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Public debt and tax cuts in a SFC model

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Abstract: *This paper proposes a SFC model, inspired by Martin (2008). It is designed to study the link between tax cuts and public debt dynamics. This model focuses on the impact of income distribution on the public debt to GDB ratio with two types of households and a progressive income tax scheme. Three different cases are explored: 1/ efficient automatic stabilisers: public spending is supposed to adjust according to private spending so as to maintain full employment; 2/ public spending is autonomous but insufficient to maintain full employment; 3/ "Maastricht" or pro-cyclical deficit case: public spending adjusts to private spending so as to ensure a target value of the public deficit to GDP ratio. Each case in assessed in the short run –i.e. with a given public debt stock– and in the long run –i.e. with an endogenous accumulation of public debt.*

Key words: fiscal policy, public debt, stock-flow consistency, debt sustainability.

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Public debt and tax cuts in a SFC model

During the Second World War, when public debts were considered as "high" in the US as in the UK, Alvin Hansen and others explained why a "high level" of public debt should not be considered as a brake to growth and prosperity (Colm, 1948, Domar, 1944, Gehrels, 1957, Hansen, 1941, 1960, Hansen and Greer, 1942, Lerner, 1943). The first Keynesian generation showed that neither the debt itself nor its "burden" –*i.e.* the interest paid on the debt– impaired the activity providing that the government adjust public expenditure so as to maintain growth at a sufficient level. The counter parts of fiscal policy and public deficits, in terms of growth and employment through the multiplier effect, can do more than balance the alleged drawbacks of an increasing indebtedness.[1]

The basic mechanism which regulates the government deficit can be summarised as follows: an increase in public spending –or a tax cut– leads to more government borrowing and, meanwhile, the multiplying effect of the fiscal policy rises the output which engenders more tax revenue. At the end of the process, the deficit and the public debt have been at least partially reduced, which explains why the ratios of debt to GDP, or of interest on public bonds to GDP, are not necessarily growing with government borrowing. For those reasons, more than sixty years ago, it has been established that the public debt has not to be repaid as such. The main Keynesian ideas on public debt are often referred to as "functional" finance advocated as such recently by Philip Arestis and Malcolm Sawyer (2010) for instance in opposition to "sound" finance (see also Wray, 1998 and Nell and Forstater, 2003). In addition to those arguments, it has been noticed very early that a private demand for public bonds exists because this kind of asset is relatively reliable: public debt itself serves indeed as a crucial support for private savings and works as a basic ingredient for portfolio diversification.

Two decades later, authors like Blinder and Solow (1973) who were facing the monetarist challenge started to integrate wealth aspects more comprehensively in the analysis of the multiplier effect of fiscal policy, and their impact on public deficits. In the same vein, the seminal article of Godley and Rowthorn (1994) is one of the first contributions proposing a "stock-flow consistent" (SFC) model to study the dynamics of public debt. It is with this type of modelling apparatus that Godley and Lavoie (2007) explore and compare the efficiency of fiscal and monetary policies in consideration of public finance constraints. Their model has been slightly modified by Martin (2008) in order to give a simple analytical exposition without simulation.

As this framework is very fruitful, the present paper proposes a SFC model, inspired by Martin's contribution. It is designed to study the link between tax cuts and public debt dynamics. For the moment, money, investment and foreign trade are not ignored but simply considered as exogenous. This version of the model focuses on the impact of income distribution on the public debt to GDB ratio with two types of households and two tax rates. It shows that tax cuts do not yield enough increase in output and hence in tax revenue to reduce *ex post* government debt to GDP ratio.[2] The relative inefficiency of tax cuts, *i.e.* the fact that they are not self-financing, can emerge because of the macro-economic context and also because of distributive considerations when a distinction between low and high income households is introduced. Three different cases are explored: 1/ efficient automatic stabilisers: public spending is supposed to adjust according to private spending so as to ensure full employment; 2/ public spending is autonomous but insufficient to maintain full employment; 3/ "Maastricht" or pro-cyclical deficit case: public spending adjusts to private spending so as to ensure a target value of the public deficit to GDP ratio. Each case is assessed in the short run –*i.e.* with a given public debt stock– and in the long run –*i.e.* with an endogenous accumulation of public debt. The first section of the paper presents the general framework, the three cases are then examined successively.

A SFC model with two types of households

The output of the economy is assumed to be entirely demand-constrained. Demand is composed of household consumption, investment, balance of trade, and public expenditure. Employment is unilaterally determined by firms, depending on the quantities to produce. The technology is assumed to be constant. Investment, foreign trade, household savings allocation, ways to finance investment and shares of public debt held by households are supposed to be exogenous. More precisely, the following hypotheses are made:[3]

- the balance of trade is a constant fraction of GDP: $BC = \delta Y$
- the ratio of investment to GDB is constant: $I = \kappa Y$
- the share of self-financed investment, noted φ, is constant:

$$\text{real wages} + \text{real dividends} = Y - \varphi I$$

- the ratio of domestic savings to public debt is constant:[4] $V = \mu B$

There are two types of households:

- “type 1” households earn law wages and have no other source of income[5]. Their revenue is proportionally taxed at a rate τ_1 and they consume their disposable income as a whole at each period.
- “type 2” households receive high wages, profits and interest on their private wealth (savings stock). As the tax system is supposed to be progressive, “type 2” households are taxed at the rate $\tau_2 = \tau_1 + \tau'$. Their consumption spending stem both from their current disposable income and from a fragment of their savings stock.

