

Eventually, assuming stable long run equilibrium, the comparative static results are presented in Table 4.

Table 4. Long run impact effects

	s_π	π	α
γ_t^{***}	0	0	+
λ_{Gt}^{***}	+	+	–
u_t^{***}	0	0	0
g_t^{***}	0	0	+
τ_t^{***}	+	+	–
r_t^{***}	–	0	+

In this framework, a change in saving or investment behaviors has no effect on the long run utilization and accumulation rates. As for the medium run, an increase in π (or in s_π) leads to a rise in the λ_G: more savings in the economy means a heighten share for public expenditures. In addition, a rise in the rate of growth of public expenditures still induces greater accumulation and profit rates. Interestingly, because of its multiplier effect on consumption and investment, the higher α reduces the share of public expenditures in aggregate demand.

4. A comparison with Serrano's Sraffian supermultiplier

It is worth noting that the present model is close to Serrano's Sraffian supermultiplier (Serrano [1995a, 1995b]). This supermultiplier also rests on the introduction of a non capacity generating autonomous demand component in a Keynesian framework. In Serrano's model, this component isn't public expenditures but the lump of capitalists' consumption and the non capacity generating part of investment[18]. Its two main features are similar to those of the above long run equilibrium: *(1)* the actual rate of capacity utilization is equal to the normal (or planned) one (u_n), and *(2)* the rate of growth of capacity and output is equal to that of the autonomous component (α). As a result, a decrease of the marginal propensity to save (which can result from a cut in the profit share) "will have a positive long-run level (on capacity

[18] In Serrano's model, the autonomous component is wholly financed out of capital which is well-founded when the autonomous component relates to capitalists expenses. But according Serrano [1995a, f.n.1], government expenditures or total exports could as well be taken into account. It seems however that such kind of developments necessitates some amendments in the model since government expenditures as well as exports are partly financed out of wages.

output), but will have no effect on the sustainable secular rate of growth of capacity" (Serrano [1995b]).

A core point of Serrano is his enlighten analysis of the saving function properties:

$$g^s = (u_t - \lambda_{Gt}) s_\pi \pi,$$

where the marginal propensity to save ($s_\pi \pi$) is exogenously given. According to Serrano, if the autonomous demand component is omitted ($\lambda_{Gt} = 0$), any change in the rate of accumulation implies a change in the utilization rate. On the other hand, with the autonomous demand component, the adjustment could take place through λ_{Gt} rather than u_t. It is then formally possible to combine saving adjustment with a rate of capacity utilization which remains at its normal level. But of course, what is possible is not necessarily what happens.

Actually, in order for the utilization rate to remain at its normal level, Serrano has to assume that "firms as a whole correctly foresee the evolution of effective demand" (Serrano [1995b]), that is to say that the expected secular rate of growth (γ) must be (on the whole and on average) equal to the rate of growth of the autonomous component (α). On the contrary, "if expectations do happen to have a systematic bias in any direction then the actual path of the economy in the long run will move systematically away from the path formed by the corresponding sequence of long-period positions, causing the average actual degree of utilization to deviate persistently from the planned degree" (Serrano [1995a, p. 87]).

Hence the question: which solution prevails? According to Serrano, it is the first one. But the author doesn't propose any formal demonstration. He only refers to "the stylized fact that (…) there seems to be, on average, a remarkable balance between the long-run trends of productive capacity and aggregate demand" (Serrano [1995a, p. 68]). In other words, what should have been a result is actually an *ad hoc* assumption: Serrano assumes the long run equality between the current and normal rates of capacity utilization, which leads him to give priority to the conclusion that "firms as a whole correctly foresee the evolution of effective demand".

Furthermore, let us note that only a part of Serrano's assertions are right. Indeed, the medium run equilibrium analyzed above where:

$$u_t^{**} = \frac{\alpha - \gamma}{\gamma_u} + u_n, \text{ and}$$

$$g_{Kt}^{**} = \alpha,$$

confirms that systematic bias in firms' expectations ($\gamma \neq \alpha$) prevents the actual utilization rate to converge toward the normal one. But these bias don't prevent the rate of accumulation to converge toward α.

