Aggregate Demand, Aggregate Supply,
and Endogenous Growth:
A Synthetic neo-Kaleckian Model

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This paper develops a neo-Kaleckian endogenous growth model that incorporates aggregate supply - demand balance and balance between labor force and employment growth. The paper explicitly models income distribution which is a critical channel whereby unemployment affects investment and growth. The model generates a growth-unemployment rate trade-off. A reduced propensity to save raises growth but it also raises the unemployment rate because of induced technological progress. This resonates with Alvin Hansen’s hypothesis. The paper contains several theoretical innovations including a new mechanism whereby unemployment affects income distribution; introduction of a Phillips curve and inflation effects; and introduction of demand growth expectation effects.

Keywords: aggregate demand, supply, unemployment, neo-Kaleckian endogenous growth theory.

JEL classification: E12, O41, O33.

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I Introduction

Keynesian growth theory has historically focused on demand-side forces, in contrast to neo-classical growth theory which focuses exclusively on supply-side forces. However, Keynesian models include capital and a production function, which means there exists a separate supply-side that needs to be reconciled and rendered consistent with the pattern of growth generated on the demand side. This has led to growing interest in developing the supply-side of the Keynesian model.

The current paper develops a synthetic growth model in the Post Keynesian neo-Kaleckian tradition that builds upon these recent developments. It includes a goods market in which growth of demand equals growth of supply so that capacity utilization is constant, and a labor market in which employment growth equals labor supply growth so that the unemployment rate is constant. An important feature of the model is the
interaction between the goods market and the labor market. Thus, unemployment conditions feed through and affect goods market conditions, while goods market developments affect the evolution of the labor market.

A critical mechanism in this looping process is the profit rate and the functional distribution of income, which makes modeling of income distribution critical. Neo-classical models focus on price signals as the drivers of economic behavior. Post Keynesian models focus on the profit share and profit rate as the drivers of behavior.

Aside from establishing a consistent structure relating goods and labor markets that includes endogenous income distribution, the paper provides a new theoretical formulation whereby unemployment affects the goods market. This channel has long been recognized by microeconomic theorists of consumption and saving behavior, but it has not been incorporated into the neo-Kaleckian growth model.

A second theoretical innovation is the introduction of inflation and the Phillips curve, which provides another channel for unemployment to influence capital accumulation. A third innovation is the introduction of expectations of demand growth. Such expectations were critical in Keynes’s General Theory (1936), but they have not been explicitly incorporated into the neo-Kaleckian growth theory. This opens a new channel for future research on how expectations are reconciled with actual outcomes.

With regard to findings, the paper reports that a fully specified neo-Kaleckian model with endogenous technological progress and a requirement that labor supply and employment grow at the same rate in equilibrium, produces a growth – unemployment rate trade-off. It also has capacity utilization and the unemployment rate moving in
opposite directions. Analytically, the cause of the trade-off is the balanced growth condition that requires technological progress be labor augmenting. Thus, faster growth caused by more rapid capital accumulation causes faster technological progress that increases the implicit supply of labor. From a policy perspective, the finding of a growth unemployment rate trade-off poses a difficult challenge.

II Relation to previous literature

The model that is presented below builds on three different strands of research. The essential core of the model is the neo-Kaleckian growth model developed by authors such as Rowthorn (1982), Taylor (1983), Dutt (1984), and Lavoie (1995). The benchmark neo-Kaleckian growth model determines the rate of growth as a function of the rate of capital accumulation and saving, with these variables in turn being influenced by the functional distribution of income. A higher saving rate lowers growth, while more rapid capital accumulation raises growth. The functional distribution of income is important because it affects both the saving rate and the rate of capital accumulation. Blecker (2002) provides an excellent survey of these first generation models.

The benchmark neo-Kaleckian model focuses exclusively on demand-side forces of growth and neglects the supply-side. This has led to growing interest in incorporating supply-side factors that are clearly critical for growth. An early contribution to this second strand of research was Palley (1996, 1997) who presents a Keynesian model with endogenous supply growth. The main innovation is the extension of the ideas of Verdoorn (1949) and Kaldor (1957) through introduction of an endogenous technical progress function in which productivity growth is positively affected by investment.
capital stock, and various proxies for the state of aggregate demand. Once a separate supply-side is introduced, aggregate demand and supply must grow at the same rate to avoid rising excess demand or supply, a feature emphasized by Cornwall (1972, 1977).

