The Keynesian Multiplier reconsidered*

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Abstract. The aim of this paper is to contribute to the theoretical discussion on the Keynesian multiplier, particularly regarding its nexus to endogenous money. Basil Moore, as a precursor of the endogenous money approach, declared the “demise of the Keynesian multiplier” due to differences in method between these two concepts. This paper analyzes Moore’s critique of the standard multiplier and the reactions it provoked. Additionally, it reviews Moore’s alternative approach, where the income velocity of money replaces the traditional multiplier. We argue that both concepts are incomplete. Nonetheless they can benefit from each other. Their useful properties may be integrated to a new version of the multiplier drawing on a time component and the reflux principle in an endogenous credit-money economy.

Keywords. Keynesian multiplier; velocity of money; endogenous money; reflux principle; credit cycles

JEL classification. B22, E12, E51, E62

1 Introduction

In several writings, Basil Moore (1988, 1994, 2006, 2008) pointed out that two basic principles of Post Keynesian Economics—endogenous credit money and the traditional multiplier analysis—are at odds concerning the role and the creation of savings. Moore, as a precursor of the endogenous money approach, therefore declared the “demise of the Keynesian multiplier.” Allin Cottrell (1994), in a reply to Moore, defended the multiplier. He argued that Moore left out important adjustment processes and that Moore’s conundrum about the identity of investment

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and savings ‘was surely resolved to most people’s satisfaction by the 1940s’ (Cottrell 1994: 113). Paul Dalziel (1996) later joined the debate to clarify Cottrell’s points.

The rough statement of this paper is in line with Gnos (2008: 181) (albeit for different reasons): ‘Moore’s criticism of the multiplier is well grounded, although it is not the end of the multiplier story.’ There are indeed inconsistencies in the standard explanation of the multiplier. Nevertheless, this does not annul the multiplier as such, if it is explained more carefully. It simply points out the need for a sound derivation that is in line with the nature of a credit money economy. Ironically enough, Moore himself made a proposal, relating the multiplier to the income velocity of money (Moore 2006: 376), yet, the approach was little-noticed in literature. This paper aims to examine Moore’s arguments concerning the standard multiplier and his alternative view. It shows that both concepts can benefit from each other.

The paper is organized as follows: The next section analyzes the key elements of the different multiplier concepts as deduced from the General Theory. It reviews and augments Moore’s critique of these concepts. The third section deals with Moore’s alternative multiplier which is related to the income velocity of money. Section four elaborates on leakages in the circuit that are absent in Moore’s concept. Section five combines these findings in a more general view on the multiplier. The final section concludes.

A short note on assumptions: Throughout this paper we consider a closed economy with underemployed resources. Inflation is mainly determined by unit costs outside the model; it is not a monetary phenomenon.

2 Two versions of the standard multiplier

Keynes considered the two components of aggregate demand in a closed economy—consumption ($C$) and investment ($I$)—to be driven by completely different forces. Investment demand is very volatile and thus the ultimate cause of economic fluctuations:

‘The theory can be summed up by saying that, given the psychology of the public, the level of output and employment as a whole depends on the amount of investment. I put it in this way, not because this is the only factor on which aggregate output depends, but because it is usual in a complex system to regard as the *causa causans* that factor which is most prone to sudden and wide fluctuation. More comprehensively, aggregate output depends on the propensity to hoard, on the policy of the monetary authority as it affects the quantity of money, on the state of confidence concerning the prospective yield of capital-assets,'
on the propensity to spend and on the social factors which influence the level of the money-wage. But of these several factors it is those which determine the rate of investment which are most unreliable, since it is they which are influenced by our views of the future about which we know so little.’
(Keynes 1937c: 221)

However, the demand for consumption goods, is given by a rather stable propensity to consume \( c = 1 - s \) out of disposable income. The multiplier formula derives from the following identities:

\[
Y \equiv C + I \quad (1)
\]
\[
Y \equiv C + S \quad (2)
\]
\[
S \equiv sY \quad (3)
\]

Substituting (3) into (2) yields

\[
C \equiv (1 - s)Y \quad (4)
\]
\[
Y \equiv \frac{1}{s}I \quad (5)
\]
\[
\Delta Y \equiv \frac{1}{s} \Delta I \quad (6)
\]

Considering \( \Delta I \) to be exogenously determined and \( s \) to be a behavioral parameter makes (6) a behavioral equation

\[
\Delta Y = \frac{1}{s} \Delta I \quad (7)
\]

where \( 1/s \) is the investment multiplier, typically denoted as \( k \).

While the multiplier formula is quite clear, the process behind is ambiguous right from the start. There are at least two different versions that have been developed out of Keynes’ works. In order to assess Moore’s criticism, we will separately investigate the textbook version of the multiplier, which we will call the *series multiplier*, and Keynes’ so called *logical multiplier* on the other hand.

