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## AGGREGATE DEMAND EXTERNALITIES, INCOME DISTRIBUTION, AND WEALTH INEQUALITY

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### ABSTRACT

We study a two-class model of growth and the distribution of income and wealth at the intersection of contemporary work in classical political economy and the post-Keynesian tradition. The key insight is that aggregate demand is an externality for individual firms: this generates a strategic complementarity in production that results in equilibrium underutilization of the economy's productive capacity and hysteresis in real GDP per-capita in balanced growth. This equilibrium inefficiency reverberates into both the functional distribution of income and the distribution of wealth: both the wage share and the workers' wealth share would be higher at full capacity. Consequently, fiscal allocation policy that achieves productive efficiency also attains a higher labor share and a more equitable distribution of wealth. Demand shocks also have permanent level effects. Extensions look at temporary growth and employment effects of fiscal policy with dynamic increasing returns, and employment hysteresis. These findings are useful as an organizing framework for thinking through the lackluster economic record of the so-called Neoliberal era, the sluggish recovery of most advanced economies following the Great Recession, and what to expect regarding the recovery from the Covid-19 shock.

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# Aggregate Demand Externalities, Income Distribution, and Wealth Inequality

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May 28, 2021

## Abstract

We study a two-class model of growth and the distribution of income and wealth at the intersection of contemporary work in classical political economy and the post-Keynesian tradition. The key insight is that aggregate demand is an externality for individual firms: this generates a strategic complementarity in production that results in equilibrium underutilization of the economy's productive capacity and hysteresis in real GDP per-capita in balanced growth. This equilibrium inefficiency reverberates into both the functional distribution of income and the distribution of wealth: both the wage share and the workers' wealth share would be higher at full capacity. Consequently, fiscal allocation policy that achieves productive efficiency also attains a higher labor share and a more equitable distribution of wealth. Demand shocks also have permanent level effects. Extensions look at temporary growth and employment effects of fiscal policy with dynamic increasing returns, and employment hysteresis. These findings are useful as an organizing framework for thinking through the lackluster economic record of the so-called Neoliberal era, the sluggish recovery of most advanced economies following the Great Recession, and what to expect regarding the recovery from the Covid-19 shock.

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# 1 Introduction

From 1980 onward the so-called “Neoliberal era” in the United States has been characterized by a slow but persistent atrophy of federal government economic activity. Figure (1) depicts the contours of what Stiglitz, Tucker, and Zucman (2020) call the “Starving State:” declining federal investment, federal consumption, and federal R&D expenditures as a share of gross domestic product, declining top marginal income tax rates, and declining effective corporate taxation. Predicated on the belief that *laissez faire* policies would deliver first-best results with respect economic growth and individual welfare—a belief abetted by macroeconomic theories depicting unemployment and under-utilization as the result of rational agents responding optimally to exogenous shocks, thereby *ex-ante* ruling out active macroeconomic management (e.g., Lucas, 1977; Kydland and Prescott, 1982)<sup>1</sup>—politicians and policymakers spent decades cutting taxes, deregulating, and divesting.

Despite the prestigious intellectual heritage of Neoliberalism’s *laissez faire* policy orientation, macroeconomic performance in the Neoliberal era has been lackluster at best.<sup>2</sup> Figure (2) depicts several macroeconomic trends characteristic of the post-1980 period. Rising wealth inequality, a decline in labor’s share in national income, decreased labor productivity growth, and a persistent downward trend in capacity utilization all obtain over the period.<sup>3</sup>

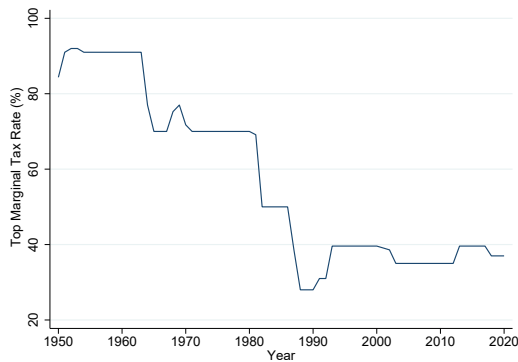
In response to the emergent macroeconomic trends depicted above, economists adopting classical-Marxian or post-Keynesian approaches to growth and distribution have largely studied: (a) the relationship between the functional distribution of income and capacity utilization (Nikiforos and Foley, 2012; Rada and Kiefer, 2015; Petach, 2020), (b) the relationship between long-run growth—both endogenous and exogenous—and the distribution of wealth (Zamparelli, 2016; Petach and Tavani, 2020), and (c) factors related to demand-driven growth and under-utilization (or under-employment) in the long-run (Allain, 2015; Setterfield, 2019; Petach and Tavani, 2019; Tavani and Petach, 2020; Fazzari, Ferri, and Variato, 2020).

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<sup>1</sup>Lucas (1977) articulates this implication of real business cycle theory quite clearly: “[B]y seeking an equilibrium account of business cycles, one accepts *in advance* rather severe limitations on the scope of governmental countercyclical policy which might be rationalized by the theory” (p. 25, italics added).

<sup>2</sup>Other familiar advocates of *laissez faire* include Nobel prize winners Milton Friedman, James Buchanan, and Friedrich Hayek. Several recent works attempt to give a history of the means by which the ideas of these thinkers grew to prominence among academics and policymakers. See MacLean (2017), Slobodian (2018), and Appelbaum (2019).

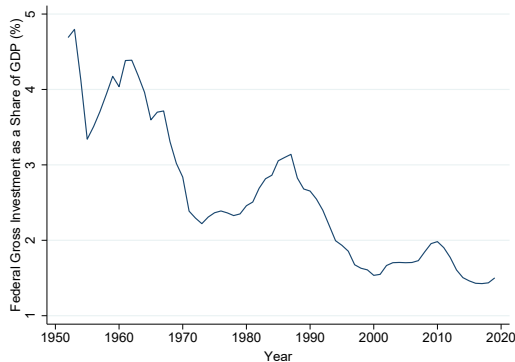
<sup>3</sup>Many authors have debated the possibility of a declining long-term trend in capacity utilization in the United States (Nikiforos, 2016, 2018, 2019; Girardi and Pariboni, 2019; Gahn and Gonzalez, 2019). Part of the so-called “utilization controversy” concerns the validity of the official Federal Reserve Board (FRB) measure of capacity utilization, due to the possibility of measurement error in the value reported by the FRB. Using two alternative measures of capacity utilization derived from the Census Bureau’s Quarterly Survey of Plant Capacity Utilization—the “Full Utilization Rate” and the “National Emergency Utilization Rate”, used in this paper—Gahn (2020) provides evidence that there is indeed a long-term downward trend in capacity utilization.



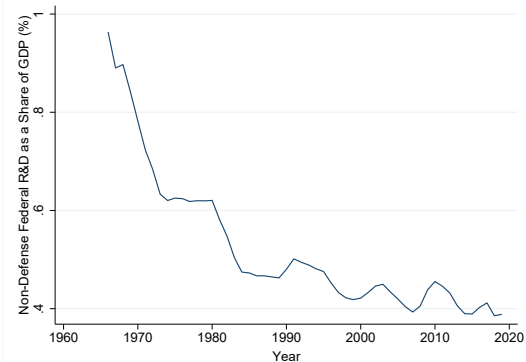
(a) Top Marginal Income Tax Rate



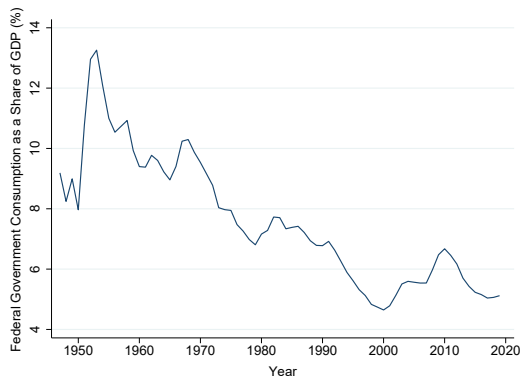
(b) Effective Corporate Tax Rate



(c) Federal Investment Spending



(d) Federal Non-Defense R&D



(e) Federal Consumption Expenditures

Figure 1: The Starving State

*Notes:* Marginal income tax rate data from the Tax Policy Center. Effective corporate tax rate measures tax receipts on corporate income as a percentage of the sum of tax receipts on corporate income and corporate profits after tax, obtained from the Federal Reserve Bank of St. Louis. Federal investment spending measures gross federal investment (FRED series A787RC1Q027SBEA) as a share of gross domestic product, obtained from the Federal Reserve Bank of St. Louis. Federal non-defense R&D measures federal investment in non-defense R&D (FRED series Y069RC1Q027SBEA) as a share of gross domestic product, obtained from the Federal Reserve Bank of St. Louis. Finally, federal consumption expenditures measures federal consumption (FRED series A957RC1Q027SBEA) as a share of GDP, also from the Federal Reserve Bank of St. Louis.