This line of research has been extended in a number of papers (see for example Hein and Tarassow, 2009; Naastepad, 2006; Naastepad and Storm, 2007) in which capacity utilization and income distribution enter as arguments in the endogenous technological progress function, thereby explicitly connecting with the benchmark neo-Kaleckian model. These papers also provide empirical evidence regarding the impact of capacity utilization and distributional variables on productivity growth. Rada (2007) also applies this logic of an endogenous technological progress function to the case of a two sector developing economy in which sector productivity growth is affected by output growth, wage growth, and employment growth.

The third line of research is a model by Dutt (2006) which introduces labor market balance as a condition of equilibrium growth. Not only must demand and supply grow at the same rate, but so too must employment and labor supply or else the unemployment rate will be unstable. To address this issue, Dutt makes the change in the rate of productivity growth a positive function of the growth of labor demand and labor supply. Thus, productivity growth accelerates when the unemployment rate is falling and slows when the unemployment rate is rising. This mechanism serves to stabilize the unemployment rate by raising employment growth when the unemployment rate is rising, and reducing employment growth when the unemployment rate is falling.
The current paper synthesizes these three strands of research in a consistent neo-Kaleckian model. The paper details the manner in which labor markets and the unemployment rate interact with the goods market. This remedies a problem with the paper by Dutt (2006) which ignores income distribution and does not trace out the economic transmission mechanism whereby the unemployment rate affects productivity growth. Rather than having the unemployment rate directly affect productivity growth (Dutt, 2006), investment is restored as the driver of productivity growth. Unemployment then affects investment via its impact on capacity utilization and the profit rate. This restores the logic of the neo-Kaleckian growth model and highlights the need to endogenize income distribution as that is the mechanism which gets firms to change their investment spending plans.

Lastly, concern with the unemployment rate - profit rate - investment nexus links to a paper by Bhaduri (2006) that also emphasizes the role of unemployment and income distribution. However, that paper has a Marxist class conflict approach to income distribution and the role of income distribution is to balance the goods market. The current paper has a Kaleckian mark-up pricing approach to income distribution and goods market balance is achieved by capacity utilization adjustment. A challenge for future research is whether these two thematically similar paradigms can be fused together in a single model.

III The model

This section presents the model economy which consists of three segments: a supply side, a demand side, and a labor market. Equilibrium requires that aggregate
demand and supply grow at the same rate, and that employment and the labor force also grow at the same rate. There is a feedback loop whereby demand side conditions affect the supply side and labor market conditions, and labor market conditions affect the demand side.

The model is neo-Kaleckian in character, which means it treats long run capacity utilization as an endogenous variable. At the outset it is recognized that such treatment has been criticized by Kurz (1986), Committeri (1986), Dumenil and Levy (1999), Skott (2008, 2010), and Skott and Zipperer (2009) who argue that equilibrium capacity utilization is not free to vary and is instead drawn to a normal rate. A defense of the neo-Kaleckian approach has been offered by Lavoie (1995), Dutt (1997, 2006), Dallery and van Treeck (2008) and Hein, Lavoie and van Treeck (2009). The resolution of this important issue remains an open issue among Post Keynesian growth theorists and a subject for future research.

**III.A The production process**

The production side of the economy is described as follows

(1) $Y^* = \min[K, AN]$

(2) $g_{Y^*} = g_k$

(3) $g_K = g_N + g_A$

where $Y^*$ = potential output, $A$ = state of technology, $K$ = capital stock, $N$ = employment, $g_{Y^*}$ = potential output growth, $g_k$ = rate of capital accumulation, $g_N$ = employment growth, and $g_A$ = labor augmenting technical progress. Technological
progress is assumed to be labor augmenting as only this is consistent with steady-state balanced growth Uzawa (1961).

Actual output is given by

\[ Y = u \min [K, AN] \quad 0 < u < 1 \]

where \( u \) = rate of capacity utilization. Capacity utilization is therefore a choice variable of firms.

**III.B Technological progress**

The process of productivity growth is similar to that described in Palley (1996), which in turn derives from the Verdoorn - Kaldor approach to productivity growth. This process is given by

\[ g_A = a(g_K) \quad a_{gK} > 0 \]

Productivity growth depends positively on the rate of capital accumulation (\( g_K \)). Other variables can also be introduced into the productivity growth function. The above specification keeps the economic model simple and transparent. The positive productivity growth effect of faster capital accumulation reflects the fact that investment embodies new ideas. It also brings with it organization and production process change, and process innovation is a key part of raising productivity.

