Asymmetric inflation dynamics, unemployment and hysteresis

Alfonso Palacio-Vera
Universidad Complutense de Madrid, Spain

Abstract: In this study we analyse the impact on the employment rate of supply and demand shocks in an economy characterized by the existence of (i) a short-run “inflation barrier” referred to as the “constant inflation employment rate” (CIER) that exhibits hysteresis effects, (ii) an inflation-targeting central bank and (iii) the presence of “asymmetric inflation dynamics” (AID) whereby, when inflation is low, the former responds more sluggishly when the output-gap is negative than when it is positive. We obtain several results. We show that the joint presence of hysteresis effects and AID in the type of economy sketched above (i) attenuates macroeconomic volatility, (ii) imparts a long-run negative bias on the unemployment rate in the wake of demand shocks irrespective of their sign (although the adverse effect is more pronounced when shocks are unfavourable) and (iii) blocks off the “reversibility” property exhibited by zero/unit root systems in the wake of supply shocks.

JEL Classification: B50, E12, E24, E50

Key words: Asymmetric inflation dynamics, hysteresis, reversibility, remanence
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1.- Introduction

In the last decade or so we have witnessed the emergence of the so-called “New Consensus” on Macroeconomics (NC hereafter). The NC has been summarised in terms of a simple model with three equations: (i) an aggregate demand equation with the output-gap typically determined by past and expected future output-gap and the ex-ante real interest rate, (ii) a short-run Phillips curve with inflation typically based on the current output-gap and past and future inflation and (iii) a monetary policy rule of the Taylor’s rule form that endogenises the setting of interest rates by the CB. Proponents of the NC approach tend to view the level of economic activity as hovering in the short run around a supply-determined equilibrium that is assumed to be largely independent of the level and time path of aggregate demand. The former is typically determined by the interaction of firms’ price-setting and workers’ wage-setting decisions. This supply-determined equilibrium is referred to as the “natural rate of unemployment” or as the NAIRU, the acronym for “non-accelerating inflation rate of unemployment”. Hence, the NC strongly suggests that (i) there is no long-run trade-off between inflation and unemployment so that monetary policy does not have permanent effects on the level of economic activity and (ii) inflation is viewed as a monetary phenomenon that can be tamed through interest rate policy without incurring in higher (or lower) unemployment in the long run. The NC approach thus contains the neutrality of money property, with inflation determined by monetary policy (that is the rate of interest), and equilibrium values of real variables being largely independent of the money supply.

As pointed out in Arestis and Sawyer (2003), inflation targeting (hereafter IT) is a policy prescription closely associated with the NC approach that involves rather more than just targeting a (low) rate of inflation as an objective of economic policy. It is taken
here to include: (i) the setting by an “independent” central bank of a numerical target range for the rate of (price) inflation; (ii) the use of interest rate adjustments to achieve that target; (iii) monetary policy only concerned with the rate of inflation and short-term output volatility. Importantly, the desirability of low inflation is closely linked with the neutrality of money property. We readily concede that an indisputable feature of the institutional framework that characterizes present-day economies is that central banks set short-term nominal interest rates to achieve an inflation target. However, we also believe that the empirical evidence that is available does not warrant the conclusion (or the assumption) that interest rates do not have permanent effects on the supply side of the economy. The most serious attack on the “neutrality of money” proposition stems from the notion of hysteresis. The former originated in the 19th century in the physical sciences to denote the persistent effects of temporary exposure of ferric metals to magnetic fields and refers to the condition that equilibria are history or path-dependent. In economics, the term hysteresis is commonly associated with dynamic linear models characterized by zero root systems (i.e., zero eigenvalue) for continuous time or by unit root systems for discrete processes (Giavazzi and Wyplosz, 1985; Amable et al., 1995).  

In such systems, it is the case that there is a continuum of equilibria and the final equilibrium reached, selected from within the continuum, depends on the features of the system. In a deterministic framework, the final equilibrium point depends on the initial conditions of the state variables as well as on the parameters describing the speed of adjustment. In a stochastic framework, shocks cumulate forever without progressively vanishing so that the position of the system at a given point in time is determined by the chronicle of exogenous shocks. According to Dutt (1997, p. 240) ‘systems of this kind can be called path-dependent systems’ since, in these models, history plays a key role in
the sense that the starting point and the time path of the economy determine the final outcome.

However, an important property of zero/unit root systems is that an initial shock followed by a second one of the same magnitude but opposite sign drives the system back to its initial position. Hence, if shocks are generated by a symmetric probability distribution, then negative and positive shocks cancel each other out in the long run. Presumably, the position taken by proponents of the NC and IT is that, since supply and demand shocks come and go and on average are zero, they do not leave any “scar” on the economy. It is disputable whether unfavourable shocks are as frequent and intense as favourable ones. Be that as it may, we do not wish to develop our assessment of IT on that feature. Nevertheless, we should like to note that the “reversibility” property has led some scholars to investigate the properties of so-called “hysteretic” systems. For instance, drawing on the work of the Russian mathematician Mark Krasnosel’skii, it has been argued that a system is “hysteretic” if it exhibits “remanence”, i.e., when the value of the output is permanently affected by an appropriate temporary change in the value of the input (see Amable et al., 1995). The crucial point is that, in such systems, a relevant exogenous force modifying the value of a given parameter $\lambda$ entails a change in the system dynamics so that if we bring parameter $\lambda$ back to the initial value, the system does not return to the original equilibrium point (Amable et al., 1995, p. 172; see also Dutt, 1997). In the context of linear zero/unit root systems, any “remanent” effect on output stemming from symmetrically distributed shocks can solely be caused by (i) an asymmetric adjustment of prices (or wages) to shocks of opposite sign or (iii) an asymmetric response by policy makers to shocks of different sign. In this study we aim to explore the former possibility. In particular, we show below that the presence of “asymmetric inflation dynamics” (hereafter AID) makes zero/unit root systems exhibit
“remanence”. We argue in section 2 that the empirical evidence for OECD countries suggests that: (i) when inflation is low, it responds more sluggishly when the output-gap is negative than when it is positive; (ii) this asymmetry grows larger as inflation declines further.