The allocation of GDP after deduction of self-financed investment is supposed to be constant among the two classes of households. The fraction going back to type 1 households (noted α) is entirely consumed. With R_k the real income of type k households, we have:

$$\begin{cases} R_1 = & \alpha(Y - \varphi I) = \alpha(1-\varphi\kappa)Y \\ R_2 = & (1-\alpha)(1-\varphi\kappa)Y + \text{perceived interest} \end{cases}$$

Savings of type 2 households is composed of a fraction of public borrowing and also of assets issued by domestic companies and from abroad. The average interest rate paying for savings of domestic households is noted i^M .

Public bonds are held both by domestic type 2 households and also by the rest of the world. Their counterpart is composed of public outlays –including interest paid on the accumulated debt– less taxes levied on households. The average cost of public debt is noted i^G ; it differs all the more from the yield of private domestic savings that the share of public debt held abroad is high. When a government borrows on international markets, it has to pay both for the interest rate prevailing abroad and for a risk premium associated to exchange rate and default risks.

Budget constraint of type 1 households

Type 1 households consume their whole after tax revenue, their real consumption is hence simply defined by:

$$C_1 = (1-\tau_1)R_1 = (1-\tau_1)\alpha(1-\varphi\kappa)Y \qquad (1)$$

Budget constraint of type 2 households

The budget constraint of those households describes the evolution of their savings stock V:

$$PV = P_{-1}V_{-1} + (1-\tau_2)\left[(1-\alpha)(1-\varphi\kappa)Y + i^M P_{-1}V_{-1}\right] - PC_2 \qquad (2)$$

In real terms:

$$V = \frac{1+(1-\tau_2)i^M}{1+\pi} V_{-1} + (1-\tau_2)(1-\alpha)(1-\varphi\kappa)Y - C_2 \qquad (3)$$

Writing $z^M = \frac{(1-\tau_2)i^M - \pi}{1+\pi}$ the real after tax yield of households savings, the equation (3) can be rewritten:

$$\underbrace{\Delta V + C_2}_{\substack{\text{additional} \\ \text{savings}} \; + \text{consumption}} = \underbrace{\left[(1-\tau_2)(1-\alpha)(1-\varphi\kappa)Y + z^M V_{-1}\right]}_{\text{real disposable income}} \qquad (4)$$

The only behavioural hypothesis of this model is about type 2 households consumption defined as:

$$C_2 = (1-s)\left[(1-\tau_2)(1-\alpha)(1-\varphi\kappa)Y + z^M V_{-1}\right] + \gamma V_{-1} \qquad (5)$$

This consumption function is thus $C_2 = (1-s)Y_2^D + \gamma V_{-1}$. Although Godley and Lavoie (2007) do not directly use such an assumption, they do indirectly. As shown by Martin (2008) indeed, they assume that there is a wealth target of the form $V^* = \varpi Y^D$ and that the one period change in wealth is described by the following adaptive process:[6]

$$\Delta V = \lambda(V^* - V_{-1})$$

They thus assume that $\Delta V = \lambda\varpi Y_2^D - \lambda V_{-1}$. Since $C_2 + \Delta V = Y_2^D$, their adaptive rule is equivalent to the consumption function $C_2 = Y^D - \Delta V = (1-\lambda\varpi)Y_2^D + \lambda V_{-1}$ with $s = \lambda\varpi$.

Government's budget contraint

The dynamics of public debt is defined by the relation:

$$PB = (1+i^G)P_{-1}B_{-1} + PG - PT \qquad (6)$$

This equation can be rewritten, after substitution, with $r^M = \frac{i^M - \pi}{1+\pi}$ the real yield of domestic savings before tax, and $r^G = \frac{i^G - \pi}{1+\pi}$ the real interest on public debt:

$$\Delta B = r^G B_{-1} + G - \tau_1 \alpha(1-\varphi\kappa)Y - \tau_2(1-\alpha)(1-\varphi\kappa)Y - (r^M - z^M)V_{-1}$$

The integration of the hypothesis of a constant ratio of domestic savings to public debt $V = \mu B$ gives:

$$\Delta B = \left(r^G - \mu(r^M - z^M)\right)B_{-1} + G - \tau_1 \alpha(1-\varphi\kappa)Y - \tau_2(1-\alpha)(1-\varphi\kappa)Y \qquad (7)$$

One can easily check that the model is stock-flow coherent by the following real transaction-flow matrix.