**III. C The labor market**

The unemployment rate is defined as

\[ U = 1 - N/L \]

where \( L \) = the labor force. The rate of change of unemployment is given by

\[ g_U = g_L - g_N \]
III.D Income distribution

Income distribution is determined by the familiar Kaleckian logic of mark-up pricing. The equations of the mark-up and income distribution given by:

(8) \( m = m(u, \psi) \quad m_u > 0, m_\psi > 0 \)
(9) \( \sigma = m/[1 + m] = \sigma(u, \psi) \quad \sigma_u > 0, \sigma_\psi > 0 \)
(10) \( \pi = \sigma u = \pi(u, \psi) \quad \pi_u > 0, \pi_\psi > 0 \)

where \( m \) = mark-up, \( \psi \) = real pricing power of firms, \( \sigma \) = profit share, and \( \pi \) = profit rate.

The mark-up is assumed to be a flexible jump variable that firms adjust in accordance with current demand pressures as reflected in the rate of capacity utilization. Higher capacity utilization raises the mark-up, the profit share, and the profit rate, reflecting the fact that higher capacity utilization corresponds to increased demand for firms’ output. Increased real pricing power of firms also increases the mark-up, the profit share, and profit rate. This exogenous shift variable can be thought of as capturing increased goods market monopoly power on the part of firms.

III.E The goods market

The goods market is a conventional Keynesian goods market in which output equals demand. The Kaleckian dimension is that saving depends on the functional distribution of income. The equations of the goods market are given by

(11) \( Y = D \)
(12) \( I/K = S/K \)
(13) \( S/K = s(u, U, \sigma, \beta) \quad s_u > 0, s_U > 0, s_\sigma > 0, s_\beta < 0 \)
(14) \( I/K = g_K = k(u, \pi) \quad k_u > 0, k_\pi > 0 \)
where \( D \) = aggregate demand, \( I \) = investment, \( S \) = saving, \( \beta \) = propensity to consume, and \( g_D \) = expected growth of demand.

Equation (11) is the production rule and has firms producing for demand. Actual output is determined by equation (4) which implies

\[
15\quad u = \frac{D}{\text{Min}[K, AN]} \quad 0 < u < 1
\]

Capacity utilization is therefore a jump variable and firms meet variations in demand by adjusting the rate of utilization. In effect, capacity utilization performs a function analogous to inventory. If demand jumps, firms increase the rate of capacity utilization as if they were drawing down inventory: if demand falls, firms reduce capacity utilization as if they were adding to inventory.

Expressing equation (11) in log form and totally differentiating yields

\[
16\quad g_Y = g_D
\]

Given the production rule, actual output growth is always equal to demand growth

Equation (12) is the saving – investment balance condition. Equation (13) is the saving function. Saving depends positively on the rate of capacity utilization, positively on the rate of unemployment, positively on the profit share, and negatively on the propensity to consume. The effects on saving of capacity utilization, the profit share, and the propensity to consume are standard. The important innovation is the effect of the rate of unemployment.

There is an extensive microeconomic literature on the effect of unemployment on saving which argues that unemployment proxies for uncertainty and increased uncertainty raises precautionary saving. Leland (1968) shows in a two period model of consumption
that an increase in future income uncertainty, defined as a mean preserving spread of income expectations, raises saving. That is because it increases the marginal utility of future income. More recently, using simulation analysis, Carroll (1992) shows that increased employment uncertainty also raises precautionary saving. The logic is increased consumer pessimism about unemployment causes increased uncertainty about future income, resulting in increased saving. This effect of unemployment on saving is critical to the workings of the model and is the channel whereby the unemployment rate affects the incentive to invest, thereby affecting the growth rate.

Equation (14) determines the rate of capital accumulation. The positive effect of capacity utilization and the profit rate on capital accumulation is standard in the neo-Kaleckian growth literature. It would be possible to have the unemployment rate directly affect capital accumulation by re-specifying the investment function as follows:

\[(14') \frac{I}{K} = g_K = k(u, \pi, U) \quad k_u > 0, \quad k_\pi > 0, \quad k_U < 0\]

In this case the unemployment rate would exert a direct negative effect on investment and capital accumulation. This is implicitly the assumption in Dutt (2006), but such a specification lacks microeconomic logic. Firms’ decisions to invest are driven by the profit rate and capacity utilization. If the unemployment rate affects investment, it does so by impacting these two latter variables. Arguments to the effect that low unemployment rates result in workers being hard to find and workers have greater bargaining power are both arguments about the effect of unemployment on the profit rate.

**IV Model solution, comparative statics, and stability**
The above model can be thought of as having two parts. The first part determines the instantaneous pattern of income distribution, the level of output, rate of capacity utilization, and rate of capital accumulation. The second half determines the steady state rate of unemployment and output growth. The short run jump variables are \( u, \sigma, \pi \) and \( m \). The state variables are the unemployment rate (\( U \)) and the capital stock (\( K \)).