Eventually and more fundamentally, Serrano's supermultiplier model doesn't pay attention enough to the model dynamics. Hence the intensive debate between Sraffian economists[19]. Focusing on dynamics allows to agree whith Trezzini [1998, p. 66] when he concludes, against Serrano, that "the determining role played by aggregate demand in the accumulation process will generally manifest itself in the variability of the average utilization of productive capacity and is therefore, even in the long run, incompatible with the assumption of normal utilization". That is the outcome of the short and medium run equilibria of the present paper.

However Serrano's outcomes are consistent with those of our long run equilibrium, but Serrano must adopt the restrictive hypothesis that "firms as a whole correctly foresee the

[19] See for instance De-Juan [2005], Palumbo and Trezzini [2001], Park [2000], Trezzini [1995, 1998] or White [2006].

evolution of effective demand" (Serrano [1995b]). Our hypothesis is less restrictive since the expected secular rate of growth (γ) can differ from the rate of growth of the autonomous component (α).The model dynamics shows how the divergence between the actual and normal rates of capacity utilization lead entrepreneurs to correct theirs expectations, and especially how this correction can restore the long run equilibrium whereas it is always connected with Harrodian instability.

5. Conclusion

This article presents a Kaleckian model including autonomous public expenditures growing at an exogenous rate. The only restrictive assumption was to suppose that tax rate adjusts for the public budget to remain balanced. The model confirms the well-known positive role of public expenditures on activity and growth.

Moreover, we have shown that these expenditures play a role of automatic stabilizer on economic growth (rather than on the activity level). Changes in, say, capitalists' propensity to save or in the profit share have a transient effect on growth: in the long run, the rate of capital accumulation is brought back to its initial value which is given by the exogenous rate of growth of public expenditures. The mechanism at stake is based on the endogenous tax rate adjustment which results in an income transfer between capitalists and government: a portion of profits which was intended for capitalists' saving is now redirected toward public expenditures via the increase of the endogenous tax rate. However, the changes in propensity to save or in income distribution have permanent effects on capital and output levels; and these effects are in the same direction than in the Kaleckian models.

We have then combined this automatic stabilizer mechanism with an entrepreneurs' accumulation adjustment behavior which is well known for its instability property. We have shown that, depending on the parameters, the former mechanism can be strong enough in order to preserve the model stability and to provide a solution to the Harrod knife-edge problem. In that case, the current rate of capital utilization converges toward its normal value. In other words, contrary to the existing literature, economic growth has not to be profit-led for the current utilization rate to converge toward the normal one.

Of course, since it depends on the parameters, this solution of the Harrod knife-edge problem remains fragile. But it opens a door that had never been opened before.

In addition to the efforts in order to improve the robustness of our conclusions, this model could be enriched in at least two directions. Firstly, its realism should be improved by relaxing the no public deficit and endogenous tax rate hypotheses. But the task is not so easy because it will be necessary to include the interests paid to capitalists, and then to take into account another dynamics: that of the public debt.

Secondly, the model could be extended to other autonomous demand components, especially to exports. Blecker [1998, 2002, 2011] among others developed the Kaleckian model in an open economy framework. But his models can't highlight the role of exports as automatic stabilizer because the share of exports in aggregate demand is assumed to be exogenously given. Some further explorations about the potential role of exports as economic growth stabilizer should also refuel the debate between Kaleckian models and other post Keynesian models such as the export-led cumulative causation or the balance-of-payment constraint models[20].

[20] See Cornwall [1977] or Setterfield and Cornwall [2002] for the former and Thirlwall [1979] for the latter. See also Blecker [2010] for a critical survey of the two approaches.

