Christian Proaño Acosta,
Department of Economics, Bielefeld University and
Macroeconomic Policy Institute (IMK) in the
Hans Boeckler Foundation, Düsseldorf, Germany

Gradual Wage-Price Adjustments,
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Christian Proaño Acosta*
Department of Economics, Bielefeld University and Macroeconomic Policy Institute (IMK) Düsseldorf, Germany
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Abstract

Contrary to the assumption of perfectly flexible labor markets commonly used in mainstream macroeconomic models, in the real world the existence of structural imperfections such as search and trading costs hinder the frictionless functioning of these markets, generally leading to outcomes of Non-Walrasian type with involuntary unemployment and open vacancies in “equilibrium”. In this paper we model the existence of labor market frictions into a Keynesian (Disequilibrium) AS-AD framework in the line of Asada, Chen, Chiarella and Flaschel (2006) through a labor search and matching function. By means of dynamic shock simulations, we find that the extent of the labor market rigidity has a great importance for the dynamics not only of employment and output, but also of wage and price inflation, and consequently also for the conduction of monetary policy.

Keywords: Labor market frictions, staggered wage and price dynamics, (D)AS-AD, monetary policy

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1 Introduction

Contrary to the assumption of Walrasian labor markets commonly used in mainstream macroeconometric models, in the real world the existence of structural imperfections such as search and trading costs hinder the frictionless functioning of the labor markets, generally leading to outcomes of Non-Walrasian type, with involuntary unemployment and unfilled vacancies in "equilibrium". Additionally, the existence of such frictions also affects the dynamics of the real side of the economy by delaying the responses of output and employment to exogenous and endogenous shocks, increasing therefore the persistence of such effects. Concerning wage and price inflation, the presence of labor market rigidities is also likely to sluggish their response to output developments through their effect on real unit labor costs (or, from another perspective, on the labor share), one of the main determinants of price inflation. Due to these different factors, the incorporation of labor market frictions in a macroeconomic framework is likely to explain to a significant extent the high degree of persistence observed in many aggregate macroeconomic indicators such as price inflation in the majority of industrialized countries, and especially in the major countries of the European Monetary Union (EMU).

Somewhat surprisingly, though, in most of the macroeconomic models developed in the last decade – including the increasingly popular DSGE (Dynamic Stochastic General Equilibrium) models in the line of Erceg, Henderson and Levin (2000) and Christiano, Eichenbaum and Evans (2005) – the existence of labor market frictions and their role in the dynamics of employment, output and inflation remained besides the research agenda, due to the almost exclusive focus of the majority of models of New Keynesian type on the existence and theoretical modelling of nominal rigidities. Nevertheless, as discussed for example in Chari, Kehoe and McGrattan (2000), a central problem of intertemporal optimizing macroeconomic models featuring only nominal rigidities is that the dynamic responses generated by them do not feature the degree of persistence observed in real data.

The empirical shortcomings of these DSGE models have led recently to the incorporation within that framework of not only nominal, but also real rigidities in form of labor market frictions in a wage bargaining setting, as done recently in Walsh (2003), Trigari (2004) and Gertler and Trigari (2006), among others. Nevertheless, the adequacy of such types of models, concerning the rational expectations assumption and the modeling strategy, still remains questioned by a large number of researchers.

The remainder of the paper can be summarized as follows. In section 2 the recent literature on staggered wage and price and employment dynamics is briefly
overviewed, with special focus on the DSGE approach. In section 3 I develop alternatively a Keynesian (Disequilibrium) AS-AD model in the line of Chen, Chiarella, Flaschel and Semmler (2006) and Proaño, Flaschel, Ernst and Semmler (2006), where I additionally introduce, by means of a search and matching labor market module, frictions in the labor markets. As it will be discussed there, this alternative modelling approach features a similar or even better ability to emulate the dynamic behavior of aggregate macroeconomic data, generating for example a higher degree of inflation persistence without the use of the highly questionable Calvo (1983) price setting mechanism often used in the DSGE framework. After identifying the local stability conditions of the resulting 4D dynamical system, in section 5 I analyze, on the basis of the previously estimated parameters, the dynamic adjustment paths of the variables of the theoretical model to monetary and aggregate demand shocks. Thereafter, in section 6 the design of monetary policy within our theoretical framework is discussed through the analysis of the dynamic adjustment and degrees of persistence of the model variables under alternative monetary policy rules. Section draws some conclusions and further research directions from this study.

2 Overview of the Literature

As stated before, labor markets are confronted to a larger extent than other types of markets with a variety of frictions such as the asymmetric or incomplete information about the state of the market, geographical and skill mismatches, as well as searching and trading costs. Indeed, as pointed out by Pissarides (2000, p.3), unlike other markets, trading in labor markets is likely to be “uncoordinated, time-consuming, and costly for both firms and workers”, itself likely to depend on the actual market conditions, that is, on the relative size of unemployed workers and vacancies.

In the last decade, nevertheless, labor markets of Walrasian type, where workers and employers do not face any type of frictions or trading costs, have been assumed in the majority of macroeconomic models discussed in the mainstream literature. Indeed, in models of New Keynesian type such as Blanchard and Kiyotaki (1987), Goodfriend and King (1997) and Rotemberg and Woodford (1997), research focused primarily on the modelling of nominal rigidities: With rational, forward looking, intertemporal utility maximizing households and profit maximizing firms, the only source of frictions is the existence of a staggered price setting mechanism à la Calvo (1983), whereafter only a fraction of firms can reset their goods prices to the mo-
nopolistically optimal level in every period. Because in that framework nominal wages are assumed to be perfectly flexible, the resulting real wage is always at the market-clearing level and therefore no involuntary unemployment exists. Because in those models the economic agents do not face any type of quantity constraint, firms and households always (and without time or monetary costs) find a proper workers and job position, respectively, and the resulting real wage in the labor markets always fulfills simultaneously the intertemporal consumption/leisure preferences of households and the profit maximization condition by the firms. The notion of the existence of Non-Walrasian labor market equilibrium situations, where households and firms might not be able to find adequate counterparts in the labor markets, and therefore where involuntary unemployment and unoccupied job positions might exist in equilibrium, remained to a large extent unconsidered in those models.

In early New Keynesian models featuring only price rigidities as e.g. Roberts (1995), the assumption of frictionless, Walrasian labor markets delivered also wide-reaching implications for the conduction of monetary policy: According to the resulting inflation adjustment equation, known as the baseline New Keynesian Phillips Curve (NKPC), where inflation is simple a function of the actual output gap and future expected inflation, stabilizing inflation is equivalent to stabilizing output. For the monetary authorities, so the conclusion of such models, there exists no trade-off between inflation and output stabilization, contrary to the experience of the majority of central bankers around the world. The absence of an output-inflation stabilization trade-off still exists in the more elaborated versions of the New Keynesian framework, for example under the hybrid NKPC developed by Galí and Gertler (1999) and Galí, Gertler and López-Salido (2001) (where actual inflation additionally depends on lagged inflation due to the assumption of price indexation), also due to its basic assumption of perfect nominal wage flexibility, as shown in Woodford (2003).

Contrarily to the abovementioned New Keynesian models, the more recently elaborated DSGE models in the line of Erceg et al. (2000), Smets and Wouters (2003) and Christiano et al. (2005), feature besides price- also nominal wage rigidities. Now, while with the inclusion of nominal wage rigidities the absence of a trade-off between output, employment and inflation stabilization (referred by Blanchard and Galí (2005) as the “divine trinity”) disappears and the dynamics of inflation predicted by the underlying theoretical model become, in a more realistic way, more persistent, the wage and price development is still solely determined by abstract stochastic

\footnote{2See Mankiw (2001), Estrella and Fuhrer (2002) and Rudd and Whelan (2005) for some critical assessments concerning the theoretical and empirical implications of the New Keynesian approach.}

\footnote{3These mentioned studies feature additionally various types of real rigidities such as habit formation in consumption, investment and adjustment costs and variable capacity utilization, but still do not incorporate labor market frictions.}
processes. Indeed, in these models, wage stickiness is introduced by assuming, in analogy to the standard modelling of optimal price setting by firms, that households – offering differentiated types of labor – possess enough indeed monopolistic power to unilaterally set (!) the level of nominal wages which allows them to maximize their intertemporal utility function. Nevertheless, just like firms in the baseline New Keynesian framework, only a constant fraction of households obtained in every period the opportunity to reset their wages optimally in a Calvo (1983) manner, see e.g. Erceg et al. (2000) and Christiano et al. (2005). Now, while this scheme of staggered wage contracts facilitates an easy and elegant incorporation of nominal rigidities in the DSGE framework, its closeness to reality is highly questionable (this criticism applies also to the standard New Keynesian Phillips Curve, which is also derived from the assumption of staggered price contracts à la Calvo (1983)): Indeed, in most industrialized countries, and especially in the members countries of EMU, most of the wages are set through a bargaining process between firms and trade unions. Even in more decentralized labor markets as in the U.S., the assumption of households setting wages in a monopolistic manner is highly questionable, due to the rather low degree of differentiation of the labor supply by the majority of the population economically active.

