The interaction between private and public physical capital accumulation in a surplus labor economy[*]

Carlândia Brito Santos Fernandes
State University of Maringá, Brazil E-mail: cbsfernandes@uem.br
&
Guilherme de Oliveira
Department of Economics and International Relations Federal University of Santa Catarina
E-mail: oliveira.guilherme@ufsc.br

Abstract

This study presents a simple dynamic development model that explores the interaction between private and public physical capital accumulation in a surplus labor economy. We introduce fiscal policy in a Lewis development framework through investments in public infrastructure. Innovatively, public infrastructure is modeled under congestion. The model shows that when both levels of public and private physical capital are relatively low there is a crowding out effect on private investment that creates the necessary conditions for the emergence of a *development trap*, from which a surplus labor economy, if left to the free play of its structural forces, may never escape. Once caught in such a trap, the economy can be potentially released through a Big Push of public or private capital or a sufficiently balanced combination of both. Our contribution also shows that overcoming underdevelopment inevitably involves a phase of strategic complementarity, or *cumulative causation*, between public and private investment in capital formation.

Keywords: Social overhead public infrastructure; capital accumulation; development trap.

JEL Codes: E60; E62; E22.

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[*] This research was supported by FAPESP (São Paulo Research Foundation) grants 2012/04600-0 and 2013/07326-9. The opinions, assumptions, conclusions, and recommendations expressed in this material are those of the authors and do not necessarily reflect the views of FAPESP. We are extremely grateful to Gilberto Tadeu Lima for useful conversations, comments, and suggestions and to Albert Fishlow for related discussions while Carlândia was a visiting scholar at Columbia University.

1 Introduction

Physical capital is one of the main engines of economic growth. But to understand both the process of economic growth and large cross-country inequalities is also needed to understand how public and private investments interact in determining physical capital accumulation. As capital restraint can be arguably viewed as the decisive barrier to economic growth in developing countries, the issue of capital formation remains a crucial point in the development macroeconomics literature.

A broad empirical literature has recently explored the role of both private and public infrastructure to reduce poverty (Calderón and Servén, 2010; Medeiros, Ribeiro, and Amaral, 2021), income inequality (Makmuri, 2017; Medeiros and Ribeiro, 2020), to improve health conditions (Bancarali, 2020), and the level of environmental quality (Erdogan *et al.*, 2020) in low- and medium-income developing countries. These growth and development-related outcomes reinforce the view that low- and medium-income developing countries face a particular challenge.

In Somalia, for instance, only 36% of the population has access to electricity, about 25% of total roads are paved, and less than one hospital bed per one thousand people is available. The deficiency in infrastructure is not limited to low-income developing countries. In Pakistan, a lower-middle-income country, about 73.9% of the population has access to electricity, 65% of total roads are paved, and the number of hospital beds by one thousand people is 0.6 (World Bank, 2019). Given that opportunities for public-private partnerships in infrastructure in low- and medium-income developing countries, in general, are limited, closing infrastructure gaps require a substantial increase in public investment (Agénor and Moreno-Dodson, 2006).

A common view is that public infrastructure services have a growth-promoting effect through a positive impact on the productivity of private inputs (Aschauer, 1989; Barro, 1990), especially when the level of infrastructure is relatively low. A rise in the productivity of capital inputs increases the profit rate, which, in turn, eventually increases the demand for private physical capital inputs. In this view, private and public investments have a complementarity effect on capital accumulation (Munnell, 1990, 1992; Easterly and Rebelo, 1993; Benos, 2009). But in the context of capital restraint, on the other hand, the level of public investment mostly depends on taxation over dynamic sectors, which depending on its impact on the net profit income, *inter alia*, can crowd out private investment itself. In this case, the levels of private and public infrastructure are inversely related, which, under certain conditions, may compromise economic growth.

This paper explores the interaction between public and private physical capital accumulation and its implications for economic growth in a simple Lewis development macromodel (Lewis, 1954; Ros, 2013). The model extends a Lewis development framework by including public capital infrastructure. The government taxes the profit income of the Modern sector, and the resulting revenue is completely invested in the provision of public infrastructure. So, in this model, the interaction between the tax collection system and the government spending on infrastructure can be defined by fiscal policy.

Our analytical contribution is related to a literature that has been addressing the role of public infrastructure within Lewis's development framework, for example, Martins-Neto and Lima (2017). While the essence of the model bears some resemblance to Martins-Neto and Lima (2017) approach, the model distinguishes at least two central points. Innovatively, we allow for public infrastructure congestion, in which public capital services has a growth-promoting effect on the Modern sector, but its productivity spillover effects are diminished by excessive use, as measured by the size of population. In a broad yet relevant sense, such an assumption is consistent with a view on the effects of congestion externalities on development macroeconomic models (Agénor, 2012; Dinkelman and Schulhofer-Wohl, 2015). In addition, the model considers public infrastructure as an accumulative factor, which allows us to explore the interaction of public and private infrastructure in capital accumulation.