**The series multiplier**

The version of the *series multiplier*, that was particularly proclaimed by Hansen (1953: 108), has found its way into nearly every macroeconomic textbook. It describes a chain of events in logical time, where an initial increase in investment in the “first round” generates additional income, which is partly spent for consumption and partly hoarded in the “second round”. The spent part induces
additional income that, again, is spent and saved in a certain proportion in “round number three”, and so on. This ‘converging series of ever diminishing waves of expenditures’ (Meade 1975: 84) yields:

\[ \Delta Y = (1 + c + c^2 + c^3 + ...) \Delta I = \frac{1}{s} \Delta I \quad \text{with} \quad 0 < c, s < 1 \]  

(8)

The process stops as soon as additional savings equal additional investment. It is the increasing income that adjusts planned savings step by step to planned investment. In other words: The lower the saving rate \( s \), the more income is generated before the capital market is in equilibrium again.

The great success of the series multiplier presumably arises from the underlying familiar process of receipts, spending and saving. This characteristic in accordance with the resulting simple formula gives it a great didactic appeal. Nonetheless, if it is supposed to have practical relevance, the model should coherently abstract from reality. Moore (1988, 1994, 2006, 2008) states that this is not the case for several reasons, that cannot be discussed at length here. Particularly, the paper does not intend to discuss whether comparative static modeling by itself is useful or realistic. Rather, the scope is to work out inconsistencies in the line of argument. Thus, we will only focus on one crucial point of the Cottrell-Moore debate, that has not sufficiently been made clear: For Cottrell

‘... “planned” or “intended” (or ex ante) \( I \) and \( S \) are not identical, and must be brought into equality by means of some mechanism or other ...’ (Cottrell 1994: 113-4) (original emphases)

In order to defend the series multiplier, this mechanism has to be discovered. The mere technical explanation by which comparative static analysis necessarily involves a stability condition is not convincing. So, why should volitional decisions of investors and savers adjust, such that \( S = I \)? Keynes provides a puzzling answer:

‘An increment of investment in terms of wage-units cannot occur unless the public are prepared to increase their savings in terms of wage-units. Ordinarily speaking, the public will not do this unless their aggregate income in terms of wage-units is increasing. Thus their effort to consume a part of their increased incomes will stimulate output until the new level [...] of incomes provides a margin of saving sufficient to correspond to the increased investment. The multiplier tells us by how much their employment has to be increased to yield an increase in real income sufficient to induce them to do the necessary extra saving ...’

Keynes (1936: 117) (emphases added; S.G.)
The passage points out the role of savings as a means of finance. Following this rationale, savings fund the initial investment, forcing them to be equal. The attempt persists in more recent Keynesian literature. For instance, Kahn (by citing Warming (1932: 215-6)) affirms that

‘He [Warming; S.G.] pointed out that it is the extra saving made out of increased income which is ‘the real source of investment’; ‘the secondary employment must continue until the total created income causes so much saving that the original investment can be paid.’

(Kahn 1984: 101)

Bailly, in his critique of the series multiplier points out:

‘In the equation \( \Delta Y = \Delta C + \Delta I \), \( \Delta I \) represents the production of investment goods, but not their purchase. The proof of this is that the income multiplication mechanism is assumed to unfold for so long as \( \Delta S < \Delta I \), that is, throughout the time when the economy is still without the financial resources for purchasing the investment goods produced.’

(Bailly 2008: 141) (original emphases)

This explanation is completely un-Keynesian, a recurrence to Say’s Law, but in fact it is very common. From that point of view, supply governs demand: At first, someone produces a capital good without facing any effective demand for it. The public still cannot afford it as long as \( \Delta S < \Delta I \). Unlike the public, the producer of the capital good is (fortunately) not constrained by a lack of savings. He pays wages that workers partly save and partly spend on consumption goods. Spending induces further income within the consumption goods sector, which is partly saved and partly spent as well. The public saves exactly as much as required for someone to eventually afford the capital good initially produced. At this point the process interrupts at once.

By trying to find a suitable model for the plausible chain of spending and income generation, the causality of \( I \) and \( S \) has been turned upside down. The multiplier is considered to explain how investment is made possible by savings, while the core of the General Theory was meant to explain that investment governs savings.¹ Keynes himself overcame this contradiction in his post-GT writings:

‘Planned investment—i.e. investment *ex-ante*—may have to secure its “financial provision” *before* the investment takes place; that is to say, before the corresponding saving has taken place. It is, so to speak, as though a particular piece of saving had to be earmarked against a

¹See Trigg (2003) for a further discussion.
particular piece of investment before either has occurred, before it is known who is going to do the particular piece of saving, and by someone who is not going to do the saving himself. [...] This service may be provided either by the new issue market or by the banks;—which it is, makes no difference. [...] But ‘finance’ has nothing to do with saving. At the ‘financial’ stage of the proceedings no net saving has taken place on anyone’s part, just as there has been no net investment. ‘Finance’ and ‘commitments to finance’ are mere credit and debit book entries, which allow entrepreneurs to go ahead with assurance.’
(Keynes 1937a: 246-7) (original emphases)

