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Keywords: Kaleckian growth model, effective demand, stationary utilization rate, endogenous capital productivity, panel estimation

JEL Classification: E12, E22, C22

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1 Introduction

The Kaleckian growth model popularized by Rowthorn (1981), Dutt (1984) and Taylor (1985) features the principle of effective demand in the long run according to which permanent demand shocks have growth effects. Since the rate of capacity utilization is the accommodating variable aligning savings to investment in the long run, the model predicts a non-stationary utilization rate. Yet, this contradicts the stylized fact of a stationary utilization rate as reported by the Fed for various US industries (cf. Skott 2012, Schoder 2012b, Taylor 2012).¹

Classical and Marxian growth models according to which economies are not demand-led in the long run are typically built around a constant long-run utilization rate with the steady-state growth rate being determined by the fundamental laws of capitalism (cf. Duménil and Lévy 1999, Shaikh 2009). In contrast to that, some variants of the Kaldorian and Robinsonian models featuring Harrodian investment dynamics put forward by Skott (1989a,b, 2010) retain the Keynesian principle of effective demand in the long run even though the long-run utilization rate is exogenous. Yet, Kaleckians may not be fully convinced by the assumption of instantaneous price adjustments in the Kaldorian model and the procyclicality of the profit share implied by the Robinsonian model, respectively.

So far, Kaleckians have been ignoring the inconsistency between the time series properties of the utilization rate as predicted by the model and as observed empirically. The present paper seeks to reconcile a stationary utilization rate and the principle of effective demand within a Kaleckian growth model by endogenizing capital productivity. We put forward theoretical arguments why one should expect capital productivity to move pro-cyclically. First, data on full-capacity output do typically not measure technically feasible output but the maximum output under normal production conditions. We argue that with indivisibilities in the production process, these normal conditions may be endogenous. Second, changes in productivities may be induced by investment. By the aid of a simple optimization problem of the firm, we argue that firms may seek to adjust capital productivity to narrow the gap between realized and desired utilization which is associated with costs. In this case, the growth rate of capital productivity is a function of the utilization gap.

We paper proceeds by providing empirical evidence for a pro-cyclical capital productiv-

¹Two more criticism have been put forward which are implicitly dealt with in the present paper as well as in Schoder (2012c): First, the model is not fully adjusted, i.e. the target utilization rate of the firm is, in general, inconsistent with the equilibrium rate. The representative firm cannot be expected to settle if its target is incongruent with the realization (Committeri 1986, Auerbach and Skott 1988). To defend the Kaleckian model against this criticism, Lavoie (1995b, 1996) and Dutt (1997, 2009) introduced an endogenous adjustment of the target rate through hysteresis effects, thus maintaining the principle of effective demand in the long run. Schoder (2012b) provides some evidence for hysteresis in the target utilization rate in the US manufacturing sector. Second, stability of the Kaleckian model requires the sensitivity of investment to changes in utilization to be lower than the utilization sensitivity of saving, also in the long run. Yet, simulation and empirical studies such as Dallery (forthcoming), respectively, suggest that the stability condition is hardly met in the long run due to a strong accelerator effect on investment. Hence, there seems to exist convincing support for a Harrodian long-run specification of investment according to which the accumulation rate is self-enforcing which is absent from the baseline Kaleckian model (cf. Skott 2012). As demonstrated by Taylor (2012) and Schoder (2012c), however, a Harrodian long-run investment function can be introduced to the Kaleckian model with distribution and debt dynamics stabilizing the system.
ity from a time series and panel estimation of the adjustment parameter for various US industries. We find an average adjustment parameter around 0.03 to 0.04.

Finally, we introduce the pro-cyclical capital productivity to a Kaleckian growth model and analyze the long-run effects of shocks to investment, saving and distribution. We find that the principle of effective demand hold but not the paradoxes of thrift and cost. Yet, adding de-stabilizing Harrodian investment dynamics to the model as well as counter-acting debt dynamics and assuming global stability reveals that the principle of effective demand as well as the paradoxes of thrift and cost hold in the long run.

The paper complements Schoder (2012a) and Schoder (2012c) by providing the rationale for an endogenous capital productivity. An endogenous capacity-capital ratio is the core implication of Schoder’s (2012a) empirical analysis of the long-run relationships between capacity output, output and demand expectations for the US manufacturing sector. There seems to exist some evidence for a small but significant long-run adjustment of capacity to realized output. Schoder (2012c) carries on the present analysis by studying the ability of the Kaleckian growth model with endogenous capital productivity, stationary capacity utilization and de-stabilizing Harrodian investment dynamics as well as stabilizing distribution and debt dynamics to generate endogenous limit cycles between utilization and distribution as well as utilization and debt as observed empirically for many industrialized countries.

The remainder of the paper proceeds as follows: Section 2 presents some theoretical considerations in support of a pro-cyclical capital productivity. Section 3 provides some empirical results for US industries confirming this view. In section 4, a simple Kaleckian model including an endogenous capital productivity is presented and demand shocks are studied. Section 5 concludes the paper.