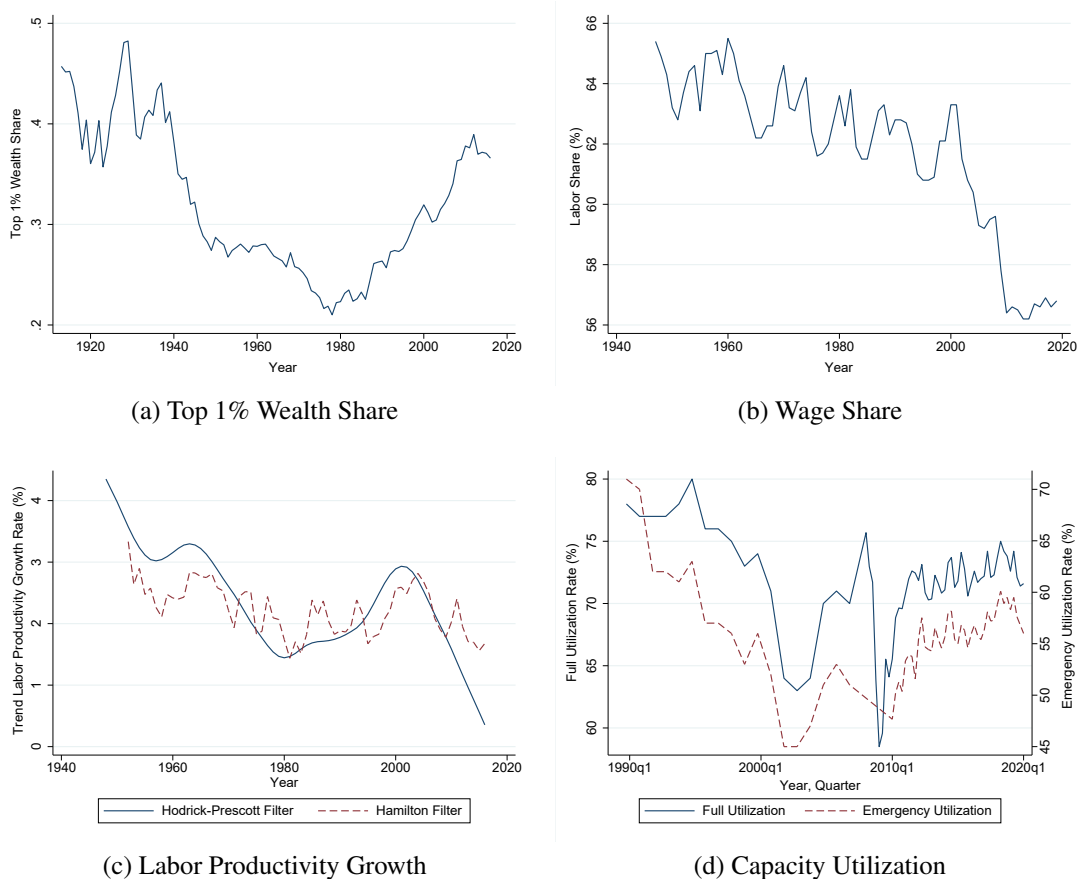


Figure 2: Macroeconomic Trends in the Neoliberal Era

*Notes:* Data on the the top 1% wealth share, wage share, labor productivity, and capacity utilization are from the World Inequality Database, the Bureau of Labor Statistics (wage share and labor productivity), and the Census Quarterly Survey of Plant Capacity Utilization, respectively. Labor productivity data plotted as the trend-component of the labor productivity series, using the Hodrick-Prescott Filter and the Hamilton Filter.

An interesting aspect of the non-mainstream literature on secular stagnation is the dichotomy, both methodological and regarding policy implications, between classical-Marxian approaches on the one hand, and post-Keynesian and Kaleckian approaches on the other. The former emphasize distributive conflict, are amenable to microeconomic foundations (see Marglin, 1984; Foley et al., 2019, for example), but ultimately assign no role to effective demand and activist policy in the long run; the latter embrace the role of effective demand and active economic management, but are usually built on ad-hoc assumptions about the behavioral grounds for individual action.<sup>4</sup>

<sup>4</sup>This point should not be over-emphasized, however. Even in micro-founded models, the choices of the key tradeoffs at stake are ultimately arbitrary. Our goal here is not to advocate for microeconomic foundations *tout court*, but to provide a *specific* microeconomic argument for the possibility of persistent under-utilization and its distributive implications.

On a more concrete level, few classical or Keynesian authors have explicitly studied either the simultaneous implications of activist fiscal policy for welfare and inequality or the relationship between the long-run underutilization of productive capacity and rising wealth inequality. Exceptions that are relevant for our analysis can be found in Ederer and Rehm (2020a,b), who study the evolution of wealth inequality in a neo-Kaleckian model. They show that in the short-run greater wealth inequality (via an increase in the capitalist share of wealth) lowers the rate of capacity utilization because of capitalists' lower marginal propensity to consume. In the long run, the distribution of wealth is endogenous, but—with exogenous income distribution—a rise in the profit share will simultaneously raise the steady-state capitalist wealth share and lower the steady-state rate of capacity utilization, such that one should expect to observe a negative reduced-form correlation between capacity utilization and wealth inequality in the data.

In this paper, we present an analytical model that bridges classical-Marxian and post-Keynesian insights and can be used to address under-utilization and its implications for income distribution and wealth inequality in the Neoliberal era. We focus on the coordinating role of active fiscal policy and its distributional effects, both on the functional distribution of income and on the distribution of wealth. In particular, we develop a stylized micro-to-macro model of utilization and accumulation, situated within modern work in the classical political economy tradition (Harris, 1978; Marglin, 1984; Michl, 2009; Foley et al., 2019), that also incorporates a role for externalities and coordination *à la* Cooper and John (1988). Building on earlier work (Petach and Tavani, 2019; Tavani and Petach, 2020), first, we introduce worker savings and the distribution of wealth, thus going beyond the typical assumption of “hand-to-mouth” workers in classical models; second, we focus explicitly on demand shocks—or alternatively shifts in Keynesian “animal spirits”—in generating path dependence or hysteresis in the economy.

Our simple framework delivers the following implications. On the one hand, and contrary to the arguments of *laissez faire* advocates, we provide strong behavioral reasons to suspect that, left to their own devices, market economies will deliver less-than-efficient outcomes with regards to economic activity and distribution—both income and wealth. Embedding the choice of utilization and Pasinetti (1962) wealth dynamics into a Goodwin (1967)-style growth cycle model we are able to study the welfare implications of activist fiscal policy and the relationship between under-utilization and wealth inequality in the long-run. On the other hand, we provide a simple way of modeling the persistence of shocks in the macroeconomy, at least in levels. This second aspect is useful in thinking not only about secular trends of the kind already discussed above, but the more recent path-dependence displayed by economies like the United States and the European Union in the aftermath of the Great Recession of 2008-09 (Fatas, 2019), as well as its distributive implications.

In a nutshell, the argument is as follows. The fact that individual firms consider aggregate

demand (i.e. aggregate capacity utilization) as an externality, without taking into account the feedback effect of their choice on the economy-wide rate of utilization, implies that equilibrium capacity utilization will be inefficiently low, or in other words that productive capacity will be under-utilized relative to its full utilization level.<sup>5</sup> Excess capacity in product markets reduces labor market tightness via a decline in the employment rate, which depresses the wage share through the usual classical Phillips curve relation. As workers' income falls, their ability to accumulate wealth is also lessened: this, in turn, increases the concentration of wealth in the hands of the capitalist class. Similar to Ederer and Rehm (2020a,b), our model predicts a negative reduced form correlation between capacity utilization and wealth inequality in the long run.<sup>6</sup> However, and unlike previous work, our model suggests both the equilibrium choice of capacity utilization *and* equilibrium distribution (income and wealth) have undesirable welfare properties. In particular, we show that inefficiently low utilization in equilibrium implies that both the labor income share and the workers' wealth share are below what they would be if the economy operated at full capacity. Accordingly, in addition to stimulating employment and production, demand management policies have an important secondary role, namely that of correcting inefficiently high equilibrium inequality by effectively redistributing toward workers. A key feature of our model is thus the elimination of the "efficiency-equity trade-off" that affects classical-Marxian growth models, where the main engine of accumulation and growth is the capitalist profit motive. Yet, this result highlights some important political economy implications: while we show that firms' profits at full utilization are higher in *levels*, both the *share* of profits in total income and the capitalist *share* in total wealth are lower: this finding may help shed some light on the aversion to activist fiscal policy to achieve full-employment by upper-income classes and businesses, which has been a constant in the US political arena at least since the reaction to the New Deal in the 1940s (Kalecki, 1943; Carter, 2020, Ch. 13). In particular, this aspect of the model reflects Kalecki (1943)'s insights on the "political aspects" of full employment, in that—despite greater profits at the full utilization level of output—capitalists may resist policies designed to achieve full employment due to concerns about the effect on their strategic bargaining position vis-a-vis workers, as proxied by changes in distributional variables.