‘There is, however, no such necessity for individuals to decide, contemporaneously with the investment-decisions of the entrepreneurs, how much of their future income they are going to save. To begin with, they do not know what their incomes are going to be, especially if they arise out of profit. But even if they form some preliminary opinion on the matter, in the first place they are under no necessity to make a definite decision (as the investors have to do), in the second place they do not make it at the same time, and in the third place they most undoubtedly do not, as a rule, deplete their existing cash well ahead of their receiving the incomes out of which they propose to save, so as to oblige the investors with “finance” at the date when the latter require to be arranging it. Finally, even if they were prepared to borrow against their prospective savings, additional cash could not become available in this way except as a result of a change of banking policy. Surely nothing is more certain than that the credit or “finance” required by ex-ante investment is not mainly supplied by ex-ante saving. [...] The ex-ante saver has no cash, but it is cash which the ex-ante investor requires. On the contrary, the finance required during the interregnum between the intention to invest and its achievement is mainly supplied by specialists, in particular by the banks, which organise and manage a revolving fund of liquid finance. For “finance” is essentially a revolving fund. It employs no savings.’
(Keynes 1937b: 664-6)

Following the endogenous money approach (that heavily draws on these articles by Keynes), finance does not stem from savings. Financial resources for investment are usually provided by banks that create credit ex nihilo. When a loan is granted, a debt and a deposit occur concurrently at the borrowers’ account. Once the borrower spends the money on newly produced capital goods, the producer receives deposits, which can be considered as temporary savings. If they are spent later on,
someone else earns and temporarily saves them. Thus, finance creates savings and not the other way round. If finance does not require savings, there is no market constraint for volitional savings to be on par with investment. If you accept the endogenous money approach, then you must reject the notion that savings finance investment.

A second look at the above citation of Keynes reveals another possible solution. One could link that passage to Keynes’ elaborations on the relation between savings and investment (Keynes 1936: 64). In this respect, the multiplier is the mechanism that brings about the necessary ex post identity of both terms. Dealing with this solution requires a short detour on different meanings of ex ante and ex post investment and saving in economics, because these concepts have been subject to confusion with regard to the multiplier. The puzzle boils down to the following: ‘Altho $S$ equals $I$ by definition, Keynes holds, at the same time, that the multiplier makes them equal.’ (Lutz 1938: 608) The root of the matter, as will be shown, is the double assignment of ex ante and ex post (i) for national accounts, and (ii) for comparative static modeling.

(i) In terms of national accounting, where historical time matters, ex post and ex ante are by-words for *actual* and *volitional*. Ex post (i.e. for any period of time, be it a year or a day) the flow of savings equals the flow of investment in a closed economy. These actual (ex post) flows of investment and saving comprise both volitional (ex ante) and non-volitional fractions. While actual savings must be identical to actual investment, there is no such constraint for volitional savings and investment. They may be equal or not, and there is neither a tendency, nor a mechanism towards equality. If volitional investment exceeds (falls short of) volitional saving, there will simply be a remainder of non-volitional saving (non-volitional investment). This is the way Moore (2006) uses these terms. He rejects the notion of a behavioral saving function. Investment, that is capital accumulation, plays the active part and actual saving is simply the accounting record of investment. Just as loans create deposits, an additional (credit-financed) investment creates additional savings, be they volitional or non-volitional: $S \equiv I$.

(ii) In terms of comparative statics only volitional acts matter. Based on logical time, ex ante and ex post mean *before* and *after* the system has adjusted to a shock. This is similar to *out of equilibrium* and *back in equilibrium*. Applied to the series multiplier: when volitional investment rises, ex ante investment is higher than ex ante savings. The system is out of equilibrium. This stirs up an adjustment process of volitional decisions to spend and save (behavioral saving function), which rises savings until they equal investment ex post. The system strives for equilibrium, where $S = I$.

Let us put things together again: If our question “Why should the decisions of investors and savers equilibrate?” is answered by “Because they have to be identical ex post!”, then this would essentially mean: Volitional saving must adjust
to volitional investment in equilibrium \((S = I)\) because actual saving and actual investment are necessarily identical\((S \equiv I)\). Clearly, this is a mix-up of the two different meanings of ex post. No law-like equality of volitional savings and investment can be derived from the identity of actual savings and investment. And even if that closure were feasible, the length of the adjustment process would be completely arbitrary, since \(S \equiv I\) for any considered period—even a single day. Moreover, a contradiction would occur: if we would suppose the multiplier to work out fully within one year, then after half a year we would still have \(S < I\), despite \(S \equiv I\) applies to half-year accounting as well. Thus, the reasoning that ex post identity arises from equilibration of ex ante terms, turns out misleading.

