The aim of this short article is to build a model in order to take into account capital scrapping (or bankruptcies) in an income distribution and growth model. The reason to introduce capital scrapping results from the intuition of some inconsistencies between theoretical predictions and empirical facts: the rate of capacity utilisation data often exhibit a greater stability than it is expected after reading theoretical models. We think that capital scrapping might contribute to stabilise the utilisation rate. The idea is as follows: an increase of the profit share implies a decrease of the rate of capacity utilisation which then involves a rise in capital scrapping (or in the rate of bankruptcies).

We briefly present some empirical facts in the first section. Section 2 is devoted to the theoretical predictions in income distribution and growth theories. We survey the existing ways to specify capital depreciation and/or capital scrapping in section 3. Our model is presented in section 4. Then, we conclude.

1. **Empirical facts**

In many countries, the rate of capital utilisation is subject to cyclical variations, related to business cycles, around a stable level. In this section, we focus on French economy.

Between 1965 and 2006, the average rate of capital utilisation is equal to 83.9%. Moreover, the convergence towards this “equilibrium level” is usually pretty fast: after the dramatically 1975 downturn ($u = 78.7\%$), the French utilisation rate goes back to 83.5% from 1976; similarly, $u$ drops to 79.8% in 1993 but reaches 82.0% in 1994 and 83.8% in 1995. The French example is particularly illustrative, but other countries display the same kind of profiles (Germany, United Kingdom, Netherlands, Spain, Italy, etc).
Moreover, from a Post Keynesian standpoint, the relative stability of the utilisation rates is all the more surprising since the most of European countries (France especially) have experienced large variations of income distribution for forty years. For example, the French profit share lost 6 points in the 1970s, then it increased of almost 8 points during the 1980s and stabilised thereafter Compared to these large fluctuations, utilisation rate variations seems quite moderate in the medium run:

2. The rate of capacity utilisation in income distribution and growth theories

The rate of capacity utilisation \( (u) \) plays a crucial role in most of the Post Keynesian growth theories for many reasons: it is the core of the accelerator effect in firms accumulation decisions; as a component of the profit rate, it contributes both to the profitability effects and to the capitalists saving decision; in addition, firms may adjust their investments to reach a normal rate of capacity utilisation \( (u_n) \). It is useful
to make a distinction between three kinds of models: Kaleckian models, the Bhaduri & Marglin [1990] specification, and the Kaldorian/Robinsonian models.\(^5\)

In their standard specification (closed economy, no workers saving), Kaleckian models are stagnationist, cooperative and wage-led at once. It means that an increase of the profit share (that we note \(\eta\) afterwards) results in a decrease of the capacity utilisation rate \((u)\), the profit rate \((r = \eta \cdot u / \nu\), where \(\nu\) stands for a constant capital coefficient) and the accumulation rate \((g = I / K\) where \(I\) and \(K\) represent investment and capital stock respectively). Hence, an exogenous shock on \(\eta\) implies a shock on \(u\) in the opposite way. Moreover, in order to be cooperative, the \(u\) variation must outweigh (in percentage) the \(\eta\) variation \((i.e. (du/u)/(d\eta/\eta) < -1)\). It is worth noting that it rarely happens (for example, only 10% of the French quarterly data exhibit the ‘good’ elasticity between 1979 and 2006). Lavoie [1996] and Lavoie, Rodriguez & Seccareccia [2004] introduce a hysteretic mechanism in order to explain the long-run convergence of \(u\) towards \(u_n\). But this mechanism is of little interest for our purpose for two reasons: firstly, it is a long-run mechanism; secondly, convergence occurs because firms adjust their normal rate through time (while the long-run rate seems to be constant).

Bhaduri & Marglin [1990] opened a new strand of models by introducing a slight difference in their specification. Assuming that the accumulation rate depends on \(u\) and \(\eta\) (instead of \(u\) and \(r\)), they show that the consequences of a shock on the profit share remain indeterminate: \(u\) and \(\eta\) may evolve either in opposite or in the same ways (stagnationist or exhilarationist regime) depending on the parameters’ value of the model. Moreover, parameters’ value is a matter of facts rather than of theoretical assumptions. It is then possible to combine large variations of \(\eta\) with small impact on \(u\). But, accepting this solution consists in admitting that income distribution has a weak impact on activity and growth. We are reluctant to accept this conclusion. This is why we try to enrich the model in order to conciliate the stability of the rate of utilisation with some significant effect of distribution on accumulation.