To the best of our knowledge, no study has yet explored the implications for unemployment of the existence of hysteresis effects and AID. A precursor to this type of analysis is Setterfield (1998) where there is an illuminating theoretical discussion about some implications of the existence of hysteresis and “adjustment asymmetries” and where the author goes through some particular examples including the existence of an asymmetry in the long-run Phillips curve stemming from the asymmetric behaviour of workers’ wage claims across the trade cycle (Setterfield, 1998, pp. 297-8). Therefore, according to us, the main contribution of this study is to analyse the dynamic properties of an economy characterized by the presence of hysteresis effects and AID. In this respect, we find that the interaction of these two features in an economy where the central bank sets interest rates with a view to achieving an inflation target: (i) attenuates macroeconomic volatility; (ii) imparts a long-run negative bias on the unemployment rate in the aftermath of demand shocks irrespective of their sign; (iii) blocks off the “reversibility” property of linear zero/unit root systems.

The study continues as follows. The following section presents some empirical evidence bearing on the existence of hysteresis effects and AID. We then present a simple model for a closed economy with a government sector. We work out its steady-growth properties and its short-run behaviour. The analysis of the long-run effects of demand and supply shocks follows and the final section summarizes and concludes.
2.- Empirical evidence on hysteresis and asymmetric inflation dynamics

Hysteresis can have different causes (see the survey in Røed, 1997). However, its main aspect is that the proportion of long-term unemployed persons rises and that the latter exert less of an influence on labour market developments and on wages in particular than do the short-term unemployed. There are many reasons why the long-term unemployed may have a hard time finding a job although the mainstream literature has typically emphasized that the human capital of the unemployed as well as their job search effort diminishes as the duration of unemployment lengthens (e.g. Blanchard and Summers, 1988; Ball, 1999). Other contributions have stressed the existence of insider-outsider relations (e.g. Blanchard and Summers, 1986; Lindbeck and Snower, 1989) and the shortage of capital (e.g. Sarantis, 1993; Rowthorn, 1999; Arestis and Biefang-Frisancho Mariscal, 2000) as a potential source of hysteresis. The latter refers to the existence of hysteresis effects due to the influence of aggregate demand on investment and thereby on the capital stock in the economy. Economists have recently used the concept of hysteresis especially in the field of unemployment theory since its properties seem to fit well the unemployment dynamics of the last decades, especially in Western Europe. For instance, Ball (1999) suggests that passive macroeconomic policies are largely to blame for the observed rise in unemployment in several OECD countries since 1985. In countries where policy shifted toward expansion after tight policy had disinflated the economy, unemployment rose only temporarily. By contrast, in those countries where monetary policy remained tight unemployment rose permanently (Ball, 1999, p. 190). Ball blames for this outcome to the presence of hysteresis effects that operated mainly through the effect on “equilibrium unemployment” of the fraction of the long-term unemployed.
Despite an initial wave of studies that claimed empirical support for the notion of hysteresis as an explanation of European unemployment (Blanchard and Summers, 1987; Cross, 1988; Franz, 1990), the subsequent emergence of controversial evidence in the nineties, especially in North America, led to an apparent decline in interest in this concept. However, several recent contributions have reopened the debate on the notion of hysteresis. For instance, based on the panel unit root test developed by Im et al. (2003), León-Ledesma (2002) tests for unit roots in the United States and the European Union and finds strong evidence in the case of the latter and weak evidence in the case of the former. Likewise, Jaeger and Parkinson (1994) apply the Kalman-filter technique to British, Canadian, German and US data and come up with significant hysteresis effects in Germany and the United Kingdom. Finally, Logeay and Tober (2006) use the Kalman-filter technique using explicit exogenous variables and come up with the finding that the dependence of the NAIRU on actual unemployment and long-term unemployment is significant for the Euro Area and Germany respectively. We believe these empirical findings are relevant enough so as to justify analysing the implications for macroeconomic performance of the existence of hysteresis in economies where the central bank regularly sets interest rates in order to achieve an inflation target.

The Japanese recession in the second half of the nineties showed that, despite the presence of a large negative output-gap for most of the period 1991-2002, inflation turned negative in the second half of the 1990s but, after 1998, core inflation remained stable at moderately negative levels reaching its trough at -0.79 percent in 2002 (De Veirman, 2007). In this respect, Krugman (1998) argues that the negative output-gap the Japanese economy exhibited in the late 1990s was largely underestimated in official statistics and suggests that it could have been as large as 8 per cent of GDP and it may have grown much larger since 1998. Likewise, Mourougane and Ibaragi (2004) estimate
Phillips curves for Japan and strong find evidence that at low or negative inflation rates, indicators of demand pressure have no statistically significant effect on price inflation. The behaviour of the Japanese inflation in that period poses a challenge to mainstream macroeconomic theory for it implies a refutation of the theory of the “natural” rate of unemployment, according to which, the inflation rate decreases whenever the output-gap is negative and vice-versa. As yet, no consensus has emerged as to the causes for the behaviour of the Japanese inflation rate in that period. In one of the scant studies on the topic, De Veirman (2007) assesses empirical evidence for three types of nonlinearity in the short-run Phillips curve to discover why large negative output gaps in Japan during the period 1998-2002 did not lead to accelerating deflation, but instead coincided with stable, albeit moderately negative inflation. He documents that this episode is best interpreted as reflecting a gradual flattening of the Phillips curve as the inflation rate decreased. His results favor the hypothesis advanced in the work of Ball et al. (1988) that declining (trend) inflation causes firms to set their prices less frequently, which would explain the observed gradual flattening in the short-run Phillips curve. However, he notes that although these models imply that the Phillips curve becomes flatter as trend inflation declines to zero they also predict that, once trend inflation becomes negative, any further decrease in trend inflation is associated with an increase in the frequency of price adjustments and, hence, in the slope of the short-run Phillips curve. In other words, this model does not predict that the risk of a deflationary spiral is negligible, as the explanations based in the ubiquitous presence of DMWR would have it. 3