In recent times, the theoretical research on the role of labor market frictions for the dynamics not only of output but also of real marginal costs and of wage and price inflation has experienced a revival, after nearly two decades where it was almost completely left aside from the academic literature on monetary economics. In a series of research papers, Walsh (2003), Trigari (2004), Christoffel and Linzert (2006a) and Gertler and Trigari (2006), among others, have started to investigate the role that labor market frictions play in the dynamics of the real economy by integrating some elements of the job search theory popularized by Mortensen and Pissarides (1994) and Pissarides (2000) (a standard approach used in labor economics to model labor market frictions) into DSGE frameworks with nominal wage and price rigidities.

The reasoning for this new modelling strategy is the following: when labor markets do not function in a frictionless manner but are confronted to real rigidities, they are not able to accommodate aggregate demand and supply shocks immediately.

\footnote{In the next section we will discuss the alternative (D)AS-AD approach to wage and price inflation dynamics by Chiarella and Flaschel (2000) and Chiarella, Flaschel and Franke (2005).}

\footnote{In the search and matching framework, the search for adequate business partners in the labor market (firms and employers) is assumed to be costly and time consuming. Vacancies and unemployed workers are assumed furthermore to be brought together by a matching function which depends on the state of the market. The use of aggregate search and matching functions in the line of Pissarides (2000) has become standard in labor economics for the analysis of the labor markets at an aggregate level, due to the high diversity in the nature of the frictions affecting labor markets. See Petrongolo and Pissarides (2001) for a survey article on the aggregate matching functions.}
Its delayed reaction to such shocks, thus, weakens the link between output and employment, making their reactions less strong and more persistent.

In order to introduce nominal rigidities in her model, Trigari (2004) assumes that, while nominal wages are re-set through a bargaining process between firms and workers in every period, a firm “obtains perhaps a phone call by Calvo himself”\(^6\) and can re-optimize its prices in a Calvo (1983) manner with additional wage indexation.\(^7\) Gertler and Trigari (2006) follow the same strategy, though in a more elaborated framework, by gathering nominal wage staggering with a multi-period wage bargaining resulting from the use of the job search theory. Alternatively, following Hall (2005), in Christoffel and Linzert (2006a) and Christoffel and Linzert (2006b) nominal rigidity is introduced by specifying a “wage norm or social consensus” after which “the actual wage level is given by a weighted average of past wage level and the equilibrium wage level”\(^8\). The basic result of these studies is that while the incorporation of labor market frictions into DSGE models implies qualitatively similar responses of output, employment and inflation to aggregate demand shocks and monetary policy shocks as in DSGE models with Walrasian labor markets, from the quantitative point of view, this modification decreases the responsiveness of output and inflation and increases their degree of persistence.

Nevertheless, as stated before, the adequacy of this type of theoretical modelling approach has been strongly questioned, despite its actual popularity in the mainstream literature, by a large number of researchers such as Mankiw (2001), Eller and Gordon (2003) and Solow (2004), precisely due to its focussing on intertemporal, “rational” and forward-looking modelling of economic agents. Solow (2004) goes one step further and rises serious doubts about the implications for economic policy advisement based on such models, primarily due to their low ability to fit real data. While elegant in their theoretical microfoundations, these types of macroeconomic models are nevertheless still far too restrictive to describe and analyze a variety of macroeconomic interactions within an economy. The microfoundations of the wage and price setting, completely oriented to the intertemporal maximization of utility and profits, disregard short run factors and dynamics which might take place even if they are not consistent with the solution of an intertemporal maximization problem. Aggregate demand pressures, as well as the state of the markets, especially of the labor markets, are not considered in models of the New Keynesian sort as the ones developed in Gali and Gertler (1999), Erceg et al. (2000) and Christiano et al. (2005).

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\(^6\)I owe this beautiful expression to Christian Merkl from the Kiel Institute of World Economics.

\(^7\)As discussed before, this Galí et al. (2001) specification leads to a hybrid type of New Keynesian Phillips Curve, where actual inflation depends on future expected and past inflation.

\(^8\)Christoffel and Linzert (2006b, p.16).
Due to these arguments, as especially due to our focus on the role of frictions in the labor markets for the dynamics of the economy as a whole, our theoretical analysis will be based on disequilibrium rather than equilibrium situations, following the theoretical modelling approach by Chiarella and Flaschel (2000), Chiarella et al. (2005) and Chen et al. (2006). This theoretical approach, which relies on the interdependent but nevertheless separate gradual adjustments of wages and prices, allows, in contrast to standard DSGE models, for disequilibrium situations in both goods and labor markets, and might therefore be more appropriate for a realistic theoretical analysis of the role of labor market frictions for the dynamics of output, inflation and income distribution. Additionally, as it will be discussed in the next section, due to the special specification of the inflation expectations, with perfectlyforeseen actual wage inflation entering in the price inflation adjustment equation and vice versa, both goods and labor markets influence in a direct manner both wage and price inflation behavior in the economy.

3 The Model

In the next section we modify and extend an alternative approach to the DSGE framework developed by Chiarella and Flaschel (2000) and Chiarella et al. (2005). Nevertheless, in contrast to similar models discussed in Chen et al. (2006) and Proaño et al. (2006), where the dynamics of the goods and the labor markets were linked in a pragmatic manner by a dynamic version of Okun’s (1970) law, I incorporate into this Keynesian (Disequilibrium) macroeconomic framework the existence of frictions in the labor markets through the additional incorporation of a search and matching module in the line of Pissarides (2000).

3.1 The Labor Markets

In a quite standard manner, we assume a single input factor technology, by which output is simply produced according to

\[ Y_t = z_t N_t^\alpha, \]  

(1)

where \( N_t \) denotes the actual (realized) level of employment and \( z_t \) represents the average labor productivity in the economy.

In the same manner, full employment output \( Y_t^f \), is simply a function of the actual level of labor supply in the economy \( L_t = \bar{L} \) (assumed for simplicity to be constant)

\[ Y_t^f = z_t L^\alpha. \]  

(2)
Firms, confronted to an aggregate demand level $Y^D$, determine their labor demand according to eq. (1), that is

$$L^D_t = \left(\frac{Y^D_t}{z_t}\right)^{1/\alpha}. \quad (3)$$

Nevertheless, due to the existence of labor market frictions, the actual level of employment $N_t$ is not necessarily consistent with the labor demand by firms $L^D_t$, so that $L^D_t = N_t$ does not hold in the normal case.

Following Hall (2005) and Shimer (2005), who find that the rise in unemployment during economic slowdowns is caused not by a higher rate of job destruction (at least in the U.S. employed workers do not get fired more frequently than in economic booms), but by a lower rate of job creation, we assume that a certain number of jobs are destroyed at an exogenous rate $\rho$ in each period. The actual number of employed workers at $t$ is then determined by the level of remaining jobs from the previous period and by the “matches” occurred at the beginning of the actual period. At $t$, the number of employees is determined by

$$N_t = (1 - \rho)N_{t-1} + m(U_t, V_t) \quad (4)$$

where $m(U_t, V_t)$ is a matching function of a standard Cobb-Douglas type

$$m(U_t, V_t) = \mu U_t^\varrho V_t^{1-\varrho}, \quad \mu \in (0, 1) \quad (5)$$

with $\mu$ representing the matching technology level, $U_t = L_t - (1 - \rho)N_{t-1}$ the number of unemployed and $V_t = L^D_t - (1 - \rho)N_{t-1}$ the number of vacancies at the beginning of period $t$ (which can be negative if the firms decide to lower their demand of labor).

By defining $u_t = U_t/L_t$ and $v_t = V_t/L_t$ as the unemployment and vacancy rates, respectively, and normalizing the total labor supply to $L_t = \bar{L} = 1$, we can reformulate eq. (4) in terms of the employment rate $e_t = N_t/\bar{L} = N_t$ as

$$e_t = (1 - \rho)e_{t-1} + m(u_t, v_t). \quad (6)$$

This specification, though quite simple, allows us to incorporate in our theoretical framework the dependency of employment on the actual labor market situation. By defining the degree of actual labor market tightness as

$$\Theta_t = v_t/u_t,$$

9While this assumption is also met by Gertler and Trigari (2006), Christoffel and Linzert (2006a) and Christoffel and Linzert (2006b), Trigari (2004) and Campolmi and Faia (2006), in contrast, assume that the job separation rate depends partly on the position of the economy within the business cycle, making the separation rate of employment partly endogenous.
we can reexpress the matching function described by eq. (5) as

\[ m(\Theta_t, v_t) = \mu \Theta_t^{-\varrho} v_t, \]

with \( m'(\Theta_t) = -\varrho \mu \Theta_t^{-\varrho - 1} v_t < 0 \). The level of employment, determined by the matching function, decreases if the ratio of labor demand (the vacancy rate) to the unemployment rate increases. Making use of the fact that

\[ L^p_t / L = \left( \frac{Y^p_t}{Y^f_t} \right)^{1/\alpha}, \]

we can rewrite eq. (6) as

\[ e_t = (1 - \rho) e_{t-1} + \mu [1 - (1 - \rho) e_{t-1}]^\varrho \left( \left( \frac{Y^p_t}{Y^f_t} \right)^{1/\alpha} - (1 - \rho) e_{t-1} \right)^{1-\varrho}, \]  

or, after reordering,

\[ e_t - e_{t-1} = \mu [1 - (1 - \rho) e_{t-1}]^\varrho \left( \left( \frac{Y^p_t}{Y^f_t} \right)^{1/\alpha} - (1 - \rho) e_{t-1} \right)^{1-\varrho} - \rho e_{t-1}, \]

which represents the law of motion of employment in discrete time. Defining generally the time lag length as \( h \), for \( h \rightarrow 0 \), we obtain the following approximate formulation for the continuous time analogous of eq. (8)

\[ \dot{e} = \mu \left[ 1 - (1 - \rho) e \right]^\varrho \left( \left( \frac{Y^p_t}{Y^f_t} \right)^{1/\alpha} - (1 - \rho) e \right)^{1-\varrho} - \rho e. \]