2 Endogenous capital productivity

In heterodox growth models, the capital productivity is typically assumed to be constant (cf. Taylor 2004, Skott 2012, Hein et al. 2012). Regarding the trend of the variable, a large body of literature analyzes if technical change is labor saving or labor augmenting in the long run, i.e. if the capital productivity tends to decrease or increase over time. In industrialized countries technical change has been found to be slightly labor saving in the long run (cf. Foley and Michl 1999, pp. 37-41 and Duménil and Lévy 2004). Since we are primarily interested in the cyclical behavior of the capital productivity, we assume its trend to be zero in the models discussed below.

Regarding the cyclical pattern, there are good reasons for a pro-cyclical capital productivity. Here, we put forward two arguments. The first one is related to the way capacity output is reported (for instance by the Fed) which is not a technological criteria but subject to profit maximization considerations. The second is based on productivity effects of capital investment.
2.1 Indivisibilities and endogenous capacity

The data on capacity utilization provided by the Fed for several US industries is typically referred to, for instance, by Skott (2012) to make a case for a stationary utilization rate. Yet, as argued by Nikiforos (2011), these data do not measure output relative to the technically feasible output but relative to the full production capability assuming a "'number of shifts, hours of plant operations, and overtime pay that can be sustained under normal conditions and a realistic work schedule'" (from the questionnaire as quoted in Nikiforos 2011, p. 13). Hence, the reported capacity output may likely be endogenous even if the capital stock is assumed to be constant.

To see Nikiforos’ point, consider a simple profit maximization problem of the firm. As argued by Kurz (1986), firms choose their desired utilization rate based on profit maximization considerations. Assume the economy comprises a number of small symmetric firms which can operate their capital stock in one, two or three shifts.
For the average firm, the cost curves, average cost curves and marginal cost curves for one, two and three shifts, respectively, are illustrated in Figure 1. The introduction of a new shift is associated with fixed costs such as hiring costs of new labor and higher maintenance costs. Given the number of shifts, raising output beyond a certain threshold is associated with rising unit variable costs since, for instance, an increasing number of workers will be required to work overtime (cf. Kurz 1986). For the average firm, marginal costs will, therefore, increase with output. Average costs first decrease because of the fixed costs but increase when rising marginal costs start to dominate. Assuming a constant price marginal revenues can be depicted by a horizontal line.

Suppose a firm operates two shifts. In this case, the profit maximizing output will be $Y^*_2$ where marginal revenues equal marginal costs. One interpretation of the full production capability as defined in the quote above is the level of output, for a given number of shifts, beyond which unit costs exceed unit revenues which are equal to marginal revenues. For the firm running two shifts full capacity will be at $Y^c_2$.

Now assume the economy is moving into a boom phase with firms expecting demand to exceed their capacity output persistently. If growth prospects are strong enough, our firm may well introduce a third shift to accommodate higher demand in the future. The new profit maximizing output and capacity output will be $Y^*_3$ and $Y^c_3$, respectively. Given a constant capital stock, a higher capital productivity measured as capacity output over capital stock will be observed. If the economy is in a recession instead with expected output being around capacity with a lower number of shifts, our firm may reduce the number of shifts to one and aim for an output of $Y^*_1$ with a capacity of $Y^c_1$. In this case, a lower capital productivity will be observed.

This simple profit maximization problem under indivisibilities in the production process provides a rationale for a pro-cyclical capital productivity following from an endogenous capacity output. Here, production capabilities refer to the highest output possible under normal conditions of production and not the highest output technically feasible. Further, it has been assumed that the capital stock is given.

Yet, in the next section we show that even a capacity output defined as the technically feasible output may be endogenous leading to a pro-cyclical capital productivity, if investment is allowed for and associated with induced technical change.

### 2.2 Investment induced technical change

Here, we motivate an endogenous capital productivity by investment induced technical change. If the aim of investing is to raise capacity in order to regain the target rate of utilization, the purchased machinery may well be labor augmenting since cost saving may not be primary objectives in this case. This can be illustrated by the following simple optimization problem of the firm.

Consider a small representative firm with a one period ahead planning horizon producing output, $Y$, according to a Leontief production function,

$$ Y = \min(\xi N, \sigma K), \quad (1) $$
where \( N \) and \( K \) are labor and capital input, respectively. \( \xi = Y/L \) and \( \sigma = Y^e/K \) are the productivities of labor and capital, respectively, with \( Y^e \) denoting full capacity output. The firm typically produces below full capacity utilization such that \( \sigma \geq Y/K \). Up to full capacity utilization the firm faces constant marginal costs.

Suppose the representative firm investing in new capital (or dis-investing for the sake of symmetry) can affect overall productivities of labor and capital by choosing the appropriate new equipment. Hence, there exists a relationship between the rates of \( \xi' \) and \( \sigma' \) and the firm’s accumulation rate, \( g \), of the form

\[
\xi' = f(\sigma')
\]

(2)

where the prime indicates the value of a variable in the next period. This function describes what the realization of \( \sigma' \) implies for \( \xi' \). One can think of several possible relationships illustrated in the following for the case of positive investment.

First, if \( \partial f(\cdot)/\partial \sigma' < 0 \) then an over-proportional rise in capacity output vis-a-vis the capital stock such that \( \sigma \) goes up requires additional labor input. Hence, for a given output, \( \xi \) decreases. There is a trade-off between labor and capital productivity. Second, it may hold that \( \partial f(\cdot)/\partial \sigma' = 0 \). Then, there is not trade-off between \( \xi \) and \( \sigma \). Installing more productive capital does neither require more nor less labor input. Finally, \( \partial f(\cdot)/\partial \sigma > 0 \) may characterize the relationship between the productivities. The additional capital stock not only increases capacity output more than proportional, it is also labor saving in the sense that, for a given output level, less labor is required with the new capital stock in place.

