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HOW LARGE ARE HYSTERESIS EFFECTS? ESTIMATES FROM A KEYNESIAN GROWTH MODEL

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ABSTRACT

This paper estimates a demand-led model of macroeconomic growth and fluctuations in which the growth rate of the economy's supply side converges to the growth rate of demand. Convergence happens because labor supply and productivity growth respond to the degree of slack in the economy. Faster demand growth reduces slack and stimulates supply (and vice-versa). We estimate the model using simulated method of moments and find statistically significant and quantitatively important hysteresis effects: the semi-elasticity of productivity and labor supply to the unemployment rate are 0.73 and 0.26, respectively. For an economy with labor market slack, these estimates imply that supply growth could accommodate a one percentage point increase in the growth rate of demand with a reasonable 0.75 percentage point reduction in the long-run unemployment rate. Additionally, we show the model replicates major features of business cycles as well the response of the economy to autonomous demand shocks, providing further validation of this approach to understanding macroeconomic dynamics.

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How large are hysteresis effects? Estimates from a Keynesian growth model*

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1 Introduction

What is the role of demand in driving macroeconomic activity? No one would deny demand is necessary to motivate production in a market economy; what is not sold, or at least expected to be sold, will not be produced. However, mainstream macroeconomic models since (at least) Modigliani (1944) relegated the role of demand to the short run, asserting that endogenous market-driven adjustments of wages and prices assure sufficient demand to purchase full-employment, potential output. These adjustments may not be instantaneous, and therefore autonomous fluctuations in aggregate demand may affect real variables for some time, but the long-run path of an economy is typically understood as independent of direct effects of spending choices, driven instead entirely by the supply side.

This perspective changed to some extent in the last two decades of the 20th century. In "new consensus" models, wise monetary policy replaced nominal adjustment as the primary instrument to restore aggregate demand to potential levels. This change elevated the importance of activist policy relative to "natural" market forces in generating convergence of output to a supply-driven path. But this perspective still relegates to the role of autonomous demand dynamics to the short run, a period now defined by the speed of the transmission of monetary policy to spending decisions.

The events following the Great Recession challenged this mainstream consensus, forcing macroeconomists to rethink the role of aggregate demand in shaping the medium- to long-run performance of an economy. In the US, output failed to regain its pre-recession trend at any point in the approximately 12 years from the beginning of the Great Recession to the COVID-19 pandemic lockdown. Some analysts, interpreting this disappointing outcome through the conventional new consensus lens, looked for some kind of a supply shock, independent of the demand collapse that caused the Great Recession (Fernald, 2015; Cette, Fernald, and Mojon, 2016). But another interpretation emerged raising the concern that weak demand can constrain the economy beyond the short run. Summers (2014), harking back to Alvin Hansen, labeled this phenomenon "secular stagnation."

In this paper, we present a simple framework to empirically study the role that autonomous aggregate demand—components of demand independent of current economic conditions—plays in shaping medium- to long-term economic performance. Our model is based on the demand-led growth model from Fazzari, Ferri and Variato (2020, hereafter FFV). The demand side of this model follows the "supermultiplier" approach pioneered by Serrano (1995) and developed in a variety of recent Post-Keynesian contributions.[1] The main innovation in FFV is an explicit model of an endogenous supply side that can accommodate a demand-led growth path. Supply accommodates demand growth (not just demand levels) through hysteresis effects. Stronger demand reduces slack in the economy, measured in the model by the unemployment rate, and induces faster growth of both labor supply and labor productivity.

We estimate this Keynesian growth model through a minimum distance estimator and assess its empirical performance along three dimensions. First, we ask whether it can match the volatility, persistence and co-movements of seven macroeconomic variables at both business-cycle and lower frequencies. Second, we compare the estimated model impulse-response functions induced by an autonomous demand shock to their empirical counterparts. Third, we test whether the key hysteresis effects present in the model, which act through the adjustment of the steady-state unemployment rate to autonomous demand, are different from zero and quantitatively significant.

While there is an extensive literature on demand-led growth models - indeed, this is a pillar of Post-Keynesian Economics (Lavoie, 2014) - there is surprisingly little empirical work estimating an explicit demand-led, dynamic macroeconomic model and comparing its predictions to the data. Recent studies have tested several implications of supermultiplier growth models (Girardi and Pariboni, 2020; Haluska, Braga and Summa, 2021; Girardi et. al, 2020). To the best of our knowledge, however, we are the first to use this kind of model to study quantitatively business cycles and growth in a unified framework and to be explicit about the structural shocks driving the data.

Recent mainstream research analysing the persistent effects of aggregate demand slumps on economic performance usually requires a lower bound on the interest rate determined by monetary policy as the reason for demand effects beyond the short run (Benigno and Fornaro, 2018; Eggertsson, Mehrotra and Robbins, 2019; Anzoategui, Comin, Gertler and Martinez, 2019). We contend, however, that monetary policy may not be fully effective at restoring full employment and potential output even when short-term nominal interest rates are not pinned at zero. We test this hypothesis by considering the robustness of our primary findings of demand-induced hysteresis on supply when the data is limited to periods when the zero lower bound was not binding. Interestingly, we find that when we exclude the zero

[1] Early contributions to the resurgence of interest in supermultiplier models include Allain (2015), Fazzari et al. (2013), Freitas and Serrano (2015), and Lavoie (2016). This literature has grown rapidly in recent years.

lower bound period our estimates imply modestly stronger effects of aggregate demand on the supply side.

Our main results are as follows. First, we find that estimating the model through simulated method of moments yields results that match the data well. In particular, our model in which structural shocks come exclusively from demand captures co-movements and volatility of supply-side variables, employment and labor productivity, relative to output. In this respect, the model outperforms real business cycle models augmented with search and matching frictions. It also yields correlations between employment, labor productivity, and unemployment that are significantly closer to the data than neoclassical models. Second, when simulating structural impulse-response functions for our model with the estimated parameters, we reproduce, at least qualitatively, most of the responses to a plausibly exogenous autonomous demand shock over a decade-long horizon, as estimated by Girardi, et al. (2020). Third, we find strong evidence for the hysteresis effects that the model identifies as necessary for the supply side to converge to the demand side. Quantitatively, we estimate that a one percentage point increase in the long-run annual growth rate of demand can be accommodated by the endogenous response of the supply side with a less than one percentage point drop in the steady-state unemployment rate, as long as the unemployment rate remains above a minimum level. This result implies it is empirically reasonable for a persistent change in aggregate demand dynamics to pull the supply side along with it, in either a positive and negative direction.

Related Literature. Our paper connects with four strands of research. First, it relates to the empirical evaluation of the supermultiplier approach to growth and distribution. This literature has proceeded in three major ways; a first group of papers attempts to establish empirical candidates for autonomous demand, and seeks to show that autonomous demand Granger causes output, as implied by the model as well as testing that autonomous demand and output are co-integrated, which is also implied by the model.[2] A second group of papers seeks to show that the investment share responds strongly and positively to an increase in the growth rate of autonomous demand, another important implication of the supermultiplier model.[3] A third set of papers estimates the effects of autonomous demand shocks on output and other macroeconomic variables, either by the use of local projections (Girardi et. al, 2020) or structural vector autoregression with recursive restrictions (Goes and Deleidi, 2022).

[2] See Girardi and Pariboni (2016); Perez-Montiel and Erbina (2020); Perez-Montiel and Manera(2022); and Perez-Montiel and Pariboni(2022)

[3] See Girardi and Pariboni (2020); Perez-Montiel and Erbina (2020); and Haluska, Braga and Summa (2021).

While this work provides important support for the implications of the supermultiplier model, none of these papers attempt to estimate the underlying economic parameters; they all use reduced-forms to test implications of the model. Our paper contributes to this literature by providing explicit estimates of economically meaningful parameters which can be used to perform quantitative exercises and used to assess the stability conditions arising from these models. In addition, we are the first to compute explicit impulse-response functions in response to an autonomous demand shock, which can then be compared to those obtained from structural vector autoregressions or local projections.

A second strand of research models demand-induced technical change in Post-Keynesian growth models. The mechanisms we present linking the supply side to the unemployment rate need not be explicitly confined to a supermultiplier model; they have been incorporated in Neo-Kaleckian growth models, for example see Chapter 8 of Hein (2014) for a thorough discussion, and Tavani and Zamparelli (2018) for a broader analysis of induced technical change in Post-Keynesian growth models.[4] The empirical counterpart to this literature usually focuses on estimates of Verdoorn's Law, which is the reduced-form correlation between output growth and productivity growth. We contribute to this literature by estimating Verdoorn's coefficient in a general-equilibrium framework along with all other model parameters, and we show how Verdoorn's coefficient can be derived as a reduced-form outcome from the supply-side model estimated here. Our results imply a Verdoorn coefficient somewhat higher than what is found for OECD countries. All of the Verdoorn effect comes from the impact of the unemployment rate on productivity growth. We cannot reject the hypothesis of a zero effect from the feedback of capital accumulation on productivity growth, which is consistent with the findings of Hein and Tarassow (2010).[5]

The third strand of related literature combines insights from endogenous growth theory with mainstream New Keynesian models to produce long-lived effects of demand shocks when interest rates are constrained by a lower bound.[6] In these models, a decline in demand when nominal interest rates are constrained from falling induces a large drop in profitability, which reduces innovation and R&D expenditures, reducing productivity growth and leaving long-

[4]A deeper discussion of the role that the supply side plays in Post Keyenesian models is given in Skott (2018) and Skott (2019).

[5]Other related research uses Okun's Law to estimate how demand dynamics may affect the "natural" rate of growth of supply (Leon-Ledesma and Thirlwall, 2002) or the path of potential output (Fontanari et. al, 2020). Mason and Jayadev (2023) interpret how supply constraints affect the transition between demand-led growth paths emphasizing hysteresis effects in both labor supply and technical change.

[6]See Benigno and Fornaro (2018); Eggertsson, Mehrotra and Robbins (2019); Anzoategui, Comin, Gertler and Martinez (2019) and Greany and Walsh (2022)

lasting effects on output growth and permanent effects on the level of output. Our model produces these same long-lasting effects of demand shocks on productivity and output, but since interest rates and the price level are not incorporated in the model, it is silent on the issue of the lower bound for interest rates.[7] We explore this issue further by estimating our model on a sub-sample of data before the zero lower bound binds in the US, and we show that our hysteresis parameters remain economically and statistically significant in that period.

The fourth strand of literature makes use of identified vector autoregressions to asses whether demand shocks have long-run effects on output. Furlanetto et. al (2021) and Maffei-Faccioli (2021) use sign restrictions on structural vector autoregressions to identify demand and supply shocks, and show demand shocks have permanent effects on the level of output. Anzoategui and Kim (2021) use a structural vector autoregression derived from a New Keynesian model to estimate the output gap, and show demand shocks have long-lasting but temporary effects on the output gap, which is a limited form of hysteresis. The framework we present in this paper provides a simple way to account for long-lasting effects of demand shocks on output growth and the supply-side of the economy, consistent with this SVAR evidence. It also shows qualitatively similar impulse-response functions to autonomous demand shocks can be generated from a simple Keynesian model.