As this labor market module is formulated, the state of the market (the labor market tightness) influences in a direct way the capability of firms to serve aggregate demand: Indeed, due to the existence of labor market frictions, firms usually do not obtain their desired level of labor demand \( L^p_t \), but \( N_t \) instead. Note that the more rigid the labor markets are, the greater will be the discrepancy between \( L^p_t \) and \( N_t \), and therefore, through the wage dynamics to be discussed below, the more sluggishly the nominal unit labor costs will react to exogenous and endogenous shocks.

As eq. (9) shows, the rate of change of the employment rate depends on the level of aggregate demand \( L^p_t / L_t = \left( \frac{Y^p_t}{Y^f_t} \right)^{1/\alpha} \), and not on the rate of change of aggregate output (which would be the case if \( Y^p_t = Y_t \), as formulated for example in Proaño et al. (2006) with the modelling of a dynamic Okun’s law. By partial differentiation, we can confirm the adequacy of the qualitative response of the employment dynamics to the aggregate demand \( Y^p_t \) and to the level of the employment rate:

\[ \frac{\partial \dot{e}}{\partial Y^p_t} = (1 - \varrho) \mu \varrho u e^{1-\varrho} \left( \frac{Y^p_t}{Y^f_t} \right)^{1/\alpha - 1} > 0 \]

\[ \frac{\partial \dot{e}}{\partial e} = \varrho \mu u^{1-\varrho} (-1 + \rho) u e^{1-\varrho} + (1 - \varrho) \mu u \varrho e^{0} (1 + \rho) - \rho < 0 \]
Note that our formulation of the employment rate dynamics differs significantly from traditional search and matching labor market models, because here the vacancies are determined basically by the goods aggregate demand pendant on the labor market (since \( L^D = (Y^D/z)^{1/\alpha} \)) and not, as usual, through a forward-looking decision process including Bellman equations and in there the cost-benefit considerations of both workers and firms. However, as discussed later on in this paper, the formulation of the employment rate dynamics delivers quite reasonable dynamics when simulated.

### 3.2 The Goods Markets

The dynamics of the goods markets in this theoretical model are still of a Keynesian type, with aggregate demand driving the level of employment and output, in this order. Indeed, due to the incorporation of labor market rigidities in our theoretical framework, the level of production in the economy is not only determined by the aggregate demand, as in standard Keynesian models, but is also influenced in a direct manner by the degree of inflexibility of the labor markets. Accordingly, we differentiate between the full-employment production level \( Y^f_t = z_t^L \alpha \) and the potential output level

\[
Y^p_t = z_t \phi L^\alpha
\]

where \( \phi \) represents a time-invariant term comprising structural labor market factors to be defined below.

We assume that excess aggregate demand (the log deviation of aggregate demand from the full employment output level) is simply determined by

\[
y^d_t = \ln \left( \frac{Y^D_t}{Y^f_t} \right) = \alpha_y y_{t-1} - \alpha_{yt}(i_{t-1} - \hat{p}_t - (i_o - \pi_o)) - \alpha_{yr}(v_{t-1} - v_o)
\]

where \( y_{t-1} \) represents the output gap (to be defined below) in the previous period, \( i_o \) denotes the steady state nominal interest rate, \( \pi_o \) the target inflation rate of the central bank (assumed for simplicity to be equal to the actual steady state inflation rate) and \( v_t - v_o \) being the deviation of the actual labor share \( v_t \) from its steady state level \( v_o \). According to eq. (11), aggregate demand is assumed to depend (i) positively (with \( 0 < \alpha_y < 1 \)) on aggregate income, (ii) negatively on the labor share (in principle this dependence could be positive, depending on whether consumption is more responsive to real wage changes than investment) and (iii) negatively on the real interest rate.

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\(^{10}\)See Proaño et al. (2006) and Franke, Flaschel and Proaño (2006) for an extensive discussion of the ambiguity of the Rose real wage channel.
Making use of eq. (7) we can express the actual output gap $y$ (defined as the log deviation of the actual, realized output level from potential output) as

$$y_t = \ln \left( \frac{Y_t}{Y_p} \right) = \ln \left( \frac{\alpha z_t^\alpha}{\alpha z_{t-1}^\alpha \phi L^\alpha} \right) = \alpha \ln \left( e_t / \phi \right),$$

$$y_t = \alpha \ln \left( \frac{(1 - \rho)e_t - 1 + \mu [1 - (1 - \rho)e_{t-1}]^\phi \exp \left( \frac{(y^d_t)^{1/\alpha} - (1 - \rho)e_{t-1}}{\frac{1}{\alpha}} \right) \right) \right)^{1-\rho}$$

Note that for $e_t = \phi = e_0$, with $e_0$ being the steady state employment rate, $y_t = 0$ holds.

As this module is formulated, aggregate goods demand determines the level of employment desired by firms, and through the search and matching function expressed by eq. (5), the actual level of employment (the employment rate). This, in turn, determines the level of production through the assumed production function, which, despite of being influenced through the existence of labor market frictions, is still demand driven and thus Keynesian in nature.

Furthermore, the growth rate of the output gap results by definition, namely

$$\hat{y}_t = y_t - y_{t-1}$$

contrarily to the dynamic IS-equation used in Asada et al. (2006), for example.

### 3.3 The Wage-Price Dynamics

As stated before, because models featuring only price rigidities are unable to generate the degree of inflation and output persistence observed in real aggregate data, the recent DSGE type of macroeconometric models in the line of Erceg et al. (2000) and Christiano et al. (2005) incorporate both staggered wage and price setting. In contrast to that framework, where the dynamics of wage and price inflation are driven only by the rational, forward-looking, profit and utility maximizing behavior of firms and households, in the Keynesian (D)AS-AD approach by Chiarella and Flaschel (2000) and Chiarella et al. (2005), on the contrary, the dynamics of wages and prices depend on the demand pressure of the relevant market, namely of the labor and the goods markets, respectively. In this theoretical framework, the measure of demand pressure in the labor markets is the deviation of the actual employment

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Note furthermore that the influence of both goods and labor market conditions on the behavior of wages and prices, present in the real world, is not considered explicitly in those types of models.
rate from the NAIRU equivalent $e_o$. In the goods markets, on the contrary, it is the output gap which measures the pressure of aggregate demand on prices.\textsuperscript{12}

The structural form of the wage-price dynamics in our framework is given by:

\begin{align*}
\dot{w} &= \beta_{we}(e - e_o) - \beta_{wu} \ln(v/v_o) + \kappa_{wp} \hat{p} + (1 - \kappa_{wp}) \pi_c + \kappa_{wz} \hat{z}, \\
\dot{p} &= \beta_{pu}(y - y_o) + \beta_{pv} \ln(v/v_o) + \kappa_{pw} (\hat{w} - \hat{z}) + (1 - \kappa_{pw}) \pi_c.
\end{align*}

(13)\textsuperscript{13}

The demand pressure terms $e - e_o$ and $u - u_o$ in the wage and price Phillips Curves are augmented by three additional terms: the log of the wage share $v$ or real unit labor costs (the error correction term discussed in Blanchard and Katz (1999, p.71)), a weighted average of corresponding expected cost-pressure terms, assumed to be model-consistent, with forward looking, cross-over wage and price inflation rates $\hat{w}$ and $\hat{p}$, respectively, and a backward looking measure of the prevailing inertial inflation in the economy (the “inflationary climate”, so to say) symbolized by $\pi_c$, and labor productivity growth $\hat{z}$ (which is expected to influence wages in a positive and prices in a negative manner, due to the associated easing in production cost pressure). Concerning the latter variable we assume for simplicity that it is always equal to the growth rate of trend productivity, namely $\hat{z} = g_z =$const.\textsuperscript{13} Note that here our approach differs again from the standard New Keynesian approach based on the work by Taylor (1980) and Calvo (1983). Instead of assuming that the aggregate price (and wage) inflation is determined in a profit maximizing manner solely by the expected future path of nominal marginal costs, or in the hybrid variant discussed in Gali et al. (2001), also by lagged inflation, in the (D)AS-AD framework it is assumed that not only the last period inflation, but also the inflationary climate where the economy is embedded is taken into account.