When both levels of public and private capital-labor ratios are relatively low, the model shows that the crowd out effect on private investment creates the necessary conditions for the emergence of a *development trap*, from which a surplus labor economy, if left to the free play of its structural forces, may never escape. Once caught in such a trap, an economy can be potentially released from it through a Big Push of public or private capital, or a sufficient combination of both. Interestingly, our contribution shows that in all the three cases, overcoming underdevelopment, inevitably involves a phase of strategic complementarity, or *cumulative causation*, between public and private investment in capital formation.

The remainder of this paper is organized as follows. The second section presents the structure of the model. The third section illustrates and analyses the behavior of the model in the long run. The paper closes with a summary of the main conclusions derived along the way.

2 The structure of the model

Consider a closed economy model with a labor surplus and two sectors, the traditional or subsistence, *S*, sector and the modern sector, *M*, that produces the same final good or service, which is used for consumption and investment. The technology of the modern sector exhibits constant returns to scale:

$$M = K^{\alpha} {L_M}^{1-\alpha} g^{\mu}, \tag{1}$$

in which *K* is the private physical capital stock, L_M is the number of workers employed in the modern sector, with $\alpha \subset (0,1) \in$ R. In turn, g is the ratio between the stock of public capital, *G*, and the total population, L, $g = \frac{G}{L}$, with $\mu \subset (0,1) \in$ R. It is supposed further that $\mu < \alpha$ and $1 - \alpha - \mu > 0$. Following an extensive literature on the role of the public sector in endogenous growth (e.g., Barro, 1990; Fisher and Turnovsky,1998), g is modeled as a service that is nonexcludable, but rivals, so that as L increases, g decreases, implying that the service obtained from the stock of public capital by the modern sector decreases. The strength of congestion is thus proportionally measured. In this sense, *g* represents what we call *social overhead public infrastructure*, that is subject to congestion. Uzawa (2005) remembers that the effectiveness of services of social common capital for each member of the society, including private firms, depends upon the extent to which other members of the society are using the same services. This mean that as highways and water systems become crowded with usage, benefits to users may diminish (Rioja, 1999).

We use the term social overhead public infrastructure in an analogous way to that of *social overhead capital (SOC)*, which is broadly defined by Hirschman (1958) as comprising those basic services without which virtuous productive activities cannot function. The concept of SOC is not easy to define in a rigorous way. Hirschman emphasizes three main broad characteristics of SOC: (i) it is capital formation in a central area which is essential to a range of virtuous economic activities; (ii) it is usually, but not exclusively, carried out by the government; and (iii) it is a non-tradable stock of capital associated with external economies. In the present model, we restrict the SOC to be formed solely by public capital, or infrastructure. We focus on specific forms of public capital subjected to congestion, and where rivalry arises from the use of the total population. The term "social" thus captures not only the nonexcludable nature of public capital, but also its use by the population. Some forms of investment in formal and technical education, housing, and transport are included in this broad set of social public infrastructure.

Robust empirical evidence emphasizes the role of social infrastructure on welfare and economic growth of low- and medium income developing countries. Duflo (2004), for example, studies the medium run consequences of an increase in the rate of accumulation of social infrastructure in Indonesia, where from 1974 to 1978, the government built over 61,000 primary schools. This program has induced a long-lasting change in the rate of human capital accumulation in the regions it affected most. In line with the prediction of a dual economy model, the rise in the productivity of the labor force was entirely absorbed by the formal sector. Banerjee, Duflo and Qian (2020), in turn, estimates the effect of transport infrastructure on the regional *per capita* gross domestic product in China. The study finds a moderately sized positive causal effect of transport infrastructure networks on *per capita* income levels across different regions. In a macroeconomic level, Sabir and Shamshir (2020) disaggregate infrastructure into economic and social infrastructure to capture its impacts on long-run economic growth of Pakistan for the period 1971–2014. The study shows that a pool of investments in social infrastructure have a positive and increasingly effect on economic growth both in short and long run.