Despite De Veirman’s conclusion, we think that “downward money wage rigidity” (hereafter DMWR) remains an important reason why the inflation rate may not behave symmetrically. As argued in Palley (1994) and Akerlof et al. (1996), the
presence of DMWR implies a convex long-run Phillips curve at low inflation rates. In particular, they claim that the lower the inflation rate, the larger is the fraction of firms which can only implement real wage cuts through a reduction in the money wage they pay to their workers. According to them, in the presence of DMWR a lower inflation rate implies that a larger fraction of firms is forced to pay real wages exceeding the wage which they deem optimal. In the model of Akerlof et al. (1996), this increases the long-run sustainable level of unemployment, an effect which becomes more pronounced as inflation decreases further below 3 percent and provides the rationale for the existence of a “grease” effect of inflation on the labour market whereby a marginal rise in the inflation rate may reduce the “equilibrium” rate of unemployment when inflation is low. By contrast, we may note that, if firms follow a markup pricing strategy and total average costs are roughly constant for a wide range of rates of capacity utilization, the presence of DMWR is more likely to manifest itself in a diminished tendency for the rate of inflation to decrease for a given (negative) output-gap as inflation falls below a certain threshold.4

There is now a strong body of empirical evidence indicating the presence of DMWR across a wide spectrum of countries (see Lebow et al., 2003, Akerlof, 2007, Holden, 2004 and Holden and Wulfsberg, 2008). Several explanations have been put forward for the existence of such rigidities, such as fairness and social norms (Bewley, 1999, and Akerlof, 2007) or labour market institutions (Holden, 2004). The combination of these factors implies that these nominal rigidities could persist for a long time even in a low inflation environment. Indeed, empirical studies for EU countries find that DMWR persists during low inflation periods (Agell and Lundburg, 2003 and Fehr and Lorenz, 2005). Finally, several studies on the U.S. labour market find that, despite a clear evidence of DMWR notwithstanding, the evidence in favour of a “grease” effect
of inflation on unemployment is weaker, that is to say, it is unclear whether the marginal benefit of inflation as “greasing” the labour market could be high when inflation is low (Groshen and Schweitzer, 1999, and Card and Hyslop, 1996). In addition to the evidence on DMWR, there is evidence that prices respond faster to cost increases than to decreases. In an important study, Peltzman (2000) examines literally hundreds of markets involving both producer and consumer goods and finds that output prices tend to respond much faster to input increases than to decreases. In particular, this tendency is found in more than two of every three markets examined and it is found as frequently in producer goods markets as in consumer goods markets. On average, the immediate response to a positive shock is found to be at least twice the response to a negative shock.

3.- The model

We now present the model we will utilize hereafter to analyze a variety of issues related to the impact on the level of economic activity of demand and supply shocks when the central bank implements a conventional IT strategy. The exposition is divided into four subsections. The first two contain the building blocks of the model, the third subsection contains the steady-growth analysis and the final one describes the behaviour of the economy in the short run.

3.1.- The supply side

Let us consider a one-sector economy with two inputs, labour and capital, and assume that (i) there is a large number of identical firms and (ii) they all utilize the same technology. If we aggregate across all firms in the economy, we may define potential output $\bar{Y}$ as:
\[ \bar{Y} = \lambda \cdot \bar{N} \leq v \cdot K \]  

(1)

where \( \bar{N} \) is the level of employment that keeps inflation constant in the absence of transitory supply shocks, \( K \) is the aggregate capital stock, and \( \lambda \) and \( v \) are respectively the productivity of labour and capital when the factors are fully utilized. The current rate of capacity utilization is:

\[ u = \frac{Y}{v \cdot K} \leq 1 \]  

(2)

where \( Y \) is the actual level of output. We postulate the existence of an “employment rate compatible with constant inflation” (hereafter CIER) which results from the conflicting income claims of workers and firms. Thus, the CIER can be interpreted as an “inflation barrier” albeit it may be affected by the level and time path of aggregate demand if, for instance, hysteresis effects are present. The presence of an “inflation barrier” entails that the central bank seeks to adjust interest rates to affect the level of economic activity so as to balance off the income claims of workers and firms. We define the rate of capacity utilization when \( Y = \bar{Y} \) as the “constant inflation capacity utilization” (CICU hereafter) which we denote by \( \bar{u} \) or:

\[ \bar{u} = \frac{\bar{Y}}{v \cdot K} = \frac{\lambda}{v} \cdot \bar{N} = \left( \frac{\lambda}{v} \right) \cdot \left( \bar{\varepsilon} \cdot L \right) \leq 1 \]  

(3)

and where we denote the by \( \bar{\varepsilon} \) the CIER, by \( L \) the labour force and where \( \bar{N} = \bar{\varepsilon} \cdot L \).

Let us now assume there is no overhead labour and firms are fully integrated, producing all the materials required for their final output so that prime costs are made up only of labour costs. If we also assume that firms practise mark up pricing, then the real (product) wage is determined by the firms’ profit-maximization objectives:

\[ \frac{w}{p} = \frac{\lambda}{m} \]  

(4)
where \( w \) is the money wage, \( p \) is the price level and \( m > 1 \) is one plus the average mark up set by firms over prime costs. Next, we assume there is DMWR across the economy so that, as inflation decreases below a threshold \( \pi > 0 \), the former responds more sluggishly when the output-gap is negative, i.e., when \( Y \ll \bar{Y} \) than when it is positive. If we also add the empirical proposition that, as in the 1998-2002 Japanese episode, the change in the inflation rate becomes negligible when the output-gap is negative and inflation decreases below a certain negative level \( (\phi L \pi CR - \phi U) / \phi U \), we have that:

\[
\hat{\pi} = \begin{cases} 
\phi U(u - \bar{u}) + \varepsilon \pi & \text{if} \quad u > \bar{u} \\
\phi L(u - \bar{u}) + \varepsilon \pi & \text{if} \quad u < \bar{u} 
\end{cases}
\]  

(5)

where \( \phi_L = \begin{cases} 
\phi_U + \phi_\pi (\pi - \pi CR) & \text{if} \quad \frac{\phi_\pi \pi^{CR} - \phi_U}{\phi_\pi} < \pi < \pi^{CR} \quad \phi_U, \phi_\pi > 0 \\
0 & \text{if} \quad \pi \leq \frac{\phi_\pi \pi^{CR} - \phi_U}{\phi_\pi}
\end{cases} \)

where \( \varepsilon_\pi \) is a variable that captures transitory cost-push shocks. The basic proposition in (5) is that, as inflation decreases below \( \pi^{CR} \), some firms will need to cut money wages if their product prices are to decrease and the proportion of firms that is subject to this constraint increases as inflation decreases further.