We should sum up our findings so far: The multiplier process

- does not show how investment is financed through savings.
- does not make volitional savings and investment end up as identical actual savings and investment.

This is also Moore’s central assault to the multiplier.\(^2\) It renders two common explanations void. But in fact it is not the whole story. Both Cottrell (1994) and Dalziel (1996) proposed alternative explanations. Cottrell draws on Keynes’ logical multiplier, which will be discussed later on. Dalziel draws on process analysis of the series multiplier.

Dalziel joined the Cottrell-Moore debate in 1996. He carefully pointed out how volitional savings could end up equal to volitional investment. Initial investment generates income of the same amount:

\[
\Delta I \equiv \Delta Y_0
\]  

(9)

The income of one round \(r > 0\) is used for consumption and volitional saving in the next round \(r + 1\):

\[
\Delta Y_r \equiv \Delta C_{r+1} + \Delta S_{r+1}
\]  

(10)

Expenditures on consumption in one round generates income of the same amount:

\[
\Delta C_r \equiv \Delta Y_r
\]  

(11)

‘Process analysis makes clear that the result is not some quirk of the underlying mathematics, or a matter of choice about assumed equilibrating mechanisms (interest rate or real income), but is the inevitable outcome of two very simple economic identities: expenditure equals income and income equals consumption plus saving.’

(Dalziel 1996: 318)

\(^2\)Besides his general notion on the flaws of equilibrium analysis, which we do not discuss here.
Of course these identities themselves tell us nothing about how the process will come to an end, even if Dalziel argues the converse:

‘Hence, it is possible to reject as an empirical matter any behavioral hypothesis about saving decisions without affecting the validity of the multiplier theory.’

(Dalziel 1996: 326)

However, equation (10) must be supplemented by a behavioral function that determines the proportion of $\Delta C_{r+1}$ to $\Delta S_{r+1}$. Otherwise volitional savings could be even negative for some rounds whereby the process might never stop. As Keynes put it:

‘My theory itself does not require my so-called psychological law as a premise. What the theory shows is that if the psychological law is not fulfilled, then we have a condition of complete instability.’

(Keynes 1973: 276)

Only if saving behavior secures a positive (but not complete) leak out of the circuit the process will eventually yield

$$\Delta I = \Delta S = \sum_{i=1}^{R} \Delta S_i.$$  \hspace{1cm} (12)

If we accept a positive leakage, e.g. a stable propensity to save, then Dalziel (1996) presents a solution that does not mix up the different ex post concepts or consider savings as finance. However, in order to calculate the multiplier two caveats apply, that are central to the remainder of this paper.

The first one concerns the role of savings: What do they represent—a growing fund or a leakage? In the first case they may be used for other purposes as well, which possibly entails an unending multiplication (Bradley 2008: 97). In the second case they are a cul-de-sac (Kahn 1931: 189), which is never disposable again. They can’t be both. We will discuss a reformulation of savings in section four.

The second caveat concerns moving from “rounds” to concrete time intervals. We do not know how long one round actually takes. Since the multiplier is a dimensionless variable, the adjustment period is undefined. Will it take a month or a decade until a multiplier of, say, 4 has come into effect? The answer can only be given outside the model which makes identifying concrete multiplier effects arbitrary. A time dependent multiplier will be subject to section three.

But before proceeding, we should turn to Cottrell’s suggestion to solve Moore’s $I-S$ conundrum. Cottrell refers to another version of the multiplier, namely, Keynes’ logical multiplier.
The logical multiplier

Keynes dropped the process analysis method when he developed his

‘... logical theory of the multiplier, which holds good continuously, without time-lag, at all moments of time [...] in every interval of time the theory of the multiplier holds good in the sense that the increment of aggregate demand is equal to the increment of aggregate investment multiplied by the marginal propensity to consume.’
(Keynes 1936: 122-3)

The logical multiplier refers to an increase of employment in the capital goods sector that forces the consumption goods sector to expect an increase in demand on a scale determined by the long run marginal propensity to consume (MPC) out of current income. The concept is static. It does not explain how savings gradually grow until they equal the change in investment. It does not refer to volitional savings at all. Savings appear simultaneously with investment, whereby the logical multiplier does not mix up those two ex post concepts described above. Moreover, savings are not needed to finance investment, whereby the logical multiplier in no way runs counter to the endogenous money approach (Gnos 2008: 191). Thus, the logical multiplier might remedy the difference in method between endogenous money and the multiplier principle. But actually, this is a trick. The contradictions of the series multiplier are merely skipped within a purely static framework. The logical multiplier does not model how the multiplier process unfolds, it simply shows its result. Keynes was aware that real processes take gradual effect and need an interval of time to unfold. Keynes (1936: 122-3) and other authors (Cottrell 1994; Bailly 2008) suppose two causes for this—either consumers do not instantly adjust their spending behavior to their new level of income, or the producers of consumption goods have not foreseen the increment in demand and need some time to increase production. Yet, the effect is the same:

‘... a temporary departure of the marginal propensity to consume away from its normal value, followed, however, by a gradual return to it.