[INSERT TABLE 1 HERE]

Where BK denotes the capital account, and W is the debt of domestic firms which can be held by domestic or foreign households.

The equilibrium on the goods market

Production equals demand which is composed of consumption of the two types of households, public expenditure, investment and balance of trade:

$$Y = C_1 + C_2 + G + I + BC$$

When the hypotheses of stability of investment to GDP and of balance of trade to GDP are integrated, the equilibrium on the goods market can be written as follows:

$$Y \quad = \quad (1\text{-}\tau_1)\alpha(1-\varphi\kappa)\mathrm{Y} + (1\text{-}\mathrm{s})\left[(1-\tau_2)(1-\alpha)(1-\varphi\kappa)Y + z^M \mu B_{-1}\right] + \gamma\mu B_{-1} + G + \kappa Y + \delta Y \qquad (8)$$

With the average tax rate noted $\bar{\tau} = \alpha\tau_1 + (1-\alpha)\tau_2$, the model describing the economy and by which the public debt evolution can be assessed is then entirely determined by the two following equations:

$$Y = \left[(1-\varphi\kappa)(1-\bar{\tau}\text{ - s}(1\text{-}\alpha)(1-\tau_2)) + \kappa + \delta\right]Y + ((1-s)z^M + \gamma)\mu B_{-1} + G \qquad (A)$$

$$B = \left[1 + r^G - \mu(r^M - z^M)\right]B_{-1} + G - \bar{\tau}(1-\varphi\kappa)Y \qquad (B)$$

This very simple model is used to show the mechanisms by which tax cuts, targeted on high incomes, can lead to the rise of the debt to GDP ratio despite their multiplier effect in the three cases mentioned in the introduction of this article.

Case 1: efficient automatic stabilisers

Following a practical hypothesis used by Godley and Lavoie (2007) and Martin (2008) it is assumed that public expenditure is determined at each period so as to reach full employment GDP noted Y^*. The model can hence be written as follows:

$$\begin{cases} G = [1-(1-\varphi\kappa)(1-\bar{\tau}-s(1-\alpha)(1-\tau_2))]Y^* - ((1-s)z^M+\gamma)\mu B_{-1} & \text{(A')} \\ B = B_{-1} + [1-(1-\varphi\kappa)(1-s(1-\alpha)(1-\tau_2))-\kappa-\delta]Y^* + [r^G - \mu(\gamma + r^M - sz^M)]B_{-1} & \text{(B')} \end{cases}$$

Short run

On the short run, i.e. with a given debt stock, GDP is independent of resource allocation and of tax policy as it corresponds, by hypothesis, to full employment GDP. The public deficit, noted *D*, depends only on the saving households tax rate and it is a decreasing function of this tax rate.

$$D = [1-(1-\varphi\kappa)(1-s(1-\alpha)(1-\tau_2))-\kappa-\delta]Y^* + [r^G - \mu(\gamma + r^M - sz^M)]B_{-1}$$

This is so because type 1 households are spending all of their disposable income. Under the automatic stabilisers rule, any increase of taxes on low income households is hence entirely compensated by a public expenditure increase of the same extent.

Public deficit increases also with the propensity to save and/or when the share of consumed national income –by low income households– is decreasing. This is simply the result of standard multiplier: when consumption increases then public expenditure required to maintain full employment is reduced. Lastly, and trivially, the burden of the debt increases the public deficit.

Evolution of the debt/GDP ratio

Assuming that the population growth rate, noted *n*, is constant and that the technology is also constant, then full employment GDP increases at the rate *n* too. The equation B' becomes:

$$\frac{B}{Y^*} = \left[1 + r^G - \mu(\gamma + r^M - sz^M)\right]\frac{B_{-1}}{Y^*} + \left[1 - (1-\varphi\kappa)\big(1 - s(1-\alpha)(1-\tau_2)\big) - \kappa - \delta\right]$$

Writing $\beta = \frac{B}{Y^*}$ the debt/GDP ratio, and knowing that $Y^* = (1+n)Y^*_{-1}$, one can write:

$$\beta = \frac{1 + r^G - \mu(\gamma + r^M - sz^M)}{1+n}\beta_{-1} + \left[1 - (1-\varphi\kappa)\big(1 - s(1-\alpha)(1-\tau_2)\big) - \kappa - \delta\right] \qquad (9)$$

On the short run, the debt ratio decreases with the type 2 tax rate and increases with the national income share of type 1 household.

The debt/GDP ratio converges if and only if:

$$r^G - \mu(\gamma + r^M - sz^M) < n \qquad (10)$$

This condition is valid for realistic values of the parameters and is also much less restrictive than the $r^G < n$ condition which is usually considered. More precisely, using the $r^G < n$ condition amounts to overlook that (1) households saving stocks and received interest are fuelling consumption, (2) interest received by households are taxed and (3) a potential gap between the average interest rate on the public debt and the average yield of households savings can exist.

