Excess Bank Reserves and Monetary Policy with a Lower-Bound Lending Rate

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Abstract

This paper proposes an analytical framework to analyze the macroeconomic effects of the unprecedented injection of excess reserves into the banking system. Central to the framework is the thesis of a lower-bound loan rate identified by the liquidity preference of banks. The loan market and the money market are integrated to determine an excess reserves-loan equation (RL equation), which is the central theoretical contribution of this work. Together with a traditional IS equation and Phillips curve, the model is solved recursively to obtain equilibrium output and inflation. The model suggests that the inflation or deflation outcome depends on a linear combination of five parameters and two probability regimes. The empirical results tend to support a deflation instead of an inflation regime.

JEL Codes: E41, E43, E52 and G21

Key words: quantitative easing, bank liquidity preference, deflation, excess reserves

1. Introduction

In the present consensus framework in monetary macroeconomics the central bank sets its policy interest rate in response to inflation and output gap developments, and money plays only a subdued role. However, concomitant with the change in the policy rate is the need to manage bank reserves (Dow, 2001). It was found that such reserve management can engender liquidity effects, which Carpenter and Demiralp (2008) note are the first step in the monetary transmission mechanism. Therefore, if the central bank reduces its interest rate target it is likely to inject liquidity so as to maintain a credible target; on the other hand, to increase the target rate the central bank must drain liquidity from the system.
While these operational aspects of monetary policy have usually been abstracted from in the theoretical literature (see e.g. Woodford, 2003), the monetary policy management by the U.S. Fed during the recent economic and financial crisis has renewed the interest in the role of bank reserves in monetary policy implementation (Gavin, 2009; Martin et al, 2011). Indeed, since the full-fledged outbreak of the recent crisis resulting from the fall of Lehman Brothers and AIG in the fall of 2008, the Federal Reserve has injected over $1.5 trillion of reserves into the U.S. banking system. However, as discussed by Keister and McAndrews (2009), most of this infusion has been held as excess reserves by banks.

Several authors have recently explored the reasons for the accumulation of unprecedented levels of excess reserves in the banking system. For instance, Heider et al (2009) attribute the hoarding of reserves to the existence of counterparty risk, while Freixas and Jorge (2008) propose asymmetric information problems as the determining factor in the significant build-up of excess reserves in the interbank market. Ashcraft et al (2009) provide a model to explain excess reserves as precautionary hoardings at the daily frequency. In a related study, Adrian and Shin (2009) examine liquidity as the ability of financial institutions to fund the steep discounts in market-based security prices during financial stress. In their set-up the shock to security prices requires reducing leverage through rapid sales of financial assets or through borrowing. A liquidity crisis ensues as many financial market participants try to so sell assets simultaneously and rapidly. This type of asset fire sale will then show up as a banking system liquidity crisis if the monetary authorities do not intervene.

In contrast, this paper investigates the implications of a long-term aggregate demand curve for excess reserves by banks for monetary policy stabilization. The framework adopts the proposition of Khemraj (2010) that a flat bank liquidity preference curve represents a minimum mark-up lending rate at which the said rate is equal to the marginal cost for making loans. However, this work goes further by examining the output and price effects of unconventional monetary policy such as quantitative easing. Similar

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1 Our aggregate model, providing a long period analysis, can be seen as complementary to the work of Heider et al (2009) and Freixas and Jorge (2008). Their analyses tend to focus on the short-term break down of the interbank market. On the other hand, the analysis herein looks at the demand for reserves using
to Khemraj (2010), the bank liquidity preference curve is identified by a model showing
the relationship between the loan interest rate and excess reserves. The liquidity
preference curve becomes flat at a loan rate substantially above zero, thus implying non-
remunerated excess reserves and interest-earning loans are perfect substitutes. Although a
small interest rate is paid to banks since October 2008, we do not believe this changes the
key proposition of the paper given that this rate is significantly below the loan interest
rate.

The mark-up interest rate can be derived from an oligopoly model of the banking
firm similar to the derivation presented by Freixas and Rochet (2008). However, in our
framework when the market loan rate rises above the threshold (the firm’s mark-up rate),
the risk adjusted loan interest rate is greater than the marginal cost; here the liquidity
preference curve is downward sloping. In this regime the demand for excess reserves is
involuntary and banks will seek to substitute interest-earning loans for excess reserves.
On the other hand, when the lower-bound threshold (the mark-up rate) is binding banks
will hold excess reserves voluntarily. Once other investment constraints are eased – such
as those associated with proprietary trading and investments in securities – the banks will
invest excess reserves in other interest-earning assets. We will not address the issue of
alternative investments in this paper but rather limit the analysis to the nexus between
excess reserves and bank loans and in turn work out the price and output implications of
large-scale quantitative easing.

Already Keynes (1936, pp. 207-208) noted the possibility that the broad monetary
aggregate and government bonds could become perfect substitutes once the bond interest
rate reaches zero. In contrast, the modern incarnation of the liquidity trap thesis holds
that expectations play a critical role in determining the effect of monetary policy at the
three decades of monthly data in order to identify threshold lending rates and model the macroeconomic
effects.