**IV. A Short run equilibrium**

The short run model reduces to two equations given by

\[
\pi = \pi(u, \psi) \\
(18) k(u, \pi) = s(u, U, \pi, \beta)
\]

Equation (18) is an IS schedule that is obtained by combing equations (12) (13) and (14). The endogenous variables are \( u \) and \( \pi \). As identified by Bhaduri and Marglin (1990), there are two possible cases in neo-Kaleckian growth models. One is when the economy is wage-led growth: the other is when it is profit-led.

The slope of the IS curve is given by

\[
\left. \frac{d\pi}{du} \right|_{\text{IS}} = \frac{su - ku}{k\pi - s\pi} > 0
\]

The slope is ambiguous because the sign of the denominator is ambiguous. The numerator is assumed to be positive \((su - ku > 0)\) reflecting the standard Keynesian multiplier stability assumption. However, the sign of the denominator depends on whether the economy is wage-led or profit-led. If it is wage-led, the denominator is negative \((k\pi < s\pi)\) and the IS is negatively sloped. If the economy is profit-led, the denominator is positive \((k\pi > s\pi)\) and the IS is positively sloped.
Figure 1 shows the determination of the instantaneous rate of profit and rate of capacity utilization in a wage-led economy. The north–east quadrant shows the IS schedule and the profit function which is denoted MM. The MM function represents equation (10) and shows the profit rate as a function of the rate of capacity utilization. The intersection of the MM function and IS schedule determine the profit rate and rate of capacity utilization that yield instantaneous goods market equilibrium. The south-west quadrant shows iso-capital accumulation contours drawn in \([\pi, u]\), with higher contours representing a faster rate of capital accumulation. Given the instantaneous equilibrium values of \([u^*, \pi^*]\), these map in to a capital accumulation contour and determine the instantaneous equilibrium rate of capital accumulation, \(g_K^*\).

< Insert Figure 1 here >

For a wage-led economy the short run solution values are:

(19) \[ u = u(U, \beta, \psi) \quad u_U < 0, u_\beta > 0, u_\psi < 0 \]

(20) \[ \pi = \pi(U, \beta, \psi) \quad \pi_U > 0, \pi_\beta > 0, \pi_\psi > 0 \]

(21) \[ g_K = k(U, \beta, \psi) \quad k_U < 0, k_\beta > 0, k_\psi > 0 \]

The signings of the solution partial derivatives can be obtained by appropriately shifting the IS and MM schedules in Figure 1. An increase in the unemployment rate reduces capacity utilization, the profit rate, and the rate of capital accumulation. The logic is higher unemployment increases saving, thereby shifting the IS leftward. An increase in the propensity to consume has the symmetric opposite effect. An increase in firms’ monopoly power raises the mark-up and shifts the MM function up. That lowers capacity
utilization and raises the profit rate, but the effect on the rate of capital accumulation is ambiguous.

For a profit-led economy the IS is positively sloped and there are two cases. One is when the IS is steeper than the MM and the other when it is flatter. For the profit-led case where the IS is steeper than the MM the short-run solution values are:

\[(19.a) \quad u = u(U, \beta, \psi) \quad u_U < 0, \quad u_\beta > 0, \quad u_\psi > 0\]

\[(20.a) \quad \pi = \pi(U, \beta, \psi) \quad \pi_U < 0, \quad \pi_\beta > 0, \quad \pi_\psi > 0\]

\[(21.a) \quad g_K = k(U, \beta, \psi) \quad k_U < 0, \quad k_\beta > 0, \quad k_\psi > 0\]

The one difference concerns the effect of an increase in firms’ bargaining power which now increases capacity utilization, the profit rate, and capital accumulation.

For the case where the IS is flatter the short run solution values are:

\[(19.b) \quad u = u(U, \beta, \psi) \quad u_U > 0, \quad u_\beta < 0, \quad u_\psi < 0\]

\[(20.b) \quad \pi = \pi(U, \beta, \psi) \quad \pi_U > 0, \quad \pi_\beta < 0, \quad \pi_\psi < 0\]

\[(21.b) \quad g_K = k(U, \beta, \psi) \quad k_U > 0, \quad k_\beta < 0, \quad k_\psi < 0\]

The case of a profit-led economy in which the IS schedule is flatter than the LM produces counter-intuitive outcomes that are inconsistent with Keynesian economic logic and the rest of the paper therefore focuses on the other two cases.

**IV.B Long run equilibrium**

The unemployment rate is a state variable and its adjustment is governed by equation (7). Rearranging equation (3) and substituting equation (5) yields the growth of employment as a function of the rate of capital accumulation:

\[(22) \quad g_N = g_K - a(g_K)\]
Substituting equation (22) into equation (7) then yields the rate of change of the unemployment rate as a function of the rate of capital accumulation

(23) \( g_U = g_L - g_K + a(g_K) \)

The rate of change of the unemployment rate is positively related to labor force growth; negatively related to the rate of capital accumulation; and positively related to the rate of labor augmenting technological progress because firms can make do with fewer workers if technological progress is rapid.