Thus, equation (5) tells us that the inflation rate adjusts differently depending on: (i) whether actual capacity utilization is higher or lower than the CICU and (ii) whether the inflation rate is higher, equal or lower than \( \pi^{CR} \). As such, parameter \( \phi_L \) may take on three different values. First, if the current inflation rate is equal or higher than the upper threshold \( \pi^{CR} \), then inflation dynamics will be symmetric in the sense that the change in the inflation rate (but not its sign) will depend solely on the \textit{absolute} value of the output-gap. If the inflation rate is lower than the upper threshold \( \pi^{CR} \) but higher than the
lower threshold \((\phi_\pi^c - \phi_\pi)/\phi_\pi\), then the adjustment of the inflation rate will be asymmetric in the sense that the change in the inflation rate will depend both on the size and the sign of the output-gap, that is to say, a positive output-gap will bring about a larger change in the inflation rate than a negative output-gap of similar size. In turn, the asymmetry will be larger the closer to the lower threshold the current inflation rate is. Finally, the inflation rate will stop decreasing when it becomes equal or lower than the lower threshold.

Finally, following a suggestion by Hargreaves Heap (1980), we assume that the dynamics of the CIER are given by:

\[
\dot{\sigma} = \zeta (\sigma - \bar{\sigma}) \quad \zeta \geq 0 \qquad (6)
\]

where the overdot denotes a time derivative and \(\zeta > 0\) measures the speed of adjustment of the CIER when it exhibits hysteresis effects. The case \(\zeta = 0\) corresponds to the case without hysteresis. Hence, we view the presence of hysteresis effects in the CIER as making it depend on the time path of the actual employment rate so that an increase in the latter tends to raise the CIER and vice-versa. To finish off, we may note that, as captured in (6), the CIER exhibits the “reversibility” property of zero/unit root systems albeit, as we show below, this property is jeopardized by the presence of AID.

3.2.- The demand side

The equilibrium condition in the goods market for a closed economy with a government sector when current output is equal to potential output is:

\[
s(r^n, z) \cdot \bar{Y} = I + PSBR \qquad (7)
\]

where \(s\) is the saving rate, \(I\) is (gross) investment, \(PSBR\) is the public sector borrowing requirements, and \(z\) is a vector of variables to be filled below. The real interest rate \(r^n\) is the “neutral” interest rate which we define here as the real interest rate that (in a closed
economy) makes planned saving at potential output equal to the sum of planned gross investment and the PSBR. It is better thought of as a long-term rate. If we divide (7) through by the (physical) capital stock $K$, denote the net investment rate by $g$, the rate of depreciation of capital by $\psi$ and make $b = PSBR/Y$, we can rewrite (7) as:

$$(s - b) \cdot \bar{u} \cdot \nu = g + \psi$$  \hspace{1cm} (8)$$

We now turn our attention to functions saving and investment functions $s$ and $g$. We assume that the saving rate $s$ is a function of the rate of growth of output $\hat{y}$, the real interest rate $r$ and a measure of shocks $\epsilon_s$ or:

$$s = s(\hat{y}, r, \epsilon_s)$$  \hspace{1cm} (9)$$

where $s_\hat{y} > 0$, $s_r > 0$, $\epsilon_s$ is a stochastic variable denoting shocks to the saving rate, and the subscripts denote the partial derivatives. The positive sign of derivative $s_\hat{y}$ stems from the so-called “disequilibrium hypothesis” (Marglin, 1984), according to which, the saving rate increases when real income rises faster than households can adapt their spending habits whereas the opposite occurs when income falls faster than households can rein in their spending. The positive sign of $s_r$ is here attributed to the presence of distribution effects. To be sure, if we assume (realistically) that the average propensity to consume of net debtors is higher than that of net creditors, then a rise in the real interest rate will redistribute income away from net debtors and towards net creditors thereby raising the aggregate saving rate.

Next, let us assume that firms have a desired rate of capacity utilization $u^* < 1$ so they expand capacity when $u > u^*$ and scale investment down when $u < u^*$. A justification for this assumption is that firms operating in imperfect goods markets keep some capacity idle in order to respond rapidly to unanticipated surges in demand and to
deter the potential entry of rivals in the industry (Spence, 1977). This assumption can be captured by defining the rate of accumulation, \( g \) as:

\[
g = v \cdot u \cdot f(u - u^*, \varepsilon_g)
\]  

(10)

where \( f_u > 0 \) is inversely proportional to the length of the capital goods construction and delivery lags, \( f(0) = \tilde{f} > 0 \) and \( \varepsilon_g \) captures exogenous shocks hitting \( g \). \( \tilde{f} \) is the ratio of net investment to output when \( u = u^* \) and captures firms’ average expected future rate of growth of demand. As for function \( b \), we assume that:

\[
b = b(\hat{\gamma}, r, \varepsilon_b)
\]  

(11)

where the partial derivatives satisfy \( b_{\gamma} < 0 \), \( b_r > 0 \) and where \( \varepsilon_b \) are exogenous shocks affecting \( b \). The negative sign of \( b_{\gamma} \) is ascribed here to the working of fiscal automatic stabilizers so its absolute value measures the stabilizing power of (non-discretionary) fiscal policy. By contrast, the positive sign of \( b_r \) captures the impact on PSBR of changes in the flow of interest payments due to the holders of government debt resulting from variations in real interest rates. Finally, \( \varepsilon_b \) captures changes in the stance of discretionary fiscal policy. All this allows us to rewrite (8) as:

\[
s(\hat{\gamma}, r^*, \varepsilon_s) - b(\hat{\gamma}, r^*, \varepsilon_b) = f(u^* - u^*, \varepsilon_g) + \frac{\psi}{\psi_u}
\]  

(12)

where (12) represents the equilibrium condition in the goods market when \( Y = \tilde{Y} \).