Moreover, we explicitly introduce the possibility of demand shocks, or shifts in animal spirits, entering the firm-level choice of capacity utilization. Contrary to the received wisdom according to which such shocks should display only temporary effects, we show that in fact

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<sup>5</sup>In previous work (Petach and Tavani, 2019; Tavani and Petach, 2020), we emphasized how this way of thinking about the firm-level choice of utilization provides a rationale for an endogenous utilization rate in neo-Kaleckian economics. See also Franke (2020) who makes a similar point in a very similar model, and the literature on the "utilization controversy" already mentioned.

<sup>6</sup>In Section(4) we show that this reduced-form correlation does indeed appear in the data.

there are permanent level effects not only on equilibrium real GDP, but also on the functional distribution of income and on the distribution of wealth. This result points to the importance of stabilization policies, in addition to allocation policy to correct for the inefficiencies already described. It also helps to shed light on the sluggish recovery from the Great Recession of 2008-09, given the documented downward revisions of potential output in many high income economies (Fatas, 2019). Furthermore, it points the attention to the likelihood of long-lasting effects of the Covid-19 shock to the world economy absent appropriate corrective policy measures: if the world economies are prone to hysteresis, the chances of a V-shaped recovery without appropriate demand stimulus are slim. Finally, by modeling aggregate demand as an externality, our paper provides a potential bridge between the non-mainstream literature and recent mainstream work on hysteresis and the persistence of aggregate demand shocks (Engler and Tervala, 2018; Farmer and Platonov, 2019; Cerra, Fatas, and Saxena, 2021)

The rest of the paper is organized as follows. Section (2) describes the economic environment. Section (3) characterizes the balanced growth path equilibrium and examines the dynamics of the model. Section (4) explores the steady state and derives some results for optimal fiscal policy. In particular, it shows that the socially efficient levels of utilization and distribution can be implemented with a simple rebate to the user cost of capital financed by lump-sum taxation, and that the optimal subsidy is set equal to the extent of strategic complementarities. Section (5) illustrates the transitional dynamics of the model following policy shocks with numerical simulations. Section (7) offers some additional discussion and concludes.

## 2 The Model

The economic environment is as follows. We assume a one-sector economy with a large number of identical, competitive firms whose “entrepreneurs” make decisions about factor demands and the rate of utilization of installed capacity, and distribute income to workers and the owner of capital stock. Similarly to Foley et al. (2019), we assume that competition among entrepreneurs implies that they earn no pure rent for their services: the entrepreneurial compensation is subsumed into the firms’ wage bill. There are then two types of households: “workers” who earn wage income and interest income on capital stock, consume and save, and “capitalists” who only earn profit income that is distributed to them by entrepreneurs, consume and save.



## 2.1 Firms

Firms in the economy produce homogeneous output according to the Leontief technology  $Y = \min\{uK, AL\}$ , where  $L$  stands for labor demand,  $A$  is labor productivity,  $K$  stands for capital stock, and  $u$  denotes the rate of utilization of installed capacity. The output-capital ratio at full capacity is normalized to one for notational simplicity.

Operating capital equipment entails a user cost, which would not be incurred if machines remained idle. The user cost function generalizes the one proposed in Petach and Tavani (2019); Tavani and Petach (2020) and depends on own utilization and the firm’s beliefs about the aggregate utilization rate (i.e. aggregate demand) in the economy, denoted by  $\tilde{u}$ , as follows:

$$\delta(u; \tilde{u}, \theta) = \frac{\beta}{\theta} u^{\frac{1}{\beta}} \tilde{u}^{-\frac{\gamma}{\beta}} \quad (1)$$

where  $\gamma, \beta$  are assumed to be positive and bounded above by 1, and  $\theta > 0$  is a parameter related the role of exogenous demand shocks or shocks to “animal spirits” in the economy, as it will become clearer below.

**Assumption 1.** (*Weak Strategic Complementarity*) Throughout this paper, we assume  $\gamma \in [0, 1 - \beta)$ .

This assumption ensures single-crossing, i.e. that the choice of utilization by the firm intersects the 45-degree line  $u = \tilde{u}$ —which is an equilibrium requirement in the model, see below—only once for strictly positive utilization rates.

Both the shock parameter  $\theta$  and the aggregate utilization  $\tilde{u}$  (demand) are taken as a given by individual firms when maximizing profits. Also, they affect the user cost in similar ways, as an increase in either lowers the user cost everything else equal. The difference between the two will show up in equilibrium: while the shock parameter remains as such, the aggregate utilization rate is an endogenous variable in the model.

The firm’s profit maximization problem requires, at each time period, to choose a rate of utilization of installed capacity to maximize the revenues minus wage costs minus the adjustment cost of utilization:

$$\Pi = Y - wL - \delta(u; \tilde{u}, \theta)K \quad (2)$$

subject to the technological constraint  $Y = \min\{uK, AL\}$  given  $\tilde{u}$  and the real wage  $w$ . Formally, the solution amounts to use the Leontief requirement that firms will set effective capital  $uK$  equal to effective labor  $AL$ —so that labor demand will be equal to  $uK/A$ —to restate the problem as choosing utilization at the margin in order to balance the marginal benefit of increasing the usage of machinery with the marginal cost of doing so. In practice, this way of thinking about the firms’ problem amounts to impose the following sequence of events. First,

the firm’s entrepreneurs choose the utilization rate given labor costs and their beliefs about aggregate utilization; then, they choose labor demand so as to equalize effective labor with effective capital. If the corresponding profit is non-negative, a firm will undertake production. Otherwise, it will remain idle.<sup>7</sup>

The choice of utilization, which replaces the constant output-capital ratio in standard classical models, is in fact a best-response function to the aggregate utilization rate (aggregate demand) in the economy. In particular, the firm will utilize more—everything else equal—if aggregate utilization increases:

$$u(\omega; \tilde{u}, \theta) = [\theta(1 - \omega)]^{\frac{\beta}{1-\beta}} \tilde{u}^{\frac{\gamma}{1-\beta}} \quad (3)$$

with  $\partial u / \partial \tilde{u} > 0$ . Notice also that the choice of utilization in this model makes the firm-level demand for labor elastic to the unit labor cost  $\omega$ , even though the underlying technology is Leontief. In fact, the firm-level labor demand is found by inserting the choice of utilization into the profit-maximization proportions of capital and labor:

$$L = \frac{u(\omega; \tilde{u}, \theta)K}{A} \quad (4)$$

which is inversely related to real unit labor costs  $\omega \equiv w/A$  and therefore to the real wage given that  $\partial u / \partial \omega < 0$ . The rationale is as follows: the choice of utilization equalizes the marginal revenue of higher utilization with the marginal user cost. If unit labor costs increase, firms’ (absolute) revenues fall, and the firm can cut back on utilization in order to reduce the user cost. This mechanism produces a feedback from real wages to labor demand despite the fixed-proportion technology that is analogous to factor substitution along a neoclassical production function.

Finally, the best-response function (3) makes it clear why the parameter  $\theta$  can be interpreted as synthetically capturing the role of exogenous shocks or Keynesian “animal spirits.” Independent on any other endogenous variable, firm-level utilization increases in  $\theta$ .