2 Although not identical to the thesis of this paper, a similar notion is found in Frost (1971). Frost proposed
a stable bank excess reserves curve that is kinked at a Treasury bill rate close to zero (between 0.3 and 0.5
percent). According to Frost, profit-maximizing banks incur brokerage fees (or transaction costs) which are
higher than the market rate earned on Treasury bills – thus the curve is kinked at this point to signal a more
elastic accumulation of excess reserves. Using an econometric procedure, Ogawa (2007) identified two
factors accounting for Japanese banks’ demand for excess reserves: (i) a near-zero short-term interest rate
and (ii) fragile bank balance sheet. See Mounts et al (2000) for an earlier survey of the literature on the
demand for excess bank reserves.
lower-bound interest rate (Krugman, 1998). In this theoretical framework monetary policy can still be effective at stimulating aggregate demand at the zero-bound policy rate once the central bank can maintain credibility by sticking to a relatively higher inflation target. However, even along the lines of this argumentation, expectations of future inflation must be backed by the ability to pay today for them to stimulate current aggregate demand. If banks hoard excess reserves, and not make loans, the ability to pay today is diminished. Furthermore, given the integration of commodity markets with financial markets and the preponderance of propriety trading desks, banks might speculate in commodity markets, pushing up commodity prices. Oligopolistic non-financial firms will then mark-up their prices over marginal cost. Thus the monetary injections could engender cost-push inflation (backward shift of the marginal cost curve) – instead of the demand-pull inflation it is intended to create – which does not solve the output problem.\(^3\)

Therefore, the implicit proposition in this paper is the loan rate is subjected to monetary policy liquidity effects over some ranges but becomes rigid when the lower-bound oligopoly mark-up rate becomes binding. At the threshold loan rate all monetary policy liquidity effects evaporate and market loan rate becomes equal to the marginal cost of funds plus the marginal cost of making loans. Banks thus accumulate excess reserves voluntarily at this point. Further, this paper provides an analytical framework for the study of the effects of bank excess reserves on aggregate output and prices once the threshold lending rate (at a flat liquidity preference curve) is binding. The paper also develops an aggregative model that links bank loanable funds with bank liquidity preference in the presence threshold interest rates.\(^4\)

The paper is organized as follows. Section 2 presents a stylized macroeconomic model with includes bank reserves. Section 3 presents empirical evidence. Section 4 provides some concluding comments.

\(^3\) As noted earlier this issue is the focus of another research paper.

\(^4\) Unifying loanable funds and liquidity preference has been an important effort in the past (see Tsiang, 1956; Ferguson and Hart, 1980). One contribution of the model in this paper is its explicit integration of banking features into loanable funds and liquidity preference.
2. A Stylized Macroeconomic Model with Bank Reserves

This section proposes a stylized macroeconomic model which links monetary policy and the demand for excess bank reserves with macroeconomic activity and inflation. As a starting point for the derivation of the mark-up loan rate we use the Cournot model as presented by Freixas and Rochet (2008). However, we augment the basic model by including the bank’s risk of being short of excess reserves.

Let us assume the representative bank could be in three excess reserves states. In state 1 there is a shortage of reserves relative to the required level, what forces the bank to borrow from the Federal funds market or the discount window. Without losing the basic conclusion we assume only one penalty interest rate for state 1 – the Federal funds rate, \( r_F \). The probability of being in a reserve deficit is denoted by \( \theta_1 \). This probability is obviously related to the risk of a systemic crisis such as a run on the banks. In state 2 there is a surplus of reserves, which allows the bank to lend in the Federal funds market. Again the bank lends at \( r_F \). The probability of being in state 2 is \( \theta_2 \). Finally, in state 3 the bank has such a large build-up of excess reserves it can hoard funds in special deposits at the central bank. The banks earn the rate of interest \( r_{SD} \) on these special deposits. The probability – which is influenced by monetary policy – of being in state 3 is \( \theta_3 \). Given that \( \theta_1 + \theta_2 + \theta_3 = 1 \), the expected return on excess reserves is \( r_e = (\theta_2 - \theta_1)F_R + \theta_3r_{SD} \).