3.3.- Steady growth analysis

Steady growth equilibrium corresponds to a period of sufficient length to enable all the variables in the economy to settle at constant rates in the absence of new shocks. In a hypothetical steady growth equilibrium we have that \( \varepsilon_s = \varepsilon_g = \varepsilon_b = 0 \), \( \hat{\gamma} = g_n = g \), \( u = \bar{u} = u^* \) and \( r = r^* \), where \( g_n = l + a \) is the “natural” rate of growth and \( l \) and \( a \) are
respectively the growth rate of labour force $L$ and labour productivity $\lambda$. We then have that, in steady growth, the following two conditions must be satisfied:

$$v \cdot u^* \cdot \tilde{f} = g_n = \hat{y} \quad (13)$$

and

$$[s(g_n, r^*) - b(g_n, r^*)] \cdot v \cdot u^* = g_n + \psi \quad (14)$$

Equation (13) tells us that, in steady growth, the rate of accumulation must equal the “natural” rate of growth. Equation (14) is the counterpart to equation (12) for the steady growth equilibrium. To obtain a solution for the steady growth “neutral” interest rate $r^*$ we assume that functions $s$ and $b$ adopt a linear form or:

$$s = \bar{s} + s_y \hat{y} + s_r r \quad (15)$$

and

$$b = \bar{b} + b_y \hat{y} + b_r r \quad (16)$$

where $\bar{s}$ is a shift term determined by individuals’ preferences, institutional factors and the average degree of liquidity preference and $\bar{b}$ denotes the stance of fiscal policy. Substituting equations (15) and (16) into (14) and re-arranging we arrive at:

$$r^* = \left[ \frac{g_n + \psi}{v \cdot u^*} - c_i - c_y g_n \right] \frac{1}{c_r} \quad (17)$$

where $c_i = \bar{s} - \bar{b}$, $c_y = s_y - b_y > 0$ and, for the sake of the argument, we assume that $c_r = s_r - b_r > 0$. Thus, $r^*$ depends on the “natural” rate of growth, the aggregate saving rate, the stance of fiscal policy and the depreciation rate, and it can be interpreted as the real interest rate where ‘all markets are in equilibrium and there is therefore no pressure for any resources to be redistributed or growth rates for any variables to change’ (see Archibald and Hunter, 2001, p. 20). Furthermore, $r^*$ also represents the real interest rate that is compatible with a neutral monetary policy in the long run and so it represents the interest rate that commonly enters into Taylor’s rule (Taylor, 1993).
3.4.- Short-run dynamics

Steady growth equilibrium is only valid for explaining a hypothetical long-run scenario where the effects of shocks and lags have already worked themselves out. Admittedly, that scenario is unrealistic because an economy is constantly being shocked away from its steady growth equilibrium. Yet, the steady growth analysis provides an equilibrium outcome around which the economy hovers in the short run and so it may offer some insights. Next, we analyse the behaviour of the economy in the short run where the former is defined as the time it takes for the actual real interest rate to affect inflation. As before, we assume that the investment function $f$ adopts a linear form or:

$$f(u - u^*, e_u) = f_u^*(u - u^*)$$  \hspace{1cm} (18)

Substituting (18), (15) and (16) into (12) and re-arranging we obtain $\hat{y}$:

$$\hat{y} = \frac{\tilde{f} - c_1 - c_r r + f_u^* (u - u^*) + \psi}{c_r v u}$$ \hspace{1cm} (19)

A solution for $r^u$ can be obtained by setting $u = \bar{u}$ in (19) and rearranging:

$$r^u = \frac{\tilde{f} - c_1 - c_r \hat{y} + f_u^* (\bar{u} - u^*) + \psi}{c_r}$$ \hspace{1cm} (20)

so we have that:

$$\frac{\partial r^u}{\partial \bar{u}} = \frac{c_2}{c_r} < 0$$ \hspace{1cm} (21)

where $c_2 = \left(f_u - \frac{\psi}{v u^2}\right)$. 

Expression (21) highlights that a change in the CICU has an ambiguous impact upon $r^u$. This is because, an increase in the CICU and, hence, in $\tilde{F}$, brings about an increase in the flow of private saving at the higher level of output that may or may not be offset by the resulting higher rate of investment. If $c_2 < 0$, the increase in the flow of saving will not be offset by the higher rate of investment and vice-versa. Hence, the
standard textbook assumption that at the margin private saving is more responsive than
investment to changes in capacity utilisation for making the Keynesian income
adjustment process stable requires imposing a negative sign on $c_2$.

4.- The effect of demand and supply shocks

We now address the consequences for macroeconomic stability of the presence
of hysteresis effects and AID. To do so, we need to derive three additional equations.
First, the presence of hysteresis means that the CIER is affected by the time path of the
employment rate and that changes in the CIER affect the CICU as shown in expression
(3). If the take logarithms in (3) and differentiate it with respect to time we get:

$$\frac{\dot{u}}{u} = \frac{\dot{c}}{c} + g_n - g$$

(22)

For the sake of convenience, we assume that the central bank knows $r^*$ and that
it sets real interest rates according to the policy rule:

$$r = r^* + \alpha (\pi - \pi^*)$$

(23)

where $\alpha$ is the response coefficient of monetary policy to changes in the inflation gap,
i.e., the difference between the actual and the target inflation rate. Therefore, the actual
long-term real interest rate is determined by a Taylor-type policy rule. Differentiation
of (2) yields:

$$\frac{\dot{u}}{u} = \dot{y} - g$$

(24)

which describes the dynamics of the actual rate of capacity utilization. Thus, the model
we use below in the simulation exercises is made up of the following equations: (5), (6),
(10), (17), (18), (19), (22), (23) and (24). Simulation results are shown in figures 1
through 22 in the appendix. Table 1 contains the parameter values, the initial conditions
and the values of the operators that describe the stability conditions. The values ascribed
to the parameters of the model are justified in the appendix. Tables 2 and 3 summarize the shocks and scenarios considered in each simulation exercise. The second column in tables 2 and 3 explain the type of shock analysed in each exercise whereas the third column identifies the parameter values in each simulation exercise that differ from the values originally reported in table 1.

