Dimensional Analysis of price-value deviations

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Abstract

This paper discusses the methodological criticism of price-value deviation measures put forward by Díaz and Osuna (2007, 2009). The authors argue that the relation between different price systems cannot be empirically quantified at all because the results would always depend on arbitrary physical units. But applying dimensional analysis shows that this conclusion is not correct. There is at least one simple and suitable measure of deviations from prices to values: the coefficient of variation.

Key words: Value, Price, Indeterminacy, Spurious correlation, Dimensional Analysis
JEL Classification: C21, C43, C52, D46

1 Introduction

During the last years, there is a growing body of empirical studies claiming that deviations from labour values to market prices are quite small.[1] The authors found correlation coefficients and coefficients of determination R^2 to be considerably larger than 0.9. Hence, they conclude that classical economists like Adam Smith (1723–1790), David Ricardo (1772–1823) and Karl Marx (1818–1883) arguing in terms of labour values were not that far away from truth as it is widely believed. Unsurprisingly, several objections have been advanced in order to doubt these outcomes.[2]

This paper only deals with Díaz and Osuna (2007, 2009). In these two articles the authors put forward a very fundamental consideration. In their view, *every* way of measuring price-value deviations is spurious due to the arbitrariness of defining the commodities unit of measurement. However, as we will see, their arguments are not convincing. To clarify the reasons it is necessary to discuss the basics of Dimensional Analysis (DA) in a first step. Afterwards we will analyse the characteristic of price-value regressions and the coefficient of variation of prices-values deviations in the light of DA.

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[1] See Shaikh (1984); Petrović (1987); Ochoa (1989); Cockshott and Cottrell (1997, 1998, 2003); Tsoulfidis and Maniatis (2002); Zachariah (2006); Tsoulfidis and Mariolis (2007); Tsoulfidis (2008); Fröhlich (2010).

[2] See Steedman and Tomkins (1998); Kliman (2002, 2005); Díaz and Osuna (2005–06, 2007, 2009).

2 The basics of Dimensional Analysis

Contrary to other disciplines like physics or engineering science, DA is not very common in economics. Most of the existing micro- and macroeconomic models (even econometric ones) are developed under the assumption that mathematical operations defined for pure numbers are still valid in case of applying economic variables. But this is in general not true. For that reason DA is regularly practised in natural science facing *quantities*: It tells us which kind of operations are valid and which are not outside of pure mathematics.

However, there *is* some economic literature on DA. The most elaborate one was written by de Jong and Quade (1967). An important paper, especially from the viewpoint of classical economics and (linear) production models, was published by Okishio (1982). Vignaux and Scott (1999) utilise DA to econometrics. Finally, a standard textbook for engineering science with some economic examples is Szirtes (2007).

To give an outline of the basics of DA it is necessary to define some fundamental concepts in the first step. We start with a definition of what is meant by the phrase *physical quantity* such as 10 metres or 30 seconds.

Definition 1. A *physical quantity* (short: quantity) is defined as the product of a numerical value $\{Q\}$ and a unit of measurement $[Q]$:

$$Q = \{Q\} \times [Q]. \tag{1}$$

Some physical quantities are additive and some are not. For instance, we can add 10 metres to 10 miles, but not 10 metres to 10 seconds. Metres and miles have the same physical dimension (length), whereas metres and seconds have not.

Definition 2. A *physical dimension* (short: dimension) is a set of additive physical quantities.

By convention, physical quantities are organised in a dimensional system built upon base quantities, each of which is regarded as having its own dimension. The world's most widely used system of measurement is the *International System of Units* (SI).[3] It has seven base quantities and dimensions which are listed in Table 1. All other dimensions and units of measurement can be derived by combining these base quantities. From an economic point of view, the SI base quantities are not complete; at least the economic quantity "money" is missing.

Definition 3. *Dimensionless physical quantities* (quantities of dimension one) are numerical values obtained as ratios of two quantities of the same dimension.

Economic examples of dimensionless quantities are wage shares and returns on sales.[4] The numerical value of such quantities is the same in all systems of units, that is dimensionless quantities do not depend on the convention of measurement.

In the next step four basic rules of DA are introduced (Szirtes, 2007, pp. 95–96 and 102–106). There are of course further rules, but for the topic of this paper it is not necessary to explain them here.[5]

[3] The abbreviation SI stands for the French *Système international d'unités*.