At last, the Kaldorian/Robinsonian models rest on the assumption that \(u = u_n\) in the long-run (Kaldor [1957], Robinson [1962]). This assumption seems to fit correctly the data, but it leaves us unsatisfied for two reasons: firstly, this equality is postulated (rather than proved); secondly the adjustments take place through the profit share variations, which is then endogenous. If this latter assumption is rejected, it is necessary to admit that \(u\) may differ from \(u_n\) (at least in the short or medium-run), and some mechanisms must be defined in order to restore the equality. Models must then get round to Harrodian instability \((i.e. when firms react to u < u_n by cutting their investments, the decrease of u worsen)\). Some solutions have been proposed to this problem\(^6\) but they are not in the scope of this article because they are based on long-run mechanisms while \(u\) seems to be stable even in the short-run.

To sum up this brief survey, theoretical predictions appear to be inconsistent with empirical data because either they assume that the utilisation rate is endogenous and exhibits wide variations or the convergence of \(u\) towards \(u_n\) is a long-run mechanism. Furthermore, if theoretical predictions are consistent with data, income distribution could have no impact on activity and growth. In the rest of the

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\(^6\) See for instance Duménil & Lévy [1999] or Skott [2008].
paper, we try to depart from these conclusions by taking capital scrapping and firms bankruptcies into account.⁷

3. Capital depreciation and capital scrapping

If accumulation depends on investment, it also depends on capital depreciation ($\delta$), that is:

$$K_{t+1} = (1 - \delta)K_t + I_t$$

In Post Keynesian income distribution and growth models, capital depreciation is usually introduced as an exogenous parameter. Nevertheless, there are two notable exceptions. On the one hand, Kurz [1991] and Bruno [1999] assume that $\delta$ is positively related to $u$ because of physical wear and tear: machines quickly damage when their use is intensive.

On the other hand, Steindl [1979] and Cassetti [2006] assume that $\delta$ is negatively related to $u$. The reason is that the expected life-time of capital is supposed to be finite and equipments are scrapped when they arrive at the end of their life.⁸ But the expected life-time might be adjusted to business cycles: firms may delay the capital scrapping when $u$ is high, and they can anticipate it when $u$ is low (or some firms bankrupt and their equipments are scrapped).

This capital scrapping has an interesting property: it contributes to the stabilisation of the rate of capital utilisation. In the case an economic slump, capital scrapping slacks the decrease of $u$ (while this decrease is intensified if firms react by cutting their investments).

We depart from Steindl’s analysis in two ways. Firstly, Steindl assumes that these capital scrapping adjustments are representative of a competitive economy whereas, in the contemporary oligopolistic regimes, “firms are more prepared to accept low long-term rates of utilisation” (p. 7). However, time series do not display the expected long waves which would result from Steindl’s conclusions. On the contrary, utilisation rates are subject to cyclical variations around a stable level. This leads us to keep the capital scrapping adjustment as a mechanism which could stabilise the rate of utilisation still now.

Secondly, neither Steindl nor Cassetti give a formal proof of the negative relation between the rate of utilisation and capital scrapping (which is postulated). Our main purpose is to find such a formal proof.

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⁷ Of course, a better way to assess the consistency between theoretical predictions and empirical data should have been to make some econometric tests. Nevertheless, our intuition is supported by a previous econometrical work (Allain & Canry [2007]) and by the fact that the results of other empirical studies hardly converge towards clear conclusions about the regime (stagnationist or exhilarationist) of real economies (see for instance Naastepad & Storm [2006] or Hein & Vogel [2007]).

⁸ The most part of Steindl’s analysis rests on the automatic variations of capital scrapping resulting from the accumulation rate changes: when growth decreases, the rise of the life-time of capital implies that capital scrapping automatically increases. Following Bhaduri [1972], Steindl compares this capital scrapping with the funds that firms constitute to replace their equipment (say $aK$). At the stationary state (growth is constant), $\delta = a$. But in a growing economy, the life-time of capital decreases ($\delta < a$) and firms replace more capital than the scrapped one.
4. The model

We propose a model in order to assess the impact of a change in income distribution on both investment and capital scrapping. It is a one period model which implies that all the economic decisions occur in the time of the model. The impact of a change in income distribution may then be analysed by comparative statics calculations.

4.1. Framework

It is necessary for our purpose to introduce heterogeneity between firms so as to make a distinction between those which will decide to produce and those which will not.

We thus assume that firms differ in regard of their labour productivity, the production \( q_j \) of the \( j^{th} \) firm being given by:

\[
q_j = \frac{l_j}{a_j}
\]

where \( l_j \) stands for the level of labour and \( a_j \) for the reciprocal of labour productivity.