## 2.2 Households: “Capitalists” and “Workers”

We revisit the Pasinetti (1962) problem and distinguish between capitalist households and worker households in order to determine the accumulation of capital stock (wealth) in the economy. Neither type of household makes decisions about the rate of utilization of capital

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<sup>7</sup>We assume that the shutdown costs are zero. This assumption is made here to simplify the analysis, but is of no consequence to what follows provided that shutdown costs are a lump-sum and do not affect the choice of utilization at the margin.

stock: they take the utilization rate chosen by the firms' entrepreneurs as a given. Capitalists earn net profit incomes  $R^c$  on their capital stock so that  $R^c = u(1 - \omega)K^c$ , have log utility from consumption  $c^c$ , and discount the future at a rate  $\rho^c > 0$ , constant.<sup>8</sup> Worker households earn wage income when active (that is when part of labor demand)  $wL = \omega u(K^c + K^w)$  and profit income on the wealth they own  $u(1 - \omega)K^w$ , have log utility from consumption  $c^w$ , and discount the future at a rate  $\rho^w > \rho^c$ . As explained above, this assumption: (a) is *de facto* equivalent to assuming workers to have a lower propensity to save than capitalists,<sup>9</sup> and (b) synthetically captures the role of wealth holdings in determining household time-impatience.<sup>10</sup>

A major focus of this contribution is the share of wealth accruing to capitalists in the economy:  $K^c/(K^c + K^w) \equiv \phi \in [0, 1]$ . Workers' income  $y^w$  each period can be written as:

$$y^w = \frac{u}{1 - \phi} [\omega + (1 - \phi)(1 - \omega)] K^w \quad (5)$$

As shown in Appendix A, intertemporal optimization under perfect foresight for both types of households, gives the two classes' consumption Euler equations:

$$\frac{\dot{c}^c}{c^c} = u(1 - \omega) - \rho^c \quad (6)$$

$$\frac{\dot{c}^w}{c^w} = \frac{u}{1 - \phi} [\omega + (1 - \phi)(1 - \omega)] - \rho^w \quad (7)$$

### 3 Balanced Growth Equilibrium and Dynamics

An equilibrium growth path is defined by: (a) sequences of consumption and capital stock such that utility is maximized given the resource constraints for both classes; (b) demands for capital and labor such that profits are maximized; (c) a rate of utilization such that profits are maximized *and firms' beliefs are realized*, so that  $u(t) = \tilde{u}(t) \forall t$ . Balanced growth requires that, d) for both classes, consumption and capital stock grow at the same rate:  $\dot{c}^i/c^i = \dot{k}^i/k^i = g^i, i = \{c, w\}$ . The equilibrium utilization rate is

$$u(\omega; \bar{\theta}) = \bar{\theta}(1 - \omega)^{\frac{\beta}{1 - \beta - \gamma}} \quad (8)$$

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<sup>8</sup>As explained above, profits are distributed to households after the user cost has been paid by entrepreneurs: that is why we consider *net* profit income  $R^c$  and  $rK^w$ .

<sup>9</sup>For empirical evidence on differential savings rates and their implications for growth, see Petach and Tavani (2021).

<sup>10</sup>The continuous time nature of our model and consequent issues with known terminal times for the households' planning horizons makes it difficult to justify differential saving propensities along the lines suggested by Michl (2009), namely that workers save for the life cycle while capitalists save for dynastic purposes. However, the difference in discount rates could be explained by appealing to the "perpetual youth" households in the Yaari (1965) or Blanchard (1985) models while letting capitalist dynasties last forever.

with the parameter  $\bar{\theta} \equiv \theta^{\beta/(1-\beta-\gamma)}$  denoting the *aggregate*—as opposed to the individual firm effect  $\theta$ —effect of exogenous shocks or “animal spirits” on aggregate demand/utilization, and  $u_\omega \equiv \partial u/\partial \omega < 0$ . Using this information, we find the two classes’ accumulation rates as:

$$g^c = u(\omega; \bar{\theta})(1 - \omega) - \rho^c \quad (9)$$

$$g^w = \frac{u(\omega; \bar{\theta})}{1 - \phi} [\omega + (1 - \phi)(1 - \omega)] - \rho^w \quad (10)$$

At a balanced growth equilibrium, the economy-wide growth rate of capital stock is a weighted average of the two classes’ accumulation rates, the weight being given by their respective shares in total wealth. From  $g = \phi g^c + (1 - \phi)g^w$ , factoring and simplifying, we find:

$$g = u(\omega; \bar{\theta}) - [\phi \rho^c + (1 - \phi)\rho^w] \quad (11)$$

### 3.1 Dynamics of the Distribution of Wealth

The capitalist share of wealth evolves through a replicator equation (Samuelson and Modigliani, 1966; Zamparelli, 2016; Ederer and Rehm, 2020a,b):

$$\dot{\phi} = \phi[g^c - g]$$

which, using (9) and (11), simplifies to:

$$\dot{\phi} = \phi [(1 - \phi)(\rho^w - \rho^c) - \omega u(\omega; \bar{\theta})] \quad (12)$$

### 3.2 Cyclical Growth Dynamics

To close the model, we embed the results of the utilization choice and the dynamics of wealth into simple growth cycle dynamics. Following Goodwin (1967), assume that the real wage grows with the employment rate  $e \equiv uK/AN$ , where  $N$  is the total labor force, according to the usual classical Phillips curve:  $\dot{w}/w = f(e)$ ,  $f(0) < 0$ ,  $f_e > 0$ ,  $f_{ee} \geq 0$ , and let labor productivity grow exogenously at a rate  $\alpha > 0$ . The wage share, then, evolves over time with the difference between the growth rate of the real wage and the growth rate of labor productivity:

$$\dot{\omega} = [f(e) - \alpha]\omega \quad (13)$$

We will use the original Goodwin specification of a linear wage-Phillips curve in what follows:  $f(e) = -\xi + \lambda e$ ,  $\xi > 0$ ,  $\lambda > 0$ . Finally, log-differentiating the employment rate  $e \equiv uK/(AN)$ , we find the following differential equation:

$$\begin{aligned}\frac{\dot{e}}{e} &= \left[ \frac{\dot{u}}{u} + g - (\alpha + n) \right] \\ &= \left\{ -\frac{\beta}{1 - \beta - \gamma} \frac{\omega}{1 - \omega} [f(e) - \alpha] + [u(\omega; \bar{\theta}) - (\phi \rho^c + (1 - \phi) \rho^w)] - (\alpha + n) \right\} \quad (14)\end{aligned}$$

We thus have a three-dimensional dynamical system tracing the evolution of the the distribution of wealth  $\phi$ , the functional distribution of income  $\omega$  and the employment rate  $e$  as described by equations (12), (13), and (14). We first characterize the steady state and draw policy implications. A detailed analysis of the local stability properties is provided in the Appendix.

## 4 Steady State and Policy

We start with characterizing the long-run distribution of wealth. Equation (12) has two steady-states: the Samuelson-Modigliani “dual” steady state  $\phi_{ss} = 0$  (Samuelson and Modigliani, 1966) and the Pasinetti (1962) two-class steady state, which is the focus of our analysis and we write in preliminary form as follows:

$$\begin{aligned}1 - \phi_{ss} &= \frac{u(\omega; \bar{\theta})\omega}{\rho^w - \rho^c} \\ &= \frac{\bar{\theta}(1 - \omega)^{\frac{\beta}{1 - \beta - \gamma}}\omega}{\rho^w - \rho^c}\end{aligned} \quad (15)$$

Notice first that the assumption of a higher rate of time-preference on behalf of workers ( $\rho^w > \rho^c$ ) guarantees that two-class wealth distribution is positive. This is analogous to the Pasinetti requirement that the workers’ saving propensity be less than the capitalists’ saving propensity. Also, for  $\phi_{ss}$  to lay between zero and one, we also need  $\rho^w - \rho^c > u(\omega_{ss})\omega_{ss}$ , which we will be assuming throughout. Furthermore, the nullcline representing the workers’ (capitalists’) wealth share is hill-shaped (U-shaped) in the wage share, because there are two forces at play here. On the one hand, the equilibrium utilization rate decreases in the wage share, but increases overall income in the economy. On the other hand, a higher wage share increases the funds available for capital accumulation by workers.<sup>11</sup>

Next, the law of motion for the wage share pins down the steady state employment rate, tied

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<sup>11</sup>Of course, the wage share is endogenous in this model: but it is nevertheless informative to think about the fact that there will be a value of the wage share of income that maximizes the workers’ wealth share. This information can be useful in thinking about policy levers that can be used in order to redistribute wealth in this economy. If the wage share was exogenous, as would be the case in a labor-abundant economy à la Lewis (1954) before the so-called “turning point,” its wealth-share maximizing value would be  $\omega^* = 1 - \frac{\beta}{1 - \gamma} \in (0, 1)$ .

up to labor productivity growth, as:

$$e_{ss} = f^{-1}(\alpha) = \frac{\xi + \alpha}{\lambda} \quad (16)$$

which, as it is standard in the literature on the growth cycle, is fully exogenous given the exogenous nature of technical change.<sup>12</sup> Finally, the law of motion for the employment rate can be used in order to solve for the long-run value of the wage share. We focus here on studying the two-class steady state. Start by considering that, imposing  $\dot{e} = 0$  at the Pasinetti steady state, we have:

$$\begin{aligned} u(\omega; \bar{\theta}) &= \phi(\rho^c - \rho^w) + \rho^w + (\alpha + n) \\ &= (1 - \phi)(\rho^w - \rho^c) + (\rho^c + \alpha + n) \end{aligned}$$

Simple manipulation using (15) then leads to the following long-run solution for income shares:

$$1 - \omega_{ss} = \left( \frac{\rho^c + \alpha + n}{\bar{\theta}} \right)^{\frac{1-\beta-\gamma}{1-\gamma}} \quad (17)$$

This solution can be plugged into equation (15) in order to solve for the long-run wealth distribution in terms of parameters only. Note that: (a) as in the original Goodwin (1967) model, the wage share increases in the capitalist propensity to save: a reduction in the rate of time preference  $\rho^c$  increases capitalist accumulation and therefore the long-run share of wages. Moreover, (b) the steady state wage share is directly related to the shock parameter  $\bar{\theta}$ .