When condition (10) is verified, the debt/GDP ratio converges towards the long term equilibrium value β_1^* defined as follows:

$$\beta_1^* = \frac{(1+n)\left[1 - (1-\varphi\kappa)\big(1 - s(1-\alpha)(1-\tau_2)\big) - \kappa - \delta\right]}{n - r^G + \mu(\gamma + r^M - sz^M)} \qquad (11)$$

Therefore, on the long run just as on the short run, tax cuts conceded to high income households increase the debt/GDP ratio when public expenditures adjust automatically so as to maintain full employment effectively. On the contrary, targeted tax cuts on low income –as for instance tax exonerations for households under a defined income threshold– have no impact.

Concerning income allocation effect, the long term debt/GDP ratio is an increasing function of the GDP share going to saving households. Through a similar multiplier effect, a rise in the saving propensity and a reduction of the propensity to spend wealth tend also to increase the debt/GDP ratio. This outline corresponds to the idea, developed notably by Alvin Hansen, that private savings "calls for" public debt.

Besides, it is interesting to emphasise that the interest rate impact on debt ratio is not clear-cut. The cost of public debt has indeed a negative impact on growth but, meanwhile, the yield of households' savings is conducive to growth.[8] The difference between public debt cost r^G and private savings yield r^M depends on quite a few heterogeneous elements: the share of public debt in private domestic savings, the level of interest rate abroad, the yield of private domestic assets compared to public bonds etc. The issue is hence more empirical than theoretical. Any way, when $r^G = r^M$ the impact of the interest rate on the debt ratio is positive, which goes along with usual results. This remark holds also for the two other cases examined bellow.

Concerning the role of domestic firms, their investment decreases the debt ratio through a multiplier effect while self-financing, for a given level of investment, increases the ratio as it reduces the share of national income going to households. The multiplier effect of foreign trade balance reduces also the ratio. The lower is the ratio of private domestic savings to public debt, the greater the debt/GDP ratio. Indeed, tax receipts on interests are greater compared to the debt cost.

Autonomous but insufficient public spending to maintain full employment

This section assumes that per capita public expenditure is exogenous and fixed at a sum noted *g* which is too low to maintain full employment. With *N* the size of the population, the model becomes:

$$Y = \left[(1-\varphi\kappa)\left(1-\bar{\tau}\text{ - s}(1\text{ - }\alpha)(1-\tau_2)\right)+\kappa+\delta\right]Y + ((1-s)z^M+\gamma)\mu B_{-1} + gN \qquad \text{(A")}$$

$$B = \left[1+r^G-\mu(r^M-z^M)\right]B_{-1} + gN - \bar{\tau}(1-\varphi\kappa)Y \qquad \text{(B")}$$

Short run

When the size of the population and the debt stock are given, GDP can be determined as follows:

$$Y \quad = \quad \frac{((1-s)z^M+\gamma)\mu B_{-1}+gN}{1-(1-\varphi\kappa)\left(1-\bar{\tau}\text{ - s}(1\text{ - }\alpha)(1-\tau_2)\right)-\kappa-\delta} \qquad (12)$$

This formulation corresponds to standard Keynesian economics. Short run GDP grows with public outlays, and with the GDP share going to type 1 households who are entirely consuming their income. Short run GDP decreases with the propensity to save and with tax rates. Public debt or its counterpart –the wealth of type 2 households– has a positive impact on GDP because, on the one hand, it induces an income –received interest– which increases global consumption and, on the other hand, a fraction of this savings stock feeds directly consumption at each period. In the same vein, the after tax interest rate has a positive impact on short run GDP.

Evolution of the debt/GDP ratio

By integrating equation (12), the debt evolution equation becomes:

$$B = \frac{1-(1-\varphi\kappa)\left(1-s(1-\alpha)(1-\tau_2)\right)-\kappa-\delta}{1-(1-\varphi\kappa)\left(1-\bar{\tau}-s(1-\alpha)(1-\tau_2)\right)-\kappa-\delta}gN + \left(\left(1+r^G-\mu(r^M-z^M)\right)-\frac{\bar{\tau}(1-\varphi\kappa)\left((1-s)z^M+\gamma\right)\mu}{1-(1-\varphi\kappa)\left(1-\bar{\tau}-s(1-\alpha)(1-\tau_2)\right)-\kappa-\delta}\right)B_{-1} \qquad (13)$$

By writing $b=\dfrac{B}{N}$, the per capita public debt ratio, one can obtain:

$$b=\frac{1-(1-\varphi\kappa)\big(1-s(1-\alpha)(1-\tau_2)\big)-\kappa-\delta}{1-(1-\varphi\kappa)\big(1-\bar{\tau}-s(1-\alpha)(1-\tau_2)\big)-\kappa-\delta}g+\left(\frac{1+r^G-\mu(r^M-z^M)}{1+n}-\frac{\bar{\tau}(1-\varphi\kappa)\big((1-s)z^M+\gamma\big)\mu}{(1+n)\big(1-(1-\varphi\kappa)\big(1-\bar{\tau}-s(1-\alpha)(1-\tau_2)\big)-\kappa-\delta\big)}\right)b_{-1} \qquad (14)$$

