Kaldorian Boom-Bust Cycles in the Housing Market

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Abstract:

We show that the Kaldor (1940) trade cycle mechanism can be meaningful applied to the market for residential housing space, since the demand for houses may be positively related to the housing price in a mid-range price domain, while it is downward sloping for house prices sufficiently small as well as sufficiently large. Confronted with the current supply of houses this gives rise to either 1 or 3 equilibria (booms, busts, and an unstable one). We then couple the employed nonlinear house demand schedule with backward-looking house price expectations and get a planar dynamics with the same range of model-consistent expectations equilibria as in the partial situation studied beforehand.

Upwardly adjusting housing supply can in the case of multiple equilibria lead to the loss of the boom equilibrium, implying that the bust equilibrium becomes a global attractor to which prices and expectations about them then converge. Such bust equilibria are however then interacting with falling housing supply which sooner or later gives rise to the disappearance of the bust equilibrium and thereafter an upward dynamics to the now again existing boom equilibrium. This process will repeat itself as long as the demand schedule for houses remains fixed. It moreover also applies to the case of myopic perfect foresight and is thus not dependent on the backward oriented expectation scheme we started from.

It is known from the literature on the Kaldorian trade cycle model that this situation can be further analyzed by methods from catastrophe theory as well as relaxation oscillations. A nonlinear demand function for houses in connection with changes in the housing stock thus initiates sudden reversals from booms into busts and vice versa which from a mathematical point of view give locally rise to fold catastrophes at the bifurcation points, with the dynamics being described by relaxation oscillations from the global perspective.

Keywords: Kaldorian trade cycles, boom-bust cycles, housing markets, relaxation oscillations.

JEL CLASSIFICATION SYSTEM: E12, E32
1 Introduction

Success breeds disregard of the possibility of failures. The absence of serious financial difficulties over a substantial period leads ... to a euphoric economy in which short-term financing of long term positions becomes the normal way of life. As previous financial crisis recedes in time, it is quite natural for central bankers, government officials, bankers, businessmen and even economists to believe that a new era has arrived. (Hyman P. Minsky, Can "It" Happen Again?, 1982, p.213).

As the bursting bubble in the US housing market was the point of departure of the current global financial crisis, it is certainly of great interest and importance to understand what forces drive the housing market and the recurrent boom-bust cycles that could be observed for the US housing market over the past decades. We want to show in this respect that there may be a specific nonlinear economic mechanism at work in the market for residential space which can explain the sudden reversal of a boom into a bust and vice versa we have observed in the past.

Our starting point for the description of such a mechanism characterizing the housing market is Kaldor’s (1940) nonlinear approach to the explanation of the trade cycle which we here reinterpret appropriately through a law of motion for housing prices and the expectations driven boom-bust cycle it can imply. In Kaldor’s original contribution the business cycle was driven by the nonlinear investment function, the dynamic multiplier process and capital accumulation, while we here adopt a mechanism that initiates boom-bust cycles in the housing market through house price dynamics, corresponding price expectations and changes in the stock of houses.

In order to relate our model to actual events in the housing market and to give some empirically motivation for the applied Kaldorian mechanism put forth, a quick overview of the US-housing market is provided in the next section.

2 The US Market for Residential Houses: Some Observations

The popular Case-Shiller house price index, which is based on quarterly data for nominal house prices throughout the US, had its peak in the second quarter of 2006. This was an all time high value which marked the top of the successively built-up housing bubble. After that point a rapid decline in the index could be observed which mirrored the bursting of the bubble and therefore the turn from a boom into a bust. No recovery in form of a halt in the downward trend of the index has been visible until summer 2009. A prediction of a further fall of even 30 percent in prices can be founded by the index. All this followed
after a boom of almost a whole decade with an acceleration of price increases since the year 2000.

Muellbauer and Murphy (2008) find in their assessment of housing markets that agents in the housing market are driven mainly by psychological forces and therefore cause inherent regularities in the housing price dynamics which seem to be in line with a systematic overvaluation. Housing prices are regarded as being mispriced and markets thus not efficient. When fundamentals exercise their full pull on prices, corrections may be accompanied by exaggerated expectations of deep falls which are partly self-fulfilling.

