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JEL Classification System: E12, E25, E62, H50.
Government Spending Composition, Aggregate Demand, Growth and Distribution

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Abstract

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1 Introduction

The deterioration of public finances in most OECD countries has been one of the adverse effects of the recent global crisis. As it is typical during economic downturns, a reduction in tax receipts caused fiscal deficits to soar, and large reductions in output—or at best sluggish growth—determined a substantial increase in public debt to GDP ratios in most advanced economies. This troubling outlook raised concerns regarding the long-term sustainability of government borrowing in several countries, thus creating a friendly environment for advocates of fiscal austerity.

Theoretical as well as empirical arguments have been put forward in order to provide justifications for fiscal restraint and debt consolidation, especially in the Eurozone. The typical Ricardian Equivalence result implies that expectations of future tax hikes by forward-looking agents would induce a reduction in private spending that could more than offset the expansionary effects of deficit-financed government spending. Conversely, the anticipation of future tax breaks could boost private spending in the face of current fiscal restraint. Empirical findings by Giavazzi and Pagano (1990); Alesina and Perotti (1997); Alesina and Ardagna (2010) seemed to provide support for these so-called ‘expansionary austerity’ claims. At the same time, Reinhart and Rogoff (2010) found that debt/GDP ratios over 90% appeared to be associated with low, or even negative growth rates. Based on these premises, EU governments have implemented strict primary fiscal surpluses at every fiscal year. Such ‘fiscal compact’ commitments by EU countries made the already restrictive Stability and Growth Pact rules (targets of 3% deficit/GDP yearly, and a long run target of 60% debt/GDP ratio) even more stringent.

The case for expansionary fiscal contractions did not seem to hold up further scrutiny. On the one hand, widely cited studies by IMF researchers (Guajardo et al., 2011; Blanchard and Leigh, 2013) have produced convincing evidence that fiscal austerity is indeed associated with recessions, as fiscal multipliers proved to be much higher than previously estimated, especially early in the crisis. On the other hand, Reinhardt and Rogoff’s analysis was dismissed after Thomas Herndon, a graduate student at UMass Amherst, found that their results were basically driven by mistakes in database handling (Herndon et al., 2014).

With the academic support for expansionary austerity faltering, arguments envisioning the relaxation of restrictive fiscal rules entered the public debate. Specifically, the idea that public investment should not be taken into account when imposing constraints on government spending has progressively gained ground. The proposed economic rationale is that, while
government consumption expenditure can affect the short-run economic performance but eventually merely crowds out private investment in the medium run, government investment may have positive, permanent, effects on potential output. From this standpoint, expansionary fiscal policies can be rescued by their positive supply-side effects. The European Commission recently took some tentative steps in promoting public investment by launching the Investment Plan for Europe, or ‘Juncker Plan’, and by granting member states some fiscal flexibility with a more lenient interpretation of the so-called ‘investment clause’ [European Commission, 2015].

In this paper, we take issue with the disposability of government consumption implied by the reasoning that sees public investment as the only useful type of government spending in the long run. Besides being a source of aggregate demand, government consumption includes wage payments to public sector employees, and affects the functional distribution of income by raising the labor share. Therefore, evaluating the growth effect of the composition of public expenditure requires a framework capable of analyzing the interplay among aggregate demand, productivity growth, and income distribution. To the purpose, we build a demand-driven growth model that incorporates an explicit role for the public sector. In particular, we look at the composition of government spending between investment in public infrastructure capital—which directly augments the production possibilities of an economy—and unproductive (although not necessarily useless) public employment—which increases the overall labor share. The main novelty in our contribution lies in combining the idea that public spending is crucial in fostering innovation and growth [Mazzucato, 2013] with the induced innovation hypothesis—a staple in the heterodox thinking about long-run growth—according to which productivity growth is an increasing function of the wage share [Poley, 2003; Julius, 2006; Dutt, 2013b]. In particular, the interaction between public infrastructure and income shares (in turn influenced by government consumption) in determining the growth rate of labor productivity over time results in a hump-shaped relationship between long-run growth and government spending composition.

In our model, private investment is a function of the realized profit rate. In the short-run, aggregate demand determines the equilibrium level of capacity utilization and the short-run accumulation rate. In the long run, the growth rate of the economy is equal to the endogenous natural growth rate, in turn equal to the sum of the growth rate of population and the growth rate of labor productivity. The latter depends on both income distribution and fiscal policy.

First, we analyze a baseline version of the model with a balanced-budget
public sector. In the short run, demand is wage-led and government spending-led, whereas growth is distribution-neutral and government spending-led. In the long run, growth is a hump-shaped function of the composition of government expenditure, whose growth-maximizing value depends on income distribution and fiscal policy; no matter the composition of government expenditure, growth is wage-led and government spending-led even in the long-run.