The microfoundations of the wage Phillips curve are of the same type as in Blanchard and Katz (1999), where the dynamics of nominal wages are determined by a wage bargaining process between the trade unions and firms. Blanchard and Katz assume, as in standard wage setting models, that the expected real wage by the trade unions is simply determined by a weighted average of the reservation real wage $\omega^\text{min}_t$ and the currency labor productivity $z_t$, augmented additionally by the state of the labor market, represented by the unemployment rate $u_t$, that is

\begin{equation}
\omega^e_t = \theta \omega^\text{min}_t + (1 - \theta) z_t - \beta_{wu} u_t
\end{equation}

\textsuperscript{12}This separate specification of the wage and price dynamics also allows to circumvent the identification problem pointed out by Sims (1987) for simultaneous estimations of wage and price equations with the same explanatory variables.

\textsuperscript{13}Even though explicitly formulated, we will assume $g_z = 0$ in the theoretical part of this paper for simplicity and leave the modeling of the labor productivity growth for future research.
The reservation real wage, in turn, is assumed to be determined by a simple rule of the form

\[ \omega_t^{\min} = a + \lambda \omega_{t-1} + (1 - \lambda) z_t. \]  

(16)

By inserting eq. (16) into (15), after some arrangements, see e.g. Flaschel and Krolzig (2006), one obtains

\[ \Delta w_t = -\beta_w u_t - (1 - \theta \lambda) \ln(v_{t-1}/v_o) + \hat{p}_t + (1 - \theta \lambda) \hat{z}_t + \theta a \]  

(17)

Eq. (17) is nearly equivalent to eq. (13) (with the unemployment gap in the place of the logarithm of the output gap) if hybrid expectations formation is additionally incorporated and additionally a zero growth rate of labor productivity is assumed.

Concerning the price Phillips curve, a similar procedure may be applied based on desired markups of firms, as discussed for example in Flaschel and Krolzig (2006). Along these lines, one in particular gets an economic motivation for the inclusion of the logarithm of the real wage (or wage share) with negative sign into the wage PC and with positive sign into the price PC, without any need for loglinear approximations. The employment gap and the output gap are thus included in these two Philips Curves, respectively, in the place of a single measure (the log of the output gap), as done for example in Woodford (2003). The wage-price module is thus consistent with standard models of unemployment based on efficiency wages, matching and competitive wage determination, and can be considered as an interesting alternative to the – theoretically rarely discussed and empirically questionable – New Keynesian form of wage-price dynamics.\(^{14}\)

Note that we assume model-consistent expectations with respect to short-run wage and price inflation, nevertheless incorporated in the above Phillips Curves in a cross-over manner, with perfectly foreseen price inflation in the wage Phillips Curve and wage inflation in the price Phillips curve. We stress that our model indeed features a forward-looking behavior here, without the need for an application of the jump variable technique of the rational expectations school in general and the New Keynesian models as e.g. in Woodford (2003, p.225), the joint evolution of wages and prices is described by the following two loglinear equations

\[ \hat{w}_t \quad w_{PC} \beta E_t[\hat{w}_{t+1}] + \beta_{wy} y_t - \beta_{w\omega} \ln \omega_t, \]

\[ \hat{p}_t \quad p_{PC} \beta E_t[\hat{p}_{t+1}] + \beta_{py} y_t + \beta_{p\omega} \ln \omega_t, \]

where \( y_t \) represents the output gap, usually calculated as the deviation of the growth rate of output from its long-term trend, and \( \omega \) represents the deviation of the real wage from its “natural” level. As it can easily be observed the expected next period wage inflation does not influence in a direct manner the price inflation and viceversa, as in eqs. (13) and (14).

\(^{14}\)For comparison, in more elaborated New Keynesian models as e.g. in Woodford (2003, p.225), the joint evolution of wages and prices is described by the following two loglinear equations

\[ \hat{w}_t \quad w_{PC} \beta E_t[\hat{w}_{t+1}] + \beta_{wy} y_t - \beta_{w\omega} \ln \omega_t, \]

\[ \hat{p}_t \quad p_{PC} \beta E_t[\hat{p}_{t+1}] + \beta_{py} y_t + \beta_{p\omega} \ln \omega_t, \]
Keynesian approach in particular as will be shown in the next section.\textsuperscript{15}

The corresponding across-markets or reduced-form Phillips curves are given by (with $\kappa = 1/(1 - \kappa_{wp}\kappa_{pw})$):

\begin{align*}
\hat{w} &= \kappa \left[ \beta_{w(e)}(e - e_o) - \beta_{w(v)} \ln(v/v_o) + \kappa_{wp}(\beta_{p(y)}(y - y_o) + \beta_{pv} \ln(v/v_o)) \right] + \kappa_{pz} \kappa_{pw} g_z + \pi_c, \\
\hat{p} &= \kappa \left[ \beta_{p(y)}(y - y_o) + \beta_{pv} \ln(v/v_o) + \kappa_{pw}(\beta_{w(e)}(e - e_o) - \beta_{wv} \ln(v/v_o)) \right] + \kappa_{pw}(\kappa_{pw} - 1) g_z + \pi_c,
\end{align*}

which represent a considerable generalization of the conventional view of a single-market price PC with only one measure of demand pressure, namely the one in the labor market.

Note that for our current version of the wage-price spiral, the inflationary climate variable does not matter for the evolution of the labor share $v = w/(pz)$, whose law of motion is given by

\begin{equation}
\hat{v} = \hat{w} - \hat{p} - \hat{z} = \kappa \left[ (1 - \kappa_{pw}) f_w(e, v) - (1 - \kappa_{wp}) f_u(u, v) + (\kappa_{pz} - 1)(1 - \kappa_{pw}) g_z \right]. \tag{20}
\end{equation}

mit

\begin{align*}
f_w(e, v) &= \beta_{w(e)}(e - e_o) - \beta_{wv} \ln(v/v_o) \quad \text{und} \\
f_p(u, v) &= \beta_{pu}(u - u_o) + \beta_{uv} \ln(v/v_o)
\end{align*}

Eq.(20) shows the ambiguity of the stability property of the real wage channel discussed by Rose (1967) which arises if despite of the incorporation of specific measures of demand and cost pressure on both the labor and the goods markets, the dynamics of the employment rate and the the output gap are linked and if inflationary cross-over expectations are incorporated in both Phillips curves. Indeed, as discussed for example in Proaño et al. (2006), a real wage increase can act itself in a stabilizing or destabilizing manner, depending on whether the dynamics of the capacity utilization rate depend positively or negatively on the real wage (i.e. if consumption reacts more strongly than investment or viceversa) and whether price flexibility is greater than nominal wage flexibility with respect to its own demand pressure measure.

Concerning the evolution of the overall inflationary expectations among the economic agents in the model economy, we follow Franke et al. (2006) and assume that

\textsuperscript{15}For a detailed comparison of our modelling approach to the New Keynesian alternative see Chiarella et al. (2005).
the dynamic behavior of the inflationary climate is described by

$$\dot{\pi}_c = \beta_{\pi_c} [\kappa_{\pi_c} (\hat{p} - \pi_c) + (1 - \kappa_{\pi_c}) (\pi_o - \pi_c)],$$

where $\beta_{\pi_c}$ is an adjustment speed parameter and $\kappa_{\pi_c}$ a weight parameter between 0 and 1, and $\pi_o$ is to be thought of as the target rate of inflation of the central bank, which is assumed to be known by the public. The degree of their confidence vis-à-vis the trend-chasing adaptive expectations component is measured by $(1 - \kappa_{\pi_c})$, which can also be referred to as the central bank’s credibility.

### 3.4 Monetary Policy

As standard in the actual theoretical literature, we do not focus on the level of money supply as the policy variable of the monetary authorities but instead use the nominal interest rate as the policy instrument. Following Svensson (1998), for example, we model the dynamics of the nominal short term interest rate through a law of motion of a Taylor rule type. Indeed, as Romer (2000, p.154-55) states, “Even in Germany, where there were money targets beginning in 1975 and where those targets paid a major role in the official policy discussions, policy from the 1970s through the 1990s was better described by an interest rate rule aimed at macroeconomic policy objectives than by money targeting.”

The target rate of the monetary authorities and the law of motion resulting from an interest rate smoothing behavior by the central bank are defined as

$$i_T = (i_o - \pi_o) + \hat{p} + \phi_p (\hat{p} - \pi_o) + \phi_y (y - y_o)$$

$$\dot{i} = \alpha_i (i_T - i).$$

The target rate of the central bank $i_T$ is here made dependent on the steady state real rate of interest $i_o - \pi_o$ augmented by actual inflation back to a nominal rate, and depends as usual on the inflation gap and the output gap. With respect to this target there is also an interest rate smoothing term with strength $\alpha_i$. Inserting $i_T$ and rearranging terms we obtain from this expression the following dynamic law for the nominal interest rate

$$\dot{i} = -\alpha_i (i - i_o) + \gamma_{ip} (\hat{p} - \pi_o) + \gamma_{iy} (y - y_o)$$

---

16 Even though the introduction of an inflationary climate term is not essential for the dynamics of the model, we introduce it now to facilitate the incorporation of open economy effects, as it is done in the companion paper to this one, Proaño (2007a). There, the inflationary climate term refers to the CPI inflation, and therefore comprises the effects of imported goods prices on the dynamics of domestic inflation.

