Heterogenous Behavioral Expectations, FX Fluctuations and Dynamic Stability in a Stylized Two-Country Macroeconomic Model

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Abstract

In this paper the role of behavioral forecasting rules of chartist and funda-
damentalist type for the dynamic macroeconomic stability of a two-country
system is investigated both analytically and numerically. The main result of
the paper is that for large trend-chasing parameters in the chartist rule used in
the FX market, not only this market but the entire macroeconomic system is
destabilized. This takes place despite of the presence of monetary policy rules
in both countries which satisfy the Taylor Principle.

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1 Introduction

In recent times a significant paradigm change has taken place in macroeconomics: After the almost exclusive development of theoretical macroeconomic models based on the assumption of forward-looking, intertemporal utility maximizing agents with rational expectations in the DSGE tradition (see e.g. Christiano, Eichenbaum and Evans (2005)), the profession has started to acknowledge the importance of different classes of heterogeneity (regarding preferences, the degree of risk aversion, information, etc.) and “behavioral” rules or attitudes at the microeconomic level for the dynamics of the economy at the aggregate level (see Akerlof (2002, 2007), and Yellen (2007) for recent and important statements in this regard).

The need for an alternative foundation of macroeconomics is perhaps most evident in the international finance literature, where the so-called “New Open Economy Macroeconomics” (NOEM) approach developed by Obstfeld and Rogoff (1995) has been the workhorse theoretical framework for the study of open-economy issues in recent years, after the long-lasting predominance of Mundell-Fleming type models. This is due to the fact that NOEM models such as Galí (2005) and Benigno and Beningno (2008), relying on the interaction of fully rational agents, seem unable to explain important stylized facts of the dynamics of the nominal exchange rate and its interaction with the real side of the economy (see Engel and West (2005) for a contrary view on this respect). Indeed, as pointed out e.g. by De Grauwe and Grimaldi (2005), Efficient Markets Rational Expectations (EMRE) models seem unable to reproduce major stylized facts on exchange rate fluctuations, such as the non-normality of returns and their volatility clustering, their apparent disconnection with macroeconomic fundamentals, as well as the occurrence of speculative bubbles, herding behavior and currency runs.

On the contrary, models based on “bounded rationality”, that is, models which feature economic agents with heterogenous beliefs and behavioral attitudes or trading schemes, seem much more successful in this task, see e.g. Frankel and Froot (1987), Allen and Taylor (1992), Cheung and Chinn (2001) and Manzan and Wetterhoff (2007). Indeed, the inclusion of such agent or beliefs heterogeneity, as well as of behavioral rules and therefore of a somewhat “boundedly rational” behavior by the economic agents, has proven quite valuable in providing insights and explanations concerning some of the “puzzles” which arise when “rationality” is assumed (see De Grauwe and Grimaldi (2006, ch.1) for an extensive discussion of the advantages of the bounded rationality approach with heterogenous agents over the rational-
expectations approach in the explanation of empirical financial market data). The analysis of this second type of models, however, has been often constrained to the foreign exchange (FX) markets, so that the effects of such non-rational behavior by FX market participants for the dynamic stability at the macroeconomic level have not been explicitly analyzed in an extensive manner so far.

In this paper an attempt is undertaken to fill in this gap in the academic literature by setting up a stylized two-country macroeconomic model with a foreign exchange market consisting of two types of traders with different beliefs concerning the future development of the nominal exchange rate: Fundamentalists and chartists. To do, so the theoretical disequilibrium AS-AD model investigated in Chen, Chiarella, Flaschel and Semmler (2006) is reformulated for the case of two large open economies, first each in isolation and then in their interaction as two subsystems within a large closed dynamical system. By means of the resulting two-country macroeconomic model, the importance of “behavioral trading” for the stability not only of the FX market, but for the whole two-country macrodynamic system will be investigated. This seems a meaningful task given the recurrent occurrence of currency crises observable in the decades and the disastrous consequences for employment and economic growth which such FX market instability has brought with.

The remainder of the paper is organized as following: In section 2 the theoretical macroeconomic framework for the case of a small open economy is described. After identifying the conditions for local stability of the one-country subsystem in an analytical manner, in section 4 the model is estimated with aggregate time series data of the U.S. economy. Section 5 in turn investigates the role of the FX markets for the dynamic macroeconomic stability of two interacting large open economies by means of numerical simulations. Section 6 draws some concluding remarks from this paper.

2 The Baseline Two-Country Model

The FX Markets

The description of model begins with the FX market due to its relative importance in the analysis of this paper. As previously stated, “boundedly rational” agents are assumed in this paper which, due to informational and/or cognitive constraints, do not calculate “mathematically rational” expectations with respect to the future dynamics of the nominal exchange rate (as it is assumed in the NOEM/DSGE framework) but use behavioral forecasting rules for this task instead.
As it is done in the majority of heterogeneous expectations models, see e.g. Manzan and Westerhoff (2007), we focus on two types of behavioral forecasting patterns used by the traders in the FX market: “Fundamentalism” and “Chartism”.

Concerning the “fundamentalist” forecasting rule, the according expected exchange depreciation rate is simply

\[ E^f_t \Delta s_{t+1} = \beta^f_s (f_t - s_t), \]  

(1)

where \( \Delta s_{t+1} = s_{t+1} - s_t \), \( s_t = \log(S_t) \) being the log nominal exchange, \( f_t \) the value of the macroeconomic fundamentals at time \( t \) and \( \beta^f_s > 0 \) a scaling factor which represents the speed of adjustment of the log nominal exchange rate towards its long-run equilibrium level \( f_t \) adopted by the fundamentalists in their FX forecasts. As it is usually done in the literature (see e.g. Froot and Rogoff (1995), Taylor and Peel (2000) and Taylor, Peel and Sarno (2001)), the PPP postulate (in its absolute form) is assumed to represent the long-run point of reference for the nominal (and real) exchange rate, that is

\[ f_t = p_t - p^*_t \]  

(2)

with \( p_t = \ln(P_t) \) and \( p^*_t = \ln(P^*_t) \) denoting the log price levels in the domestic and foreign economies, respectively.\(^1\) Inserting this expression in eq.(1) delivers

\[ E^f_t \Delta s_{t+1} = \beta^f_s (p_t - p^*_t - s_t) \]

\[ = \beta^f_s (-\eta_t) \]  

(3)

where \( \eta_t \) is the log of the real exchange rate at time \( t \) and \( \eta_t = 0 \) would represent its PPP-consistent level.

Concerning the second forecasting strategy, it is assumed that the respecting expected nominal exchange rate depreciation for \( t + 1 \) is determined by

\[ E^f_t \Delta s_{t+1} = \beta^f_s \Delta s_t, \]  

(4)

\(^1\)Note that this is not the only possible specification for the fundamentalist rule: The “fundamentalist” expected nominal exchange rate depreciation could also be determined by the PPP in its relative form, that is \( E^f_t (\Delta s_{t+1}) = \Delta p_t - \Delta p^*_t \) (\( \Delta p \) and \( \Delta p^* \) being the domestic and foreign price inflation rates). The Uncovered Interest Rate Parity represents another obvious possibility for the modeling of \( E^f_t (\Delta s_{t+1}) \): However, the implied dynamic behavior of the nominal exchange rate in response to changes in the interest rate differential – large immediate jumps of the nominal exchange rate up to the level consistent with the future expected depreciation rate determined by the UIP – does not seem to be concordant with real world data: Indeed, as pointed out by Eichenbaum and Evans (1995, p.976), “the maximal effect of a contractionary monetary policy shock on U.S. exchange rates is not contemporaneous; instead the dollar continues to appreciate for a substantial period of time [a finding which] is inconsistent with simple rational expectations overshooting models of the sort considered by Dornbusch (1976).”
being therefore a forecasting rule of a “chartist” or “technical analysis” type (with $\beta^c_s > 0$).

