IMK
Institut für Makroökonomie
und Konjunkturforschung
Macroeconomic Policy Institute

Mai 2012 93

Working Paper

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Real-Financial Market Interactions with Heterogeneous Expectations Dynamics

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Keywords: Output dynamics, Portfolio Adjustments, Opinion Dynamics, Viability, Policy Measures

JEL classications: E12, E24, E31, E52.

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Towards Keynesian DSGD(isequilibrium) Modelling: Real-Financial Market Interactions with Heterogeneous Expectations Dynamics

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*We thank participants of the International POLHIA Conference on 'Rethinking Economic Policies in a Landscape of Heterogeneous Agents', October 13-15, 2011 Catholic University, Milan, Italy for useful comments. Of course usual caveats apply.

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1. Introduction

> *So here's what I think economists have to do. First, they have to face up to the inconvenient reality that financial markets fall far short of perfection, that they are subject to extraordinary delusions and the madness of crowds. Second, they have to admit – and this will be very hard for the people who giggled and whispered over Keynes – that Keynesian economics remains the best framework we have for making sense of recessions and depressions. Third, they'll have to do their best to incorporate the realities of finance into macroeconomics.*
> *Many economists will find these changes deeply disturbing.*

Paul Krugman, New York Times, September 6, 2009

Financial crises are an important phenomenon in market economies: are recurrent, can be extremely disruptive and costly, and they raise important issues for theorists and policy makers alike. The ruling paradigm of Dynamic Stochastic General Equilibrium (henceforth, DSGE) in macroeconomics has done a rather poor job in explaining financial crises and especially the recent global downturn, as admitted also by proponents of the DSGE approach, such as Charie et al. (2009). Arguably, this unsatisfactory performance has not been the result of a lack of mathematical sophistication. Rather, it derives from the adoption of an equilibrium approach coupled with the assumption of Rational Expectations, which seem methodologically and empirically questionable. Indeed, De Grauwe (2010) has attempted to build DSGE models without Rational Expectations by assuming agents to have limited cognitive abilities. Tovar (2009) has also argued that it is necessary to incorporate various transmission mechanisms that are absent in the DSGE literature, but are nonetheless crucial to understand monetary market economies.

This paper proposes a number of departures from DSGE methodology, which can be seen as the building blocks of a new approach in the Keynesian tradition, which we call Dynamic Stochastic General Disequilibrium (henceforth, DSGD). We construct an integrated macrodynamic model which incorporates some important feedback channels from the real to the financial sector (and vice versa), and in which markets are not assumed to jump to their equilibrium positions, but dynamic adjustment processes take place. Further, unlike in much of the macrodynamic literature out of the DGSE approach, we analyse *microfounded* expectation processes on financial markets by incorporating an innovative concept of *animal spirits* developed by Franke (2011) instead of the standard rational expectation apparatus.

To be precise, we consider a one-good economy where output moves according to a dynamic multiplier approach which considerably simplifies the Metzlerian inventory accelerator mechanism of the Keynes-Metzler-Goodwin model of Chiarella and Flaschel (2000). Since our focus in this paper is on financial markets and their specific sources of instability, the real side of the economy is kept as simple as possible. However, we assume that stock markets have real effects by influencing the agents' state of confidence, and so their investment and consumption decisions.

Three types of assets are traded on financial markets: first, a capital stock asset which is directly owned by households who supply the means of financing to firms. The second asset is a short-term, fix-price government bond, whose rate of interest is set by the Central Bank which issues the third asset, money M. A portfolio approach based on Tobin (1982) is employed to address disequilibrium adjustment processes on financial markets. This allows us to identify the feedbacks between financial and real markets, via Tobin's q, here given by the market price of capital K. In particular, we consider the effects of real variables on capital gain expectations, which represent a key element of the expected rate of return on stocks.

Focusing only on three assets (and only one risky one) is appropriate in order to identify the key dynamic mechanisms and real/financial feedbacks. But it is important to stress that this is just a simplifying assumption and the model can be extended to include, for example, equities E, long-term bonds B^l and bank loans Λ as in Charpe et al. (2011), Chiarella et al. (2012) and in Malikane et al. (2009).

One of the key contributions of the paper, however, is the explicit incorporation of opinion dynamics in financial markets populated by heterogeneous agents, which allows us to examine the effects of herding and speculative behaviour. More precisely, we adopt the distinction between *chartists* and *fundamentalists* proposed by Brunnermeier (2008). Chartists behave like speculators and can be seen as technical traders who adopt a simple adaptive expectation mechanism. In contrast, fundamentalists focus on basic economic data and expect variables to return to steady state values with a certain adjustment speed. Chartists tend to exert a destabilising influence on the economy, whereas the presence of fundamentalists is stabilising.

Albeit simple, this description of agent heterogeneity on financial markets is consistent with studies analysing expectational heterogeneity (see, for example, Menkhoff et al., 2009), and agents' behaviour on financial or foreign exchange markets (see, for example, De Grauwe and Grimaldi, 2005 and recently Proaño, 2011), and sufficient to examine some of the core features of financial markets that have played a prominent role in the current crisis.[1] Overall *market* expectations are here a function of individual fundamentalist and chartist expectations, and of the relative weight of each group in the market.

When heterogeneity is introduced in macroeconomic models, the agents' type is normally exogenously given and constant. In this paper, we analyse a dynamic mechanism that endogenously determines agents' type and therefore the sizes of the different populations of traders. To be specific, we adopt the microfounded notion of *animal spirits* recently formalised by Franke (2011), in the context of his analysis of business sentiments. We assume that at every moment in time there is a positive probability of each agent changing their status, from chartist to fundamentalist, or vice versa. This probability depends on the key variables of the economy (output, expected capital gains, asset prices), but also on the composition of market traders itself, which allows us to capture herding processes.

The model economy thus constructed contains two potential sources of instability: the feedbacks between real and financial markets via Tobin's q, and the endogenous opinion dynamics produced by the interaction of heterogenous agents on asset markets. Thus, it allows us to investigate a key question emerging from the current financial crisis, namely whether unfettered, interconnected markets with heterogeneous agents are able to absorb external shocks, or rather tend to amplify them.

We prove that the 4D dynamic system describing the evolution of the economy always has a steady state, but even though various subdynamics of the model can be stable, the complete system may be locally unstable around the equilibrium. Given the complexity of the 4D nonlinear system, though, it is difficult to draw more precise conclusions on the overall dynamics. Therefore, we adopt numerical methods to explore the properties of the considered DSGD model.