There are now two regimes to consider, the wage-led regime and the profit-led regime. The first case considered is the wage-led regime. Substituting equation (21) into equation (23) yields

(24) \( g_U = g_L - k(U, \beta, \psi) + a(k(U, \beta, \psi)) \)

\[ k_U < 0, \quad k_\beta > 0, \quad k_\psi > 0, \quad a_k > 0 \]

\( = V(g_L, U, \beta, \psi) \)

where \( V_U = dV_U/dU = [a_k - 1]k_U > 0 \)

\( V_{g_L} = dV_U/dg_L = 1 > 0 \)

\( V_\beta = dV_U/d\beta = [a_k - 1]k_\beta > 0 \)

\( V_\psi = dV_U/d\psi = [a_k - 1]k_\psi > 0 \)

Stability requires \( V_U < 0 \), the necessary condition for which is \( a_k - 1 > 1 \). The logic is as follows. A higher unemployment rate lowers capacity utilization and the profit rate, inducing less capital accumulation. Lower accumulation directly reduces employment growth and also reduces technological progress which reduces implicit labor force growth. The net effect on the direction of change of the unemployment rate is therefore ambiguous. For the labor market adjustment mechanism to be stable it is necessary that
reduced implicit labor force growth exceeds reduced employment growth. Absent this, higher unemployment rates will cause cut backs in accumulation that further raise the unemployment, thereby causing the unemployment rate to explode – or go to zero in the case where the economy starts below the equilibrium unemployment rate. Figure 2 shows the phase diagram for the case of a stable unemployment rate adjustment mechanism.

< Insert Figure 2 here >

Assuming the adjustment mechanism is stable it is possible to sign some comparative statics. An increase in the rate of population growth increases the steady state unemployment rate. In terms of Figure 2 it shifts the phase line right. Somewhat surprisingly, an increase in the propensity to consume ($\beta$) also increases the steady state unemployment rate. This is because increased consumption increases the rate of capital accumulation, which increases the rate of labor augmenting technical progress. Since $a_k > 1$, the implicit labor supply effect outweighs the direct employment growth effect.

Lastly, the effect of an increase in firms’ monopoly pricing power is ambiguous. If it assumed increased monopoly pricing power is contractionary ($k_\psi < 0$) then $V_\psi < 0$, so that the phase line shifts down and the equilibrium unemployment rate falls. This somewhat surprising result reflects the fact that capacity utilization and the unemployment rate may move in opposite directions in a fully specified neo-Kaleckian model. The logic is increased pricing power increases the profit share which decreases capacity utilization and investment (recall, the economy is wage-led). However, reduced capital accumulation reduces the rate of labor augmenting technological progress which lowers the unemployment rate.
Given these comparative statics, the steady state equilibrium unemployment rate and rate of growth are given by

\begin{align}
(25) \quad U^* &= U(g_L, \beta, \psi) \quad U_{g_L} > 0, \ U_\beta < 0, \ U_\psi > 0 \\
(26) \quad g_y^* &= g_K^* = k(U^*, \beta, \psi) \quad k_{U^*} < 0, \ k_\beta > 0, \ k_\psi > 0 \\
&= k(g_L, \beta, \psi) \quad k_{g_L} < 0, \ k_\beta > 0, \ k_\psi > 0
\end{align}

Increased labor force growth lowers steady state capital accumulation and output growth because it increases the unemployment rate, which lowers the profit rate and capacity utilization, thereby lowering the rate of investment. An increase in the propensity to consume increases capital accumulation and growth, while the growth and accumulation affects of an increase in firms’ monopoly pricing power is ambiguous.

The second case is that of a profit-led regime. The unemployment rate adjustment dynamics remain governed by equation (24). As with the wage-led case the necessary stability condition is $a_k - 1 > 0$. The only change from the wage-led case concerns the partial derivative $k_\psi$ that is now positive. The comparative static regarding labor supply growth and the propensity to consume are the same as in the wage-led case. However, an increase in firms’ monopoly pricing power unambiguously increases the steady state unemployment rate while also increasing the rate of capital accumulation and growth. The logic is that increased monopoly power raises the profit rate and capacity utilization, which increases the rate of capital accumulation. That increases growth, but it also increases the unemployment rate because of the effect on labor augmenting technological progress (i.e. implicit labor supply growth).
In sum, faster capital accumulation in the neo-Kaleckian model with endogenous technological progress raises growth, but it also raises the unemployment rate. This result has some resemblance to Alvin Hansen’s (1932) hypothesis regarding technologically induced unemployment. The economic reasoning is that increases in the rate of investment spending raise the rate of labor augmenting technical progress, thereby increasing implicit labor supply growth and the steady state unemployment rate. This feature means that there is a growth – unemployment rate trade-off. A second feature is that the unemployment rate and capacity utilization can move in opposite directions. Higher capacity utilization spurs capital accumulation which causes higher unemployment. Both of these features follow from the balanced growth requirement that technological progress be labor augmenting rather than being an oddity of the neo-Kaleckian model.

