

The Normal Rate of Capacity Utilization as a Convention: Genuine Hysteresis and Harrodian Instability in the Kaleckian Model*

Tarek Fahim[†]

October 2026

Abstract

To contain Harrodian instability, part of the Kaleckian tradition endogenizes the normal rate of capacity utilization by means of an adaptive expectations rule. This rule fails to reproduce the genuine hysteresis that the recent literature attributes to the normal rate, and which only micro-founded, agent-based modelling had so far managed to integrate into a dynamic model. This paper argues that an aggregate model is sufficient. Drawing on the mathematical theory of hysteresis operators, it replaces the adaptive rule with a *simple play operator*: the normal rate is revised only when the gap between the actual and the normal rate crosses a threshold. The Kaleckian taming of Harrodian instability does not survive this substitution: beyond the threshold, the normal rate never catches up with the actual rate, and the system diverges in the absence of a correcting mechanism. Finally, the framework reconciles two requirements that are usually opposed: the inertia of conventions, which justifies the ordinary practice of Kaleckian models of treating them as given, and their adaptation to changes in economic policy, the premise of the Lucas critique. It rejects the conclusion of that critique, however, since an aggregate rule is sufficient to account for this adaptation.

Keywords: normal rate of capacity utilization, genuine hysteresis, hysteresis operators, simple play, Harrodian instability, Kaleckian models.

JEL: B50, E11, E12, C62.

1 Introduction

The problem of the instability inherent in the growth paths of capitalist economies dates back to the birth of macroeconomics. Harrod (1939) was the first to state it clearly: any gap between the actual and the expected rate of accumulation triggers a revision of expectations which, far from closing the gap, widens it, so that the growth equilibrium lies on a *knife-edge*. Yet real economies do not display the explosive paths that this mechanism, left to itself, would imply: observed utilization rates move within bounded ranges. The debate therefore gradually shifted to the *correcting mechanisms* capable of containing Harrodian instability, and it is over the nature of these mechanisms that the heterodox traditions part ways. The first critiques of Kaleckian models, which object that they contain no mechanism bringing the utilization rate back towards its normal rate, date from the mid-1980s (Committeri, 1986; Auerbach & Skott, 1988); Skott (1989) subsequently developed a Harrodian alternative. For classical and Marxian

*Paper prepared for the 30th FMM Conference, Berlin, 22–24 October 2026. I thank my thesis co-supervisor, Dany Lang (Université Sorbonne Paris Nord), for reading the manuscript and for his comments and suggestions. Any remaining errors and omissions are my own.

[†]ACT (Analyse des Crises et Transitions), Université Sorbonne Paris Nord. PhD candidate at Sorbonne Université.

authors (Serrano, 1995; Duménil & Lévy, 1999), as for some post-Keynesians of Harroddian persuasion (Skott, 2010), the correction brings the actual rate u back towards an *exogenous* normal rate u_n , which thus serves as an anchor, while the adjustment takes place elsewhere: through changes in the average propensity to save or in the profit share, through economic policy, or through pressures in the labour market. For Kaleckian authors (Lavoie, 1995, 1996; Dutt, 1997, 2010; for a survey, see Hein et al., 2012), the correction runs instead through the *endogenization of the normal rate itself*: u_n adjusts adaptively to the actual rate, a revision which, under certain conditions, closes the gap and tames Harroddian instability. These authors describe this endogenization of the normal rate as hysteresis (Setterfield & Avritzer, 2020, p. 2, n. 1). Following Bassi et al. (2022, p. 198), we reserve the term for *genuine hysteresis*, as described by the theory developed in the physical sciences and introduced into economics by Amable et al. (1993, 1994) and Cross (1993): a shock followed by an opposite shock of the same size leaves a remanence, whereas a unit-root process returns to its starting point. The expression is used in this sense throughout the paper. This disagreement has far-reaching implications: depending on whether the normal rate is exogenous or endogenous, the paradoxes of thrift and of costs either vanish in the long run or persist there.

Skott (2012) levelled against the Kaleckian position a three-part critique that has structured the debate ever since.¹ The canonical Kaleckian closure

$$\dot{u}_n = \omega(u^* - u_n), \quad \omega > 0, \quad (1)$$

would lack, in his view, (i) behavioural foundations, (ii) an explanation of the conventional character of u_n , and (iii) a justification of the speed and extent of the adjustment. Recent contributions have answered this critique convincingly; we fully subscribe to the behavioural foundation they propose, and it is its formalization that we discuss. Setterfield & Avritzer (2020) answer Skott's three objections. To the first, they oppose a Steindlian foundation: under fundamental uncertainty, firms deliberately hold excess capacity in order not to lose market share in the face of unforeseen variations in demand, and this buffer is all the larger the more volatile demand is. To the second, they reply that the normal rate is a convention informed by the facts: on US data, the volatility of utilization rises in the crises that bring long expansions to an end, and the normal rate falls when this volatility rises. It follows that the normal rate moves in the same direction as the actual rate, which makes Lavoie's equation a sensible first approximation. Since the relationship is discontinuous from one phase to the next, however, they propose a stylized representation of it in the form of a *non-ideal relay*, borrowed from the theory of strong hysteresis (Cross, 1993; Amable et al., 1994; Lang, 2008): firms switch to a low normal rate when volatility exceeds an upper threshold, and return to the high rate only when it falls back below a lower threshold. To the third objection, finally, they reply that in their representation the normal rate varies neither quickly nor without limit: it takes only two values, between which it switches by jumps rather than continuously, and these switches are rare, confined to episodes of extreme volatility. They consider, however, that their one-sector representation does not do justice to the structure of strong hysteresis, which requires thresholds that differ across sectors or firms, and they therefore call for a micro-founded model. Firm heterogeneity has indeed become, in the recent literature, a taming mechanism for Harroddian instability in its own right, as the survey by Lavoie (2022, §6.5.7, pp. 430–431) shows, drawing in particular on Setterfield (2019) and Franke (2021). Bassi et al. (2022) took up this programme. On 25 US manufacturing sectors, they test the properties of hysteresis and identify sector-specific volatility thresholds for 21 of them; they then build an agent-based model which, subjected to an imposed cycle of volatility, generates an aggregate Ewing loop.

For Bassi *et al.*, however, this hysteresis is inseparable from its microfoundation: in order to account for sector-specific dynamics and for genuine hysteresis, aggregate structural models

¹Setterfield and Avritzer count four criticisms; the second, aimed at the formulation of Dutt (1997) only, is not taken up here.

should in their view “always” be augmented by an agent-based block akin to theirs (Bassi et al., 2022, p. 209). Without such a block, one-sector aggregate models are confined to linear approximations, of which Lavoie’s equation (1) is the canonical example: it does produce a permanent effect of transitory shocks, but through a unit root, not through genuine hysteresis (Bassi et al., 2022, p. 198). Bassi *et al.* do not, for all that, give up one-sector models. They propose one, made of two linear differential equations, in which two adjustments operate simultaneously: the normal rate moves towards the actual rate, according to Lavoie’s equation, and the actual rate moves towards the normal rate, in what they elsewhere call the classical regime. Following a shock, the normal rate then absorbs only a fraction of the initial change in the actual rate. This fraction, which they call the “degree of hysteresis”, would be zero if the normal rate were exogenous and equal to one under Lavoie’s equation alone; their model gives it an intermediate value, which reproduces the partial adjustment obtained in their simulations. They relate this model to one of the models of Lavoie (1996, pp. 138–142) and conclude that the case for one-sector models is somewhat stronger than the “hybrid” or pragmatic position of Setterfield (2009) suggested, according to which unit-root systems may serve as a first approximation even though they do not give rise to genuine hysteresis (Bassi et al., 2022, pp. 198 and 210). They acknowledge, however, that this model approximates hysteresis as a unit- or zero-root process (Bassi et al., 2022, p. 210). We share their diagnosis of the usual Kaleckian closure entirely; we contest only that a one-sector model is condemned to choose between simplicity and genuine hysteresis.

Our contribution is to show that this detour through microfoundations is unnecessary: a one-sector aggregate model can incorporate genuine hysteresis in the normal rate, provided that the differential equation (1) is replaced not by other linear equations but by a hysteresis operator. We adopt a *symmetric simple play*, which Krasnosel’skii & Pokrovskii (1989, p. vii) present, together with the stop, as a simple example of a hysteron, and which Brokate & Sprekels (1996, Remark 2.4.16) regard as preferable to the relay for representing continuous hysteresis processes. This operator takes the path of $u^*(\cdot)$ as its input and delivers the path of $u_n(\cdot)$ as its output; the latter remains constant as long as u^* moves within an *inaction band*, and adjusts only on contact with the boundaries of the band. This band receives a decision-theoretic foundation: it corresponds to the range of non-revision of Shackle (1969), within which the realized value of a variable does not lead to a revision of expectations. Five main results follow from this substitution.

First, the amended system has a unique global solution under standard assumptions, by the Picard–Lindelöf theorem for systems with hysteresis (Brokate & Sprekels, 1996, Theorems 3.1.1 and 3.1.3). Closing the model with a hysteresis operator raises no formal obstruction.

Second, the hysteresis thus obtained is indeed genuine. Subjected to a shock followed by the opposite shock of the same size, the normal rate does not return to its initial value, whereas Lavoie’s closure brings it back exactly; and the loop that the model traces under an imposed cycle does not depend on the speed at which this cycle is traversed (Section 5). A transitory demand episode, neutral in the long run under the linear closure, therefore displaces the convention lastingly.

Third, the long-run *continuum* of equilibria of Lavoie (1996), the line $\alpha = A u_n$, is *preserved* by the amendment, but it consists of unstable equilibria, any gap to the continuum being amplified in the inaction regime; the simple play does not alter it, but endows it with a dynamic *inaction band* inside which the normal rate remains frozen. Restricted to the interval documented by Bassi et al. (2022), this continuum appears as the *densely populated range of variation* of u_n in their Figures 6–8.

Fourth, the dynamics of the system break down into three piecewise regimes (inaction, upper boundary, lower boundary), each with its own stability conditions. In the boundary regimes, Harrodian instability is neither extinguished nor explosive: from exponential, the divergence becomes linear, the trajectory drifting parallel to the continuum without ever reaching it. The

taming achieved by Lavoie's closure therefore rests on the linear form of his equation. The paradoxes of thrift and of costs are preserved as instantaneous effects; but in the boundary regimes the system reaches no stationary equilibrium, so that comparative statics between long-run equilibria loses its meaning. This result is established for the reference case in which the convention is revised one-for-one at the boundaries of the band, and the resulting drift is unbounded: on both counts, our operator differs from the relay of Setterfield and Avritzer, whose adjustment is bounded, and we return to this in Sections 4.6 and 4.7.

Fifth, this framework makes it possible to take a stand on the critique of Lucas (1976). A hysteretic convention is locally invariant: as long as the gap remains within the band, it behaves like a parameter, which justifies the Kaleckian practice of treating it as given; but it is revised beyond a threshold, which partly concedes Lucas's premise that behavioural rules change with the conduct of economic policy. Since an aggregate rule is sufficient to describe this adaptation, his conclusion, that only micro-founded models with rational expectations could account for it, is refuted. The model is only a first sketch; the conclusion indicates how to complete it.

The paper is organized as follows. Section 2 recapitulates the debate and identifies precisely the missing formal step. Section 3 provides the Shackleian foundation of the inaction band, introduces the symmetric simple play and calibrates its band on the saturation levels of the aggregate Ewing loop that Bassi *et al.* obtain by simulation. Section 4 states the amended system, the existence and uniqueness theorem and the set of long-run equilibria, then describes the piecewise dynamics and their stability conditions, and discusses possible extensions of the model. Section 5 presents three numerical simulations, including the Ewing loop that the play produces under exogenous forcing, and discusses the fate of the paradoxes in the long run. Section 6 concludes. All formal proofs are relegated to a separate mathematical appendix, available on request, as is the code that reproduces the figures.