As in Lewis (1954), the subsistence sector uses traditional production techniques that are labor-intensive and exhibits constant returns to scale:

$$S = W_S L_S, \tag{2}$$

where L_S is the labor input and W_S is the wage in the traditional sector, and thus the average product of labor is constant. For the sake of simplicity and tractability of the dynamic system described more below, we neglect the impact of social overhead public infrastructure on the output of the subsistence sector. Therefore, it is supposed that, although available to workers in the subsistence sector, the services provided by the stock of public capital generate spillovers effects only on the sector that accumulates capital, in an analogous way of Romer (1986). If the subsistence sector is confined to a rural region, while the modern sector to an "urban" region, it is also possible to understand the specific social overhead public effect in terms of an *unbalancing growth* strategy. According to Hirschman (1958), no developing country has sufficient endowment to enable it to invest simultaneously in all sectors of the economy in order to achieve a *balanced growth*. For Hirschman (1958), the social overhead capital must be supplied in adequate volume to support and stimulate the growth of industry. All in all, a rise in the productivity of labor force induced from a change in g is entirely absorbed by the modern sector (Duflo, 2004).

These two sectors will coexist until the excess of labor remains, that is, whereas the economy is underdeveloped (not mature at the development level, not industrialized or with structural dualism). When the economy reaches the mature phase, the traditional sector will no longer exist.

In turn, the government, by assumption, operates on a balanced budget, that is, the government spends all its tax revenue to provide the social overhead public infrastructure, as an input to the modern sector. Then, through the tax and spending policies, detailed more below, the government, has a fundamental role in economic development and growth since the economy will not be able to develop if the government does not provide the supply of the public infrastructure in a proper flow. In fact, as discussed in what follows, a connection, or cycle, which may be vicious or virtuous depending on the phase of economic development, will occur.

2.1 Wages, profit rate and social public investment

In the subsistence sector, the average product of labor determines wages, so wages are constant and will remain so as long as there is labor surplus in this economy. In the modern sector, on the other hand, the wage is determined by the wage in the subsistence sector plus a wage premium that it has to pay to attract workers from the other sector. As in Lewis (1954), this difference is resulted from the higher cost of living that workers would have from the moment that they choose to get a job outside of their original sector. The wage premium, $f-1$, is constant so that, as long as the two sectors coexist, the modern sector pays a real wage, W_M, given by:

$$W_M = fW_S. \quad (3)$$

For simplicity and tractability, we supposed that the wage premium is equal to zero (*i.e f = 1*). Then, we have that $W_M = W_S = 1$, without it making any significant qualitative difference in the analysis. In addition, it is supposed perfect competition in both product and factors markets. Meanwhile, when maturity as a developed economy is reached, labor supply becomes inelastic, and the wage is equal to its marginal product. Hence, from the first order conditions, the wage paid in the modern sector in the mature phase, under $L_S = 0$, is given by the following equation:

$$W_M = (1 - a)\left(\frac{K}{L_M}\right)^a g^{\mu}. \quad (4)$$

The wage of the modern sector in mature phase is positive affected by the private capital stock, since the higher K is, the higher the labor demand is and the higher the wages are. Alternatively, the higher the employment share of the modern sector is, the smaller the wage is, which happens when the modern sector faces an excess supply of labor. The social overhead public infrastructure also has a positive effect on wage of the modern sector, as this service generates a positive effect on the capital used by the modern sector, leading to an increase in the labor demand and thus in wage income.

The profit rate on private physical capital, *r,* is the marginal product of private capital in both development phases and is expressed by:

$$r = a\left(\frac{L_M}{K}\right)^{1-a} g^{\mu}\text{s}. \quad (5)$$

The social overhead public infrastructure positively affects the profit rate on private stock of capital. A rise in the productivity of private capital inputs increases the profit rate, which, in turn, increases the demand for private physical capital inputs. However, the productivity effects of public infrastructure services are diminished by excessive use, as measured by the size of population. In turn, the level of private physical capital is negatively related to the profit rate, as expected.

Regarding social overhead public infrastructure (*g*) the government finances it by a tax aliquot, $\tau \subset (0,1) \in$ R, exogenously determined, on the profits of the modern sector.

$$I_G = \tau r K. \quad (6)$$

This is a permanent balanced budget, with, I_G, representing the public investment, the government thus spends all its revenue by investing in social public infrastructure (Fisher; Turnovsky, 1998). The next section describes how the economy moves over time, with the state variables of interest being the private physical capital-labor ratio, *k*, and the social overhead public infrastructure, *g*.

3 The behavior of the model in the long run

In the long run, it is supposed that the short-run equilibrium values of the variables are always attained with the economy moving in time due to changes in *K*, *G*, and *L.* Regarding *G*, we suppose that the social public infrastructure depreciates at a rate of $\delta \subset (0,1) \in$ R. Then, from (6), the level of public infrastructure changes in time according to the difference between the flow of public investment and public capital depreciation (Carboni; Medda, 2011):

$$\frac{dG}{dt} = \tau r K - \delta G. \tag{7}$$

Hence, we can write the growth rate of the social overhead public infrastructure as:

$$\hat{g} = \tau r k g^{-1} - (n + \delta), \tag{8}$$

in which $n \in$ R is the population growth rate, taken as exogenous and fixed.