V Extending the model

It is now possible to extend the basic model to include some additional factors. The first extension concerns the Phillips curve and inflation expectations. The second extension concerns expected demand growth.

V. A Inflation and the Phillips curve

Tobin (1965) argues that inflation increases portfolio demands for equity (i.e. capital), which in turn increases capital accumulation via the Tobin q channel. Additionally, inflation can have a positive effect on investment and consumption via spending acceleration effects (Neary and Stiglitz, 1983; Palley, 2010). In anticipation of higher prices, households and business bring forward their expenditures.
These effects can be incorporated in the Kaleckian model by adding a Phillips curve and amending the saving and investment functions to incorporate the effect of inflation expectations as follows:

(25) \( \Pi = \Pi(U) \) \( \Pi_U < 0 \)

(26) \( \Pi = \Pi^e \)

(27) \( S/K = s(u, U, \sigma, \beta, \Pi^e) \) \( s_u > 0, s_U > 0, s_\sigma > 0, s_\beta < 0, s_{\Pi^e} < 0 \)

(28) \( I/K = g_K = k(u, \pi, \Pi^e) \) \( k_u > 0, k_\pi > 0, k_{\Pi^e} > 0 \)

where \( \Pi \) = inflation and \( \Pi^e \) = expected inflation. Equation (25) is the long run negatively sloped Phillips curve in which inflation is a negative function of the inflation rate. Equation (26) has inflation expectations equal actual inflation. Equations (27) and (28) are the saving and investment functions, amended to take account of inflation expectations which negatively impact saving and positively impact investment.

There are two important features about this specification. First, unemployment now has a stronger positive effect on saving described by

\[ \delta S/\delta U = S_U + S_{\Pi^e} \Pi^e_U > 0 \]

There is the direct effect on precautionary saving \( (S_U) \) and there is an additional inter-temporal consumption acceleration effect \( (S_{\Pi^e} \Pi^e_U) \). Second, inflation now affects capital accumulation via the investment acceleration effect, providing a channel for unemployment to affect investment as follows

\[ \delta k/\delta U = k_{\Pi^e} \Pi^e_U > 0 \]

After substituting for inflation and inflation expectations in the saving and investment functions, the short run IS-MM model reduces to two equations given by
(29) \( \pi = \pi(u, \psi) \)

(30) \( k(u, \pi, U) = s(u, U, \pi, \beta) \)

The MM function is unchanged, but the effect of unemployment on the IS schedule is larger. In terms of Figure 1, an increase in unemployment leads to a larger leftward shift of the IS owing to the additional effects of lower inflation on investment and saving.

The solutions for the endogenous variables \( \pi, u \) and \( k \) have the same functional form and signing as in equations (19) – (21) for the wage-led regime and (19.a) – (21.a) for the profit-led regime. The only change is that the magnitude of the derivatives \( \pi_U, u_U \) and \( k_U \) are larger.

The long run steady state dynamics remain governed by equation (24) and the necessary stability condition remains \( a_k - 1 > 0 \). However, the partial derivative \( k_U \) is now larger in absolute magnitude because of the additional effect of the unemployment rate operating via the effect of inflation on consumption and investment. Higher unemployment lowers the inflation rate leading to a larger reduction in capacity utilization and the profit rate, which in turn leads to a larger fall in investment spending. In terms of Figure 2 that makes the phase line steeper (i.e. more negatively sloped).

V.B Expectations of demand growth

A second expansion of the model is to introduce expected demand growth into the investment function, which is then given by

(31) \( I/K = g_{K} = k(u, \pi, \Pi^e, g^d) \quad k_u > 0, \ k_\pi > 0, \ k_{\Pi^e} > 0, \ k_{gd} > 0 \)

where \( g^d \) = expected demand growth. The positive effect of expected demand growth reflects the fact that firms need to put capital in place in expectation of future sales.
Consequently, there are now two demand pressure variables – demand growth and capacity utilization. The effect of expectations on investment is emphasized by Keynes (1936) in his *General Theory*, and demand growth is also emphasized by Kaldor:

“…economic growth is demand induced and not resource constrained… - i.e. that it is to be explained by the growth of demand which is exogenous to the industrial sector and not by the (exogenously given) technical progress over time (Kaldor, 1975, 895).”