As we focus on proportions (rate of accumulation, rate of bankruptcies, etc.), the scale of the economy (number of firms, capital stock, etc.) has no impact on the results of the model. So, we suppose that an exogenous number of firms have decided to enter the market. When they have taken this decision, firms knew the \( a_j \) distribution law, i.e. \( a_j \) is uniformly distributed between \( \sigma - \sigma \) and \( \sigma + \sigma \) (where \( \sigma < \sigma \) is a constant) but they did not know their own \( a_j \). As a firm entered the market, it had to invest \( i_0 \) units of (fixed) capital (whatever could happen afterwards).

In a second step, we suppose that labour productivity level is disclosed to each firm. From this information, each firm has two decisions to take: firstly, whether to produce or not; secondly, which additive capital it has to accumulate in order to produce. We suppose that firms decide to produce, according to their productivity, if production allows them to get a minimum profit rate \( r_{min} \). Firms pulling a very low labour productivity stop producing and lose their capital investment \( i_0 \), which is scrapped. We say that these firms bankrupt.

The goods market is considered to be monopolistically competitive. The demand to the \( j^{th} \) firm is specified as:

\[
q_j = k_j \left( \frac{p_j}{p} \right)^{1/\eta}
\]

where \( \eta \) stands for the exogenous demand elasticity, \( p \) (\( p_j \)) for the endogenous market’s (firm’s) price of consumption goods and \( k_j \) the quantity of capital the firm \( j \) finally decide to accumulate.

We assume that, to remain competitive and keep customers, all the firms are actually compelled to set their own price as the leading firm, i.e. the firm getting the lower unit labour cost, i.e. the higher productivity level \( 1/(a-\sigma) \). The leader maximises their profits by applying a mark-up (depending on \( \eta \)) on its unit labour cost, that is:
\[ p = \frac{1}{1 - \eta} (\bar{a} - \sigma) w \]

Note that \( \eta (< 1) \) represents the mark-up or the profit share (in variable costs) of the leading firm. This parameter is actually the degree of monopoly on the goods market (the lower \( \eta \), the more competitive is the goods markets). For other firms, profit share is given by:

\[ \eta_j = 1 - (1 - \eta) \frac{a_j}{\bar{a} - \sigma} \]

Without loss of generality, we set \( \sigma \) so as to have \( \eta_j = 0 \) for the less productive firm \( (a_j = \bar{a} + \sigma) \). So, we get:

\[ \sigma = \frac{a\eta}{2 - \eta} \]

Finally, every producing firm sets its price to \( p \), whatever its own \( a_j \). So, the profit condition to produce can be written as:

\[
\begin{align*}
r_j &\geq r_{\text{min}} \\
\Rightarrow \eta_j u_j &\geq r_{\text{min}} \\
\Rightarrow \left(1 - (1 - \eta) \frac{a_j}{\bar{a} - \sigma}\right) \frac{u_j}{\nu} &\geq r_{\text{min}}
\end{align*}
\]

We add two hypotheses:

- We suppose that the capital coefficient \( \nu \) is equal to one.
- More fundamentally, we suppose that all producing firms have the same rate of capacity utilization. In our model, the accumulation rate by firms is related to their profitability (which is itself related to different \( a_j \)), so that the denominator of \( u \) will differ from one firm to another: very productive firms will fuel the goods markets more than less productive firms thanks to a higher capital accumulation. Nevertheless, we suppose that, given this height effect, the global demand is uniformly spread over producing firms: \( u_j = u \) (for every producing firm; non producing firm have a utilisation rate equal to zero). Finally, the profit condition gives the value of \( a_j \) which splits producing firms from failing ones (remember that labour productivity is decreasing with parameter \( a_j \)).

\[ a^* \leq \frac{1}{r_{\text{min}}} \frac{1}{(1 - \eta)(\bar{a} - \sigma)} u \]

We call \( \eta^* \) the profit share of the “last” producing firm, i.e. firm with a labour productivity of \( 1/a^* \): firms which have a profit share inferior to \( \eta^* \) will bankrupt, whereas firms with higher profit share will produce. At the equilibrium, we thus have:

\[ \eta^* = \frac{r_{\text{min}}}{u} \] \hspace{1cm} (1)
This “production condition” equation establishes a negative relation between $\eta^*$ and $u^*$. Finally, it is very easy to show that $\eta^*/\eta$ is the bankruptcy rate of this economy.

4.2. Investment behaviour

In this model, we have assumed that every firm on the market has “already” invested the initial “entry” capital $i_0$.

Firms which are not profitable enough stop investing, so that their accumulation (which is given by $i = i_0$) involves a revenue effect but has no impact on aggregate capital stock.