3There is a third version of the multiplier, developed by Hartwig (2004, 2008), which closely relates to the logical multiplier. His structural multiplier is based on Marxian reproduction schemes in a model with two departments—one producing capital goods, the other producing consumption goods. ‘It designates the proportion of department II relative to department I that is necessary for completely successful reproduction and that must be sustained in an expanding economy to ensure the identity between saving and investment’ (Hartwig 2008: 23). With the structural multiplier $c/(1 - c)$ the entrepreneurs of department II have a guideline on how much to increase their production when they face a certain increase of production in department I. Without underestimating the unique features of Hartwig’s approach we may attach it to the logical multiplier for the scope of this paper.
when the deferred consumption is enjoyed, the marginal propensity to consume rises temporarily above its normal level, to compensate for the extent to which it previously fell below it, and eventually returns to its normal level;’
(Keynes 1936: 123-4)

This is in fact a dynamic adjustment process of the MPC, that can be modeled by a damped oscillation. As savings instantaneously rise with investment, the MPC out of increasing income must initially equal zero to compensate for

\[ \Delta Y = \Delta I = \Delta S. \]  \hspace{1cm} (13)

Until then, no multiplying process has happened. The multiplier sets in by assuming that the MPC reaches back to its normal or long run level—the average propensity to consume (APC). Moore (1994: 127) doubts whether such a normal level actually exists. According to him, the unpredictable, path-dependable future does not allow for assuming any equilibrium value. As long term expectations change continuously along the time path, so do equilibrium values. He concludes: ‘Ergo, there can be no Keynesian income multiplier’ (Moore 1994: 127). However, for the mere existence of the logical multiplier it is irrelevant whether the MPC follows a long run trend. Once it eventually exceeds zero, there will be at least some multiplying effect while, certainly, nothing is said about the concrete value of the multiplier then. In other words: If you drop the assumption that the MPC equals the APC, then you have to find another suitable value for \( s \) in equation (7).

But let us, for the sake of the argument, suppose the MPC to be a damped oscillation that reaches back to the APC. Still we don’t know how long it takes to spring back. Basically, the same question occurs concerning the series multiplier, where we do not know how long “one round” actually takes. Both approaches entail a dimensionless multiplier that does not reveal at which point in time it has worked out half or fully. This calls for a time-dependent multiplier which is subject to the next section.

But before proceeding, we should briefly summarize this section: The well-known series multiplier is a comprehensible way to model the process of expenditures and receipts stemming from an initial demand for capital goods. But in order to find a position of rest, usually there is an unsound explanation how the stability condition (volitional savings equal volitional investment) comes into being. Either it draws on a mix-up of different meanings of ex ante and ex post, or on a mix-up of savings and finance. When assessing the process more carefully one can overcome these contradictions. However, two other questions come to the fore: What are these savings that leak out of the circuit and how does a dimensionless multiplier apply to real time processes? Keynes’ logical multiplier is supposed to avoid these questions by dropping process analysis. But it turns out that some adjustment process must
be employed to account for time lags. If done so, the MPC becomes an oscillating variable that somehow reaches back to its long run level. Questionable as this may be, it definitely requires a time-dependent multiplicand (as does the series multiplier) to apply to real processes.

3 The velocity multiplier

Moore (1988: 305; 2006: 376; 2008: 125) develops an alternative approach, where the income velocity of money replaces the MPC as the multiplicand. It derives from the Quantity Equation as follows: The whole volume of trading $T_t$ in a certain period of time equals the amount of money that flows from hand to hand. This amount is decomposed by the stock of money $M_t^*$ and it’s velocity $V_t^*$. Then the truism holds that

$$ T_t \equiv M_t^* V_t^* $$

(14)

which resembles the Quantity Equation for the whole volume of trading. By focussing on the amount of money that is used to buy currently produced commodities and services, the standard Quantity Equation comes up:

$$ Y_t \equiv M_t V_t $$

(15)

A change in aggregate demand ($\Delta Y_{t+1}$) comes along with changes in the stock and/or in the velocity of money. Totally differentiating (15) yields

$$ \Delta Y_{t+1} \equiv \Delta M_{t+1} V_t + \Delta V_{t+1} M_t + \Delta M_{t+1} \Delta V_{t+1} $$

(16)

So much for definitions. To predict next period’s expected income ($\Delta Y_{t+1}^e$) Moore (2008: 125) assumes the following:

1. The expected change of velocity of money is zero ($\Delta V_{t+1}^e = 0$)
2. Changes in investment spending are totally financed by bank lending ($\Delta I_{t+1} = \Delta L_{t+1}$)
3. Bank lending finances nothing else but investment expenditures ($\Delta L_{t+1} = \Delta M_{t+1}$)

In the strict sense, assumption # 3 is not homogeneous in dimensions (which Moore does not mention). While $\Delta L_{t+1}$ measures a change in a flow comparing two periods, $\Delta M_{t+1}$ measures a change in a stock comparing the beginning and the end of one period. To bypass that problem we may relate the change in the flow to the very beginning of the period under consideration. This way, (16) becomes

$$ \Delta Y_{t+1}^e = V_t \Delta I_{t+1} $$

(17)
where the income velocity of money for a certain period resembles the Keynesian multiplier. ‘In each period the Keynesian multiplier is the income velocity of money!’ (Moore 2008: 126). This conclusion is remarkable. While Moore repeatedly announced the ‘demise of the Keynesian multiplier’ he develops his own version, which we may call the velocity multiplier from now on.