Outline The rest of the paper is organized as follows: Section 2 presents a simpler version of FFV, which we use to develop intuition and characterize how the model responds to autonomous demand shocks in the presence of hysteresis. Section 3 presents the full model we use for estimation, which is a stochastic version of FFV. Section 4 describes the data, our definition of autonomous demand, and the minimum distance estimator. Section 5 presents the main empirical results, including parameter estimates, a comparison between the moments generated by the model and those present in the data, and impulse-response functions. Section 6 concludes and outlines avenues for future research.

[7]Some Post-Keynesian research is skeptical about the ability of monetary policy to eliminate real effects of demand dynamics even if the interest rate is not constrained by a lower bound. See Fazzari (2020) for a summary of these arguments.

2 A simple demand-led growth model

This section presents a simple, stylized version of the model that delivers the key theoretical results: even in the long-run, macroeconomic performance depends on demand, shocks to demand can have permanent effects on output and productivity, and the supply side accommodates the growth rate of demand. In addition, the simple version allows us to explicitly characterise the impulse-response functions to an autonomous demand shock.

In this simple model, there is no capital and labor supply is fixed and normalized to 1. In addition, while the simple model incorporates the dynamics of the supply side, we assume resources constraints are not binding, in the sense that the unemployment rate remains above a minimum level, which implies output at any point in time is determined by demand. The more general model in the next section relaxes these assumptions. However, the key results derived here continue to hold in a more general environment.

2.1 Steady-state results

We work in discrete time. Consumption depends on income, which equals output:

$$C_t = (1-s)Y_t \tag{1}$$

Where s is the marginal propensity to save, C_t is consumption and Y_t is output. Demand also includes an *autonomous* component, Z_t, independent from current and past output. Related research has considered many possible sources of such exogenous demand growth. Military spending is a natural candidate, since it can be argued that such expenditures are de-coupled from national economic performance (Allain, 2015). In small open economies, another natural candidate is exports, since at least part of the secular trend in exports is explained by the rate of growth of the rest of the world (Nah and Lavoie, 2017). Other authors propose that residential investment could be considered autonomous because it is largely determined by financial conditions, especially mortgage interest rates, rather than the current level of income (Fiebiger, 2018; Perez-Montiel and Pariboni, 2021). At this stage we do not take a stand on what constitutes autonomous demand; we discuss this issue further when we turn to estimating the model.

Autonomous demand has a constant exponential growth rate[8] g_z:

[8]While many macroeconomic models include autonomous demand components like government spending, the assumption that autonomous demand grows at an exogenous rate is less common. Supermultiplier models often make this assumption to generate analytical results. But when interpreting the relevance of the models

$$Z_t = Z_0 e^{g_z t} \tag{2}$$

Equilibrium in the goods market equates output (Y_t) to demand (Y_t^D):

$$Y_t = Y_t^D = C_t + Z_t \tag{3}$$

which implies:[9]

$$Y_t = \frac{1}{s} Z_t. \tag{4}$$

Taking log differences and using equation 2:

$$g_Y = g_z \tag{5}$$

where g_Y denotes the growth rate of output. This is a simple dynamic extension of the even more simple textbook Keynesian model. Equilibrium output at any point in time is autonomous demand times a multiplier that depends on the propensity to save. If the multiplier is constant, then the growth rate of demand-determined output equals the growth rate of autonomous demand.

What about the supply side of the economy? What guarantees that a balanced growth path exists such that supply accommodates the demand-led growth path? To answer these questions, begin by assuming that the ability of the economy to produce output, that is, supply-determined potential output, is a linear function of the state of technology (A_t) and labor (l_t).

$$Y_t^S = A_t l_t \tag{6}$$

The linearity assumption is made here for convenience; there is no loss of generality by considering a diminishing-returns technology. In addition, the size of the population is fixed to 1, and all workers are either employed or unemployed. Hence:

$$1 = u_t + l_t \tag{7}$$

for empirical analysis, authors discuss variations in autonomous demand growth (see Girardi et al. 2020 and FFV, section 6, for example). In the empirical analysis to follow we measure autonomous demand in the data and do not impose a constant growth rate.

[9]In this simple model without capital investment, it is evident that saving equals autonomous demand. If autonomous demand consists of debt-financed government spending, one can think of saving as leading to accumulation of government debt. Alternatively, the saving rate could be replaced by a proportional tax rate on income, without loss of generality, if autonomous government spending is tax-financed.

The key assumption made in FFV, also present in Aghion and Howitt (1994) and Dutt (2006), is that productivity growth depends on the state of the economy, measured by the unemployment rate. Following Ester Boserup (1965), one could argue that necessity is the mother of invention, and labor-saving innovations depend on the tightness of the labor market and the effect of the labor market on wage costs. Alternatively, the degree of slack in the economy, partially determines cash flows, and if firms are financially constrained, cash flows impact investment in R&D (Brown, et al., 2009). These structural channels motivate a reduced-form link between unemployment and productivity growth:

$$g_A = \phi_0 - \phi_1 u_t. \tag{8}$$

With fixed labor supply, g_A determines the growth rate of Y_t^S.

For the simple model to deliver a balanced growth path, aggregate supply and demand must grow at the same rate. In steady state, demand growth is determined by the growth rate of autonomous spending. To equate supply growth to g_z use equation 8 to solve for the steady-state rate of unemployment:

$$u^* = \frac{\phi_0 - g_z}{\phi_1} \tag{9}$$

The unemployment rate along the balanced growth path depends on the growth rate of demand. Consider a positive shock to g_z. Faster growth of demand lowers the unemployment rate, which raises the growth rate of productivity, causing supply to accommodate the acceleration of demand. The lower bound on unemployment determines the maximum feasible growth rate: faster demand growth can increase supply growth only up to the point where there are no more hands or minds willing and able to work. In this simple version of the model and with a zero lower bound on the unemployment rate, the maximum growth rate is ϕ_0, the intercept of the productivity growth equation. But equilibrium unemployment will be above the minimum rate if demand growth falls short of ϕ_0. Equilibrium conditions do not determine a long-run unemployment rate independent of the demand side; supply constraints only impose a lower bound on the unemployment rate.

The diagram below in the (u, g) space illustrates the steady-state in this model, and the Keynesian effects of an increase in autonomous demand growth, when the lower bound on unemployment is not binding. The AD schedule is determined by constant autonomous demand growth only. The AS schedule slopes downward because a lower unemployment rate stimulates faster productivity growth. Acceleration in demand growth from g_z to g^{**} shifts the AD curve upward. Demand stimulus reduces unemployment and the stronger economy with less slack raises supply growth to match the faster rate of demand growth. To sustain faster supply growth in equilibrium, the unemployment rate must remain lower than its initial value, that is, faster demand growth permanently reduces the unemployment rate.

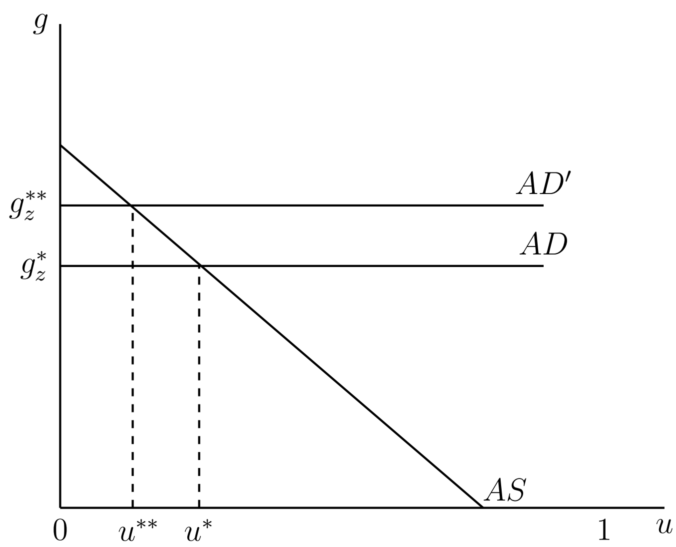


Figure 1: An increase in the rate of growth of demand

2.2 The effects of permanent demand shocks

This simple model can also be used to study macroeconomic fluctuations and persistent changes in the output and employment paths. To do so, suppose log autonomous demand evolves according to a random walk:

$$\ln Z_t = g_z + \ln Z_{t-1} + \varepsilon_t \tag{10}$$

with the demand shock, ε_t as a white noise process. This specification implies that a unitary shock to log autonomous demand induces a permanent one percentage point increase in the level, but not the growth rate, of future autonomous demand. Using this specification, we show in Appendix 1 that the time-series processes for output, unemployment, and the growth rate of productivity are:

$$\ln Y_t = g_z + \ln Y_{t-1} + \varepsilon_t \tag{11}$$

$$u_t = \phi_0 - g_z + (1 - \phi_1)u_{t-1} - \varepsilon_t \tag{12}$$

$$g_{A,t} = \phi_1 g_z + (1-\phi_1) g_{A,t-1} + \phi_1 \varepsilon_{t-1} \tag{13}$$

Therefore, log-output is a random walk with a drift, the unemployment rate is an AR(1) process, and log-productivity is an ARIMA(1,1,1) process. Given these simple univariate time-series process, it is straightforward to compute their impulse response functions after a temporary demand shocks. Let $IRF^h_{y,\varepsilon}$ be the impulse response function at horizon h of the endogenous variable y after being shocked, at time $t = 0$, with a unit shock on ε. Then, we have:

$$IRF^h_{\ln Y,\varepsilon} = 1 \quad \forall h > 0 \tag{14}$$

$$IRF^h_{u,\varepsilon} = (1-\phi_1)^h \quad \forall h > 0 \tag{15}$$

$$IRF^h_{g_A,\varepsilon} = \phi_1 (1-\phi_1)^h \quad \forall h > 1 \tag{16}$$

Hence, a transitory demand shock has a permanent effect on the log-output, and a transitory effect on the level of the unemployment rate. It also has a transitory effect on the *growth rate* of labor productivity, but it has a permanent effect on its level. It is important to stress that these hysteresis effects are not a result of the demand process following a random walk: if $\phi_1 = 0$, then the level of productivity is not affected by demand shocks - as can be readily checked by examining the impulse response function for labor productivity. These predictions are partially consistent with the empirical results obtained by Furlaneto et. al (2021), where, using a structural VAR with both sign and long-run restrictions, they find that a contractionary demand shock permanently reduces output and unemployment; however, they find that demand shocks have little effect on labor productivity.