In an IMF working paper before the downturn, Schnure (2005) for example gave the following assessment of the market situation:”...while some of these gains reflect catch-up following slow appreciation in previous years, recent increases have been particularly rapid, and may be ahead of fundamentals.” A change in the structure of the mortgage market from on-balance sheet lending of banks to a system of securitization of mortgages and loans was intended to smooth the stop-go credit cycle in housing finance and via several transmission channels to moderate output volatility of the whole economy. For an investigation of diverse feedback channels between the housing market and the macroeconomy see Goodhart and Hofmann (2008).

This liberalization of mortgage markets meant a shift from the local banks to national and global financial markets. Subprime lending became a really important business. A huge amount of clients endowed with seemingly cheap credits in order to purchase their own homes, had then a propensity to consume larger than one. It is a simple economic truth that this is not not sustainable for ever. Risks were no longer assessed in a reasonable regard of the specific borrower. This led to a tremendous underestimation of default risk and became later the reason for breaking credit chains and an enduring distrust of banks among themselves. Problems came in when the Fed had to increase interest rates and the monthly rates could no longer be afforded by a large part of the banking clients.

A variety of explanations were given at that time why the boom could sustain or at least be followed by a slow appreciation. One reason which was mentioned was the level of house supply. Its level was as low as the one where house prices stabilized in previous boom-bust cycles. Moreover it was stated that housing supply was less speculative than in the decades before the 1990s, indicated by a low level of inventories in relation to the number of house sales (Schnure 2005). This fact should support the view that a decline in housing demand would not necessarily lead to a large downswing in prices. But all these optimistic scenarios have been proven wrong.

Instead of a price volatility moderation by improved credit allocation, markets were swept with liquidity. Cheap finance combined with an illegitimate practice of lending to private clients without sufficient income fired the price rally first and laid the foundation for the following breakdown. One has to conclude that the pattern of boom-bust cycles in the housing market had been by far not obsolete, but then stroke back with a vengeance. The following model should penlight a mechanism at work in this particular market.
3 Kaldorian Boom-Bust Cycles in the Housing Market

We explore a highly stylized model of the housing market which features a nonlinear demand function for private housing. The kind of nonlinearity is drawn from the Kaldorian investment function used in his model of the trade cycle. The nonlinear demand schedule is characterized by two shifts in the relationship to the housing price. This will be a prerequisite to establish later on boom-bust cycles with the help of movements in the housing stock. We show that the Kaldor (1940) trade cycle mechanism can be meaningfully applied to the market for private housing space, since the demand for houses may be positively related to the housing price in a mid-range price domain (while it is downward sloping for house prices sufficiently small as well as sufficiently large). Confronted with the current supply of houses this gives rise to 1 or 3 equilibria (booms, busts, and an unstable one). We couple the nonlinear house demand schedule with purely backward-looking house price expectations and get a planar dynamics with the same range of model-consistent expectations equilibria as in the partial situation studied beforehand. But first the model has to be set up via laws of motion for the price formation and the expectations about it. The demand function must be characterized formally as well.

Figure 1: Boom / Bust Situations: Single and Multiple Equilibria (for given price expectations)

The demand for houses depends, as one would have expected, negatively on the price for houses at areas where the stock of houses is low or high. In the range in between, for a
medium domain of the stock of houses, the relationship is however reversed. Rising prices here induce increasing demand. One can interpret that for high and low stocks we have the expected normal reaction with respect to house price movements. In the area where the demand function shows a positive slope the speculative motive prevails. The increase in housing prices there causes buyers to expect even higher prices and therefore increasing values of the stock of houses that currently exists. If houses are not mainly purchased for residential purposes, but are held as a kind of asset, we experience a kind of wealth effect. As a consequence rising prices lead to rising demand for some time. At a certain level houses demand reaches its peak and then starts to fall again with prices going up further. The market becomes saturated because the speculative motive slows down and demand for residential reasons becomes the dominating motive again. This is a verbal description of the housing market demand schedule which should be now formalized: We denote by \( p_h \) the price of houses, by \( p_e^h \) its expected value, by \( H^d(p_h, p_e^h) \), \( H_1^d \geq < 0, H_2^d > 0 \) the stock demand for houses and by \( H \) the stock supply. We assume that this stock demand is not fully active at each point in time, but only a portion of it, measured by the parameter \( \alpha_h \). As percentage excess demand function we therefore postulate the relationship