Following Ros (2013), the dynamics of the *per capita* private capital accumulation can be defined as follows:

$$\hat{k} = s(1 - \tau)r - (n + \delta), \tag{9}$$

in which is supposed that firm-owner capitalists save a given fraction, $s \subset (0,1) \in$ R, of their net profits, and private physical capital depreciates at the same as public infrastructure. As usual, the rate of change of private physical capital per unit of labor is the difference between actual net investment per unit of labor and the breakeven even investment $(n + \delta)$.

A full description of the long-run behavior of the model depends on the profit rate of the modern sector in the labor surplus phase (when $L_S > 0$ and $W_M = W_S = 1$) and the mature one (when $L_S = 0$ and W_M is represented by (4)). Then, using these definitions in (5), the profit rate in both development phases can be rewritten, respectively, as:

$$r_U = \alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}} g^{\frac{\mu}{\alpha}}, \tag{10}$$

$$r_D = \alpha k^{\alpha-1} g^{\mu}. \tag{11}$$

Note that when $L_S > 0$, the profit rate does not depend on the level of private capital-labor ratio. When $L_S = 0$, the profit rate decreases as k increases, since α is lower than unity, which indicates the presence of decreasing returns to capital in technology production of the modern sector. However, in the two phases the profit rate increases as the social overhead public infrastructure expands. This result is in line with common view that public infrastructure services have a growth-promoting effect through a positive impact on the productivity of private inputs (Aschauer, 1989; Barro, 1990). A rise in the productivity of capital inputs increases the profit rate, which, in turn, eventually increases the demand for private physical capital inputs. However, the profit rate increases at a diminishing rate with the level of *g*.

Now, that we have the profit rates is necessary to compute the rates of accumulation of private physical capital, $\hat{k}$, and the social overhead public infrastructure, $\hat{g}$, for the different development phases. From equation (10) into (8) and (9) the two-dimensional non-linear dynamical system in the labor surplus phase is respectively given by:

$$\hat{g} = \tau\theta k g^{\frac{\mu-\alpha}{\alpha}} - (n+\delta), \tag{12}$$

$$\hat{k} = s(1-\tau)\theta g^{\frac{\mu}{\alpha}} - (n+\delta), \tag{13}$$

in which $\theta = \alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}}$. From (12), the rate of change of social overhead public infrastructure increases with *k*, while *g* negatively affects $\hat{g}$, once $\mu < \alpha$. From (13), the rate of change of private physical capital per unit of labor, $\hat{k}$,increases with *g*. All the causal channels are being driving by market and external effects on the profit rate (10). The dynamical system (12)-(13) has a unique pair of economically relevant (i.e., strictly positive) equilibrium values, (k_U^*, g_U^*) (Appendix).

The properties of the system can be qualitatively analyzed using a Taylor's expansion on the neighborhood of the unique pair equilibrium values:

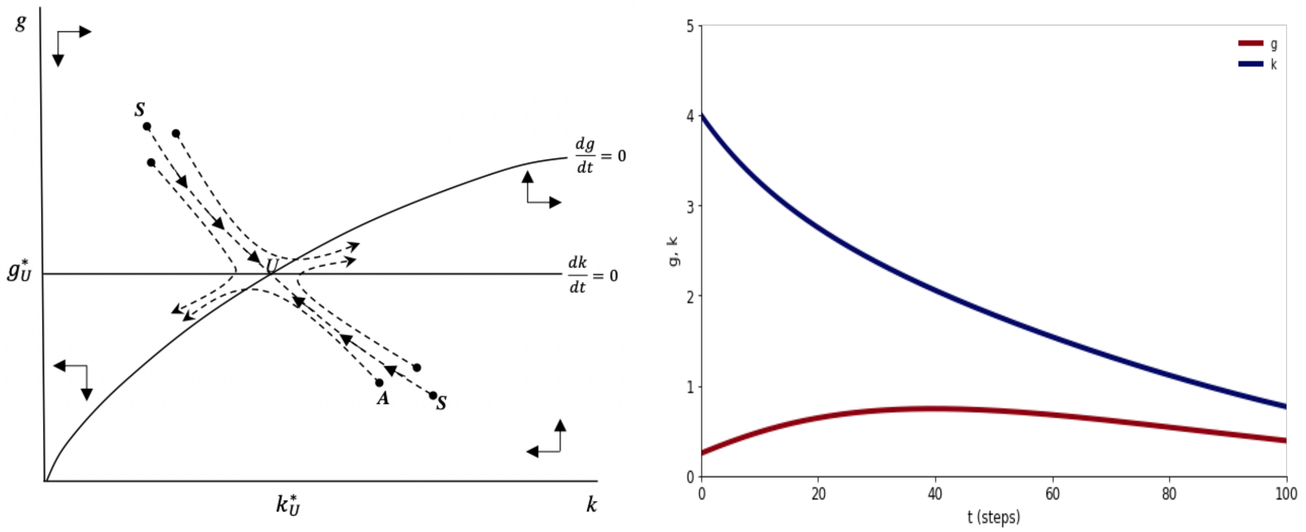
$$\begin{bmatrix} \frac{dg}{dt} \\ \frac{dk}{dt} \end{bmatrix} \cong \begin{bmatrix} \frac{dg_U^*}{dt} \\ \frac{dk_U^*}{dt} \end{bmatrix} + \begin{bmatrix} (\mu-\alpha)\alpha^{-1}\tau\theta k_U^* g_U^{*\,(\mu-\alpha)\alpha^{-1}} & \tau\theta g_U^{*\,\mu\alpha^{-1}} \\ \mu\alpha^{-1}s(1-\tau)\theta k_U^* g_U^{*\,(\mu-\alpha)\alpha^{-1}} & 0 \end{bmatrix} \begin{bmatrix} g - g_U^* \\ k - k_U^* \end{bmatrix}.$$