The idea breaks away from Keynes’ original reasoning that the amount of investment is the causa causans of aggregate demand just because it is its most volatile component. In this sense, Moore doesn’t make a distinction between investment and consumption. What makes the difference between $I$ and $C$ is how they are paid for. While $C$ is usually paid out of current or past income, $I$ is mostly financed by newly created bank credit or by drawing on overdraft facilities. Thus, investment drives aggregate demand precisely because it requires additional credit money. In principle, it is the additional amount of circulating credit money that initiates the multiplier effect. This gives rise to a deeper understanding how endogenous money and the multiplier are intertwined.

An additional amount of money—created by someone who is willing to borrow and by a bank that is willing to lend—induces a succession of incomes and receipts, just like the process of the series multiplier suggests. The difference is: there is no resting point where the system gravitates towards; nothing leaks out of the circuit. Instead of that, the process happens in real time, which allows to measure the laps of the money in the circuit. The question: “How large is the multiplier in a certain period of time?” boils down to: “How often will the additional money go round during that certain period?” The velocity multiplier has an infinite value for an infinite period, but a definite value for a definite period of, say, 4 per year or 1 per quarter. The series multiplier is the antipode: It yields a finite value for an infinite succession of rounds, while it does not define how long one round actually takes.

This is a great advantage of the velocity multiplier. As the income velocity of money has a concrete time dimension, it applies to processes in historical time. Additionally, volitional savings do not need to adjust to volitional investment, whereby the model gets rid of a stability condition, without loosing the ability to yield a solution. Savings are not needed to finance investment. Thus, the approach is coherent with the concept of endogenous credit-money. As consumption and investment differ in the way they are usually paid for, but not in general in the way they influence further expenses, an artificial division can be dropped: Within the standard multiplier framework initial investment exclusively causes consumption, which in turn exclusively causes further consumption and so on. By no means is investment induced by additional aggregate demand; except for the
initial change, investment remains constant throughout the process.\textsuperscript{4} Within the velocity multiplier framework initial investment causes further expenditures which are not limited to consumption, but may also enclose further investment. It does not solely refer to the marginal propensity to consume (MPC), but to the more general term that we will call the \textit{marginal propensity to expend} (MPE) from now on.\textsuperscript{5}

However, the approach has at least three distinct weaknesses. The first one concerns the assumption that velocity is stable along the time span of measurement. Moore (2008: 125) states: ‘The velocity of money is continually changing. But since it has a unit root, the best estimate of next period’s velocity is the current period’s velocity, so the expected change in velocity is zero.’ Moore uses the very same reasoning for two diametrically opposing conclusions: While rejecting the proposition of a stable MPC over calendar time due to its unit root, he assumes a stable velocity of money because it has a unit root. Basically, assuming a stable velocity is as binding as assuming a long run stable MPC. This is particularly problematic, since the velocity of money in Moore’s multiplier plays exactly the same part as the MPC for the logical multiplier. However, they differ in a crucial detail: While the MPC initially has to fall to zero and rise afterwards to depict the time lag, the velocity of money does not need to oscillate as the time dimension is already enclosed. Additionally, $\Delta V_{t+1} = 0$ only refers to a short time horizon, while the MPC has to be stable in the long run. In conclusion, stability of velocity is a less stringent assumption than stability of the MPC.

The second weakness relates to measurement. Figures of the velocity of money are only taken ex post by dividing a certain money aggregate by income. It is just a residual to fulfill the Quantity Equation. Depending on the money aggregate under consideration, $V_t$ is higher or lower when $M_t$ is defined narrower or broader. If velocity is regarded as a behavioral parameter, it must draw on microeconomic data.\textsuperscript{6}

The third (and most crucial) weakness concerns the assumption, that the additional volume of credit money $\Delta M_t$ is fully kept within the circuit during the whole period under consideration. As already mentioned, the velocity multiplier does not model a leakage, whereby the MPE equals unity. The additional credit money will never leak out of the circuit. Hoarding is just a temporary phenomenon embodied in a certain velocity of money, but the hoards will be used again sooner or later.\textsuperscript{7} This is the reason why an endless period implies an infinit multiplier.