17 See also Clarida and Gertler (1997).

18 All of the employed gaps are measured relative to the steady state of the model, in order to allow for an interest rate policy that is consistent with it.
where we have $\gamma_i p = \alpha_i (1 + \phi p)$, i.e., $\phi p = \gamma_i p / \alpha_i - 1$ and $\gamma_i y = \alpha_i \phi y$. Furthermore, the actual (perfectly foreseen) rate of inflation $\hat{p}$ is used to measure the inflation gap with respect to the inflation target $\pi_o$ of the central bank. Note finally that we could have included (but have not done this here yet) a new kind of gap into the above Taylor rule, the labor share gap, since we have in our model a dependence of aggregate demand on income distribution and the labor share, since the state of income distribution matters for the dynamics of our model and thus should also play a role in the decisions of the central bank.

### 3.5 The 4D Dynamical System

Taken together, our theoretical model consists of the four laws of motion, which together form the following autonomous 4D dynamical system

\[
\dot{e} = \mu \left( 1 - (1 - \rho) e \right) \left( \exp(y^d)^{1/\alpha} - (1 - \rho) e \right)^{1 - \rho} - \rho e \tag{25}
\]

\[
\dot{i} = -\alpha_i (i - \hat{i}_o) + \gamma_i p (\hat{p} - \pi_o) + \gamma_i y \tag{26}
\]

\[
\dot{v} = \kappa [(1 - \kappa_{pw}) f_w(e, v) - (1 - \kappa_{wp}) f_u(u, v) + (\kappa_{wz} - 1)(1 - \kappa_{pw}) g_z] \tag{27}
\]

\[
\dot{\pi}_c = \beta \pi_c \left[ \kappa_{\pi_c} (\hat{p} - \pi_c) + (1 - \kappa_{\pi_c}) (\pi_o - \pi_c) \right] \tag{28}
\]

with

\[
y^d = \alpha_y y - \alpha_{yt}(i - \hat{p} + (i_o - \pi_o)) - \alpha_{yv}(v - v_o) \]

\[
y = f_y(e)
\]

and $\hat{p}$, according to eq. (19), to be inserted in several places.

The Jacobian of the 4D dynamic system (which comprises the first partial derivatives of the dynamical endogenous variables), calculated at the interior steady state described in the previous section, is characterized by the following sign structure:

\[
J = \begin{pmatrix}
\partial \dot{e} / \partial e & \partial \dot{e} / \partial i & \partial \dot{e} / \partial v & \partial \dot{e} / \partial \pi_c \\
\partial \dot{i} / \partial e & \partial \dot{i} / \partial i & \partial \dot{i} / \partial v & \partial \dot{i} / \partial \pi_c \\
\partial \dot{v} / \partial e & \partial \dot{v} / \partial i & \partial \dot{v} / \partial v & \partial \dot{v} / \partial \pi_c \\
\partial \dot{\pi}_c / \partial e & \partial \dot{\pi}_c / \partial i & \partial \dot{\pi}_c / \partial v & \partial \dot{\pi}_c / \partial \pi_c 
\end{pmatrix} = \begin{pmatrix}
\pm & - & \pm & + \\
+ & - & \pm & + \\
\pm & 0 & - & 0 \\
+ & 0 & \pm & - 
\end{pmatrix}.
\]

This representation of the interaction between the different variables within the economy by means of partial derivatives allows us to examine in more detail the different channels through which the different variables act in a stabilizing or destabilizing manner.
Due to the formulation of our model and the role that labor market frictions play in it, the labor markets affect the output determination in a twofold manner: In the first place, through the restrictions they impose concerning the level of output the firms can actually produce, and in the second place, due to the effect they have on the reduced form of the price Phillips curve described by eq. (19). The qualitative direction of this influence, nevertheless, is unambiguously positive. In contrast, as the dynamics of the labor markets and more specifically of the employment rate are formulated, the effect of the employment rate on its own rate of change is ambiguous (∂\dot{e}/∂e): On the one side, a high level of macroeconomic activity (high employment rate) influences \dot{e} positively, but on the other hand, due to the specification of the matching process of labor, the level of the employment rate itself affects the tightness of the labor market, decreasing therefore the rate at which workers and vacancies are matched.  

The rate of change of the employment rate is also influenced (through the aggregate goods and labor demand) by the nominal (and real) interest rate, as well as by the wage share (the real wage), whose influence we assume here to be negative (due its cost effect on the firms’ profits and investment), but could also be positive if alternatively the income effect on consumption turns out to be predominant. Concerning the real interest rate, note that, through our formulation of the reduced form Phillips curve equation to be inserted in eq. (25), price inflation in determined not only by the goods, but also by the labor market situations and the weighting coefficients of both wage and price Phillips curves equations concerning the cross-over expectations mechanism. We thus have here a quite more complex (and realistic) theory for the price inflation dynamics and its interactions with the real side of the economy than other theoretical approaches.

As our model is formulated, it also features additional potentially (at least partially) destabilizing feedback mechanisms concerning the influence of the wage share on aggregate demand, employment and output (in this order), due to the presence of the Mundell-effect in the dynamics of the goods-market and the opposing Blanchard-Katz error correction terms in the reduced form price Phillips curve given by eq. (19). As formulated there, the (log of the) labor share, the Blanchard-Katz error correction terms, affects aggregate price inflation and the inflationary expectations in an ambiguous manner, through its opposing influence on the structural wage and price Phillips curve equations given by eq. (13) and (14). Note that since the net effect of the Blanchard-Katz terms on aggregate price inflation depends on the values of κ_{wp},

\[19\text{In the next section we will show that the relative size of these effects is central for the stability of the system.}\]

\[20\text{See Chiarella et al. (2005), as well as Proaño et al. (2006) and Franke et al. (2006) for a detailed discussion of the Rose real wage channel.}\]
and $\kappa_{pw}$, the cross-over expectation formation by the economic agents determines to a greater extent the direction of the labor share’s influence on output.

Concerning the dynamics of the labor share ($\frac{\partial \dot{v}}{\partial e} < 0$), the joint, net effect of the goods and labor market dynamics depends on the signs and values of the parameter estimates of the two structural Phillips curves and therefore, again, on the cross-over expectations formation of the economic agents. On the contrary, the influence of the (log of the) wage share on its rate of growth is unambiguously negative, according to eq.(20).

Additionally to these channels, our model also incorporates the Mundell inflationary expectations channel, which affects positively the dynamics of all other dynamic variables of the system through its positive influence on price inflation, as well as on wage inflation and from there on employment rate and the output gap, as it can be observed in the last column of the Jacobian of the 4D dynamical system. This unambiguous property of the Mundell expectations channel, discussed extensively in Chiarella et al. (2005), is more destabilizing the higher $\beta_\pi$ is. These and the remaining feedback channels and interactions of our theoretical D(isequilibrium)AS-AD model are sketched in figure 1.

3.5.1 Steady State Solution

We now determine the unique steady state of the 4D dynamical system in a sequential manner as follows. Concerning the goods markets, these are in equilibrium when
\( i = i_o = 0, \hat{p} = \pi_o \) and \( y^d = y = 0 \). Inserting this values in the dynamic law of the employment rate delivers
\[
\dot{e} = \mu [1 - (1 - \rho)e]^{\rho} [1 - (1 - \rho)e]^{1 - \rho} - \rho e \\
= \mu (1 - (1 - \rho)e) - \rho e
\]
since \( \exp(0)^{1/\alpha} = 1 \). For \( \dot{e} = 0 \), we obtain
\[
e_o = \frac{\mu}{(1 - \rho)\mu + \rho}.
\]  
(29)

If \( e = e_o \), the labor markets are in equilibrium and do not exert any pressures on the dynamics of wages and prices.

As it can be easily observed, the steady state rate of employment is determined purely by structural factors concerning the labor markets, namely the labor separation rate \( \rho \) and the matching technology \( \mu \), with
\[
\frac{\partial e_o}{\partial \rho} = -\mu [(1 - \rho)\mu + \rho]^{-2}(1 - \mu) < 0 \\
\frac{\partial e_o}{\partial \mu} = \frac{[(1 - \rho)\mu + \rho]^{-1} - \mu (1 - \rho)[(1 - \rho)\mu + \rho]^{-2}}{[(1 - \rho)\mu + \rho]^{-1} - \mu (1 - \rho)[(1 - \rho)\mu + \rho]^{-2}} = \frac{\rho}{[(1 - \rho)\mu + \rho]^2} > 0
\]

Note that these factors not only influence the level of the steady state employment rate, but also the dynamic response of the labor markets to exogenous shocks, as shown in figure [2].