In the Jacobian matrix, note that as $\alpha > \mu$, J_{11} is negative. A rise in the level of social overhead public infrastructure causes a reverse effect on the rate of change of g. In turn, a rise in the level of capital-labor ratio increases the profit rate, which in turn increases the mass of gross profits, and thus the tax revenues that finance public infrastructure accumulation, thus justifying $J_{12} > 0$. In turn, the social overhead public infrastructure increases the rate of capital accumulation, $J_{21} > 0$. As discussed above, g has a growth-promoting effect through a positive impact on the productivity of private physical capital, so that a rise in the productivity of the private capital input increases the profit rate, which, in turn, increases capital accumulation. Also note that when $L_S > 0$, the profit rate (10) does not depend of the level of private capital-labor ratio, so that $J_{22} = 0$.

Given that both k_U^* and g_U^* are strictly positive, it is readily seen that the determinant of the Jacobian matrix is unambiguously negative. Then, the equilibrium at the labor surplus phase is saddle-point unstable, which is represented in Figure 1. In panel (a), the axis are the respective levels for g and k, with the two demarcation curves intersecting at the equilibrium point U^* and dividing the phase portrait into four distinct regions. When the level of the social overhead public infrastructure is continuously increasing (g-axis), the rate of change of g undergoes a steady decrease, so that $\frac{dg}{dt}$ is positive (negative) below (above) the $\frac{dg}{dt} = 0$ isocline. Moreover, when the level of g is continuously increasing (g-axis), the rate of change of the private capital-labor ratio steady increase, with $\frac{dk}{dt}$ being negative (positive) below (above) the $\frac{dk}{dt} = 0$ isocline. Note also that the stable arm of the saddle point, the separatrix *SS*, is negative sloped in the plane.

All the area to the left of such a separatrix constitutes a development trap, in which one or both g and k experience a steady decrease. When the economy starts at the left of *SS* there are no endogenous forces capable of reverting such a steady decrease in both state variables. In panel (b), for instance, we present an illustrative simulation of trajectories of the dynamical system when the initial conditions are settled around point *A*, in panel (a), with a relatively high level of private physical capital per unit of labor than social overhead public infrastructure. Both trajectories diverse negatively. The economic implications of this trap are explored more below.

Figure 1: Saddle-point instability in the surplus labor phase.



(a) *Phase portrait*. (b) *Simulation*.

Note: Panel (a) presents an illustration of the phase portrait of the dynamical system (12)-(13) in the neighborhood of the unique pair of economic relevant equilibrium values. For reasonable parameters values, panel (b) presents a simulation of the unstable trajectories of the system with initial conditions illustrating the behavior of the economy around point *A*, in panel (a). The set of parameters values are defined as: $s = 0.17$; $\alpha = 0.36$; $\mu = 0.15$; $\tau = 0{,}10$; $n = 0.025$; $\delta = 0.015$.

When the subsistence sector eventually disappears ($L_S = 0$) the economy becomes a mature/developed economy. From equation (11) into (8) and (9) the proportional rate of change for the social overhead public infrastructure and private physical capital per unit of labor form the following two-dimensional non-linear dynamical system:

$$\hat{g} = \tau \alpha k^{\alpha} g^{\mu-1} - (n + \delta), \tag{14}$$

$$\hat{k} = s(1-\tau)\alpha k^{\alpha-1} g^{\mu} - (n + \delta). \tag{15}$$

From (14), the rate of change of the social overhead public infrastructure increases with k and decreases with g, as the term $(\mu - 1)$ is negative. From (15), we can observe that the accumulation rate of private physical capital per unit of labor increases with g, but reduces with k, the latter reflecting the diminishing marginal returns to capital, while the former the positive spillovers of public infrastructure, both effects being captured throughout the profit rate (11).