[4] Sometimes counts ("The number of people in the room") are also considered to be dimensionless quantities.

[5] See Szirtes (2007, chapter 5 and 6) for details.

Table 1: SI base quantities and dimensions.

Base quantity	Symbol for quantity	Symbol for dimension	SI base unit	Symbol for unit
Length	l, x, r, etc.	L	metre	m
Time	t	T	second	s
Mass	m	M	kilogram	kg
Electric current	I, i	I	ampere	A
Thermodynamic temperature	T	Θ	kelvin	K
Amount of substance	n	N	mole	mol
Luminous intensity	I_v	J	candela	cd

Source: Bureau International des Poids et Mesures (2006, pp. 105, 116).

Rule 1. *Every theoretically derived equation must be homogeneous in dimensions. This means that both sides of the equation have identical dimensions.*

Rule 1 is rather self-evident. Different kinds of quantities are incommensurable, hence they cannot be equal.

Rule 2. *If either one or both sides of a theoretically derived equation has/have more than one member joined by addition or subtraction, then all of these members must have identical dimensions.*

Rule 2 is tautologically linked to Definition 2: Apples can only be added to apples, not oranges.

Rule 3. *Physical quantities can be multiplied or divided without dimensional restrictions.*

The product or ratio of physical quantities is a derived quantity. It is often a new physical quantity (e.g. square metre) or, as mentioned above, a dimensionless quantity.

Rule 4. *Exponents and arguments of transcendental functions must always be pure numbers or dimensionless quantities.*

Rule 4 is a consequence of Rule 2. Because we will come back to it later on, the fourth rule deserves some notice. Consider, for example, the exponential function $y(t) = \mathrm{e}^t$. Its Taylor Expansion is

$$y(t) = \mathrm{e}^t = 1 + t + \frac{t^2}{2!} + \frac{t^3}{3!} + \cdots . \tag{2}$$

Obviously, due to dimensional homogeneity, t must be a dimensionless number, because otherwise Rule 2 would be infringed. Since $y = \mathrm{e}^t$ and $\log y = t$, the same statement holds in case of logarithm (Szirtes, 2007, pp. 104, 108).[6] To put it precisely: Any logarithm is only defined in case of pure numbers. Given some quantities a and b, this implies that $\log(ab)$ may exist even though

[6] Throughout this paper $\log z$ denotes the natural logarithm of an arbitrary positive real number z.

$\log a$ and $\log b$ do not (de Jong and Quade, 1967, pp. 188–189). Hence, $\log(ab) = \log a + \log b$ is correct *if and only if* both a and b are dimensionless quantities.

What would happen if we ignore Rule 4? Suppose that t in equation (2) is a time variable measured in seconds. For $t = 5$ s, we then get $y = 148.41$. Changing the unit of measurement from seconds to minutes leads to $y = 1.09$ instead. Although the underlying quantity is still the same ($t = 5\,\text{s} \approx 0.083\,\text{min}$), the results differ. In a physical sense, therefore, the outcome of (2) is just meaningless.

3 Indeterminacy in price-value regression analysis vs. dimensional homogeneity

Classical economists, especially marxian authors, state that exchange ratios of commodities are driven by the corresponding labour values, i.e. the sum of direct and indirect labour time socially needed to (re-)produce a commodity.[7] Suppose that there are n commodities and no joint production. Equation (3) expresses the so-called *law of value*:

$$\frac{p_i}{p_j} = \frac{\lambda_i}{\lambda_j}, \; i \neq j \text{ and } i, j = 1, \ldots, n, \tag{3}$$

where p_i is the unit market price of commodity i and λ_i is its unit labour value. With recourse to Input-Output (IO) tables (3) becomes

$$\frac{p_i x_i}{p_j x_j} = \frac{\lambda_i x_i}{\lambda_j x_j}, \; i \neq j \text{ and } i, j = 1, \ldots, n. \tag{4}$$

In this modified expression, x_i denotes the quantity of commodity i. Clearly, the exchange ratios given in (4) do not depend on the arbitrary definition of physical units because all units cancel out. Therefore, both sides of equation (4) consists of dimensionless quantities.