Per capita public debt stabilises on the long run if

$$r^G-\mu(r^M-z^M)-\frac{\bar{\tau}(1-\varphi\kappa)}{(1+n)\big(1-(1-\varphi\kappa)\big(1-\bar{\tau}-s(1-\alpha)(1-\tau_2)\big)-\kappa-\delta\big)}\mu\big((1-s)z^M+\gamma\big)<n \qquad (15)$$

This condition is more restrictive than the condition (10) which can be rewritten:

$$r^G-\mu(r^M-z^M)-\mu\big((1-s)z^M+\gamma\big)<n \qquad (10')$$

In both cases, a sufficient condition is that $r^G-\mu(r^M-z^M)<n$. In the specific case where $\mu=1$ and $r^G=r^M$, a sufficient condition for convergence is thus that the real after tax interest rate is lower than the growth rate.

When the condition (15) holds, the debt per capita converges towards its fix point defined by:

$$b^*=\frac{(1+n)\big(1-(1-\varphi\kappa)\big(1-s(1-\alpha)(1-\tau_2)\big)-\kappa-\delta\big)}{\big(n-r^G+\mu(r^M-z^M)\big)\big(1-(1-\varphi\kappa)\big(1-\bar{\tau}-s(1-\alpha)(1-\tau_2)\big)-\kappa-\delta\big)+\bar{\tau}(1-\varphi\kappa)\big((1-s)z^M+\gamma\big)\mu}g \qquad (16)$$

Per capita GDP noted *y* converges then towards its long run value y^* defined by:

$$y^*=\frac{n-r^G+\mu\big(\gamma+r^M-sz^M\big)}{\big(n-r^G+\mu(r^M-z^M)\big)\big(1-(1-\varphi\kappa)\big(1-\bar{\tau}-s(1-\alpha)(1-\tau_2)\big)-\kappa-\delta\big)+\bar{\tau}(1-\varphi\kappa)\big((1-s)z^M+\gamma\big)\mu}g \qquad (17)$$

The first result is that, in the long run, public debt and GDP grow at the same rate as population, which implies notably a constant unemployment rate. This result could be generalised by supposing that public expenditures grow at a constant rate but different from *n*. Under this hypothesis, public debt and GDP would then grow at the same rate as public outlays. In this framework, autonomous public expenditures are actually the driving force of growth.

Per capita public debt increases of course with public spending through deficits accumulation. It grows also along with the propensity to save and with the GDP share going to type 2 households: both reduce GDP and tax receipts. Lastly, per capita public debt decreases with both tax rates as they have a direct influence on tax receipts.

Long run per capita GDP is an increasing function of public outlays and, of the share of GDP going to type 1 households and, of the propensity to spend wealth through a standard multiplier effect. But it is also an increasing function of the after tax interest rate as it is a source of income and consumption. Per capita GDP is decreasing with both tax rates and with propensity to save.

Altogether, tax cuts increase both long run per capita GDP and public debt. Their impact on the debt/GDP ratio is *a priori* undetermined and depends on the size of their multiplier effect.

The long term debt/GDP ratio can be easily determined from (16) and (17):

$$\beta_2^* = \frac{(1+n)\left[1-(1-\varphi\kappa)\left(1-s(1-\alpha)(1-\tau_2)\right)-\kappa-\delta\right]}{n-r^G+\mu(\gamma+r^M-sz^M)} \qquad (18)$$

On the long run, this ratio is independent of per capita public expenditures but it can be reduced by an increase of taxes on high income households, who are saving a share of the wages, profits and interests they receive. Contrary to the standard idea that taxes should be cut to stimulate the activity, and eventually public spending reduced to limit public deficits, the model indicates that tax cuts increase public debt relatively to GDP, in other words: whatever the level of public spending, a reduction of the tax rate on high income households increases the debt to GDP ratio.[7]

Note that the long run debt/GDP ratio is the same here as in the previous case where the objective of fiscal policy was to maintain full employment:

$$\beta_2^* = \beta_1^*$$

The dominant idea that ambitious growth support by public spending policies should be abandoned is not advocated by this model, which emphasises on the contrary that the pursuit of full employment through public expenditure has no long term effect on the relative weight of public debt. Any government that intends to improve both the growth level –and hence employment– and the debt to GDP ratio should both increase public outlays and taxes on high income households. Investment and its self-financing, foreign trade, and domestic savings to public debt ratio have the same impact on the debt/GDP ratio than in case 1.