The numerical simulations show that the 4D system is indeed *viable*: all

[1] See Proaño (2011) for the incorporation of heterogenous expectations in a two-country model along the lines of the disequilibrium approach presented here.

trajectories remain in an economically meaningful subset of the state space. In this sense, the model shows the somewhat surprising result that unfettered markets with possibly accelerating real-financial feedback mechanisms have some in-built stabilising mechanism (based on opinion dynamics) that prevent the economy to move on an infeasible path. Moreover, despite the trivial dynamics of the 2D subsystems, the full 4D dynamics can exhibit irregular and even complex motions. Hence, if the steady state is locally unstable, the system can exhibit persistent real-financial market fluctuations. The considered opinion dynamics is therefore capable of ensuring upper and lower turning points in the real-financial market interactions, but the generated persistent fluctuations may still be too large to be acceptable from the societal point of view.

We consider therefore various policies that may act as stabilisers of the private sector. Because markets are highly interconnected, we follow Minsky (1982) and consider multiple policy instruments: we show that a Tobin-type tax on capital gains together with a capital market oriented monetary policy rule (the only 'risky asset' of the model) can stabilise the economy.

2. Framework

The main purpose of this paper is to analyse the specific sources of instability induced by financial markets and by feedback mechanisms between the financial and the real sector. Therefore, we simplify the real part of the Turnovsky (1995) model by ignoring inflation and growth, and by representing the quantity adjustment process by means of a dynamic multiplier approach. This simplifies the Metzlerian inventory accelerator mechanism of the real-side oriented Keynes-Metzler-Goodwin model of Chiarella and Flaschel (2000), thus suppressing it as a source of instability.[2] As a result, the real part of the economy is always stable (from this partial perspective), provided the propensity to spend is less than one. However, we assume that stock markets have real effects on investment and consumption.

To be precise, we assume that output moves according to a standard dynamic multiplier process, except that the state of confidence of the economy – which influences investment and consumption decisions – is measured by the price of the capital stock, p_k, instead of short-term interest rates. Formally,

[2]The instability induced in the KMG approach by the wage-price spiral is also ignored.

the law of motion of output (denoted by Y) is[3]

$$\dot{Y} = \beta_y(Y^d - Y) = \beta_y(a_y Y + a_k(p_k - p_{ko})K + A - Y), \tag{1}$$

where Y^d is aggregate demand; A is autonomous expenditure; K is the total capital stock; β_y is the speed of adjustment concerning goods-market disequilibria; $a_y \in (0,1)$ is the propensity to spend; $a_k > 0$ measures the reaction of investment and consumption demand to deviations between the actual and the steady state value of the capital stock.[4]

There is only one risky asset traded in the economy: K comprises the various forms of ownership claims on the physical capital stock (such as equities, corporate bonds, and credit), and its quantity is exogenously given. We assume that the market for K is imperfect, owing to information asymmetries, adjustment costs, or institutional restrictions, and therefore prices do not move instantaneously to clear markets. Let b denote the (given) profit share and let π_k^e denote the expected change in stock prices.[5] Then, assuming that profits are entirely distributed as dividends, the expected rate of return on the capital stock, ρ_k^e, is given by

$$\rho_k^e = \frac{bY}{p_k K} + \pi_k^e. \tag{2}$$

Let W denote nominal private wealth and let $f(\rho_k^e - \rho_{ko}^e)$ be the private sector's demand function for the capital stock *per unit of wealth*. We suppose that f is differentiable and strictly increasing in ρ_k^e, so that $f'(\rho_k^e - \rho_{ko}^e) > 0$, and $f(0) = 1$. Following Asada et al. (2011), we postulate a dynamic

[3]For any dynamic variable x, $\dot{x}$ denotes its time derivative, $\widehat{x}$ denotes its rate of growth, and x_o denotes its steady state value.

[4]The flow-consistency background of such a Kaldorian dynamic multiplier process is considered in detail in ch. 5 of Chiarella and Flaschel (2000) and extended towards a Metzlerian treatment in their ch. 6. Note that firms (owned by households) hold inventories of goods and money in this framework which are passively changed through windfall profits or losses if goods demand exceeds (falls short of) output. It is assumed that investment decisions are made by the management of firms which are thus owned, but not directed by the household sector. Note finally that we do not yet consider capital depreciation, growth and budget deficits in our model.

[5]We assume a constant profit share throughout the paper. This is consistent with the assumptions of constant output prices, wages, and labour productivity.

disequilibrium adjustment process for stock prices:[6]

$$\hat{p}_k = \beta_k \alpha_k \frac{f(\rho_k^e - \rho_{ko}^e)W - p_k K}{p_k K}, \quad \alpha_k \in (0,1). \tag{3}$$

In other words, only a fraction α_k of current aggregate excess demand for the capital *stock* $f(\rho_k^e - \rho_{ko}^e)W - p_k \bar{K}$ actually enters the asset market owing to the existence of adjustment costs. Thus, $1/\alpha_k$ represents the delay with which agents wish to clear any stock imbalance $f(\rho_k^e - \rho_{ko}^e)W - p_k \bar{K}$. As Asada et al. (2011) have argued, this approach is necessary in an open economy where *flow* rather than stock imbalances enter the capital account of the balance of payments. But it is also plausible in closed economies, in a continuous time setup, to assume that adjustment processes on the financial markets are somewhat gradual. The flow processes on asset markets are then translated into asset price changes by the speed parameter β.

In Charpe et al. (2011), the stock of wealth is made of money M and short-term fix-price bonds, B.. For the sake of simplicity, we assume here that households are focused on the market for real capital to such an extent that their perceived nominal wealth can be approximated by the value of the only risky asset $W = p_k K$.

Equations (1)-(3) represent the baseline model. In this economy, Tobin's q is measured by p_k, and it plays a key role in breaking down the real/financial dichotomy. Real markets influence asset markets via the role of output as the main determinant of the rate of profit of firms, and thus of the rate of return on real capital. Financial markets feedback onto the real side via the impact of Tobin's q on aggregate demand (either via a consumption or an investment effect).

In order to focus on the stability characteristics of the Tobin feedback channel in isolation, assume capital gain expectations to be stationary, so that the corresponding 2D system describing the dynamics of Y, p_k is only subject to a Tobin-type accelerator mechanism. The Jacobian J of the real-financial market interaction is:

$$J = \begin{pmatrix} - & + \\ + & - \end{pmatrix}.$$

[6]Note that the reference rate ρ_{ko}^e is a parameter of the model which will be equal to the steady state rate by assumption.