Next, we generalize the model by allowing the public sector to run fiscal deficits and accumulate debt. We assume that investment function depends on the profit rate net of interest payments on public debt. Thus, accumulation of public debt has crowding out effects on both capacity utilization and growth, but only in the short run. The long-run growth rate of the economy is independent of the size of public debt, and it retains the wage-led and government spending-led properties found in the basic model. Still, government debt may become a source of instability when the interest rate is not kept sufficiently below the growth rate of the economy. This result highlights the important role played by monetary authorities in accommodating fiscal policy in order to sustain growth.

The paper is organized as follows. Section 2 relates our analysis with the existing literature. Section 3 outlines the basic feature of our model. Section 4 describes the balanced-budget model, while Section 5 studies the effects of government deficits and debt. Section 6 concludes. Proofs of analytical results are presented in the Appendices.

## 2 Related Literature

Our paper lays at the intersection of several strands of literature. First, there is a copious amount of articles in the Post-Keynesian and Neo-Kaleckian tradition investigating the linkages between income distribution, aggregate demand, and growth. Early contributions by Dutta (1990) and Rowthorn (1982) have argued that aggregate demand and growth are wage-led, in that a redistribution toward wages fosters economic activity. The seminal paper by Marglin and Bhaduri (1990), on the other hand, has highlighted the relevance of profit-led demand and growth, in order to explain a paradigmatic shift in the growth trajectory of the United States after the oil shocks of the 1970s. Blecker (1989) has shown that openness to trade makes it more likely for aggregate demand to turn profit-led, while Blecker (2002) provides a survey of open economy features of aggregate demand and growth. From an empirical standpoint, Barbosa-Filho and Taylor (2006) and Tavani et
al. (2011) argued that US demand-driven cycles appear to be profit-led. On the other hand, Hein and Vogel (2009) and Stockhammer and Onaran (2012) have found wage-led results. Nikiforos and Foley (2012) have shown that non-linearities are of great importance, while Rada and Kiefer (2015) find empirically that, in accordance with numerical results by von Arnim et al. (2014), global demand looks wage-led even though individual countries appear to be profit-led. From a theoretical standpoint, whether demand and growth are wage-led or profit-led (or neither) in the short run depends on the shape of the investment function. In our paper, distribution is neutral on growth in the short run. However, growth is wage-led in the long run thanks to the induced innovation hypothesis, and not because of the investment function.

Second, our paper relates to the non-mainstream literature on the role of government spending on growth. Our analysis of aggregate demand and government spending composition builds on Dutt (2006) and Dutt (2013a). In particular, the focus of these contributions is not limited to the short-run features of demand-driven growth, but includes an analysis of long-run adjustments to the (endogenous) natural growth rate. Dutt (2013a) studies a Keynesian growth model with government consumption and investment where public investment, besides contributing to aggregate demand, has the desirable properties of crowding in private investment and increasing labor productivity growth. In equilibrium, contrary to our main result, growth is always increasing in the investment share of government spending. Also, income distribution is absent in Dutt (2013a). Conversely, factor shares are central in Tavani and Zamparelli (2015), where the focus is on the productive and redistributive role of the government in a growth model with two classes. Yet, there is no role for aggregate demand in that framework.

Third, beginning with the seminal work by Barro (1990), the role of productive government investment has been thoroughly investigated in the mainstream endogenous growth literature. The general result is that growth is a hump-shaped function of the tax rate, with the maximum occurring when the tax rate equals the elasticity of output to government spending; several generalizations of the basic framework, however, have provided examples when growth may be maximized by different choices of the tax rate (see Irmen and Kuehnel 2009 for a survey). With respect to this literature, our model produces completely different results: both in the short and in the long run, growth monotonically increases in the tax rate. Endogenous growth theory has also analyzed the composition of government spending, though seldom with purely unproductive expenditure. In such cases, growth-maximizing strategies require any unproductive spending to be set to zero.
Our results differ from these conclusions, in that a growth-maximizing strategy will typically require to set government consumption strictly above zero.

Fourth, our paper relates to the Classical-Marxian tradition on technical change. In a recent survey, Dutt (2013b) shows that rising labor productivity growth may be modeled as the outcome of two general forces: increasing returns to scale and rising labor share. The first idea goes back to the Smith-Marshall-Young notion that the division of labor and labor productivity are limited by the size of the economy; along these lines, Kaldor’s technical progress function (Kaldor, 1961) assumes an explicit dependence of labor productivity growth on per capita accumulation. The second mechanism is already present in Marx (1876) and found a modern formalization with the induced innovation literature. Pioneering analyses can be found in Kennedy (1964) and Drandakis and Phelps (1966). Neoclassical revivals of this literature focusing on microeconomic foundations appear in Funk (2002) and Acemoglu (2002), while Foley (2003), Julius (2006), and Zamparelli (2015) apply the framework to Classical labor-constrained models of growth. None of these contributions features a role for the public sector, which is central in our analysis.