\[
\alpha_h \frac{H^d - H}{H} = \alpha_h H^d(p_h, p_e^h)
\]

and assume that this determines the growth rate \( \hat{p}_h \) of the housing prices \( p_h \) with speed \( \beta_h \). This basic component of the demand schedule is shown in figure 1 for given values of house price expectations and various levels of the stock of houses. Depending on the level of the housing stock \( H \) the shown situation can exhibit up to three price equilibria.

We thus assume that the housing market is characterized by the following situations: At equilibrium 1 and 3 we have a normal type of housing demand, while around 2 the demand for houses is increasing with their price \( p_h \), as in the actual situation in the US housing market some time ago. Ignoring \( p_e^h \), equilibria 1 and 3 are stable, while point 2 is unstable.

We now extend the above situation to the two laws of motion by going from static expectations to backward looking adaptive ones.

\[
\hat{p}_h = \beta_h \alpha_h H^d(p_h, p_e^h) \tag{1}
\]

\[
\hat{p}_e^h = \beta_e \left( \frac{p_h}{p_e^h} - 1 \right) \tag{2}
\]

which add the dynamic of house price expectations to the one that drives actual house price formation. Equation (2) can be rewritten as

\[
\hat{p}_e^h = \beta_e (p_h - p_e^h) \tag{3}
\]

. We thus have a growth law for housing prices that is coupled with a differential equation for house price expectations. The study of this system of differential equations is the subject of the following section.
4 Local Stability Analysis

Under suitable parameter constellations in an economic meaningful range, the postulated laws of motions will give rise to three equilibria when the intensive housing demand function $h^d$ equals zero and expected prices are the same as the actual ones. The intensive housing excess demand function $h^d$ is zero when supply is exactly met by demand. This causes the price dynamics to come to a halt and the market is put onto the curved isocline. For all right price expectations the housing market becomes located at the 45-line. The intersections of both curves deliver the steady state points of the whole system. Whether the points of rest are stable or not, has to be investigated by evaluating the Jacobians at the respective equilibria. Obviously, see also figure 2, the system has three steady states, where $p^0_h = p^{eq}_h$ holds true. The Jacobian reads at the steady states:

$$J_0 = \begin{pmatrix} \beta_h \alpha_h \frac{H^d_1}{H^d_2} p^0_h & \beta_h \alpha_h \frac{H^d_1}{H^d_2} p^0_h \\ \beta_e & -\beta_e \end{pmatrix} = \begin{pmatrix} \pm & + \\ + & - \end{pmatrix}$$

We assume that

$$H^d_1(p_{ho}, p^{eq}_{ho}) + H^d_2(p_{ho}, p^{eq}_{ho}) < 0 \text{ if and only if } \frac{-H^d_1(p_{ho}, p^{eq}_{ho})}{H^d_2(p_{ho}, p^{eq}_{ho})} > 1 \ (\ast)$$

holds in the cases 1 and 3, which states that the positive effect of changing house price expectations is dominated by the actual and negative housing price effect as far as marginal changes are concerned. This sum is of course $> 0$ in the equilibrium 2. For $\det J$ there holds:

$$\det J = \beta_h \beta_e \alpha_h \frac{p^0_h}{H} \left| \begin{pmatrix} H^d_1 & H^d_2 \\ 1 & -1 \end{pmatrix} \right| = -\beta_h \beta_e \alpha_h \frac{p^0_h}{H} [H^d_1 + H^d_2]$$

i.e., we get $\det J > 0$ in the equilibria 1 and 3 and $\det J < 0$ in the equilibrium 2, which in the latter case gives a saddle point.