Suppose the representative firm operates in an environment of oligopolistic competition facing the danger of losing market shares to potential market entrants. As shown by Spence (1977), firms may hold excess capacity in the optimum in order to deter potential competitors from entering the market.\(^2\) Following this line of argument, we take a short cut and assume an optimal rate of capacity utilization, \( \bar{u} < 1 \) beyond which the realized sales will be lower than a benchmark value, \( \bar{Y} \), which the firm takes as given and may be determined by the average growth rate of output across all firms. This is because with high utilization firms are less flexible to cater possible demand shocks allowing entrants to gain ground in the market. In particular, we assume

\[
Y' = \bar{Y} - f(u^e - \bar{u}) \quad \text{with} \quad \begin{cases} 
    f(\cdot) = 0, & \text{if } u^e \leq \bar{u} \\
    f(\cdot) > 0 \text{ and } \partial f(\cdot)/\partial u^e > 0, & \text{otherwise}
\end{cases}
\]

(3)

where \( u^e = Y^e/(\sigma'K') \) with \( Y^e \) denoting the expected sales in the next period. Only if the expected sales imply an expected utilization rate that exceeds the benchmark level, \( \bar{u} \), market shares will be lost compared to the average across all firms present in the market.

Having described technology, i.e. the relationship between \( \xi \) and \( \sigma \), as well as the demand to be accommodated in the next period, we now state the firm’s objective which is profit

\(^2\)Kurz (1986) derives the optimal utilization rate from a cost minimization problem. Both below and beyond this rate unit costs increase due to maintenance and fixed costs of capital and wage premia for over time and night shifts, respectively. Here, the optimal utilization rate is motivated as a strategic device to deter market entry.
maximization. Note that the accumulation rate is predetermined. One may well argue that investment should be taken as a control variable as in Skott (1989a). Yet, as put forward by many post-Keynesians such as Davidson (1972), the presence of fundamental uncertainty regarding future states of the world may not allow for a sensible characterization of firm’s investment behavior by an inter-temporal optimization problem. For the purpose of the present paper, we follow most of the post-Keynesian literature in deriving an investment function from stylized facts and some theoretical consideration rather than strict micro-foundations.

Given the rate of accumulation, the technology characterized in (1) and (2) as well as expected output in (3), the firm seeks to maximize the next period’s expected profits by choosing an optimal $\sigma'$. With the expected price normalized to unity, revenues are equal to expected output, $Y'$. Costs arise from expected interest payments, $i\lambda K'$, with $i$ denoting the interest rate and $\lambda = L/K$ the debt-capital ratio, as well as expected labor costs, $wY'/\xi'$ with the wage rate, $w$, constant. The firm’s problem, therefore, is

$$\max_{\sigma'} Y' - i\lambda K' - wY'/\xi'$$

s.t. $\xi' = f(\sigma')$

$$Y' = \bar{Y} - h\left(\frac{Y_e}{\sigma'K'} - \bar{u}\right)$$

The first-order optimality condition is

$$\frac{w}{f(\sigma'^*)^2} \frac{\partial f(\cdot)}{\partial \sigma'} \left(\bar{Y} - h\left(\frac{Y_e}{\sigma'^*K'} - \bar{u}\right)\right) + \left(1 - \frac{w}{f(\sigma'^*)} \frac{\partial h(\cdot)}{\partial \sigma'} \frac{Y_e}{(\sigma'^*)^2K}\right) = 0.$$  

(5)

There are two regimes of interest.

First, suppose the firm’s market power is strong and there is much danger of rival market entry, i.e. $\partial h(\cdot)/\partial \sigma'$ is low on the relevant domain. Assume, there is a trade-off between $\xi$ and $\sigma$, i.e. $\partial f(\cdot)/\partial \sigma' < 0$. In this case, we obtain a corner solution for the optimal capital productivity, i.e. $\sigma'^* = Y'/K'$. Since capital productivity does not affect output growth much through (3) while a lower labor input reduces costs, the firm seeks to choose equipment that boosts labor productivity as much as possible at the expense of capital productivity. The firm will seek to operate at full capacity utilization.

Second, suppose the firm’s standing in the market is weak and a lack of flexibility due to a utilization rate too high is associated with large losses of demand, i.e. $\partial h(\cdot)/\partial \sigma'$ is high. Further, the trade-off between $\xi$ and $\sigma$ is weak, i.e. $\partial f(\cdot)/\partial \sigma' < 0$ but low in absolute terms. Since a rise in capital productivity causes only little sacrifices in terms of labor productivity and, hence, labor costs, the optimal capital productivity will be close to $Y'/(\bar{a}K')$ implying the utilization gap, which is associated with large costs in terms of output loss, to be closed.