3 Basic Features of the Model

3.1 Production Technology

As it is standard in the post-Keynesian and Neo-Kaleckian literature, we distinguish between potential output \( Y^P \) and current output \( Y \). Following Tavani and Zamparelli (2015), we assume that potential output depends on both public capital \( X \) and private capital \( K \), that they are imperfect substitutes, and that they enter production in linearly homogeneous fashion. Accordingly,

\[
Y^P = H(X, K) = KH \left( \frac{X}{K}, 1 \right) = Kh(\chi)
\]

where \( \chi \equiv X/K \) is the composition of aggregate capital stock. The basic idea behind equation (1) is that private firms need public infrastructure to bring their products to market. Current output may differ from potential output because of the degree of capacity utilization \( u \), so that \( Y = uY^P \). We assume further that aggregate capital stock and labor \( (L) \) are perfect complements. Denoting (the time-varying) labor productivity by \( A > 0 \), we have that \( uY^P = AL \) at all times.
3.2 Government Spending Composition

The government levies proportional taxes on overall profits $\Pi$ at a rate $\tau \in (0,1)$ in order to fund its expenditures. Government spending $G$ has two uses: on the one hand, it finances the accumulation of public capital; on the other hand, it is used to finance government consumption $C_G$. Government consumption is used to pay wages to public employees. Furthermore, we suppose following [Dutt (2013a)] that the government allocates a fraction $\theta \in [0,1]$ of its total spending to public investment, and the remaining fraction to government consumption. Hence, $\dot{X} = \theta G$, and $C_G = (1 - \theta)G$.

3.3 Income Shares

As far as income shares are concerned, we denote the share of profits in production by $\pi$, which is a parameter in our model. Regarding wages, on the other hand, there are two types of workers: public and private employees. The labor market is not segmented, and workers in the two sectors earn the same wage. It will be labor demand in the two sectors that adjusts to the existing wage conditions. The overall labor share in the economy, denoted by $\tilde{\omega}$, is made up by the sum of the labor share in production and the share of public sector wages in GDP:

$$\tilde{\omega} = 1 - \pi + (1 - \theta)G/Y.$$  \hfill (2)

3.4 Labor Productivity Growth

It is a long-standing tradition in heterodox economics to relate the growth rate of labor productivity to the labor share in GDP. This is known as induced technical change hypothesis: a higher wage share induces firms to direct technical change towards labor saving innovations ([Kennedy, 1964; Drandakis and Phelps, 1966; Funk, 2002; Foley, 2003; Julius, 2006; Zamparelli, 2015]). It would be quite natural, then, to relate the growth rate of labor productivity to the overall labor share $\tilde{\omega}$. However, recent influential work by Mariana Mazzucato ([Mazzucato, 2013]), has emphasized that the public sector plays a fundamental role in the innovation process. On the one hand, individual private firms are in a better position to innovate when public infrastructures work well (a stock argument). On the other hand, the public sector directly finances innovation through public funds (a flow argument).

In our framework, the natural candidate to analyze the public role in the innovation process is the relative size of public capital to private capital stock, that is $\chi$. The composition of capital stock acts together with unit
labor costs in fostering labor productivity growth. Accordingly, we assume that the growth rate of labor productivity $g^A \equiv \dot{A}/A$ is log-linear in the composition of capital stock and the overall labor share:

$$g^A = \chi^\delta \tilde{\omega}^{1-\delta},$$

with $\delta \in (0, 1)$. The sum of the endogenous growth rate of labor productivity $g^A$, and the exogenous growth rate of the labor force $n$, anchors accumulation in the long-run of our model.

4 Balanced Budget

If the public sector runs a balanced budget, its total spending satisfies:

$$G = T = \tau \Pi = \tau \pi Y,$$

where $T$ stands for total tax revenues. With a balanced budget, a fiscal policy is fully defined by the couple $(\tau, \theta)$, that is government expenditure and its composition.

4.1 Savings and Investment

A distinctive feature of PK economics is an investment function that is separate from savings. Many different investment functions have been proposed in the literature, but all of them relate investment demand to profitability in one way or another. Here, we suppose that investment demand (normalized by the size of private capital stock) depends on an autonomous component $\gamma$ –the investors' animal spirits– and on the profit rate in production $r \equiv \Pi/K$:

$$g^i \equiv \frac{I}{K} = \gamma + \eta r = \gamma + \eta \pi h(\chi)$$

Autonomous investment will change over time, in response to economic conditions. We look at the evolution of $\gamma$ in Section 4.3.

We also assume the typical two-class structure of the economy. The private sector is made of workers, who consume all of their labor incomes, do not pay taxes and do not save, and firm-owners (capitalists), who earn profit incomes, pay taxes, and consume/save out of their disposable income. Public sector workers also do not save. Denoting the capitalists’ propensity to save by $s \in (0, 1)$, the growth rate of private capital stock allowed by savings is:

$$g^s \equiv \frac{S}{K} = s(1 - \tau) \pi u h(\chi)$$
4.2 Short-Run Equilibrium

The rate of capacity utilization adjusts so as to attain equilibrium in the goods market. Equating $g^i$ and $g^s$ and solving for $u$, we obtain:

$$u^* = \frac{\gamma}{\pi h(\chi)[s(1 - \tau) - \eta]},$$  \hspace{1cm} (7)

where we assume the typical restriction requiring the (net) propensity to save to be greater than the propensity to invest out of profits to be satisfied. Observe that such requirement restricts the tax rate to be strictly less than $1 - \eta/s$. Notice furthermore that the equilibrium utilization rate is inversely related to the share of profits in production. As is well-known in the literature, this is due to the postulated dependence of investment demand on the profit rate. Finally, there is an inverse relationship between the utilization rate and the potential output/capital ratio $h(\chi)$: with output determined on the demand side, greater productivity of capital implies a lower degree of capital utilization.