In the spirit of De Graauwe and Grimaldi (2006), the last-period one-period earnings of investing one unit of domestic currency in the foreign asset are defined as

$$\psi^j_{t-1} = [S_{t-1}(1+i_t^s) - (1+i_{t-1})S_{t-2}] \text{ sgn} [E^j_{t-2}\Delta s_{t-1}] \quad j = c, f$$

(5)

with

$$\text{sgn} [E^j_{t-2}\Delta s_{t-1}] = \left\{ \begin{array}{ccc} 1 & \text{for } E^j_{t-2}\Delta s_{t-1} > 0 \\ 0 & \text{for } E^j_{t-2}\Delta s_{t-1} = 0 \\ -1 & \text{for } E^j_{t-2}\Delta s_{t-1} < 0 \end{array} \right. $$

According to eq. (5), if a FX market trader expects a domestic currency depreciation ($E^j_{t-1}\Delta s_t > 0$) and this indeed takes place, the trader makes an profit of $S_t(1 + i_t^s) - (1 + i_t)S_{t-1}$ or an analogous loss if $E^j_{t-1}\Delta s_t < 0$.

At every $t$, the relative share of both forecasting rules within the FX market (the so-called “market mood” in Dieci, Foroni, Gardini and He (2005)) is represented by the variable $\omega_t$, which, in the spirit of Brock and Hommes (1997, 1998), is determined by

$$\omega_t = \frac{\exp[\gamma(\psi^j_{t-1} - \sigma^2_{j,t-1})]}{\exp[\gamma(\psi^j_{t-1} - \sigma^2_{j,t-1})] + \exp[\gamma(\psi^c_{t-1} - \sigma^2_{c,t-1})]}$$

with

$$\lim_{\psi^j_{t-1} \to -\infty} \omega_t = 1 \quad \text{and} \quad \lim_{\psi^j_{t-1} \to 0} \omega_t = 0,$$

and

$$\sigma^2_{j,t-1} = (E^j_{t-2}s_{t-1} - s_{t-1})^2$$

being the last period’s squared forecast error of the behavioral forecasting rule $j$, and $\gamma$ measuring the intensity with which traders revise their choice of the forecasting rules. The evolution of the market mood variable $\omega_t$ is thus assumed to be determined by the relative profitability resulting from the forecast rules $E^j_{t-1}\Delta s_t = -\beta^j_s \eta_{t-1}$ and $E^c_{t-1}\Delta s_t = \beta^c_s (\Delta s_{t-1})$ by the fundamentalists and chartists, respectively. The FX market traders are thus assumed to choose between the two forecasting rules according to their relative profitability in the previous period.

The factual evolution of the log nominal exchange rate is assumed to be deter-
\[
\Delta s_{t+1} = -\omega t \beta s \eta_t + (1 - \omega t) \beta s^c (\Delta s_t) \\
= \frac{-\exp[\gamma (\psi^f_{t-1} - \sigma^2_{\psi,f_{t-1}})] \beta s^c \eta_t + \exp[\gamma (\psi^c_{t-1} - \sigma^2_{\psi,c_{t-1}})] \beta s^c (\Delta s_t)}{\exp[\gamma (\psi^f_{t-1} - \sigma^2_{\psi,f_{t-1}})] + \exp[\gamma (\psi^c_{t-1} - \sigma^2_{\psi,c_{t-1}})]},
\]
that is, by a weighted average of the expected nominal exchange rate depreciation rates of the two forecasting rules, with the relative weight of these two terms being determined by eq.(6). In other words, in this formulation the dynamics of the log nominal exchange rate are thus determined by the relative importance of the two discussed forecasting rules in the FX market (the “market mood”), which depends in turn in a nonlinear manner on the relative profitability of the latter and therefore, indirectly, on the nominal interest rate differential \(i - i^*\).

It should be noted that eq.(7) indeed opens up the possibility for a regime switching behavior of the log nominal exchange (determined by the relative profitability and interplay of the two forecasting rules), with periods of large persistence in the nominal exchange rate (and of deviations of the real exchange rate from the PPP level) as well as nonlinear adjustments of the nominal exchange rate, documented empirically by Taylor and Peel (2000) Taylor, Peel and Sarno (2001), among others.

The Macroeconomy

In order to keep this exposition as transparent as possible, the real side of the economy is modeled in a quite parsimonious manner. Accordingly, the output dynamics are represented by the change in the capacity utilization rate of firms \(u\),

\[
\Delta u_t = u_t - u_{t-1},
\]

assumed to be determined by

\[
\Delta u_t = -\alpha_u (u_{t-1} - u_o) - \alpha_{ur} (i_{t-1} - \Delta p_t - (i_o - \pi_o)) - \alpha_{ur} (v_{t-1} - v_o) + \alpha_g \eta_{t-1}
\]

where \(u_o\) denotes the steady state log capacity utilization rate, \(i_{t-1}\) the short-term nominal interest rate (\(i_o\) being the steady state nominal interest rate), \(\Delta p_t\) the price inflation rate (\(\pi_o\) being the steady state inflation rate), \(v - v_o\) the deviation of the log wage share from its steady state level.\(^2\)

\(^2\)Note that in the aggregate the influence of the wage share on the output dynamics can be either positive or negative, depending on whether the consumption or the investment (and net exports) reaction to wage share increases is the predominant, see e.g. Franke, Flaschel and Proaño (2006). We will, however, not engage into this debate here but rather adopt the most traditional view according to which \(\partial \Delta u / \partial v\) is unambiguously negative.
With respect to the link between the goods and the labor markets, the validity of Okun’s (1970) law is assumed for simplicity, whereas

\[ e_t/e_o = (Y/Y^\text{pot})^\beta \approx u_t^{\alpha}\epsilon_o \iff e_t = u_t^{\alpha}\epsilon_o, \]  

(9)

with \( Y \) as the actual level of output and \( Y^\text{pot} \) as its potential level.\(^3\)

Nominal wages and prices are assumed to adjust only gradually to disequilibrium situations in the real markets, following the theoretical disequilibrium approach proposed by Chiarella and Flaschel (2000) and Chiarella, Flaschel and Franke (2005).\(^4\)

According to this modeling approach, the structural form of the wage-price dynamics can be expressed by:

\[
\begin{align*}
\Delta w_t & = \beta_w e_t(e_{t-1} - e_o) - \beta_{ww}(v_{t-1} - v_o) + \kappa_{w}\Delta p_t + (1 - \kappa_{wp})\xi_t^c + \kappa_{wz}g_z, \\
\Delta p_t & = \beta_{pp}(u_{t-1} - u_o) + \beta_{pw}(v_{t-1} - v_o) + \kappa_{pw}(\Delta w_t - g_z) + (1 - \kappa_{pw})\xi_t^c. 
\end{align*}
\]