The profit function, taken to be concave in loans (\( L_i \)) and deposits (\( D_i \)), of the \( i \)th bank is given by eq.(1). The bank’s balance sheet identity is given by eq.(2), and \( R_i = \) excess reserves, \( zD_i = \) required reserves (where \( z = \) required reserve ratio), and \( D_i = \) deposits. The inverse function forms \( r_e(L) \) and \( r_{SD}(D) \) are used in the derivation process.

\[
\Pi_i = r_e(L)L_i + r_eR_i - r_{SD}(D)D_i \tag{1}
\]

\[
zD_i + R_i + L_i = D_i \tag{2}
\]

\[\text{5 This is the contemporary situation where banks are paid interest on special deposit of excess reserves at the Federal Reserve (Keister and McAndrews, 2009).}\]
Solving the balance sheet constraint for $R_i$ and substituting into eq.(1) gives the profit function in eq.(3). In the Cournot equilibrium the $i$th bank maximizes profit by taking the volume of loans and deposits of other banks as given. In other words, for the $i$th bank, the pair $(L_i', D_i')$ solves eq.(3). The aggregate quantity of loans and deposits, respectively, demanded by the entire banking sector are described in eq.(4).

$$\Pi_i = [r_L(L) - r_{E_L}]L_i - [r_D(D) - r_{E_D}(1-z)]D_i$$

$$L = L_i + \sum_{i \neq j} L_j; D = D_i + \sum_{i \neq j} D_j$$

The first-order conditions after maximizing the profit function are described by eq.(5). The market demand curve the bank faces is downward sloping, hence the elasticity of demand denoted by eq.(5-2). The symbol $a$ is the elasticity of demand for loans. There is a unique equilibrium in which bank $i$ assumes $L_i' = L' / N$, where $N$ denotes the number of commercial banks that makes up the banking sector. The expression $r_L'(L)$ represents the first derivative of the loan rate with respect to $L$ and it is simply the inverse of $L'(r_L)$.

$$\frac{d\Pi_i}{dL_i} = r_L(L) + r_L'(L)L_i - r_{E_L} = 0$$

$$r_L'(L) = \frac{1}{L'(r_L)} \quad (5-1)$$

$$a = r_L \cdot L'(r_L) / L \quad (5-2)$$

Substituting eqs. (5-1) and (5-2) into eq.(5) gives the expression (6) from which the minimum threshold rate ($r_L$) is obtained. The mark-up is dependent on the inverse of the product of $N$ and the market elasticity of demand ($a$) for loans. $N=1$ describes the case of a monopoly where the mark-up is highest, while as $N \to \infty$ one bank has an infinitesimal share of the market; the equilibrium approaches the purely competitive state in which the mark-up approaches zero.

In the lower bound loan rate equation, $r_F$ is subjected to liquidity effects and therefore can be written as $r_F(R)$ with the effect being measured by the slope $r_F'(R)$.

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6 The use of $N$ weighs each bank equally. This is clearly an unrealistic assumption for the purpose of making the mathematics tractable. Nevertheless, the simplification does not change the conclusion of the model.
\[ r_L(1 + \frac{1}{aN}) = r_E \quad \text{or} \quad r_L(1 + \frac{1}{aN}) = (\theta_2 - \theta_1)r_F + \theta_3 r_{SD} \] (6)

\[ r_T = \frac{(\theta_2 - \theta_1)r_F + \theta_3 r_{SD}}{(1 + \frac{1}{aN})} \] (7)

Figure 1 shows that the lower bound for the loan rate is given by \( r_T \), which in turn is determined and in fact becomes the effective supply of loans. The demand curve for excess reserves is given by \( R_D \) and it becomes flat at \( r_T \), which represents the effective supply curve (or threshold supply curve) of loans. Moreover, \( r_T \) represents the rate at which all liquidity effects have been exhausted by the central bank’s monetary expansion. It is postulated here that the rate is determined by banks that possess market power. On the other hand, households and firms accept the rate as given. The commercial banks must, in turn, consider the marginal cost of funds, risk and liquidity conditions.

![Diagram showing excess reserves and loanable funds](image)

**Figure 1.** The threshold rate and loanable funds

The supply of reserves by the central bank is given by \( R_s \). When \( R_D = R_s \) the equilibrium quantity of reserves is determined as \( R^* \). The demand for loans is denoted
by $L_D$. The downward sloping curve reflects the idea that an increase in the lending rate decreases the present value of future profit flows of businesses. The converse occurs when the loan rate falls. It also reflects that households’ discounted future cash flows fall when the mortgage rate (or the rate on consumer credit) increases. A decline in the mortgage rate has the opposite effect on households. Substituting $r_T$ into $L_D$ gives the equilibrium level of credit ($L^*$).

On the surface, the horizontal depiction of the loan supply curve might suggest borrowers can obtain all credit at the lower bound rate $R_D = r_T$ – something that would be inconsistent with a credit crunch. However, the horizontal line indicates an asymmetric determination of the lending rate, which the banks determine by mark-up and the public unconditionally accepts. This oligopolistic feature was found in the empirical literature (see Neumark and Sharpe, 1992). Consumers do not determine credit card rates; small and medium sized businesses do not determine the rate at which they borrow. In other words, the banks set the rate at which credit is offered and market demand responds accordingly. An upward shift in the line is an indication of a credit crunch as it leads to an upward movement along the loan demand curve. In fact, all borrowers do not obtain credit at the pre-determined mark-up rate. Only those who can pay the established mark-up over the lower bound loan rate will be able to borrow at a level determined at the point where the rate intersects the demand curve.