The corresponding short-run growth rate of capital stock is

$$g = \left[\frac{s(1 - \tau)}{s(1 - \tau) - \eta}\right] \gamma = \left[\frac{1}{1 - \frac{\eta}{s(1 - \tau)}}\right] \gamma,$$  \hspace{1cm} (8)

where the second equality is written to highlight the role played by the autonomous spending multiplier on the growth rate. Notice that short-run growth is independent of income distribution, but increasing in government spending: recall that, the government running a balanced budget, $\tau$ is government spending per unit of incomes taxed. The neutrality of distribution on growth in the short-run is also due to the postulated shape of the investment function. We will see below that distribution does play a role in shaping the long-run growth rate of the economy.

4.3 Dynamics

We are now set to consider the evolution over time of the capital stock composition $\chi$ and of the autonomous investment component $\gamma$. Regarding the former, we have the following differential equation:

$$\frac{\dot{\chi}}{\chi} = \frac{\dot{X}}{X} - \frac{\dot{K}}{K} = \theta \tau \pi u h(\chi) - s(1 - \tau) \pi u h(\chi).$$

Evaluating the utilization rate at its short-run equilibrium value, we obtain:

$$\dot{\chi} = \frac{\gamma}{s(1 - \tau) - \eta} \left[\frac{\tau \theta}{\chi} - s(1 - \tau)\right] \chi, \hspace{1cm} (9)$$
Next, following Dutta (2006, 2013a) we postulate that autonomous investment responds to differences between the growth rate of labor demand \( \dot{L}/L \) and the growth rate of labor supply \( n \). When labor demand grows faster than labor supply there is a reduction in unemployment rate; if lower unemployment empowers workers and enables them to increase the wage share, it is plausible to assume that a reduction in autonomous investment will follow. With negative speed of adjustment \(-\lambda\), we can write:

\[
\frac{\dot{\gamma}}{\gamma} = -\lambda \left[ \frac{\dot{L}}{L} - n \right]
\]

Factoring and substituting \( g \) from (8), we find:

\[
\frac{\dot{\gamma}}{\gamma} = -\beta \left[ \frac{s_\gamma}{s(1 - \tau)} - \left( \chi^\delta \omega^{1-\delta} + n \right) \right] \gamma,
\]

where \( \beta \equiv \lambda/(1 + \lambda) \). The steady state value of capital stock composition \( \chi_{ss} \) is readily found as:

\[
\chi_{ss} = \frac{\theta \tau}{s(1 - \tau)},
\]

increasing in government spending and in the share of it that goes to public investment, while decreasing in the private sector’s propensity to save.

On the other hand, the steady state value of autonomous investment is such that \( g_{ss} = g_{ss}^A + n \), and can be preliminary written as

\[
\gamma_{ss} = \left[ \frac{s(1 - \tau) - \eta}{s(1 - \tau)} \right] \left( \chi_{ss}^\delta \omega^{1-\delta} + n \right)
\]

Appendix A is dedicated to showing that the steady state formed by (11) and (12) is stable, and that convergence to it is monotonic.

\(^1\)As the calculations leading to equation (10) below show, this is tantamount to postulating that autonomous investment changes with the difference between the short run growth rate \( g \) and the long-run growth rate \( g_A + n \).
4.4 Long Run Growth, Government Spending Composition, and Factor Shares

One of the interesting features of this simple model is that the long-run growth rate of the economy depends non-monotonically on the composition of government spending. In fact, plugging the steady state value \( \chi_{ss} \) into the growth rate of labor productivity \( g_A \), and using the definition of the overall labor share \( \tilde{\omega} \), we see that:

\[
g_{ss} = g_{ss}^A + n = \left[ \frac{\tau \theta}{s(1-\tau)} \right]^\delta \left[ (1-\theta)\tau \pi + (1-\pi) \right]^{1-\delta} + n. \tag{13}\]

On the one hand, \( \theta \) has a positive effect on the long-run composition of capital stock \( \chi_{ss} \). On the other hand, \( \theta \) has a negative effect on the labor share. As a result, long-run growth is first increasing and then decreasing in the share of government spending that goes to public investment. Hence, there may be an interior, growth-maximizing government spending composition \( \theta^* \in (0, 1) \). In Appendix B we show that this is indeed the case, and that

\[
\theta^* = \delta \left[ 1 + \frac{1-\pi}{\tau \pi} \right], \tag{14}\]

with the economic interpretation that the share of investment in government spending exceeds the elasticity of labor productivity growth to the composition of capital stock by an amount that varies directly with the share of labor in production. Notice that \( \theta^* < 1 \) requires \( \delta < \frac{\tau \pi}{1-\pi(1-\tau)} \); if the labor share is very high, the growth-maximizing government consumption may be null, as the productivity gains achievable by raising the wage share may be small as compared to investing in public capital accumulation.