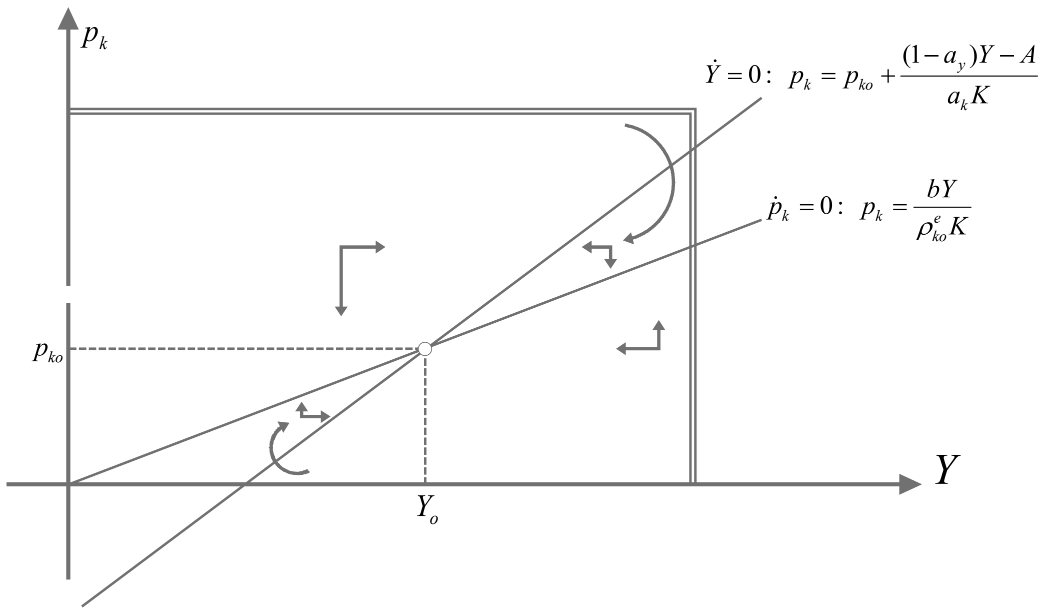

Figure 1: Asymptotically stable real-financial market interaction.

The trace of J, trJ, is unambiguously negative. Then, it is not difficult to prove that if a_y is sufficiently close to (but smaller than) one and a_k is sufficiently small, the determinant of J, $\det J$, is positive, and the system is locally asymptotically stable around the steady state. This case is illustrated in Figure 1. If the above restrictions on a_y and a_k do not hold, then it is possible to have $\det J < 0$, and so the system displays saddle-point dynamics around the steady state. If one assumes that policy is able to reduce the parameters a_y and a_k, at least far off the steady state, then Figure 1 suggests that global stability may obtain. These stabilising forces may however be absent in a neighbourhood of the equilibrium. In this case the steady state is a repeller and Figure 1 suggests the existence of a limit cycle within the compact box depicted.

These conclusions only concern the interaction of real and financial adjustment processes and do not depend on the presence of behavioural traders on the financial markets, which are introduced in the next section.

3. Capital gain expectations

We consider financial markets with heterogeneous agents and, following Brunnermeier (2008), distinguish between *fundamentalists*, f, and *chartists*, c. Fundamentalists expect capital gains to converge back to their steady state position (zero in our model). Chartists instead adopt a simple adaptive mechanism to forecast the evolution of capital gains $\dot{\pi}_k^e$. Formally:

$$\begin{aligned} \dot{\pi}_{kf}^e &= \beta_{\pi_{kf}^e}(0 - \pi_{kf}^e), \\ \dot{\pi}_{kc}^e &= \beta_{\pi_{kc}^e}(\hat{p}_k - \pi_{kc}^e). \end{aligned}$$

To be sure, more complex expectation formation mechanisms can be adopted for each type of agent, including forward looking rules. Yet, our formulation has the virtue of analytical simplicity, and it allows us to draw a sharp distinction with respect to Rational Expectation models.

Given that agents have heterogeneous expectations, it is not obvious a priori what *market* expectations should be. In standard equilibrium models with efficient markets, heterogeneous information and beliefs are spontaneously aggregated and made uniform under the pressure of market forces. This is clearly not the case in our framework. As a first step, suppose that the population shares of chartists and fundamentalists, $\nu_c, (1 - \nu_c)$, respectively, are constant.[7] It may be tempting to argue that the market expectation is the weighted average of the expectations of chartists and fundamentalists:

$$\pi_k^e = \nu_c \pi_{kc}^e + (1 - \nu_c)\pi_{kf}^e.$$

It is not clear, however, that this is the theoretically appropriate way of capturing the formation of market expectations. For market expectations π_k^e may actually reflect what both types of agents *think* will emerge from the process of aggregation of fundamentalist and chartist expectations. In other words, market expectations may reflect the agents' view about the 'average' opinion. And this need not be the exact, weighted average of the individual expectations. In turn, the law of motion of market expectations may be the product of what on average agents think the average opinion and its rate of change will be.

In this paper, we consider the following law of motion for aggregate capital gain expectations:

$$\dot{\pi}_k^e = \beta_{\pi_k^e}[\nu_c \hat{p}_k(Y, p_k, \pi_k^e) - \pi_k^e], \tag{4}$$

[7] Population shares are endogenised in the next section.

where $\beta_{\pi_k^e} > 0$ represents an adjustment speed parameter and where share price inflation only enters expectations with the weight ν_c of the chartists (since the change in their number is not foreseen). To be sure, this is only one possible formalisation of the dynamics of aggregate expectations in markets with heterogeneous agents, and alternative approaches can be proposed (see, for example, the approach adopted by De Grauwe and Grimaldi, 2005, in their analysis of the behaviour of agents on foreign exchange markets). Yet, we regard equation (4) as a very parsimonious way of capturing *both* the influence of aggregate observed variables *and* the role of heterogeneity and self-driving forces in expectation formation.

In order to analyse the dynamics of this economy, note that if the weight of chartists in average expectation is zero, the Jacobian of the 3D system (1), (3), (4) at the steady state becomes

$$J = \begin{pmatrix} - & + & 0 \\ + & - & + \\ 0 & 0 & - \end{pmatrix}.$$

with $J_{33} = -\beta_{\pi_k^e}$, so that a negative eigenvalue is added to the system. Therefore if a_y is sufficiently close to (but smaller than) one and a_k is sufficiently small, the steady state of the expectations-augmented real-financial interaction process is, again, locally stable. Hence, given the continuity properties of eigenvalues, if a_y is sufficiently close to (but smaller than) one and a_k is sufficiently small, then the steady state of the Tobin dynamics (1)-(3), augmented by the capital gain expectations rule (4), remains locally asymptotically stable even if the weight of chartists in average expectations formation is positive but sufficiently small. Intuitively, fundamentalists – if sufficiently dominant – may counteract any destabilising tendencies that chartists may create.