3 A more complete model for estimation

The simple model transparently illustrates the theoretical intuition for how demand leads the economy and how the supply side adjusts to the demand-led output path. We now extend the previous results to include capital, investment, endogenous labor supply, and expectations. Our purpose is to develop a model that is rich enough to be compared with time-series moments in macroeconomic data and can generate empirical parameter estimates to test the main implications of a demand-led growth model.

3.1 The demand side

The demand side is identical to FFV (2020) and will be summarized here. Agents take decisions based on adaptive expectations of output growth:

$$Eg_t = (1-\alpha)g_{t-1} + \alpha Eg_{t-1}. \tag{17}$$

Agents update their forecasts with a weighted average of their previous growth rate expectation and the most recent observation of actual growth. This specification is equivalent to a forecast based on geometrically declining weights of past output growth. Although simple adaptive expectation models have been criticized, we interpret this specification as an approximation to a more general model of learning.[10]

The consumption equation now takes the form[11]:

$$C_t = (1-s)(1+Eg_t)Y_{t-1} \tag{18}$$

Business investment in period t (I_t) becomes productive capital in period $t+1$ (K_{t+1}) and targets an evolving capital-output ratio ($\hat{v}_t$) :

$$I_t = K_{t+1} - (1-\delta_K)K_t = \hat{v}_t(1+Eg_t)^2 Y_{t-1} - (1-\delta_K)K_t \tag{19}$$

where δ_K is the geometric depreciation rate of the capital stock. The period-by-period target capital output ratio ($\hat{v}_t$) evolves according to a partial adjustment rule to eventually reach the long-run, technologically determined ratio (v^*)[12]:

$$\hat{v}_t = (1-\lambda)v_{t-1} + \lambda v^*. \tag{20}$$

The parameter λ controls the speed of adjustment as in Freitas and Serrano (2017). Autonomous demand (Z_t) grows at a rate g_z and follows the stochastic process:

$$\ln Z_{t+1} = g_z \cdot t + \rho \ln Z_t + \varepsilon_t \tag{21}$$

[10] We draw support for this interpretation from Lawrence Summers who stated during a March 22, 2022 interview with Ezra Klein that "a consensus view would be that people learn from the past, because what else would you learn from? And so people form their expectations based on what they've observed recently and based on what they think is going to happen in the future" https://www.nytimes.com/2022/03/29/podcasts/transcript-ezra-klein-interviews-larry-summers.html.

[11] Taxes are not modelled explicitly. A proportional income tax rate can be included in s without loss of generality.

[12] This specification may be interpreted as a reduced-form from a capital adjustment cost model.

where $\varepsilon_t \sim i.i.d.N(0, \sigma^2)$. These white-noise shocks, ε_t, along with the propagation mechanisms inside the model, account for all the cyclical and low-frequency fluctuations generated by the model.[13]

Aggregate demand is the sum of the demand components, and output equals demand (unless demand exceeds the supply-determined potential to produce):

$$Y_t = Y_t^D = C_t + I_t + Z_t \tag{22}$$

Substituting specifications for the demand components into the current output equation and dividing by lagged output yields the law of motion for output growth:

$$1 + g_t = (1 - s)(1 + Eg_t) + \hat{v}_t(1 + Eg_t)^2 - v_t(1 - \delta)(1 + g_t) + z_t(1 + g_t) \tag{23}$$

where $g_t = (Y_t/(Y_{t-1}) - 1,$, $v_t = K_t/Y_t$, $z_t = Z_t/Y_t$. If autonomous demand grows at a constant rate g_z, the model has a steady state given by:

$$g_t = Eg_t = g_z \tag{24}$$

$$v_t = \hat{v}_t = v^* \tag{25}$$

$$z_t = z^* = s - v^*(g_z + \delta) \tag{26}$$

Note that the steady-state share of autonomous demand in output is just the difference between the share of saving and the share of investment. This steady state is unique; and, as shown in FFV(2020) the dynamics are stable for a wide range of empirically plausible parameter values.

3.2 The supply side

The economy's potential output now depends on capital as well as labor and the Leontief production function becomes

$$Y_{S,t} = \min\{A_t N_t, \frac{K_t}{\hat{v}_{t-1}}\}. \tag{27}$$

[13] Note that, in contrast to section 2, here we allow the process for autonomous demand to be either trend stationary ($(\rho < 1)$) or to be a random walk with a drift ($\rho = 1$) which is slightly more general than the pure random walk assumption used in section 2.

where N_t is labor supply. Since we now relax the assumption that labor supply is fixed, the resource constraint for labor is:

$$N_t = L_t + U_t \tag{28}$$

or, after some re-arranging, $L_t = (1 - u_t)N_t$. With an efficient choice of technique, that is, with no redundant labor, $Y_t = A_t L_t$. Capital may or may not be fully utilized.

Hysteresis effects connect supply growth to demand dynamics through two structural channels. First, labor force growth (g_t^{LS}) depends on the strength of the economy as measured by the unemployment rate:

$$g_{LS,t} = \theta_0 - \theta_1 u_{t-1} \tag{29}$$

Second, the growth rate of the labor productivity term (g_t^A) depends, again, on the current state of the economy through the unemployment rate as well as the rate at which the capital stock is renewed by new investment ($g_{t-1}^K + \delta$, the growth rate of the capital stock plus the depreciation rate):

$$g_{A,t} = \phi_0 - \phi_1 u_{t-1} + \phi_2 (g_{t-1}^K + \delta) \tag{30}$$

FFV (2020) and Mason and Jayadev (2023) present more motivation and extensive references to justify these supply-side equations.[14]

3.3 Steady-State Unemployment and Hysteresis

The key result from FFV (2020) is that supply growth will accommodate the demand growth path as long as the effect of unemployment on supply growth is positive ($\theta_1 + \phi_1 > 0$). Because demand growth is g_z in steady state, equations 29 and 30 solve for the steady-state unemployment rate that equates supply growth to demand growth:

$$u^* = \frac{\theta_0 + \phi_0 - g_z(1 - \phi_2) + \phi_2 \delta}{\theta_1 + \phi_1} \tag{31}$$

These results have several interesting implications. As in the simple model, there is no "natural" rate of unemployment defined independently of the demand side of the economy.[15]

[14] Work in progress (González, 2023) shows that this specification for productivity can be derived from a simple learning-by-doing model.

[15] The idea that there is no natural rate of unemployment independent of demand dynamics has been

The engine of long-run demand growth (g_z) affects steady-state unemployment. As is also the case in the simple model, the minimum unemployment rate limits how fast the economy can grow, but this maximum growth rate is in no sense an equilibrium. Sluggish demand growth can trap the economy in a steady state with unemployment persistently higher than the level imposed by labor supply constraints.

The steady-state equation for the unemployment rate provides a framework to measure hysteresis effects. Suppose the growth rate of demand, g_z, increases. What will happen to the steady-state rate of unemployment? The derivative:

$$\frac{du}{dg_z} = -\frac{(1-\phi_2)}{\theta_1 + \phi_1} \tag{32}$$

measures the response of the unemployment rate to the growth rate of demand. The intuition is simple: if there is a rise in autonomous demand growth, the unemployment rate falls and the supply-side of the economy reacts to this decrease in unemployment by accelerating productivity and labor supply growth. The empirical magnitude of this derivative is a central focus of this paper, and it is critical for understanding the practical relevance of demand-led growth. If it is small in absolute value, because the hysteresis effects are large, then the possibility of supply accommodating demand with modest changes in the unemployment rate is plausible. [16]

Does this result mean the economy can grow unboundedly faster by just increasing demand? Is the supply side, then, largely irrelevant? No. There is a lower bound on the unemployment rate; it cannot be negative. Suppose the unemployment rate is normalized so its minimum value is zero. Then, there is an upper bound on demand growth, $\bar{g}_z$ the supply side can accommodate, given by:

$$\bar{g}_z = \frac{\phi_0 + \theta_0 + \theta_2\delta}{(1-\phi_2)} \tag{33}$$

This equation shows that the exogenous growth rates of technical progress and labor supply determine the upper bound, or a *ceiling*, of the growth rate. If these parameters are too low, then autonomous demand growth will have little space to stimulate the economy. It is then natural to ask what is a reasonable empirical value for this ceiling, a question we tackle in

proposed, in a different context, by Stockhammer (2008).

[16] Mason and Jayadev (2023) propose that the supply side constrains the speed at which output can adjust to changes in demand. This interpretation of hysteresis corresponds closely to the size of the parameters ϕ_1 and θ_1 in our model. A large value of $\theta_1 + \phi_1$ implies supply adjust quickly to demand shocks, and vice-versa.

our empirical exercise.[17]

These results also lead to an original interpretation of Verdoorn's law. Verdoorn (1949) found a statistical relationship between the growth rate of labor productivity and the growth rate of output, which was made famous by Kaldor (1966). This reduced form, in our notation, is:

$$g_A = \alpha + \beta g_Y \tag{34}$$

where β is the Kaldor-Verdoorn coefficient. The β coefficient sometimes been interpreted as measuring the degree of demand-induced technical progress and the degree of increasing returns to scale present in the economy. Therefore, one could be tempted to use this statistical relationship as a building block of a macroeconomic model. However, as noted by McCombie and Spreafico (2015) and Basu and Budhiraja (2021), this equation is a statistical relationship, which must be derived from the underlying economic theory.

To see the connection between the Kaldor-Verdorn coefficient and our model, consider the steady-state version of the productivity growth equation:

$$g_A = \phi_0 - \phi_1 u^* + \phi_2(g_K + \delta) \tag{35}$$

Along the balanced growth path, $g_K = g_Y = g_z$. Plugging in the equation for the steady-state unemployment rate, and some tedious but straightforward algebra yields:

$$g_A = \frac{\phi_0\theta_1 - \phi_1\theta_0 + \phi_2\delta(1 - \theta_1 - \phi_1)}{\theta_1 + \phi_1} + \frac{\phi_1 + \phi_2\theta_1}{\theta_1 + \phi_1} g_z \tag{36}$$

Therefore, along the balanced-growth path, the mapping between the Kaldor-Verdorn coefficient and our hysteresis parameters is:

$$\beta = \frac{\phi_1 + \phi_2\theta_1}{\theta_1 + \phi_1} \tag{37}$$

Note that β takes a value of zero if $\phi_1 = \phi_2 = 0$, even if $\theta_1 > 0$. That is, $\beta = 0$ if the growth rate of productivity is exogenous even if there is some hysteresis in labor supply. In contrast to Basu and Budhiraja (2021), our model economy implies that if the Kaldor-Verdoorn coefficient is strictly positive, then there is some influence of the demand side on the supply side of the economy. Furthermore, a positive value of β need not imply static increasing

[17] In an open economy with a balance-of-payment constraint Thrilwall's Law could also impose an upper bound on growth that may well bind at a lower rate than the $\bar{g}_z$ constraint.

returns to scale: our Leontief production function assumes static constant returns. It is also clear that this statistical relationship is not informative about which parameters that govern the relationship between the supply side and the demand side are quantitatively important. Estimation of all the economically meaningful parameters that govern supply-side hysteresis is needed, a task to which we now turn.