The maximal growth rate corresponding to \( \theta^* \) is

\[
g^* = \left\{ \frac{1-\pi(1-\tau)}{\tau s(1-\tau)} \right\}^\delta \left\{ (1-\delta)\left[ (1-\pi(1-\tau)) \right] \right\}^{1-\delta} + n. \tag{15}\]

Differentiating \( g_{ss} \) with respect to the share of profits in production we can show that long-run growth is wage-led in this model, regardless the composition of government expenditure. In fact,

\[
\frac{\partial \ln g_{ss}^A}{\partial \pi} = - (1-(1-\theta)\tau) \frac{1-\delta}{\tilde{\omega}} < 0.
\]

The reason has to be found in the role played by induced bias in shaping the pattern of technical change in the economy. As the share of labor in
production increases, so does the overall labor share. Everything else equal, the growth rate of labor productivity increases. It is worth emphasizing once more that the wage-led character of economic growth is a long-run feature of the model, and it does not occur through short-run adjustments in capacity utilization.

Growth is also government spending-led in the long-run. Again, this is true whatever the composition of public expenditure. In fact, differentiating $g_{ss}$ with respect to the tax rate yields:

$$\frac{\partial \ln g_{ss}^A}{\partial \tau} = \frac{(1-\delta)(1-\theta)}{\tilde{\omega}} + \frac{\delta}{\tau(1-\tau)} > 0.$$ 

Both the composition of aggregate capital stock $\chi_{ss}$ and the overall labor share $\tilde{\omega}$ are increasing in the tax rate: hence, labor productivity growth rises with $\tau$. This important feature of the model is also independent of the short-run multiplier on government spending. Rather, it depends on the long-run role played by government spending in both forces at play in fostering labor productivity growth, namely public capital stock and induced technical change.

5 Government Debt

We now extend the framework presented above in order to incorporate a role for government debt $D$. We assume that the interest rate on government debt $i$ is an exogenous variable. In line with part of the Post-Keynesian literature on the subject [Lavoie 2014; Rochon 1999; Smithin 1996], it will be the money supply to adjust in order to meet the interest rate targeted by the monetary authority. The public sector’s budget constraint is:

$$T + \dot{D} = G + iD.$$ 

Government debt is owned by capitalists, and it provides an income stream $iD$ per period. For simplicity, we assume further that all income accruing to capitalists is taxed at the same rate $\tau$. In this context it is important to consider that, for an exogenously given wage share in production, if income shares are to add up to one in the presence of government debt we must have:

$$\pi \equiv \frac{\Pi}{Y} = \frac{rK + iD}{Y}. \quad (16)$$

Such an accounting restriction matters not much to saving, but to investment behavior. In fact, the growth rate of capital stock allowed by savings is given
by equation (6) as before. Conversely, if as it is plausible capitalists only look at the profit share net of interest income $\Pi - iD$ (but still gross of taxes) when formulating investment plans, then the investment function takes the form:

$$g^i = \gamma + \eta \left[ \pi u h(\chi) - iD \right], \quad (17)$$

where $d$ denotes the public debt to private capital ratio. Such an investment function gives a flavor of the mainstream-favorite notion of crowding-out of private investment by public spending. However, we shall see that crowding-out is at most a short-run phenomenon, and does not affect the long-run growth rate of the economy.

Since the public sector can issue debt, its spending on public capital and public consumption is not bound by taxes. With total tax revenues given by $T = \tau \pi Y$ as before, we assume $G = \alpha T$, where $\alpha$ can exceed unity, but $\alpha \tau < 1$ as $G$ cannot be higher than total profits. Therefore, the government deficit normalized by the size of capital stock satisfies:

$$\frac{G - T}{K} = \tau(\alpha - 1)\pi u h(\chi).$$

Just like before, a fraction $\theta$ of total government spending goes to financing the accumulation of public capital: $\dot{X} = \theta G$. A fiscal policy is now defined by the triple $(\alpha, \tau, \theta)$, which pins down government deficit, total government expenditure, and the composition of the latter.

Finally, the goods market equilibrium condition modifies to:

$$g^s - g^i = \frac{G - T}{K}, \quad (18)$$

which ensures that the overall excess demand in the economy is equal to zero.