2 The debate on the normal rate and the formal impasse of closure (1)

2.1 The minimal Kaleckian model and Harroddian instability

We consider the standard one-sector post-Keynesian Kaleckian model in continuous time, simplified to the bare minimum in order to isolate the adjustment mechanism of the normal rate. Let u be the rate of capacity utilization, u_n the normal rate, g the rate of accumulation, g^s the saving rate relative to the capital stock, h the profit share, v the capital-capacity ratio and s_P the propensity to save out of profits. The saving function is

$$g^s = s_P \frac{h}{v} u = A u, \quad A \equiv s_P \frac{h}{v}, \quad (2)$$

where A , the slope of the saving function, measures the amount of saving, relative to the capital stock, released by each additional point of utilization. This magnitude has a Harroddian reading that will serve throughout the paper: at normal utilization, the capital required per unit of output is v/u_n and saving per unit of income is $s_P h$, so that $A u_n = s_P h u_n / v$ is the *warranted* rate of growth in the sense of Harrod (1939), evaluated at the convention, that is, the pace of accumulation that saving can finance when the economy operates at the rate firms deem normal. The Kaleckian investment function reads

$$g = \alpha + \beta(u - u_n), \quad (3)$$

where α represents the expected trend rate of growth of sales (the *animal spirits*) and $\beta > 0$ the sensitivity of investment to the gap between the actual and the normal rate. The Keynesian stability condition reads $\Delta \equiv A - \beta > 0$: Δ measures by how much saving responds more than

investment to a rise in the utilization rate. The short-run equilibrium of the goods market gives

$$u^* = \frac{\alpha - \beta u_n}{\Delta}, \quad g^* = A u^*. \quad (4)$$

The inverse of Δ is thus the short-run multiplier: a one-point rise in expectations α raises the utilization rate by $1/\Delta$ points. One quantity will be used throughout:

$$z \equiv \alpha - A u_n, \quad u^* - u_n = \frac{z}{\Delta}. \quad (5)$$

The gap z measures the excess of expected growth α over the growth warranted at the convention, $A u_n$; the second equality, which follows from (4), shows that it is proportional to the gap between the actual and the normal rate. When $z > 0$, the economy operates above its norm, and conversely.

Harroddian instability, introduced by Harrod (1939), was formalized within the Kaleckian framework by Lavoie (1995, pp. 805–806), following Committeri (1986), then in continuous time by Lavoie (1996, p. 139); Hein (2014, §11.2) takes up this formulation. It arises from the adaptive feedback of animal spirits on the utilization rate:

$$\dot{\alpha} = \chi(g^* - \alpha), \quad \chi > 0. \quad (6)$$

When the actual rate of accumulation g^* exceeds expectations α , firms revise their estimate of the trend growth rate upwards; this shifts the investment function upwards, raises u^* , and hence $g^* = A u^*$ even further, so that the revision reopens the gap $g^* - \alpha$ it was supposed to close. By (4), a one-point rise in expectations α raises actual accumulation g^* by A/Δ points: A/Δ is the multiplier linking expected growth to actual accumulation, and it exceeds one since $\Delta = A - \beta < A$. Any upward revision of expectations therefore raises actual accumulation by more than itself, which makes the adjustment self-defeating: the process is cumulatively unstable. At a given normal rate, nothing stops it, and the gap between u^* and u_n grows without limit. The whole question is which mechanism stops it; in the amended model (Section 4), this gap grows only until it reaches the threshold of the inaction band, where the convention starts following the actual rate, but the drift of the economy carries on.

2.2 The adaptive Kaleckian closure

To contain Harroddian instability while preserving the Kaleckian mechanism, Lavoie (1996, pp. 138–142) proposes to make u_n itself endogenous to the actual rate through equation (1). The system {(1), (6)} then has a continuum of long-run equilibria located on the line

$$\alpha = A u_n \quad (7)$$

in the (u_n, α) plane. The stability condition, established by Lavoie (1996, p. 140) and restated by Hein (2014, §11.6, p. 459), reads

$$\frac{\chi\beta}{\omega} < A, \quad (8)$$

that is, geometrically, the transition path (of slope $\chi\beta/\omega$ in the (u_n, α) plane) must be flatter than the continuum of equilibria (of slope A).

2.3 The diagnosis of Bassi *et al.*: closure (1) does not capture genuine hysteresis

Bassi *et al.* (2022, p. 198) show that closure (1), although it produces permanent effects of transitory shocks, is mathematically equivalent to a unit-root process. Writing the discrete version $\Delta u_n = \omega \Delta t \cdot (u - u_n)$ and setting $u_t = u_{nt-1} + \varepsilon_t$ with ε_t a white noise, one obtains

$$u_{nt} = u_{nt-1} + \omega \Delta t \cdot \varepsilon_t,$$

which is precisely a unit-root process. Yet, as Amable et al. (1993, p. 128) and Amable et al. (1994) have shown, a unit-root process *does not* deliver genuine hysteresis. It lacks the thresholds and the inaction band: its linear structure rules out any path memory properly so called, since the system is sensitive to the integral of the shock, but neither to the order in which thresholds have been crossed nor to the sequence through which the input has passed.

2.4 Three missing bridges

A complete answer requires bringing together three ingredients, drawn from three distinct contributions.

Setterfield & Avritzer (2020) provide the *Steindlian behavioural foundation*. The idea, inherited from Steindl (1952, pp. 9–14), is that firms deliberately hold a buffer of idle capacity ($u_n < 1$) as a reserve against an unforeseen rise in demand, so as not to surrender market share to new competitors during a boom. Under fundamental uncertainty in Keynes’s sense, where the optimal response cannot be computed, this normal rate is a *convention*; not, they stress against Nikiforos (2016), a “mere” arbitrary convention, but a convention informed by actual events, in this case the observed volatility of demand. The mechanism is then made of two relations. On the one hand, the volatility of demand σ_u^2 varies inversely with the actual utilization rate: crises come with high volatility, booms with relative tranquillity. On the other hand, the normal rate varies inversely with this volatility: the greater the uncertainty, the more firms widen their reserve by lowering u_n . By composition, the normal rate comes to move in the same direction as the actual rate.

This twofold mechanism takes the form of a *non-ideal relay* between two states u_{nH} and u_{nL} (borrowed from the theory of strong hysteresis: Cross, 1993; Amable et al., 1994; Lang, 2008), whose two distinct switching thresholds generate an inaction zone: starting from the high state u_{nH} , firms switch to u_{nL} only if volatility exceeds an upper threshold σ_{uH}^2 (triggered by a crisis, $u < u_L$), and then remain at u_{nL} as long as volatility has not fallen back below a lower threshold $\sigma_{uL}^2 < \sigma_{uH}^2$. Setterfield and Avritzer finally stress the *asymmetric* character of this revision: the fall from u_{nH} to u_{nL} is fast, governed by current volatility σ_{ut}^2 and triggered by the crisis shock; but the recovery from u_{nL} to u_{nH} is slow, because it is governed not by current volatility but by its average $\frac{1}{n} \sum_{t=1}^n \sigma_{ut}^2$ over a “reaction period” which they explicitly relate to the instability principle of Harrod (1939): it takes time for firms that have been burnt to convince themselves that the crisis is over and that a new lasting boom is under way. A statistically brief crisis can thus leave a lasting behavioural imprint. But their relay is only a stylized one-sector representation, and they explicitly acknowledge that its integration into a structural macrodynamic model remains to be done (Setterfield & Avritzer, 2020, p. 20).

Bassi et al. (2022) provide the *aggregation through heterogeneity*: they show by agent-based simulation that, under an imposed cycle of volatility, discrete micro-level jumps produce a continuous Ewing loop at the aggregate level, with empirically calibratable saturation levels. But their macro reduction (Section 5.2) amounts to a system of two linear differential equations, $\dot{u}_n = \beta_B(u - u_n)$ and $\dot{u} = -\gamma_B(u - u_n)$, which they themselves acknowledge approximates hysteresis as a unit- or zero-root process (Bassi et al., 2022, p. 210). This linear reduction loses precisely the genuine hysteresis that their agent-based model had brought out.

Lavoie (1996) provides the *structural Kaleckian framework*: the coupled equations (1) and (6), the continuum of equilibria, the stability condition (8). But his closure (1) is precisely the one that Bassi et al. show fails to capture genuine hysteresis.

The missing step consists in providing a *one-sector macro reduced form* that (i) fits within Lavoie’s Kaleckian framework, (ii) captures the genuine hysteresis required by Bassi et al. (2022), and (iii) remains faithful to the behavioural foundation of Setterfield & Avritzer (2020). That is the programme of this paper.

3 A macro-aggregate hysteresis operator

3.1 From the relay to the operator: two levels of description

The non-ideal relay of Setterfield & Avritzer (2020) and of Bassi et al. (2022, Fig. 1) is a *two-state* object: $u_n^{(i,j)} \in \{u_{nL}^{(i,j)}, u_{nH}^{(i,j)}\}$ for firm i in sector j . At the micro level, it describes a discrete jump between two conventions. At the aggregate level, the object is different: aggregation over 20,000 heterogeneous firms produces what Bassi et al. (2022, Fig. 4) display as a *continuous curve* in the (σ, u_n) plane, traversed hysteretically according to whether volatility σ rises or falls.

Rather than letting hysteresis emerge from the aggregation of individual relays, we propose to posit directly, at the aggregate level, a hysteresis operator whose loop, drawn in the (u^*, u_n) plane, plays the role of the one Bassi et al. (2022) obtain in the (σ, u_n) plane. The input of the operator is therefore no longer the volatility of demand, but the utilization rate u^* itself. The status of this operator calls for a clarification. Strictly speaking, the aggregation of relays with distributed thresholds yields a Preisach operator: strong hysteresis in the sense of Amable et al. (1993), multi-branch and with a remanence that rises with the amplitude of the cycle. The simple play is an elementary representation of it, with a remanence independent of the amplitude. We do not adopt the Preisach operator because its implementation would require specifying a density on the half-plane of thresholds, which the aggregate saturation levels of Bassi et al. (2022) alone do not allow us to identify: one would have to go back down to the level of firms, that is, to microfound again. For dynamics with a single dominant cycle, such as those they study, we conjecture that the two operators are qualitatively equivalent (appendix, Section A.7). This choice extends a reduction that Setterfield & Avritzer (2020) already perform. They first note that the volatility of utilization is lower the higher utilization is, long booms being tranquil and crises volatile, a relation they infer from the comparison of boom and crisis phases (their Table 1) and present as a first approximation of the stylized facts:

$$\sigma_u^2 = f(u), \quad f' < 0.$$

They then estimate, on annual data, a decreasing relation between the normal rate, measured by the filtered trend of utilization, and the variance of its cyclical component, in line with the Steindlian motive (their Tables 4 and 5):

$$u_n = g(\sigma_u^2), \quad g' < 0.$$

From these they deduce that the normal rate moves in the same direction as the actual rate, $u_n = h(u) = g(f(u))$ with $h' > 0$, a reduced form of which they judge Lavoie's linear closure to be a "sensible first approximation" (p. 17). We refine this first approximation by retaining in it the thresholds that their analysis brings to light.

Replacing volatility with the utilization rate does not make these thresholds disappear. In Setterfield and Avritzer, firms lower their normal rate when volatility exceeds an upper threshold σ_H^2 , and raise it when volatility falls back below a lower threshold σ_L^2 . Since f is decreasing, each of these events is equivalent to a threshold crossing by utilization:

$$\sigma_u^2 > \sigma_H^2 \iff u < u_L = f^{-1}(\sigma_H^2), \quad \sigma_u^2 < \sigma_L^2 \iff u > u_H = f^{-1}(\sigma_L^2).$$

The revision rule can therefore be stated indifferently in terms of volatility or of utilization. Setterfield and Avritzer themselves perform this translation for the upper threshold, writing $\sigma_H^2 = f(u_L)$ (p. 18). The rule thus rewritten retains the two properties that define hysteresis: the revision depends on the crossing of thresholds, not on the speed at which they are reached, and a round trip of utilization leaves a lasting trace on the normal rate. This equivalence concerns the relay of Setterfield and Avritzer, whose thresholds are fixed; the simple play, whose band moves with the convention, is an aggregate representation of it, in the sense discussed in the previous paragraph.

It leaves open, however, the size of the revision once the threshold has been crossed: the simple play assumes that the convention then follows the actual rate one-for-one. We discuss this choice in Section 4.7, after establishing its role in the dynamics.

Finally, this reduction is a simplification. In Setterfield and Avritzer, volatility is computed over a past reaction period: fluctuations of utilization that remain inside the band may, by raising this volatility, suffice to trigger a revision. Our operator, which reacts only to the crossing of the boundaries of the band, ignores such fluctuations. Two reasons nevertheless justify this choice. First, our model is deterministic: it determines the level of utilization at each date, but contains no random element, so that volatility does not exist in it as a variable; to preserve the mechanism of Setterfield and Avritzer, it must be expressed by means of a variable that the model determines, and the relation $\sigma_u^2 = f(u)$ points to the utilization rate. Second, in the Kaleckian model, investment responds to the gap between u^* and u_n : by taking u^* as its input, the operator becomes part of a loop internal to the model, since the normal rate it produces alters investment, hence utilization, which becomes its input again. With volatility as the input, an additional equation would be needed to determine it, which the model does not contain; this, incidentally, is why Bassi et al. (2022) treat it as exogenous.