The kind of adjustment to the asymptotically stable equilibria can be stated more precisely. One can show that the stable equilibria are nodes, not spirals by showing that the discriminant $\Delta = \left(\frac{1}{2} tr A\right)^2 - \det A$ is of the form $(a - b)^2$ and subsequently larger than zero.

The Implicit Function Theorem allows to calculate the slope of the $\dot{p}_h = 0$-isocline as follows:

$$p^{eq}_h(p_h) = -\frac{H^d_1}{H^d_2} > 0 \text{ in points 1 and 3 and } p^{eq}_h(p_h) < 0 \text{ in point 2.}$$

The implied situation is depicted in the form of a phase diagram of the dynamics in the figure 2. This figure expands what is shown in figure 1 if assumption $(\ast)$ holds and if $H^d(0, 0) > 0$ is assumed, in which case the origin of the phase space is a saddlepoint. Note however that the basins of attraction of the two stable equilibria are no longer easy to determine though one can conjecture that they are separated by the stable arms of the saddlepoint dynamics.
5 Global Boundedness and Basins of Attraction

After having shown that local stability exists at least for two of the equilibria, our concern now is the viability of the global dynamics. We will restrict our analysis here to graphical means that will allow for a proper treatment nonetheless. The assumptions made above imply the following global situation in the case where three stationary housing market equilibria exists. Figure 2 shows the dynamics of housing prices and of adaptively adjusting housing price expectations. The drawn box reduces the range of possible values for the variables on economic grounds. A sensitive analysis is needed to restrict the phase space to economically meaningful values of the price variables. At once all negative values can be excluded from the analysis. Furthermore all dynamics on the boundary of the found compact subdomain of the phase space points into this domain. It is fairly easy to see, since the housing stock $H$ has not started to move yet and thus is still a given parameter. The phase diagram as such offers sufficient insight as an analytical device for the moment. Any trajectory starting inside the indicated box will stay inside it and converge to one of the two attracting equilibria. The basins of attraction are divided by a separatrix which is equivalent as one might conjecture to the stable arm of the saddle equilibrium's dynamics. The global dynamics are certainly not attracted by this equilibrium. Its attracting domain is of measure zero in the domain of all converging trajectories. Therefore an arbitrarily chosen starting point within the box leads necessarily to one of the locally stable equilibria, because one is either in the basin of attraction of the lower or the upper equilibrium.
6 The Dissolution of Boom and Bust Equilibria through Slowly Adjusting Housing Supply

Our consideration so far was concerned with the short run where the housing stock remained fixed. When we take into account that this variable starts moving in time, the situation gives rise to permanently changing regimes with respect to booms and busts similar to what has been investigated for the Kaldorian trade cycle in Varian (1979), see also Flaschel (2009, ch.s 3,7). The driving force for these regime changes is the continuously moving stock of houses. Given that the above made assumption regarding the direction of the stock adjustment is true, the increase of housing supply associated with a boom phase gives way for the disappearance of the upper equilibrium. A reduction of the housing supply in connection with an unaltered demand schedule in a bust, let the lower equilibrium vanish at one point. In order to leave the analysis tractable in the easiest way without restricting the results, the dissolution of the equilibria shall be done under a perfect foresight expectation scheme which saves us one law of motion, since we have already shown before that the qualitative characteristics with regard to existence and stability of the boom and bust equilibria do not depend of the employed expectation formation scheme.

![Figure 3: The Ideal Limit Case: Price Adjustment under Myopic Perfect Foresight](image)

The situation shown in figure 2 has a well-defined limit case, shown in figure 3, the case of myopic perfect foresight. In this limit case the economy is always on the 45 degree line (which is so to speak approached vertically with infinite speed). We note that the curved isocline is shifting upwards with increasing housing supply (and thus in the boom phase of this market) and downwards in the bust phase. The housing market is therefore characterized by a tendency that leads to the disappearance of boom equilibria in the boom phase as well as the dismantling of bust equilibria in the bust phase in the course of time. We thus get that the naive adaptive expectations mechanism leads to results that do not differ qualitatively from the results obtained under the assumption of myopic perfect foresight.
Figure 4: Bust-driven downward shifting housing supply and the emergence of a single globally attracting boom equilibrium