4 Estimating a Demand-Led Growth Model

The model presented in the previous section proposes simple relationships to describe the dynamics of consumption, investment, and autonomous demand, along with adjustment of the capital stock and expectations. Equations 29 and 30 introduce key hysteresis effects that create structural channels for demand dynamics to lead the supply side. In this section we develop a strategy to estimate the model parameters and quantitatively assess how well it can explain major features of US macroeconomic data.

4.1 Parameter Estimation

4.1.1 Data

We use data on the components of U.S. GDP for the period 1959:Q1 - 2019:Q4 obtained from the Federal Reserve Bank of St. Louis FRED data site. Nominal data are deflated by the same price index, the chain-weighted deflator for personal consumption expenditure. This index is the Federal Reserve's preferred measure of aggregate inflation. Using a common deflator is appropriate for studying demand-led dynamics because the objective is to capture expenditure flows. So-called "real" series for separate GDP components such as business fixed investment and personal consumption expenditure attempt to adjust for quality changes. For example, personal computer price indexes decline dramatically more than the actual purchase price of these machines reflecting the fact that they are faster and have more memory and storage than earlier products. These quality adjustments may reflect that new computers are somehow "better" than their earlier counterparts, but higher quality is not reflected, other things equal, in expenditure.

Our model identifies three components of demand that drive output: induced consumption, induced business investment, and autonomous demand. Business investment adds directly to productive capacity and therefore our business investment variable excludes residential construction and inventory changes. The definition of autonomous demand is central to the logic of the FFV model we study, but there are ambiguities in defining what is indeed "autonomous."

Any definition of autonomous demand includes consumption and investment spending by federal, state, and local governments (Allain, 2015). In addition, our autonomous demand definition includes direct government spending on social programs. In the U.S., this spending is almost entirely for health care (Medicare and Medicaid). [18] In the national accounts, this spending is treated as a transfer to the private sector and then added, dollar-for-dollar, to personal consumption expenditure. But it is clear these expenditures are not induced by cash income flows to households. Rather, they are large injections of demand by government and are just as "autonomous" as other parts of government spending. We also add Social Security, government transfer payments primarily to retirees. These payments are surely autonomous, in the sense they are steady, defined-benefit expenditures that do not depend on the current state of the economy. Ambiguity arises, however, because Social Security income is not necessarily spent and therefore does not add dollar-for-dollar to demand like government health care spending. However, Social Security is large and much of it goes to households of limited means. Most of this spending likely supports demand and that demand is largely autonomous.[19]

Exports are also part of autonomous demand (Nah and Lavoie, 2017). For almost any country, exports are likely independent of domestic economic shocks. The U.S. is large enough that its own exports could be affected by domestic shocks because those domestic shocks could spill over in non-trivial ways to its trading partners. Nonetheless, there is a good case that even US exports are largely autonomous, especially at lower data frequencies.

Similar to other empirical work based on supermultiplier models, we also include residential construction in autonomous demand. Other authors argue that residential construction is driven fundamentally by credit flows and the accumulation of debt; hence, especially in recent U.S history, it has been largely detached from current output (Fiebiger, 2018; Pérez-Montiel and Pariboni, 2021). The same authors have also argued that, in the long-run, the growth rate of residential investment must be determined by the rate of population growth, which they presume as exogenous.

It is important to note that while many supermultiplier theoretical models assume, for an-

[18] To avoid induced government spending, we exclude cyclical components of spending: unemployment insurance, supplemental nutrition (SNAP), and refundable tax credits.

[19] Because we include government social spending in autonomous demand, we subtract the identical amount from consumption to avoid double counting.

alytical purposes, that autonomous demand grows at a constant real rate, our approach generalizes the econometric specification of autonomous demand as an AR(1) stochastic process that nests trend- or difference- stationary processes.

Supply-side data come from the labor market. We use time series on the unemployment rate and employment, measured as total workers. Labor productivity is output divided by employment.

4.1.2 A Minimum Distance Estimator

The model described in section 3 poses numerous challenges for estimation. Because the dynamics are described by a system of non-linear stochastic difference equations, it is not possible to solve for a closed-form expression that links the endogenous variables to the lags of themselves and the stochastic term. Additionally, the model contains variables likely measured with error, including the capital stock, or which are unobserved, such as agents' expectations.

We address this issue by using a minimum distance estimator, in particular, the Simulated Method of Moments (SMM) estimator proposed originally by McFadden (1989) and Pakes and Pollard (1989) and extended to time-series models by Lee and Ingram (1991) and Duffie and Singleton (1993). To understand the SMM estimator consider, a fully-specified model with parameter vector Θ, of dimension q. The econometrician has a sample y_t of weakly stationary data, used to compute sample moments $T^{-1}\sum_t m(y_t)$, such as correlations between variables in the data. The economic model defined by Θ is used to simulate a data vector S and moments $m(y_s(\Theta))$, then a natural way to estimate the model is to minimize:

$$\min_{\theta} Q(\Theta) = M(\Theta)' W M(\Theta) \tag{38}$$

Where W is a positive-definite weighting matrix, and

$$M(\Theta) = T^{-1}\sum_{t=1}^{T} m(y_t) - S^{-1}\sum_{s=1}^{S} m(y_s(\Theta)) \tag{39}$$

That is, the vector of estimated parameters minimizes a measure of the distance between the moments generated by the model and those present in the data. As long as the moment conditions are correctly specified at the true parameter vector, and the vector of data y_t is weakly stationary, the estimator is consistent and asymptotically normal.

This estimator is powerful in our context: it allows consistent estimation when some variables are measured with error or unobserved, it does not require the economic model to have an explicit solution, only that it can be simulated, and it allows for model misspecification in the sense that we do not require all the implications of the model to be true, only those moment conditions which will be used for estimation. This last feature is especially attractive since we do not claim our model is sufficiently rich to be a full theory of growth and fluctuations; however, we do propose that it captures important transmission mechanisms of aggregate demand to the rest of the economy.

We estimate the full vector of parameters implied by the model, with the exception of g_z and δ. The growth rate of autonomous demand is set to equal the average quarterly growth rate of output over our sample, $g = 0.03/4$. In addition, we fix $\delta = 0.025$ at the quarterly level, as is conventional in much of the literature. The rest of the parameter vector $\Theta = (\sigma, \rho, s, \lambda, \alpha, v^*, \theta_0, \theta_1, \phi_0, \phi_1, \phi_2)$ is estimated by matching 23 moments, which contain information about business cycle fluctuations and long-run growth. To match business-cycle fluctuations, we use the cyclical correlation between output and consumption, investment, autonomous demand, the unemployment rate, total employment, and labor productivity; the first-order autocorrelation of these variables, and the relative volatility between output and the remaining variables. To match the long-run behavior of the data, the moments include the average consumption share in output, which identifies s, the average investment share in output, which identifies v^* conditional on g and δ, the average unemployment rate, and the average growth rate of labor productivity.

Because SMM requires stationary data, we obtain business-cycle statistics by extracting frequencies between 6 and 32 quarters with the Baxter-King (1999) filter on both actual and simulated data. We do not apply this filter to the long-run moments, we simply compute sample averages of these ratios and growth rates.[20] Since the model has implications for the data at all frequencies, we also report estimates extracting frequencies at 32-80 quarters and between 6-80 quarters. These estimates correspond to what Comin and Gertler (2006) define as the 'medium-term' business cycles, and is a natural benchmark to understand how endogenous productivity and other dynamics behave in our environment over longer horizons.

[20]Note that assuming these shares are stationary in the literature, or more stationary than variables in levels, is consistent with a wide class of growth models that imply a balanced growth path, including the one presented here. Stock and Watson (1999) present evidence to support this approach.

Because our objective function is highly non-linear, we obtain point estimates numerically with the calibrated parameters from FFV (2020) as starting values. We obtain confidence intervals and non-linear functions of our parameters by Monte Carlo simulations, simulating the path induced by a stochastic demand shock 1,000 times, and for each path, we obtain an estimator. The confidence intervals reported below correspond to the relevant percentiles of the empirical distribution function of these simulated estimates.

5 Empirical Results

5.1 Parameter Estimates and Second Moments

Table 1 presents our baseline estimation results, where we display the model estimated at business-cycle frequencies, the "medium-run" frequencies, and both frequencies taken together. It is instructive to compare the results obtained here with those obtained in FFV through calibration and single-equation, reduced-form regressions. Starting with the demand side, the values for the savings rate and the capital-output ratio are remarkably close. Note that the FFV model was calibrated using annual data, while we have estimated the model using quarterly data; therefore, we multiply or divide the parameters by four where appropriate. The annualized value of the capital output ratio varies between 0.99 and 1.04 depending on the frequency; while FFV calibrated a value of 1.2. The main differences are for the value of λ and α; our value for λ is not statistically different from 0 at the 5% significance level; however, the confidence intervals are wide. They include a value of $\lambda = 0.10$, which would imply 34% of adjustment between v_t and v^* over a year. This implies that our estimates are not very informative when it comes to λ, a point to which we return below. The point estimate for the adaptive expectation parameter $\alpha = 0.94$, implies a yearly value[21] of 0.784, which implies expectations are less persistent than what was found in FFV, where $\alpha = 0.9$.

It is important to stress that our point estimate of the marginal propensity to save, 0.44, while not substantially different from what was found in FFV, might seem high for the U.S economy. However, one can interpret this parameter as incorporating the effect of income taxation and the fact that some induced consumption will be imported rather than domestically produced. That is, the s parameter estimated here is better interpreted as one minus the propensity to consume domestic production out of GDP.