3.2 Shacklean foundation: the inaction band as a range of non-revision

The choice of an operator with an inaction band is not a mere technical convenience: it rests on the decision theory of Shackle (1949, 1969), from which we draw the decision-theoretic foundation of the convention u_n under fundamental uncertainty. Shackle holds any *distributional* uncertainty variable, that is, any probability, to be in principle inappropriate for the analysis of decision (Shackle, 1969, preface to the first edition), whether frequentist, logical or subjective, to use the classification of Weckstein that he discusses (Shackle, 1969, p. 104). Among his reasons, two matter here. The rival outcomes a decision-maker can imagine cannot be drawn up into a complete list: he must allow for a “residual hypothesis”, a closed door behind which anything whatever may lie; and since the share of these unknown outcomes cannot be known, that of the listed ones cannot be determined either, which Shackle calls the arithmetical obstacle to any distribution (Shackle, 1969, p. 49). And such decisions are not serial: a firm whose survival depends on the success of an investment cannot treat it as one trial among others, for lack of the frequency record that would allow a probability to be attached to it (Shackle, 1969, p. 58). The object Shackle constructs is therefore a “*non-distributional uncertainty variable*” (Shackle, 1969, p. 68), *potential surprise*: the degree of surprise the agent would feel if the hypothesis under consideration came true. Zero potential surprise means that the hypothesis is deemed perfectly possible; maximal surprise $\bar{y}$, that it is rejected as impossible.

The property that radically separates this variable from a probability is its *non-distributional* character: “*Zero potential surprise can be assigned to each of an unlimited number of rival, mutually exclusive, hypotheses all at once*” (Shackle, 1969, p. 70). Where the summation axiom ($\sum p_i = 1$) forces rival hypotheses to share a fixed mass, potential surprise allows an *entire continuum* of incompatible outcomes to be held perfectly possible simultaneously. A whole zone of current rates u^* around the convention can thus be deemed perfectly possible by firms at the same time: none of these rates would surprise them. A probabilistic representation of uncertainty cannot found such a zone; a representation by potential surprise does so effortlessly.

The typical shape of the potential surprise curve of an investment scheme is described by Shackle with precision (Shackle, 1969, pp. 140–141), and Figure 1 reproduces it: a *flat inner segment* where surprise is zero on either side of the neutral outcome G_N , flanked by two *sloping segments* where it rises with distance, up to *plateaus* of absolute rejection $\bar{y}$; this is the profile of a “*vertical section through an ordinary flat-bottomed enamel basin*”. Shackle further chooses to treat possibility and potential surprise as continuous variables, so that the curve moves from zero to the maximum through sloping segments rather than jumps (Shackle, 1969, p. 142): between the perfectly possible and the rejected outcome, surprise rises by degrees.

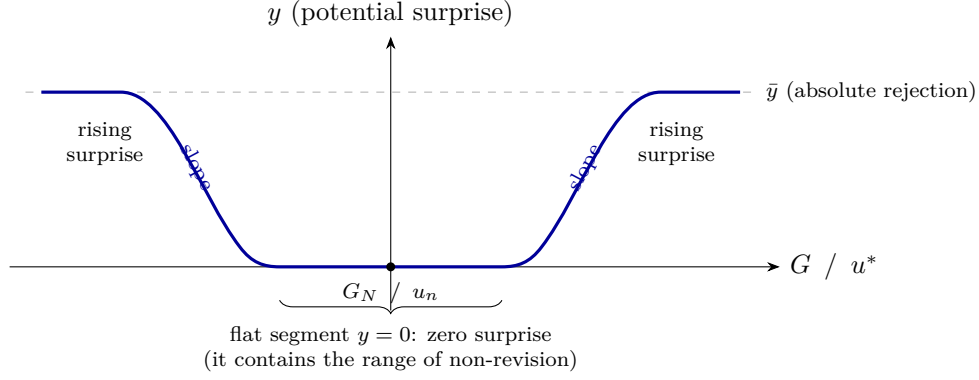


Figure 1: The potential surprise curve of Shackle (1969, pp. 140–142): the “flat-bottomed enamel basin” profile. Flat inner segment ($y = 0$, perfectly possible hypotheses), sloping segments (surprise rising by degrees), plateaus of absolute rejection ($\bar{y}$). Double labelling: Shackle’s variables (G , hypothetical outcome; G_N , neutral outcome) and their macroeconomic transposition (u^* , actual rate; u_n , convention).

This profile, however, describes an *ex ante* judgement, passed on the hypothetical outcomes of an action at the moment of decision; Shackle stresses that it cannot be compared with the surprise actually felt later, when the event occurs (Shackle, 1969, p. 69). Our band, by contrast, concerns *realized* values of the current rate. The relevant Shackle concept is therefore another one: the *range of non-revision*, which he introduces in the chapter devoted to changes in expectations (Shackle, 1969, pp. 218–220). As long as the realized value of a variable falls within this range, expectations remain unchanged; if it falls outside, the potential surprise curve is revised. This range lies wholly within the segment of zero surprise, without necessarily coinciding with it: a realization may cause no surprise and nevertheless lead to a revision of expectations (Shackle, 1969, pp. 266–267). Shackle doubts that a “calculable dynamics”, which would make entrepreneurs’ expectations depend on a few past aggregates, is a well-founded ambition (Shackle, 1969, pp. 264–265). He nevertheless concedes, to comply with economists’ habit of reasoning with functions of a single variable, that expectations could be made to depend on the excess of the realized value over the bound of the range of non-revision, a variable that is zero “as long as the actual quantum falls inside the range of non-revision” (Shackle, 1969, p. 267). Our model belongs to this concession, which we accept as such. This is the structure of the simple play: a zone where nothing is revised, then a revision governed by the overshooting of the boundary. Our inaction band is the range of non-revision of the convention u_n , and its half-width γ measures its extent. The correspondence is thus as follows:

Shackle (1969)	Formalization by the simple play
neutral outcome G_N	convention u_n
hypothetical outcome G	actual rate u^*
flat segment $y = 0$	actual rates deemed perfectly possible
range of non-revision (pp. 218–220)	inaction band $ u^* - u_n < \gamma$
excess over the bound of the range (p. 267)	sliding at the boundaries, $\dot{u}_n = \dot{u}^*$
sloping segments	not represented (intensity of surprise)
extreme plateaus $y = \bar{y}$	saturation levels u_{nL}, u_{nH} (Bassi et al., 2022)

As long as $|u^* - u_n| < \gamma$, the actual rate remains within the range of non-revision and the convention is not revised: this inaction does not reflect resistance to change, an adjustment cost, but the *absence of any reason to change*. When the actual rate reaches a boundary and keeps moving away from it, revision sets in, and its size equals the overshooting of the boundary, in keeping with the variable Shackle envisages. Three caveats delimit the scope of this transposition. First, the plateaus of absolute rejection correspond to the saturation

levels obtained by Bassi et al. (2022), which bound the calibration but lie outside the operator (Section 3.4); a play with saturating boundaries would be its extended formalization. Second, the simple play does not represent the sloping segments: it distinguishes the band and its boundaries, but ignores that surprise rises by degrees with the gap; a generalized play, with non-linear boundaries, would make it possible to incorporate it. Third, the play does not represent the sudden changes in expectations that some news brings about, which Shackle likens to “*a twist of the hand upon a kaleidoscope*” (Shackle, 1969, p. 232), nor, more radically, unexpected events, never imagined, which wreck the picture of the world on which expectations rested (Shackle, 1969, pp. 282–284); these would call for an exogenous resetting mechanism, beyond the scope of the present paper. Non-distributional uncertainty itself is presupposed by our formalization, which does not reproduce its axiomatics.

This transposition has precedents that deserve to be named. The idea of a range of utilization rates within which firms do not react goes back to Dutt (1990, pp. 58–60) and to Lavoie (1992, pp. 327–332 and 417–422), and Setterfield (2019) has recently grounded it in Simon’s satisficing behaviour; Harrod (1972, p. 129), in an essay in honour of Shackle, already rejected the knife-edge image in favour of a “fairly flat shallow dome”. Above all, Dutt (2010, pp. 242–243) proposed, drawing explicitly on Shackle’s potential surprise, a band of values within which firms do not alter their behaviour, which Lavoie (2022, p. 412) calls a corridor of stability, a zone of inertia. What the simple play adds to these intuitions is their articulation with the revision of the convention: the same rule describes inertia inside the band and sliding at its boundaries, together with the path memory that results. Lavoie (2022, p. 427) observes that it is “not always easy to disentangle” the inertia of the normal rate within a range from its endogeneity to the actual rate; the hysteresis operator is precisely what makes it possible to hold the two together.

3.3 The play operator

The operator we adopt is the symmetric simple *play*, a simple example of a continuous hysteresis operator (Krasnosel’skii & Pokrovskii, 1989; Brokate & Sprekels, 1996). Its principle can be stated without formalism. The output, here the convention u_n , lives in a band of fixed half-width $\gamma > 0$ around the input, here the actual rate u^* . As long as the actual rate stays inside the band, the convention is frozen: this is inaction. When the actual rate reaches a boundary of the band and keeps moving away from it, the convention is dragged along: it slides and follows the actual rate at distance γ . The operator can therefore be read as a case-by-case revision rule:

$$\dot{u}_n = \begin{cases} 0 & \text{if } |u^* - u_n| < \gamma \quad (\text{inaction band}), \\ \dot{u}^* & \text{if } |u^* - u_n| = \gamma \quad (\text{boundary}). \end{cases} \quad (9)$$

Inside the band, the gap between the actual rate and the convention causes no surprise and nothing is revised (Section 3.2); on a boundary, the convention follows the actual rate, keeping the gap at γ . It slides only if the actual rate keeps moving away, rising on the upper boundary ($u^* - u_n = \gamma$), falling on the lower one; if the actual rate turns back into the band, the convention freezes again. This rule is not a loose paraphrase: for an absolutely continuous input, which is the case of u^* between two shocks, it characterizes the play exactly, almost everywhere (Brokate & Sprekels, 1996, Lemma 2.3.8).

Two consequences can be read directly from the rule. First, fluctuations of the actual rate whose amplitude is smaller than the width 2γ of the band leave the convention unchanged, except possibly during an initial phase: the operator eliminates them (Brokate & Sprekels, 1996, p. 48). Second, it produces remanence (Figure 2): if the actual rate rises beyond the band and then returns to its initial value, the convention, dragged along on the way up, remains frozen on the way down and does not return.

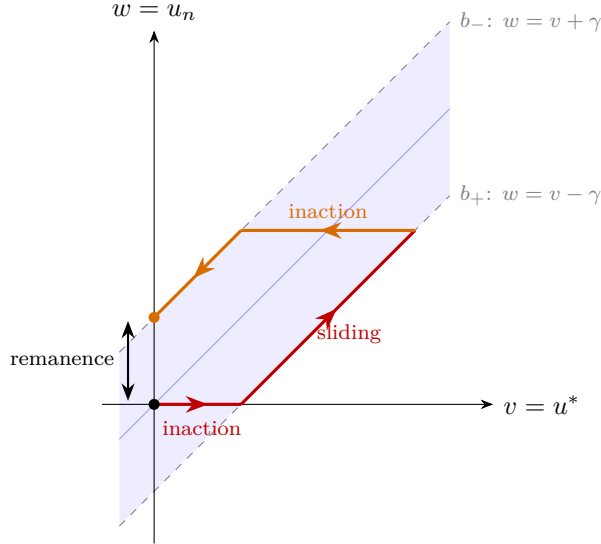


Figure 2: Elementary loop of the simple play in the input–output plane $(v, w) = (u^*, u_n)$, for $\gamma = 1$ (arbitrary units). The actual rate rises from 0 to 3, then returns to 0. On the way up, the convention first remains frozen (inaction), then slides along the boundary b_+ ; on the way back, it remains frozen until it reaches the boundary b_- , then slides. It ends at 1, not at 0: this gap is the remanence.

The play is not new to economics. Göcke (2002) made it the core of a family of “band of inaction” models (*play hysteresis*) applied to foreign trade, employment and investment (Belke et al., 2013; Göcke & Werner, 2015), in which the band arises from sunk entry and exit costs; Cross et al. (2009) give a general presentation. Closer to our problem, Lamba et al. (2020) and Krejčí et al. (2023) replace the rational inflation expectations of a standard model with a “sticky” response modelled by a play, and show that the unique equilibrium gives way to a whole segment of path-dependent equilibria. Our contribution differs from these works on three counts: the band does not result from adjustment costs but from a convention, in the sense of Shackle’s range of non-revision (Section 3.2); the operator is applied to the normal rate of the Kaleckian framework, where it meets Harroddian instability; and the continuum of equilibria pre-exists, inherited from Lavoie (1996), without being reached.