We suppose that each producing firm accumulates additional capital. To estimate this additional capital accumulation behaviour, we take the $i_0$ as the “initial” capital stock of these firms: $k_j = k_0$. We then suppose a very standard accumulation behaviour (the only original point is that this is actually a microeconomic behaviour equation):

$$\frac{\Delta k_j}{k_j} = i_j, r_j + \eta u$$

4.3. Saving behaviour

Only firms which produce make profits. We suppose that capitalists who earn these firms want to save a portion $s_p$ of these profits. The saving behaviour can be written:

$$\frac{s_j}{k_j} = s_p r_j$$

4.4. Macroeconomic investment

If we suppose that the initial investment $i_0$ is the denominator in the capital accumulation function, we can write the total investment function as:

$$\frac{i_j}{k_j} = \frac{i_0 + \Delta k_j}{k_j} = 1 + i_j, r_j + \eta u \quad \text{for producing firms},$$

$$\frac{i_j}{k_j} = \frac{i_0}{k_j} = 1 \quad \text{for non-producing firms}. $$

So the macroeconomic investment function is given by:

$$\frac{I}{K} = \frac{1}{2\sigma} \left( \int_{\alpha}^{\sigma} \int_{\alpha}^{\sigma} \frac{\Delta k}{k} da + \int_{\alpha}^{\sigma} \frac{\Delta k}{k} da \right)$$

which can be written as:

$$\frac{I}{K} = 1 + \left(1 - \frac{\eta^*}{\eta} \right) \left( \frac{1}{2} i_j (\eta + \eta^*) + \eta \right) u$$
4.5. **Macroeconomic saving**

Similarly, we can calculate the macroeconomic saving function:

\[
\frac{S}{K} = \frac{1}{a^*-a+\sigma} \int_{a^*-\sigma}^{a^*} s_p \pi(a_j) u \, da_j
\]

which can be simplified as:

\[
\frac{S}{K} = \frac{1}{2} s_p (\eta + \eta^*) u
\]

4.6. **Macroeconomic equilibrium**

The macroeconomic goods market equilibrium \( \frac{I}{K} = \frac{S}{K} \) allows eventually to determine \( u \), the capacity utilisation for producing firms:

\[
u^* = \frac{1}{1 - \frac{\eta^*}{\eta}} \left( \frac{1}{2} \left( \eta + \eta^* \right) \left( S - i_r - i_u \right) \right)
\]

(2)

This equation corresponds to the IS curve of our model. At this stage, we must combine equations (1) and (2). We thus eventually get a system of two equations:

\[\begin{align*}
\eta^* &= f(u, \eta^*) \\
u^* &= g(\eta, \eta^*)
\end{align*}\]

From this system, we can get an increasing relation between the profit share \( \eta \) and the bankruptcy rate \( \eta^*/\eta \), i.e. the proportion of failing firms increases as \( \eta \) increases. This pretty counterintuitive result can be explained easily: as the economy is stagnationist, an increase of \( \eta \) involves a more than proportional decrease in \( u \). Finally the average profit rate of the economy decreases and a greater proportion of firms do not fulfil the minimum profit rate condition anymore.

5. **Conclusion**

The main conclusion of our model is that the macroeconomic accumulation rate decreases with profit share, but this accumulation slowdown goes with some kind of production concentration on the goods market: as the bankruptcy rate increases, production is shared between less (remaining) firms.

To understand this conclusion, it is worth noting that, in our model, if all firms had the same profit share \( \eta \) (and thus if there is no bankrupt: \( \eta^* = 0 \)), the IS curve (equation (2)) would give the same rate of capacity utilisation (\( u^* \)) as in the standard Kaleckian model. Our crucial result is that \( u^* \) is now the product of two distinct terms which vary in opposite directions as \( \eta \) increases: on the one hand, the “standard Kaleckian \( u^* \)” (second term) encompasses the stagnationist nature of the economy (i.e. a negative relation between \( u^* \) and \( \eta \)); on the other hand, the first term indicates that the utilisation rate of producing firms is an increasing function of the bankruptcy.

\[\text{Calculations are available upon request to the authors.}\]
rate $\eta^*/\eta$. This second effect thus lessens --without offsetting-- the negative effect of the profit share on the utilisation rate. We think that it contributes to explain the utilisation rate stability in data.

6. References


LAVOIE M., RODRIGUEZ G., SECCARECCIA M. [2004], “Similitudes and Discrepancies in Post-Keynesian and Marxist Theories of Investment: A Theoretical and Empirical