The MM function is unchanged, while the new IS schedule is obtained by combining equations (25), (26), (27) and (31). This yields a short run IS-MM model given by

(32) $\pi = \pi(u, \psi)$

(33) $k(u, \pi, U, gd) = s(u, U, \pi, \beta)$

The only change from the previous section is the inclusion of the expected demand growth argument in the investment function. The short run endogenous variables are $u$ and $\pi$. The state variables are $U$ and $gd$.

The general form solutions for the short run endogenous variables are:

(34) $u = u(U, \beta, \psi, gd)$  $u_U < 0, u_\beta > 0, u_\psi < 0, u_{gd} > 0$

(35) $\pi = \pi(U, \beta, \psi, gd)$  $\pi_U > 0, \pi_\beta > 0, \pi_\psi > 0, \pi_{gd} > 0$

(36) $gK = k(U, \beta, \psi, gd)$  $k_U < 0, k_\beta > 0, k_\psi > 0, k_{gd} > 0$

There are now two state variables, the unemployment rate and expected demand growth. Unemployment rate dynamics remain governed by equation (23), while the adjustment of demand expectations is governed as follows

(37) $g_{gd} = G(gY - gd)$  $G(0) = 0, G’ > 0$
where \( g_{gd} \) = rate of change of growth of demand. The logic is that demand growth expectations converge to actual output growth. When \( g^d \) equals \( g_Y \) expected demand growth equals output growth which equals actual demand growth.

Substituting for \( g_Y \) and \( g_K \) in equations (23) and (37) yields a simultaneous set of differential equations given by

\[
(38) \quad g_U = g_L - k(U, \beta, \psi, g^d) + a(k(U, \beta, \psi, g^d)) \quad k_U < 0, k_\beta > 0, k_\psi < 0, k_{gd} > 0, a_k > 0
\]

\[
\quad = V(U, g_L, \beta, \psi, g^d) \quad V_U < 0 \text{ if } a_k > 1; \quad V_{gL} > 0
\]

\[
\quad V_\beta, \quad V_{gd} > 0 \text{ if } a_k > 1; \quad V_\psi < 0
\]

\[(39) \quad g_{gd} = G(k(U, \beta, \psi, g^d) - g^d) \quad G_U < 0, G_\beta > 0, G_\psi > 0, G_{gd} > 0 \text{ if } k_{gd} > 1
\]

The Jacobian of a linearized version of this system is

\[
|J| = |
\begin{array}{cc}
V_U & V_{gd} \\
G_U & G_{gd}
\end{array}|
\]

where

\[
V_U = [a_k - 1]k_U
\]

\[
V_{gd} = [a_k - 1]k_{gd}
\]

\[
G_U = G'k_U
\]

\[
G_{gd} = G'[k_{gd} - 1]
\]

The necessary and sufficient conditions for stability are \( V_U < 0 \) and \( |J| > 0 \). The former holds if \( [a_k - 1] > 1 \) and the latter holds if \( [k_{gd} - 1] < 0 \). This last condition is the analogue of the Keynesian expenditure multiplier stability condition. It prevents faster expected demand growth generating ever faster potential output growth and expected
demand growth. Figure 3 shows the expected demand growth adjustment mechanism under the assumption of stability.

The two adjustment equations governing the unemployment rate and expected demand growth can be combined in a phase diagram to determine the long run general equilibrium outcome and its stability and comparative static properties. The phase plane diagrams show points of equilibrium obtained by setting equations (38) and (39) equal to zero. Dynamic equilibrium obtains in the goods market when \( g_D = g_Y \) and in the labor market when \( g_L = g_N \). Dynamic general equilibrium obtains when both conditions are satisfied simultaneously. This yields the conditions

\[
(40) \ V(g_L, U, \beta, \psi, g^d) = 0 \\
(41) \ G(U, \beta, \psi, g^d) = 0
\]

Differentiating equations (40) and (41) with respect to \( g_D \) and \( U \) yields the slopes of the expected demand growth and unemployment rate phase planes. These phase planes are denoted \( GG \) and \( VV \) respectively and the unemployment and growth rates are constant along them. Their slopes are given by

\[
dg^d/dU_{|VV} = -\frac{V_U}{V_{gd}} > 0 \\
dg^d/dU_{|GG} = -\frac{G_U}{G_{gd}} < 0
\]

Figure 4 shows the phase diagram for this pattern of adjustment. In regions above the \( GG \) schedule, demand growth exceeds potential output growth so that demand growth slows. The reverse holds for regions below the \( GG \) schedule. In regions to the left of the \( UU \) schedule, the unemployment rate is so low that labor-augmenting technical progress is
rapid causing the unemployment rate to rise. The reverse holds for regions to the right of the UU schedule. If the economy is stable it converges cyclically to an equilibrium of \([U^*, gD^*] \).