The law of value is usually tested by the following double-log regression model (see Shaikh (1984); Ochoa (1989); Díaz and Osuna (2007, p. 391) for further information):

$$\log\left(\frac{p_i x_i}{p_j x_j}\right) = \alpha + \beta \log\left(\frac{d_i x_i}{d_j x_j}\right) + u_{ij}, \; E(u_{ij}) = 0, \; i \neq j \text{ and } i, j = 1, \ldots, n, \tag{5}$$

with regression coefficients α and β and error term u_{ij}. We refer to direct prices d_i instead of labour values λ_i. These specific prices are proportional to labour values and they are used because in most cases IO tables provide only information on wages but not on labour time. Technically, direct prices are obtained by computing a neoricardian price vector with zero profit rate. However, Díaz and Osuna (2007, p. 392) present equation (5) in a different way. Since log-relationships occur they suggest the regression model could be written as follows:

$$\begin{aligned} \log\left(\frac{p_i}{p_j}\right) + \log\left(\frac{x_i}{x_j}\right) &= \alpha + \beta \log\left(\frac{d_i}{d_j}\right) + \log\left(\frac{x_i}{x_j}\right) \\ &\quad + \beta \log\left(\frac{x_i}{x_j}\right) - \beta \log\left(\frac{x_i}{x_j}\right) + u_{ij}. \end{aligned} \tag{6}$$

[7] For details see, for instance, Pasinetti (1977) or Fröhlich (2010).

Rearranging the last equation yields

$$\log\left(\frac{p_i x_i}{p_j x_j}\right) = \alpha + \beta \log\left(\frac{d_i x_i}{d_j x_j}\right) + (1-\beta)\log\left(\frac{x_i}{x_j}\right) + u_{ij}. \tag{7}$$

Díaz and Osuna claim that regression results depend on physical units in $\log\left(\frac{x_i}{x_j}\right)$. In their view, this constitutes the fundamental problem of indeterminacy in price-value correlation measures mentioned above.

But why is there a difference between (5) and (7)? To understand the reason, one has to remember that arguments of transcendental functions must be dimensionless (Rule 4). Now let us have a look on (4) and (5). Examining expression (4) shows that all elements are dimensionless quantities. Hence, we may log-transform and write it like (5). Still, merely dimensionless scalars appear and, as a consequence, no dependence on units occurs. Yet, converting (5) into (6) and (7) destroys dimensional homogeneity. Relative prices should be added to relative quantities in that case. But this is impossible (Rule 2).

The mistakes arises because the authors disregard the fact that any logarithm is only defined in case of pure numbers. Equation (6) and (7) do not fulfil this prerequisite. In fact, facing unit dependencies is essentially a strong hint on dimensional heterogeneity. Or, rephrased, every equation which is dimensionally homogeneous is formally independent of the choice of units (de Jong and Quade, 1967, p. 28). Therefore, Díaz's and Osuna's criticism provides no argument for deciding whether price-value correlations are indeterminate or not.

4 An alternative measurement

In Díaz and Osuna (2009) the authors try to generalise their methodological criticism by showing that not only price-value regressions but *all* measurement procedures of deviations from prices to values would *always* lead to indeterminate results. Because "the central theoretical issues in capital theory, etc., turn only on how *relative* prices differ from *relative* values, i.e. on how the directions of the two vectors differ from one another" (Steedman and Tomkins, 1998, p. 379), they use a geometric approach in their paper. It is not necessary to review all of the measurement procedures presented by Díaz and Osuna since the discussion of Steedman's and Tomkin's approach will show that there is at least one meaningful possibility to measure price-value deviations without getting into dimensional trouble, contrary to what is suggested by Díaz and Osuna.

Steedman and Tomkins (1998, p. 381) consider a neoricardian price system for an economy with n sectors and a uniform period of production. Each sector is producing a single output and the economy is assumed to be described by a linear, constant-returns-to-scale technology:

$$\boldsymbol{p} = w\boldsymbol{\lambda} + r\boldsymbol{p}\boldsymbol{H}, \tag{8}$$