In the short run, a tight trade-off between labor and capital productivity may prevail since the existing equipment is not changed completely. Yet, over the course of an entire investment project, the representative firm may adjust its capital stock such that capital productivity can be increased while leaving labor productivity constant. If feasible, the investment project may well aim at increasing both productivities.
Assuming that there is no trade-off between the productivities of capital and labor, the firm seeks to adjust $\sigma'$ such that the utilization gap is immediately closed. Yet, the desired capital productivity may not be feasible if the new capital investment is insufficient. Hence, the realized capital productivity can be expected to be adjust over time as long as the utilization gap remains. This motivates the endogenous capital productivity as

\[ \dot{\sigma} = \sigma \phi_\sigma (u - \bar{u}) \]  \hspace{1cm} (6)

with $\partial \phi_\sigma (\cdot)/\partial u > 0$.

3 Empirical evidence

Schoder (2012a) estimate a Cointegrated VAR model for the US manufacturing sector from 1955Q1 to 2012Q2 in output, capacity and a composite leading indicator and provide evidence that production capacities adjust slowly to current output in the long run. In particular, two cointegrating relationships have been identified between the three variables: the first one between output, capacity and a constant; the second one between output and the composite leading indicator. Output is found to be fully characterized by the composite leading indicator, since output is error-correcting to the output relation while it is not error-correcting to the utilization relation. Moreover, capacity seems to be error-correcting to the utilization relation, i.e. to be endogenously adjusting. The composite leading indicator has been found to be weakly exogenous. Note that even though the variables are not normalized by the capital stock the observed endogenous capacity is not the ordinary capacity effect of investment present in any heterodox model. This is because the effect of output on capacity is a long-run and not a short-run phenomenon.

In the present paper, we directly estimate (6) assuming a linear relationship between the growth rate of capital productivity and the utilization gap for various US industries.

First, we use quarterly data from 1960Q3 to 2012Q2 for the manufacturing sector and from 1967Q1 to 2012Q2 for total industries. Data on the rate of utilization and industrial capacity can be obtained from the Fed. Yet, there is no quarterly data on capital stock available for the two sectors considered. Assuming that the growth rates of the sectoral capital stock and of the capital stock of the total economy are not too different over the cycle, the quarterly capital stock data provided by the OECD Economic Outlook database can be used as a proxy.

Second, we estimate (6) for a panel of 17 industries using annual data on utilization and capacity provided by the FED and annual data on fixed assets for the industries considered provided by the US Bureau of Economic Analysis.

Note that in both regression analyses the trend of capital productivity growth and the target utilization rate are approximated by an HP-filter. We check the robustness of our results with respect to different smoothness parameters below. Capital productivity growth and the target utilization rate are approximated by an HP-filter. We check the robustness of our results with respect to different smoothness parameters below.
Figure 2: The cyclical components of the utilization rate and the growth rate of capital productivity for the US manufacturing sector

3.1 Time series estimates for manufacturing and total industry

Figures 2 and 3 plot the cyclical movement of the growth of capital productivity and utilization, i.e. the cyclical element of the HP-filtered variables for a smoothness parameter $\lambda = 160000$ for manufacturing and total industry, respectively. A slight pro-cyclical behavior of capital productivity can be observed.
To reconfirm this observation by means of statistical inference, we estimate

$$\hat{\sigma}_t - \bar{\sigma}_t = \alpha + \beta(u_{t-1} - \bar{u}_{t-1}) + \varepsilon_t. \quad (7)$$

where $\varepsilon_t$ is a stationary disturbance term. We use the first lag of the utilization gap in order to reduce endogeneity problems arising from feedback effects of the dependent variable on the regressor. Note that $\alpha$ captures the remaining trend growth rate of capital productivity not taken care of by the HP-filter. Due to serial correlation in the residuals, we compute autocorrelation and heteroscedasticity-robust Newey-West standard errors.

Table 1: Regression output of time series data

<table>
<thead>
<tr>
<th></th>
<th>$\lambda = 16000$</th>
<th>Manufacturing</th>
<th>$\lambda = 160000$</th>
<th>Total industry</th>
<th>$\lambda = 1600000$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$L=1$</td>
<td>$L=2$</td>
<td>$L=1$</td>
<td>$L=2$</td>
<td>$L=1$</td>
</tr>
<tr>
<td>intercept</td>
<td>0.001</td>
<td>-0.001</td>
<td>0.001</td>
<td>-0.002</td>
<td>0.001</td>
</tr>
<tr>
<td>(u - $\bar{u}$)$_{t-1}$</td>
<td>0.024***</td>
<td>0.040***</td>
<td>0.031***</td>
<td>0.043***</td>
<td>0.038***</td>
</tr>
<tr>
<td>(u - $\bar{u}$)$_{t-2}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>obs.</td>
<td>206</td>
<td>205</td>
<td>206</td>
<td>205</td>
<td>206</td>
</tr>
</tbody>
</table>

Notes: The dependent variable is the growth rate of the capital productivity minus its trend. $u$ denotes the rate of capacity utilization and $\bar{u}$ is the HP-filtered $u$ with a smoothing parameter of $\lambda$. Newey-West standard errors are in parenthesis. $L$ is the number of lags of the independent variable used. ***, ** and * denote the level of significance at 0.01%, 0.05% and 0.1%, respectively.

Table 1 reports the regression results for different choices of $\lambda$ and lag structures. The estimates for the adjustment parameters is typically positive and significant for smoother trends. For the manufacturing sector the parameter seems to be somewhere between 0.02 and 0.04 for the manufacturing sector and between 0.01 and 0.03 for the total industry depending on the smoothness of target utilization rate chosen.