In order to draw policy implications, we must establish whether the economy operates at full capacity. The analysis below shows that: (i) this is not the case, and (ii) it has implications for both wealth and income distribution.

## 4.1 Full Utilization

Consider a benevolent planner solving the choice of utilization under the additional constraint that  $u = \tilde{u}$ , that is internalizing the aggregate demand externality. The resulting, full (efficient) utilization is

$$u^*(\omega) = \bar{\theta} \left( \frac{1 - \omega}{1 - \gamma} \right)^{\frac{\beta}{1-\beta-\gamma}} \quad (18)$$

and it is always higher than equilibrium utilization provided that  $1 - \beta > \gamma > 0$  as per Assumption 1. This finding has implications not only for the long-run functional distribution of income, but also for the long-run distribution of wealth. In fact, consider first the long-run

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<sup>12</sup>See Section 6.2 for an extension of the model that relaxes this conclusion.

worker's share of wealth at full utilization:

$$1 - \phi_{ss}^* = \left( \frac{1}{1 - \gamma} \right)^{\frac{\beta}{1 - \beta - \gamma}} \left[ \frac{\bar{\theta}(1 - \omega)^{\frac{\beta}{1 - \beta - \gamma}} \omega}{\rho^w - \rho^c} \right] \quad (19)$$

Next, the wage share at full utilization can be found, using the same procedure as above, from:

$$1 - \omega_{ss}^* = (1 - \gamma)^{\frac{\beta}{1 - \gamma}} \left( \frac{\rho^c + \alpha + n}{\bar{\theta}} \right)^{\frac{1 - \beta - \gamma}{1 - \gamma}} \quad (20)$$

We can then state the following result, proven in the Appendix.

**Proposition 1.** *At a steady state of the model, both the wage share and workers' wealth share are inefficiently low. Moreover, both the wage share and the workers' share of wealth permanently increases following a positive demand shock.*

A first implication of this result is that—by operating at less than full capacity—the long-run position of this economy is also characterized by inefficiently high wealth inequality and an inefficiently low wage share. Thus, policies that push the economy toward the full capacity equilibrium will not only have the effect of raising real GDP, but also of increasing the workers' share of both income and wealth in the economy. This conclusion seems puzzling at first glance: since both the wage and profit share on the one hand, and the capitalists' and workers' wage share on the other sum up to one, it must necessarily be the case that the higher wage share and workers' wealth share that this economy can attain along the efficient path would occur at the detriment of the capitalists in the economy. However, one must notice that, at the efficient utilization rate, the economy's *total profit income* will be higher than in equilibrium, despite their shares in income and wealth being lower. This is easily seen by calculating the firm's profit function in equilibrium and at the efficient choice of utilization. Using (8) evaluated at (17) and (18) evaluated at (20) we find:

$$\Pi = (1 - \beta \bar{\theta}^{\frac{1 - \gamma - 2\beta}{\beta}}) (\rho^c + \alpha + n) K \quad (21)$$

$$\Pi^* = \left( \frac{1}{1 - \gamma} \right)^{\frac{\beta^2}{(1 - \gamma)(1 - \beta - \gamma)}} \Pi \quad (22)$$

with  $\Pi^* > \Pi$  for  $\gamma \in (0, 1 - \beta)$ . Resistance to full-employment policy is therefore insufficiently explained by appeals to profitability, as total profit income is *lower* in equilibrium. Instead, one might infer from our model that—in resisting fiscal stimulus—capitalists and entrepreneurs are expressing concern about the implications of distributional changes for their strategic bargaining position vis-a-vis workers. Indeed, the model rationalizes Kalecki (1943)'s insight that

although “profits would be higher under a regime of full employment than they are on the average under *laissez-faire*” capitalists nonetheless resist this regime because “the social position of the boss would be undermined” and “‘discipline in the factories’ and ‘political stability’ are more appreciated than profits by business leaders” (p. 3).

A secondary implication of the model is that—similar to Ederer and Rehm (2020a,b)—there exists an inverse reduced-form correlation between capacity utilization and wealth inequality in the long-run, insofar as policies that push the economy toward full utilization also reduce the capitalist wealth share (and vice-versa). Figure (3) plots the share of wealth held by the top 1% against two alternative measures of capacity utilization<sup>13</sup>. In both cases capacity utilization and wealth inequality are negatively correlated. The slope of the naïve OLS estimates suggests that a one percentage-point increase in the rate of capacity utilization reduces the top 1% wealth share between 0.33 and 0.45 percentage points.

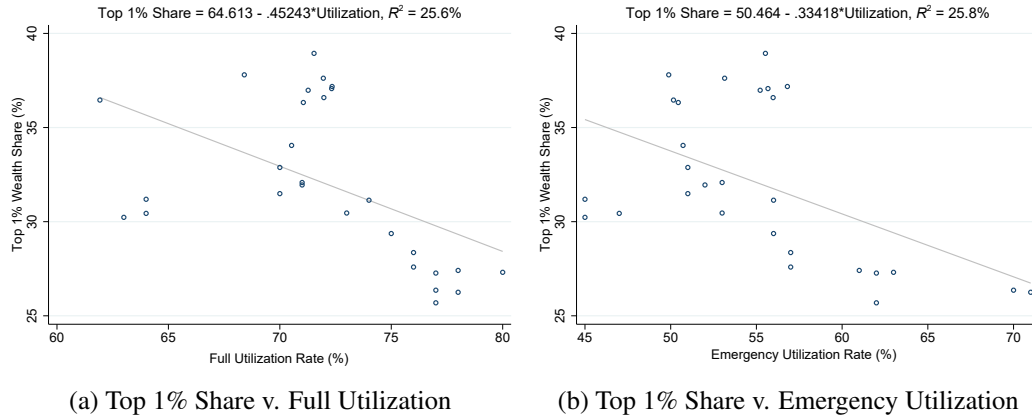


Figure 3: Wealth Inequality and Capacity Utilization, 1989-2016

Notes: Capacity utilization data from the Census Quarterly Survey of Plant Capacity Utilization. Top 1% wealth share data from the World Inequality Database.

A third implication is that positive demand shocks have progressive and permanent level effects not only on real GDP, but also on the distribution of income and wealth. Mechanically, an increase in the parameter  $\bar{\theta}$  shifts the best-response function (3) upward, implying higher equilibrium utilization. This in turn lowers the capitalist wealth share, given that the two are inversely related. Moreover, it increases the long-run share of wages in national income. The intuition is the following: everything else equal, an increase in  $\bar{\theta}$  fosters capital accumulation and employment in the short run. But the long-run growth rate, equal to the growth rate of labor productivity  $\alpha$ , is constant: restoring the balanced growth condition  $g = \alpha + n$  requires the profit rate to fall, which can only happen through an increase in the wage share. Given

<sup>13</sup>Full Utilization and Emergency Utilization from the Census Quarterly Survey of Plant Capacity Utilization.



that workers' labor income has increased, their funds available to accumulation also increase, which leads to an increase in the workers' wealth share.