A monetary contraction from $R_1$ to $R_2$ leads to an increase in the lending rate above threshold to $r_T$. This implies the central bank’s liquidity management has liquidity effects only above $r_T$. These liquidity effects were uncovered empirically by Carpenter and Demiralp (2008). Therefore, embedded in the threshold loan rate is the policy interest rate – $r_T$. A decrease in the target $r_T$ is followed by an expansion of bank reserves in order to defend the target. On the other hand, when the $r_T$ target is increased the central bank must diminish bank reserves to keep the target credible. The shocks to excess reserves are demonstrated by a movement of a vertical reserve supply curve (figure 1).

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7 As an aside, albeit an important one, borrower surplus – bounded by the area under the loan demand curve and above $r_T$ – increases when the demand for credit shifts outward. However, the surplus would diminish as the interest rate rises above the threshold as liquidity conditions tighten.
along the demand curve\(^8\). Consequently, credit is contracted from \(L\) to \(L'\). On the other hand, a monetary expansion from \(R_s\) to \(R_{s1}\) leads to no further decrease in the lending rate as the minimum threshold rate is now binding. Credit expansion stops at \(L'\) and excess reserves are accumulated voluntarily. Therefore, once the threshold rate is reached credit intermediation would require that policies directly stimulate the demand for loans along this rate. The demand curve for loans shifts out from \(L_d\) to \(L_{d1}\).

Given the stylized facts and the diagrammatic exposition (see figures 3 and 4), it is reasonable to express the banks’ demand for excess reserves as the following reciprocal model

\[
r_L = r_r + \beta \left( \frac{1}{R'} \right) \tag{8}\]

Note that the threshold minimum rate is the asymptote. \(\beta\) is a coefficient and \(R'\) is the equilibrium level of excess reserves as shown in figure 1, from which we can form the following relationship between excess reserves and the demand for loans.

\[
r_r + \beta \left( \frac{1}{R'} \right) = -ar_L + bY \tag{9}\]

The demand for loans is given by the following simple double-log function \(L_d = -ar_L + bY\), which is chosen for the purpose of algebraic convenience. \(a\) = the public’s elasticity of demand for loans; \(b\) = the public’s income elasticity of demand for loans; and \(Y\) = aggregate output. By inserting eq.(7) in eq.(9) we obtain

\[
\frac{(\theta_2 - \theta_1)r_f(R') + \theta_r r_{sd}}{(1 + \frac{1}{aN})} + \beta \left( \frac{1}{R} \right) = -ar_L + bY \tag{10}\]

Eq.(10) can be solved for \(r_L\) to obtain the RL equation in terms of \(Y\), \(R'\) and the exogenous parameters of the model. One of the attractive features of this equation is that it introduces a microeconomic term into a macroeconomic function.

\(^8\) Later in the paper where we present the econometric analyses, we will show that the shocks or liquidity effects can be measured by the derivative of a reciprocal function, which models the liquidity preference curve.
In order to analyze the implications of a lower bound in the loan interest rate at the macroeconomic level, we use a traditional IS equation which links the expected real interest rate with the level of economic activity

\[ Y_t = A - \alpha_r (r_L - \pi_{t-1}) \]  \hspace{1cm} (11)

From equation 11, \( A \) is determined by the autonomous components of consumption and government spending, \( \alpha_r \) represents the interest rate elasticity of output and \( \pi_{t-1} \) represents aggregate price inflation in the previous time period. We assume that the equation of motion for aggregate price inflation is determined by the following Phillips curve relationship

\[ \pi_t = \lambda \pi_{t-1} + \gamma y_t \]  \hspace{1cm} (12)

Where \( \pi_t \) denotes the inflation rate in the present period and \( y_t \) is the output gap between trend output (\( \bar{Y} \)) and equilibrium output, which we will solve for later in the paper; thus \( y_t = Y^* - \bar{Y} \), with \( \lambda \) representing the degree of inflation persistence in the economy.

Solving for \( r_L \) in eq.(10) and setting it equal to the IS equation gives the reduced form solution for aggregate output \( Y^* \). The equilibrium output is given by eq.(13), which shows that excess reserves influence aggregate output via changes in the funds rate and the composite elasticity

\[ Y^* = \left( \frac{\theta_2 - \theta_1}{a} \right) r_f (R^*) + \frac{\theta_1 r_{SD}}{a(1 + \frac{1}{aN})} + \left( \frac{\beta}{(b + \frac{1}{a}) (a + \frac{1}{\alpha_r})} \right) R^{*-1} + \left( \frac{1}{\alpha_r (b + \frac{1}{a})} \right) A + \left( \frac{1}{b + \frac{1}{a}} \right) \pi_{t-1} \]  \hspace{1cm} (13)