This rule has a compact formulation, which is the usual definition of the play and the one on which our proofs rely. Denote by $v = u^*$ the input, by $w = u_n$ the output, and by $\gamma > 0$ the half-width of the band. The play operator $\mathcal{F}_\gamma$ associates with any continuous function $v \in C[0, t_E]$ and any initial state $w_{-1} \in \mathbb{R}$ such that $|v(0) - w_{-1}| \leq \gamma$ an output function $w = \mathcal{F}_\gamma[v; w_{-1}] \in C[0, t_E]$ characterized by

$$w(t) = \max\{v(t) - \gamma, \min\{v(t) + \gamma, w(\tau)\}\} \quad (10)$$

at every instant t such that v is monotone on $[\tau, t]$, with $w(0)$ fixed by the initial state. Geometrically, w remains inside the inaction band $[v - \gamma, v + \gamma]$ and is dragged by v only when the input reaches one of the two boundaries $b_-(v) = v + \gamma$ or $b_+(v) = v - \gamma$.

Proposition 1 (Lipschitz continuity of the play, Brokate & Sprekels, 1996, Theorem 2.3.2). *For every $\gamma \geq 0$, $\mathcal{F}_\gamma : C[0, t_E] \times \mathbb{R} \rightarrow C[0, t_E]$ is Lipschitz continuous: for all $v_1, v_2 \in C[0, t_E]$ and $w_{-1,1}, w_{-1,2} \in \mathbb{R}$,*

$$\|\mathcal{F}_\gamma[v_1; w_{-1,1}] - \mathcal{F}_\gamma[v_2; w_{-1,2}]\|_\infty \leq \max\{\|v_1 - v_2\|_\infty, |w_{-1,1} - w_{-1,2}|\}.$$

The simple play is the operator used throughout this paper. A generalization could be considered, namely the *generalized play operator* of Brokate & Sprekels (1996, §2.4), which allows

non-parallel boundaries and a variable half-width, but none of the results that follow requires it: they are all established for the symmetric simple play, and the extension to the generalized case would call for a separate analysis (see the discussion of asymmetry in Section 4.8).

Figure 3 summarizes the geometry of the operator in the state plane (u_n, u^*) and the typical sequence of regimes.

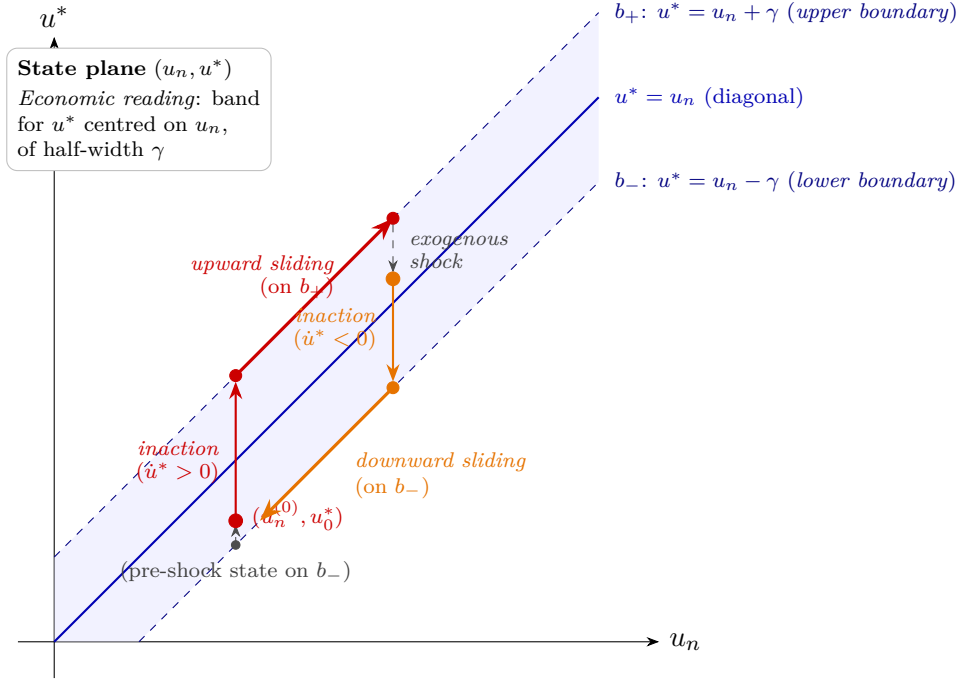


Figure 3: Geometry of the simple play in the state plane (u_n, u^*) . *Diagonal $u^* = u_n$* : states where input and output coincide (corresponding to Lavoie’s continuum via $\alpha = A u_n$). *Boundaries (dashed)*: the upper boundary b_+ ($u^* = u_n + \gamma$, above the diagonal) and the lower boundary b_- ($u^* = u_n - \gamma$, below it), the bounds of the tolerance band of u^* around the convention u_n (blue area). *Example trajectory* in four phases: (1) inaction with $\dot{u}^* > 0$, u_n frozen; (2) upward sliding along b_+ ; (3) downward inaction after the loading is reversed by an exogenous shock (the return from sliding to inaction is not endogenous); (4) downward sliding along b_- .

3.4 Calibration on the aggregate Ewing loop of Bassi *et al.*

Bassi *et al.* (Bassi *et al.*, 2022, Fig. 4) obtain by agent-based simulation, averaged over 200 runs, an aggregate Ewing loop for the US economy whose saturation levels are

$$u_{nH} = 86.4\%, \quad u_{nL} = 73.6\%.$$

These two values are the plateaus reached by the aggregate normal rate when all firms have switched, respectively, to the upper or to the lower arm of their individual relay.

To calibrate our operator on this loop, we proceed in three simple steps in the symmetric case.

Step 1. The *centre* of the band is placed halfway between the saturation levels:

$$u_n^{\text{centre}} = \frac{u_{nH} + u_{nL}}{2} = 80\%.$$

This centre serves only as a geometric reference point for positioning the two parallel boundaries $b_{\pm}(v) = v \mp \gamma$; it plays no dynamic role.

Step 2. The *half-width* of the band is chosen so that the vertical distance between the two branches of the loop generated by the play, namely 2γ , the ascending branch following

$b_+(v) = v - \gamma$ and the descending branch $b_-(v) = v + \gamma$, coincides with the distance between the two empirical saturation levels. This gives

$$\gamma = \frac{u_{nH} - u_{nL}}{2} = 6.4\%.$$

The macro-aggregate inaction band, calibrated on the US economy over 1972–2018 by Bassi *et al.*, thus has a half-width of about 6.4 percentage points of the utilization rate. Two caveats are in order. These saturation levels are obtained by ignoring the correlations between sectors (Bassi *et al.*, 2022, Fig. 4). And the sectoral normal rate used to calibrate them is measured by a moving average of actual utilization, a measure of which Lavoie (2022, p. 429) observes that it need not be a proper estimate of the normal rate, since, in a Kaleckian model, actual utilization may depart durably from the norm. Our model makes this caveat quantifiable: the gap between the two rates always lies within the band, so that the error made in taking an average of actual utilization for the normal rate is bounded by γ .

Step 3. The initial state w_{-1} is calibrated on the historical value of the normal rate observed at the initial date of the simulation. For the United States in 1972, Setterfield & Avritzer (2020, Table 1) report a mean detrended utilization rate of $u^{DT} \approx 79.79\%$ over 1960–1972. One may therefore set $w_{-1} \approx 80\%$, a value very close to the centre u_n^{centre} , which initially places the system at the heart of the inaction band.

This calibration captures the *amplitude* of the inaction band, but ignores the boom-slump asymmetry documented by Setterfield & Avritzer (2020, p. 19) and reflected in Figures 6–8 of Bassi *et al.* (2022), where some sectors populate the upper part of the band more densely and others the lower part. Section 4.8 shows how this limitation can be lifted by moving to $\gamma_+ \neq \gamma_-$.

4 The amended Lavoie model: existence, equilibria and dynamics

4.1 The amended system

We replace the linear closure (1) of Lavoie (1996) with the symmetric simple play. The amended system reads:

$$u_n(t) = \mathcal{F}_\gamma[u^*](t) \tag{11}$$

$$\dot{\alpha}(t) = \chi\beta \cdot \frac{\alpha(t) - A u_n(t)}{\Delta} \tag{12}$$

$$u^*(t) = \frac{\alpha(t) - \beta u_n(t)}{\Delta} \tag{13}$$

where $\Delta = A - \beta > 0$ (Keynesian stability condition).

Equation (11) replaces Lavoie’s differential equation (1) with a hysteresis functional. Equation (12) is the Harrodian instability equation (6) in reduced form, kept unchanged. Equation (13) is the algebraic short-run equilibrium relation of the goods market, identical to (4).

Let us pre-empt an objection at once. Growth expectations α are, just as much as the normal rate, a convention formed under fundamental uncertainty; one could therefore argue that they too should be revised hysteretically, rather than according to the adaptive rule (6). We acknowledge this. This model is a theoretical model, whose purpose is to examine how a macrodynamic model behaves once genuine hysteresis is introduced into it; we introduce it into the closure of the normal rate alone, keeping the Harrodian rule for α , in order to isolate its effects. There is a deeper reason for this choice, specific to Lavoie’s structure: both conventions respond to the same gap, since $g^* - \alpha = \beta(u^* - u_n)$. If both were made hysteretic, nothing would move as long as this gap remained within the narrower of the two bands, and the model would be frozen. Lifting this limitation requires a third equation; we return to it in the conclusion.

4.2 Existence and uniqueness theorem

Theorem 1 (Global existence and uniqueness). *Under the following assumptions:*

- (i) Keynesian stability: $\beta < A$, which guarantees $\Delta := A - \beta > 0$;
- (ii) symmetric simple play: the hysteresis operator has boundaries $b_-(v) = v + \gamma$ and $b_+(v) = v - \gamma$ for a constant half-width $\gamma > 0$;
- (iii) admissible initial condition: $(\alpha_0, u_n^{(0)}) \in \mathbb{R}^2$ with $|u_n^{(0)} - u^*(0)| \leq \gamma$, where $u^*(0) = (\alpha_0 - \beta u_n^{(0)})/\Delta$;

the system (11)–(13) has a unique solution $(\alpha, u_n) \in W^{1,\infty}(0, t_E) \times W^{1,\infty}(0, t_E)$ on the whole interval $[0, t_E]$.

The proof is given in the mathematical appendix (Theorem A.11). It rests on the Picard–Lindelöf theorem for systems with hysteresis (Brokate & Sprekels, 1996, Theorems 3.1.1 and 3.1.3), whose linear growth conditions are trivially satisfied here, and on two specific properties of the symmetric simple play, Masing’s rule (simultaneous translation of input and output) and an algebraic reduction identity, both of which require the linearity and the symmetry of the boundaries. It is this twofold requirement that restricts the theorem to the simple play: the case of non-linear or genuinely asymmetric boundaries calls for a separate analysis (see Section 4.8).

4.3 The set of long-run equilibria

In Lavoie’s original model, the long-run equilibria $\dot{u}_n = \dot{\alpha} = 0$ lie on the line $\alpha = A u_n$ (equation 7). The amended system preserves this structure, as the following proposition establishes.

Proposition 2 (Preservation of the continuum of equilibria). *A state (u_n^{**}, α^{**}) is a stationary equilibrium of the system (11)–(13) if and only if the following two conditions hold simultaneously:*

- (i) $\alpha^{**} = A u_n^{**}$ (goods-market equilibrium at the normal rate),
- (ii) $b_+(u_n^{**}) \leq u_n^{**} \leq b_-(u_n^{**})$ with $u_n^{**} = u_n^{**}$ (admissible memory state of $\mathcal{F}_\gamma$).

Proof. Condition (i) follows from $\dot{\alpha} = 0$ in (12). Condition (ii) follows from $\dot{u}_n = 0$ in (11): in the inaction regime of the operator $\mathcal{F}_\gamma$, u_n remains constant as long as it stays strictly inside the band $[b_+(u^*), b_-(u^*)]$, that is, as long as $|u^* - u_n| < \gamma$. At the equilibrium point, $u^{**} = u_n^{**}$ by (13) with $\alpha^{**} = A u_n^{**}$. Condition (ii) therefore rewrites as $b_+(u_n^{**}) \leq u_n^{**} \leq b_-(u_n^{**})$, i.e. $|u_n^{**} - u_n^{**}| \leq \gamma$, which is trivially satisfied for every u_n^{**} . See the appendix, Proposition A.14. $\square$

For the symmetric simple play, condition (ii) holds for every u_n^{**} : the set of long-run equilibria in the (u_n, α) plane is therefore exactly the line $\alpha = A u_n$, the continuum of Lavoie (1996), unchanged. The amendment does not alter this set; what the play adds is a dynamic *inaction band* around this line, whose consequences will appear in the piecewise dynamics (Section 4.4). The restriction to a bounded segment, $u_n \in [73.6\%, 86.4\%]$ in the calibration of Section 3.4, does not stem from the formalism but from the saturation levels obtained by Bassi et al. (2022); it is therefore *external* to the model and belongs to the calibration. Figure 4 depicts this geometry in the (u_n, α) plane.