Instead, if the number of chartists, ν_c, the responsiveness of asset prices to disequilibria, β_k, and / or the responsiveness of the demand for capital stocks to expected returns, $f'(0)$, are sufficiently high, then one may obtain $J_{33} > 0$ and even $tr J > 0$. In this case, if the upper 2×2 minor satisfies the Routh-Hurwicz stability conditions, the system becomes unstable by way of Hopf-bifurcations, i.e., in general, by the death of a stable corridor around the steady state or by the birth of stable persistent fluctuations around it. The dynamic system (1), (3), (4) can thus provide a theory of business fluctuations caused by the interaction of real and financial markets.

To be sure, the previous argument and the existence of Hopf bifurcations is only based on a local analysis. Yet one may expect the presence of chartists to lead to explosive dynamics in general, if the speed of adjustment on financial markets or the responsiveness of the demand for capital stock are sufficiently high. This explosiveness may be tamed far off the steady state if nonlinear changes in behaviour or policy reduce β_k and/or $f'(0)$ enough to make the system globally stable, thus ensuring that all trajectories remain within an economically meaningful bounded domain. We do not analyse this conjecture further here. Rather, in the next section, we explore the possibility that endogenous changes in the agents' populations, ν_c, reduce the influence of chartists far off the steady state and thereby create turning points in the evolution of capital gain expectations.

4. Opinion dynamics

Even if one rejects the assumption of Rational Expectations, agents in financial markets do learn and they may change their behaviour endogenously in response to changes in the key economic variables. In this section, we adopt a version of the herding and switching mechanism developed by Lux (1996) and Franke (2011), which provides behavioural foundations to the agents' attitudes in the financial market. Unlike in standard DSGE models, we do not start from individual optimisation programmes. The switching mechanism is arguably more realistic than DSGE and it is a very elegant way of capturing *both* rational behaviour *and* purely speculative effects and herding. In fact, agents decide whether to take a chartist, or a fundamentalist, stance depending on the current status of the economy (captured by the key variables Y, p_k), on expectations on the evolution of financial gains (π_k^e), and also on the current composition of the market (captured by the variable x, defined below).

Formally, suppose that there are $2N$ agents in the economy. Of these, N_c are chartists and N_f are fundamentalists so that $N_c + N_f = 2N$. Let $n = \frac{N_c - N_f}{2}$. Following Franke (2011), we describe the distribution of chartists and fundamentalists in the population by focusing on the difference in the size of the two groups (normalised by N). To be precise, we define

$$x = \frac{n}{N} \in [-1, 1], \quad 1 - x = \frac{N_f}{N}, 1 + x = \frac{N_c}{N}, \tag{5}$$

where, as in Franke (2011), N is assumed to be large enough that the intrinsic noise from different realisations when individual agents apply their random mechanism can be neglected. Formally, as in Franke (2011), given the continuous time setting, we take the limit of x as N tends to infinity.

Let $p^{f\to c}$ be the transition probability that a fundamentalist becomes a chartist, and likewise for $p^{c\to f}$. The change in x depends on the relative size of each population multiplied by the relevant transition probability.

$$\dot{x} = (1-x)p^{f\to c} - (1+x)p^{c\to f}.$$

The key behavioural assumption concerns the determinants of transition probabilities: we suppose that they are determined by a *switching index* s, summarising the expectations of traders on market performance. The switching index depends positively on itself (capturing the idea of herding, see Franke and Westerhoff (2009, p.7), and on economic activity, and negatively on the market value of the capital stock and on average capital gain expectations. Formally:[8]

$$\begin{aligned} s &= s(x, Y, p_k, \pi_k^e), \qquad (6) \\ & s_x > 0, s_y > 0, s_{p_k} < 0, s_{\pi_k^e} < 0, \quad s(x_o, Y_o, p_{ko}, \pi_{ko}^e) = 0. \end{aligned}$$

An increase in s is assumed to increase the probability that a fundamentalist becomes a chartist, and to decrease the probability that a fundamentalist becomes a chartist. More precisely, assuming that the relative changes of $p^{c\to f}$ and $p^{f\to c}$ in response to changes in s are linear and symmetric:

$$p^{f\to c} = \beta \exp(as), \qquad (7)$$
$$p^{c\to f} = \beta \exp(-as). \qquad (8)$$

Given the above assumptions, the complete dynamic system becomes:

$$\dot{Y} = \beta_y[(a_y - 1)Y + a_k(p_k - p_k^o)K + A] \qquad (9)$$
$$\hat{p}_k = \beta_k \alpha_k (f(Y, p_k, \pi_k^e) - 1) \qquad (10)$$
$$\dot{\pi}_k^e = \beta_{\pi_k^e}[\frac{1+x}{2}\hat{p}_k(Y, p_k, \pi_k^e) - \pi_k^e] \qquad (11)$$
$$\dot{x} = \beta[(1-x)\exp(as(x, Y, p_k, \pi_k^e)) - (1+x)\exp(-as(x, Y, p_k, \pi_k^e))] \qquad (12)$$

[8]The details of the approach are in Lux (1996) and Franke (2011).

Equations (9)-(12) represent a DSGD model. All variables are dynamic in the sense that their evolution over time is described by gradual adjustment processes, and no algebraic equilibrium condition is involved. Markets are essentially interconnected and there are various feedback mechanisms from the real sector to financial markets, and vice versa. Finally, microeconomic processes play a crucial role in market expectation formation.

The key theoretical and policy question is, whether the unfettered market economies described by the DSGD model, where real/financial feedbacks play a prominent role and expectation formation may be affected by herding behaviour, display explosive trajectories, or rather they contain some inherent stabilising mechanisms. As a first step, note that the dynamic system (9)-(12) always has the following class of steady states:

$$Y_0 = A/(1-a_y), \tag{13}$$
$$p_{ko} = bY_o/(\rho^e_{ko}K), \tag{14}$$
$$\pi^e_{ko} = 0, \tag{15}$$
$$x_o = 0. \tag{16}$$

If $s_x \leq 1/a$ then this class of steady states is unique. If $s_x > 1/a$, then there are two additional steady state values for $x_o : e_f, e_c$ (see Figure 2), one where chartist are dominant and one where the opposite holds (all other steady state values remain unchanged). This is due to the backward-bending shape of the $\dot{x} = 0$-isocline.

