## The paradoxes of fundamentalists' profits

G. Galanis ${ }^{1}$, J. Lustenhouwer ${ }^{2}$ and G. Ricchiuti ${ }^{3}$

Berlin, FMM, 2023

[^0]
## Motivation

- Fundamentalists are a standard type in models with HIAs
- Fundamentalists act according to information (or beliefs) about the value of a variable
- Their expectation is
- consistent if everyone is fundamentalist
- not necessarily consistent when HIAs are assumed


## Questions:

(1) Does it matter if expectations are non consistent with 'reality'?
(2) If no, does it make sense for agents to pay to be fundamentalists?

## Overview

- Study the baseline version of Brock and Hommes (1998)
- Asset pricing model with two types:
- fundamentalists (costly strategy)
- chartists (or trend followers)
- Focus on profits (change in wealth) of strategies
- Counterintuitive insights or 'paradoxes':
(1) Fundamentalists gain less on average
(2) Fundamentalists' profits are higher when price is far from the fundamental value
(3) Decreasing marginal profits with costs


## Asset pricing model with two types

- Agents are mean variance wealth maximisers
- Two types of strategies: fundamentalist and trend following
- Two types of assets:
- Risk free, perfectly elastically supplied, paying interest $R$
- Risky pays stochastic (IID) dividend $y_{t}$ and is sold at price $p_{t}$
- Assuming zero supply of outside shares, define as the fundamental value of the asset:

$$
\begin{equation*}
p^{*}=y /(R-1) \tag{1}
\end{equation*}
$$

## Price deviations and expected profits

- Let $x_{t}=p_{t}-p^{*}$ denote the deviation from the fundamental value
- Agents who pay $C$ have access to the fundamental value and expect $x_{t}=0$
- Trend followers expect that $x_{t}$ follows a trend $g$ compared to its value in the previous period.
- Expected profits of fundamentalist and trend following strategies are

$$
\begin{equation*}
\pi_{f, t}=\frac{1}{a \sigma^{2}} R x_{t-1}\left(R x_{t-1}-x_{t}\right)-C \tag{2}
\end{equation*}
$$

where $a$ is assumed risk aversion and $\sigma^{2}$ is the excess return variance, and

$$
\begin{equation*}
\pi_{c, t}=\frac{1}{a \sigma^{2}}\left(x_{t}-R x_{t-1}\right)\left(g x_{t-2}-R x_{t-1}\right) \tag{3}
\end{equation*}
$$

## Dynamics

Assuming a logit framework with scale parameter $1 / \beta$, regarding choices
the fractions of fundamentalists and trend followers in each period are given by

$$
\begin{equation*}
n_{f, t}=\frac{e^{\beta \pi_{f, t}}}{e^{\beta \pi_{f, t}}+e^{\beta \pi_{f, t}}} \tag{4}
\end{equation*}
$$

and

$$
\begin{equation*}
n_{c, t}=\frac{e^{\beta \pi_{c, t}}}{e^{\beta \pi_{f, t}}+e^{\beta \pi_{f, t}}} \tag{5}
\end{equation*}
$$

with

$$
\begin{equation*}
m_{t}=n_{f, t}-n_{c, t}=\tanh \left[\frac{\beta}{2}\left(\frac{g x_{t-2}\left(R x_{t-1}-x_{t}\right)}{a \sigma^{2}}-C\right)\right] \tag{6}
\end{equation*}
$$

and

$$
\begin{equation*}
R x_{t}=n_{c, t-1} g x_{t-1} \tag{7}
\end{equation*}
$$

## Summary

Prices can deviate from fundamentals

Bifurcation diagram


## Relative average profits

- We run the model for $N(10000)$ periods
- Calculate the average profits for each of the strategies for the whole period:

$$
\hat{\pi}_{h}=\frac{1}{N} \sum_{t=1}^{N}\left[\pi_{h, t}\right] \quad h=f, c
$$

- Calculate $\hat{\pi}_{f}-\hat{\pi}_{c}$ for different values of
- costs $C$
- intensity of choice $\beta$


## Fundamentalists gain less



- When price $x_{t}=0, \hat{\pi}_{f}-\hat{\pi}_{c}=C$
- $\hat{\pi}_{f}-\hat{\pi}_{c}$ increases as $x_{t}>0$, decreases in part of the chaotic region


## Trend followers' average profits



Trend followers also lose $\rightarrow$ not the driver of difference in profits

## Fundamentalists' profits



The difference is driven by the profits of fundamentalists

## Profitability paradoxes?

- Is it paradoxical that fundamentalists gain less on average?


## Profitability paradoxes?

- Is it paradoxical that fundamentalists gain less on average?
- Not necessarily as agents change strategies over time
- Fundamentalist strategy is less often profitable with not high profits
- Fundamentalists bring information to the market and then others can use this


## Profitability paradoxes?

- Is it paradoxical that fundamentalists gain less on average?
- Not necessarily as agents change strategies over time
- Fundamentalist strategy is less often profitable with not high profits
- Fundamentalists bring information to the market and then others can use this


## Paradox 1

As steady state price moves away from its fundamental value, fundamentalists gain relatively more

## Profitability paradoxes?

- Is it paradoxical that fundamentalists gain less on average?
- Not necessarily as agents change strategies over time
- Fundamentalist strategy is less often profitable with not high profits
- Fundamentalists bring information to the market and then others can use this


## Paradox 1

As steady state price moves away from its fundamental value, fundamentalists gain relatively more

## Paradox 2

When non trivial price dynamics emerge, fundamentalists relative profits are reduced

What if costs are lower?
Average difference in profits for $C=0.01$


## One more paradox?

- Can fundamentalists gain more on average?
- If $C>0$, then no
- If $C=0$, only marginally


## Paradox 3

Marginal average profits of fundamentalists decrease when costs go down

## Conclusion

What we have learned:
(1) If you are a fundamentalist it's good to be wrong
(2) Driving the price to its fundamental value reduces profits
(3) Inconsistency between fundamentalists' beliefs and actual prices It's not good to be right for long

Next steps:

- Understand the paradoxes
- Include higher levels of reasoning
- What if some agents knew the dynamics ex ante?


[^0]:    ${ }^{1}$ Queen Mary, University of London
    ${ }^{2}$ University of Heidelberg
    ${ }^{3}$ Università degli Studi di Firenze; Complexity Lab in Economics (CLE), Università Cattolica del Sacro Cuore, Milano