In equation (8) $\boldsymbol{p}$ is the $(1 \times n)$-vector of prices of production, $\boldsymbol{\lambda}$ denotes the $(1 \times n)$-vector of labour values, w is the wage rate, r the profit rate and $\boldsymbol{H}$ symbolises the vertically integrated $(n \times n)$-matrix of input coefficients. Further manipulation gives

$$w^{-1}\boldsymbol{p}\boldsymbol{\Lambda}^{-1} = \boldsymbol{i} + r\left(w^{-1}\boldsymbol{p}\boldsymbol{\Lambda}^{-1}\right)\left(\boldsymbol{\Lambda}\boldsymbol{H}\boldsymbol{\Lambda}^{-1}\right), \tag{9}$$

where $\boldsymbol{i}$ is the $(1 \times n)$-vector of dimensionless unit elements and $\boldsymbol{\Lambda} := \operatorname{diag}(\lambda_1, \ldots, \lambda_n)$. The expression $\left(w^{-1}\boldsymbol{p}\boldsymbol{\Lambda}^{-1}\right)$ is therefore another dimensionless vector and $\left(\boldsymbol{\Lambda}\boldsymbol{H}\boldsymbol{\Lambda}^{-1}\right)$ is a matrix of dimensionless quantities, too. For notational convenience we additionally define $\bar{\boldsymbol{p}} := w^{-1}\boldsymbol{p}\boldsymbol{\Lambda}^{-1}$. Now the angle θ between $\bar{\boldsymbol{p}}$ and $\boldsymbol{i}$ is an appropriate way to measure the deviations from prices to values because it is only based on the *direction* of the relevant vectors under consideration. Or, rephrased, we are dealing with relative prices. Hence, θ must be independent of any physical unit and it is also not affected by the arbitrary choice of numéraire. Applying the inner product immediately gives

$$\sqrt{n(\bar{\boldsymbol{p}}\bar{\boldsymbol{p}}^T)}\cos\theta = \bar{\boldsymbol{p}}\boldsymbol{i}^T. \tag{10}$$

As before, n indicates the number of sectors and commodities. Now it is possible to use the mean $\mu_{\bar{\boldsymbol{p}}}$ and the standard deviation (SD) $\sigma_{\bar{\boldsymbol{p}}}$ to rewrite (10) as

$$\sqrt{\mu_{\bar{\boldsymbol{p}}}^2 + \sigma_{\bar{\boldsymbol{p}}}^2}\cos\theta = \mu_{\bar{\boldsymbol{p}}}. \tag{11}$$

Since, by elementary trigonometry, $\tan\theta = \left(\sqrt{1-\cos^2\theta}\right)(\cos\theta)^{-1}$, it follows that

$$\tan\theta = \frac{\sigma_{\bar{\boldsymbol{p}}}}{\mu_{\bar{\boldsymbol{p}}}}. \tag{12}$$

From equation (12) it becomes obvious that $\tan\theta$ equals the coefficient of variation (CV) of $\bar{\boldsymbol{p}}$. From an econometric point of view, CV is a measure of the percentage error of the linear regression model

$$\bar{p}_j = \beta i_j + u_j,\ E(u_j) = 0,\ j = 1, \ldots, n, \tag{13}$$

where again β is the regression coefficient and u_j is the error term. The (uncentered) coefficient of determination is given by $R^2 = \cos^2\theta$.[8] As mentioned above, neither θ nor CV depends on the choice of numéraire or physical units. Additionally, contrary to calculating relative prices given by (5), this approach can be directly applied to IO tables (Steedman and Tomkins, 1998, p. 382).

Díaz and Osuna (2009, p. 435) agree with the statement that CV is not effected by the choice of numéraire or the choice of measurement units. But they disagree with the conclusion that CV is a suitable measure to compare different kind of price systems because there would be $(n-1)$ degrees of freedom in calculating the mean $\mu_{\bar{\boldsymbol{p}}}$. Therefore, computing CV would yield no benefit compared to other flawed ways of estimation.

But this reasoning is not convincing. Of course there is no unique solution of $\bar{\boldsymbol{p}}$ without defining a numéraire. However, switching between different numéraires changes the commodity of measurement. But because the ratio of prices and values is a dimensionless quantity, this has no effect on $\bar{\boldsymbol{p}}$. At the same time the numerical values of $\boldsymbol{p}$ and $w\boldsymbol{\lambda}$ are scaled with an identical scalar due to the changing numéraire. As before, this effect cancels out while estimating $\bar{\boldsymbol{p}}, \sigma_{\bar{\boldsymbol{p}}}$

[8] Centered data is obtained by shifting a data sample by its mean. Usually, a centered R^2 is estimated when running a linear regression like (5). But for regressions without a constant term this procedure does not make sense.

and $\mu_{\bar{\boldsymbol{p}}}$. As a consequence, CV is always the same and can be applied regardless of any specific numéraires.