\textsuperscript{4}Of course there are multiplier-accelerator models that employ an investment function as well, but this is not directly related to the multiplier. See Hicks (1959) for an early model.

\textsuperscript{5}It could also be called \textit{marginal propensity to spend}, but the consequential abbreviation MPS might be conflated with the marginal propensity to save.

\textsuperscript{6}See Machlup (1939) for a comprehensive discussion of the matter.

\textsuperscript{7}See the dynamic model of Andresen (2006) for an application of the velocity multiplier.
But in fact there are leakages in the circuit. They are subject to the next section.

4 The leakages in the circuit

The standard multiplier assigns the role of the leakage to savings which are defined as *money not consumed*. The usual way to teach this to undergraduates is to put savings on a level with hoarding. Machlup was one of the first to point out the muddle with that term:

‘Many of the critics of Multiplier theory were not able to interpret the leakage as anything but “hoarding.” Accumulation of idle cash balances (and cancellation of bank deposits through debt repayment) was the only answer which these critics had for their query as to the nature of the leakages. This identification of the leakages with hoarding is liable to make full-blooded Keynesians furious. They usually react to it with an explanation of the meaninglessness of the concept of hoarding in the Keynesian language—but they do not tell their misinterpreters “what happens to the leaked-out funds.” They confine themselves to the contention that all that matters is the fact that these amounts are not spent on consumption. This answer, in turn, is apt to make their opponents furious. As a matter of fact, it is irrelevant for the immediate effect what the nature of the leakage really is. It is true that it does not make any immediate difference “what happens to the leaked-out funds.” But the critics have nevertheless a perfect right to know what happens if the funds are not “hoarded”.’

(Machlup 1939: 19)

Machlup is right that the mere existence of the multiplier is not challenged by the nature of the leakage as long as at least something leaks out. But in order to assess the size of the multiplier it is worth dealing with the nature of leakages in more detail.

In an electronic money economy hoardings are deposits on bank accounts which banks may use to a large part to do further business. Moore (2006: 366) terms this *convenience lending* by the depositor. It may be questionable to which part these hoards are used for further productive expenditures. In any case, they may not leak out fully, which, in our framework implies that the MPE exceeds the MPC. In a more common sense this is equal to *crowding-in* or accelerator effects.

Kahn’s *cul-de-sac* metaphor in general does not fit to a credit money economy as it implies something like a growing treasure. In fact the leakage is quite the opposite, namely, a depletion of the amount of credit money that circulates in the productive sphere of the economy. Three different channels may be investigated here. Firstly, Kahn (1931: 176) suggested *saving on the dole* if the initial
investment, be it public or private, provides additional employment. Technically speaking, this is an automatic stabilizer that reduces the net effect of an additional investment.

A second channel concerns money fading from the productive to the financial sphere. To be sure, this does not refer to buying a financial asset that already exists, which would make a zero sum game. But convenience lending may also be used to buy newly created financial assets from banks. The net inflow to the financial sphere has been largely positive throughout the last decades as the literature on financialisation has pointed out (Palley 2008). Thus, convenience lending may be largely used to buy newly created financial assets yielding an increase in income but only a negligible multiplication of employment. Strictly speaking, this does not constitute a leakage in the income dimension but a loss in transmission to the labor market dimension.

The third (and maybe most important) leakage relates to the reflux principle (Rochon 2008; Lavoie 1999; Kaldor and Trevithick 1981). Consider the following: Someone takes out a loan in order to finance an additional investment. At the beginning a debt and a deposit occur concurrently at the borrowers’ account. Once the borrower spends the money on newly produced capital goods, the producer receives deposits, while the initial borrower is left over with a tangible asset and a debt. The tangible asset, e.g. a machine, produces goods that generate prospective receipts. Once money flows in, the initial borrower (at least partly) uses it to repay the debt, which is a deliberate decision, since debit interest rates usually exceed credit interest rates. This is the heart of the reflux principle. Excess credit money is repaid if there is no application (Lavoie 1999: 106). The reflux principle constitutes a leak of credit money out of the circuit. The amount of circulating money decreases by debt settlement.

Suppose a slight variation in the way credit is granted: Given there was not an initial loan but an agreement on overdraft facilities between the bank and its customer. Then, the borrower does not have separate accounts for loans and deposits, but only one single account where inflows automatically decrease debt. Then the reflux mechanism works automatically.

Suppose another variation, which is actually a generalization: Given the succession of receipts and expenditures encounters any net debtor’s account (and not explicitly the original borrower’s). No matter who is the debtor receiving the money, she will deliberately or automatically use (at least some part of) the receipts to settle her debt. This points out, that the reflux is not tied to the initial loan but relates to the general level of debt.