(10)

(11)

where \( w \) denotes the log nominal wage, \( p \) the log producer price level, \( v \) the log wage share and \( g_z = \text{const.} \) the trend labor productivity growth. The respective demand pressure terms in the wage and price Phillips Curves \( e_{t-1} - e_o \) and \( u_{t-1} - u_o \) in the market for labor and for goods \( (e \text{ represents the rate of employment on the labor market and } e_o \text{ the NAIRU-equivalent level of this rate, and similarly } u \text{ the rate of capacity utilization of the capital stock and } u_o \text{ the normal rate of capacity utilization of firms}) \), respectively,\(^5\) are thus augmented by three additional terms:

1) the deviation of the log wage share \( v \) or real unit labor costs from its steady state level (the error correction term discussed in Blanchard and Katz (1999, p.71)),
2) a weighted average of corresponding expected cost-pressure terms, assumed to be model-consistent, with forward looking, cross-over wage and price inflation rates \( \Delta w \) and \( \Delta p \), respectively; and a backward-looking measure of the prevailing inertial

---

\(^3\)Despite of being largely criticized due to its “lack of microfoundations”, in a large number of microfounded, “rational expectations” models such as Taylor (1994), Okun’s law is used to link production with employment.

\(^4\)This is a common characteristic between the approach pursued here and advanced New Keynesian models such as Eggert, Henderson and Levin (2000) and Woodford (2003). However, even though the resulting structural wage and price Phillips Curves equations to be discussed resemble to a significant extent those included in those theoretical models, their underlying modeling philosophy (based on neoclassical microfoundations) is completely different, see Chiarella, Flaschel and Franke (2005) for an extensive discussion of this issue.

\(^5\)As pointed out by Sims (1987), such strategy allows to circumvent the identification problem which arises in econometric estimations where both wage and price inflation equations have the same explanatory variables.
inflation in the economy (the “inflationary climate”, so to say) symbolized by $\pi^e$, and, finally, 3) trend labor productivity growth $g_z$ (which is expected to influence wages in a positive and prices in a negative manner, due to the associated easing in production cost pressure).}

The inflationary climate $\pi^e_t$ is assumed to be determined by CPI inflation, defined as

$$\Delta p^e_t = \xi \Delta p_t + (1 - \xi)(\Delta s_t + \Delta p^*_t),$$

with $\xi$ as the share of imported goods in the CPI goods basket.

Because of the uncertainty linked with nominal exchange rate movements, I assume for both workers and firms’ decision taking processes, that CPI inflation based inflationary climate $\pi^e_t$ is updated in an adaptive manner according to

$$\Delta \pi^e_t = \beta_w(\Delta p^e_t - \pi^e_{t-1}) = \beta_w(\xi \Delta p_t + (1 - \xi)(\Delta s_t + \Delta p^*_t) - \pi^e_{t-1}).$$

It should be pointed out that, as the wage-price mechanisms are formulated, the development of the CPI inflation does not matter for the evolution of the domestic log wage share $v = (w/p)/z$, measured in terms of producer prices, the law of motion of which is given by (with $\kappa = 1/(1 - \kappa_{wp}\kappa_{pw})$ and $\varepsilon$ given by eq.(9)):

$$\Delta v_t = \kappa \left[(1 - \kappa_{pw}) f_w(u, v) - (1 - \kappa_{wp}) f_p(u, v) + (\kappa_{pw} - 1)(1 - \kappa_{pw}) g_z\right].$$

with $f_w(u, v) = \beta_w(u_{t-1} - e_o) - \beta_w(v_{t-1} - v_o)$ and $f_p(u, v) = \beta_{pw}(u_{t-1} - u_o) + \beta_{pw}(v_{t-1} - v_o)$.

As eq.(14) clearly shows, due to the particular specification of the wage and price inflation dynamics, the labor share depends on both the situations in the goods and labor markets, on the state of income distribution in the economy as well as on the relative weights of cross-over inflation expectations in the wage and price inflation adjustment equations.

Exploiting the fact that the structural wage- and price inflation adjustment equations given by eqs. (10) and (11) also imply the following reduced-form Price Phillips

\[ \Delta p_t = \kappa \left[(1 - \kappa_{pw}) f_w(u, v) - (1 - \kappa_{wp}) f_p(u, v) + (\kappa_{pw} - 1)(1 - \kappa_{pw}) g_z\right]. \]

\[ f_w(u, v) = \beta_w(u_{t-1} - e_o) - \beta_w(v_{t-1} - v_o) \quad \text{and} \quad f_p(u, v) = \beta_{pw}(u_{t-1} - u_o) + \beta_{pw}(v_{t-1} - v_o). \]

\[ \Delta p^* = \beta_{we}(\Delta p^* - \pi^e_{t-1}) = \beta_{we}(\xi \Delta p_t + (1 - \xi)(\Delta s_t + \Delta p^*_t) - \pi^e_{t-1}). \]

\[ \Delta v_t = \kappa \left[(1 - \kappa_{pw}) f_w(u, v) - (1 - \kappa_{wp}) f_p(u, v) + (\kappa_{pw} - 1)(1 - \kappa_{pw}) g_z\right]. \]

\[ f_w(u, v) = \beta_{we}(u_{t-1} - e_o) - \beta_{we}(v_{t-1} - v_o) \quad \text{and} \quad f_p(u, v) = \beta_{pw}(u_{t-1} - u_o) + \beta_{pw}(v_{t-1} - v_o). \]
Curve:

\[ \Delta p_t = \kappa [f_p(u, v) + \kappa_{pw} f_w(u, v) + \kappa_{pw}(\kappa_{wz} - 1)g_z] + \pi^*_t, \]

(15)

the following equation for the evolution of the log real exchange rate can be obtained 
in conjunction with the law of motion for the log nominal exchange rate given by 
eq \text{eq.}(7) \text{ (assuming } \Delta p^*_t = \bar{\pi}^* = \text{const.}):

\[ \Delta \eta_t = \Delta s_t + \bar{\pi}^* - \Delta p_t \]

\[ = \frac{-\exp(\psi^f_{t-1}) \beta^f \eta_{t-1} + \exp(\psi^f_{t-1}) \beta^s (\Delta s_{t-1})}{\exp(\psi^f_{t-1}) + \exp(\psi^s_{t-1})} + \bar{\pi}^* - \Delta p_t \]

(16)

Monetary Policy

Concerning monetary policy, as it is usual in standard modern macroeconometric 
models, I assume that the short-term nominal interest rate is determined by a classical Taylor rule:

\[ i^T_t = i_o + \phi_x (\pi_t - \pi_o) + \phi_y (u_t - u_o). \]

(17)

where the target nominal interest rate of the central bank \( i^T_t \) is assumed to depend 
on the steady state nominal rate of interest \( i_o \), on the inflation gap \( \Delta p_t - \pi_o \) (with 
a reaction strength \( \phi_x \)) – for now assumed to be defined in terms of producer prices 
inflation – and on the output gap (with a reaction strength \( \phi_y \)).\(^9\)

3 Local Stability Analysis

In this section the model’s stability conditions are investigated in an analytical manner. 
We focus our analysis on the stability properties of the small-open economy 
subsystem, assuming that the foreign economy is and remains at its steady state 
level \((u^* = u^*_o = 1, \ v^* = v^*_o = 1, \\ v^* = v^*_o).\(^{10}\)

\(^9\)It should be pointed out that no interest rate smoothing term is included in \text{eq.}(17), implying 
that nominal interest rate is at every point in time equal the target level of the domestic monetary 
authorities. Indeed, there is an ongoing and still unsolved debate in the academic literature about 
whether there is an interest smoothing parameter in the monetary policy reaction rule of the central 
banks or whether the observed high autocorrelation in the nominal interest rate is simply the result 
of highly correlated shocks or only slowly available information, see e.g. Rudebusch (2002, 2006) 
and for a thorough discussion of this issue.