## 4.2 Decentralization

Let us introduce a government authority that taxes firms lump-sum at the rate  $\tau$ , and rebates the tax proceedings in the form of a user cost rebate  $\sigma$ . Firms now maximize profits given by  $\Pi = uK(1 - \omega) - \delta(u; \tilde{u})(1 - \sigma)K - \tau$ . Assume further that the government runs a balanced budget at all times, so that  $\sigma\delta(u; \tilde{u}) = \tau$  always. The firm-level choice of utilization now fulfills

$$u(\omega; \tilde{u}, \sigma, \theta) = \theta \left( \frac{1 - \omega}{1 - \sigma} \right)^{\frac{\beta}{1 - \beta}} \tilde{u}^{\frac{\gamma}{1 - \beta}}$$

and is increasing in the policy parameter  $\sigma$ . Imposing the equilibrium condition  $u = \tilde{u}$  gives aggregate utilization as a function of government spending (and the wage share) as:

$$u(\omega; \sigma, \bar{\theta}) = \bar{\theta} \left( \frac{1 - \omega}{1 - \sigma} \right)^{\frac{\beta}{1 - \beta - \gamma}} \quad (23)$$

which makes it clear that setting  $\sigma = \gamma$  achieves the full utilization rate. The previous findings have already shown that an economy operating at full utilization also features a higher wage share and a higher workers' wealth share. Also, as shown in Tavani and Petach (2020), we can recover the fiscal multiplier as the ratio of the equilibrium response to the policy over the individual response. We can summarize these results in the following proposition.

**Proposition 2.** *A fiscal authority can implement the full utilization rate through a user cost subsidy  $\sigma = \gamma$ . The policy increases the wage share of income and the workers' share of wealth in the economy. The balanced budget fiscal multiplier is equal to  $m = (1 - \beta)/(1 - \beta - \gamma) > 1$ .*

The result that fiscal policy that increases capacity utilization in the economy also has progressive effects on the distribution of income is similar to the findings in Rada and Kiefer (2015). Their estimates suggest the so-called *demand regime*—the long-run dependence of utilization on the labor share—to be downward sloping or profit-led, and the same is true in our model as per equation (8). They also estimate the so-called *distributive curve*, that is the long-run relation between the labor share and utilization, and find it displays profit-squeeze in that it is upward sloping. The combination of profit-led demand and profit-squeeze distribution delivers progressive effects of a fiscal expansion. In our model, utilization is profit-led as per equation (8), but distributive curve is flat as per equation (17). The progressive effects of fiscal policy arise through the Harroddian balanced growth condition that requires the accumulation rate to equal the growth rate of the effective labor force:  $g = \alpha + n$ . A fiscal expansion that

achieves the full rate of utilization puts pressure on the accumulation rate  $g$ , but the long run growth rate  $\alpha + n$  is constant: the wage share must increase in order to restore balanced growth.

Furthermore, the welfare properties of the equilibrium path and the possibility of decentralizing it through taxes and subsidies have some interesting political economy implications. In fact, while decentralizing the efficient rate of utilization makes profit-earning households better off in *absolute* terms—they will earn higher profits, after all—it ultimately implies a worsening of their *relative* position both income- and wealth-wise—their shares in both income and wealth will be lower. This result paraphrases the argument in Kalecki (1943), by highlighting a reason for the elites to resist government intervention to achieve full utilization of the economy’s productive resources, given that it will worsen their relative status in the economy.

## 5 Dynamics and Numerical Simulations

Appendix C provides a formal analysis of the local stability properties of the two-class steady state of the model, and shows that it is locally stable under mild conditions on parameters. This finding marks a difference with the benchmark Goodwin (1967) model where the struggle between the two classes leads to perpetual cycles in employment and distribution. As highlighted by Shah and Desai (1981) and van der Ploeg (1985), the disappearance of the Goodwin cycle in the long run arises when firms can counter the workers’ wage demands following increases in employment through changes in the technique of production that substitute labor for capital. This mechanism is guaranteed in our model by the choice of utilization which, as shown above, makes labor demand elastic to the wage despite the Leontief technology.<sup>14</sup>

While the framework is simple enough that it can be studied analytically, it is interesting to visualize the transitional dynamics numerically in simulations. Appendix D provides details on the calibration used for these simulation rounds. Importantly, these exercises are meant to showcase the qualitative features of the transitional dynamics and *not* to accurately replicate or predict the quantitative behavior of actual economies. Two numerical exercises are of interest: (1) the effect of a demand shock, and (2) a decentralization policy along the lines of Section 4.2 taking place at time zero.<sup>15</sup> The left panel of Figure 4 displays the transitional dynamics following of a 2.5% demand shock occurring at time zero; while the right panel plots the transitional dynamics of implementing the decentralization policy outlined above, again at time zero, supposing that the system was operating in *laissez faire* before the policy. In both plots, the economy starts in balanced growth equilibrium; and the three variables of interest are plotted

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<sup>14</sup>The ultimate dampening of the cycle does not mean, however, that there won’t be oscillations at all: the simulations shown below show that oscillations do in fact occur in the aftermath of a policy or demand shock, before the system converges to its new steady state.

<sup>15</sup>The simulations are coded in *Mathematica* 12, and are available at <https://danieletavani.com/code>.

as ratios of their value after the shock over their pre-shock value. The dotted blue line is flat at 100% to facilitate visualization.

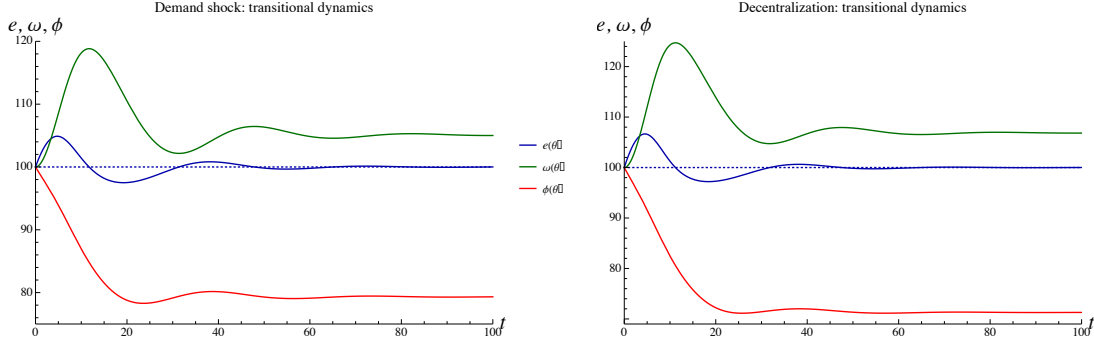


Figure 4: The effect of a demand shock (left) and of a fiscal policy shock (right).

## 6 Extensions

### 6.1 Temporary Growth and Employment Effects of Demand Policy with a Kaldor-Verdoorn Law

A popular specification of technical change in post-Keynesian economics is the one based on the idea by Verdoorn (1949) and Kaldor (1957) that capital accumulation can stimulate labor productivity growth through dynamic increasing returns. The corresponding Kaldor-Verdoorn law translates into our model in the following specification for the growth rate of labor productivity:

$$\alpha = \mu g, \quad \mu \in (0, 1) \quad (24)$$

where  $g$  is given by equation (11) along an equilibrium growth path. In balanced growth, the accumulation rate  $g$  must equal the Harrod rate  $\alpha + n$ , so that the long-run growth rate of labor productivity is of the *semi-endogenous* kind: it is derived within the model but policy-invariant (Jones, 1995). Solving, we find the following values for long-run labor productivity growth and employment:

$$\alpha = \frac{n}{1 - \mu}; \quad e_{ss} = \frac{(1 - \mu)\xi + n}{(1 - \mu)\lambda} \quad (25)$$

While there are no policy effects nor effects of demand shocks on the growth rate (nor employment) in the long run, this extended model produces temporary growth effects along the transitional dynamics. We explore the effects of a demand shock in simulations: the plots corresponding to an exogenous demand shock and the decentralization are displayed in Figure 5.

The dynamics are quite similar to the benchmark model. Long-run employment does not vary, since it is tied up to a semi-endogenous growth rate: in fact, it is easy to verify from (25) that  $\partial e_{ss}/\partial \mu = 0$ .

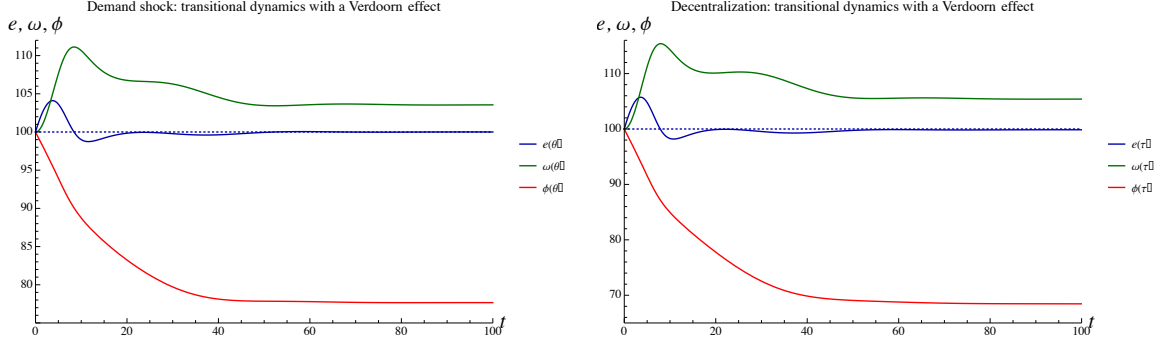


Figure 5: The effect of a demand shock (left) and of a fiscal policy shock (right) with a Verdoorn effect.