This geometry ties in with the reading of Bassi et al. (2022, Fig. 9), who distinguish three possible adjustment regimes after a shock: classical, in which the actual rate returns towards the normal rate; Harroddian, in which it moves cumulatively away from it; and hysteretic Kaleckian, in which it is the normal rate that adjusts. Our model chains two of them, the Harroddian regime inside the band and the hysteretic Kaleckian regime on its boundaries (Section 4.4); the classical regime does not appear, for want of a mechanism bringing the actual rate back towards the normal rate.

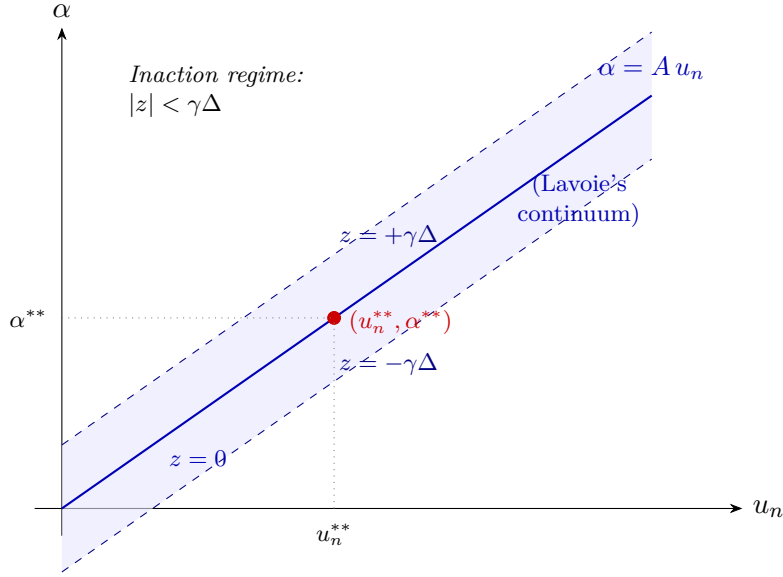


Figure 4: Continuum of equilibria and inaction corridor in the (u_n, α) plane. The line $\alpha = A u_n$ is the continuum of stationary equilibria (Proposition 2): every point on this line is an admissible equilibrium. The inaction corridor (blue area) is the band $\{|z| < \gamma\Delta\}$ with $z = \alpha - A u_n$; its boundaries (dashed) correspond to contact with one of the two boundaries of the play. Inside, the inaction regime; on the boundaries, the sliding regime.

4.4 The three regimes

The system (11)–(13) has three dynamic regimes, depending on the position of the memory state of the operator $\mathcal{F}_\gamma$. Recall that $z = \alpha - A u_n$, defined in (5), measures the excess of expectations over the growth warranted at the convention; geometrically, it is the vertical distance between the state of the system and the continuum of equilibria in the (u_n, α) plane. Since $u^* - u_n = z/\Delta$, the inaction band $|u^* - u_n| < \gamma$ reads $|z| < \gamma\Delta$.

Regime I (inaction): if $b_+(u^*(t)) < u_n(t) < b_-(u^*(t))$, then $\dot{u}_n = 0$ and the system reduces to

$$\dot{\alpha} = \chi\beta \cdot \frac{z}{\Delta}, \quad u_n = \text{const.}$$

At fixed u_n , this is a linear ODE in α . The numerator z varies linearly with α at constant u_n , so that $\dot{\alpha} = (\chi\beta/\Delta)\alpha + \text{const.}$ This ODE is *unstable*: the coefficient of α is strictly positive. This is the pure Harrodian instability of the basic model of Hein (2014, §11.2).

Regime II (upper boundary): if $u_n(t) = b_+(u^*(t))$ and the system stays there, then u_n slides along the upper boundary and $\dot{u}_n = b'_+(u^*) \cdot \dot{u}^*$. For the simple play, $b'_+ = 1$, so that $\dot{u}_n = \dot{u}^*$: the convention and the actual rate move together, at the constant distance γ . Substituting $u_n = b_+(u^*)$ into (13) and then into (12) yields a scalar ODE in u^* , the sign of whose derivative determines whether the boundary remains active or whether the system leaves Regime II.

Regime III (lower boundary): symmetric, with $u_n = b_-(u^*)$.

4.5 Piecewise stability

Proposition 3 (Instability of Regime I). *In the inaction regime, at fixed u_n , the dynamics of α are unstable: any gap $|z(0)| > 0$ grows exponentially at the rate $\lambda_I = \chi\beta/\Delta > 0$.*

Proof. At fixed u_n , $\dot{z} = \dot{\alpha} = \chi\beta z/\Delta$, hence $z(t) = z(0)e^{\lambda_I t}$ with $\lambda_I = \chi\beta/\Delta > 0$. See the appendix, Proposition A.16. $\square$

Proposition 4 (Exit conditions from Regime I). *Let $(\alpha_0, u_n^{(0)})$ with $b_+(u^*(0)) < u_n^{(0)} < b_-(u^*(0))$ and $z(0) > 0$. The system leaves Regime I in finite time $t_+ < +\infty$ through contact with the upper boundary, that is, $u^*(t_+) = b_+^{-1}(u_n^{(0)})$. Symmetrically, with $z(0) < 0$, for the lower boundary.*

Sketch. Under the instability of Regime I, $z(t) = z(0)e^{\lambda_I t}$ grows without bound. Now $u^*(t) = (\alpha(t) - \beta u_n^{(0)})/\Delta$ grows linearly with $\alpha(t) = A u_n^{(0)} + z(t)$. Hence $u^*(t) - u_n^{(0)} = z(t)/\Delta \rightarrow +\infty$, which forces contact with $b_+(\cdot)$ in finite time. See the appendix, Proposition A.17. $\square$

Proposition 5 (Reduced dynamics in the sliding regime). *In Regime II (sliding along the upper boundary, $u_n = b_+(u^*)$ with $\dot{u}^* > 0$), the reduced dynamics of u^* read*

$$\dot{u}^* = \frac{\chi\beta(\alpha - A b_+(u^*))}{\Delta(\Delta + \beta b'_+(u^*))},$$

and symmetrically in Regime III with b_- in place of b_+ . The quantity $z = \alpha - A u_n$ then satisfies

$$\dot{z} = \mu(b'_+) z, \quad \mu(b'_+) = \frac{\chi\beta(1 - b'_+)}{\Delta + \beta b'_+} :$$

sliding is destabilizing if $b'_+ < 1$ ($|z|$ grows exponentially), stabilizing if $b'_+ > 1$ ($|z|$ decays exponentially), and marginal if $b'_+ = 1$ (z constant). The proofs are given in the appendix (Propositions A.19 and A.21).

The simple play is the marginal case of this proposition. For it, $b'_+(u^*) = 1$ identically, hence $\mu = 0$: z is constant throughout the sliding interval, equal to its entry value $z_e = \pm\gamma\Delta$. The reduced dynamics simplify to $\dot{u}_n = \dot{u}^* = \chi\beta z_e/(\Delta A)$ (appendix, Remark A.20): u^* , u_n and α drift at constant speed, and the trajectory in the (u_n, α) plane is the line $\alpha = A u_n + z_e$, parallel to the continuum of equilibria and at a fixed vertical distance $|z_e|$ that it never crosses. Sliding therefore neither dampens the gap to the continuum nor widens it: the conventional revision of the normal rate exactly offsets the growth of animal spirits, with neither amplification nor catching up. The system converges to no stationary equilibrium and drifts indefinitely: Harrodian divergence, exponential in the inaction regime, becomes *linear* in the sliding regime, but it persists.

4.6 Typical trajectory

Propositions 4 and 5 together fully determine the structure of the trajectory in the absence of any correcting mechanism (appendix, Proposition A.24):

1. *Unstable Harrodian inaction phase:* u_n remains constant at $u_n^{(0)}$, the gap z grows exponentially at the rate $\lambda_I = \chi\beta/\Delta$, until contact in finite time with one of the two boundaries (the upper one if $z(0) > 0$, the lower one if $z(0) < 0$).
2. *Perpetual rectilinear sliding phase:* from the moment of contact, the system slides along the boundary it has reached with z constant ($z_e = \pm\gamma\Delta$); u^* , u_n and α drift at constant speed along a line parallel to the continuum of equilibria, without ever reaching it or returning to the inaction regime.

The return from sliding to inaction is not endogenous: it requires an external event (a negative shock to α , a parametric shock) that reverses the sign of the loading. In the absence of such a shock or of a damping mechanism, the system can neither converge to a stationary equilibrium nor produce a closed endogenous cycle. At the speed calibrated in Section 5.1, 0.32 percentage point per year, the convention would in fact leave the interval documented by Bassi

et al. (2022) in about twenty years starting from its centre. This drift must be taken for what it is: the behaviour of a theoretical model, a first sketch that calls for more complete models, endowed with saturation at the observed extremes or with boundaries of slope greater than one (appendix, Remark A.25).

Genuine hysteresis shows in the fact that a *transitory* shock ($\alpha \rightarrow \alpha + \Delta\alpha$ and back) does not return u_n to its initial level: the operator exerts no restoring force, and the normal rate keeps the value reached during sliding as long as the system remains in the inaction regime. The system is *path-dependent*, in keeping with the definition of genuine hysteresis given in the introduction.

4.7 Scope of the reference case: the slope of the boundaries

The translation of volatility thresholds into utilization thresholds (Section 3.1) leaves open the size of the revision once the threshold has been crossed, and Proposition 5 shows that everything hinges on it. In the simple play, the convention, once dragged, follows the actual rate point for point: along a boundary, $du_n/du^* = 1$, which amounts to assuming a constant tolerance margin, measured in points of utilization, around the convention. Nothing dictates this value. The relevant magnitude is the slope of the branches of the hysteresis loop expressed as a function of utilization, that is, the response of the normal rate to volatility along a branch, multiplied by the sensitivity $f'(u)$ of volatility to utilization. It is not to be confused with the slope of Setterfield and Avritzer's reduced form, $h'(u) = g'(f(u)) f'(u)$, which describes a single-valued relation, without a loop. To our knowledge, none of these slopes has been estimated: Setterfield and Avritzer estimate a single-valued relation between the normal rate and volatility, but not the relation between volatility and utilization, and Bassi et al. (2022) estimate sector-specific thresholds and regime levels, not a slope. Bassi et al. do approximate, in a one-sector model, the relation between the normal and the actual rate by $\Delta u_n = \alpha_B \Delta u$, with $0 < \alpha_B < 1$, where α_B is the fraction of a change in the actual rate that is passed on to the normal rate (Bassi et al., 2022, pp. 209–210). They obtain it by coupling Lavoie's equation, $\dot{u}_n = \beta_B(u - u_n)$, with a reversion of the actual rate towards the normal rate, $\dot{u} = -\gamma_B(u - u_n)$, which gives $\alpha_B = \beta_B/(\beta_B + \gamma_B)$ (the subscript B distinguishes their parameters from ours). But this coefficient measures the total adjustment between two equilibrium positions of a linear system, without threshold or inaction zone, whereas the slope of a boundary of the play measures the response of the normal rate during sliding; it therefore provides no direct information about the latter, and has not been estimated either. Yet the slope of the boundaries determines the nature of the sliding: a response stronger than one-for-one would dampen it, a weaker response would amplify it (Proposition 5). The simple play, with unit slope, is the reference case separating these two configurations, and it is for this case that our results are established; the estimation of this slope determines their empirical scope.

4.8 Extensions of the basic model

Two extensions of the formalism, which remain within the framework of hysteresis operators, seem to us particularly promising. Neither is developed here; the point is to indicate what they would bring, and under which conditions they are within reach.

An asymmetric revision of conventions. Setterfield & Avritzer (2020, p. 19) document a boom-slump asymmetry: the convention is revised quickly downwards when a crisis breaks out, slowly upwards when the recovery sets in, and Figures 6–8 of Bassi et al. (2022) bear its trace. The play extends naturally to this case by allowing two distinct half-widths, γ_+ and γ_- , that is, $b_+(v) = v - \gamma_+$ and $b_-(v) = v + \gamma_-$: the convention then tolerates an actual rate at most γ_+ above it and at most γ_- below it. A lower tolerance downwards than upwards, $\gamma_- < \gamma_+$, makes the convention be revised earlier in crises and later in recoveries, and places it on average below the actual rate, which ties in with the Steindlian idea of a deliberate reserve of capacity. This

asymmetric linear play reduces to the simple play by a mere translation: setting $\bar{\gamma} = (\gamma_+ + \gamma_-)/2$ and $c = (\gamma_- - \gamma_+)/2$, the boundaries read $b_{\pm}(v) = v + c \mp \bar{\gamma}$, a simple play of half-width $\bar{\gamma}$ shifted by c . Theorem 1 therefore applies, given a trivial redefinition of the initial condition (appendix, Remark A.32). The set of equilibria remains Lavoie’s continuum, but the band surrounding it becomes wider on one side than on the other, and, under an imposed input cycle, the loop becomes asymmetric as well, which would illustrate the “*slow process of forgetting the last crisis*” of Setterfield & Avritzer (2020, p. 19, n. 21). Representing the asymmetry of speeds itself, a fast downward and slow upward revision, would require boundaries with different slopes: this is the genuinely asymmetric case, which calls for a separate proof, through the implicit hysteresis operator of Brokate & Sprekels (1996, Proposition 2.11.10), under the condition $\beta < \Delta$, which is stronger than Keynesian stability, or through a markedly heavier regularization (appendix, Proposition A.31).