\(^{10}\)Note that since the same structure is assumed for both economies, the conditions for local 

stability of “domestic economy” subsystem also hold for the “foreign economy”. 

8
Based on the notion that the qualitative dynamics and stability properties of a macroeconomic model should not depend on whether it is formulated in continuous- or discrete time,\textsuperscript{11} we use a continuous-time representation of the model for the following local stability analysis. For this the underlying period length is defined in general terms as $\Delta t$, so that for $\Delta t \to 0$, the following continuous time approximation for the output dynamics equation can be formulated

$$\dot{u} \approx \Delta u_{t-\Delta t} \approx -\alpha_u (u - u_o) - \alpha urinary (i - \dot{p} - (i_o - \pi_o)) - \alpha uv (v - v_o) + \alpha y\eta \eta$$  (18)$$

with $\dot{p} \approx \Delta p_{t-\Delta t}$ and

$$\dot{p} = \kappa [\beta pu (u - u_o) + \beta pv (v - v_o) + \kappa pw (\beta urc (u^{\alpha u} - 1) - \beta uv (v - v_o))] + \pi c$$,  (19)

see eq.(15), and $e$ given by eq.(9) and $e_o = 1$.

Analogously, for the wage share given by eq. (14) (setting $g_z = 0$ for notational simplicity), we obtain

$$\dot{v} = \kappa [(1 - \beta pu) f w (u, v) - (1 - \beta wp) f_p (u, v)].$$  (20)

With respect to the nominal exchange rate dynamics expressed by eq.(7), due to the non-differentiability of eq.(5) and the significant nonlinearity comprised in eq.(6), see De Grauwe and Grimaldi (2006, p.25ff), as well as to the fact that this will be investigated extensively in the next section by means of impulse-response analysis, for now I prescind from the dynamics of $\omega$, the market mood variable, and assume it to remain constant at its steady state level $\omega = \omega_o$.

Under this simplifying assumption, it follows for the dynamics of the log nominal exchange rate

$$\dot{s} = -\omega_o \beta_s^f \eta + (1 - \omega_o) \beta_s^c \dot{s},$$

where $\dot{s} \approx \Delta s_t = \Delta s_{t-\Delta t}$ is assumed to hold. Reordering delivers

$$\dot{s} = \frac{-\omega_o \beta_s^f \eta}{1 - (1 - \omega_o) \beta_s^c}.$$  (21)

For the continuous-time approximation of the log real exchange rate dynamics described by eq.(16), it follows (with $\Delta p^* = const. = \bar{\pi}^*$)

$$\dot{\pi} = \dot{s} + \bar{\pi} - \dot{p} = \frac{-\omega_o \beta_s^f \eta}{1 - (1 - \omega_o) \beta_s^c} + \bar{\pi} - \dot{p},$$  (22)

\textsuperscript{11}As pointed out by Foley (1975, p.310), “No substantive prediction or explanation in a well-defined macroeconomic period model should depend on the real time length of the period”, see also Flaschel and Proao (2009).
and for the dynamics of the inflationary climate

\[ \dot{\pi}^c = \beta_\pi^c(\xi \dot{p} + (1 - \xi)(\dot{s} + \bar{\pi}^*) - \pi^c). \]  

(23)

In sum, the 4D dynamical system is given by

\[
\begin{aligned}
\dot{u} &= -\alpha_{uu}(u - u_o) - \alpha_{ur}((\phi_\pi - 1)(\dot{p} - \pi_o) + \phi_y(u - u_o)) - \alpha_{av}(v - v_o) + \alpha_{au}\eta \\
\dot{v} &= \kappa [(1 - \kappa_{pu})f_w(u, v) - (1 - \kappa_{wp})f_p(u, v)] \\
\dot{\eta} &= \frac{-\omega_o\beta_{\eta}^f \eta}{1 - (1 - \omega_o)\beta_s^c} + \bar{\pi}^* - \dot{p}, \\
\dot{\pi}^c &= \beta_\pi^c\left(\xi (\dot{p} - \pi^c) + (1 - \xi) \left(\frac{-\omega_o\beta_{\eta}^f \eta}{1 - (1 - \omega_o)\beta_s^c} + \bar{\pi}^*\right) - \pi^c\right)
\end{aligned}
\]

with eq. (19) to be inserted in several places.

The corresponding Jacobian of this 4D dynamical subsystem is given by

\[
J_{3D} = \begin{bmatrix}
\frac{\partial \dot{u}}{\partial u} & \frac{\partial \dot{u}}{\partial v} & \frac{\partial \dot{u}}{\partial \eta} & \frac{\partial \dot{u}}{\partial \pi^c} \\
\frac{\partial \dot{\eta}}{\partial u} & \frac{\partial \dot{\eta}}{\partial v} & \frac{\partial \dot{\eta}}{\partial \eta} & \frac{\partial \dot{\eta}}{\partial \pi^c} \\
\frac{\partial \dot{\pi}^c}{\partial u} & \frac{\partial \dot{\pi}^c}{\partial v} & \frac{\partial \dot{\pi}^c}{\partial \eta} & \frac{\partial \dot{\pi}^c}{\partial \pi^c}
\end{bmatrix}
= \begin{bmatrix}
J_{11} & J_{12} & J_{13} & J_{14} \\
J_{21} & J_{22} & J_{23} & J_{24} \\
J_{31} & J_{32} & J_{33} & J_{34} \\
J_{41} & J_{42} & J_{43} & J_{44}
\end{bmatrix},
\]

with

\[
\begin{aligned}
J_{11} &= -\alpha_{uu} - \alpha_{ur}((\phi_\pi - 1)\kappa(\beta_{pu} + \kappa_{pu}\beta_{we}\alpha_{eu}) + \phi_y) < 0 \\
J_{12} &= -\alpha_{uv} - \alpha_{ur}(\phi_\pi - 1)\kappa(\beta_{pv} - \kappa_{pu}\beta_{uv}) \\
J_{13} &= \alpha_{au} > 0, \quad J_{14} = -\alpha_{ur}(\phi_\pi - 1) \\
J_{21} &= \kappa((1 - \kappa_{pu})\beta_{we}\alpha_{eu} - (1 - \kappa_{uv})\beta_{pu}) \\
J_{22} &= -\kappa((1 - \kappa_{pu})\beta_{uv} + (1 - \kappa_{wp})\beta_{pv}) < 0, \\
J_{23} &= 0, \quad J_{24} = 0, \\
J_{31} &= -\kappa(\beta_{pu} + \kappa_{pu}\beta_{we}\alpha_{eu}), \quad J_{32} = -\kappa(\beta_{pv} - \beta_{uv}\kappa_{pu}), \\
J_{33} &= -\frac{\beta_{\eta}^f \omega_o}{1 - \beta_{\beta}^c(1 - \omega_o)} < 0 \quad J_{34} = -1 \\
J_{41} &= \beta_\pi^c\xi_\kappa(\beta_{pu} + \kappa_{pu}\beta_{we}\alpha_{eu}), \quad J_{42} = \beta_\pi^c\xi_\kappa(\beta_{pv} - \beta_{uv}\kappa_{pu}), \\
J_{43} &= -\frac{\beta_{\eta}^f(1 - \xi)\omega_o \beta_{\eta}^f}{1 - \beta_{\beta}^c(1 - \omega_o)} < 0, \quad J_{44} = -\beta_\pi < 0.
\end{aligned}
\]