Before analysing the dynamics of the complete system, it is interesting to consider the properties of the opinion dynamics and the expectational part of the model in isolation. For this purpose, we make two simplifying assumptions. First, we consider a linear functional form for f:

$$f(\rho^e_k - \rho^e_{ko}) = 1 + c(\rho^e_k - \rho^e_{ko}). \tag{17}$$

This is only for the sake of concreteness and similar results can be derived with different specifications of f. Second, suppose that the switching index has the following form:

$$s = s_x x + s_y(Y - Y_o) - s_{p_k}(p_k - p_{ko})^2 - s_{\pi^e_k}(\pi^e_k)^2. \tag{18}$$

This switching index assumes – besides the herding term and the role of economic activity as in Franke (2011) – that the deviations of share prices and capital gain expectations from their steady state values (in both directions) favour opinion making in the direction of the fundamentalists, because doubts concerning the macroeconomic situation become widespread. This change can be interpreted as a change in the state of confidence, whereby agents believe that increasing deviations from the steady state eventually become unsustainable. A similar approach concentrating on price p_k misalignment is used in Franke and Westerhoff (2009, eq.6).

In order to investigate the properties of the expectational dynamics in isolation, assume first output and dividend payments to be fixed at their steady state values. This yields the following 2D system:

$$\dot{\pi}_k^e = \beta_{\pi_k^e}[\frac{1+x}{2}\beta_k\alpha_k c - 1]\pi_k^e, \quad (19)$$

$$\dot{x} = \beta[(1-x)\exp(as(x,\pi_k^e)) - (1+x)\exp(-as(x,\pi_k^e))]. \quad (20)$$

First, note that x always points inwards at the border of the $x-$domain $[-1,1]$. Then, it can be conjectured that there must be an upper and a lower turning point for π_k^e in the economically relevant phase space $[-1,1] \times [-\infty,+\infty]$ and that, if the steady state $(0,0)$ is unstable, the generated cycle stays in a compact subset of this phase space. The expectational herding mechanism would thus be bounded, if taken by itself.

Franke (2011) shows this conjecture to be correct in the context of a formally similar 2D system. Here we simply note that $\dot{x}$ approaches infinity if there is an unlimited increase, or decrease, in the capital gains inflation rate π_k^e. However, as x approaches zero from above or from below, $\dot{x}$ would go to zero if it did not cross the vertical axis at $x = 0$. This is a contradiction and therefore there must always be an upper or lower turning point for capital gain inflation or deflation.

The phase space of the 2D system (19)-(20) is shown in figure 2. The diagram is drawn under the assumption that $s_x > 1/a$, and so there are three steady states (e_f, e_o, e_c). The horizontal axis is an invariant set of the dynamics which cannot be left (or entered) in finite time. Focusing on this part of the $\dot{\pi}_k^e = 0$-isocline we see that both the fundamentalist and the chartist steady state (e_f, e_c) are attracting, but that this only holds for the fundamentalist equilibrium, when the economy is subject to non-zero capital gain expectations.

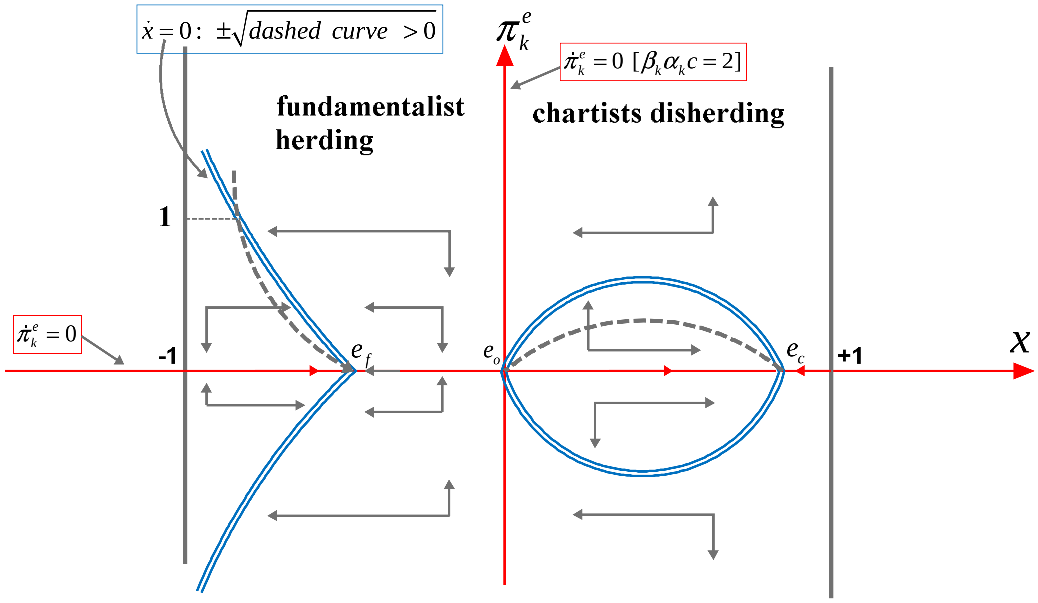

Figure 2: Bounded herding Behaviour.

The $\dot{x} = 0$ isocline is:

$$\pi_k^e = \pm\sqrt{\frac{s_x x - \ln\sqrt{(\frac{1+x}{1-x})/a}}{s_{\pi_k^e}}},$$

and it is attracting with respect to x, since x falls whenever π_k^e is above the isocline and it rises if π_k^e is below it. Note that this isocline is not defined for values of x that make the numerator inside the square root negative. Figure 2 displays some innovative features, as compared to the 2D phase diagrams in the literature, though the outcome of the 2D subdynamics is a fairly trivial one, since only the equilibrium where fundamentalists dominate is by and large a stable one. The figure also suggests that the economy remains in a bounded subset of the state space, if capital gains depart too much from their steady state value (which is zero), due to the strong effects this has on opinion dynamics.

However, because the law of motion of expected capital gains is not easily mapped onto figure 2, it is difficult to analyse the properties of the full 4D system. One should expect the local dynamics to be unstable without policy intervention, since the real-financial markets interaction, in connection with opinion dynamics, is likely to be of centrifugal nature. This raises the issue of the global viability of the unfettered market economy. For based on the analysis of the 2D systems, we cannot conclude that the trajectories of the full 4D dynamic system will always remain in an economically significant subset of the state space.