The housing stock $H$ can therefore be interpreted in correspondence to Kaldor (1940) as a slow, continuously changing variable which causes shifts of the $\dot{p}_h = 0$ isocline. As these shifts can result eventually in the loss of one of the stable equilibria and the saddle point, we then get stable booms or busts for a given stock of houses in specific points in time. Adjustment to the then unique equilibrium points is ensured by the fast price and price expectation variables. But as the movement of $H$ goes on, e.g. in a boom situation, the market becomes saturated and the supply of houses decreases again. Maybe no new houses are constructed by builders or house owners decide not to sell their residents anymore. This supply squeeze causes sooner or later a shift of the isocline back into the direction where three equilibria are reestablished. After a while a point could be reached when the shortage in housing supply becomes so severe in connection with slightly falling prices now, that the isocline moves far enough again to lose the upper equilibrium and the unstable saddle equilibrium.

A sudden downward jump in housing prices and subsequently expectations about them occurs. The following process could be characterized by slowly upward moving prices and expectations. Excess capacities are decumulated and the bust reaches its floor. If this process goes on with sufficient strength, the $\dot{p}_h = 0$-isocline will intersect with the $\dot{p}_e = 0$-isocline again in the three equilibrium points. The bust continues as long as all three equilibria are kept in existence. The bust turns into a boom when supply scarcity makes a jump in $p_h$ upwards necessary initiated by the loss of the lower stable equilibrium. Figure 2 shows that there should exist a compact subdomain of the whole phase space where all dynamics point inwards everywhere on the boundary. The drawn box only contains values of $p_h$ and $p_e > 0$. Moreover it should be restricted to non-catastrophic values, since $p_h$ and $p_e^c = 0$ are economically not justifiable and might deliver nonreliable
Figure 5: Boom-driven increasing housing supply and the return of a globally attracting bust equilibrium

results. The subdomain must be limited to price values that are economically reasonable and empirically observable. This would allow to construct a stable limit cycle in the $p_h,p_h^*$-dynamics. Then this limit cycle would attract all relevant trajectories. No trajectory that starts in the box could leave it anymore. But the analysis of the limit cycle that result must be left here to a graphical treatment.

7 Fold Catastrophes and Relaxation Oscillations in the Housing Market

In order to further simplify the analysis we assume myopic perfect foresight of agents in the housing market with respect to the evolution of prices. Then we are left with just one law of motion for the price level. We can regard the housing supply as a parameter in the short run which becomes a variable in the long run.

With the help of catastrophe theory the local behavior of the system dynamics can be studied in more detail at the critical points where the boom phase ends and the switching to a bust occurs, respectively the bust ceases and the boom starts. These transitions must be executed by jumping processes of the fast variable, here the house price level, from one area of the state space to another. These jumps are called catastrophes. We will investigate here the simplest form of catastrophe, the so-called fold catastrophe. Having classified the stock of houses as the slow variable and the price level as the fast one, we can track the interaction of the upper and lower short run equilibrium with the long-term
dynamics in the $H,p_h$-phase space.

Figure 6: Singularities, fold catastrophes and relaxation oscillations in the myopic perfect foresight case

The points $p_{ho}$ and $p_{ho}^*$ are stable equilibria in the short run, as long as $H$, the housing stock, does not evolve. But in time $H$ is no more a given magnitude and starts to move. Boom situations are associated with upward adjusting housing supply, whereas busts interact with downward corrections of the supply. Nothing happens until the housing stock arrives at a sensible value that allows no more to maintain the prevailing equilibrium, but gives rise to a bifurcation point. The bifurcation set of the catastrophe manifold are exactly the short-run equilibrium positions $p_{h1}$ and $p_{h2}$ where the qualitative characteristic of the economic situation turns over. These short-run equilibria have continuously moved to these states as a result of the slow motion of $H$. Taking a look at the lower equilibrium $p_{h*}$, shows that the point of rest has moved that far downwards that no movement in the same direction is possible anymore along the housing demand curve. Thus the downward shifting supply of houses enforces the loss of the stable point here and necessitates a jump in $p_h$ to the right. A new stable equilibrium is established on the housing demand curve’s falling right arm. The market jumps from a bust into a boom. When you are initially in a boom, the description applies just the other way around.