A revision sensitive to the intensity of surprise. This second extension bears directly on the central question of the paper. In Shackle, surprise rises by degrees with the gap (Section 3.2); the simple play ignores this, since it distinguishes only the band and its boundaries. The generalized play of Brokate & Sprekels (1996, §2.4), with non-linear boundaries, would represent it through a revision that accelerates with the gap, the boundaries steepening as the actual rate moves away from the convention. Now Section 4.7 has shown that the nature of the sliding depends on the slope of the boundaries: beyond one, sliding is dampened and the trajectory converges, under a uniform stability condition, to a fixed point (appendix, Remark A.23). A revision that is all the sharper as surprise is stronger could therefore restore a form of taming of Harroddian instability, no longer through the linear form of Lavoie’s equation, but through the very structure of the convention; this is the question that this extension would make it possible to settle. Its existence is not covered by Theorem 1 and calls for the same routes as the asymmetric case. Its calibration also requires a precaution: saturating boundaries must remain reachable over the whole empirical range. If, for instance, the upper boundary levelled off at 0.80 while the normal rate stood at 0.82, it would never be reached: the convention would never be revised and Harroddian divergence would set in unchecked (appendix, Remark A.18). The upper boundary must therefore be able to exceed u_{nH} and the lower boundary to fall below u_{nL} .

Finally, the extension of hysteresis to growth expectations α , anticipated in Section 4.1, is discussed in the conclusion.

5 Numerical simulations and the long-run fate of the paradoxes

5.1 Calibration of the macroeconomic parameters

For the simulations we adopt the following parameters, whose orders of magnitude are usual in the empirical post-Keynesian Kaleckian literature (Stockhammer & Onaran, 2013; Hein, 2014), the unit of time being the year:

$$s_P = 0.5, \quad h = 0.3, \quad v = 2.5, \quad \beta = 0.03, \quad \chi = 0.10.$$

The propensity to save out of profits, the profit share and the capital–capacity ratio fix $A := s_P h / v = 0.06$: the macroeconomic saving rate relative to the capital stock equals 6 % of the utilization rate, so that at the initial equilibrium $u_n^{(0)} = 0.80$ the rate of accumulation is $g^* = 4.8\%$ per year, a plausible order of magnitude. The sensitivity $\beta = 0.03$ of investment to the utilization gap gives $\Delta = A - \beta = 0.03 > 0$: Keynesian stability holds. The speed of revision of expectations $\chi = 0.10$ implies an inaction instability rate $\lambda_I = \chi \beta / \Delta = 0.10$ per year, that is, a doubling time of the gap z of about seven years: the Harroddian drift is slow, on the scale of a decade, which is consistent with a long-run conventional revision dynamics in the spirit of the “reaction period” of Setterfield & Avritzer (2020). The symmetric band is calibrated on the saturation levels of Bassi et al. (2022), $u_{nL} = 0.736$ and $u_{nH} = 0.864$, hence $\gamma = 0.064$

(Section 3.4); this yields a sliding speed $\chi\beta\gamma/A = 0.0032$ per year: in the boundary regime, the convention is revised by 0.32 percentage point per year, a slow revision, in keeping with the intuition of a convention that yields only gradually. This being a theoretical paper, we do not carry out a systematic sensitivity analysis; the closed-form formulas given below nevertheless allow the reader to recompute speeds and durations for other parameter values.

5.2 Three numerical simulations

The trajectories that follow are computed *exactly*, without approximate numerical integration: in each regime the system reduces to a linear dynamics with an analytical solution, and the transition times are solved in closed form. Let us spell out these formulas, which allow the reader to recompute every value reported below. In the *inaction regime*, u_n is constant and the gap $z = \alpha - Au_n$ follows $z(t) = z(0)e^{\lambda_I t}$ with $\lambda_I = \chi\beta/\Delta$ (appendix, Proposition A.16); contact with a boundary occurs when $|z|$ reaches $\gamma\Delta$, that is, after a time $\lambda_I^{-1} \ln(\gamma\Delta/|z(0)|)$. In the *sliding regime*, z is held at $\pm\gamma\Delta$ and the drift is affine: $\dot{u}_n = \dot{u}^* = \pm\chi\beta\gamma/A$ and $\dot{\alpha} = \pm\chi\beta\gamma$ (appendix, Remark A.20 and Proposition A.21). The shocks to α are calibrated so that the instantaneous jump they induce in u^* , equal to $\Delta\alpha/\Delta$, remains strictly smaller than γ : the input of the play thus remains within the tolerance band at the moment of the shock, and the operator never has to handle an input that would throw it outside its admissible region.²

Table 1 summarizes the three shock protocols and their effects; each simulation is then detailed in a single format: the *protocol* (which shock, on which variable, with which economic interpretation), the *course* (the chronology of the regimes traversed) and the *result* (what the simulation demonstrates).

	Imposed shock	What is tested	Observed effect
1	<i>Net-zero</i> pulse on α : +3% at $t = 0$, fully withdrawn at $t = 10$ years	Path memory; status of inaction plateaus	Irreversible drift: u_n moves from 0.80 to 0.823 (transitory plateau), then 0.874 at $t = 40$; no return
2	The same pulse, applied to Lavoie's system ($\omega = 0.5$)	Sensitivity to the <i>cumulative sum</i> vs the <i>order</i> of shocks	Exact round trip: $0.80 \rightarrow 0.827 \rightarrow 0.80$; strictly zero net effect
3	Triangular cycle <i>imposed</i> on α ($0.048 \rightarrow 0.0521 \rightarrow 0.0451 \rightarrow 0.048$)	Response to an exogenous cycle; <i>rate-independence</i>	Parallelogram-shaped Ewing loop, whose area is independent of the speed of the cycle

Table 1: The three simulations: shock protocols and effects.

Simulation 1: transitory shock to α and path memory. *Protocol.* We start from the equilibrium $(\alpha_0, u_n^{(0)}) = (0.048, 0.80)$ on the continuum and apply a *net-zero* pulse: α is raised by +3% ($\Delta\alpha = 0.00144$) at $t = 0$, then lowered by the same amount at $t = 10$ years. Economically, this pulse represents a transitory episode of optimism, an upward revision of the expected trend growth of sales, ten years of euphoria, fully reabsorbed afterwards. The protocol transposes the test by which Amable et al. (1993, p. 128) distinguish hysteresis from mere persistence: subject the system to a shock, then to the opposite shock of the same size, and see whether it returns to its initial state. A unit-root system does, whatever the size of the shock; a hysteretic system retains a remanence.

Course (Figure 5; the numbers refer to the phases in panel (a), and panel (b) shows the gap $z = \alpha - Au_n$ that governs everything):

1. *Inaction*, $t \in [0, 2.9]$. At the moment of the shock, the utilization rate jumps from 0.80 to 0.848. The gap with the convention, 4.8 points, remains within the tolerance of 6.4 points:

²These jumps are exogenous shocks that we produce ourselves in the simulation. The play, defined here on continuous inputs, extends to such inputs through its update formula (Krasnosel'skii & Pokrovskii, 1989, ch. 2); each shock can also be read as a fast ramp, without any change in the results.

firms do not regard it as abnormal and do not revise their norm, which stays at 0.80. But optimism feeds on itself: actual accumulation exceeds expectations, which firms therefore revise upwards, which raises utilization further. In terms of z , the shock brings the gap to $\Delta\alpha = 0.00144$, below the threshold $\gamma\Delta = 0.00192$; z then grows exponentially at the rate $\lambda_I = 0.1$ (pure Harrodian instability) and reaches the threshold after $\lambda_I^{-1} \ln(4/3) \approx 2.9$ years: the actual rate touches the upper boundary of the band.

2. *Sliding*, $t \in [2.9, 10]$. From now on, any further rise in utilization leaves the range of non-revision: firms raise their norm at the same pace, so that the actual rate and the convention rise together, at the constant distance γ ($\dot{u}_n = \dot{u}^*$). The gap z stays pinned at the threshold, and the convention is revised at the constant speed $\chi\beta\gamma/A = 0.0032$ per year. At the moment of withdrawal, $u_n = 0.823$.
3. *Withdrawal of the shock, inaction plateau*, $t \in [10, 23.9]$. Optimism collapses at once: the utilization rate falls by 4.8 points and moves back inside the band. The convention does not follow: the gap having become tolerable again, it stays at 0.823. But expectations remain slightly above the growth warranted at this new convention: z falls back to 0.00048, inside the corridor but strictly positive. This plateau is therefore not an equilibrium: Harrodian instability slowly restarts the growth of z , which must quadruple before touching the threshold again, i.e. after $\lambda_I^{-1} \ln 4 \approx 13.9$ years.
4. *Resumption of sliding*, $t \in [23.9, 40]$. The actual rate touches the upper boundary again and the convention starts rising once more, at the same speed: $u_n \approx 0.874$ at $t = 40$, and the drift would continue beyond, the forty-year horizon having no particular significance.

Result. Two properties stand out: *path memory*, since the operator exerts no restoring force towards $u_n^{(0)} = 0.80$, so that ten years of reabsorbed euphoria leave a lastingly displaced convention; and the *transitory* character of any inaction plateau in the absence of a correcting mechanism (appendix, Proposition A.24), since the residual tension between animal spirits and the convention, however small, eventually restarts the cumulative process. Panel (c) shows the geometric signature of the central result: the sliding phases follow lines *parallel* to the continuum, at a vertical distance $\gamma\Delta$, without ever rejoining it.

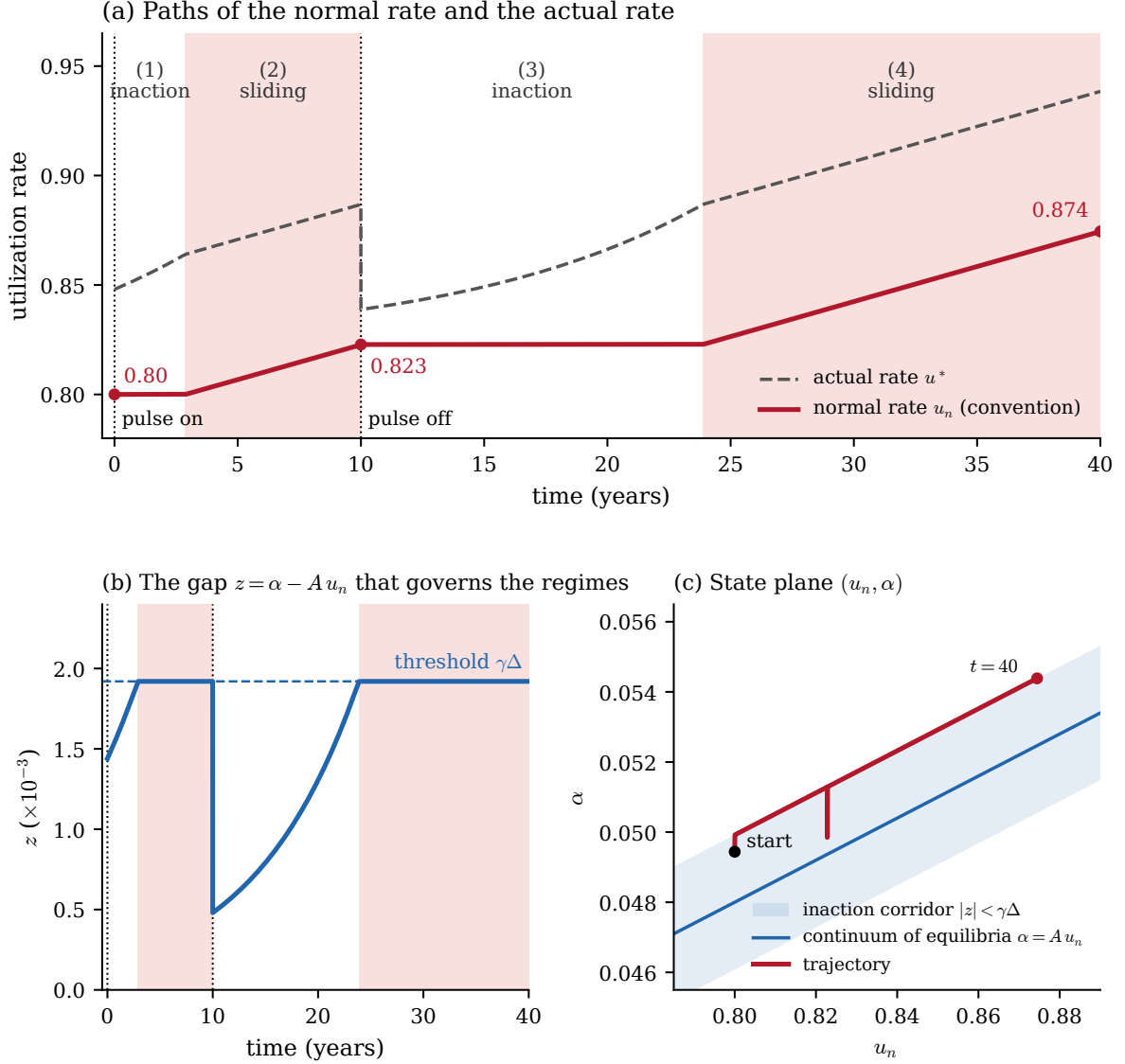


Figure 5: Simulation 1: net-zero pulse on α (+3% at $t = 0$, withdrawn at $t = 10$). (a) Paths of u_n and u^* , with the four numbered phases (shaded bands: sliding phases). (b) The gap $z = \alpha - A u_n$ that governs the transitions: exponential growth in inaction, pinning at the threshold $\gamma\Delta$ during sliding, drop at the withdrawal of the shock. (c) State plane (u_n, α) : vertical inaction phases, sliding along the boundary of the corridor, parallel to the continuum. Exact regime-wise solutions; parameters of Section 5.1.