Given the complexity of the integrated nonlinear full 4D dynamics, we shall address these questions by means of numerical simulations.

5. Numerical simulations

Some stability properties of the integrated 4D system (9)-(12) can be deduced from the Jacobian J:

$$J = \begin{pmatrix} - & + & 0 & 0 \\ + & - & + & 0 \\ + & - & \pm & 0 \\ + & - & - & \pm \end{pmatrix},$$

where $J_{44} > 0$ if and only if $as_x > 1$. Then, it is immediate to show that *even if* the Tobin feedback mechanism with capital gains expectations in the upper 3D principal minor is stable, the system is unstable whenever $as_x > 1$, so that $J_{44} > 0$ and $\det J > 0$.

As for the investigation of the global dynamics, we reformulate the model in discrete-time in order to simulate it for different parameter sets. We use a standard Euler discretisation to re-write the model.[9] In the following, different types of shocks to the economy are considered. The following parameters are, however, held constant throughout the numerical analysis that follows: $\beta_{\pi_k^e} = 4$, $\beta_k = 2.723$, $\alpha_k = 0.5$, $c = 2$, $b = 0.28$, $s_{\pi_k^e} = 0.5$, $a = 1$, $\beta_y = 2$, $a_y = 0.6$, $a_k = 0.35$, $A = 1$, $K = 1$, $\rho_0^e = 0.25$, $s_y = 0.1$. We stress that they just provide an example around which the dynamics can be very sensitive with respect to parameter changes, providing outcomes which we consider interesting from the perspective of the simple decoupled 2D dynamics.

In the first simulation, we start off by assuming $s_x = 1.6$ and $s_{p_k} = 0.1$ and the population of chartists and fundamentalists to be constant by setting the parameter $\beta = 0$. Then, the system traverses – after a longer transient period – to a stable limit cycle generated by a Hopf-bifurcation displaying

[9]See Flaschel and Proaño (2009) for a detailed consideration of this procedure.

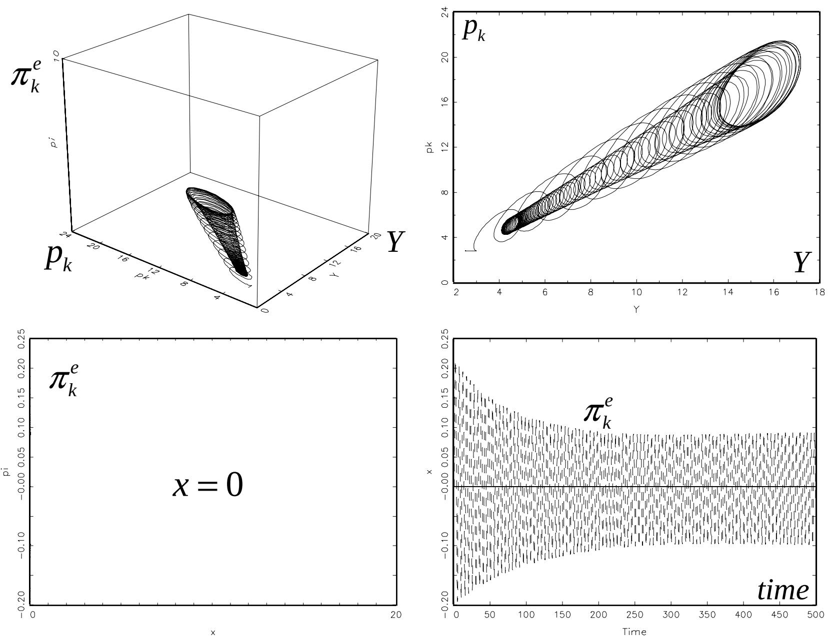

Figure 3: An attracting limit cycle (in the figure top-right in the left part of this figure).

persistent fluctuations, as shown in Figure 3. This result arises if the dynamics of the system (1),(3),(4) is described by figure 1, since the determinant of the present 3D dynamics must then always be negative, see section 3. The fact that a stable limit cycle is born, via a supercritical Hopf-bifurcation, by making the third law of motion sufficiently pronounced, can however only be demonstrated numerically as is well-known.

If we keep $s_x = 1.6$ and $s_{p_k} = 0.1$ and add the opinion dynamics by setting $\beta = 0.1$, our second simulation in figure 4 shows that the limit cycle then disappears and is replaced by a point attractor with most agents switching to fundamentalism along the way to it. Note her that the system is always started in the steady state $x_o = 0$, which is disturbed by a shock at time $t = 1$.

The third simulation shows, in figure 5, that if we increase the responsiveness of population shares to $\beta = 0.2$ and the responsiveness of the switching index to deviations of asset prices from the steady state to $s_{p-k} = 0.04$, and reduce the responsiveness of the switching index to the composition of agents to $s_x = 0.6$, then a unique, unstable steady state is given at $x = 0$.

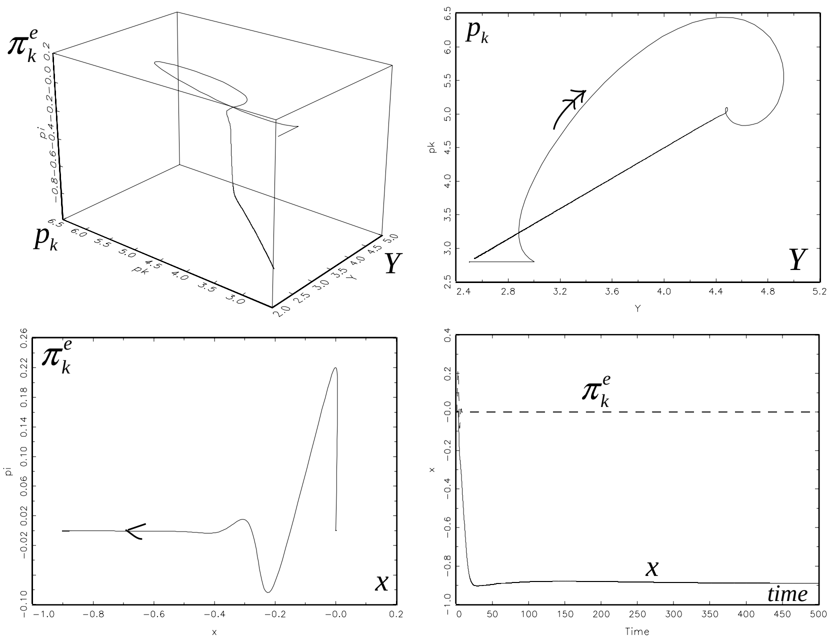

Figure 4: Fundamentalist herding and convergence.