### 3.2 Estimates for a panel of industries

Assuming group-specific fixed effects, we estimate an error-component model of the form

$$\hat{\sigma}_{i,t} - \bar{\sigma}_{i,t} = \alpha + \beta(u_{i,t-1} - \bar{u}_{i,t-1}) + \mu_i + \varepsilon_{i,t} \quad (8)$$

10
where $\mu_i$ is an unobserved industry-specific independent growth rate of $\sigma$ as a difference to the average growth rate across all industries captured by $\alpha$. Again, the lagged utilization gap has been included as a regressor in order to avoid endogeneity problems. The model is fitted to the data employing the fixed effects estimator.

<table>
<thead>
<tr>
<th>$\lambda = 16000$</th>
<th>$\lambda = 160000$</th>
<th>$\lambda = 1600000$</th>
</tr>
</thead>
<tbody>
<tr>
<td>intercept</td>
<td>$-2.864^{***}$</td>
<td>$-2.865^{***}$</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.001)</td>
</tr>
<tr>
<td>$(u - \bar{u})_{t-1}$</td>
<td>0.046*</td>
<td>0.048*</td>
</tr>
<tr>
<td></td>
<td>(0.025)</td>
<td>(0.025)</td>
</tr>
<tr>
<td>obs.</td>
<td>828</td>
<td>828</td>
</tr>
</tbody>
</table>

Notes: The dependent variable is the growth rate of the capital productivity minus its trend. $u$ denotes the rate of capacity utilization and $\bar{u}$ is the HP-filtered $u$ with a smoothing parameter of $\lambda$. Newey-West standard errors are in parenthesis. $^{***}$, $^{**}$ and $^*$ denote the level of significance at 0.01%, 0.05% and 0.1%, respectively.

The estimation results for the sub-sectors of manufacturing and mining combined are reported in Table 2 using different smoothness parameters. The estimated adjustment parameters for the 17 sub-industries are around 0.047 and significant at the 10% level.

4 Effective demand and constant long-run utilization in a Kaleckian growth model

4.1 The short-run Kaleckian model

We consider a closed economy comprising a representative household and a firm. Production is characterized by Leontief-type of production function in capital and labor with a fully elastic labor supply and a, in the long run, endogenous capital-coefficient as discussed below. There is only one good used for consumption and investment. The household provides labor for which it receives a wage and owns the firm for which it receives dividends. The firm receives profit income and invests.

Tables 3 and 4 report the stocks and flows, respectively, normalized by the capital stock of the economy considered here which is kept as simple as possible. Since prices and price expectations are irrelevant for the purpose of the present paper, we assume price expectations to be correct and prices to be equal to unity. As shown in Table 3, the household’s assets are money and deposits held at the bank, $\mu$ which equal the household’s net worth, $\omega_H$. The firm’s capital stock is matched by the sum of its debt-capital ratio, $\lambda$, and its net worth, $\omega_F$. The bank’s assets is the outstanding debt, $\lambda$, and its liabilities are money, $\mu$, and net worth $\omega_B$.

---

3Since annual data are used, we only consider one lag of the regressor.
4This sub-section draws from Schoder (2012c).
5For Keynesian macro-dynamics derived from price expectations, see Taylor (2012).
Table 3: Balance sheets

<table>
<thead>
<tr>
<th></th>
<th>Households</th>
<th>Firms</th>
<th>Banks</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu$</td>
<td>$\omega_H$</td>
<td>1</td>
<td>$\lambda$</td>
</tr>
<tr>
<td>$\omega_F$</td>
<td></td>
<td></td>
<td>$\mu$</td>
</tr>
<tr>
<td>$\omega_B$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: $\mu$ is money over capital, $\lambda$ debt over capital and $\omega_i$ is net worth over capital for sector $i$.

The sources and uses of the corresponding flows are reported in Table 4. Let $u = Y/Y^c$ and $\sigma = Y^c/K$ denote the rate of capacity utilization and the capacity-capital ratio, respectively, with $K$, $Y$ and $Y^c$ being capital, output and full-capacity output, respectively. Output equals household consumption, $c$, and investment, $g^I$. The production of output generates wage income (normalized by capital), $\psi u \sigma$, and profit income, $(1 - \psi) u \sigma$, where $\psi$ is the wage share. Apart from wage income, households also earn interest income, $i \mu$. The part of this income saved, $g^s_H$, is used for new deposits, $\dot{M}/K$. A part of the firm’s profit income is distributed to the bank, $i \lambda$, and the rest is saved. The firm’s savings plus the funds acquired from obtaining new loans, $\dot{L}/K$, are used for investment. The bank’s savings is the the part of the interest income not used for interest payments on deposits. The newly granted loans, $\dot{L}/K$, are matched by these savings plus new money, $\dot{M}/K$.