5.1 Short-run Equilibrium

The equilibrium utilization rate and growth rate in the short-run are, respectively:

$$u^* = \left[ \frac{1}{s(1 - \tau) - \eta - (\alpha - 1)\tau} \right] \frac{\gamma - \eta i d}{\pi h(\chi)}, \quad (19)$$

and

$$g = \left[ \frac{1}{1 - \frac{\eta + (\alpha - 1)\tau}{s(1 - \tau)}} \right] (\gamma - \eta i d). \quad (20)$$
The main effect of government debt is that there is an interaction between the financial side and the real side of the economy. Both the debt-to-capital ratio and the interest rate reduce the equilibrium level of activity, so that there is a potential negative feedback from the accumulation of public debt to economic growth. At the same time, however, the size of government deficit to GDP ratio, which is governed by $(\alpha - 1)\tau$, increases the value of the growth multiplier of autonomous spending everything else equal. Thus, it is not necessarily true that the presence of government debt lowers the growth rate relative to the balanced budget case.

The analysis of the dynamical system below sheds light on the interaction between public debt, autonomous investment, and the composition of capital stock.

**5.2 Dynamics**

The introduction of government debt into the picture gives rise to the following three-dimensional dynamical system:

\[
\dot{\chi} = \left[ \frac{\gamma - \eta id}{s(1 - \tau) - \eta - (\alpha - 1)\tau} \right] \left[ \frac{\theta \alpha \tau}{\chi} - s(1 - \tau) \right] \chi, \tag{21}
\]

\[
\dot{\gamma} = -\beta \left\{ \frac{s(1 - \tau)(\gamma - \eta id)}{s(1 - \tau) - \eta - (\alpha - 1)\tau} - \left( \chi \delta \omega^{1-\delta} + \eta \right) \right\} \gamma, \tag{22}
\]

\[
\dot{d} = \left( \frac{\dot{D}}{K} \right) d = \frac{\tau (\alpha - 1)(\gamma - \eta id)}{s(1 - \tau) - \eta - (\alpha - 1)\tau} - (g - i)d. \tag{23}
\]

As before, we first look at the steady state, where:

\[
\chi_{ss} = \frac{\alpha \tau \theta}{s(1 - \tau)} \tag{24}
\]

\[
\gamma_{ss} = (\chi_{ss} \delta \omega^{1-\delta} + \eta) \left[ s(1 - \tau) - \eta - (\alpha - 1)\tau + \frac{\eta (\alpha - 1)}{\chi_{ss} \delta \omega^{1-\delta} + \eta} \right] \tag{25}
\]

\[
d_{ss} = \left[ \frac{\tau (\alpha - 1)}{\chi_{ss} \delta \omega^{1-\delta} + \eta} \right] \left( \chi_{ss} \delta \omega^{1-\delta} + \eta \right). \tag{26}
\]

As equation (24) shows, the long-run composition of capital stock is independent of the size of government debt. This result has important implications, as will be illustrated in the next subsection.

As far as stability is concerned, Appendix C is dedicated to show analytically that a sufficient condition for stability of the steady state is $i < g_{ss}(1 - s(1 - \tau))$, with the interpretation that the growth rate of the economy must be greater than the ratio: interest rate on government debt/net
marginal propensity to consume out of profits. On the contrary, we show that \( i > g_{ss} \) is sufficient to make the system unstable. The implication is quite strong: if the monetary authority is able and willing to keep the interest rate on government bonds sufficiently low, it can ensure the stability of the growth path, and basically make the size of public debt irrelevant. When maintaining the interest rate low enough is not possible, the system may not approach the steady state and the economy may depart from its long-run growth rate. From this point of view, monetary policy is non-neutral as it affects the dynamics of the economy. Also, this result emphasizes the importance of sovereign central banks. If a country loses its monetary policy independence, say either through a currency peg or a currency union, it may lose the ability to stabilize the economy’s growth path towards its long-run equilibrium.

Finally, equation (26) provides a standard result on the dynamics of public debt: a positive value of \( d_{ss} \) requires either a primary deficit \((\alpha > 1)\) and a growth rate higher than the interest rate \((\chi_{ss} \tilde{\omega}^{1-\delta} + n > i)\), or a primary fiscal surplus and a growth rate lower than the interest rate. Otherwise, the public sector would become a net-lender in steady state: a practice seldom observed in real economies. Notice that, however, the previous discussion on stability points out that a primary deficit coupled with growth rate higher than the interest rate is the only possibility for a positive and stable \( d_{ss} \).

### 5.3 Long-run Growth, Government Spending Composition, and Factor Shares

The steady state growth rate satisfies:

\[
g_{ss} = \left[ \frac{\alpha \tau \theta}{s(1 - \tau)} \right]^\delta \left[ (1 - \theta) \alpha \tau \pi + (1 - \pi) \right]^{1 - \delta} + n. \tag{27}
\]

As a first observation, long-run growth is independent of government debt, because the long-run composition of capital stock is. This is one of the most important results of our analysis: crowding out is at most a short-run phenomenon, because long-run growth is not affected by the size of government debt, but only by the composition of capital stock and the determinants of the labor share. Moreover, observe that this result is quite robust, in that it does not depend on the exogeneity of the interest rate. In this respect, the model is more general than the seemingly restrictive case in which an accommodating monetary authority can credibly keep the interest rate on government bonds at the desired rate independently of the size of public
debt. Still, our discussion on instability imposes caution, in that if the interest rate shoots above the growth rate, the long-run growth rate as in (27) might not be attained.