Simulation 2: the same shock, applied to Lavoie’s closure (1). *Protocol.* We subject the original Lavoie system $\{(1), (6)\}$ with $\omega = 0.5$ to the same net-zero pulse, and Figure 6 superimposes the two responses. This simulation serves as a control: it isolates what, in the response of Simulation 1, is due to the operator rather than to the rest of the model.

Course. With the linear closure, the convention does not wait: from the moment of the shock, it starts following the actual rate, without threshold, at a speed proportional to the gap. Since the convention is revised faster than expectations here (stability condition), the gap closes quickly: z decays at the rate $(A\omega - \chi\beta)/\Delta = 0.9$ per year, and the convention settles at 0.827. When the shock is withdrawn, the movement reverses exactly: the convention comes back down to 0.80. The reason is that the system is linear. The quantity $\alpha - (\chi\beta/\omega) u_n$ remains constant between two shocks and alone determines the final equilibrium; the shock raises it by $\Delta\alpha$, the withdrawal lowers it by the same amount. The response to the pulse is therefore the superposition of the

response to the shock and of the delayed, opposite response to its withdrawal, so that a pulse whose cumulative sum is zero has a strictly zero asymptotic effect. This is the signature of linear zero-root dynamics, the continuous-time version of the unit root (Amable et al., 1994), which is sensitive only to the cumulative sum of the shocks. Lavoie (1996, p. 141, n. 27) acknowledges it himself: in his model, “there will be irreversibility if the reaction parameters change through time”, and not otherwise.

Result. The simple play, by contrast, *retains*: u_n never comes back down, and even drifts further. The difference between the two closures is not quantitative but qualitative: only the second possesses a memory of the order of the crossings. The significance of the contrast goes beyond mechanics: within the model, a transitory demand episode, a stimulus followed by its withdrawal, has no long-run effect on the convention under the linear closure, but displaces it lastingly under genuine hysteresis. Any evaluation of demand policies that presupposes the reversibility of conventional effects therefore silently inherits the linear form of the closure adopted.

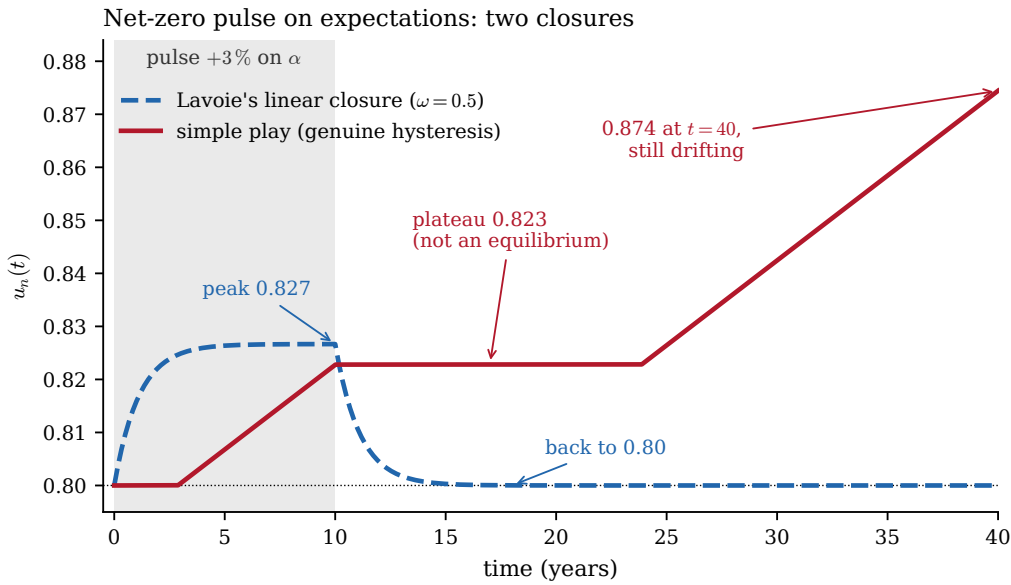


Figure 6: Simulation 2: the same pulse, two closures. Lavoie’s linear closure ($\omega = 0.5$, blue dashes) returns exactly to $u_n^{(0)} = 0.80$: zero net effect. The simple play (red line) keeps what was gained during sliding and continues its drift: genuine remanence.

Simulation 3: Ewing loop under exogenous forcing. *Protocol.* A loop between input and output may arise from genuine hysteresis, but also from a mere lag; only the former persists when the input cycle is traversed more and more slowly. To trace the loop of the model, we impose on α a triangular cycle ($0.048 \rightarrow 0.0521 \rightarrow 0.0451 \rightarrow 0.048$), the own dynamics of α being switched off for the duration of the simulation, just as Bassi et al. (2022) impose a volatility cycle on their firms, and we trace the response in the (u^*, u_n) plane (Figure 7).

Course. Expectations first rise: the actual rate climbs from 0.80 to 0.864 without the convention moving, then, once the upper boundary is reached, both rise together up to $(u^*, u_n) = (0.900; 0.836)$. When the cycle turns, expectations fall: the actual rate comes down, but the convention stays frozen at 0.836 as long as the gap remains tolerable, that is, until the actual rate reaches the lower boundary at 0.772; both then fall together down to $(0.720; 0.784)$. When expectations rise back to their initial value, the convention stays frozen at 0.784 and the actual rate rises alone, up to 0.816. The path traces a parallelogram-shaped Ewing loop: two oblique sliding branches along the boundaries b_+ and b_- , two horizontal crossings of the inaction band. At the end of the cycle, expectations are back at their starting value, but the convention is not: it ends at 0.784, a remanence of 1.6 points.

Result. The linear closure (1) would admittedly also produce a loop under cyclical forcing, but a mere *lag* loop, which would vanish in the quasi-static limit of an arbitrarily slow forcing. The loop of the play is, on the contrary, *rate-independent*, like that of any hysteresis operator (Brokate & Sprekels, 1996, p. 37): its area does not depend on the speed at which the input cycle is traversed, and it persists at zero frequency, which is the operational marker of genuine hysteresis as opposed to lag effects. The economic reading of the parallelogram ties in with Section 3.2: each oblique branch is a regime of conventional revision, triggered by the overshooting of the range of non-revision, and each horizontal crossing is that range itself, where the convention holds firm. The figure is thus the macro-formal analogue, elementary in the sense specified in Section 3.1, of the aggregate loop that Bassi et al. (2022, Fig. 4) obtain by simulating twenty thousand individual relays. It must be stressed that this loop is obtained by *exogenous forcing* of the input cycle, as in Bassi et al. (2022): it illustrates the hysteretic response of the operator to an imposed cycle, not a cycle that the dynamics of the system would generate by themselves, since the amended model produces no endogenous cycle.

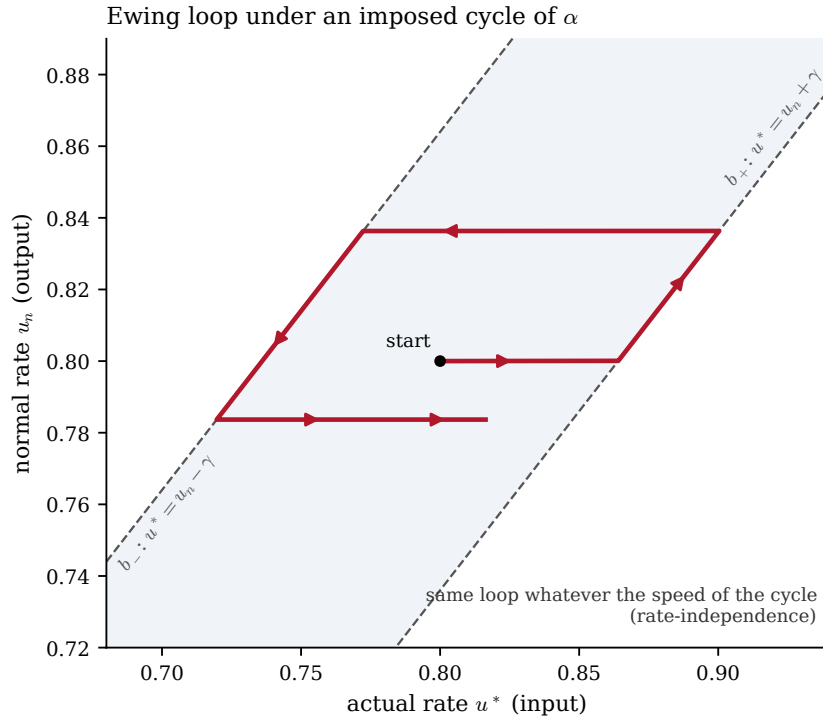


Figure 7: Simulation 3: Ewing loop in the (u^*, u_n) plane under an imposed triangular cycle of α . Sliding branches along b_+ (ascent) and b_- (descent), horizontal inaction crossings. The input cycle is exogenous: the system produces no endogenous cycle.

5.3 The long-run fate of the paradoxes

The paradoxes of thrift and of costs in the classical Kaleckian model rest on the variability of u in the long run. In the amended system, their status splits in two: the instantaneous effect survives, comparative statics between long-run equilibria disappears.

Proposition 6 (Paradoxes in the inaction regime). *In the inaction regime, at fixed u_n , a fall in s_P or in h raises u^* and g^* , and these effects persist as long as the system does not leave the inaction band. The paradoxes of thrift and of costs are therefore preserved, as instantaneous effects, in the inaction regime (appendix, Propositions A.27 and A.28).*

Proposition 7 (Boundary regime: absence of stationary equilibrium). *In a boundary regime (II or III), the system reaches no stationary equilibrium: the normal rate slides along the boundary without ever catching up with the actual rate, and the system drifts indefinitely along a parallel to the continuum of equilibria (appendix, Propositions A.21 and A.24). The question of comparative statics between stationary equilibria therefore does not arise in this regime (appendix, Proposition A.29).*

The amendment by a hysteresis operator therefore does not overturn the Kaleckian paradoxes: a fall in the propensity to save or in the profit share raises utilization and growth at the moment of the shock, inside the inaction band as well as on a boundary, where the sliding it triggers runs in the same direction. What it removes is the possibility of stating them as a comparison between long-run equilibria, since in a boundary regime the system leaves every stationary equilibrium and diverges in the Harroddian sense, in the absence of a correcting mechanism. Far from producing a stable synthesis between the classical and Kaleckian regimes, the amendment thus reveals that the taming of Harroddian instability achieved by the linear closure of Lavoie (1996) hinges on the particular form of its adjustment equation: as soon as the latter is replaced by hysteretic dynamics, Harroddian instability re-emerges at the boundaries.

6 Conclusion

This paper has shown that a one-sector aggregate model is sufficient to account for genuine hysteresis in the normal rate of capacity utilization, without the detour through micro-founded modelling that Setterfield and Avritzer called for and that Bassi *et al.* regard as indispensable. It suffices to endogenize u_n not through the linear differential equation of Lavoie (1996), but through a symmetric *simple play*, a hysteresis operator posited directly at the macro-aggregate level.