Table 4: Social accounting matrix

<table>
<thead>
<tr>
<th></th>
<th>Output</th>
<th>Current expenditures</th>
<th>Changes in claims</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>cost</td>
<td>Households</td>
<td>Firms</td>
</tr>
<tr>
<td>Output</td>
<td>$c$</td>
<td>$g^I$</td>
<td>$\dot{y}$</td>
</tr>
<tr>
<td>Incomes</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Households</td>
<td>$\psi u \sigma$</td>
<td>$i \mu$</td>
<td>$y_H$</td>
</tr>
<tr>
<td>Firms</td>
<td>$(1 - \psi) u \sigma$</td>
<td>$i \lambda$</td>
<td>$y_F$</td>
</tr>
<tr>
<td>Banks</td>
<td></td>
<td>$i \lambda$</td>
<td>$y_B$</td>
</tr>
<tr>
<td>Flow of funds</td>
<td></td>
<td>$g^s_H$</td>
<td>$\dot{g}^I$</td>
</tr>
<tr>
<td>Households</td>
<td></td>
<td>$g^s_F$</td>
<td>$-g^I$</td>
</tr>
<tr>
<td>Firms</td>
<td></td>
<td>$g^s_B$</td>
<td>$-\dot{L}/K$</td>
</tr>
<tr>
<td>Total</td>
<td>$y$</td>
<td>$y_H$</td>
<td>$y_F$</td>
</tr>
</tbody>
</table>

Notes: $u = Y/Y^c$ and $\sigma = Y^c/K$ denote the rate of capacity utilization and the capacity-capital ratio, respectively, with $K$, $Y$ and $Y^c$ being capital, output and full-capacity output, respectively. $\psi$ is the wage share, $y_i$ and $g_i^s$ the income and savings, respectively, for sector $i$ over $K$, $c$ consumption over $K$, $g^I$ the accumulation rate of capital, $i$ the interest rate, $\lambda$ debt, $L$ over $K$ and $\mu$ money, $M$ over $K$. $X$ indicates the change of $X$.

In the short run, our economy is fully characterized by the above stock-flow relations and two behavioral assumptions: First, an investment function,

$$g^I = g'(\rho, u, \psi)$$

(9)
where $\rho$ is the autonomous part of investment. In the long-run analysis below its change will be made dependent on utilization and debt. For now, we assume it to be constant such that the investment function is consistent with the standard specification in the Kaleckian literature.\(^6\) Second, a constant saving rate of the household, $s$, leading to an aggregate saving function of the form
\[
g^s = -(1 - s)i\mu + (1 - (1 - s)\psi)u\sigma,
\]
then following from Table 4. The macroeconomic balance condition,
\[
g^i = g^s,
\]
then implies a short-run equilibrium of utilization, $u^*(\psi, \sigma)$, as a function of the wage share and the capital productivity with $\partial u^*(\cdot)/\partial \psi > 0$, i.e wage-led demand, and $\partial u^*(\cdot)/\partial \sigma < 0$. Keynesian stability holds if $\partial (g^*(\cdot)\sigma)/\partial u > \partial (g^*(\cdot)\sigma)/\partial u$. Note that the paradox of thrift holds, i.e. $\partial u^*(\psi, \sigma)/\partial s < 0$. The paradox of cost may or may not hold.\(^7\)

This simple model has been extended extensively in the literature. First, various dynamics have been added to the model, among others, by Taylor (2004, ch. 9) to study the dynamic interaction of productivity, distribution and growth. Taylor (2012) and Schoder (2012c) added de-stabilizing Harrodian investment dynamics to the model. Second, open economy issues have been discussed, among others, by Blecker (1989). Third, the phenomenon of financialization has been recently discussed by Lavoie (1995a) and Hein (2007) and Hein and Schoder (2011).

As can be readily seen, the model predicts a permanent change of the equilibrium utilization rate as a response to a permanent change of any parameter in the investment or saving function. If the model was empirically valid for the long run, the enormous shifts in the saving rates of US households since the 1980’s, for instance, should have lead to permanent shifts in the utilization rate. Yet, as emphasized by Skott (2012), the utilization rates reported by the FED for many US industries are strongly mean-reverting.

Within a Kaleckian framework, a constant $u$ requires either dropping the principle of effective demand such that $Y/K$ is pre-determined in the long-run or endogenizing $\sigma$ since $Y/K = u\sigma$. Given the theoretical and empirical arguments put forward above, we follow the latter approach and analyze below the effects of demand shocks on steady state growth in a long-run variant of the Kaleckian model featuring pro-cyclical capital productivity.

### 4.2 A long-run closure of the Kaleckian growth model with constant utilization and effective demand

Several long-run closures of the short-run model seeking to establish a constant long-run utilization rate have been proposed in the literature (cf. Duménil and Lévy 1999, Shaikh\(^6\) See Rowthorn (1981), Dutt (1984), Amadeo (1986) and Bhaduri and Marglin (1990).

\(^7\)The effects of distribution on capacity utilization and growth within this framework have been studied empirically by, among others, Naastepad and Storm (2006-07), Stockhammer et al. (2009) and Hein and Vogel (2008).
2009, Skott 2010). All of them can be interpreted in terms of long-run shifts and rotations of the investment and savings functions such that the long-run equilibrium is realized at the target utilization rate. A detailed discussion of these proposals can be found in Hein et al. (2011).

4.2.1 Introducing long-run features

The core feature of the present model which gives rise to the principle of effective demand in the long run is an endogenous capital productivity, $\sigma$, motivated theoretically and empirically above. Here, the differential equation is reproduced for convenience:

$$\dot{\sigma} = \sigma \phi_{\sigma}(u - \bar{u})$$

(12)

with $\partial \phi_{\sigma}(\cdot)/\partial u > 0$.