4.1.- The impact on macroeconomic volatility

Figures 1 through 3 below show the time-path of the inflation rate, capacity utilization and the employment rate in the aftermath of an adverse shock to the inflation rate when the economy exhibits neither hysteresis effects nor AID. For the set of parameter values reported in table 1 the economy exhibits self-sustained oscillations. The former can be interpreted as the result of the interplay in a non-linear context of an (unstable) multiplier-accelerator-type mechanism and the policy-induced stabilizing behaviour of the real interest rate. Figures 4 through 6 show the time-path of the same variables when the economy exhibits hysteresis. It can be seen that the presence of hysteresis effects dampens the oscillations. This is because, as the actual employment rate decreases, the CIER also decreases along with it and this, in turn, cushions the downward pressure on the inflation rate and the real interest rate. Conventional wisdom suggests that this “cushion” effect should weaken rather than reinforce macroeconomic stability because, as inflation decreases, the CB lowers the real interest rate and this, in turn, spurs aggregate spending thereby helping to reverse the initial shock. However, the simulation exercise shows that, at least in the context of our model, the presence of hysteresis ameliorates macroeconomic volatility. The exercise also revealed that the higher parameter $\zeta$ is the more dampened oscillations become. Furthermore, figures 7
through 9 show that the presence of AID also dampens oscillations. Thus, the presence of hysteresis and AID appears to reinforce the stabilizing power of monetary policy.

4.2.- The impact of shocks to the inflation rate

As is well-known, the presence of hysteresis effects implies that persistent but nevertheless transitory shocks may have permanent effects on the economy. For instance, figure 6 shows that a disinflation process in the wake of an adverse supply shock that initially raises the actual inflation rate above its target \( \pi_0 = 0.035 > \pi^* \) imposes on the economy a permanently lower employment rate. However, as pointed out above, systems exhibiting (unit-root) hysteresis possess the “reversibility” property. Following with the previous example this means that, if the economy is now subject to a shock of the same intensity but opposite sign (e.g. \( \pi_0 = 0.005 < \pi^* \)), the employment rate stabilizes at a higher level in the long run (see figure 16). Importantly, the long-run decrease in the employment rate brought about by the unfavourable shock is roughly equal to the long-run increase brought about by the favourable one.

Next, we address the long-run consequences for unemployment of supply shocks when the economy exhibits hysteresis effects and AID.\(^8\) In figure 14, the disinflation process brought about by an adverse supply shock makes the employment rate settle at 0.884 in the long run, well below its initial level at 0.9. By contrast, in the wake of a shock of equal intensity but opposite sign the employment rate stabilizes at 0.908 (see figure 15). The existence of a negative bias to the employment rate is confirmed by the stochastic simulation exercise depicted in figures 17 through 19. Figure 17 shows the time path of the employment rate when the economy is subject to a sequence of random supply shocks \( \varepsilon_x \) where the latter were generated by a normal probability distribution of zero mean and standard deviation equal to 0.0096 (see figure 19). In the absence of
AID, the employment rate rapidly drops off and converges to zero. Figure 18 shows the result of the replication of the exercise when the economy exhibits hysteresis effects and AID. The employment rate now fluctuates around a decreasing trend. Therefore, as far as inflation is concerned, the presence of hysteresis effects and AID blocks off the “reversibility” property exhibited by zero/unit root systems by imparting a long-term negative bias on the employment rate in the sense that, if the economy is initially at equilibrium and is subjected to a unfavourable supply shock that raises the inflation rate and this is subsequently followed by a second shock of the same intensity but in the opposite direction then the employment rate does not return to the initial equilibrium but stabilizes at a lower level.

4.3. The impact of demand shocks

We now investigate the long-run effect on the employment rate of demand shocks when the CIER exhibits hysteresis effects. The presence of an inflation-targeting central bank implies that the short-run impact of demand shocks will be, at least partially, offset by changes in real interest rates. This is because the demand shock affects \( r^* \) and, if the central bank periodically updates its estimate of \( r^* \), the impact on aggregate demand is offset to some extent by a change in the real interest rate.\(^9\) Figures 10 and 11 depict the long-run impact on the employment rate of a favourable and an unfavourable demand shock respectively. The simulation exercise confirms that, in the absence of AID, demand shocks do not affect the employment rate in the long run which is equal to the initial (supply-determined) one.

Next, we analyse how the previous results vary when there is both hysteresis effects and AID. Figure 12 shows the case of a favourable demand shock whereas figure 13 shows the case of an unfavourable one. In both cases, the employment rate settles
below its initial level at 0.9, albeit the long-term decrease is clearly more pronounced when the demand shock is unfavourable. Therefore, the presence of AID makes demand shocks have an *adverse* long-term impact on the employment rate even when the former are expansionary. This result is confirmed in the stochastic simulation exercise depicted in figures 20 through 22. As with supply shocks, we subject the aggregate saving rate to random shocks drawn from a normal probability distribution of zero mean and standard deviation equal to 0.0084 (figure 22). Figure 20 shows the time path of the employment rate when the economy only exhibits hysteresis effects whereas figure 21 shows the time path when the economy also exhibits AID. In the first case, the employment rate hovers in the short run around a *stationary* trend whereas, in the second case, the employment rate exhibits a *declining* trend. In short, demand shocks are, at least partially, neutralized by changes in real interest rates so they do affect the employment rate in the short run but do *not* affect it in the long run unless the economy exhibits AID; in this latter case they affect it adversely even when shocks are expansionary.

5.- Summary and conclusion

In this study we analysed the long-run impact on the employment rate of supply and demand shocks in an economy characterized by the existence of: (i) a short-run “inflation barrier” referred to as the “constant inflation employment rate” (CIER) that exhibits hysteresis effects, (ii) a central bank that sets real interest rates to achieve an inflation target and (iii) asymmetric inflation dynamics (AID). For that purpose, we postulated a macroeconomic model for a closed economy with a government sector that incorporates the above-mentioned features. We obtained several results. First, we found that, in an economy characterized by the above features but AID, demand shocks (even when they are permanent) do not affect the employment rate in the long run. We argued
that this occurs because demand shocks bring about a change in the steady growth neutral interest rate and this, in turn, is passed through into the actual real interest rate by virtue of the monetary policy rule of the CB. Second, we showed that the presence of hysteresis effects and AID in the type of economy sketched above: (i) attenuates macroeconomic volatility and thus reinforces the stabilizing power of monetary policy, (ii) imparts a long-run negative bias on the employment rate in the aftermath of demand shocks irrespective of their sign (although the negative effect is more pronounced when shocks are contractionary) and (iii) blocks off the “reversibility” property exhibited by linear zero/unit root systems in the wake of supply shocks.