Eventually, the above model doesn't take account of labor market and unemployment. It thus provides no solution for the 'first' Harrod problem, except arguing that government sets the rate of growth of public expenditures in accordance with demographic growth in order to stabilize or eliminate unemployment. This question will deserve more attention in further researches.

6. Bibliography

Allain O. [2009] "La modération salariale : le point de vue des (néo-)kaleckiens", *Revue Economique*, 60(1), pp. 81-108.

Allain O., Canry N. [2008] *Growth, Capital Scrapping, and the Rate of Capacity Utilization*, Working Paper presented at the 12th conference of the Research Network Macroeconomics and Macroeconomic Policies, Berlin.

Blecker R.A. [1998], "International Competitiveness, Relative Wages, and the Balance-of-Payments Constraint," *Journal of Post Keynesian Economics*, 20(4), pp. 495-526.

Blecker R.A. [2002] "Distribution, Demand, and Growth in Neo-Kaleckian Macro Models," in M. Setterfield (ed.), *The Economics of Demand-Led Growth: Challenging the Supply-Side Vision of the Long Run*, Edward Elgar, pp. 129-152.

Blecker R.A. [2010] "Long-Run Growth in Open Economies: Export-Led Cumulative Causation or a Balance-of-Payments Constraint?", in G.C. Harcourt and P. Kriesler (eds), *Handbook of Post-Keynesian Economics*, Oxford University Press (forthcoming).

Blecker R.A. [2011] "Open economy models of distribution and growth", in E. Hein and E. Stockhammer (eds.), *A modern guide to Keynesian macroeconomics and economic policies*, Edward Elgar, 2011, pp. 215-239.

Chatelain J.B. [2010] "The Profit-Investment-Unemployment Nexus and Capacity Utilization in a Stock-Flow Consistent Model", *Metroeconomica*, 61(3), pp. 454-472.

Commendatore P., Panico C., Pinto A. [2005] "Government Debt, Growth and Distribution in a Postkeynesian Approach", *in* N. Salvadori and R. Balducci (eds) *Innovation, Unemployment, and Policy in the Theories of Growth and Distribution*, Edward Elgar, pp. 223-37.

Cornwall J. [1977] *Modern Capitalism: Its Growth and Transformation*, New York: St. Martin's.

Dallery T. [2007] *Kaleckian Models of Growth and Distribution Revisited: Evaluating their Relevance through Simulations*, Working Paper presented at the 11th conference of the Research Network Macroeconomics and Macroeconomic Policies, Berlin.

Dallery T., van Treeck T. [2011] "Conflicting Claims and Equilibrium Adjustment Processes in a Stock-flow Consistent Macroeconomic Model", *Review of Political Economy*, 23(2), pp. 189-211.

De-Juan O. [2005] "Paths of Accumulation and Growth: Towards a Keynesian Long-Period Theory of Output", *Review of Political Economy*, 17(2), pp. 231-252.

Duménil G., Lévy D. [1999] "Being Keynesian in the Short Term and Classical in the Long Term: The Traverse to Classical Long-Term Equilibrium", *The Manchester School*, 67(6), pp. 684-716.

Dutt A.K. [2011] "Growth and income distribution: a post-Keynesian perspective", in E. Hein and E. Stockhammer (eds.), *A modern guide to Keynesian macroeconomics and economic policies*, Edward Elgar, 2011, pp. 61-87.

Hein E., Lavoie M., van Treeck T. [2011] "Some instability puzzles in Kaleckian models of growth and distribution: a critical survey", *Cambridge Journal of Economics*, 35(3), pp. 587-612.

Hein E., Lavoie M., van Treeck T. [2012] "Harrodian Instability and the 'Normal Rate' of Capacity Utilization in Kaleckian Models of Distribution and Growth—a Survey", *Metroeconomica*, 63(1), pp. 139-169.

Kaldor N. [1955-56] "Alternative Theories of Distribution", *Review of Economic Studies*, 23, pp. 83-100.