As shown there, the higher the value of the matching technology parameter \( \mu \), the higher is the response of the employment rate to an exogenous labor demand shock, and the quicker is the return of the employment rate to its steady state level (which in turn also depends on the value of \( \mu \)). Ceteris paribus, a low matching technology leads thus to a low response of the actual employment to labor demand shocks that is also more persistent, a result which is in line with other studies featuring aggregate matching functions in the labor markets, see e.g. Amable and Ernst (2006).

Concerning the dynamics of wages and prices at the steady state, it should be clear that in equilibrium none of them is confronted to any demand pressures either from the labor nor from the goods markets.

### 3.5.2 Reduced 3D-Feedback Guided Stability Analysis

Having explicitly defined the unique steady state of the economy, we now turn to the analysis of the local asymptotic stability properties of the interior steady state
Figure 2: Employment rate response to an exogenous 1% labor demand shock in $t = 2$ for different values of the matching technology $\mu$

of the 4D dynamical system given by eqs. (25)-(28) (with eqs. (11), (12) and (19) inserted wherever needed) through partial considerations from the feedback chains that characterize this empirically oriented baseline model of Keynesian dynamics.

We have employed reduced-form expressions in the above system of differential equations whenever possible. We have thereby obtained a dynamical system in four state variables that is in a natural or intrinsic way nonlinear (due to its reliance on growth rate formulations). We note that there are many items that reappear in various equations, or are similar to each other, implying that stability analysis can exploit a variety of linear dependencies in the calculation of the conditions for local asymptotic stability.

In order to focus more specifically on the role of the labor market friction for the stability of the economy, we decouple the dynamics of the inflation expectations and their influence on the rest of the system by setting $\beta_{\pi_c} = 0$, reducing so our dynamical system by one dimension.\[21\]

According to the Routh-Hurwitz stability conditions for a 3D dynamical system, asymptotic local stability of a steady state is fulfilled when

$$a_i > 0, \quad i = 1, 2, 3 \quad \text{and} \quad a_1a_2 - a_3 > 0,$$

\[21\]Note that in this case the inflationary expectations become static, with the growth rate of labor productivity $g_\ell$ entering in both wage and price Phillips curve equations with a weight coefficient equal to one.

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where \( a_1 = -\text{trace}(J) \), \( a_2 = \sum_{k=1}^3 J_k \) with
\[
J_1 = \begin{bmatrix} J_{22} & J_{23} \\ J_{32} & J_{33} \end{bmatrix}, \quad J_2 = \begin{bmatrix} J_{11} & J_{13} \\ J_{31} & J_{33} \end{bmatrix}, \quad J_3 = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix}.
\]

and \( a_3 = -\det(J) \). Our reduced 3D dynamical system is stable around its interior steady state, if following propositions are fulfilled:

**Proposition 1:**

Assume the validity of the inequality
\[
|\mu(1 + \rho) + \rho| > (1 - \varrho) \mu \left( \frac{\alpha \alpha_y}{\epsilon_o} + \frac{\alpha \gamma_i}{\epsilon_o} \right),
\]
that assures the unambiguous stability of the employment rate dynamics at least with respect to its own influence. Then: The trace of the implied 3D dynamical system (assuming \( g = 0 \) for simplicity)
\[
\dot{e} = \mu (1 - (1 - \rho) e)^\varrho \left( \exp(y^d)^{\alpha} - (1 - \rho) e \right)^{1-\varrho} - \rho e,
\]
\[
\dot{i} = -\alpha_i (i - i_o) + \gamma_i p (\beta - \pi_o) + \gamma_i y f_e(e)
\]
\[
\dot{v} = \kappa [(1 - \kappa_{pw})(\beta_{we}(e - e_o) - \beta_{uv} \ln(v/v_o)) - (1 - \kappa_{wp})(\beta_{pa} f_y(e) + \beta_{pw} \ln(v/v_o))]
\]
around its interior steady state is unambiguously negative. 

**Sketch of Proof:**

Evaluated at the unique steady state, the 3D Jacobian of the system is given by
\[
\begin{pmatrix}
-\rho + \mu(\rho - 1) + (1 - \varrho) \mu \left( \frac{\alpha \alpha_y}{\epsilon_o} + \frac{\alpha \gamma_i}{\epsilon_o} \right) - \frac{\alpha \gamma_i}{\epsilon_o} (1 - \varrho) \mu \left( \frac{\beta_{pw} - \beta_{we} \kappa_{pw}}{\epsilon_o} \right) \\
\frac{\alpha \gamma_i}{\epsilon_o} + \gamma_i p \kappa \left( \frac{\alpha \beta_{pw}}{\epsilon_o} + \beta_{we} \kappa_{pw} \right) - \alpha_i \gamma_i p \kappa \left( \frac{\beta_{pw} - \beta_{we} \kappa_{pw}}{\epsilon_o} \right) \\
\kappa (1 - \kappa_{pw}) \left( \beta_{we} - \frac{\alpha \beta_{pw}(1 - \kappa_{wp})}{\epsilon_o} \right) 0 \kappa (1 - \kappa_{pw}) \left( -\beta_{uv} - \beta_{pw} (1 - \kappa_{wp}) \right)
\end{pmatrix}
\]

Under Proposition 1, which assures a stable dynamic behavior of the employment rate with respect to its own dynamics, the sign structure of the Jacobian is
\[
\begin{pmatrix}
- & - & \pm \\
+ & - & \pm \\
\pm & 0 & -
\end{pmatrix},
\]
and \( \text{tr}(J) < 0 \) unambiguously holds.
Proposition 2:

Assume that $|\alpha_{yv}| > \alpha_{yi} (\frac{\beta_{pv} - \beta_{wu} \kappa_{pw}}{v_o})$, and additionally, that $\beta_{we} \gtrless \beta_{py}$ hold. Under these conditions, $a_2 > 0$, the second Routh-Hurwitz local stability condition for a 3D dynamical system,

$$a_2 = \begin{vmatrix} J_{22} & J_{23} \\ J_{32} & J_{33} \end{vmatrix} + \begin{vmatrix} J_{11} & J_{13} \\ J_{31} & J_{33} \end{vmatrix} + \begin{vmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{vmatrix},$$

is always fulfilled.

Sketch of Proof:

Under the validity of Proposition 1, but without the need of further assumptions, $J_1$ and $J_3$ in are unambiguously positive. If Proposition 2 additionally holds, then

$$J_2 = \left[ \mu(1 - \rho) + \rho - (1 - \varrho)\mu \left( \frac{\alpha_{yy} + \alpha_{yi} \kappa (\beta_{pv} e_o + \beta_{wu} \kappa_{pw})}{\alpha} \right) \right],$$

$$\left( \frac{\beta_{we} + \beta_{pw}(1 - \kappa_{wp})}{v_o} \right) - \frac{(1 - \varrho)\mu}{\alpha} \left( -\alpha_{yve} + \alpha_{yi} \kappa \left( \beta_{pv} - \beta_{wu} \kappa_{pw} \right) v_o \right),$$

and the sum of the minors of order 2, $a_2$, is positive.

Under the validity of Propositions 1 and 2, the full set of Routh-Hurwitz conditions are always fulfilled, since $\det J = -a_3$ is unambiguously negative. In this case, the interior steady state reduced 3D dynamical system is locally stable.

4 Model Calibration and Stochastic Simulation

After having identified the stability conditions of the dynamical system, we analyze now the effects of labor market frictions for the dynamics of the system after different types of shocks by means of computer simulations. To do so, we use a discrete time version of the 4D continuous time system discussed in the previous section, using nevertheless alternative labor market specifications to the search framework described in section 3 in order to evaluate the dynamic properties of the model. On the one hand, we analyze a scenario where the labor markets function frictionless,
so that $L_t^D = N_t$ and $Y_t^D = Y_t$ always hold (and the dynamic law of motion of the employment rate based on the aggregate matching function is replaced simply by the production technology equation). On the other hand, we model the link between the goods and labor markets by means of a dynamic version of Okun’s Law, as done in Asada et al. (2006), Proaño et al. (2006) and Franke et al. (2006), according to which the growth rate of the employment rate is determined by

$$
\dot{e}_t = \alpha_{ey1} \dot{y}_{t-1} + \alpha_{ey2} \dot{y}_{t-2} + \alpha_{ey3} \dot{y}_{t-3}.
$$

(30)

The parameter values used in the different specifications are shown in Table 1.