Appendix

For the sake of the argument, we make a number of simplifying assumptions. First, and foremost, we assume that the central bank knows \( r^* \). Second, we impose the condition \( \tilde{f} = (g_n/v \cdot u^*) \). The former means that firms’ output growth expectations are firmly anchored to its secular growth rate so we remove any instability that may result from changes in firms’ (volatile) profit expectations. Admittedly, the former provides the economy with a built-in stabilizing mechanism but it makes the system much easier to handle. Third, we assume that \( r^* \) is positive and high enough so as to make the zero lower bound constraint on nominal interest rates not bind. Finally, the analysis focuses on the local stability of the economy. We may obtain the singular points of the system by setting \( \dot{\pi} = \dot{u} = \ddot{u} = 0 \) which yields two singular points:

\[
\begin{align*}
\pi_{1,2}^* &= \frac{\tilde{f}v - v f_u u^* \pm \sqrt{(-\tilde{f}v + v f_u u^*)^2 + 4v f_u g_n}}{2v f_u} \\
\pi_{1,2}^* &= \frac{-\tilde{f} - c_1 + f_u (u-u^*) - c_2 r^* + c_3 \pi^* + \frac{\psi}{\psi u} - c_3 \tilde{f} u^* + c_3 v f_u \pi^* + c_3 v f_u u^*}{c_n + c_2 \alpha}
\end{align*}
\]
where the only one with economic meaning is:

\[ P^* = (\pi^*, u^*), \]

The simulation exercise was aimed at exploring the implications for the long-run behaviour of the employment rate of the existence of hysteresis effects and AID. Table 1 reports the values of the parameters of the model, including \( r^* \) and \( \tilde{f} \), and the initial conditions, whereas tables 2 and 3 report the values of those parameters and initial conditions whose values differ in successive simulation exercises from those reported in table 1. The parameters and/or initial conditions responsible for the single shock are underlined. The values of the parameters were chosen according to the values typically reported in the literature. For instance, the inflation target for many central banks is 2 percent. The literature usually reports that the technical output-capital ratio \( v \) is about 0.3. Studies for the U.S. economy suggest that the CICU is about 82 percent (Garner, 1994 and Corrado and Mattey, 1997) and the value assigned to \( \phi_U \) stems from results in McElhattan (1985) who finds that, for each percentage point that capacity utilization exceeds 82 percent, inflation accelerates by about 0.15 percentage points. As for \( \pi^{CR} \), we follow Akerlof et al. (1996) and assume that the presence of DMWR starts to bite when the inflation rate is less than 3 percent. This implies that, if \( \phi_x = 4 \) and \( \phi_U = 0.15 \), then the inflation rate ceases to decrease when it is equal to -0.0075 which is roughly the level at which the Japanese inflation rate settled after 1998. Next, the resulting value for the short-term real interest rate — taking into account that \( r^* \) is a long-term interest rate and so we need to subtract a risk/term premium to obtain the former — is roughly the value of the \( \textit{real} \) federal funds rate over the 1960-1998 period in the U.S. if we assume the term premium is about 1 percent: 2.55 percent (Reifschneider and Williams, 2000, p. 950). As for the parameters in the saving and investment function, we set them
so as to render the economy stable and set $\bar{f}$ equal to its steady growth value to make the model more manageable. The standard deviation of the normal distributions we used to generate a set of random supply and demand shocks were taken from Orphanides and Wieland (2000, p. 1373) who estimate a NC-type model for the Euro area for the period 1976-1998 and report a standard deviation of supply and demand shocks equal to 0.96 and 0.84 respectively. Finally, the subscripts of the variables in table 1 denote the values that parameters adopt in a series of simulation exercises.

$$f_{\nu,0} = 0.015, \quad c_r = 2.3, \quad \zeta_2 = 0.3, \quad e_0 = \bar{e}_0 = 0.9$$

$$f_{u,1} = 0.3, \quad u^*_0 = 0.8, \quad g_n = 0.03, \quad \alpha = 0.5$$

$$\pi^* = 0.02, \quad \pi^{CR}_0 = 0, \quad \phi_U = 0.15, \quad u_0 = \bar{u}_0 = 0.8$$

$$c_1 = 0.14, \quad \pi^{CR}_1 = 0.03, \quad \phi_{\pi,0} = 0, \quad \bar{f} = 0.125$$

$$c_2 = -0.03, \quad \zeta_0 = 0, \quad \phi_{\pi,1} = 4, \quad \psi = 0.035$$

$$c_{\tilde{\gamma}} = 1.5, \quad \zeta_{\tilde{\gamma}} = 0.1, \quad \nu = 0.3, \quad r^* = 0.037$$

### Table 1: Parameter values, initial conditions and operators

<table>
<thead>
<tr>
<th>Fig.</th>
<th>Single Shock</th>
<th>Parameter values (when they differ from those reported in table 1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fig. 1</td>
<td>Unfavourable supply shock</td>
<td>Absence of hysteresis and AID: $\pi_0 = 0.035$</td>
</tr>
<tr>
<td>Fig. 2</td>
<td>Unfavourable supply shock</td>
<td>Absence of hysteresis and AID: $\pi_0 = 0.035$</td>
</tr>
<tr>
<td>Fig. 3</td>
<td>Unfavourable supply shock</td>
<td>Absence of hysteresis and AID: $\pi_0 = 0.035$</td>
</tr>
<tr>
<td>Fig. 4</td>
<td>Unfavourable supply shock</td>
<td>Hysteresis, no AID: $\pi_0 = 0.035$ and $\zeta_1 = 0.1$</td>
</tr>
<tr>
<td>Fig. 5</td>
<td>Unfavourable supply shock</td>
<td>Hysteresis, no AID: $\pi_0 = 0.035$ and $\zeta_1 = 0.1$</td>
</tr>
<tr>
<td>Fig. 6</td>
<td>Unfavourable supply shock</td>
<td>Hysteresis, no AID: $\pi_0 = 0.035$ and $\zeta_1 = 0.1$</td>
</tr>
<tr>
<td>Fig. 7</td>
<td>Unfavourable supply shock</td>
<td>AID, no hysteresis: $\pi_0 = 0.035$, $\pi_1^{CR} = 0.03$ and $\phi_{\pi,1} = 4$</td>
</tr>
<tr>
<td>Fig. 8</td>
<td>Unfavourable supply shock</td>
<td>AID, no hysteresis: $\pi_0 = 0.035$, $\pi_1^{CR} = 0.03$ and $\phi_{\pi,1} = 4$</td>
</tr>
</tbody>
</table>
Table 2: Summary of parameter values and initial conditions in the simulation exercises with a single shock