The dynamical system (14)-(15) has a unique pair of economically relevant (i.e., strictly positive) equilibrium values, (k_D^*, g_D^*) (Appendix). As in the labor surplus phase, the properties of the system in the developed phase can be qualitatively analyzed using a Taylor's expansion on the neighborhood of the unique pair equilibrium values:

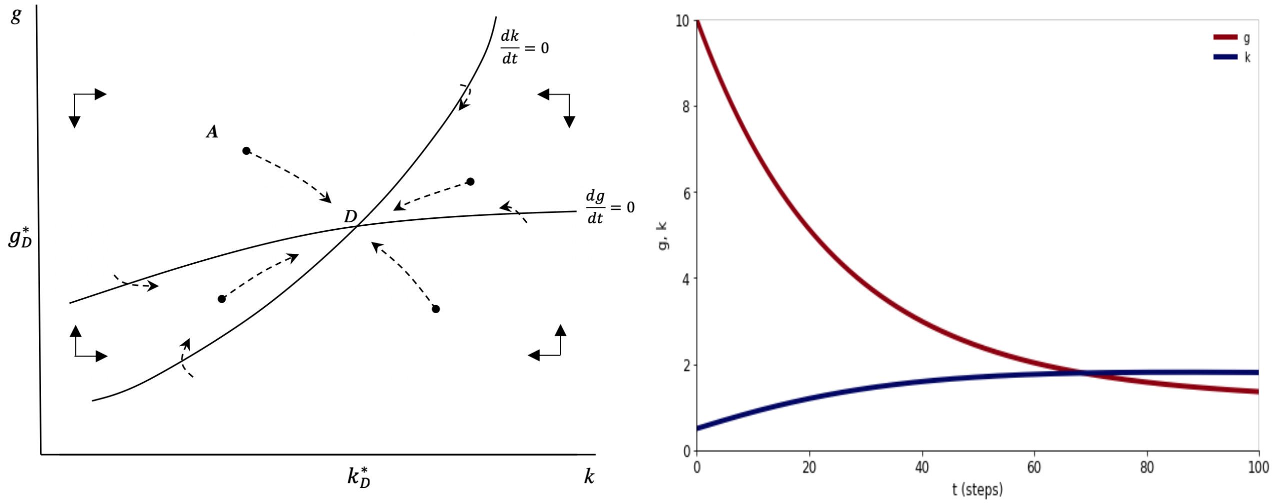
$$\begin{bmatrix} \frac{dg}{dt} \\ \frac{dk}{dt} \end{bmatrix} \cong \begin{bmatrix} \frac{dg_D^*}{dt} \\ \frac{dk_D^*}{dt} \end{bmatrix} + \begin{bmatrix} (\mu-1)\alpha\tau {k_D^*}^{\alpha} {g_D^*}^{\mu-1} & \alpha^2\tau {k_D^*}^{\alpha-1} {g_D^*}^{\mu-1} \\ \mu s(1-\tau)\alpha {k_D^*}^{\alpha} {g_D^*}^{\mu} & (\alpha-1)s(1-\tau)\alpha {k_D^*}^{\alpha-1} {g_D^*}^{\mu} \end{bmatrix} \begin{bmatrix} g-g_D^* \\ k-k_D^* \end{bmatrix}.$$

The effect of g on $\frac{dg}{dt}$ in the developed phase remains the same as in the labor surplus phase, $J_{11} < 0$, as well the effect of the private physical capital per unit of labor on the rate of change of the level of social overhead public infrastructure, which remains positive $J_{12} > 0$. Despite that J_{21} remains positive, its magnitude is now smaller than in the underdevelopment phase. The main reason for this change is the influence of diminishing marginal returns to private capital, which now are a binding constraint. The diminishing marginal returns to private capital in development phase also explains $J_{22} < 0$, so that a rise in the level of k has a stabilizing effect over $\frac{dk}{dt}$.

As it is observed, given that both k_D^* and g_D^* are strictly positive, the trace of the Jacobian matrix is unambiguously negative, a necessary condition for asymptotic stability. Regarding the determinant of the Jacobian matrix, we have $(1-\alpha-\mu)\alpha^2 s\tau(1-\tau){k_D^*}^{2\alpha-1}{g_D^*}^{2\mu-1}$, which is always positive when the parametric restriction of the production function (1) is satisfied, *i.e.,* $\alpha + \mu < 1$. Therefore, the long-run equilibrium in the development phase is asymptotically stable, which is represented in Figure 2.