In order to obtain the output dynamics, take the total differential of eq.(13). Note that \( r_f^t (R^*) \) is the slope of the bank liquidity preference curve in the Federal funds market. This slope is given by \(-\alpha / R^2\). Express \( dY^* \), \( dR^* \), \( dA \) and \( d\pi_{t-1} \) in discrete form, respectively, as follows \( \Delta Y^*, \Delta R^*, \Delta A \) and \( \Delta \pi_{t-1} \); and note that \( \Delta Y^* = Y_t - Y_{t-1} \). We can assume a partial adjustment framework as follows: \( Y_t - Y_{t-1} = \delta (\bar{Y} - Y_{t-1}) \). Substituting the slope and the discrete forms and taking into consideration the partial adjustment mechanism give eq.(15).
\[ dY^* = \frac{(\theta_2 - \theta_1)\gamma(R^*)}{a\left(\frac{b}{a} + \frac{1}{\alpha_r}\right)(1 + \frac{1}{aN})} \cdot dR^* - \left(\frac{\beta}{a\left(\frac{b}{a} + \frac{1}{\alpha_r}\right)}\right) R^{*-2} \cdot dR^* + \left(\frac{1}{\alpha_r\left(\frac{b}{a} + \frac{1}{\alpha_r}\right)}\right) dA + \left(\frac{1}{b + \frac{1}{\alpha_r}}\right) d\pi_{t-1} \] 

(14)

\[ Y_t = (1 - \delta)Y_{t-1} + \delta \bar{Y} - \left(\frac{\alpha(\theta_2 - \theta_1)}{a\left(\frac{b}{a} + \frac{1}{\alpha_r}\right)(1 + \frac{1}{aN})} + \frac{\beta}{a\left(\frac{b}{a} + \frac{1}{\alpha_r}\right)}\right) \frac{1}{R^{t-2}} \Delta R_t^* + \left(\frac{1}{\alpha_r\left(\frac{b}{a} + \frac{1}{\alpha_r}\right)}\right) \Delta A_t + \left(\frac{1}{b + \frac{1}{\alpha_r}}\right) \Delta \pi_{t-1} \] 

(15)

Let

\[ \alpha_1 = -\left(\frac{\alpha(\theta_2 - \theta_1)}{a\left(\frac{b}{a} + \frac{1}{\alpha_r}\right)(1 + \frac{1}{aN})} + \frac{\beta}{a\left(\frac{b}{a} + \frac{1}{\alpha_r}\right)}\right) \frac{1}{R^{t-2}} \]

\[ \alpha_2 = \frac{1}{\alpha_r\left(\frac{b}{a} + \frac{1}{\alpha_r}\right)} \]

\[ \alpha_3 = \frac{b + 1}{a} \]

\[ Y_t = (1 - \delta)Y_{t-1} + \delta \bar{Y} - \alpha_1 \Delta R_t^* + \alpha_2 \Delta A_t + \alpha_3 \Delta \pi_{t-1} \]

(16)

Eq.(16) allows us to solve for the time path of \( Y_t \) and to derive the dynamic multipliers to study the effect of excess bank reserves and the autonomous components.

For an initial value of output \( (Y_0) \) and \( 1 - \delta < 0 \) we obtain the following solution by the recursive method (eq.17). The dynamic multiplier showing the effect on output (for \( s \) future periods) given a change in \( \Delta R \) is given by eq.(18). The equation has an interesting feature because the equilibrium level of reserves, \( R^* \), stays in the equation. This allows us to calculate the effect of liquidity injections (or contractions) at the lower bound level.

\[ Y_t = \bar{Y} + (1 - \delta)Y_0 - \alpha_1 \sum_{i=0}^{t-1} (1 - \delta)^i \Delta R_{t-i} + \alpha_2 \sum_{i=0}^{t-1} (1 - \delta)^i \Delta A_{t-i} + \alpha_3 \sum_{i=2}^{t-1} (1 - \delta)^i \Delta \pi_{t-i} \] 

(17)
\[
\frac{\partial Y_s}{\partial (\Delta R)} = -\alpha_i (1-\delta) = \left\{ \frac{\alpha (\theta_2 - \theta_1)}{a \left( b + \frac{1}{a} \right) (1 + \frac{1}{aN})} + \frac{\beta}{a \left( b + \frac{1}{a} \right)} \right\} \left( 1 - \frac{\Delta}{R^2} \right) (1-\delta)^i \tag{18}
\]

Another feature of eq.(18) is the effect of the reserve injection depends on two regimes. First, there is regime 1 in which \( \theta_2 < \theta_1 \). In this regime bank lending is strong and there is a greater likelihood of a reserve deficit. Here if \( N \) is sufficiently large (we have competition rather than oligopoly) and \( \alpha \) is also large enough, we can have the situation in which the stimulation of bank reserves feeds through to higher output. Otherwise, regime 2, whereby \( \theta_2 > \theta_1 \), holds. There is weak lending and surplus of excess reserves. Therefore, the excess reserves are consistent with a decreased in output and therefore employment. Figure 2 below shows the simulation of these possibilities for different levels of excess reserves in the two regimes over eight time periods (\( s = 8 \)). The diagram suggests, given the same parameters, in each regime increasing reserves will diminish the response once the threshold interest rate is binding.