There is a large negative shock with respect to the chartist population which however becomes reversed in time and then gives rise to irregular, but bounded and persistent output and share price fluctuations. The amplitude of the large fluctuations shown in figure 5 varies quite a lot. Note also that the transient period towards the establishment of such fluctuations is quite long and it comes to an end through the significant increases in the number of chartists (which however is already shrinking again before the fluctuations in particular in capital gain expectations really become powerful. Once set in motion they however do not die out again, but even significantly peak from time to time.

The fourth simulation, in figure 6, shows that if we keep $\beta = 0.2$, but increase both parameters in the switching index function to $s_x = 0.8$ and $s_{p_k} = 0.06$, then we still obtain a unique steady state at $x = 0$, but irregular fluctuations emerge in a significant way. This is an extremely interesting result, for it shows that our model economy can generate in a continuous-time framework complex dynamics. Instead they show that even smooth systems can run into certain 'bottlenecks' where their behaviour apparently 'spontaneously' changes (a result that does not – as all other numerical simulations

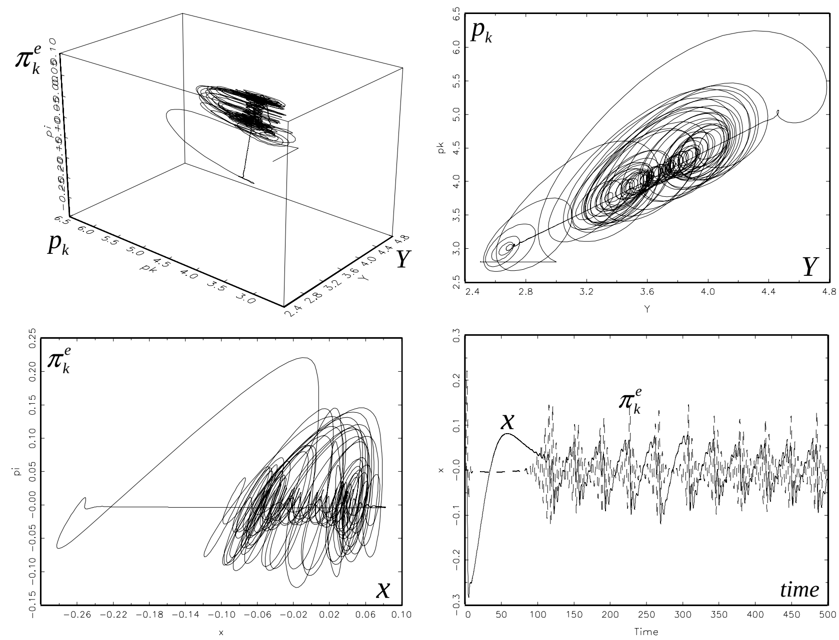

Figure 5: Persistent fluctuations in the real-financial market interactions.

in this section – depend on the step size chosen for the iteration procedure which is of course discrete in time).

The final simulation exercise, in figure 7, shows that if we keep $\beta = 0.2$ and $s_{p_k} = 0.06$, but set $s_x = 1.6$ and if we assume $\beta_{\pi_k^e} = 0.4$, $\beta_k = 2$, $b = 0.26$, then persistent herding of chartists can in fact emerge for quite a while, though the instability of the economy is increasing too. However, this is only a 'temporary' phenomenon: after a (considerable) while, the dynamics become unstable to such a degree that population shares switch quickly into a fundamentalist position and finally come to rest at a fundamentalist equilibrium which also stabilises the rest of the economy.

Summing up, we can state that the simulations demonstrate the global viability of the fully integrated 4D dynamics. The $\dot{x}$ mechanism is clearly pointing inside and presents a crucial part of the model that keeps the behaviour of the system bounded. Yet the steady states of the dynamics may be locally unstable, and the economy may face severe booms and busts along its business fluctuations. The next section therefore briefly discusses some policy proposals to address such issues.

It remains to be said that the simulations only provide examples of the

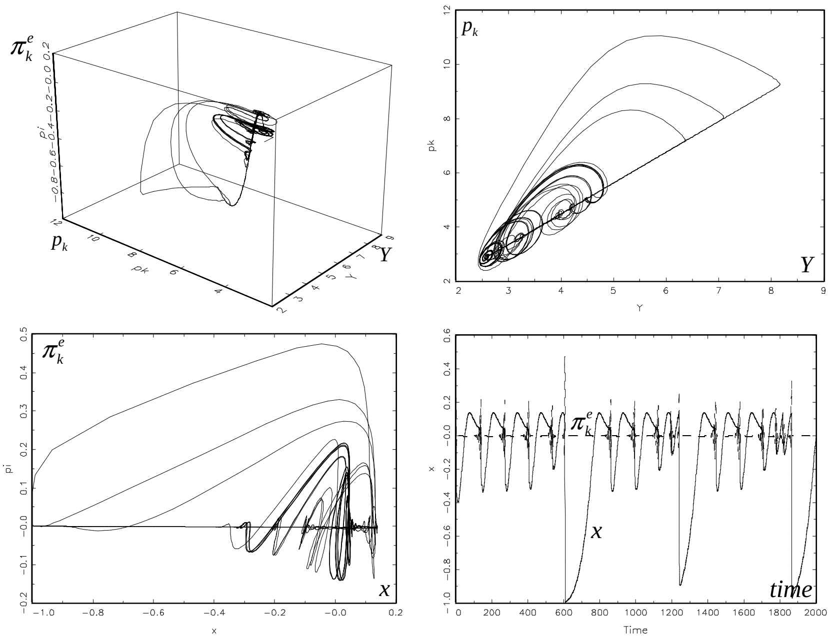

Figure 6: Irregular outbursts of extreme share price booms.