In panel (a), the axis are the respective levels for g and k, with the two demarcation curves intersecting at the equilibrium point D^* and dividing the phase portrait into four distinct regions. When the level of the social overhead public infrastructure is continuously increasing (g-axis), the rate of change of g undergoes a steady decrease, so that $\frac{dg}{dt}$ is positive (negative) below (above) the $\frac{dg}{dt} = 0$ isocline. Moreover, when the level of g is continuously increasing (g-axis), the rate of change of the private capital-labor ratio steady increase, with $\frac{dk}{dt}$ being negative (positive) below (above) the $\frac{dk}{dt} = 0$ isocline. A difference regarding surplus labor phase is that the slope of both isocline is positive.

Figure 2: Asymptotically stable equilibrium in the mature phase.



(a) *Phase portrait.* (b) *Simulation.*

Note: Panel (a) presents an illustration of the phase portrait of the dynamical system (12)-(13) in the neighborhood of the unique pair of economic relevant equilibrium values. For reasonable parameters values, panel (b) presents a simulation of the asymptotically stable trajectories of the system with initial conditions illustrating the behavior of the economy around point *A*, in panel (a). The set of parameters values are defined as: $s = 0.17$; $\alpha = 0.36$; $\mu = 0.15$; $\tau = 0{,}10$; $n = 0.025$; $\delta = 0.015$.

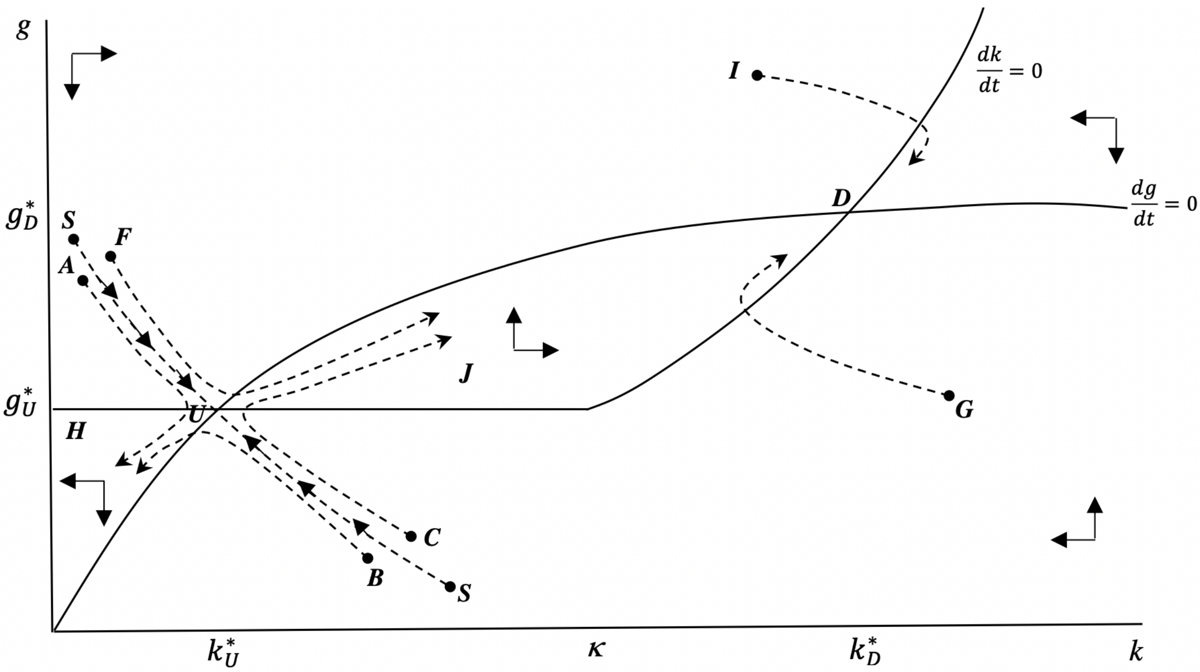
In panel (b), we present an illustrative simulation of the asymptotically stable trajectories of the dynamical system when the economy eventually overcomes the surplus labor phase and starts maturity around point *A*, in panel (a). Around point *A* the economy is characterized by a relatively high level of social overhead public infrastructure than private physical capital per unit of labor. Both trajectories converge to the equilibrium by monotonically substituting some level of *g* by *k*. The economic implications of such substituting behavior are explored next section.

3.1 Multiple equilibrium analysis: The development trap

This section provides a further qualitative and illustrative representation of the dynamical system in (12)–(13) and (14)–(15) by combining the unique economically relevant equilibrium in each phase of development. Indeed, there is a set of parameters for which a joint representation of Figures 1 and 2 exists. We focus here on an economic interpretation of a possible development trap configuration represented in Figure 3.