Four main results emerge. *Formally*, the amended system has a unique global solution under standard assumptions, by the Picard–Lindelöf theorem for systems with hysteresis. *Qualitatively*, the hysteresis obtained is genuine: a shock followed by the opposite shock leaves a remanence where Lavoie’s closure returns exactly to its starting point, and the loop of the model does not depend on the speed of the cycle; a transitory demand episode therefore displaces the convention lastingly. *Structurally*, the long-run continuum of equilibria of Lavoie (1996) is preserved by the amendment, but it consists of unstable equilibria, any gap to the continuum being amplified in the inaction regime, and the amendment endows it with a dynamic inaction band; the dynamics break down into three piecewise regimes whose stability conditions are explicitly characterized. *Economically*, the paradoxes of thrift and of costs are preserved as instantaneous effects, but can no longer be stated as a comparison between long-run equilibria, since in a boundary regime the system leaves every stationary equilibrium and diverges in the Harroddian sense, linearly rather than exponentially, drifting parallel to the continuum without ever reaching it: the taming of Harroddian instability achieved by the linear closure of Lavoie (1996) turns out to hinge on the particular form of its adjustment equation. This last result holds for the reference case of a one-for-one revision at the boundaries of the band; a stronger response would dampen the sliding, and the estimation of this slope determines the empirical scope of our conclusions.

This paper brings together three distinct contributions. The Steindlian behavioural foundation of Setterfield & Avritzer (2020) provides the micro-behavioural justification of genuine hysteresis. The aggregation through heterogeneity of Bassi *et al.* (2022) provides the empirical and computational demonstration that a macro hysteresis loop does emerge. The mathematical theory of hysteresis operators of Brokate & Sprekels (1996) provides the rigorous object that makes it possible to formalize this path memory at the aggregate level and to couple it with the structural Kaleckian framework of Lavoie (1996). What we do is to join them.

This framework nevertheless comes at a high formal cost. The proof of the existence and uniqueness theorem, which takes up a large part of the mathematical appendix, is tedious; it

also rests on properties specific to the symmetric simple play, Masing’s rule and an algebraic reduction identity, so that extensions with non-linear or genuinely asymmetric boundaries each require a separate proof. The two extensions discussed in Section 4.8 nevertheless seem to us particularly promising. An asymmetric revision of conventions would make it possible to represent the speed of downward revisions in crises and their slowness upwards in recoveries, which Setterfield & Avritzer (2020) document. A revision sensitive to the intensity of surprise, through a generalized play whose boundaries steepen with the gap, would make it possible to settle the question this paper leaves open: since the taming of Harrodian instability depends on the slope of the boundaries, a convention that is all the quicker to be revised as surprise is stronger could restore it through its very structure. Two further lines of work deserve mention. The generalization to a multisectoral framework, with a simple play $\mathcal{F}_\gamma^{(j)}$ specific to each sector j , would capture the sectoral diversity empirically documented by Bassi et al. (2022, Fig. 6–8). The integration of an investment function sensitive to the interest rate and to debt, along the lines of Chapter 9 of Hein (2014), would open the way to an analysis of monetary policy in the presence of hysteresis in the normal rate, a topic for which Duménil & Lévy (1999) and Lavoie (2022, §6.5.4, pp. 420–422) have already sketched avenues that our formalism makes it possible to pursue rigorously. Lavoie notes in particular that a Phillips curve with a horizontal segment defines a range of utilization rates within which the central bank does not react, so that the Kaleckian results continue to hold there (Lavoie, 2022, p. 422): an inaction band of monetary policy, which the play would make it possible to formalize.

Finally, the reconciliation of the inertia and the adaptation of conventions allows us to take a stand on the critique of Lucas (1976). Its premise is that a change in the conduct of economic policy alters the formation of expectations; its conclusion, that only micro-founded models with rational expectations can account for this. We accept the premise in part: a hysteretic convention is *locally invariant*, since, as long as the gap between the actual and the normal rate remains within the band, it behaves like a parameter, which justifies the ordinary practice of Kaleckian models of treating it as given; but it is revised when this gap reaches a threshold, which concedes that agents alter their behavioural rules when their environment changes sufficiently. We reject the conclusion: a change in the formation of expectations following a change in economic policy can be accounted for, provided it is done realistically, through a conventional revision rule at threshold crossings rather than through re-optimization under rational expectations. The present work is only a first sketch: it shows what a rigorous macroeconomic model that partially takes Lucas’s premise into account might look like, and an explicit policy rule remains to be introduced. The same hysteresis should eventually apply to growth expectations α , which are a convention in the same way as the normal rate. Within Lavoie’s framework, this cannot be done without freezing the dynamics: both conventions respond to the same gap $u^* - u_n$, and as long as it remains within the narrower of the two bands, neither is revised and the utilization rate does not move. The model then stays in the ordinary Kaleckian equilibrium, which is, in a way, a justification of common Kaleckian practice, Harrodian instability awakening only beyond the thresholds. A third equation is needed. The Kaleckian supermultiplier of Lavoie (2022, §6.5.8, pp. 432–438), after Allain (2015) and Lavoie (2016), provides it: the ratio of autonomous demand to capital, z , evolves there according to $\dot{z} = z(g_z - g^*)$, where g_z is the exogenous growth rate of this demand, and it keeps moving the utilization rate when both conventions are frozen, while anchoring them to the growth of autonomous demand. With g_z as the growth rate of government expenditure, as in Allain (2015), this three-equation framework would provide the policy rule that is missing here, and would make it possible to carry the refutation of Lucas’s conclusion through to the end. This is the subject of future work.

References

- Allain, O. (2015). Tackling the instability of growth: A Kaleckian-Harrodian model with an autonomous expenditure component. *Cambridge Journal of Economics*, 39(5), 1351–1371.
- Amable, B., Henry, J., Lordon, F. & Topol, R. (1993). Unit root in the wage-price spiral is not hysteresis in unemployment. *Journal of Economic Studies*, 20(1/2), 123–135.
- Amable, B., Henry, J., Lordon, F. & Topol, R. (1994). Strong hysteresis versus zero-root dynamics. *Economics Letters*, 44(1–2), 43–47.
- Auerbach, P. & Skott, P. (1988). Concentration, competition and distribution. *International Review of Applied Economics*, 2(1), 42–61.
- Bassi, F., Bauermann, T., Lang, D. & Setterfield, M. (2022). Is capacity utilization variable in the long run? An agent-based sectoral approach to modeling hysteresis in the normal rate of capacity utilization. *Structural Change and Economic Dynamics*, 63, 196–212.
- Belke, A., Göcke, M. & Günther, M. (2013). Exchange rate bands of inaction and play-hysteresis in German exports: Sectoral evidence for some OECD destinations. *Metroeconomica*, 64(1), 152–179.
- Brokate, M. & Sprekels, J. (1996). *Hysteresis and Phase Transitions*. Applied Mathematical Sciences vol. 121, Springer-Verlag.
- Committeri, M. (1986). Some comments on recent contributions on capital accumulation, income distribution and capacity utilization. *Political Economy: Studies in the Surplus Approach*, 2(2), 161–186.
- Cross, R. (1993). On the foundations of hysteresis in economic systems. *Economics and Philosophy*, 9(1), 53–74.
- Cross, R., Grinfeld, M. & Lamba, H. (2009). Hysteresis and economics: Taking the economic past into account. *IEEE Control Systems Magazine*, 29(1), 30–43.
- Duménil, G. & Lévy, D. (1999). Being Keynesian in the short term and classical in the long term: The traverse to classical long-term equilibrium. *Manchester School*, 67(6), 684–716.
- Dutt, A. K. (1990). *Growth, Distribution and Uneven Development*. Cambridge University Press.
- Dutt, A. K. (1997). Equilibrium, path dependence and hysteresis in post-Keynesian models. In P. Arestis, G. Palma & M. Sawyer (Eds.), *Markets, Unemployment and Economic Policy: Essays in Honour of Geoff Harcourt, Volume Two* (pp. 238–253). Routledge.
- Dutt, A. K. (2010). Equilibrium, stability and path dependence in post-Keynesian models of economic growth. In A. Birol, D. Foley, H. D. Kurz, B. Schefold & I. Steedman (Eds.), *Production, Distribution and Trade: Alternative Perspectives. Essays in Honour of Sergio Parrinello* (pp. 233–253). Routledge.
- Franke, R. (2021). Pure Harrodian dynamics: Heterogeneous expectations and the loss of three established propositions. Working paper, DOI: 10.13140/RG.2.2.17746.07365.
- Göcke, M. (2002). Various concepts of hysteresis applied in economics. *Journal of Economic Surveys*, 16(2), 167–188.
- Göcke, M. & Werner, L. (2015). Play hysteresis in supply or in demand as part of a market model. *Metroeconomica*, 66(2), 339–374.

- Harrod, R. F. (1939). An essay in dynamic theory. *Economic Journal*, 49(193), 14–33.
- Harrod, R. F. (1972). Uncertainty and dynamic axioms. In C. F. Carter & J. L. Ford (Eds.), *Uncertainty and Expectations in Economics: Essays in Honour of G. L. S. Shackle* (pp. 122–135). Basil Blackwell.
- Hein, E. (2014). *Distribution and Growth after Keynes: A Post-Keynesian Guide*. Edward Elgar.
- Hein, E., Lavoie, M. & van Treeck, T. (2012). Harroddian instability and the ‘normal rate’ of capacity utilization in Kaleckian models of distribution and growth: A survey. *Metroeconomica*, 63(1), 139–169.
- Krasnosel’skii, M. A. & Pokrovskii, A. V. (1989). *Systems with Hysteresis*. Springer-Verlag.
- Krejčí, P., Kwame, E., Lamba, H., Rachinskii, D. & Zagvozdkin, A. (2023). A continuum of path-dependent equilibrium solutions induced by sticky expectations. *Applications of Mathematics*, 68(6), 751–793.
- Lamba, H., Krejčí, P. & Rachinskii, D. (2020). The global stability of a class of history-dependent macroeconomic models. *Mathematical Modelling of Natural Phenomena*, 15, article 49.
- Lang, D. (2008). Why economists should choose their inheritance: Physics and path-independence in economic systems. *Review of Political Economy*, 20(3), 405–420.
- Lavoie, M. (1992). *Foundations of Post-Keynesian Economic Analysis*. Edward Elgar.
- Lavoie, M. (1995). The Kaleckian model of growth and distribution and its neo-Ricardian and neo-Marxian critiques. *Cambridge Journal of Economics*, 19(6), 789–818.
- Lavoie, M. (1996). Traverse, hysteresis and normal rates of capacity utilization in Kaleckian models of growth and distribution. *Review of Radical Political Economics*, 28(4), 113–147.
- Lavoie, M. (2016). Convergence towards the normal rate of capacity utilization in neo-Kaleckian models: The role of non-capacity creating autonomous expenditures. *Metroeconomica*, 67(1), 172–201.
- Lavoie, M. (2022). *Post-Keynesian Economics: New Foundations* (2nd ed.). Edward Elgar.
- Lucas, R. E. (1976). Econometric policy evaluation: A critique. In K. Brunner & A. Meltzer (Eds.), *The Phillips Curve and Labor Markets*, Carnegie-Rochester Conference Series on Public Policy, 1, 19–46.
- Nikiforos, M. (2016). On the ‘utilisation controversy’: A theoretical and empirical discussion of the Kaleckian model of growth and distribution. *Cambridge Journal of Economics*, 40(2), 437–467.
- Serrano, F. (1995). Long period effective demand and the Sraffian supermultiplier. *Contributions to Political Economy*, 14, 67–90.
- Setterfield, M. (2009). Path dependency, hysteresis and macrodynamics. In P. Arestis & M. Sawyer (Eds.), *Path Dependency and Macroeconomics* (pp. 37–79). Palgrave Macmillan.
- Setterfield, M. (2019). Long-run variation in capacity utilization in the presence of a fixed normal rate. *Cambridge Journal of Economics*, 43(2), 443–463.
- Setterfield, M. & Avritzer, J. D. (2020). Hysteresis in the normal rate of capacity utilization: A behavioral explanation. *Metroeconomica*, 71(4), 898–919.

- Shackle, G. L. S. (1949). *Expectation in Economics*. Cambridge: Cambridge University Press.
- Shackle, G. L. S. (1969). *Decision, Order and Time in Human Affairs*, 2^e éd. Cambridge: Cambridge University Press.
- Skott, P. (1989). *Conflict and Effective Demand in Economic Growth*. Cambridge University Press.
- Skott, P. (2010). Growth, instability and cycles: Harrodian and Kaleckian models of accumulation and income distribution. In M. Setterfield (Ed.), *Handbook of Alternative Theories of Economic Growth* (pp. 108–131). Edward Elgar.
- Skott, P. (2012). Theoretical and empirical shortcomings of the Kaleckian investment function. *Metroeconomica*, 63(1), 109–138.
- Steindl, J. (1952). *Maturity and Stagnation in American Capitalism*. Oxford: Blackwell.
- Stockhammer, E. & Onaran, Ö. (2013). Wage-led growth: Theory, evidence, policy. *Review of Keynesian Economics*, 1(1), 61–78.