**Figure 2.** Dynamic multipliers (DM) showing output response for two lending/reserves regimes
We now need to examine how inflation will adjust given changes in excess reserves. This requires substituting eq.(13), the equilibrium output, into the equation which shows price adjustments, eq.(12). From eq.(13) let

\[
M = \frac{(\theta_2 - \theta_1) r_f(R^*) + \theta_S r_{sd}}{a(b + \frac{1}{a\alpha})(1 + \frac{1}{aN})}
\]

and

\[
\phi = \frac{b}{a(b + \frac{1}{a\alpha})}
\]

Therefore, eq.(13) can be rewritten as

\[
Y_i^* = M + \phi R_i^{-1} + \alpha_2 A + \alpha_3 \pi_{t-1}
\]

(20)

Substituting eq.(20) into (12) gives

\[
\pi_t = \gamma(M - \bar{Y}) + (\lambda + \alpha_3 \gamma) \pi_{t-1} + \gamma \phi R_i^{-1} + \alpha_2 A
\]

(21)

Alternatively eq.(21) can be rewritten as

\[
\pi_t = \gamma \left(\frac{(\theta_2 - \theta_1) r_f(R^*) + \theta_S r_{sd}}{a(b + \frac{1}{a\alpha})(1 + \frac{1}{aN})} - \bar{Y}\right) + (\lambda + \alpha_3 \gamma) \pi_{t-1} + \frac{\gamma b}{a(b + \frac{1}{a\alpha})} R_i^{-1} + \alpha_2 \gamma A
\]

(22)

Eq.(22), a dynamic equation, allows us to analyze the effect of excess reserves on inflation. The dynamic multiplier (DM) can be visualized by the product of the following two derivatives. As noted earlier, \(r_f'(R^*) = -\alpha / R^{22}. Therefore,

\[
DM = \begin{pmatrix}
\frac{\partial \pi_t}{\partial R_f'} & \frac{\partial \pi_t}{\partial \pi_{t-1}} \\
\frac{\partial \pi_t}{\partial \pi_{t-1}} & \frac{\partial \pi_t}{\partial \pi_{t-1}}
\end{pmatrix} = -\left(\frac{1}{a(b + \frac{1}{a\alpha})} \begin{pmatrix}
\alpha(\theta_2 - \theta_1) \\
(1 + \frac{1}{aN})
\end{pmatrix} + \beta \begin{pmatrix}
\gamma \\
R_i^{-2}
\end{pmatrix}\right) (\lambda + \alpha_3 \gamma)'
\]

(23)

Eq.(23) suggests four possible price level outcomes in the two regimes. These are the outcomes given a specific amount of increase or decrease in excess or bank reserves. Table 1 below indicates the possible scenarios, which are dependent on the relative size of the various parameters.
Table 1. Price level outcomes given an increase in excess reserves

<table>
<thead>
<tr>
<th></th>
<th>Regime 1: $\theta_2 &lt; \theta_1$</th>
<th>Regime 2: $\theta_2 &gt; \theta_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0 &lt; \lambda + \alpha_3 \gamma &lt; 1$</td>
<td>Inflationary event but price level returns to equilibrium</td>
<td>Deflationary event but price level returns to equilibrium</td>
</tr>
<tr>
<td>$\lambda + \alpha_3 \gamma &gt; 1$</td>
<td>Aggregate prices explode; hyperinflation</td>
<td>A deflationary spiral results; prices explode downward</td>
</tr>
</tbody>
</table>

Crucial to the inflation or deflation dynamic is the term $\lambda + \alpha_3 \gamma$, which implies that five parameters are important for driving the inflationary or deflationary process. These parameters are $a$, $b$, $\alpha_r$, $\lambda$ and $\gamma$. As noted earlier, $a$ = the public’s elasticity of demand for loans; $b$ = the public’s income elasticity of demand for loans; $\alpha_r$ is the output sensitivity to the lending rate; $\lambda$ = the degree of inflation persistence in the economy; and $\gamma$ = a measurement of the output gap and inflation relationship. The linear combination of these parameters together with the two probability regimes provide insight into the extent to which the unprecedented expansion of bank reserves can engender aggregate output and price changes.

3. Empirical Evidence

Let us now take a look at the empirical evidence for the existence of a lower-bound loan interest rate. Figure 3 presents a scatter plot of the prime loan interest rate and excess reserves. As it can be easily observed, the relationship between these two variables features a significant nonlinearity as the one suggested by the theoretical model discussed in the previous section. Accordingly, the picture suggests the existence of a lower bound loan rate of approximately 7%, around which point the curve becomes flat. This stylized fact becomes particularly striking when the sample is enlarged to include the recent crisis period where the threshold occurs at around 4%\(^9\).