"Maastricht" (Stability and Growth Pact –GSP) or pro-cyclical deficit case

Let's imagine that public expenditure adjusts to private spending so as to ensure a target value of the public deficit to GDP ratio, noted $\frac{D}{Y} = d^*$. Public spending is then specified by the relation bellow:

$$d^* = \frac{G + \left(r^G - \mu(r^M - z^M)\right)B_{-1} - \bar{\tau}(1-\varphi\kappa)Y}{Y} \Leftrightarrow G = (d^* + \bar{\tau}(1-\varphi\kappa))Y - \left(r^G - \mu(r^M - z^M)\right)B_{-1} \qquad (19)$$

Integrating the definition (18) into the goods market equilibrium equation (A) gives the following model:

$$Y = \frac{\mu\left(\gamma + r^M - sz^M\right) - r^G}{1 - \left[1 - s(1-\alpha)(1-\tau_2)\right](1-\varphi\kappa) - \kappa - \delta - d^*} B_{-1} \qquad \text{(A')}$$

$$B = d^* Y + B_{-1} \qquad \text{(B')}$$

Short run

With a given debt stock, any reduction in the tax rate on high income households reduces GDP. This result can appear as counter intuitive because a lower tax rate encourages consumption. But in the present case, any tax cut forces the government to reduce also public expenditures which produces a greater negative impact than the positive effect of the tax cut because it goes only partially to consumption. It must be noted that a targeted cut in the low income households tax rate, which would leave unchanged the tax rate on high income households, would have no impact on the GDP; the reduction of public expenditure required

to meet the target deficit criteria would indeed exactly compensate for the effect of tax cut on consumption. For this reason, the tax rate on low income households has no impact on GDP.
On the contrary, any rise in the deficit to GDP target ratio increases GDP as it enables the government to increase its outlays without changing the tax rates.

Evolution of the debt/GDP ratio

Integrating (A') into (B') leads to the equation of debt evolution:

$$B=\left(1+d^{*}\frac{\mu\left(\gamma+r^{M}-sz^{M}\right)-r^{G}}{1-\left[1-\mathrm{s}(1-\alpha)(1-\tau_{2})\right](1-\varphi\kappa)-\kappa-\delta-d^{*}}\right)B_{-1} \qquad (20)$$

It appears that the debt dynamics is explosive. Since the output level is proportionnal to the level of public debt inherited from the previous period, this explosive dynamics is reflected in the GDP one, which implies that in this framework, GDP increase is led by the debt as in Gehrels (1957). More precisely, public debt and GDP grow at the rate ρ defined by:

$$\rho=d^{*}\frac{\mu\left(\gamma+r^{M}-sz^{M}\right)-r^{G}}{1-\left[1-\mathrm{s}(1-\alpha)(1-\tau_{2})\right](1-\varphi\kappa)-\kappa-\delta-d^{*}} \qquad (21)$$

This rate is smaller than the population growth rate when the target deficit ratio is too low, more precisely when:

$$d^{*}<\frac{n\left\{1-\left[1-\mathrm{s}(1-\alpha)(1-\tau_{2})\right](1-\varphi\kappa)-\kappa-\delta\right\}}{n-r^{G}+\mu\left(\gamma+r^{M}-sz^{M}\right)}$$

When the government adjusts public expenditures in order to respect a given deficit to GDP ratio, the weakness of the growth rate of the economy relatively to the population growth is all the more likely to happen that the tax rate on high incomes is low.
Besides those arguments, the growth rate of the economy increases with the ratio of domestic private savings to public debt because tax receipts on perceived interests by domestic households reduce the debt cost. At the same time, growth is driven by a rise in the GDP share of low income households, investment and balance of trade through a multiplier

mechanism. On the other hand, for a given level of investment, a rising share of self-financed investment reduces the share of revenue going back to households which reduces growth.

As GDP and public debt grow at the same rate, the debt/GDP ratio is constant:

$$\beta_3^* = d^* + \frac{1 - \left[1 - s(1-\alpha)(1-\tau_2)\right](1-\varphi\kappa) - \kappa - \delta - d^*}{\mu\left(\gamma + r^M - sz^M\right) - r^G} \quad (22)$$

A reduction in the tax rate on high income households increases the debt to GDP ratio. In the "Maastricht" framework, any tax cut implies to reduce public spending: the multiplier effect of tax cuts is insufficient to compensate the induced expenditure cut. Note that this result holds in a model with a single type of agents saving a share of their income ($\alpha = 0$), which means that the weakness of the tax cut multiplier is also true when allocation effects are ignored; allocation effects simply amplify this phenomenon.

In accordance with the multiplier, the debt/GDP ratio diminishes with the share of GDP going to households consuming all their income; it shrinks also with the domestic savings/public debt ratio and with the shares of investment and trade balance in the GDP. On the other hand, for a given level of investment, the debt ratio increases with self-financing investment as it reduces the income share going to households. The debt/GDP ratio rises also with the debt cost but decreases with the yield of domestic savings. Lastly and less intuitively, any increase of the targeted deficit/GDP ratio would reduce the debt/GDP ratio as it would enable the government to increase public expenditure and hence to encourage growth. In other words, the tight deficit target ratio is not steadily compatible with a debt to GDP target ratio of the "Maastricht" framework, as emphasised also by Schlicht (2006) with a different type of model.