[21] To annualize these estimates, we use the formula $0.941^4 = 0.784$

Table 1: Baseline Parameter Estimates

	6-32 Quarters		32-80 Quarters		6-80 Quarters	
Parameter	Coef.	C.I	Coef.	C.I	Coef.	C.I
σ	0.030	[0.003, 0.040]	0.040	[0.001, 0.040]	0.032	[0.001, 0.040]
ρ	0.989	[0.688, 0.990]	0.985	[0.081, 0.990]	0.990	[0.616, 0.989]
θ_0	0.008	[0.006,0.009]	0.008	[0.006,0.011]	0.010	[0.006,0.011]
θ_1	0.073	[0.053, 0.106]	0.087	[0.070, 0.140]	0.113	[0.076, 0.137]
ϕ_0	0.020	[0.012,0.024]	0.015	[0.006,0.020]	0.019	[0.014,0.024]
ϕ_1	0.261	[0.227, 0.324]	0.193	[0.162, 0.266]	0.258	[0.223, 0.328]
ϕ_2	0.0006	[0.000,0.197]	0.001	[0.000,0.247]	0.004	[0.000,0.104]
v^*	4.113	[3.508, 4.736]	3.982	[3.513, 4.723]	4.158	[3.525, 4.610]
s	0.441	[0.428, 0.500]	0.400	[0.391, 0.466]	0.419	[0.405, 0.475]
λ	0.003	[0.000, 0.132]	0.003	[0.000, 0.141]	0.008	[0.000, 0.131]
α	0.941	[0.925, 0.990]	0.950	[0.932, 0.996]	0.948	[0.926, 0.990]
J statistic	50.5		52.9		53.9	

There are also important differences in the supply side. The calibration in FFV along with the single-equation estimation methods suggested that hysteresis in labor supply and productivity were equal in importance. The SMM estimates of the full model yield a productivity effect (ϕ_1) roughly three times greater than the labor supply effect (θ_1). In this sense, hysteresis in labor productivity is quantitatively much more important than hysteresis in labor supply to explain the adjustment of the supply side of the economy to movements in demand. Another important difference is that the effect of the accumulation rate on productivity growth ϕ_2 is not statistically different from zero. As discussed before, this result does not imply that the Kaldor-Verdoorn coefficient in this economy is zero, but rather that the effect of the *level* of economic activity on growth is more important than the feedback of effect of growth on growth.

It is interesting to note that for the estimated model to reproduce realistic fluctuations, the volatility of autonomous demand must be rather large. The process itself also needs to be very persistent to match the data, which means the propagation mechanisms inside the model are not very strong.

A possible criticism of the estimates of θ_1 and ϕ_1 is that they arise from temporary changes in labor supply and productivity due to short-run business cycles rather than true supply effects. For example, labor hoarding would imply a decline in productivity during recessions. To explore this possibility, we estimate the model at long frequencies (32-80 quarters).

The effect of unemployment on productivity (ϕ_1) is somewhat lower at longer frequencies, but it remains statistically and economically significant: a one percentage point rise in the unemployment rate would decrease productivity growth by roughly 0.8 percentage points (annualized) at low frequencies. The labor supply estimate (θ_1) rises slightly at longer frequencies. Are these long-run effects plausible? Previous research found very strong negative co-movements between the unemployment rate and productivity growth; see, for example Benigno, Ricci, and Surico (2015) and Müller and Watson (2018). Our model explains those co-movement directly as the result of demand-induced technical change; and, as Müller and Watson (2018) note, "A long-run one-percentage-point increase in the rate of growth of productivity is associated with an estimated one-percentage-point decline in the long-run unemployment rate. We are unaware of an economically compelling theoretical explanation for the large negative correlation (p.799)". We propose that a demand-induced theory of technical change provides a compelling explanation of this fact.[22]

Due to the similarity of the estimates at different frequency ranges, in what follows we will use the estimates at business cycle frequencies to compare the moments predicted by the model to those found in the data, compute impulse-response functions, and analyze some interesting non-linear functions of the parameter estimates.

To assess how well the model performs along various dimensions, tables 2 and 3 compare the business-cycle and growth statistics estimated from the model with those in the data. Our rather parsimonious model (just 11 parameters for 23 moments) matches all the business cycle statistics qualitatively, and it also matches them reasonably well quantitatively. The model generates the high persistence in all macroeconomic variables present in the data, it matches the large correlations between employment and output, and labor productivity and output, rather well. It also matches the fact that investment is highly volatile over the cycle, as well as matching closely the relative volatilities of employment and labor productivity relative to output. Finally, the correlation between autonomous demand and output, as well as its relative volatility, are well explained by the model, which implies that a simple autoregressive process is a reasonable approximation for possibly more complex dynamics that underlie autonomous demand. The empirical success of this generalization of the autonomous demand process, compared with models assuming autonomous demand grows a constant rate, helps address criticisms raised by Nikiforos (2018) and Skott (2019), among

[22] In addition, González (2023) shows that if one derives this productivity equation from a conventional learning-by-doing model, then these productivity effects are within bounds of microeconomic estimates derived from firm-level and industry-level data on learning-by-doing.

others.

Table 2: Business Cycle Statistics

	Correlations		Persistence		Volatility	
Variable	Data	Model	Data	Model	Data	Model
ln Y	1.00	1.00	0.93	0.96	1.00	1.00
ln C	0.90	0.96	0.95	0.96	1.20	1.05
ln I	0.74	0.83	0.94	0.95	3.04	3.26
ln Z	0.58	0.62	0.92	0.90	0.83	0.85
u	-0.83	-0.57	0.94	0.93	0.50	0.62
ln L	0.77	0.77	0.94	0.93	0.68	0.68
ln A	0.74	0.74	0.91	0.97	0.65	0.65

The dimensions along which the model is less successful at matching the data are all related to the unemployment rate: the correlation between unemployment and output is 30% below what we find in the data. Likewise, the volatility of unemployment estimated from model is somewhat higher than in the data. The volatility of consumption and the correlation between investment and output are also roughly 10% different from the data. The other differences between the model and the data are well below 10%. Somewhat paradoxically, although the unemployment rate moments match the data less well, the labor productivity and employment moments display an excellent match. These moments are arguably the most important in our study because they determine the critical transmission from demand dynamics to the supply side.

Table 3: Great Ratios and Growth Rates

Variable	Data	Model
C/Y	49%	56%
I/Y	13%	14%
u	6.0%	6.0%
g_A	0.4%	0.4%

Table 3 displays our great ratios and growth rates of key variables used for estimation. The model does an excellent job of matching the average unemployment rate over the sample; the share of investment in output, and the average growth rate of productivity. The share of consumption in total output, however, is matched less successfully. Recall that we do not filter these moments, since we presume that these great ratios are stationary along the balanced growth path. The same holds for the growth rate of productivity and the unemployment share.

5.2 Identification

As is well know, non-linear models are often plagued by identification issues; in other words, there could be multiple linear combinations of the parameters that deliver the same match between the model and data. To address this issue, we check conditions for local identification. The Cragg-Donald (1993) statistic checks that the Hessian of our objective function is of full rank:

$$\text{rank}\left[E\left(\frac{\partial m(x_s(\theta))}{\partial\theta}\right)\right]=q \tag{40}$$

Hence, the Hessian is simply the Jacobian of the simulated moments with respect to the parameter estimates. Around our parameter estimates the matrix is indeed full rank, which demonstrates that our model is locally identified.

In addition to checking local identification, we can get a sense of how strong the identification is by plotting the percentage change of the objective function against the percentage change in parameter values relative to the point estimates. The graphs in figures 2 and 3 show substantial curvature around most parameter values; in particular, the hysteresis parameters on productivity and labor supply are sharply identified. Two parameters seem to be locally unidentified: the embedded technical progress coefficient (ϕ_2), and the adjustment speed of the capital stock (λ). This outcome is not surprising, given that we estimate these two parameters to be very close to zero, which means the point estimates are on the boundary of the parameter space. As we discussed earlier, our confidence intervals for λ were very wide and were compatible with the capital-output ratio being either a random walk or it having rather fast adjustment. The empirical connection between capital adjustment and economic conditions is a topic for further research. Also, the persistence of the demand shock parameter, though locally identified, has another local minimum between 0.93 and 0.96. Overall, identification issues that plague many dynamic nonlinear stochastic models (see Canova and Sala, 2009) are not pervasive in our application.

Figure 2: Identification: Autonomous Demand and Supply Side

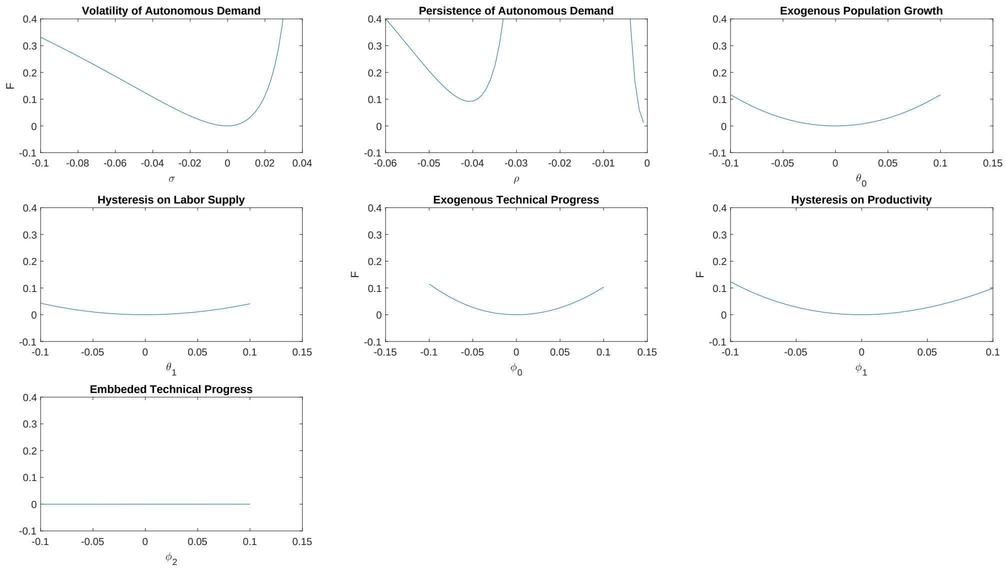
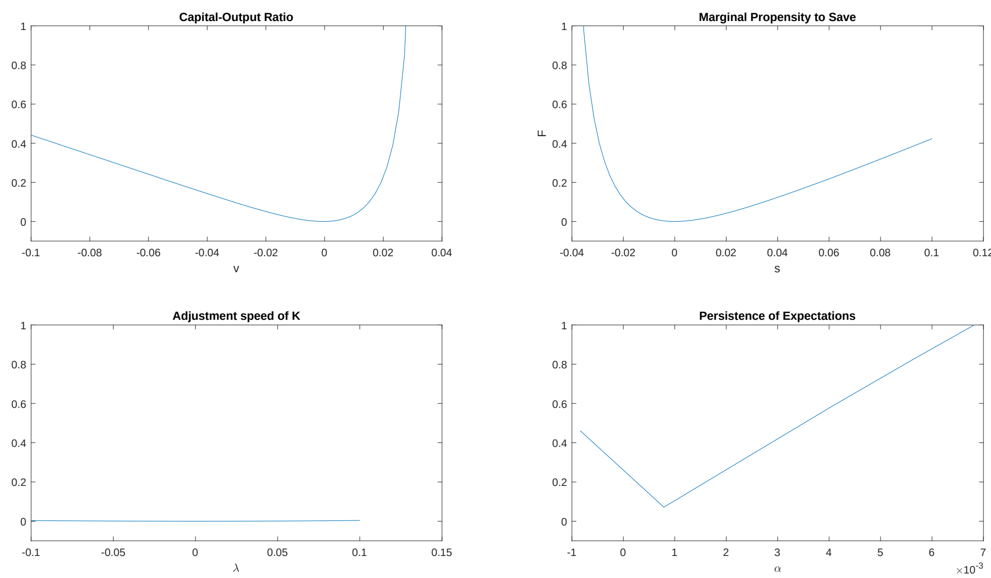


Figure 3: Identification: Demand Side



5.3 Hysteresis Effects and the Growth Ceiling

A key question in research on demand-led growth models is the extent to which demand can lead supply, that is, how strong are the hysteresis effects? In our context the answer to this question depends on the long-run relationship between the growth rate of autonomous demand and the steady-state rate of unemployment. In particular, total differentiation of equation (15) gives:

$$\frac{du^*}{dg_z} = -\frac{(1-\phi_2)}{\theta_1 + \phi_1} \tag{41}$$

A plug-in estimator for this quantity can be obtained by replacing the parameters with our estimated values. Table 4 shows the results for this exercise for our baseline estimates, along with confidence intervals. The point estimate for quarterly data is -3.05; an annualized estimate is obtained by dividing by 4, which gives -0.76, remarkably similar to the calibrated value of -0.7 obtained by FFV (2020). Furthermore, FFV reports a rather wide range of values for this effect (-0.3 to -1.7) by simulating across a plausible range of the parameter space. Our statistical estimate of the 95% confidence intervals reported in table 4 are much tighter.