Proposition 1:

Let $\beta_u^c = 0$ (with $\pi^c = 0$), as well as $\alpha_{uu} = 0$ and $\beta_{ev} = \beta_{pu} = 0$. This decouples the dynamics of the log wage share $v$ and of the inflationary
climate $\pi^c$ from the rest of the system, which comes 2D and given by
\[ \dot{u} = -\alpha_{uu}(u - u_o) - \alpha_{ur}((\phi_\pi - 1)(\dot{p} - \pi_o) + \phi_y(u - u_o)) + \alpha_{ur}\eta \]
\[ \dot{\eta} = \frac{-\omega_0\beta^c_\eta}{1 - (1 - \omega_o)\beta^c_s} - \kappa[\beta_{pu}(u - u_o) + \kappa_{pu}\beta_{we}(u^{\alpha_{eu}}e_o - e_o)]. \]

Assume for this reduced subsystem that the destabilizing influence of the chartists in the FX market is not predominant, i.e., that (i) $\beta^c_\eta(1 - \omega_o) < 1$ holds. Additionally, assume that (ii) $\phi_\pi > 1$, that is, the Taylor Principle, holds.

Then: The Routh-Hurwitz conditions are fulfilled and the unique steady state of the reduced 2D dynamical system is locally asymptotically stable.

Proof:

According to the Routh-Hurwitz stability conditions for a 2D dynamical system, asymptotic local stability of a steady state is fulfilled when
\[ a_1 = -\text{trace}(J_{2D}) > 0 \text{ and } a_2 = \det(J_{2D}) > 0. \]

As already stated, as long as the destabilizing influence of the chartists in the foreign exchange market is not predominant (condition (i) in the above Proposition), the dynamics of the nominal exchange rate are asymptotically stable. In this case $\partial\dot{\eta}/\partial u < 0$, and the trace of $J_{2D}$ is unambiguously negative (and $a_1 > 0$ holds), since
\[ \text{tr}(J_{2D}) = J_{11} + J_{33} \]
\[ = -\alpha_{uu} - \alpha_{ur}((\phi_\pi - 1)\kappa(\beta_{pu} + \kappa_{pu}\beta_{we}\alpha_{eu}) + \phi_y) - \frac{\omega_0\beta^c_\eta}{1 - (1 - \omega_o)\beta^c_s} < 0. \]

Additionally, condition (i) ensures that $\partial\dot{\eta}/\partial u < 0$ and therefore, that
\[ \det(J_{2D}) = J_{11}J_{33} - J_{13}J_{31} \]
\[ = \frac{[\alpha_{uu} + \alpha_{ur}((\phi_\pi - 1)\kappa(\beta_{pu} + \kappa_{pu}\beta_{we}\alpha_{eu}) + \phi_y)]\omega_0\beta^c_\eta}{1 - (1 - \omega_o)\beta^c_s} + \alpha_{u\eta}\kappa(\beta_{pu} + \kappa_{pu}\beta_{we}\alpha_{eu}) > 0. \]

In other words, the steady state of the 2D dynamic system is asymptotically stable if, on the one hand, the intrinsic dynamics of the nominal exchange rate are not all-too explosive ($\beta^c_\eta < (1 - \omega_o)^{-1}$) and, on the other hand, if the conduction of monetary policy is aggressive enough to bring about price inflation and output
stability ($\phi_y \geq 0$ and $\phi_\pi > 1$). Note that these two conditions must \textit{jointly} hold, being by no means substitutes from each other: If $\beta_\pi > (1 - \omega_\nu)^{-1}$, not only the FX market but also the real economy is subject to explosive forces which would make the steady state unstable, and this for irrespective values of $\phi_\pi$ and $\phi_y$. This result thus implies that an exclusive focus on price (and output) stability might not be sufficient to achieve macroeconomic stability if the FX (and in more general terms, the financial markets) are highly unstable.

\textbf{Proposition 2:}

Let $\beta_\pi = 0$, but allow $\alpha_{uv}, \beta_{uv} \text{ and } \beta_{pw} > 0$. This reintegration of the dynamics of the log wage share into the dynamical system, which becomes 3D. In addition to conditions (i) and (ii) of Proposition 1, assume furthermore that $\kappa_{pw}$ is of a sufficiently small dimension, so that (iii) $\beta_{pw} > \kappa_{pw}\beta_{uv}$ and $(1 - \kappa_{pw})\beta_{uv}\alpha_{uv} > (1 - \kappa_{uv})\beta_{uy}$. Then: The Routh-Hurwitz conditions are fulfilled and the unique steady state of the new 3D dynamical system of the dynamical variables $u, v$ and $\eta$ is locally asymptotically stable.

\textbf{Proof:}

According to the Routh-Hurwitz stability conditions for a 3D dynamical system, asymptotic local stability of a steady state is fulfilled when

$$a_i > 0, \quad i = 1, 2, 3 \quad \text{and} \quad a_1a_2 - a_3 > 0,$$

where $a_1 = -\text{trace}(J_{3D}), a_2 = \sum_{k=1}^{3} J_k$ with

$$J_1 = \begin{vmatrix} J_{22} & J_{23} \\ J_{32} & J_{33} \end{vmatrix}, J_2 = \begin{vmatrix} J_{11} & J_{13} \\ J_{31} & J_{33} \end{vmatrix}, J_3 = \begin{vmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{vmatrix}.$$ 

and $a_3 = -\det(J)$.

As already stated, as long as the influence of the chartists in the foreign exchange market is not predominant (condition (i) in Proposition 1), the dynamics of the nominal exchange rate are asymptotically stable, so that $\partial \eta/\partial \eta < 0$. Condition (ii) ($\phi_\pi > 1$) ensures that $\partial u/\partial u$ is unambiguously negative, and condition (iii), on the other hand, that $\partial \nu/\partial \nu < 0$. 

12
Under these conditions, the trace of $J_{3D}$ is thus unambiguously negative (and $a_1 > 0$ holds), since

$$\text{tr}(J_{3D}) = J_{11} + J_{22} + J_{33} < 0.$$ 

Finally, condition (iii) - linked $\phi_\pi > 1$ - guarantees that $\partial \hat{u}/\partial v < 0$, as well as $\partial \hat{v}/\partial v < 0$.