As discussed in the previous section, the benchmark interest rate (assumed here to be represented by the effective federal funds rate) is embedded in the lower bound loan

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\(^9\)The liquidity preference curves are all extracted from scatter plots using the method of locally weighted regressions with a smoothing parameter of 0.4 (see Cleveland, 1993; 1979). Two outliers were removed – those are September 2001 and August 2003. Removing the outliers does not affect the pre-2007 interest thresholds. Instead including the outliers makes the threshold rate more conspicuous.
rate. If the loan interest rate is a mark-up over the marginal cost of funds – the Federal funds rate – then we should observe a threshold behavior when examining a scatter plot between excess reserves and the funds rate. This possibility is illustrated by figure 4. It is clear that the flat segment of the pre- and post-crisis curves occurs below the threshold obtained when the prime lending rate is used (figure 3). This implies a stable relationship between the loan-funds rate spread and excess reserves.

![Figure 3](http://research.stlouisfed.org/fred2/)

**Figure 3.** Loan market liquidity preference – monthly data 1980:1–2006:12 (left panel) and 1980:1-2011:5 (right panel). Data source: Federal Reserve Economic Data (http://research.stlouisfed.org/fred2/)

![Figure 4](http://research.stlouisfed.org/fred2/)

**Figure 4.** Federal funds market liquidity preference – monthly data. 1980:1 – 2006:12 (left panel), 1980:1 to 2011:5 (right panel). Data source: Federal Reserve Economic Data (http://research.stlouisfed.org/fred2/)
The theoretical framework indicates that the response of output and the price level to changes in bank reserves is nonlinear and regime-dependent. Accordingly, a proper econometric estimation of the theoretical model would entail the use of time-varying or regime-dependent estimation techniques. Such an investigation, however, would go beyond the scope of this paper. Here, instead, we pursue an indirect strategy to account possible nonlinear effects. The reciprocal functional form of the liquidity preference curve allows us to have a slope of $r'_t(R) = -\alpha / R^2$. This slope represents a structural measure of liquidity effects. Changes in this slope reflect central bank open market operations or more aggressive policies such as quantitative easing. Changes in bank liquidity owing to monetary policy, therefore, result in movements along the reciprocal liquidity preference curve but not a shift of this curve. On the other hand, shifts in the liquidity preference curve will reflect shocks to bank liquidity preference. The shift in the curve may be endogenous to various macroeconomic variables and it can also exert influences. However, changes in the slope are reflective of central bank policy actions and therefore they are exogenous. Some may argue that the liquidity effect is responding to or anticipating the business cycle, therefore this variable should be endogenous. Therefore, in addition to the bivariate regression results we present impulse response functions from a VAR system motivated by figure 1 (see Appendix 1).

The approach of this paper requires an approximation for $\alpha$, which can be estimated by an empirical reciprocal regression function: $r' = r + \alpha / R + \epsilon^D$. $\epsilon^D$ is the measure of the shifts or shocks of bank liquidity preference. We estimated $\alpha$ by least squares for the period 1980: Jan – 2011: May$^{10}$. The coefficient estimate was found to be 3.3916 with a t-value of 7.74. The federal funds market threshold ($r_T$) was estimated to be 2.086 with a t-statistic of 4.44 (robust standard errors). Once $\alpha$ is estimated we can approximate the liquidity effect (LIQ) for each time period. Since LIQ is exogenous we can include this measure in a series of bivariate regressions to measure its influence.

The following bivariate regression is estimated.

$$X_t = \epsilon_0 + \sum_{i=1}^g c_i X_{t-i} + dLIQ_t + \epsilon_t$$

$^{10}$ In future research we can estimate this coefficient with a time-varying method such as rolling regressions or the state space method.
The regression results are presented in table 2 for two time periods – 1980: Jan to 2006: Dec and 2007: Jan to 2011: May. $X_t$ represents the set macroeconomic variables reported in table 2. We do not report the coefficient of the lagged dependent variables. The final parsimonious regression model was decided based on Wald F-tests on a general model with lags of both the dependent and key independent variable (LIQ). Robust standard errors were calculated in each case. In both periods the liquidity effect is associated with price decrease instead of inflation measured as consumer price index (CPI) and producer price index (PPI). This result is statistically significant for each period. The deflation of prices, captured by the higher negative coefficient, is faster for the post-2006 period.