Table 4: Estimates of Hysteresis Effect

	Coef.	95% C.I
6-32 Quarters	-0.75	[-0.83,-0.57]
32-80 Quarters	-0.89	[-0.99,-0.61]
6-80 Quarters	-0.67	[-0.91,-0.53]

These results strongly support the hypothesis of demand-led growth followed by accommodating supply. Parameter estimates that are too high (in absolute value) would imply an excessive change in unemployment would be necessary for supply to accommodate a modest change in the autonomous demand growth rate. However, if the estimate is too low it would imply that supply is excessively sensitive to demand. With our estimates, if yearly autonomous demand growth were to increase by one percentage point, the long-run unemployment rate would decline by 0.76 percentage points, which would bring down the sample average from 6% TO 5.2%. We believe this effect reflects a plausible quantitative magnitude. It implies that there may be substantial potential faster demand growth to allow the U.S economy to grow above what has been achieved historically.

As we discussed above, given our parameter estimates for the hysteresis effects, we can compute the Verdoorn coefficient implied by our model, β in the notation of equation (34) and (37). This yields:

$$\hat{\beta} = \frac{\hat{\phi}_1 + \hat{\phi}_2\hat{\theta}_1}{\hat{\theta}_1 + \hat{\phi}_1} = 0.786 \tag{42}$$

With confidence intervals in the range (0.72,0.83). This point estimate is somewhat higher than what is commonly found in the literature that uses reduced-form methods with estimates ranging between 0.3 and 0.6 (Basu and Budhiraja, 2021). Importantly, we show the effect of capital accumulation on labor productivity (ϕ_2) - a key ingredient of the early formulations of Kaldor's technical progress function (Kaldor 1957, 1961) - has a point estimate near zero, with the upper end of the confidence interval still reasonably small near 0.2. This is the case even though there is substantial induced technical progress arising from the feedback from the unemployment rate to the growth rate of supply. Note that this outcome is also true at all frequencies - both across the 1.5 and 8 years that are usually thought as the typical duration of the business cycle, and in periods between 8 and 20 years, where one might expect to see some effect of capital accumulation on technical progress.

The fact that hysteresis effects are of a reasonable magnitude implies that the supply side adjusts in response to higher demand growth. But how much growth can the supply side actually accommodate? What growth rates are feasible in our economy? In our environment, the upper bound on growth, or the 'ceiling' growth rate is given when the economy reaches full employment. In particular, since we know that $u_t \geq 0$, we can use our steady-state condition to derive the following relationship:

$$\bar{g}_z = \frac{\phi_0 + \theta_0 + \theta_2 \delta}{1 - \phi_2} \tag{43}$$

Where $\bar{g}_z$ now denotes the ceiling, or the growth rate of output consistent with full employment. Of course, it is rather extreme identify full employment with an unemployment rate of literally zero. Suppose instead, that there is a minimum rate of unemployment that the economy can reach, $\hat{u}$. In this case, the growth ceiling will be given by:

$$\bar{g}_z = \frac{\phi_0 + \theta_0 + \theta_2 \delta - \hat{u}(\phi_1 + \theta_1)}{1 - \phi_2} \tag{44}$$

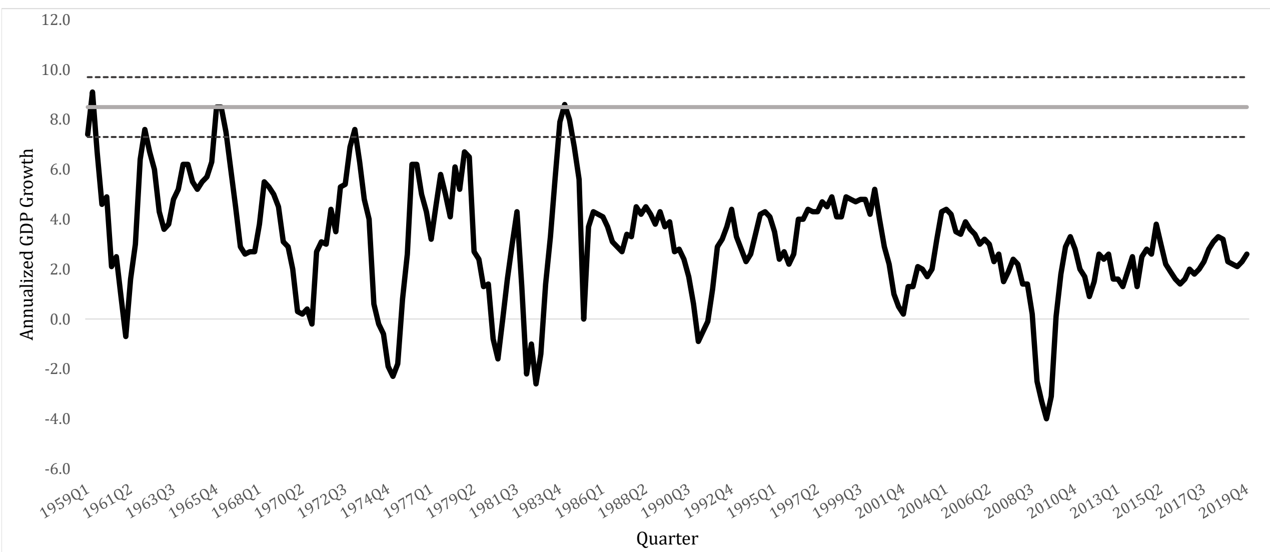
Table 5 shows the point estimates of the annualised growth ceiling along with confidence intervals for different values of the possible "minimum" unemployment rates. Our confidence intervals are surprisingly tight in each case.

Table 5: Estimates of the growth ceiling

$\hat{u}$	$\bar{g}_z$	95% C.I
0	0.112	[0.097,0.132]
0.02	0.085	[0.073,0.097]
0.04	0.059	[0.049,0.063]

To get a sense of how reasonable these estimates are, we plot in Figure 4 below the annualized quarterly growth rate of output, along with the confidence interval for the ceiling, assuming the minimum unemployment rate is 2%. We choose this minimum unemployment as a compromise between what we saw as an exceedingly unrealistic lower bound (a 0% unemployment rate), and a minimum value that has been reached multiple times over the sample, and that implies a maximum growth rate that would have been surpassed many times (4%). A minimum unemployment rate of 2%, however, would imply that the U.S economy only reached the growth ceiling on four occasions during the whole sample: During 1959Q1-1959Q2, 1965Q4-1966Q2, 1973Q1 and then finally during 1983Q4-1984Q2.

Figure 4: The growth ceiling



There a are a number of important caveats with these estimates. First, our model does not provide a theory of the minimum unemployment rate therefore there is substantial uncertainty over the actual growth ceiling. Second, the intercepts and hysteresis parameters governing productivity growth and labor supply growth could be time-varying. In particular, as figure 4 shows, peaks of the cyclical growth rate have declined since the middle 1980s and the maximum feasible rate may have also declined as well. Third, the linear hysteresis relationships in our model could, in reality, be non-linear, particularly as the economy pushes up against a minimum unemployment constraint. Fourth, the growth ceiling $\bar{g}_z$ is derived from steady-state conditions; it does not bind at a quarterly frequency. Nevertheless, our estimates suggest that stronger demand growth - say, an increase of one percentage point - would not hit the growth ceiling, and hence could be accommodated with a lower rate of unemployment.

5.4 The Zero Lower Bound

As discussed in the introduction, a number of recent contributions have combined New Keynesian models with some elements of endogenous growth theory to produce long-run effects of demand shocks on output. An important aspect of these models is that the economy must be constrained by a lower bound on interest rates to exhibit these long-run effects of demand shocks (Benigno and Fornaro, 2018; Eggerston, Mehrota and Robbins, 2019 and Michau, 2018). Since the model discussed here does not feature interest rates - or any relative price whatsoever - we cannot discuss this issue theoretically. However, to asses whether the strength of our hysteresis channels is mostly driven by the episodes where the zero lower bound on interest rates binds, we estimate the model on a sub-sample that excludes the zero lower bound. Table 6 below shows our parameter estimates for this subsample, the implied estimate for du/dg and the Kaldor-Verdoorn coefficient, along with our baseline estimates from Table 1 for the sake of convenience.

Table 6: Estimates excluding ZLB: 1959Q1 - 2008Q1

	Excluding ZLB		Whole Sample	
Parameter	Coef.	C.I	Coef.	C.I
σ	0.026	[0.002, 0.040]	0.030	[0.003, 0.040]
ρ	0.969	[0.755, 0.990]	0.989	[0.688, 0.990]
θ_0	0.009	[0.007,0.010]	0.008	[0.006, 0.009]
θ_1	0.101	[0.078, 0.126]	0.073	[0.053, 0.106]
ϕ_0	0.022	[0.018,0.024]	0.020	[0.012, 0.024]
ϕ_1	0.300	[0.262, 0.344]	0.261	[0.227, 0.324]
ϕ_2	0.0001	[0.000,0.07]	0.0006	[0.000, 0.197]
v^*	4.021	[3.537, 4.564]	4.113	[3.508, 4.736]
s	0.421	[0.413, 0.477]	0.441	[0.428, 0.500]
λ	0.0001	[0.000, 0.134]	0.003	[0.000, 0.132]
α	0.939	[0.924, 0.987]	0.941	[0.925, 0.990]
du/dg	-0.62	[-0.713, -0.527]	-0.75	[-0.83, -0.57]
KV Coef.	0.75	[0.728, 0.785]	0.78	[0.72,0.83]

Results for this subsample period show that most of our parameters are essentially the same as when we estimate the model using the whole sample, with an important exception: our hysteresis parameters ϕ_1 and θ_1, which measure the extent to which the unemployment rate influences productivity growth and labor supply growth, are larger when we exclude the ZLB episode from the sample. This implies that the unemployment rate adjusts by somewhat less (du/dg is -0.62, versus -0.76, percentage points for a one percentage point increase in autonomous demand growth). Overall, this result implies that when we exclude the zero lower bound part of the sample, supply adjusts more strongly to demand than in the whole sample, and the hysteresis effects are stronger. We interpret this as evidence that the exclusive focus previous literature has put on the zero lower bound to generate persistent effects of demand on the macroeconomy may be somewhat misplaced. This issue is an important topic for further research with an extended model that explicitly incorporates interest rates and monetary policy.