## 6.2 Employment Hysteresis

A shortcoming of the analysis we detailed thus far is that hysteresis effects manifest only on the rate of capacity utilization but not on the employment rate which, as exemplified in Stockhammer (2008), is basically a classical-Marxian version of an exogenously given NAIRU. A simple way to capture employment hysteresis amounts to extend the classical Phillips curve  $\dot{w}/w = f(e)$  in order to incorporate a feedback from utilization to real wage growth. To economize on notation, we assume that the intercept term in the wage-Phillips curve now depends on utilization as follows:<sup>16</sup>

$$\frac{\dot{w}}{w} = -\xi(u) + \lambda e, \quad \xi_u > 0 \quad (26)$$

so that the steady state employment rate, always found by setting the change in the wage share equal to zero in equation (13), now depends directly on the utilization rate:

$$e_{ss} = \frac{\xi[u(\omega_{ss}; \bar{\theta})] + \alpha}{\lambda} \quad (27)$$

Now, equilibrium unemployment will also be inefficiently low, and demand shocks will have permanent effects on employment, too, given the feedback from utilization. The effects of a

<sup>16</sup>The term  $\xi$  in the Phillips curve can be thought of as capturing the effect of price-inflation on real wages. In fact, the corresponding equation can be written as relating the growth rate of *nominal* wages  $\dot{w}/w + \xi = \lambda e$  when  $\xi$  is taken as the inflation rate. Rendering  $\xi = \xi(u)$  amounts to synthetically capture the role of stronger aggregate demand in determining price increases.

demand shock on this version of the model are illustrated in the simulations in Figure 6. For these simulations, we used a linear  $\xi(u)$  function:  $\xi(u) = \xi_0 + \xi_1 u$ . In both cases, long-run

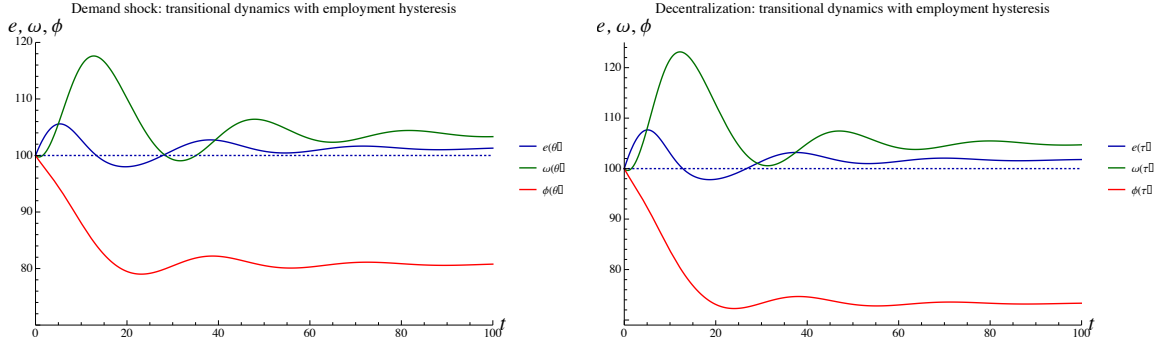


Figure 6: The effect of a demand shock (left) and of a fiscal policy shock (right) with employment hysteresis.

employment permanently increases following both a time-zero positive demand shock and a fiscal policy shock that decentralizes the full (efficient) utilization rate. Finally, a similar result would be obtained by postulating a dependence of labor productivity growth on the utilization rate, say  $\alpha = \alpha(u)$ ,  $\alpha_u > 0$ , with the important difference that in this case hysteresis would not affect just the long-run employment rate as here, but also the economy long-run growth rate.

## 7 Conclusion

In this paper, we study a growth and distribution model at the intersection of contemporary work in classical political economy and post-Keynesian economics. The intersection amounts to introducing a role for aggregate demand as an externality in an otherwise standard two-class model featuring workers' saving and the distribution of wealth (Foley et al., 2019, Ch. 17). Additionally, by incorporating aggregate demand as an externality, our model provides a potential bridge between the non-mainstream literature and recent mainstream work on hysteresis and animal spirits (Engler and Tervala, 2018; Farmer and Platonov, 2019; Cerra, Fatas, and Saxena, 2021). The resulting framework produces equilibrium under-utilization coexisting with capital accumulation; and the inefficiency reverberates on both income and wealth distribution. Similar to Cooper and John (1988), we highlight the coordinating role of allocation policy. Moreover, we showed that a fiscal package that decentralizes the efficient utilization rate will have progressive distributional effects. We motivated our contribution with the retreat of the State from allocation policy during the Neoliberal era: our analysis suggests that market economies, left on their own, are likely to be prone to stagnation and inequality, and that the

government can play an important role in coordinating economic activity while at the same time taming the concentration of wealth and ameliorating the workers' distributional position.

We suggested that, despite the possibility of achieving allocational efficiency and a more equitable distribution of both income and wealth, the fact that both the wage share and the workers' wealth share increase through fiscal policy that push the economy toward full capacity may prevent the owners of capital stock from signing off on such policies, even in the face of higher total profits. As such, our argument can be seen as offering—to paraphrase Kalecki (1943)—insights on the “political aspects of full utilization.”

Finally, we looked at the effects of aggregate demand shocks, or equivalently shocks to Keynesian “animal spirits,” and show that they have important distributional effects of the same sign on both the long-run wage share and the workers' wealth share. In the baseline model, there are no long-run employment effects of either allocation policy nor demand shocks: but a simple extension allowing for employment hysteresis points to the long-run effect of both kinds of shocks on steady-state employment. Given the lackluster economic recovery following the Great Recession, our hope is that this simple model provides an organizing framework for thinking through the persistent effects of shocks to economic activity—such as the Covid-19 recession—and the importance of countervailing economic measures.

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## A Dynamic Optimization

The capitalist's optimization program that leads to equation (6) in the body of the paper has already been studied in Tavani and Petach (2020). As for the workers' problem, crucial assumptions are: that neither workers nor capitalist households are responsible for the utilization choice, so they take  $u(t)$  as a given at all times; and that each worker household is negligible enough not to be able to internalize its influence either on the economy-wide wealth or income distribution. Hence, the typical worker household takes  $\phi(t)$  and  $\omega(t)$  as a given at all times. It solves the following problem:

$$\begin{aligned}
 & \text{Given } \{u(t), \omega(t), \phi(t)\} \forall t, \\
 & \text{Choose } \{c^w(t)\}_{t \in [s, \infty)} \quad \text{to max } \int_s^\infty \exp\{-\rho^w(t-s)\} \ln c^w(t) dt \\
 & \text{s. t. } \quad \dot{K}^w(t) = \frac{u(t)}{1-\phi(t)} [\omega + (1-\phi(t))(1-\omega)] K^w(t) \\
 & \quad \quad K^w(s) \equiv K_s^w > 0, \text{ given} \\
 & \quad \quad \lim_{t \rightarrow \infty} \exp\{-\rho(t-s)\} K^w(t) \geq 0
 \end{aligned} \tag{28}$$

This problem involves a strictly concave objective function to be maximized over a convex set. Thus, with co-state variable  $\mu(t)$ , the standard first-order conditions on the associated current-value Hamiltonian

$$\mathcal{H} = \ln c + \mu \left\{ \frac{u}{1-\phi} [\omega + (1-\phi)(1-\omega)] \right\} K^w$$

will be necessary and sufficient for an optimal control. They are:

$$c^{-1} = \mu \tag{29}$$

$$\rho\mu - \dot{\mu} = \mu \left\{ \frac{u}{1-\phi} [\omega + (1-\phi)(1-\omega)] \right\} \tag{30}$$

$$\lim_{t \rightarrow \infty} \exp\{-\rho^w t\} \mu(t) K^w(t) = 0 \tag{31}$$

To obtain the Euler equation for consumption, differentiate (29) with respect to time and use (29) and (30) to get:

$$\frac{\dot{c}^w}{c^w} = \frac{u}{1-\phi} [\omega + (1-\phi)(1-\omega)] - \rho^w$$

Imposing a balanced growth path where consumption and capital stock grow at the same rate gives equation (7). As it is standard, what ensures that workers' consumption and capital stock grow at the same rate is that workers' consume a constant fraction of their end-of-period wealth, the fraction being equal to the discount rate  $\rho^w$ .