Second, and similarly to the balanced-budget case, the long-run growth rate of the economy depends non-monotonically on the composition of government spending. In Appendix [B] we show that the growth-maximizing government spending composition is

$$\theta^* = \delta \left[ 1 + \frac{1 - \pi}{\alpha \tau \pi} \right].$$

(28)

Once again, the labor share in production plays a role in determining the amount by which the growth-maximizing spending composition exceeds the elasticity of the technical progress function to public capital. The maximal growth rate corresponding to $\theta^*$ is

$$g^* = \left[ -\frac{\delta}{\pi s(1 - \tau)} \right]^\delta (1 - \delta)^{1 - \delta}[\alpha \tau \pi + 1 - \pi] + n.$$

(29)

Third, equation (27) shows that the positive growth effects of government expenditure are not confined to the short-run only. While in the short-run government deficit increases aggregate demand through the multiplier, long-run growth is increasing in the government expenditure share of profits ($\alpha \tau$) as it raises both components of labor productivity growth: the public-to-private capital ratio, and the overall labor share in the economy. This is true no matter the composition of public expenditure $\theta$.

Fourth, the presence of public debt does not change the wage-ledness of long-run growth. Differentiating $g_A^*$ with respect to the share of profits yields

$$\frac{\partial \ln g_A}{\partial \pi} = -\frac{1 - \delta}{\omega} [(1 - \theta)\alpha \tau] < 0.$$  

6 Conclusion

In this paper, we presented a simple demand-driven growth and distribution model in order to analyze the role of the composition of government spending in an economy’s growth process. The main hypothesis in our analysis is that both private incentives such as growing unit labor costs and public capital stock are essential in shaping up the productivity growth trajectory of the economy.

We studied both a simplified case in which the public sector runs a balanced budget, and a more realistic version of the model including government...
debt. In both cases, our strongest results pertain to the long-run implications of our framework.

First, we showed that long-run growth is hump-shaped in the composition of government spending \( \theta \), and that the growth-maximizing composition \( \theta^* \) depends on income shares. In particular, it rises with the share of labor in production. Second, even though the short-run growth rate of the economy is independent of income distribution, we showed that long-run growth is wage-led, both with and without government debt. Third, in both cases we showed that the size of government spending, independent of its composition, has a positive effect on growth in the long run. Fourth, we showed that the crowding out effects of government debt are at most a short-run phenomenon. Government debt might lower growth in the short-run, because of its adverse effects on investment demand; but the long-run growth rate of the economy is independent of the size of public debt. Fifth, we provided a sufficient condition for the stability of the long-run equilibrium of the model with government debt: the economy must grow at a rate greater than the ratio of the interest rate over the net marginal propensity to consume out of profits.

To qualify our results, it must be said first that the short-run features of our model are sensitive to the postulated shape of the investment function, which is an open issue among Post-Keynesians. The same is not true, however, for the long-run growth rate: both the steady state composition of capital stock and the total labor share are independent of whether utilization is wage-led or profit-led. Therefore, our long-run results are robust to alternative specifications of the investment function. Second, our paper does not speak to the issue of whether capacity utilization should adjust in the long-run to a desired rate, be that endogenous or exogenous. This is an open controversy among Post-Keynesian economists: competing views can be found in [Lavoie 2014, Chapter 6] on the one hand, and [Skott 2012] on the other. In this respect, our contribution falls within the standard Neo-Kaleckian approach: with an exogenous desired utilization rate, the steady state of the model would be independent of aggregate demand.

Finally, our analysis of the role of government debt only applies to countries whose central bank can freely set the nominal interest rate. We showed that the growth process becomes unstable as the interest rate shoots above the long-run growth rate of the economy. This result highlights the important role played by sovereign central banks in stabilization policy.
A Stability Analysis: 2D case

Linearization of the dynamical system formed by equations (9) and (10) around its steady state position yields the following Jacobian matrix:

\[
J(\chi_{ss}, \gamma_{ss}) = \begin{bmatrix}
\dot{\chi} \\
\dot{\gamma}
\end{bmatrix}_{ss} = \begin{bmatrix}
-\frac{g_{ss}}{\chi_{ss}} & 0 \\
\frac{\beta g_{ss} - n}{\chi_{ss}} & -\beta g_{ss}
\end{bmatrix}
\]

with negative trace and positive determinant. Hence, the Jacobian matrix has two distinct eigenvalues with real parts that add up to a negative number and are of the same sign. This can be true only if the two eigenvalues are negative, which is necessary and sufficient for stability of the steady state.