As it turns out below, de-stabilizing Harrodian investment dynamics are required for the model to feature the possibility of the paradoxes of thrift and cost. De-stabilizing forces, however, need counter-forces to obtain global stability. Hence, we also introduce stabilizing debt dynamics. We model the part of investment which is autonomous in the short run as

$$\dot{\rho} = \phi_{\rho}(u - \bar{u}) - \phi_{\psi}(\lambda - \bar{\lambda})$$

(13)

with $\partial \phi_{\rho}(\cdot)/\partial u > 0$ and $\partial \phi_{\psi}(\cdot)/\partial \psi > 0$. If the representative firm faces an inconsistency of the realized utilization rate with the target, it will keep adjusting the rate of accumulation until the gap disappears. Empirical evidence for such a behavior is provided by Skott (2012) and Schoder (2012b). Loosely following Minsky (1976), the firm may also have a target debt-capital ratio which they try to achieve by adjusting the accumulation rate accordingly. On the one hand, a high debt-capital ratio bears the risk of bankruptcy and the firm may attempt to de-leverage by slowing down accumulation. On the other hand, with a low debt-capital ratio additional profits outweigh the risk of default associated with an acceleration of accumulation.

4.2.2 Dynamics excluding Harrodian instability

Let us consider the simple case without Harrodian dynamics, i.e. $\dot{\rho} = 0$, first. To obtain the dynamics of $u$, we log-differentiate (10), use (12) as well as (9) to (11) characterizing the short-run equilibrium growth rate and rearrange leading to

$$\dot{u} = -\phi_{\sigma}(u - \bar{u})u$$

(14)

which is a stable differential equation fully characterizing the adjustment to the steady state $\bar{u}$.

Figure 4 illustrates several demand shocks. Panel (a) depicts the adjustment to the new steady state after a shock to investment. Starting from an equilibrium consistent with the target utilization rate, an upward shift of the investment function will increase the equilibrium utilization rate given the saving function with the economy moving from A to B.
Figure 4: The response of the economy to various demand shocks without Harrodian instability

Now, long-run mechanism start to operate. With no other forces in place, (12) will cause the saving function to rotate counter-clockwise until the equilibrium ends up at the new steady state consistent with $\bar{u}$. Obviously, point C implies a higher growth rate than the starting point A since capital productivity has increased.

Now, let us consider a drop in the saving rate, illustrated in panel (b). Equilibrium utilization goes up, so does the accumulation rate. The economy moves to point B in the short run with the paradox of thrift holding. Yet, in the long run, capital productivity increases rotating the saving function counter-clockwise. Since the investment function has not changed and the saving function returned to its previous position, the new steady state in C equals the old one in A. Hence, the paradox of thrift does not hold.

The long-run effect of a change in distribution on growth is not so straightforward. As shown by Bhaduri and Marglin (1990), both growth and utilization can be wage or profit-led
summing up to four different regimes in the short run. Here we consider only the two cases with growth and distribution responding in the same direction to a change in distribution. Panel (c) illustrates the response of a profit-led economy to an increase in the wage share. The investment function shifts downwards and the saving function rotates clockwise. At the short run equilibrium in B, both growth and utilization are lower. This triggers a reduction in capital productivity in the long run which causes the saving function to rotate clockwise until the target utilization rate is reached. The paradox of cost does not hold since the growth rate is lower than before the rise in the wage share. Panel (d) illustrates the equivalent response of a wage-led economy. After the shifts of the functions, the new short-run equilibrium features higher growth and higher utilization. This causes capital productivity to rise, i.e. the saving function to rotate counter-clockwise until the equilibrium is consistent with the target utilization rate. The growth rate will be lower than before the shock. Hence, the paradox of cost does not hold.

4.2.3 Dynamics including Harrodian instability

Let us derive and analyze the dynamics of the full system. Note that investment can be financed internally, \((1 - \psi)u\sigma - i\lambda - d\epsilon\) and by new debt, \(\lambda\hat{L}\). Hence,

\[
g = (1 - \psi)u\sigma - i\lambda + \lambda\hat{L}. \tag{15}
\]

Using \(\hat{L} = \hat{\lambda} + g\), (15) implies together with the short-run equilibrium characterized by (9) to (11) that

\[
\dot{\lambda} = (1 - \lambda)\left( (1 - (1 - s)\psi)u\sigma - (1 - s)i\mu \right) + i\lambda - (1 - \psi)u\sigma. \tag{16}
\]

Note that (16) evaluated at \(\bar{u}\) and \(\bar{\psi}\) implies for the steady-state value of \(\lambda\) to be a function of the steady-state value of \(\sigma\), i.e.

\[
\bar{\lambda}(\sigma^{**}) = \frac{g^{**} - (1 - \psi)\bar{u}\sigma^{**}}{g^{**} - i} \tag{17}
\]

with \(g^{**} = s(\bar{\psi}, \bar{u}, \sigma^{**})\bar{u}\sigma^{**}\). Hence, we assume implicitly the target level of the debt-capital ratio to be endogenous.