Concluding discussion

The SFC model presented in this paper is able to reproduce the main elements of the standard Keynesian analysis of public debt based on the idea that the multiplier effect of fiscal policy offsets the growth of public debt in absolute value. It attempted to examine more accurately the differentiated effects of public outlays and tax receipts. The tax receipts side of the model has been slightly more developed than usual through the integration of progressive taxation among two types of agents and through the role of national income allocation. The monetary and financial aspects of the economy are exogenous to the analysis but they are not ignored; these aspects require further special developments. Three cases have been examined: 1/ automatic stabilisers are efficient: public spending is supposed to adjust according to private spending so as to maintain full employment; 2/ public spending is autonomous but insufficient to maintain full employment; 3/ "Maastricht" or pro-cyclical deficit case: public spending adjusts to private spending so as to ensure a target value of the public deficit to GDP ratio.

The main general result is that the debt/GDP ratio always stabilises on the long run for realistic values of the parameters. More precisely, the case 1 of this model meets up with Godley and Lavoie's (2007) "surprising result" obtained by simulation and determines explicitly a possible solution. They established that the real rate of interest has not necessarily to be lower than the real rate of growth to stabilise the debt ratio. The present model specifies this argument in equation (10), which condition is valid for realistic values of the parameters. The usual condition requiring that the real interest rate has to be lower than the growth rate overlooks indeed that (1) households' saving stocks and received interest are fuelling consumption, (2) interest received by households are taxed and (3) a potential gap between the average interest rate on the public debt and the average yield of households savings can exist. In case 2, the evolution of the debt ratio cannot be directly examined. For this reason, the per capita public debt is analysed. The equation (15) gives the condition for per capita debt convergence which is more restrictive than in case 1 but the real rate of interest has not anyway to be lower than the real rate of growth to stabilise per capita debt and thus also the debt/GDP ratio. In case 3, there is no convergence problem as the debt/GDP ratio is constant.

The second and probably most original outcome of this model is that public spending and taxes have not a similar effect on the debt/GDP ratio. Public expenditures have a strong influence on growth but no impact on the long run debt/GDP ratio. On the contrary, tax cuts or tax increases have less consequence on growth but contribute decisively to determine the level of debt/GDP ratio.

A third general result is that, in all examined cases, tax cuts (or increases) conceded (imposed) to high income households give rise to an increase (fall) in the long term debt/GDP ratio whereas targeted tax cuts (or increases) on low income households have no effect what so ever on the ratio. When public outlays are autonomous, tax cuts on high income households have the usual Keynesian positive impact on short run GDP whereas under the GSP constraint they reduce GDP in the short run because they compel government to reduce also its expenditures.

Fourth result, in all cases, the debt/GDP ratio is an increasing function of the national income going to saving households; in other words, all things being unchanged, an increasing debt/GDP ratio can be interpreted as an expression of the rise in income inequality in the society. As inequalities have a macroeconomic impact through savings behaviour, a predictable corollary of the previous result is that higher saving propensity leads to higher debt/GDP ratio, *i.e.* private savings "calls for" public debt.

The first case, exploring the conditions for efficient automatic stabilisers, gives similar results as Godley and Lavoie (2007) and Martin (2008) and displays the same long term debt/GDP ratio as in the second case where public expenditure is autonomous but insufficient to maintain full employment. The second case represents however the most usual features of the Keynesian analysis, and it is particularly appropriate to study per capita public debt and long run GDP. Per capita public debt increases with deficits accumulation and decreases with both tax rates. Similarly, both tax rates also influence negatively per capita GDP whereas only the type 2 households' tax rate has an influence on the debt/GDP ratio. As the after tax interest rate is a source of income and consumption, it influences positively GDP per head. Lastly, public spending appears as the driving force of long run per capita GDP.

The third case, where public expenditure adjusts to private spending so as to ensure a target value of the public deficit/GDP ratio, gives even more original insights. On the short run, it shows the following counter intuitive result: a lower tax rate on high incomes reduces GDP. As the deficit target ratio defined by the GSP has indeed to be respected, a reduction in tax receipts forces the government to reduce also its outlays which produces a greater negative impact on GDP than the positive one resulting of the initial tax cut. On the other hand, a rise in the deficit/GDP target ratio gives leeway to the government to increase its outlays without changing tax rates which has a positive impact on GDP. The long term analysis of the debt/GDP ratio under GSP shows another original result: if the target deficit ratio is too low then the growth rate of the activity can be lower than the population growth. In particular, this lack of economic expansion can be the result of an insufficient tax rate on high income households. Besides, as in the previous cases, the debt/GDP ratio depends negatively on the tax rate on high incomes. Noteworthy and less intuitive, the last result is that an increase in the target deficit ratio reduces the debt ratio because it enables the government to raise public outlays which stimulates growth. This result concurs with Schlicht (2006), through a different model, on the internal contradictions of the GSP which places contradictory limits to public finances and hence creates macroeconomic instability.