It can be easily confirmed using the second-order minors of $J$, $J_1$, $J_2$ and $J_3$

$$J_1 = J_{22} \cdot J_{33} - J_{32} \cdot J_{23} = \frac{\omega_0 \beta_4^2 \kappa (\beta_{uv}(1 - \kappa_{pu}) + \beta_{pu}(1 - \kappa_{wp}))}{(1 - \beta_2^2(1 - \omega_\alpha))} > 0$$

$$J_2 = J_{11} \cdot J_{33} - J_{31} \cdot J_{13} = \frac{\omega_0 \beta_4^2 \alpha_{uu} + \alpha_{ur} (\phi_y + \phi_\pi \kappa (\beta_{pu} + \kappa_{pu} \kappa_{uw} \alpha_{eu}))}{(1 - \beta_2^2(1 - \omega_\alpha))} + \alpha_{uw} \kappa (\beta_{pu} + \kappa_{uw} \kappa_{pu} \alpha_{eu}) > 0$$

$$J_3 = J_{11} \cdot J_{22} - J_{21} \cdot J_{12} = [\alpha_{uu} + \alpha_{ur} ((\phi_\pi - 1) \kappa (\beta_{pu} + \beta_{wp} \kappa_{pu} \alpha_{eu}) + \phi_y)] \kappa (\beta_{uv}(1 - \kappa_{pu}) + \beta_{pu}(1 - \kappa_{wp}))$$

$$+ \kappa ((1 - \kappa_{wp}) \beta_{uw} \alpha_{eu} - \beta_{pu}(1 - \kappa_{wp})) [\alpha_{uv} + \alpha_{ur} (\phi_\pi - 1) \kappa (\beta_{pu} - \kappa_{uw} \beta_{uv})] > 0$$

that $a_2 = \sum_{k=1}^3 J_k > 0$ and $a_3 = -\det(J_{3D}) > 0$, as well as the critical condition $a_1 a_2 - a_3 > 0$ for local asymptotic stability of the steady state of the system hold under the assumed parameter dimensions.

**Proposition 3:**

Finally, let $\beta_{\pi\pi} > 0$, but of a sufficiently small dimension. This reintegrates the dynamics of the inflation climate into the dynamical system, which becomes 4D again. Assume furthermore the validity of conditions (i) - (iii). Then: The Routh-Hurwitz stability conditions for a 4D dynamical system are fulfilled and the unique steady state of the system is locally asymptotically stable.

**Sketch of Proof:**

Given the adaptive nature of the dynamics of the inflationary climate $\pi^c$ described by eq. (23), as long as conditions (i)-(iii) are fulfilled and the adjustment speed $\beta_{\pi\pi}$ is sufficiently low, the dynamics of $\pi^c$ do not alter the local stability of the system’s steady state.\textsuperscript{12}

\textsuperscript{12}Note that conditions (ii)-(iii) are necessary conditions for stability: If for example $\beta_4^2(1 - \omega_\alpha) < 1$ or $\phi_\pi > 1$ are no longer fulfilled, the dynamics of $\pi^c$ would not only not be able to bring about
It should be pointed out, however, that these results (and in general the local stability analysis just performed) are based on the (simplifying) assumption that the market mood (represented by $\omega$) remained constant and at its steady state level $\omega = \omega_0 = 1/2$, so that the interaction between the macroeconomic dynamics of a small open economy and the profitability of the assumed behavioral forecasting rules has been still not investigated. This is done by means of a numerical analysis in section 5.

4 Model Estimation

In this section the theoretical model just discussed is estimated with aggregate time series data of the U.S. economy.\textsuperscript{13} The aim of this exercise is twofold: On the one hand, we attempt to determine the empirical plausibility of the theoretical model just discussed. On the other hand, we intend to obtain parameter values needed for the numerical simulations of the next section.

I will not attempt, however, to estimate all parameters of the theoretical model discussed in the previous section, since a proper estimation of some parameters like $\beta_{n^*}$ (given its unobservable nature), as well as $\beta^f$ and $\beta^c$ (due to their actual state-dependent and time-varying character) would be out of the scope of this paper.\textsuperscript{14} Instead, I estimate only the wage-price inflation adjustment- as well as the goods market equations (which I expect to be the most stable relationships across time) and calibrate the remaining parameters as discussed in the next section.

4.1 Data Description

The empirical data of the corresponding time series for the U.S. economy stem from the Federal Reserve Bank of St. Louis data set (http://research.stlouisfed.org/fred2/), the OECD and the Euro Area-Wide Model (http://eabcn.org/area-wide-model) databases. The data is quarterly, seasonally adjusted and concern the period from 1980:1 to

\textsuperscript{13}See Proaño, Flaschel, Ernst and Semmler (2006) for a comparison between the U.S. and the euro area based on the estimation of the original closed-economy model by Chen et al. (2006).

\textsuperscript{14}The monetary policy rule given by eq.(17) will also be calibrated since, as pointed out by Orphanides (2001), estimations of monetary policy rules not based on real time datasets might deliver biased coefficients, providing thus wrong conclusions concerning the actual strategy pursued by the monetary authorities.
Table 1: Data Set

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description of the original series</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c$</td>
<td>Employment rate</td>
</tr>
<tr>
<td>$u$</td>
<td>Capacity utilization: Manufacturing, percent of capacity</td>
</tr>
<tr>
<td>$w$</td>
<td>Nonfarm Business Sector: Compensation per hour, 1992=100 (in logs)</td>
</tr>
<tr>
<td>$p$</td>
<td>Gross Domestic Product: Implicit Price Deflator, 1996=100</td>
</tr>
<tr>
<td>$p^*$</td>
<td>World Demand Deflator: Composite indicator</td>
</tr>
<tr>
<td>$z$</td>
<td>Nonfarm Business Sector; Output per hour of all persons, 1992=100 (in logs)</td>
</tr>
<tr>
<td>$v$</td>
<td>Nonfarm Business Sector: Real compensation per output unit, 1992=100 (in logs)</td>
</tr>
<tr>
<td>$i$</td>
<td>Federal Funds rate</td>
</tr>
<tr>
<td>$s$</td>
<td>EUR/USD Nominal exchange rate (in logs)</td>
</tr>
</tbody>
</table>

For the following estimation the (actually unobservable) inflationary climate variable $\pi_t^\epsilon$ of the theoretical part of this paper is approximated by a linearly declining moving average of price inflation rates with linearly decreasing weights over the past twelve quarters, denoted as $\pi_t^{12}$.

Taking this into account, the equations to be estimated are given by

\[
\begin{align*}
\Delta w_t & = \beta_{wu}(e_t-1 - e_o) - \beta_{wu}(v_{t-1} - v_o) + \kappa_{wp}\Delta p_t + \kappa_{wp\pi}\pi_t^{12} + \kappa_{wz}\Delta z_t \\
\Delta p_t & = \beta_{pu}(u_t-1 - u_o) + \beta_{pv}(v_{t-1} - v_o) + \kappa_{pw}(\Delta w_t - \hat{z}_t) + \kappa_{pw\pi}\pi_t^{12} \\
u_t & = u_{t-1} + \alpha_{wu}(u_{t-1} - u_o) - \alpha_{uv}(u_{t-1} - \Delta p_t - \kappa_{u\pi}\pi_t^{12}) - \alpha_{uv}(v_{t-1} - v_o) + \alpha_{u\pi}\pi_t^{12}
\end{align*}
\]

with $\Delta z_t = z_t - z_{t-1}$ denoting the growth rate of labor productivity - introduced here as an exogenous variable -, and sample means being characterized by a subscript $o$.