The extension to the situation of backward-looking behavior is straightforward. The difference is that the singularities to look at, lie on the house market equilibrium curve in the $p_h, p_e$-space. Then the housing supply would only in the back determine the long-run position. But no new insights could be gained from it. As the analysis is of purely local nature, it only tells us that at the bifurcation points singularities exist which are of a fold catastrophe type and give way to qualitative changes of the economic characteristics.

As we did before, instantaneous adjustment to the housing market equilibrium curve, the $p_h = 0$-isocline, is assumed for the time being. This means that one of the two laws
of motion collapses into an equilibrium condition. For infinitely fast adjustment of the price level expectations, the steady state condition $p_h = p_{he}$ is fulfilled in every point of time. The parameter $\beta_e$ approaches infinity in this extreme case. With only one law of motion left for the short-run, we are able to study the global dynamics of the housing market system in an easy way. Global dynamics are restricted to a compact domain in the $p_h, p_{he}$-space. All dynamics from outside this domain are attracted by it. All dynamics within the domain, are bound in it and therefore cannot leave it. This is guaranteed by the motion of the housing stock which is in the short run a given parameter. But as time goes on the stock becomes a slowly changing variable. It continuously moves upward in association with boom states and shrinking in bust environments. This more long-run oriented view on the action in the housing market shows the switching processes inherent to the specific market.

**Figure 7:** The long-run volatility of housing prices from the perspective of relaxation oscillations

The movement of $H$ causes the prevailing (short-run)equilibrium to vanish eventually. Booms suddenly turn into busts, busts immediately turn into booms. But from a long-run perspective the process is stable in the sense that it repeats itself as long as the demand schedule remains fixed. Mathematically speaking the interaction of the short-run equilibria with the very sluggish adjustment in the supply of houses and subsequently the housing stock, gives rise to a stable relaxation oscillations limit cycle. Relaxations oscillations here come about because the reduction in dimension of the model due to the transition of a dynamic equation to an algebraic one. Being noted that it is necessary to demonstrate that the characteristics of the dynamics are not affected by this procedure. The method is only appropriate if the dynamics are structurally stable, meaning equivalent for finite adjustment speeds and the limit case.

Plotting these dynamic patterns against time for the housing price delivers ongoing wave-like fluctuations. Overtime an extremum is obtained, the price falls sharply down or strives up rapidly. Figure 7 captures the course of the time pattern of $p_h$. 

14
8 Conclusions

We have shown in this paper that a demand function for residential space which is upward sloping in housing prices in a midrange area of the price space (and as usual downward sloping outside of this range) can give rise to multiple equilibria under static housing price expectations. Moreover, making such expectations endogenous through a simple adaptive adjustment of them, we could show the existence of two stable and one (midrange) unstable equilibrium for the phase space composed of housing prices and their expected values. This situation also extends to the case of myopic perfect foresight as the natural limit case of adaptive expectations that adjust with ever increasing speed.

We then investigated the basins of attraction of the obtained boom and bust situations in more detail and also their change in the case of a sluggishly adjusting supply of residential space, which lead to fold catastrophes for sufficiently large changes in the supply of houses. Combining these isolated boom and bust situation again in a joint phase diagram we could finally show from the global point of view the emergence of so called relaxation oscillations in the housing market where booms suddenly change into bust situations and vice versa when critical thresholds in the supply of residential space are passed, giving rise to the existence of boom-bust cycles in planar systems for myopic perfect foresight and in a 3D phase space in the case of adaptively formed house price expectations.

Future work along these lines should integrate a debt financing process (mortgages) into such boom-bust cycle generating mechanisms and consider the specific contributions of debt accumulation and of changes in the loan rate in the formation of such boom-bust cycles.
References