## B Proof of Proposition 1

Both results can be proven directly, by comparing the equilibrium solutions with the efficient solution. Start with the wealth share. Equation (19) can be rearranged to obtain

$$\frac{1 - \phi_{ss}^*}{1 - \phi_{ss}} = \left( \frac{1}{1 - \gamma} \right)^{\frac{\beta}{1 - \beta - \gamma}} > 1$$

which shows that the workers' wealth share at full utilization is higher than in equilibrium. For the wage share of income, equation (20) implies that

$$\frac{1 - \omega_{ss}^*}{1 - \omega_{ss}} = (1 - \gamma)^{\frac{\beta}{1 - \gamma}} < 1$$

which proves that the profit (labor) share at full utilization is lower (higher) than in equilibrium.

## C Local Stability Analysis

Linearization of the dynamical system formed by (14), (13), and (12) around the two-class steady state with  $\phi_{ss} \in (0, 1)$  given by (15) gives a Jacobian matrix with the following sign structure:

$$J_{ss} = \begin{bmatrix} J_{11} & J_{12} & J_{13} \\ (-) & (-) & (+) \\ J_{21} & 0 & 0 \\ (+) & & \\ 0 & J_{32} & J_{33} \\ & (\pm) & (-) \end{bmatrix}$$

given that:

$$\begin{aligned} J_{11} &= \left. \frac{\partial \dot{e}}{\partial e} \right|_{ss} = -\frac{\beta}{1 - \beta - \gamma} \frac{\omega_{ss}}{1 - \omega_{ss}} \lambda e_{ss} < 0 \\ J_{22} &= \left. \frac{\partial \dot{\omega}}{\partial \omega} \right|_{ss} = u_{\omega} e_{ss} < 0 \\ J_{13} &= \left. \frac{\partial \dot{e}}{\partial \phi} \right|_{ss} = (\rho^w - \rho^c) e_{ss} > 0 \\ J_{21} &= \left. \frac{\partial \dot{\omega}}{\partial e} \right|_{ss} = \lambda \omega_{ss} > 0 \\ J_{32} &= \left. \frac{\partial \dot{\phi}}{\partial \omega} \right|_{ss} = -\phi_{ss} [u(\omega_{ss}) + \omega_{ss} u_{\omega}] \begin{matrix} \geq 0 \\ < 0 \end{matrix} \\ J_{33} &= \left. \frac{\partial \dot{\phi}}{\partial \phi} \right|_{ss} = -(\rho^w - \rho^c) \phi_{ss} < 0 \\ J_{22} &= J_{23} = J_{31} = 0 \end{aligned}$$

The only entry in the matrix that is in principle ambiguous in sign is  $J_{32}$ , given that it reduces to

$$-\phi_{ss}\bar{\theta}(1-\omega_{ss})^{\frac{\beta}{1-\beta-\gamma}}\left(1-\frac{\omega_{ss}}{1-\omega_{ss}}\frac{\beta}{1-\beta-\gamma}\right)$$

and can be positive or negative depending on parameter values for  $\beta, \gamma$ . In order to check for local stability, we need to evaluate the Routh-Hurwitz conditions, namely:

- $Tr J_{ss} < 0$ , which is clearly satisfied, given that  $J_{11} + J_{33} < 0$ .
- $Det J_{ss} < 0$ . We have that:

$$\begin{aligned} Det J_{ss} &= J_{21}(J_{13}J_{32} - J_{12}J_{33}) \\ &= \phi_{ss}\omega_{ss}(\rho^w - \rho^c)\lambda e_{ss}[u_{\omega,ss}(1-\omega_{ss}) - u(\omega_{ss})] \\ &= \phi_{ss}\omega_{ss}(\rho^w - \rho^c)\lambda e_{ss}\left[u(\omega_{ss})\left(\frac{\beta}{1-\beta-\gamma} - 1\right)\right] \end{aligned}$$

whose sign depends on whether the last term in the multiplication is positive or negative. A necessary and sufficient condition for negativity of the determinant is that  $\frac{\beta}{1-\gamma-\beta} < 1$ , or  $\beta < (1-\gamma)/2$ .

- $\sum_{j=1}^3 PmJ_j > 0$ , where  $PmJ_j$  is the principal minor obtained removing row  $j$  and column  $j$  from the whole matrix. This sum is equal to  $-J_{12}J_{21} + J_{11}J_{33} > 0$  as required.
- $-\sum_{j=1}^3 PmJ_j + \frac{Det J_{ss}}{Tr J_{ss}} > 0$ . This condition boils down to:

$$-J_{11}J_{33} + \frac{J_{21}}{J_{11} + J_{33}}(J_{11}J_{12} + J_{13}J_{32}) < 0$$

The first term is unambiguously positive. Since the ratio  $J_{21}/(J_{11} + J_{33})$  is *negative*—the numerator is positive while the denominator is the trace of the Jacobian, which we just shown to be negative—a sufficient condition for this fourth requirement to be satisfied is that the term in parentheses  $J_{11}J_{12} + J_{13}J_{32}$  be *positive* instead. Now,  $J_{11}J_{12}$  is certainly *positive*, given it is the product of two negative numbers. If we can identify a sufficient condition such that the second addendum  $J_{13}J_{32} > 0$  as well, we are done.<sup>17</sup> We have that:

$$J_{13}J_{32} = \phi_{ss}(\rho^w - \rho^c)e_{ss}[u(\omega_{ss}) + \omega_{ss}u_{\omega,ss}]$$

whose sign depends on the sign of:

$$u(\omega_{ss}) + \omega_{ss}u_{\omega,ss} = \bar{\theta}(1-\omega_{ss})^{\frac{\beta}{1-\beta-\gamma}}\left[1 - \frac{\omega_{ss}}{1-\omega_{ss}}\frac{\beta}{1-\beta-\gamma}\right]$$

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<sup>17</sup>Checking necessary conditions is more complicated.

which in turn is positive if the term in brackets is positive. This will be the case provided that  $\beta < (1 - \omega_{ss})(1 - \gamma)$ . Even though this involves the value of an endogenous variable, as long as  $1 - \omega_{ss} > 1/2$ , which is certainly satisfied for high-income economies such as the United States, the condition for the determinant to be negative identified above is sufficient for this stability requirement to be satisfied.

We conclude that  $\beta < (1 - \gamma)/2$  is sufficient for local stability. Note finally that, although the steady state is locally stable, the stability analysis above does not rule out the occurrence of (damped) oscillations around the steady state. These oscillations actually occur in the simulations presented in the paper.

## D Parameter Calibration

In order to calibrate the parameters of the model, we use the following strategy. For the user cost parameters  $\beta, \gamma$  we take the point estimates provided in Petach and Tavani (2019), which satisfy the restriction required for local stability, namely  $2\beta < 1 - \gamma$ . For the capitalist discount rate  $\rho^c$ , we set a value of 5% which is standard in the literature. The growth rate of labor productivity  $\alpha$  and the population growth rate  $n$  are set at 2% and 1% respectively, also standard. For the slope parameter of the Phillips curve  $\lambda$ , we use the naïve estimates provided in intermediate macro textbooks such as Blanchard (2017), namely .73. We are then left with three parameters to calibrate internally. To calibrate the Phillips curve intercept  $\xi$ , we solve for the value required to return a steady-state employment rate  $e_{ss} = 94\%$ , which is in line with a long-run unemployment rate of 6%, in equation (16). To calibrate the workers' discount rate, we solve for the value required to obtain a capitalist wealth share of 40%, in line with the current estimates of the top 1% wealth share in Saez and Zucman (2016). Finally, we solve for the value of  $\bar{\theta}$  required to obtain a wage share of .6, conforming with the estimates provided in Figure 2. Table 1 summarizes the baseline parameter calibration.

Parameter	Moment to match	Source
$\beta$	0.1	Petach and Tavani (2019)
$\gamma$	0.45	Petach and Tavani (2019)
$\alpha$	0.02	Standard
$n$	.01	Standard
$\bar{\theta}$	$\omega_{ss} = .6$	Calibrated internally
$\rho^c$	.05	Standard
$\rho^w$	$\phi_{ss} = .4$	Saez and Zucman (2016)
$\lambda$	.52	Blanchard (2017)
$\xi$	$e_{ss} = .94$	Calibrated internally

Table 1: Parameter calibration.

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