To obtain the dynamics of \(u\), we log-differentiate (10), use (9) differentiated with respect to time, (12) and (15) to (17) as well as (9) to (11) characterizing the short-run equilibrium growth rate and rearrange which leads to

\[
\dot{u} = \frac{\phi_p(u - \bar{u}) - \phi_p(\lambda - \bar{\lambda})}{(1 - (1 - s)\psi)\sigma} - \phi_s(u - \bar{u})u \tag{18}
\]

Equations (12), (16), (17) and (18) characterize a three-dimensional dynamical system.
in \( u, \lambda \) and \( \sigma \). The corresponding Jacobian Matrix is

\[
J = \begin{bmatrix}
\frac{\partial \phi_u(\cdot)}{\partial u} - \left( \phi_u(\cdot) + u \partial \phi_u(\cdot) / \partial u \right) (1 - \lambda)(1 - (1 - s)\psi) \sigma - (1 - \psi)\sigma \frac{\partial \phi_u(\cdot)}{\partial u} & (1 - \lambda)(1 - (1 - s)\psi) \sigma - (1 - \psi)\sigma \\
\frac{-\partial \phi_u(\cdot)}{\partial \lambda} \left( (1 - (1 - s)\psi) \sigma \right) & -(1 - (1 - s)\psi) u\sigma + (1 - s)i\mu + i \\
\frac{-\phi_u(\cdot) - \phi_u(\cdot)}{\left( (1 - (1 - s)\psi) \sigma \right)^2} & 0
\end{bmatrix}
\]

(19)

The signs of the Jacobian are the following:

\[
J = \begin{bmatrix}
+ & + & + \\
- & - & 0 \\
+ & 0 & 0
\end{bmatrix}
\]

(20)

According to the Routh-Hurwitz conditions, local stability of the 3-dimensional dynamical system evaluated at the steady state requires

1. \( \text{tr}(J) < 0 \)
2. \( \det(J_1) + \det(J_2) + \det(J_3) > 0 \)
3. \( \det(J) < 0 \)
4. \( - \text{tr}(J)( \det(J_1) + \det(J_2) + \det(J_3) ) + \det(J) > 0 \).

Given the signs of the Jacobian as illustrated in (20), the first condition is satisfied if the accelerator effect in the investment function is not too large as compared to the self-stabilizing own-effect of the debt-capital ratio which is favored by a low interest rate. The second condition is also likely to hold with plausible parameter values: The first principal minor is always positive; the second will be positive if the debt effect on investment is strong; and the third is always negative but low in absolute terms. The third condition always holds, given the signs of the Jacobian as in (20). Given the first and second conditions hold and the determinant of the Jacobian being sufficiently close to zero, the fourth condition is also met and local stability at the steady state is achieved.

Figure 7 illustrates the same demand shocks considered before but now including the additional long-run dynamics for the stable case. In panel (a), the economy has been hit by a rise in the animal spirits with the economy moving, again, to B in the short run. Yet, in the long run, not only does the savings function rotate counter-clockwise, the investment function starts shifting upwards. Since the debt-capital ratio also starts increasing putting downward pressure on investment, the economy will settle at the target rate of utilization with a higher steady growth rate. Hence, with Harrodian dynamics the long-run impact of a demand shock is more pronounced than without. Panel (b) illustrates the effects of a

\(^8\)Note that \( \partial \lambda / \partial u > 0 \) requires the saving rate to be high and the debt-capital ratio to be low. As shown by Schoder (2012c), however, introducing a plausible pro-cyclical interest rate rule relaxes this restriction as a positive derivative becomes more likely. Yet, for simplicity we do not consider such a rule here.
Figure 5: The response of the economy to various demand shocks with Harrodian instability drop in the saving rate. From the new short-run equilibrium in B, the investment function shifts upwards and settles due the negative effect of increasing debt while the saving function rotates back beyond the initial position. The paradox of thrift now holds. Panel (c) and (d) show the dynamic adjustment of the economy in response to a rise in the wage share. In both the profit and wage-led case, the effects are more pronounced with Harrodian dynamics than without.

5 Concluding remarks

The present paper has sought to reconcile an exogenous long-run utilization rate and the principle of effective demand within a Kaleckian growth model by endogenizing capital productivity. We have put forward theoretical arguments why one would expect capital pro-
ductivity to move slightly pro-cyclically. On the one hand, data on full-capacity output do
typically not measure technically feasible output but the maximum output under normal
production conditions. We have argued that with indivisibilities in the production process,
these normal conditions may be endogenous. On the other hand, changes in productivities
may be induced by investment. By the aid of a simple optimization problem of the firm, we
have argued that firms may seek to adjust capital productivity to narrow the gap between
realized and desired utilization which is associated with costs. In this case, the growth rate
of capital productivity is a function of the utilization gap.

We have then provided empirical evidence for a pro-cyclical capital productivity from a
time series and panel estimation of the adjustment parameter for various US industries. We
have found an average adjustment parameter around 0.03.

Finally, we have introduced the pro-cyclical capital productivity to a Kaleckian growth
model and have analyzed the long-run effects of shocks to investment, saving and distribution.
We have found that the principle of effective demand holds but not the paradoxes of thrift
and cost. Yet, adding de-stabilizing Harrodian investment dynamics to the model as well as
counter-acting debt dynamics has revealed that the principle of effective demand as well as
the paradoxes of thrift and cost may hold in the long run.

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