5.5 Untargeted Moments

We can compute a variety of other statistics of interest from our estimates. In particular, we can compute moments in the data that were not used in our parameter estimation - moments that were not targeted by our objective function - and ask how well the estimated model replicates them. In particular, we focus on two moments that have received widespread attention in the business cycle literature: the correlation between labor productivity and employment, and the correlation between unemployment and labor productivity. Matching the cyclical co-movement between employment and labor productivity has been perceived as, in the words of Christiano and Eichenbaum (1992),*"a litmus test for aggregate economic models"*. The old vintage of Keynesian models proposed between 1930 and 1970, which invoked wage rigidities to explain the effects of aggregate demand shocks, predicted counter-cyclical real wages, while the data show real wage are either a-cyclical or weakly pro-cyclical. In contrast, standard RBC models usually imply that the real wage is strongly pro-cyclical, since positive technology shocks shift labor demand outward along an upward-sloping labor supply curve, increasing employment and wages (and vice-versa for negative technology shocks). In any economic model where real wages are proportional to average labor productivity, as is the case with competitive neoclassical factor markets under a Cobb-Douglas production technology, these results carry over to the cyclical behavior of output and labor productivity. A large literature in the 1990s extended the standard RBC model to match the cyclical behavior of average labor productivity and real wages with moderate success; prominent examples that also summarize the research in this period are Hansen and Wright

(1992) and Christiano and Eichenbaum (1992).

The correlation between unemployment and labor productivity has also received prominent attention in recent decades, starting with an observation by Shimer (2005) that search and matching labor markets in the tradition of Diamond-Mortensen-Pissarides usually produce a correlation between unemployment and labor productivity equal to -1, while this correlation is much weaker in the data. This observation has led to a rich literature extending or modifying the baseline model to account for this observation (Hall, 2005; Pissarides, 2009; Gertler and Trigari, 2009, among others). It thus seems natural to ask whether our model produces reasonable predictions for these correlations, and it hence describes the joint dynamics of productivity and unemployment better. Table 6 presents the results for both moments from our estimates and prominent estimates from real business cycle models.

Table 7: Untargeted Moments - Neoclassical vs Post-Keynesian

Moment	Data	Neoclassical	Post-Keynesian
$\sigma_{A,L}$	0.14	0.57*	0.14
$\sigma_{A,u}$	-0.28	-0.95**	0.12

*=Christiano & Eichenbaum (1992); **=Shimer (2005)

The correlations produced by our model regarding employment and labor productivity are remarkably close to those implied by the data. Indeed, they are much closer than the correlation of one implied by a standard RBC model, but it also performs better than the various extensions proposed by Christiano and Eichenbaum (1992) and Hansen and Wright (1992).[23] These results carry over to some extent to the correlation between unemployment and labor productivity: a correlation of 0.12 is much closer to -0.28 than -1. Nevertheless, there is still some room for improvement along this dimension.

The reason why our model produces an almost zero correlation between employment and labor productivity is simple. Suppose the economy is hit by a demand shock. Upon impact, production expands, which increases employment and hence decreases unemployment. This decrease in unemployment accelerates productivity growth (and its level) in the next period: it is lagged unemployment that affects productivity growth, not contemporaneous unemployment. A natural interpretation for this is that newly employed workers must take some time to learn before they increase their productivity, as was suggested in our simple model.

[23] In those papers, the lower end of the correlations reported in tables 4 and 3, respectively for each paper, are of 0.57 and 0.49. Hence, our model outperforms these specifications as well.

5.6 The effects of autonomous demand shocks

Impulse-response functions (IRFs) following an autonomous demand shock provide insights into the implications of our estimates and further validation for the results. In particular, we compare our IRFs to estimates from local projections by Girardi, Paternesi and Stirati (2020). This provides another informal test of our estimated parameters.

To calculate impulse response functions, we initialise the model at the steady state, and compute a path of the vector of endogenous variables, $\mathbf{Y}$, in absence of shocks. We then initialise the model again at the steady state, shock the the first period, i.e, $\varepsilon_1^D = \Delta$ and $\varepsilon_t^D = 0 \quad \forall t > 1$, and compute an alternative path of the endogenous variables. The IRF vector is the difference between these two simulated paths, that is, $E[\mathbf{Y}_{t+h}|\varepsilon_1^D = \Delta] - E[\mathbf{Y}_{t+h}|\varepsilon_1^D = 0] = IRF_h$. The size of the shock Δ is calibrated to one percent of period one output. Confidence intervals are constructed by a Monte-Carlo procedure, using 1,000 simulations, where each one is based on a draw of the parameter vector from the joint estimated parameter distribution.

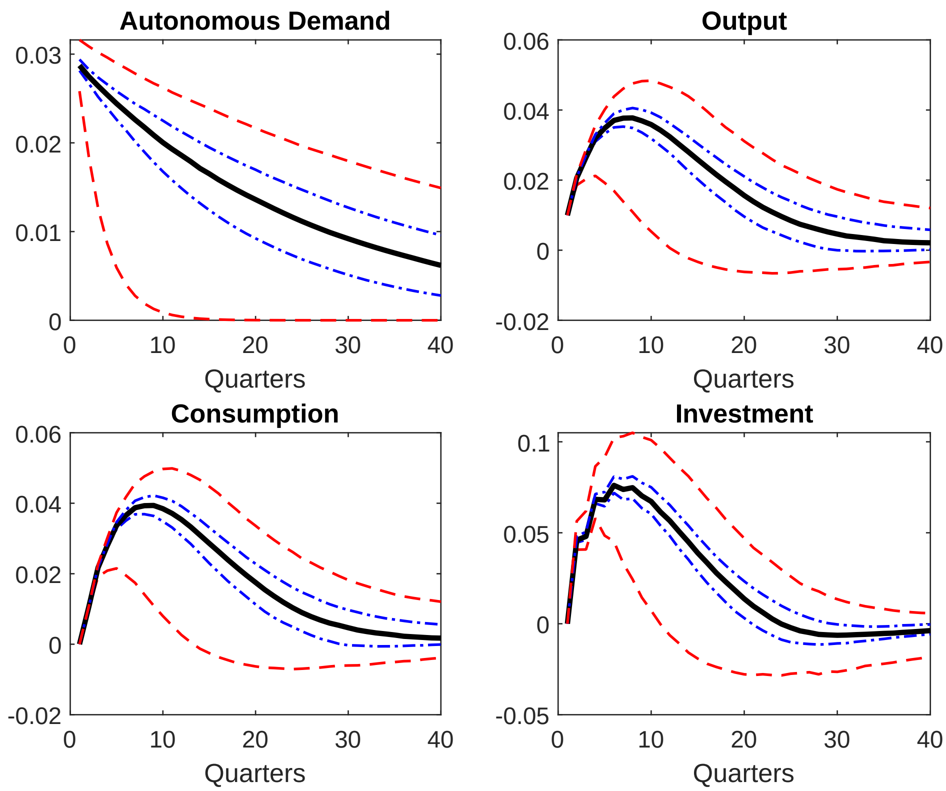
Figures 5 and 6 show the median response of the demand and supply side of the economy to an autonomous demand shock. The figures also plot 68% and 95% confidence intervals. The peak effect of autonomous demand on output, consumption, and investment is reached at roughly eight quarters and then the effect monotonically declines. The effect of a temporary autonomous demand shock on all model variables is zero in steady state. But the large estimated persistence of autonomous demand keeps output above steady state for years, qualitatively similar to results from Girardi et al.[24] While the initial strong effects of autonomous demand on output are consistent with the findings of Girardi, et al. they obtain more persistent effects beyond five years, which would also arise from our model if the autonomous demand impulse in our IRFs did not decay in the second half of the time horizon.[25] Our hump-shaped estimates show autonomous demand shocks can generate the boom-and-bust patterns characteristic of business cycles. Also, the components of induced demand and output co-move strongly.

The response of the supply side also mimics the hump-shaped response that we find in the

[24] Note that Girardi et al. (2020) estimate IRFs from international panel data following very large autonomous demand shocks. Nonetheless, their estimates share many features with the IRFs we obtain from US data.

[25] Because we have modeled the autonomous demand shock as AR1 and estimate an autoregressive effect less than one, our model cannot produce permanent effects on autonomous demand from an initial temporary shock. The estimated autonomous demand IRF from Girardi et al. remains approximately constant at 70 percent of its initial magnitude from year five to year ten.