< Insert Figure 4 here >

The phase diagrams can now be used to examine the comparative statics of the model. There are two cases to be considered: a profit-led economy and a wage-led economy.

*Comparative statics for a profit-led economy*

The first experiment is an increase in the marginal propensity to consume. Differentiating equations (40) and (41) with respect to \(\beta\) yields:

\[
\begin{align*}
\frac{dgD}{d\beta}|_{GG} &= G_\beta > 0 \\
\frac{dU}{d\beta}|_{VV} &= V_\beta > 0
\end{align*}
\]

An increase in the propensity to consume shifts the GG schedule up. The logic is an increased propensity to consume raises capital accumulation, which raises output growth and expected demand growth. It also shifts the VV schedule right. The logic is that faster capital accumulation raises labor augmenting technical progress, causing the unemployment rate to rise. The net result is unemployment increases but the effect on growth is ambiguous.

The second experiment is an increase in the real pricing power of firms. Differentiating equations (40) and (41) with respect to \(\psi\) yields:

\[
\begin{align*}
\frac{dgD}{d\psi}|_{GG} &= G_\psi > 0 \\
\frac{dU}{d\psi}|_{VV} &= V_\psi > 0
\end{align*}
\]
The direction of shift of the GG and VV schedules is therefore the same as for an increase in the propensity to consume. The GG shifts up because greater monopoly pricing in a profit-led economy increases capital accumulation, output growth, and expected demand growth. The VV shifts left because faster capital accumulation increases labor-saving technical progress, which increases the unemployment rate.

The third experiment is an increase in the rate of growth of the labor force. Differentiating equations (40) with respect to $g_L$ yields:

$$-\frac{dU}{dg_L}\bigg|_{VV} = V g_L > 0$$

The GG schedule is unaffected while the VV schedule shifts right. The net result is growth falls and the unemployment rate rises.

*Comparative statics in a wage-led economy*

The shifts of the GG and VV schedules in response to an increase in the propensity to consume spend, expected demand growth, and labor force growth are the same as in the profit-led regime. The only difference concerns firms’ monopoly pricing power, for which the shifts are:

$$\frac{d\beta}{d\beta}\bigg|_{GG} = G \psi < 0$$

$$\frac{dU}{d\beta}\bigg|_{VV} = V \psi < 0$$

The GG schedule shifts down because increased pricing power in a wage-led economy lowers capital accumulation, output growth, and expected demand growth. The VV shifts left because lower capital accumulation slows labor augmenting technical progress, which lowers the unemployment rate.
VI Conclusion

This paper has presented a synthetic endogenous growth model in the neo-Kaleckian tradition that incorporates both goods market and labor market balance conditions. The model incorporates a number of theoretical innovations including allowing unemployment to influence capacity utilization and income distribution via its effect on saving; introducing a Phillips curve that allows unemployment to influence saving and investment via its effect on inflation; and expectations of demand growth that influence the rate of capital accumulation.

Somewhat surprisingly, a fully specified neo-Kaleckian balanced growth model that includes technological progress and requirement that labor supply and employment grow at the same rate in equilibrium, produces a growth – unemployment rate trade-off. A second surprising result is that capacity utilization and the unemployment rate move in opposite directions.

Analytically, this means an increase in the propensity to consume raises the growth rate but it also raises the equilibrium unemployment rate. Increased spending stimulates capital accumulation which raises growth, but it also contributes to faster labor augmenting technological that raises unemployment. Second, in a wage-led economy increasing the wage share spurs capital accumulation via its impact on capacity utilization, but faster accumulation also raises the unemployment rate. In a profit-led economy increasing the worker share has the reverse effect. From a policy perspective, the finding of a growth unemployment rate trade-off poses a difficult challenge.
References


Skott, P., “Theoretical and empirical shortcomings of the Kaleckian investment function,” Working paper 2008-11, Department of Economics, University of Massachusetts, Amherst, MA.


Figure 1. Determination of short run equilibrium in the wage-led case ($g_K^* > g_K^1$)

Figure 2. The unemployment adjustment mechanism

$g_U = g_L + g_A - g_k$
Figure 3. Mechanism governing demand growth adjustment: \( g_{gD} = G(g_Y - g^d) \)

Figure 4. Phase diagram governing the adjustment of the labor market and expectations of demand growth.