<table>
<thead>
<tr>
<th>Table 1: Baseline calibration parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Goods Markets</strong></td>
</tr>
<tr>
<td>-------------------</td>
</tr>
<tr>
<td>Wage Phillips Curve</td>
</tr>
<tr>
<td>Price Phillips Curve</td>
</tr>
<tr>
<td>Monetary Policy Rule</td>
</tr>
<tr>
<td>Labor Markets</td>
</tr>
<tr>
<td>-------------------</td>
</tr>
<tr>
<td>$0.7$</td>
</tr>
<tr>
<td>$0.210$</td>
</tr>
<tr>
<td>$0.7$</td>
</tr>
<tr>
<td>$0.70$</td>
</tr>
</tbody>
</table>

**Labor Markets** In the baseline scenario I set the matching technology factor $\mu = .6$ (Christoffel and Linzert (2006b) choose $\mu = 0.4$). For the job separation rate $\rho$ (exogenous in our model), a value of 0.1 was chosen, which is consistent with the empirical findings (on quarterly frequency) by Hall (1995), Hall (2005), Shimer (2005). For the choice of the Cobb-Douglas parameter of the the search and matching function $\varrho$, I set a lower value ($\varrho = 0.2$) than Walsh (2005), who sets this parameter equal to 0.4 (Setting a value of $\varrho = 0.4$ does not change the qualitative reactions of the model, but delivers too low responses compared with the scenario featuring Okuns’ Law). Note that in the scenarios featuring the labor search function the steady state employment rate is not one, as in the other two scenarios, but is instead determined by the structural labor markets parameters according to eq. (29).

**Goods Markets** For the choice of the parameters $\alpha_{yy}$, $\alpha_{yi}$ and $\alpha_{yw}$, we rely on the system GMM parameter estimates of Proaño et al. (2006) and Franke et al. (2006), which are consistent with other studies as Goodhart and Hofmann (2005) who also perform system GMM estimations of Phillips curve and IS equations, using nevertheless also expected values of future variables.

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Wage-Price Dynamics Concerning the parameters in the wage and price Phillips curve equations (assumed to be equal across all labor market specifications), we use the empirical parameter estimates obtained for the U.S. economy by Proaño et al. (2006) and Franke et al. (2006) using system GMM estimation techniques. These estimated parameter values, even though obtained by means of the GMM methodology, are consistent with related studies on wage and price inflation dynamics as Chen and Flaschel (2006) and Flaschel and Krolzig (2006).

Monetary Policy Following Taylor (1999) and all the related literature on monetary policy rules, we set the responsiveness of the short term interest rate to the inflation gap equal to 1.5, and to the output gap, 0.5, and we set the interest rate smoothing parameter value equal to 0.7 as in Walsh (2005) in order to take into account the high degree of inertia observable in nominal interest rate time series data.24

In order to evaluate the empirical plausibility of our theoretical framework under this parameter calibration we simulated the model over 1000 quarters, assuming that in each quarter the economy is hit by an aggregate demand, a labor productivity and a monetary policy shock. For the first two shocks we assume a first order autoregressive process with an autoregressive parameter of $\rho_y = \rho_z = 0.7$, as usually done in the literature, see e.g. Smets and Wouters (2003).

Based on the estimations by Juillard, Karam, Laxton and Pesenti (2006), we set the standard deviations of output, labor productivity and the nominal interest rate equal to $\sigma_z = 0.0089$, $\sigma_z = 0.0039$ and $\sigma_z = 0.0039$, respectively (we assume though normally distributed shocks, while Juillard et al. (2006) assume that these are inverse gamma distributed).

Table 2: Actual (1980: 1–2005: 4) and Simulated Standard Deviations (in %)

<table>
<thead>
<tr>
<th></th>
<th>Output</th>
<th>Employment Rate</th>
<th>Price Inflation</th>
<th>Wage Inflation</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S.</td>
<td>2.2</td>
<td>1.5</td>
<td>0.525</td>
<td>0.075</td>
</tr>
<tr>
<td>Euro Area</td>
<td>1.7</td>
<td>1.4</td>
<td>0.650</td>
<td>0.850</td>
</tr>
<tr>
<td>Model</td>
<td>1.9</td>
<td>2.4</td>
<td>0.600</td>
<td>0.610</td>
</tr>
</tbody>
</table>

Given the rather parsimonious formulation of my model and the fact that in our simulations only three types of shocks (compared with the eight of Juillard et al.

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24As discussed in Rudebusch and Wu (2003) and Franke et al. (2006), there is also some empirical and theoretical arguments supporting the modelling of monetary policy impulses as autoregressive processes due, for example, the uncertainty of the monetary authorities at time $t$ concerning the actual state of the economy. We choose nevertheless to model the interest rate inertia by means of a smoothing parameter in the Taylor rule, in order to remain consistent with the theoretical framework of the previous section.
are assumed to hit the economy in each period, the performance of the model concerning second moments seems to be quite acceptable, even though precisely the volatility of the employment rate seems somewhat high when compared with actual data.

5 Dynamic Adjustments under Different Labor Market Schemes

We analyze the dynamic properties of our model by simulating the responses of the economy with respect to two different types of exogenous shocks: a monetary policy shock and an aggregate demand shock. The underlying time unit is a quarter, but to ease the interpretation of the simulation results the data is annualized.

Figure 3 shows the dynamic response of our artificial economy to a 1% increase in the nominal interest rate. As it can observed, our model delivers quite reasonable qualitative responses which are furthermore also in line with the predicted reactions of other macroeconomic models with labor frictions, such as Walsh (2003) and Walsh (2005), even though our approach does not rely on intertemporal utility and profit maximizing behavior by households and firms assumed there, nor on the highly questionable Calvo (1983) staggered wage and price setting behavior.

Figure 3: Impulse-response functions to a 1% monetary shock for different employment rate dynamic adjustment mechanisms (annualized inflation rates, in quarters, percent values): The solid line represents the baseline calibration with labor market frictions and a matching technology factor of $\mu = .6$. The dashed line represents the scenario with $\mu = .1$, that is, with a lower degree of labor market rigidity. The dotted line shows the case where the labor markets function frictionless, and the dashed-dotted line represents the specification with a dynamic Okun’s law.
As expected, after an increase in the nominal interest rate, both output and employment decrease, leading also to a fall in wage and price inflation. The reaction of the wage share, on the contrary, is much more complex, because it depends strongly not only on the relative sizes of the output and employment growth but also on the weighting parameters in the wage and price Phillips curve equations. However, we observe a uniform negative response of the labor share in all labor market scenarios, with the perfectly flexible labor markets scenario, as expected, showing the largest response. Concerning specifically the role of labor market frictions for the dynamic response of the economy to shocks, as the impulse response functions in figure 3 show, the more rigid the labor markets are the weaker is the responsiveness of employment (and output) to exogenous shocks. This, in turn, affects the reaction of price and wage inflation to exogenous developments and therefore also the reaction of the wage share dynamics, as discussed in the previous section.

The second scenario we study is the response of the economy to an aggregate demand shock, say, due to an expansionary fiscal policy impulse (shown in figure 4). Again, the aggregate variables of our model deliver quite reasonable dynamic responses, with employment and output increasing in response to the higher aggregate demand, and price and wage inflation, as well as the nominal interest rate, following the former variables (the response of the nominal interest rate is due to the Taylor rule specification, obviously). Note again that the degree of labor market rigidity plays a key role for the extent as well as for the persistence of the impulse responses generated by the model, and that the procyclical reaction of the labor share is by no means trivial but rather the result of the formulation of the cross-over expectations.
in both wage and price Phillips Curves.

Finally, we simulate again the reaction of the economy to a monetary policy shock for the case of higher credibility of the price stability commitment (represented by a lower value of $\kappa_{\pi_c}$ in the inflation climate adjustment mechanism) by monetary authorities.

Figure 5: Impulse-response functions to a 1% monetary shock with higher credibility.

By comparing the dynamic responses of the economy depicted in figures 3 and 5, we find that in the higher credibility scenario the duration and extent of such reactions are lower than in the alternative case, where the inflation climate depends on a more significant manner on the past inflation rates than on the actual inflation target of the monetary authorities. This result is quite intuitive, showing that higher inflation inertia leads to an overall slower adjustment of all variables of the economy to exogenous and endogenous shocks.

6 Output Stabilization and Monetary Policy Rules

After the analysis of the dynamic properties of the model under different labor market-schemes and degrees of search efficiency we focus now on the role of monetary policy and more specifically on the monetary policy targets for the dynamics of the economy.

In order to investigate the dynamic response of the model economy to a monetary policy shock under different monetary policy rules, we simulate the reaction of the model under different rules, whereas as the benchmark rule we use Taylor’s (1993)
The alternative monetary policy rules as well as the corresponding target weights values are shown in Table 3.