Kaldor N. [1957] "A Model of Economic Growth", *Economic Journal*, 67, pp. 591-624.

Lavoie M. [1996] "Traverse, Hysteresis and Normal Rates of Capacity Utilization in Kaleckian Models of Growth and Distribution", *Review of Radical Political Economics*, 28(4), pp. 113-147.

Lavoie M. [2000] "Government Deficits in Simple Kaleckian Models", in H. Bougrine (ed.), *The Economics of Public Spending: Debts, Deficits and Economic Performance*, Edward Elgar, pp. 122-134.

Lavoie M. [2012] "History and methods of post-Keynesians economics", in E. Hein and E. Stockhammer (eds.), *A modern guide to Keynesian macroeconomics and economic policies*, Edward Elgar, pp. 1-33.

Nikiforos M. [2011] *On the Desired Rate of Capacity Utilization*, The New School for Social Research, Working Paper 16/2011, 43 p.

Palumbo A., Trezzini A. [2001] "Growth without Normal Capacity Utilization", *The European Journal of the History of Economic Thought*, 10(1), pp. 109-135.

Park M.-S. [2000] "Autonomous Demand and the Warranted Rate of Growth", *Contributions to Political Economy*, 19, pp. 1-18.

Robinson J. [1956] *The Accumulation of Capital*, London, Macmillan.

Robinson J. [1962] *Essays in the Theory of Economic Growth*, London, Macmillan.

Sawyer M. [2012] "The Kaleckian analysis of demand-led growth", *Metroeconomica*, 63(1), pp. 7-28.

Serrano F. [1995a] "Long Period Effective Demand and the Sraffian Supermultiplier", *Contributions to Political Economy*, 14, pp. 67-90.

Serrano F. [1995b] *The Sraffian Supermultiplier*, PhD dissertation, Faculty of Economics ans politics, University of Cambridge.

Setterfield M., Cornwall J. [2002] "A neo-Kaldorian Perspective on the Rise and Decline of the Golden Age", in M. Setterfield (ed.), *The Economics of Demand-led Growth: Challenging the Supply-side Vision of the Long Run*, Edward Elgar, pp. 67-86.

Shaikh A. [2007] *A Proposed Synthesis of Classical and Keynesian Growth*, Schwartz Center for Economic Policy Analysis (SCEPA), Working Paper, The New School.

Shaikh A. [2009] "Economic Policy in a Growth Context: a Classical Synthesis of Keynes and Harrod", *Metroeconomica*, 60(3), pp. 455-494.

Skott P. [2010] “Growth, Instability and Cycles: Harrodian and Kaleckian Models of Accumulation and Income Distribution”, in M. Setterfield (ed.), *Handbook of Alternative Theories of Economic Growth*, Edward Elgar.

Skott P. [2012] “Theoretical and Empirical Shortcomings of the Kaleckian Investment Function”, *Metroeconomica*, 63(1), pp. 109-138.

Thirlwall A. P. [1979] “The Balance of Payments Constraint as an Explanation of International Growth Rate Differences”, *Banca Nazionale del Lavoro Quarterly Review*, 128, pp. 45-53.

Trezzini A. [1995] “Capacity Utilisation in the Long Run and the Autonomous Components of Aggregate Demand”, *Contributions to Political Economy*, 14, pp. 33-66.

Trezzini A. [1998] “Capacity Utilisation in the Long Run: Some further Considerations”, *Contributions to Political Economy*, 17, pp. 53-67.

White G. [2006] “Demand-Led Growth and the Classical Approach to Value and Distribution: Are they Compatible?”, in N. Salvadori (ed.) *Economic Growth and Distribution: On the Nature and Causes of the Wealth of Nations*, Edward Elgar, pp. 148-178.

You J.-I., Dutt A.K. [1996] “Government Debt, Income Distribution and Growth”, *Cambridge Journal of Economics*, 20(3), pp. 335-351.