In order to be able to rule out oscillations around the steady state, we must show that the two eigenvalues \(\varepsilon_{1,2}\) of the Jacobian matrix do not have imaginary parts. First, we write down the characteristic equation:

\[
\text{Det}[J(\chi_{ss}, \gamma_{ss}) - \varepsilon I] = \left(-\frac{g_{ss}}{\chi_{ss}} - \varepsilon\right) \left(-\beta g_{ss} - \varepsilon\right) = 0
\]

which solves for

\[
\varepsilon_{1,2} = -g_{ss} \left(\frac{1}{\chi_{ss}} + \beta\right) \pm \sqrt{\frac{g_{ss}^2}{\chi_{ss}} \left(\frac{1}{\chi_{ss}} + \beta\right)^2 - 4\beta g_{ss}^2 \chi_{ss}}
\]

To rule out oscillations, we have to show that the discriminant is positive, that is

\[
g_{ss}^2 \left(\frac{1}{\chi_{ss}} + \beta\right)^2 - 4\beta g_{ss}^2 \chi_{ss} > 0.
\]

Expanding the squared term and collecting items, we see that this boils down to

\[
\chi_{ss}^{-2} - 2\beta/\chi_{ss} + \beta^2 \chi_{ss}^{-2} = (\chi_{ss}^{-1} - \beta)^2 > 0,
\]

which is always true.

Hence, given any initial condition, the system converges monotonically to its steady state position.

B Growth-maximizing Government Spending Composition \(\theta^*\)

Taking logs of the growth rate of labor productivity in (13), we have:

\[
\ln g_A = \delta[\ln \tau + \ln \theta - \ln s - \ln(1 - \tau)] + (1 - \delta) \ln[(1 - \theta)\tau \pi + (1 - \pi)].
\]
Differentiating with respect to $\theta$ and setting the derivative equal to zero we find:

$$
\frac{\delta}{\delta \theta} \left( \frac{1 - \delta}{(1 - \theta)\tau \pi + (1 - \pi)} \right) = 0,
$$

from which (14) follows immediately.

Proceeding in the same way for the government debt case, where $g_A = \left[ \frac{\alpha \tau \theta}{s(1 - \tau)} \right]^{\delta} \left[ (1 - \theta)\alpha \tau \pi + (1 - \pi) \right]^{1 - \delta}$, yields (28).

C Stability Analysis: 3D Case

The Jacobian matrix evaluated at the steady state is

$$
J(\chi_{ss}, \gamma_{ss}, d_{ss}) = \begin{bmatrix}
\dot{\chi} & \dot{\gamma} & \dot{d} \\
\hat{\chi} & \hat{\gamma} & \hat{d} \\
\chi_{ss} & \gamma_{ss} & d_{ss}
\end{bmatrix}
$$

where $\Gamma \equiv s(1 - \tau) - \eta - (\alpha - 1)\tau$. The Routh-Hurwitz necessary and sufficient conditions for stability are:

1. $Tr.J < 0$. A sufficient condition is $g_{ss}(1 - s(1 - \tau)) > i$.

2. $Det.J < 0$. We have that $Det.J = -\frac{g_{ss}}{\chi_{ss}} \beta g_{ss} (g_{ss} - i)$. Again, $g_{ss}(1 - s(1 - \tau)) > i$ is sufficient for a negative determinant, because it implies $g_{ss} > i$.

3. $Pm.J > 0$, where $Pm.J$ stands for the sum of the principal minors (that is, the determinants of the sub-matrices obtained by removing the first row and column, then the second row and column, and finally the third row and column) of the Jacobian matrix. We have that

$$
Pm.J = \beta g_{ss}^2 \chi_{ss} + \left\{ \frac{\tau(\alpha - 1)\eta_i [g_{ss}(1 - s(1 - \tau)) - i]}{\Gamma(g_{ss} - i)} + (g_{ss} - i) \right\} \frac{g_{ss}}{\chi_{ss}} + \beta g_{ss} (g_{ss} - i).
$$

Once again, a sufficient condition is $g_{ss}(1 - s(1 - \tau)) > i$.

4. $-Pm.J + \frac{Det.J}{Tr.J} < 0$. This boils down to

$$
-\beta g_{ss}(\beta g_{ss} + \Omega) - \left( \frac{g_{ss}^2}{\chi_{ss}} + \frac{g_{ss}}{\chi_{ss}} \Omega \right) \left( \frac{g_{ss}}{\chi_{ss}} + \beta g_{ss} + \Omega \right) < 0,
$$
where $\Omega \equiv (g_{ss} - i) + \frac{\tau(\alpha-1)g_{ss}(1-s(1-\tau)) - i}{\Gamma(g_{ss} - i)} > 0$ under $g_{ss}(1 - s(1 - \tau)) > i$. We conclude that a sufficient condition for local stability of the system is $g_{ss}(1 - s(1 - \tau)) > i$.

C.1 Instability

Observe that $g_{ss} < i$ is sufficient for instability, because $\text{Det}J = -\frac{g_{ss}}{\chi_{ss}} \beta g_{ss}(g_{ss} - i)$ turns positive, and so the second necessary and sufficient Routh-Hurwitz condition above is violated.

References


Mazzucato, M., 2013: The Entrepreneurial State. Anthem Press.