Table 3: Alternative Monetary Policy Rules: Weighting Parameters

<table>
<thead>
<tr>
<th>Rule</th>
<th>II. Strict Inflation Targeting</th>
<th>III. Flexible Inflation Targeting with Employment Target</th>
<th>IV. Flexible Inflation Targeting with Wage Inflation Target</th>
<th>V. Flexible Inflation Targeting with Wage Share Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weights</td>
<td>φ̂_p = 2, φ_y = 0</td>
<td>φ̂_p = 1, φ_y = 0.5</td>
<td>φ̂_p = 1, φ_y = 0.5</td>
<td>φ̂_p = 1, φ_y = 0.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>φ̂_p = 1, φ_y = 0.5</td>
<td>φ̂_p = 1, φ_y = 0.5</td>
<td>φ̂_p = 1, φ_y = 0.5</td>
<td>φ̂_p = 1, φ_y = 0.5</td>
</tr>
<tr>
<td></td>
<td>φ̂_p = 0, φ_y = 0.5</td>
<td>φ̂_p = 0, φ_y = 0.5</td>
<td>φ̂_p = 0, φ_y = 0.5</td>
<td>φ̂_p = 0, φ_y = 0.5</td>
</tr>
<tr>
<td></td>
<td>φ̂_p = 0, φ_y = 0</td>
<td>φ̂_p = 0, φ_y = 0</td>
<td>φ̂_p = 0, φ_y = 0</td>
<td>φ̂_p = 0, φ_y = 0</td>
</tr>
</tbody>
</table>

The (absolute) cumulated dynamic responses of the model sketched in figure 6 deliver three important insights on the importance of the choice of the targets and the relative weighting values in objective function of the monetary authorities: In the first place they highlight the multi-dimensionality of monetary policy conduction that comes to light once the baseline New Keynesian model with (solely) price stickiness is abandoned and nominal wage as well as labor market rigidities are incorporated. As figure 6 and Table 3 clearly show, monetary policy rules that achieve an efficient stabilization of price inflation are not necessarily as effective concerning other variables. Indeed, it can be easily observed, Taylor’s (1993) original specification (I) and the flexible inflation targeting with nominal wage growth target (IV) outperform the other three rules (II) (strict inflation targeting), (III) (flexible inflation targeting with an employment target) and (V) (flexible inflation targeting with employment and with wage share target), concerning the overall extent of the dynamic reaction of the simulated variables.

In the second place, we find that the dynamic responses under rules I and IV (Taylor’s (1993) specification and flexible inflation targeting with a nominal wage inflation target) have an almost identical performance concerning the cumulated dynamic response of the economy: According to our model, a pure flexible inflation targeting rule with a weight φ̂_p = 1.5 is nearly equivalent to a flexible inflation targeting with weights φ̂_p = 1 and φ̂_p = 0.5 (both with φ_y = 0.5).
Figure 6: Cumulated impulse-response functions under alternative monetary policy rules.

Table 4: Cumulated Responses at Different Horizons

<table>
<thead>
<tr>
<th>Monetary Policy Rule</th>
<th>Output</th>
<th>Employment Rate</th>
<th>Price Inflation</th>
<th>Wage Inflation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>8 Quarters</td>
<td>16 Quarters</td>
<td>24 Quarters</td>
<td>32 Quarters</td>
</tr>
<tr>
<td>I</td>
<td>0.3191</td>
<td>0.3700</td>
<td>0.3855</td>
<td>0.3926</td>
</tr>
<tr>
<td>II</td>
<td>0.3215</td>
<td>0.3704</td>
<td>0.3945</td>
<td>0.4039</td>
</tr>
<tr>
<td>III</td>
<td>0.3246</td>
<td>0.3915</td>
<td>0.3970</td>
<td>0.4017</td>
</tr>
<tr>
<td>IV</td>
<td>0.3183</td>
<td>0.3686</td>
<td>0.3831</td>
<td>0.3899</td>
</tr>
<tr>
<td>V</td>
<td>0.3229</td>
<td>0.3923</td>
<td>0.3908</td>
<td>0.3959</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Employment Rate</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I</td>
<td>0.4273</td>
<td>0.4954</td>
<td>0.5162</td>
<td>0.5257</td>
</tr>
<tr>
<td>II</td>
<td>0.4305</td>
<td>0.4960</td>
<td>0.5283</td>
<td>0.5408</td>
</tr>
<tr>
<td>III</td>
<td>0.4346</td>
<td>0.5242</td>
<td>0.5315</td>
<td>0.5379</td>
</tr>
<tr>
<td>IV</td>
<td>0.4261</td>
<td>0.4936</td>
<td>0.5129</td>
<td>0.5220</td>
</tr>
<tr>
<td>V</td>
<td>0.4323</td>
<td>0.5119</td>
<td>0.5233</td>
<td>0.5301</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Price Inflation</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I</td>
<td>0.4095</td>
<td>0.5728</td>
<td>0.6291</td>
<td>0.6331</td>
</tr>
<tr>
<td>II</td>
<td>0.3255</td>
<td>0.5718</td>
<td>0.6149</td>
<td>0.6243</td>
</tr>
<tr>
<td>III</td>
<td>0.3265</td>
<td>0.5966</td>
<td>0.6775</td>
<td>0.6900</td>
</tr>
<tr>
<td>IV</td>
<td>0.3227</td>
<td>0.5709</td>
<td>0.6280</td>
<td>0.6318</td>
</tr>
<tr>
<td>V</td>
<td>0.3256</td>
<td>0.5874</td>
<td>0.6584</td>
<td>0.6667</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wage Inflation</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I</td>
<td>0.3552</td>
<td>0.5790</td>
<td>0.6277</td>
<td>0.6309</td>
</tr>
<tr>
<td>II</td>
<td>0.3577</td>
<td>0.5772</td>
<td>0.6125</td>
<td>0.6205</td>
</tr>
<tr>
<td>III</td>
<td>0.3590</td>
<td>0.6047</td>
<td>0.6773</td>
<td>0.6895</td>
</tr>
<tr>
<td>IV</td>
<td>0.3545</td>
<td>0.5770</td>
<td>0.6266</td>
<td>0.6300</td>
</tr>
<tr>
<td>V</td>
<td>0.3579</td>
<td>0.5947</td>
<td>0.6577</td>
<td>0.6661</td>
</tr>
</tbody>
</table>

Lastly, our dynamic simulations show that the strict inflation targeting rule (II), while the most effective in shortening the adjustment duration of the economy, is
not the most effective when it comes to the reduction in the response variability of all macroeconomic variables besides price inflation: Indeed, as shown in figure 6, due to the overshooting that takes place when only price inflation is targeted, the adjustment process of the economy after a monetary policy shock is much more volatile than in the alternative cases, a result which is in line with the theoretical considerations of Woodford (2003, ch.4).

In a related paper, though within a DSGE framework, Faia (2006) performs similar experiments which are all in all in line with our findings. There is though one important difference between her conclusions and ours: Since in her framework the evolution of the real marginal costs depend solely on unemployment, targeting the output gap is suboptimal towards targeting the unemployment gap, since the latter is the variable which comprises the source of the inefficiency in the economy. In our framework, though, the choice of the optimal strategy is not so straightforward since the dynamics of the labor share (the real marginal costs) are not only driven by the unemployment gap but also by the disequilibrium in the goods markets due to our cross-over specification in the wage-price dynamics. In fact, a strategy which targets both output and gap as rule IV seems to be less efficient than a Taylor rule with standard coefficients as rule I, as shown in figure 6.

7 Concluding Remarks

Despite of the high degree of technological process in the industrialized countries, their labor markets are and will probably also in the future be confronted to a variety of real imperfection which will hinder the complete clearing of the market in equilibrium. In this paper we studied the role of structural labor market frictions for the dynamics of the economy by incorporating in a theoretical framework in the line of Chen et al. (2006) a labor market module containing basic search-and-matching elements. As our model was formulated, the degree of labor market rigidity affected not only the dynamics of the employment rate, but also of the output gap (and consequently also wage and price inflation) through the restrictions it imposed on the ability of firms to find adequate workers and serve aggregate demand.

This straightforward modification of the baseline (D)AS-AD framework delivered some interesting results concerning the dynamic responses of the economy to various exogenous shocks. On the one hand, we found that the degree of rigidity in the labor markets has an important effect on the dynamics of output and inflation: The more rigid the labor markets are, the smaller is the response of employment, output and inflation to exogenous shocks. On the other hand, though still concerning this first result, we found that the dynamic response of the labor share, which in turn
influences the aggregate demand as well as price inflation, depends dramatically on the relative responses of production and employment to shocks, and therefore on how and through which channels shocks are transmitted within the economy. Since nowadays the predominant view (represented also by the DSGE approach) is that the real marginal costs (the labor share) are the main force driving price inflation, we think that our approach delivers an interesting alternative to the DSGE modelling approach, which considers the real marginal costs as being determined primarily by intertemporal profit maximization under imperfect competition.

Concerning the role of monetary policy, our dynamic simulations show that a flexible inflation targeting rule in the line of Taylor (1993), where price inflation as well as output are targeted, and/or a flexible inflation targeting rule with an additional wage inflation target have a better performance than flexible inflation targeting rules where employment or the wage share besides the output gap are targeted, or strict inflation targeting where solely the inflation gap is targeted.

On more real world-related grounds, if one takes into account the significant differences in the characteristics of the labor markets across the EMU countries, the findings of this paper might deliver some interesting insights on the recent inflation developments in those economies, with countries as Germany or Austria with persistently low inflation rates compared to other countries such as Spain and Ireland, as discussed for example in Honohan and Lane (2003), Angeloni and Ehrmann (2004), Fritsche, Logeay, Lommatzsch, Rietzler, Stephan and Zwiener (2005) and Proaño (2007b). In order to analyze the implications of labor market rigidity for as well as the effectiveness of alternative monetary policy rules in a monetary union with heterogenous national labor market characteristics such as the EMU, in a companion paper (Proaño 2007a) I incorporate to the theoretical framework developed here open economy issues such as the import price inflation, relative competitiveness and foreign aggregate demand.
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