There is another point of critique put forward by Díaz and Osuna. They state that results based on (12) or (13) are nonetheless indeterminate because it is always possible to convert the regression model (including all other estimation procedures presented by them)

$$p_j = \beta v_j + u_j,\ E(u_j) = 0,\ j = 1, \dots, n, \tag{14}$$

$$v_j := w\lambda_j, \tag{15}$$

to that given in (13) by defining the physical units of v_j such that $v_j = i_j = 1$ ($j = 1, \dots, n$). This would lead to

$$p_j = \beta i_j + u_j,\ E(u_j) = 0,\ j = 1, \dots, n. \tag{16}$$

Since (14) and (16), respectively, are obviously sensitive to changes in physical units and yield always indeterminate results, the same would hold for (12) (Díaz and Osuna, 2009, p. 436).

Again, this argument is not valid. The regression models (13) and (16) have the same numerical values but not the same dimensions. As mentioned above, (13) is based on dimensionless quantities (a percentage number) whereas (16) has the dimension "money per quantity" and the quantity units arbitrarily cancel out all variations in the vector $\boldsymbol{v} = (v_1, \dots, v_n) = (1, \dots, 1)$. But this does not mean that all estimations of price-value deviations lead to indeterminate results. Instead, it means that model (16) yields correct numerical values as a special exception. To say that model (13) and CV would be also affected by this problem is a converse error.

5 A numerical example

Díaz and Osuna present a numerical example to exemplify their argument. For reasons of clarity, it is useful to reproduce their considerations at this point.

The authors picture an economy with two single products (bread and eggs) and the associated prices p_i and values v_i. There are two different sets of units. The first set corresponds to regression model (14) and the second one to model (16) (see Table 2). Table 3 shows deviations from prices to values based on the data given in Table 2. The angle θ is always measured in radians.

Now consider the estimations related to (14) and (16). As might have been expected, the outcome depends on the choice of units sets. But preferring one of the two results would be a completely arbitrary decision. This is the reason why Díaz and Osuna state that price-value deviations are always indeterminate. On the other hand, glancing at the results based on equation (13) makes clear that now both sets of units show identical deviations from prices to values, that is, a change in units sets has no effect on the outcome. It is important to realise that Díaz's and Osuna's argument only holds with respect to regression models (14) and (16), but not with respect to estimations based on model (13). As mentioned above, this is due to the fact that (13) and (16) coincide in numerical values but differ in dimensions.

Table 2: Two price and value systems with different sets of units.

First units set	Price (p)	Value (v)	Measurement units
Bread	6	4	kg/$
Eggs	2	12	kg/dozen
Second units set	Price (p)	Value (v)	Measurement units
Bread	3/2	4	$/piece (1 kilogram = 4 pieces)
Eggs	1/6	12	$/unit (1 dozen = 12 units)

Source: Díaz and Osuna (2009, p. 437).

Table 3: Price-value deviations for the data set given in Table 2.

Equation	Deviation measure	First units set	Second units set
(13)	$\tan\theta$	0.80	0.80
	$R^2 = \cos^2\theta$	0.61	0.61
(14), (16)	$\tan\theta$	1.33	0.80
	$R^2 = \cos^2\theta$	0.36	0.61

Source: Díaz and Osuna (2009, p. 438) (14), own calculations (12).

6 Summary

In this paper two methodological criticism of price-value deviation measures were discussed. The basics of Dimensional Analysis were introduced in a first step. It was shown that price-value regressions do not depend on physical units if the regression model is homogeneous in dimension. Subsequently, it was argued that the angle between the vector of price value ratios and the vector of unit elements is a simple and appropriate way to compare different price systems because it does not depend on either numéraire or physical unit choices. All in all, the arguments given by Díaz and Osuna (2007, 2009) do not seem to hold.

As a matter of course, other points of critique (Díaz and Osuna (2005–06); Díaz and Osuna (2007, pp. 395–398)) are not necessarily affected by the considerations in this paper. But going into detail would lead us to different topics which are already discussed elsewhere.[9]

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