Table 2. Liquidity effect on selected macroeconomic variables

<table>
<thead>
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<tbody>
<tr>
<td>Inflation – CPI</td>
<td>-0.0051; t-stat = -4.22; p-val = 0.000; two significant lagged dependent terms</td>
<td>-0.1057; t-stat = -2.15; p-val = 0.035; two significant lagged dependent terms</td>
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<tr>
<td>Inflation – PPI</td>
<td>-0.0053; t-stat = -2.52; p-val = 0.0121; two significant lagged dependent terms</td>
<td>-3.448; t-stat = -1.72; p-val = 0.092; two significant lagged dependent terms</td>
</tr>
<tr>
<td>Unemployment</td>
<td>-0.0018; t-stat = 1.13; p-val = 0.257; one significant lagged dependent term</td>
<td>0.334; t-stat = 1.53; p-val = 0.130; one significant lagged dependent term</td>
</tr>
<tr>
<td>∆(Loans)</td>
<td>0.1282; t-stat = 3.55; p-val = 0.000; one significant lagged dependent term</td>
<td>-35.7; t-stat = -2.47; p-val = 0.016; one significant lagged dependent term</td>
</tr>
<tr>
<td>∆(Deposits)</td>
<td>0.0376; t-stat = 2.19; p-val = 0.028; one significant lagged dependent term</td>
<td>8.45; t-stat = 2.06; p-val = 0.045; one significant lagged dependent term</td>
</tr>
<tr>
<td>Mortgage rate</td>
<td>-0.0033; t-stat = -0.87; p-val = 0.385; one significant lagged dependent term</td>
<td>-0.1902; t-stat = -3.28; p-val = 0.002; one significant lagged dependent term</td>
</tr>
<tr>
<td>Moody_AAA</td>
<td>-0.0022; t-stat = -0.82; p-val = 0.42; one significant lagged dependent term</td>
<td>-0.0723; t-stat = -1.48; p-val = 0.146; one significant lagged dependent term</td>
</tr>
<tr>
<td>Moody_BAA</td>
<td>-0.0031; t-stat = -1.3; p-val = 0.194; one significant lagged dependent term</td>
<td>-0.048; t-stat = -0.87; p-val = 0.389; one significant lagged dependent term</td>
</tr>
</tbody>
</table>

For the period 1980 to 2006 the liquidity effect appears to reduce unemployment but by an amount that is very small and statistically insignificant. However, the unemployment rate tends to be positively related with LIQ in the period 2007 to May.
2011. This result is consistent with the deflation instead of inflation regime. LIQ tends to increase lending, \( \Delta(\text{Loans}) \), in the pre-2007 period, while lending declined substantially given the liquidity effect. There is a consistent increase in deposits, \( \Delta(\text{deposits}) \), albeit a stronger increase after 2006. One of the justifications of quantitative easing, which has the effect of increasing bank reserves significantly, is that it diminishes interest rates at the longer end of assets. We find evidence in support of this whereby the mortgage rate, Moody’s AAA and Moody’s BAA bond yields have declined substantially after 2006 given changes in the measured liquidity effect.

4. Conclusion

This paper analyzed the effect of bank reserve expansion in the presence of aggregate bank liquidity preference and a mark-up threshold loan rate. Although there is a large literature on various monetary transmission mechanisms, to our knowledge this line of exploration does not exist in the present literature. Moreover, this paper takes the loan rate as being determined as a mark-up rate instead of a competitive loanable funds mechanism, thus making this work more rooted in the institutional feature of banking. This line of analysis comes in the presence of unprecedented expansion of excess bank reserves by the Federal Reserve in spite of the conventional wisdom which holds that the Federal Reserve uses the Federal funds rate as its main instrument since the late 1980s (Meulendyke, 1998).

The paper examined the effects of reserve expansion when a lower bound lending rate, identified by a flat bank liquidity preference curve, is binding. Therefore, instead to focusing on the demand for broad monetary aggregates, this study underscores that the behavior of banks, as it relates to interest rate mark-up and liquidity preference, is crucial for the functioning of the monetary transmission mechanism. Moreover, using a long data set, long-term liquidity preferences are identified; thus distinguishing this study from those which perform short period analyses of the liquidity build-up in the interbank markets of Europe and the United States. The paper proposed a model which helps us to understand whether quantitative easing will engender a short period deflation episode, a deflationary spiral, inflation or hyperinflation. In addition, the model shows how liquidity preference and loanable funds can be integrated at the level of the banking firm. The
empirical results suggest that excess reserves are associated with a deflationary episode instead of an inflationary environment.

References


Appendix 1

In view of the model shown by figure 1, we estimate a baseline six variable VAR using monthly data from 1980: Jan to 2011: Aug. The VAR model is estimated with five lags which the standard AIC and Schwartz information criteria suggest to be best. The VAR method is helpful because it gives us the impulse response functions (IRFs) for unemployment and price level (CPI) given a shock to excess bank reserves. The IRFs are identified using generalized impulses that are invariant to the ordering of the variables (Pesaran and Shin, 1998). The IRFs are also provided for the effective Federal funds rate, the loan rate and loan quantity. Figure A1 confirms the existence of the liquidity effect similar to Carpenter and Demiralp (2008), except we show that it is also applicable to the loan interest rate.

Figure A1. Loan and money market liquidity effects.

Figure A2 shows results consistent with the deflation scenario. The positive shock to excess reserves is associated with an increase in unemployment and decline in the price level as measured by CPI. Figure A3 shows that expansion of bank reserves do not lead to expansion of lending; instead lending declines. This is inconsistent with the money multiplier thesis. One possibility is the marginal excess reserves are invested in securities. This is consistent with the more institutional and historical literature focusing on the interaction among security dealers, banks and the Fed (Mehrling, 2011).
**Figure A2.** Confirming the deflation scenario

**Figure A3.** The deflation scenario