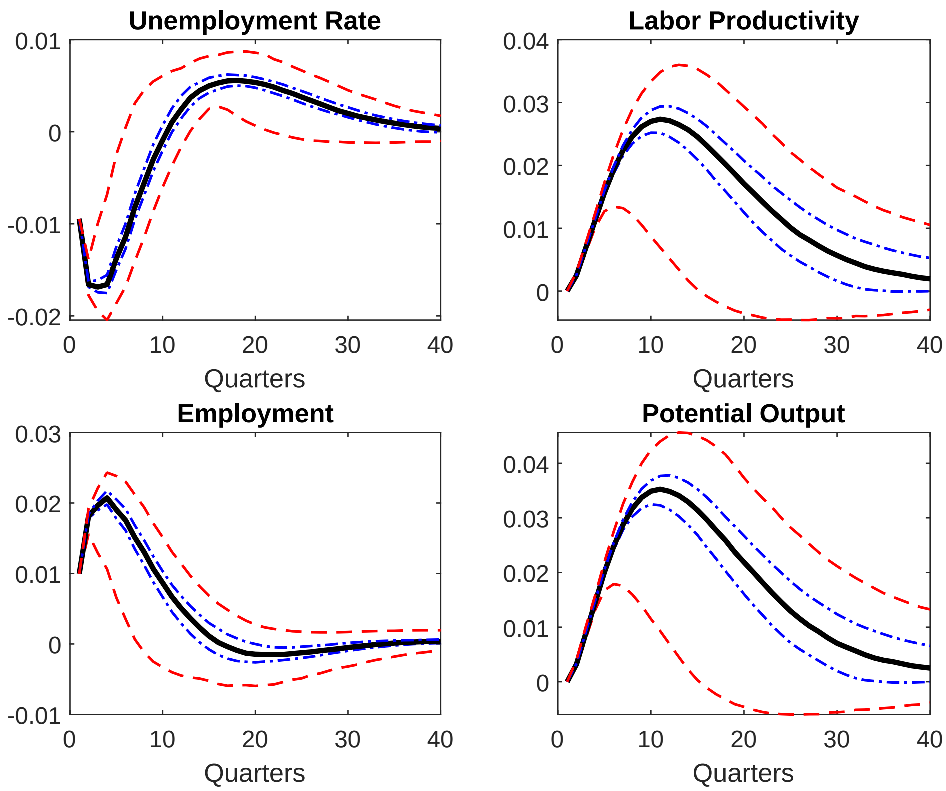
Figure 5: IRF of Demand Side



demand side. The peak response of unemployment and employment, however, come sooner, roughly 2 to 4 quarters after the demand shock hits. Note that the unemployment rate overshoots its steady-state value slightly after 10 quarters, and the economy experiences some slightly elevated unemployment before decreasing monotonically to the steady-state. The response of labor productivity is zero upon impact; however, as unemployment decreases sharply, demand-induced technical change emerges slowly and the peak response occurs roughly at 10 quarters, which coincides with the peak response of the demand side. After that, productivity decreases monotonically to the steady-state. Overall, the supply side reacts in a way that mimics business-cycle co-movements after a demand shock. It is interesting to note that our IRF for unemployment has the same qualitative shape as the one for the long-term unemployed estimated by Girardi et. al. Interestingly, we find a stronger supply-side response, evaluated at the peak value of the IRFs, than what was found by Girardi et al. Our peak supply-side response is also faster than what they found using international data.

The fact that the effects of the autonomous demand shock dies out does not imply that

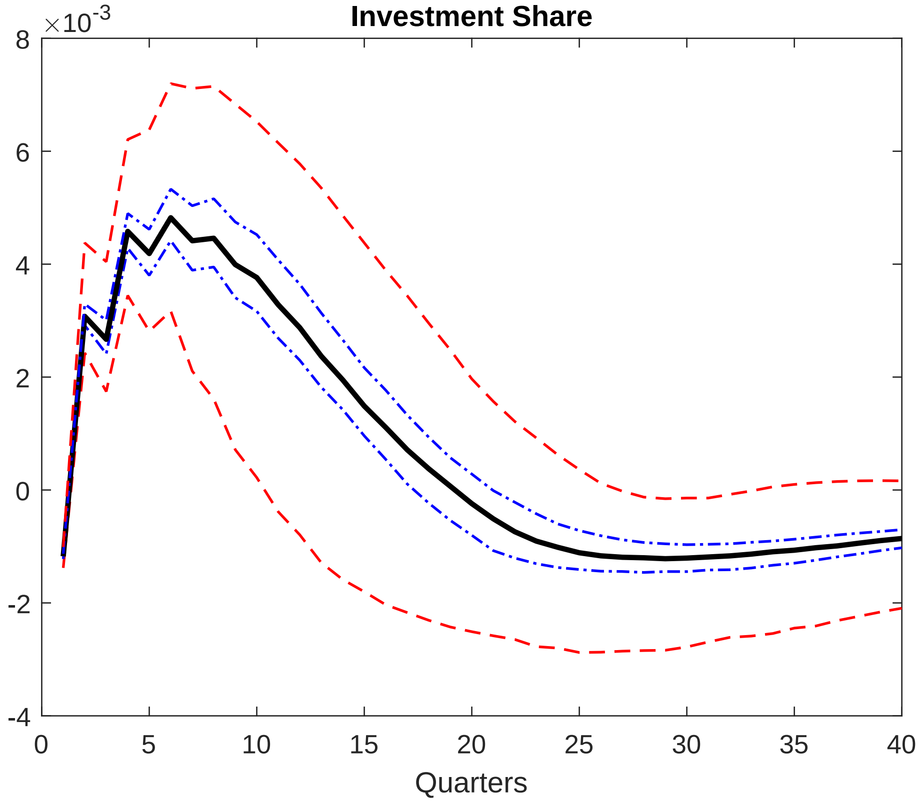
Figure 6: IRF of Supply Side



the supply side does not respond to the demand side in the long run. These IRFs answer the question of what happens if autonomous demand rises unexpectedly for a period and then return to its steady-state value. Hence, it summarises the transitional dynamics of the economy in response to a demand shock. A permanent shock to the level of autonomous demand would have a permanent effect on the level of output and potential output, with the supply side accommodating higher demand through the strong hysteresis effects we estimate (see table 1).

Girardi and Pariboni (2016) show that a positive response of the share of investment in output helps distinguish the empirical implications of demand-led supermultiplier models from other models of aggregate growth. The response of the investment share to an autonomous demand shock implied by our model is depicted in figure 6. The investment share displays a hump-shaped response to autonomous demand shocks, rising until the sixth quarter and then declining monotonically as autonomous demand returns to its steady-state path. This evidence is qualitatively similar to results reported in Girardi and Pariboni (2016), which provides further validation of our estimates.

Figure 7: IRF of the Investment Share



6 Conclusions and Further Research

This paper presents a simple Post-Keynesian growth model where the growth rate of autonomous demand determines the growth rate of output and the steady-state level of unemployment. We ask whether a simple demand-led model can reproduce salient features of the U.S business cycles, as measured by the second moments of seven key macroeconomic variables. We find that the model performs at least as well as a real business cycle model with search and matching frictions along these dimensions.

Furthermore, our model incorporates endogenous responses of labor supply and labor productivity to the demand side of the economy. Stronger demand reduces economic slack which induces faster supply-side growth. We find strong empirical support for these hysteresis effects. The estimates imply that modest reductions in the unemployment rate (or possibly other measures of economic slack) can lead the supply side of the economy to accommodate substantial permanent increases in demand growth. These results imply there is no equilibrium growth rate or unemployment rate that is independent of demand dynamics. Strong demand pulls the supply side up with it, while weak demand can lead to "stagnation traps" along the lines proposed by Benigno and Fornaro (2018).

Numerous topics for further research are suggested by the analysis here; we mention several here. Our estimates of the speed of capital stock adjustment following a demand shock is imprecisely estimated. More refined models of capital and investment dynamics may provide better information about this important economic process. While results presented in section 5.3 offer some information about the extent to which the supply side can accommodate demand growth, further exploration of the limits on growth is warranted, as emphasized in research by Leon-Ledesma and Thirlwall (2002), Coibion et al. (2018) and Fontanari et al. (2020) among others. Also, while our results in section 5.4 provide some interesting findings on the possibility that demand dynamics affect longer run economic performance even when the zero lower bound for interest rates is not binding, this issue needs further study in a model with explicit channels through which interest rates affect demand.

Two other issues deserve discussion. First, given that we find empirical support for the idea that demand dynamics exert an important influence in the long-run, a natural question that arises is how fiscal stimulus affects the long-run performance of the economy. In an ongoing project, Fazzari, Ferri, and González (2023), we study how the growth rate of fiscal expenditures impacts long-run macroeconomic performance, and whether such fiscal stimulus is sustainable. We find that the growth rate of government expenditures affects the growth rate of the economy in the long-run, and debt-financed fiscal stimulus does not raise the long-run ratio of public debt to output as long as the unemployment rate is not constrained by its minimum value. Second, since hysteresis estimates rely on a model without explicit microfoundations, use of the model for policy analysis is subject to the Lucas critique. Are there preferences and technology such that this specification arises as a reduced-form from a model with optimizing microfoundations? In ongoing work, González (2023) shows this is indeed the case, at least in a simplified version of the model without labor supply.

We hope these results stimulate further refinement and extensions of this Keynesian growth framework in which demand dynamics can have an economically significant impact on output and employment beyond the short run.

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A The effects of permanent demand shocks

This section derives fully equations (x) through (xx) in section 2. We start from the assumption that autonomous demand is a random walk in logs:

$$(1-L)\ln F_t = g_z + \varepsilon_t \tag{45}$$

Where L is the lag operator, i.e, $Ly_t = y_{t-1}$. We first derive the time series process for output; we pre-multiply equation (z) by $(1 - L)$ to get:

$$(1 - L) \ln Y_t = (1 - L)(-\ln s_t + \ln F_t) \tag{46}$$
$$= g_z + \varepsilon_t \tag{47}$$

This shows that output is a random walk with a drift, and it follows exactly the same process as autonomous demand. We next derive the stochastic process for unemployment. Start from the log-differenced production function:

$$(1 - L) \ln Y_t = (1 - L) \ln A_t - (1 - L)u_t \tag{48}$$

Replace inside the equations for learning by doing, as well as the stochastic process for output to get:

$$g_z + \varepsilon_t = \phi_0 - \phi_1 u_{t-1} - u_t + u_{t-1} \tag{49}$$

Re arrange in terms of the unemployment rate to get:

$$u_t = \phi_0 - g_z + (1 - \phi_1)u_{t-1} - \varepsilon_t \tag{50}$$

This shows that the unemployment rate is an AR(1) process; hence, permanent demand shocks have some persistent, but finite lived, effects on the unemployment rate. Finally, we solve out for by writing the process for the unemployment rate as:

$$[1 - (1 - \phi_1)L]u_t = \phi_0 - g_z - \varepsilon_t \tag{51}$$

Now take the learning by doing equation in growth from and pre-multiply it by $(1 - L\phi_1)$ to get:

$$[1 - (1 - \phi_1)L]g_{A,t} = [1 - (1 - \phi_1)L](\phi_0 - \phi_1 u_{t-1}) \tag{52}$$

$$g_{A,t} = (1-(1-\phi_1))\phi_0 + (1-\phi_1)g_{A,t-1} - \phi_1(\phi_0 - g_z - \varepsilon_{t-1}) \tag{53}$$
$$= \phi_0\phi_1 + (1-\phi_1)g_{A,t-1} - \phi_0\phi_1 + \phi_1 g_z + \phi_1\varepsilon_{t-1} \tag{54}$$
$$= \phi_1 g_z + (1-\phi_1)g_{A,t-1} + \phi_1\varepsilon_{t-1} \tag{55}$$
$$\tag{56}$$

This shows that log-productivity is an ARIMA(1,1,1) process. To prove that the new shock $u_t = \phi_1\varepsilon_t$ has no serial correlation, note that since it was assumed that ε_t is i.i.d, it follows that $E[\varepsilon_t\varepsilon_{t-j}] = 0 \quad \forall j = 1, ..., T$. Therefore, $E[u_t u_{t-j}] = E[\phi_1^2\varepsilon_t\varepsilon_{t-j}] = 0$, which completes the proof.

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