In order to check the stationarity of the analyzed time series, Phillips-Perron unit root tests were carried out in order to account for residual autocorrelation (as done by the standard ADF Tests), and also for possible residual heteroskedasticity. The Phillips-Perron test specifications and results are shown in Table 2.

As Table 2 shows, the applied unit root tests reject the hypothesis of a unit root for all series at the 10% level, with the sole exception of the real exchange rate. However, due to the general low power of the unit root tests for small samples, we interpret the corresponding t-statistic as only providing a hint for the strong autocorrelation present in the real exchange rate.
Table 2: Phillips-Perron Unit Root Test Results. Sample: 1980:1-2004:4

<p>| | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Δw</td>
<td>-</td>
<td>-7.529</td>
<td>0.0000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Δp</td>
<td>-</td>
<td>-3.119</td>
<td>0.0278</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Δu</td>
<td>-</td>
<td>-5.348</td>
<td>0.0000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>e - e_o</td>
<td>-</td>
<td>-1.849</td>
<td>0.0615</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>v - v_o</td>
<td>-</td>
<td>-1.813</td>
<td>0.0665</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>i</td>
<td>-</td>
<td>-1.933</td>
<td>0.0512</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>η</td>
<td>-</td>
<td>-2.159</td>
<td>0.2223</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>


4.2 Estimation Results

In order to account for possible regressor endogeneity, the model is estimated by means of Three-Stage-Least-Squares (3SLS), using the past values of the endogenous variables as instruments.

Table 3: 3SLS Parameter Estimates: Two-Country System

<p>| | | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>w_t</td>
<td>β_w</td>
<td>β_v</td>
<td>κ_w</td>
<td>κ_wα</td>
<td>κ_wαβ</td>
<td>R^2</td>
<td>DW</td>
</tr>
<tr>
<td>U.S.</td>
<td>0.603</td>
<td>-0.189</td>
<td>0.032</td>
<td>0.902</td>
<td>0.387</td>
<td>0.335</td>
<td>1.994</td>
</tr>
<tr>
<td></td>
<td>[18.368]</td>
<td>[-4.780]</td>
<td>[0.417]</td>
<td>[15.430]</td>
<td>[13.830]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>p_t</td>
<td>β_p</td>
<td>β_v</td>
<td>κ_p</td>
<td>κ_pα</td>
<td>κ_pαβ</td>
<td>R^2</td>
<td>DW</td>
</tr>
<tr>
<td>U.S.</td>
<td>0.017</td>
<td>0.042</td>
<td>0.055</td>
<td>0.589</td>
<td>0.774</td>
<td>1.165</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[1.969]</td>
<td>[2.287]</td>
<td>[3.372]</td>
<td>[27.709]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>u_t</td>
<td>α_u</td>
<td>α_u</td>
<td>α_u</td>
<td>α_u</td>
<td>α_u</td>
<td>R^2</td>
<td>DW</td>
</tr>
<tr>
<td>U.S.</td>
<td>-0.077</td>
<td>-0.094</td>
<td>-0.073</td>
<td>0.000</td>
<td>0.925</td>
<td>1.077</td>
<td></td>
</tr>
</tbody>
</table>

As Table 3 shows, the obtained parameter estimates by and large support the theoretical formulation of the model discussed in the previous section. On the one hand the influence of the market specific demand pressure terms (the capacity utilization in the price- and the employment rate in the wage Phillips curve equations) seems to be corroborated by the model estimation, as well as the fact that wage flexibility is larger than price flexibility (concerning their respective demand pressure measures), a result concordant with the findings of Chen and Flaschel (2006) and Flaschel and Krolzig (2006)). Furthermore, Table 3 delivers statistical evidence for the significance of the Blanchard and Katz (1999) error correction terms v - v_o in both the wage and price adjustment equations of the U.S. economy. Concerning
the cross-over formulation of the inflationary expectations, it cannot be statistically rejected in the price inflation equation, but only the inflationary climate term $\pi^{12}$ seems to enter significantly (though with a coefficient near to 1) in the wage inflation equation.

Concerning the estimated open economy IS equation, the 3SLS estimation shows, as expected, the negative influence of the expected real interest rate on the dynamics of capacity utilization. The same holds true for the effect of $v - v_o$ the deviation of the labor share from its steady state level, showing that a relatively high labor share (or real average unit labor costs) has a negative impact on the domestic rate of capacity utilization. And finally, as expected, a positive influence of the log real exchange rate on the capacity utilization of the economy was also found.

Having obtained empirical estimates of the model parameters, as next I focus on the stability properties of the two-country model concerning variations in the parameter values of the FX markets.

5 Behavioral FX Dynamics, Two-Country Interactions and Macroeconomic Stability

As previously shown by the analytical calculation of the local stability conditions of the one-country submodule of section 3, the specific values of the behavioral FX forecasting rules play a key role in the macroeconomic stability of a small open economy. In this section this previous analysis is further extended by the incorporation of two-country interactions, this time by means of numerical simulations. This done also in a deterministic setup as throughout the paper, in order to analyze the intrinsic dynamics of the model without the influence of permanent stochastic shocks.$^{15}$

In order to assure laboratory conditions, two identical large open economies (countries $A$ and $B$) are also assumed in this section and these are calibrated the wage- and price Phillips Curve, as well as the goods market equations of the resulting two-country system with the U.S. parameter estimates discussed in the previous section. Respecting the remaining coefficients, the parameter values summarized in

$^{15}$See Franca (2009) for an attempt in this direction using a one-country model along the lines of Rudebusch and Svensson (1999). As it is shown there, in a stochastic environment with behaviorally determined nominal exchange rate dynamics, important stylized facts on the nominal exchange rate such as the disconnection of its dynamics from the macroeconomic fundamentals, as well as the non-normality of returns and their volatility clustering can be generated.
Table 4 are used in the following simulations.  

<table>
<thead>
<tr>
<th>Monetary Policy</th>
<th>FX Markets</th>
<th>Labor Markets</th>
<th>CPI Inflation Climate</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_x = 1.5$</td>
<td>$\beta^M = 0.5$</td>
<td>$\alpha_w = 0.3$</td>
<td>$\beta_{w^c} = 0.75$</td>
</tr>
<tr>
<td>$\phi_y = 0.5$</td>
<td>$\beta^s = 1.5$</td>
<td></td>
<td>$\xi = 0.1$</td>
</tr>
<tr>
<td>$\gamma = 1$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

For the monetary policy rule, I chose the standard parameters assumed for monetary policy rules of the Taylor (1993) type. Concerning the link between the goods and the labor markets, that is Okun’s law (eq. (9)), the value of 0.3 also common in the literature is assumed. Given the insights of the local stability analysis of section 3, I choose FX market parameter values which should not imply an unstable behavior of the nominal exchange rate in the baseline scenario, namely $\beta^M = 0.5$, $\beta^s = 1.5$ and $\gamma = 1$. Finally, with respect to the CPI inflationary climate adjustment parameter, a value of $\beta_{w^c} = 0.75$ is chosen, and for the share of the foreign goods in the CPI basket $\xi = 0.15$, following Rabanal and Tuesta (2006).