Suppose yet another variation, where the circulating money flows to someone who buys newly issued bonds. The issuer may take the money to deliberately or automatically settle her debt to the bank. In this case, buying newly issued bonds
refinances an outstanding credit, which lowers the amount of credit money.\footnote{The example shows that we have to be careful concerning our definition of money. The bonds may act as a means of payment for some transactions as well, albeit they lack general acceptance.}

Above we raised the question whether savings can play a double role, both as a means of financing investment and as a definite leak out of the circuit. Being aware of the reflux principle, the conundrum can be resolved. Savings may truly appear step by step, as the series multiplier claims. But they should not be considered in their common sense—\textit{income not consumed}. Savings should be defined as \textit{net wealth accumulation} (Moore 2006: 161) which includes buying newly issued bonds and settling debt as well. Consequently, the multiplier does not show, how an initial investment is financed or paid by savings. What it does show is the process until an additional amount of debt is refinanced or repaid. This is more than a simple renaming. It allows for a crucial understanding of the multiplier process. As long as the money is kept within the circuit it is used for additional effective demand. Once it is used by anyone to refinance or repay a debt, it leaks out. When the amount of debt returns to its former level the multiplier process has come to its end.

5 Some early considerations on an alternative multiplier

We may now combine our findings of the last two sections to formulate some early ideas on a new version of the multiplier, which is both time dependent and allows for leakages. There are two important channels:

1. The velocity of money that buys newly produced goods and assets

2. The magnitude of the leakage within a certain period as determined by the MPE

The higher the velocity of money the higher is the multiplier, and the more intense the leakage the lower is the multiplier. Both determinants are related to time, and so must be the multiplier. Both are subject to fluctuations, which entails fluctuations in aggregate income through the multiplier. Multiplier analysis resembles business cycle analysis in this respect.

As the last section has shown, the leakage is not tied to the initial loan but generally concerns a change in the level of circulating credit money. An increasing level of credit money provides the basis for a higher level of income. In contrast, deleveraging may drag down the level of economic activity. In this sense the multiplier relates to a more general discussion on credit cycles and their influence on the business cycle (Fisher 1933; Biggs et al. 2009; Keen 2010).
In addition it shows, that the multiplier is a rather loose concept than a mechanical process. The reflux may occur automatically, but that does not necessarily stop the process. In other words: Even if an inflow mechanically lowers debt, it may entail demanding a new credit in turn. The MPE is a variable, not a fixed parameter. Moreover, it is not constrained to values between zero and unity. A dimensionless multiplier needs such an assumption as a stability condition. A time-dependent multiplier does not. An MPE larger than unity is conceivable when credit creation induces further credit creation. Accelerator effects would occur. Likewise effects may stem from an acceleration of the velocity of money. We may refer to these variations as horizontal, since they refer to a concrete time span, such as a certain phase of the business cycle. In other words: The very same amount of credit creation may have different effects in upturns and downturns as they provide different environments.

We may consider vertical variations as well. They relate to the efficiency of different measures with the same initial credit impulse at the same time. Employing the impulse-response method to a certain increase in the amount of circulating money may help to estimate its effect. However, it assigns average values of the whole economy to the parameters of a certain project. That may not be perfectly wrong, but definitely it is far from being perfectly correct.

Both the MPE and the velocity of money are still rather vague concepts that need to be defined more precisely. Modeling these relations will be subject to future research.

6 Conclusion

Even though the multiplier does not demise, Moore has pointed out the shortcomings in common explanations of the multiplier process. The Cottrell-Moore debate has revealed the need for a sound explanation. Process analysis helps, but it uncovers two further questions that must be answered to apply the multiplier to real processes:

1. How long does one round of the process and the whole process actually take?

2. What happens to the money, that is “saved”?

Basically, Moore’s velocity multiplier provides an answer to the first question. Even if money velocity is a rather vague concept, it renders a time dependent multiplier possible. However, the velocity multiplier disregards the second question. It simply abstracts from any leakages. Section four has proven this assumption ineligible.

9The very same applies to the standard multiplier, where the APC is usually taken to show the outcome of a certain project.
There are leakages in the circuit. The reflux principle relates the leakage in a credit money economy to the settlement of debt. In a narrow interpretation the multiplier unfolds until an initial loan is refinanced or repaid. In a broad interpretation (or better: on a macro level) the multiplier works from take-off to landing at a certain level of debt. This is how credit cycles and business cycles intertwine with the multiplier.

The discussion has shown that two main channels influence the multiplier—the velocity of money and the magnitude of the leakage. These channels are by themselves influenced by other determinants, such as animal spirits, bank behavior, liquidity preference, central bank reactions and so on. To put it with Machlup’s words:

“The theory of the Multiplier, if it is to be of use to those who wish to know the possible and the probable effects of public works, must renounce the attractive appearance of neatness and preciseness. The two variables which seem to play the main parts in the play of the Multiplier must be decomposed into the all too large number of variables which play the important roles in the real world.”
(Machlup 1939: 27)

Modeling these relations will be subject to future research.

References


Biggs, M., T. Mayer and A. Pick (2009), Credit and economic recovery, DNB working paper 218, Amsterdam.