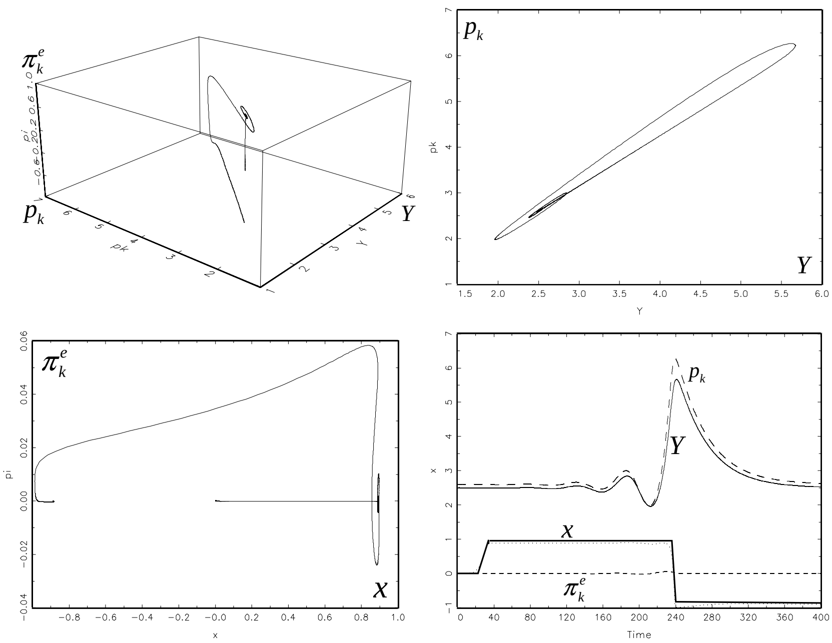

Figure 7: Multiple equilibria and temporary chartist herding.

manifold outcomes that this simple model of real-financial market interaction and opinion dynamics can generate.

6. Tobin Taxes and Quantitative Easing

Both the analysis of the Jacobian and the numerical simulations have shown that, albeit globally stable, the economy may display a locally unstable equilibrium and thus face persistent, and potentially severe fluctuations. The question therefore arises as to the possible policy measures that may stabilise the system and eliminate, or at least reduce the recurrence and severity of downturns.

The model highlights *two* sources of instability in the economy: the interconnection between real and financial markets, and the destabilising role of chartists in asset markets. Therefore, a *bundle* of policy measures may be necessary to tackle the channels of instability. In this section, we briefly consider two specific proposals.

First of all, as many authors since Keynes (1936) and Minsky (1982, 1986) have stressed, the main function of stock markets should be to ensure the efficient allocation of savings, and gambling activities should be constrained. It is therefore appropriate to consider a *Tobin type tax (or subsidy)* on capital gains at rate τ_k, such that total tax revenue is equal to:

$$\tau_k \alpha_k f(\rho_k^e - \rho_{ko}^e)\dot{p}_k K.$$

Therefore, given equations (10) and (17), the law of motion for capital gain expectations (11) can be re-written as:

$$\dot{\pi}_k^e = \beta_{\pi_k^e}[\frac{1+x}{2}(1-\tau_k)\alpha_k \hat{p}_k - \pi_k^e],$$

and Tobin taxes indeed have a stabilising effect by weakening the impact of chartists on the process of market expectation formation.

The second potential source of instability is the Tobin accelerator, and one way of stabilising the saddle point dynamics in the real-financial interaction subsystem might be a sort of ‘Quantitative Easing’, whereby the Central Bank directly intervenes on asset markets in response to the state of the economy: it increases aggregate demand for capital during downturns and reduces it during booms, thus affecting the price of capital which in turns affects consumers’ and investors’ decisions.

Formally, we may assume that the Central Bank sets a policy parameter $m_k > 0$ that represents its responsiveness to the output gap. Then, using (17), and setting $\pi_k^e = \pi_{ko}^e = 0$, the real-financial subsystem (1)-(3) becomes:

$$\begin{aligned} \dot{Y} &= \beta_y[(a_y - 1)Y + a_k(p_k - p_k^o)K + A], \\ \hat{p}_k &= \beta_k \left[\alpha_k c \left(\frac{bY}{p_k K} - \rho_{ko}^e \right) - m_k \left(\frac{Y}{K} - \frac{Y_o}{K} \right) \right]. \end{aligned}$$

The Jacobian of this system has, again, the following structure:

$$J = \begin{pmatrix} - & + \\ + & - \end{pmatrix},$$

but the determinant of J is now more likely to be positive, thanks to the policy m_k. Thus, this type 'Quantitative Easing' has indeed a stabilising effect by counteracting the unstable spiral of positive reinforcement.

7. Conclusions

This paper presents a Dynamic Stochastic General Disequilibrium model in the Keynesian tradition that as alternative to the received DSGE paradigm. In our model, the assumptions of market clearing and rational expectations are dropped. Instead a set of gradual, dynamic adjustment processes take place on highly interconnected real and financial markets. A Tobinian accelerator process describes the evolution of real macroeconomic activity. Financial markets influence the state of confidence of the economy, as measured by Tobin's q, and thus consumption and investment decisions. In turn, the performance of the real sector influences agents' decisions on financial markets. We show that this interaction need not be stable. Further, we introduce heterogeneous expectations on financial markets populated by chartists and fundamentalists, and show that chartist behaviour is another potential source of instability in the economy.

The key theoretical, empirical, and policy question, then, is whether unregulated market economies contain some mechanisms ensuring the stability of equilibria, or rather centrifugal forces prevail, making the equilibrium unstable and, potentially, the system unviable. Numerical simulations show that global stability can be ensured if, far off the steady state, opinion dynamics induces fundamentalist behaviour during booms and busts which ensures that there are turning points in both of these situations. However, both the local analysis and the simulations suggest that market economies can be plagued by fluctuations and recurrent crisis phenomena. We show that two policy measures often advocated in the Keynesian literature, namely Tobin taxes, here on capital gains, and quantitative easing, can mitigate these problems.

We close this paper with some remarks on our treatment of expectation formation, which suggest interesting lines for further research. First, one may argue that the theoretical expectation rules characterising chartists and fundamentalists, and the process of formation of market expectations should be replaced by more sophisticated backward- and forward-looking rules based on econometric estimation techniques. It would certainly be interesting to analyse the impact of different expectation rules on the system. But we do not expect these changes to significantly affect the key conclusions of our analysis. Second, in our formalisation of market expectations, we suppose the agents' guessing process is stopped after one step: market expectations are what agents think they will be on average. We consider this as a first step into the analysis of more complex processes of aggregate expectation formation. Once one drops the assumption of Rational Expectations, other possibilities can be explored, including Keynes' (1936) celebrated 'third degree' process, where agents try to anticipate what *average opinion* expects average opinion to be. We leave this suggestion for further research.

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Publisher: Hans-Böckler-Stiftung, Hans-Böckler-Str. 39, 40476 Düsseldorf, Germany
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ISSN: 1861-2199

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