<table>
<thead>
<tr>
<th>Fig.</th>
<th>Shocks</th>
<th>Parameter values (when they differ from those reported in table 1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fig. 17</td>
<td>Stochastic supply shocks</td>
<td>Hysteresis, no AID: $\pi_0 = 0.02$ and $\zeta_2 = 0.3$</td>
</tr>
<tr>
<td>Fig. 18</td>
<td>Stochastic supply shocks</td>
<td>Hysteresis and AID: $\pi_0 = 0.02$, $\pi_1^{CR} = 0.03$, $\phi_{\pi,1} = 4$ and $\zeta_1 = 0.1$</td>
</tr>
<tr>
<td>Fig. 19</td>
<td>Stochastic supply shocks</td>
<td>Drawn from a normal distribution</td>
</tr>
<tr>
<td>Fig. 20</td>
<td>Stochastic demand shocks</td>
<td>Hysteresis, no AID: $\pi_0 = 0.02$ and $\zeta_1 = 0.1$</td>
</tr>
<tr>
<td>Fig. 21</td>
<td>Stochastic demand shocks</td>
<td>Hysteresis and AID: $\pi_0 = 0.02$, $\pi_1^{CR} = 0.03$, $\phi_{\pi,1} = 4$ and $\zeta_1 = 0.1$</td>
</tr>
<tr>
<td>Fig. 22</td>
<td>Stochastic shocks to the saving rate</td>
<td>Drawn from a normal distribution</td>
</tr>
</tbody>
</table>

Note: The parameters underlined in the third column are the ones that trigger the shock.
Fig. 1: Inflation rate

Fig. 2: Capacity utilization

Fig. 3: Employment rate

Fig. 4: Inflation rate
Fig. 5: Capacity utilization

Fig. 6: Employment rate

Fig. 7: Inflation rate

Fig. 8: Capacity utilization
Fig. 9: Employment rate

Fig. 10: Employment rate

Fig. 11: Employment rate

Fig. 12: Employment rate
Fig. 13: Employment rate

Fig. 14: Employment rate

Fig. 15: Employment rate

Fig. 16: Employment rate
Fig. 17: Employment rate

Fig. 18: Employment rate

Fig. 19: Supply shocks

Fig. 20: Employment rate
Fig. 21: Employment rate

Fig. 22: Demand shocks

References


* E-mail: apv@ccce.ucm.es. Correspondence address: Alfonso Palacio-Vera, Departamento de Economía Aplicada III, Facultad de Ciencias Económicas y Empresariales, Campus de Somosaguas, Universidad Complutense de Madrid, Pozuelo de Alarcón, 28223 (Madrid), Spain. Acknowledgements: the author would like to thank Engelbert Stockhammer for his useful comments and suggestions on a previous draft of this study. The usual disclaimer applies.

1 Detailed expositions of the NC approach can be found in Clarida et al. (1999) and Meyer (2001) and two recent critical reviews are in Arestis and Sawyer (2006, 2008).

2 However, as noted in Setterfield (1998, p. 283), ‘the unit root approach is based on the assumption of linear functional forms which, although convenient in its simplicity, is ultimately arbitrary. Postulating non-linearities in economic relationships can quickly give rise to hysteresis regardless of the existence of unit roots’.

3 Surprisingly, he rejects outright the explanation based on DMWR by citing one study which supports the presence of DMWR in Japan for the period 1992-1998 and a more recent study by the very same authors which finds no evidence for DMWR in the nominal wages of full-time Japanese workers during the period 1998-2001 (De Veirman, 2007, p. 11).

4 Unlike in a perfectly competitive economy, in an imperfectly competitive one the price-setting curve may not be negatively-sloped so that, for the relevant range of the curve, there may be a neutral or even a positive relationship between the employment rate and the real (product) wage. The former implies that, in the presence of DMWR, an increase in the real (product) wage paid by the firms when the inflation rate is low or negative may not necessarily result in a higher level of “equilibrium unemployment”.

5 The non-accelerating inflation rate of unemployment or NAIRU is hence equal to 1 − 1 . In this study, we assume that the CIER is equivalent to the notion of “equilibrium unemployment”. The latter refers to a rate of unemployment that is determined at the intersection of a price-setting function and an upward-sloping wage-setting function in real wage-employment space. Admittedly, a substantive difference between these two concepts is that the NAIRU is typically defined as a situation where the rate of inflation is constant, whereas the only requirement of the concept of “equilibrium unemployment” is that
real wages are constant (when there is no productivity growth) or else grow at the same pace than productivity. Yet, the two approaches can be reconciled by grafting money wage and price dynamics onto the static approach (see Lindbeck, 1993, Appendix B),

6 In general, an asterisk denotes the steady growth value of the variable. However, in the case of $u$ its steady growth value coincides with the desired rate of capacity utilization.

7 An interest rate policy rule like this can be derived from a loss function according to which the central bank seeks to minimize the deviation of the actual inflation rate and actual level of output from their respective target levels (see, for instance, Carlin and Soskice, 2005).

8 For the sake of simplicity, we assume that supply shocks affect the inflation rate but do not affect the CIER.

9 In this study we assume that the central bank estimates the steady growth neutral interest rate correctly in order to frame our results in the most favourable scenario for the implementation of monetary policy decisions. Of course, we readily admit that central banks face enormous practical difficulties when estimating it (see, for instance, Weber et al., 2008).