The ongoing so called “public finance crisis” has a close relation to the main arguments presented here. The analysis of the European puzzle through this model remains to be done but the aim of this paper is also to understand public debt dynamics under “normal” functioning conditions. Lastly, this paper has focused only on the direct impact of high income households tax cuts on public debt/GDP ratio nevertheless, some insights can be drawn about their indirect effect on the debt ratio through several channels considered as exogenous in this model. For example, tax cuts can be interpreted by the "markets" as a higher default risk which can induce a higher interest rate on public debt. This indirect effect would then worsen the initial increase of the debt ratio. The same is true of the indirect impact the balance of trade: lowering taxes on high incomes can increase importations which would then increase the debt ratio even more.

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Appendix A : notation

Y = real GDP

C_1 = real consumption of type 1 households

C_2 = real consumption of type 2 households

I = investment with $I = \kappa Y$

φ = share of self-financed investment (GDP – wages – dividends = φI)

BC = balance of trade with $BC = \delta Y$

BK = capital account without interests flows

G = real government expenditure

V = real domestic private wealth (savings stock)

B = public debt (bonds) measured in real term with $\mu = V/B$

W = debt of domestic firms measured in real terms

α = share of type 1 households in national income

r = real interest rate

i = nominal interest rate

τ_i = tax rate on income of type i households, for $i=1,2$

s = propensity to save private revenue (only type 2 households)

γ = share of wealth used for consumption (only type 2 households)

P = price level

π = inflation rate

n = population growth rate

i^G / r^G = average nominal/real interest rate on public debt

i^M / r^M = average nominal/real interest rate on domestic savings

z^M = real after tax interest rate on domestic savings

Appendix B: composition of type 2 households' savings

Domestic households' savings is composed of public bonds (V^P), of assets issued by domestic firms to finance a fraction of their investment (V^E) and of assets issued abroad (V^F).

We assume that the savings composition is stable: $V^P = a_P V$, $V^E = a_E V$ and $V^F = a_F V$.

Public debt is held both by domestic households ($B^N = V^P$) and by the rest of the world (B^F), its allocation is supposed to be stable and also exogenous: $B^N = \theta B$ and $B^F = (1-\theta)B$.

We can deduce from this that: $V = \dfrac{V^P}{a_P} = \dfrac{\theta}{a_P} B \Rightarrow V = \mu B \; with \; \mu = \dfrac{\theta}{a_P}$.

The interest rates of those different assets are noted i^P, i^E and i^F, and i^* is the rate at which the international capital market lends to domestic government.

The yield of domestic private savings is hence $i^M = (a_P i^P + a_E i^E + a_F i^F)$. The real interest rate on domestic savings r^M and the real after tax interest rate on domestic savings z^M are respectively noted $r^M = \dfrac{i^M - \pi}{1+\pi}$ and $z^M = \dfrac{(1-\tau)i^M - \pi}{1+\pi}$.

Notes

1 The drawbacks of fiscal policy are often designed as various types of "crowding out" effects, see Arestis and Sawyer (2004).

2 Note that doubts about tax cuts and their alleged "trickle down effect" are also expressed in a neoclassical framework by Yang (2007) about capital income tax cuts.

3 See appendix for a list of all notations.

4 See appendix for more precisions about the composition of households' savings and public debt financing.

5 The unemployment benefits correspond to an insurance mechanism, as for instance in Germany and in France, which explains why it is not included in public spending or in households' income. Besides, the stabilizing effect of labour "solidarity" or aid outlays is neglected here.

6 On the issue of *current* versus *lifetime* hypothesis about disposable income, see Martin (2008), p. 652.

7 Godley and Rowthorn (1994) obtain the same result with a unique tax rate.

8 Note that $r^M - sz^M = \frac{(1-s(1-\tau_2))i^M - (1-s)\pi}{1+\pi}$.

Table 1: transaction-flow matrix

	Type 1 households	Type 2 Households	Firms	Government	Foreign sector	Sum
Exchanges on the goods market	$-C_1$	$-C_2$	$Y-I$	$-G$	$-BC$	0
GDP income distribution	$\alpha(Y-\varphi I)$	$(1-\alpha)(Y-\varphi I)$	$-(Y-\varphi I)$			0
Interests		$r^M V_{-1}$	$-r^E W_{-1}$	$-r^G B_{-1}$	$-r^F\left[V_{-1}^F-(W_{-1}-V_{-1}^E)-B^F\right]$	0
Taxes	$-\tau_1\alpha(Y-\varphi I)$	$-\tau_2(1-\alpha)(Y-\varphi I)$ $-\tau_2\frac{i^M}{1+\pi}V_{-1}$		$\left[\tau_1\alpha+\tau_2(1-\alpha)\right](Y-\varphi I)$ $+\tau_2\frac{i^M}{1+\pi}V_{-1}$		0
Wealth/Debt variation		$-\Delta V$	ΔW	ΔB	$-BK$	0
Sum	0	0	0	0	0	