Using the calibrated two-country system, the dynamic reactions of the model to asymmetric shocks are analyzed.  

To begin and in order to illustrate the dynamics of the two-country model for given parameter values of $\beta^M$ and $\beta^s$, Figure 1 shows the dynamic reactions of the two economies to an asymmetric shock, namely an aggregate demand shock solely in country $A$.

As it can be observed, following a positive aggregate demand shock in country $A$ (and the resulting increase in price inflation) the nominal interest rate in that country increases according to the assumed monetary policy rule. This leads to an increase in the nominal interest rate differential, and therefore to a differentiated profitability of the two behavioral trading rules, which in turn leads to variations in the FX market mood, i.e., in the relative share of fundamentalists and chartists in that market. As illustrated in the third panel of the first row of Figure 1, the

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16 In section 3 the stability conditions of the small open economy subsystem were identified. Since the two economies are assumed to be identical and both economies also fulfill the stability criteria discussed there, instability should only arise from the FX markets or the interaction between the two economies via the real exchange rate channel.

17 It should be intuitive that for the case of symmetric countries, symmetric real shocks do not have any effects on the level of the nominal exchange rate, due to the fact that the monetary policy in both countries is assumed to react in an identical manner. Accordingly, in this case the resulting nominal interest rate differential after the occurrence of the shocks is zero in all periods.
reaction of the nominal exchange rate is driven primarily by the fundamentalist rule, after which the price of country B’s currency in terms of country A’s currency (the nominal exchange rate) should rise following a higher price inflation in country A, for the real exchange rate $\eta$ to return to its PPP level.

These dynamics are in stark contrast with the ones resulting from NOEM models: On the one hand, the nominal exchange rate depreciation following the nominal interest rate increase in country A does not occur instantly as assumed e.g. in Gál and Monacelli (2005), but – more accordingly with empirical data, see again Eichenbaum and Evans (1995) – occurs with a certain delay. And on the other hand, in contrast to rational expectations NOEM models (and the myopic perfect foresight model) which assume for the nominal exchange rate (the “jump variable”) a monotonic convergence towards equilibrium after its initial, overshooting reaction to shocks according to the Uncovered Interest Rate Parity, this model generates a non-monotonic, fluctuating convergence towards equilibrium which results from the interaction of inflation, output, monetary policy and the behavioral FX market trading.

As next I investigate the influence of $\beta^f$ and $\beta^p$ for the dynamic stability of the two-country system. For this purpose, I make use of an intuitive and straightforward procedure which allows me to analyze in a clear and transparent manner the effects
of that parameter for the dynamic stability of the two-country system. I simply recalculate the dynamic responses of both economies to the same exogenous aggregate demand shock under the ceteris paribus variation of one single FX market parameter at a time.

![Graphs](image_url)

Figure 2: Dynamic responses of FX markets and real economy to a one-time domestic monetary policy shock for varying values of $\beta^f \in (0, 1)$ (with $\beta^s = 1$)

Figure 2 shows the resulting 3D diagrams of the dynamic reactions of the two-country system for ceteris paribus variations of $\beta^f$, the “fundamentalism” FX market parameter. As it can be clearly observed there, for larger values of $\beta^f$ the dynamic reactions of the two-country system are not only not destabilized, but they are even of smaller amplitude, and also of shorter duration, showing the clearly stabilizing influence of “fundamentalistic” behavior in the FX markets at the macroeconomic level. This result is quite intuitive, since we would have expected $\beta^f$ to act in an intrinsically stabilizing manner at the macroeconomic level also in the two-country case, because it represents (indirectly, through the influence of the “fundamentalist” rule on the market outcome) the speed of convergence of the log nominal exchange rate towards its PPP consistent level. As Figure 2 shows, the interaction of two “large” economies does not seem to undermine this behavior.

Figure 3 in turn shows similar 3D graphs for varying values of the “chartism” parameter $\beta^s$.

As it can be clearly observed, increasingly extrapolative expectations of the chartist rule (represented by larger values of $\beta^s$) lead to an increasingly overshooting behavior not only of the factual nominal (and real) exchange rate, but also of output.
and domestic price inflation in both countries, in part due to the procyclical reaction of monetary policy (whereafter nominal interest rate rise as a response to higher price inflation rates and fall after a decrease in inflation) in both countries, which lead to increasing nominal interest rate differentials and thus to a potentiation of the nominal (and real) exchange rate fluctuations.

These diverging two-country dynamics illustrated in Figure 3 highlight in a clear and straightforward fashion the destabilizing influence of the “chartism” rule (with large extrapolative expectations) also at the macroeconomic level, corroborating also the analytical investigation of the one-country case discussed in section 3, where it was shown that the steady state of the one-country subsystem was locally stable only for sufficiently small values of $\beta^c_s$, the trend-extrapolating parameter in the chartism rule, and this despite of how aggressive monetary policy reacted to domestic price inflation developments.

6 Concluding Remarks

In contrast to the predominant NOEM/DSGE modeling approach which is based on the assumption of homogenous, forward-looking, intertemporal utility maximizing agents with rational expectations, in this paper a behavioral heterogenous agents approach à la De Grauwe and Grimaldi (2005) for the modeling of the nominal exchange rate dynamics was incorporated into a stylized two-country macroeconomic
model along the lines of Chen et al. (2006).

Since system stability in the resulting model was not imposed by a-priori by the rational expectations assumption as it is done in the NOEM/DSGE approach, a particular focus of this paper was set on the analysis of the stability conditions of the two-country system and in that context the role of the behaviorally determined nominal (and real) exchange rate dynamics. Through an analytical and numerical analysis undertaken in this paper it was possible to corroborate on the one hand not only the standard notion concerning the destabilizing influence of the chartists for the FX dynamics, but on the other hand also that for a large enough chartist parameter \( \beta_c \), not only the FX market, but also both economies – due to the interaction of the nominal (real) exchange rate, macroeconomic fundamentals and the respective monetary policy rules – could exhibit an unstable behavior.

While the analysis of this paper was restrained to the FX markets, its implications extend to other financial markets, due to the inherent uncertainty and subjectivity of economic agents interacting in real world markets. Indeed, the analysis of this paper shows that the existence of “behavioral rules” used for decision making in the financial markets, together with the economy’s reaction at the macroeconomic level, can lead to unstable (but possible) scenarios not controllable under standard Taylor type monetary policy rules. Furthermore, if we take into account that the market’s behavior and perception (in our model represented by the parameters \( \beta_n^u \) and \( \beta_n^d \) are not likely to be constant but instead state-dependent (and thus varying over time), the occurrence of such unstable scenarios (to be found, among other things, in stock market bubbles) seems more likely than it is suggested by rational expectations models. Therefore, the incorporation of behavioral features in macroeconometric models used for policy advice seems to be a necessary task to be undertaken if economic policy indeed intends to bring under control not the the perfect, by rational expectations driven markets, but the real world markets where behavioral rules or trading schemes play an important role.
References