Consider, for example, that the initial conditions are such that the economy starts around point *B,* where the level of *k* is relatively high, but the level of *g* is relatively low. In this region, the level of congestion is relatively high, so that the physical capital productivity spillover effects of public infrastructure services are further diminishing the profit rate. The flow of profit is insufficient to finance both private physical capital and public infrastructure accumulation. The mass of gross profit first generates a sufficient amount of tax revenues to increase the rate of growth of social overhead public infrastructure, but the remaining mass of net profit is insufficient to cover the breakeven of private investment, so that private capital accumulation per unit of labor is decreasing. Recall that the profit rate increases at a diminishing rate with the level of g. Then, as k is falling, g is raising at a decreasing marginal rate so that at some point, eventually, the flow of public investment turns insufficient to cover the breakeven of public investment. The economy eventually ingresses in a region of cumulative decline of public infrastructure and private physical capital accumulation, H. Although in this area there is a labor surplus and then, the wage is constant this is not a sufficient condition to reverse the negative tendency of both public and private capital accumulation.

Figure 3: A possible multiple equilibria configuration along the development path.



Note: This figure presents a further qualitative and illustrative representation of the dynamic system in (12)–(13) and (14)–(15) by combining the unique economically relevant equilibrium in each phase of development. Indeed, there is a set of parameters for which a joint representation of Figures 1 and 2 exists. Note that with $L_S >$, $\frac{dk}{dt} = 0$ is horizontal, so that a joint parametric illustration requires the locus $\frac{dk}{dt} = 0$ of both development phases be equal at point κ. A similar parametric restriction is required for the joint representation of $\frac{dg}{dt} = 0$. Given the format of both isoclines detailed in Appendix A, for illustrative purposes, we represent $\frac{dk}{dt} = 0$ as a smooth differentiable concave curve.

Conversely, consider that the initial conditions are such that the economy starts around point *A*, where the level of social overhead public infrastructure is relatively high, but the level of private physical capital per unit of labor is relatively low. This combination produces a relatively high profit rate, but as *k* is low the mass of gross profit is also low. On the one hand, as the level of *g* is relatively high, the amount that the government collects by taxing the modern sector is insufficient to maintain investments in public infrastructure and to breakeven public investment, so the rate of change of *g* falls and the level of *g* experiences a steady decrease. On the other hand, the rate of growth of private physical capital per unit of labor is increasing, but this increase is insufficient to revert the process and to put *g* in a trajectory of rising. In fact, the fall in both the rate of *g* and its level produce the congestion effect plus the diminishing and eventually negative spillover effects on the marginal productivity of private capital per unit of labor. These external diseconomies effects eventually turn sufficiently high to further decrease the flow of profits to a level that is insufficient to finance both public infrastructure and private capital accumulation. The economy eventually ingresses in the region of cumulative declined of both *g* and *k, H*.

At both points, *A* and *B*, the underdevelopment problem is the absence of a minimum level of social overhead public infrastructure and/or private physical capital per unit of labor, which prevents the economy from starting the convergence towards the mature phase, somewhere to the right of the separatrix *SS*. The crowding-out relationship between public and private flow of investment, thus creates the necessary condition to the emergence of a development trap, from which this surplus labor economy, if left to the free play of its structural forces, may never overcome. The relative size of the trap depends on the level of the surplus labor long-run equilibrium point, *U*. Even under the presence of a productive government, this economy has insufficient endowment of resources as to enable it to invest simultaneously in both forms of capital to achieve a trajectory towards the mature phase.

In line with the arguments of Rosenstein-Rodan (1943) and Lewis (1960) the situation of such development trap would be facilitated by external financial assistance, which would allow a faster growth of the national income, providing conditions for taxation. In other words, once caught in the development trap, a Big Push of public infrastructure or private physical capital can potentially release this surplus labor economy from it. Suppose again that the economy starts to the left of the separatrix *SS* around point *A* or *B* in Figure 3. The economy can overcome the development trap by means of an exogenous shock of investment that horizontally pushes the system to the right (a big push of private physical capital per unit of labor), or vertically pushes the economy to up (a big push of social overhead infrastructure). These two development strategies are both represented by the regions around points *C* and *F*, to the right of the separatrix *SS*.

If the system is on the right of the separatrix *SS,* around point *C* or *F*, the economy has overcome the development trap and, in this region, the economic forces are such that eventually drive the system to a region of virtuous circle, around point *J*. The tax revenue becomes sufficient to ensure the rise of the social public infrastructure, as the modern sector profit encourages the process of capital accumulation. Additionally, because of the spillover effects of the public infrastructure, the modern sector accumulates even more capital, causing positive feedback in the industrialization and development process.

The notion that a surplus labor economy must choose specific sectors to invest dates back at least to Hirschman (1958), and his unbalanced growth theory. Hirschman divides the pool of investment between SOC and directed productive actives, DPA. An initial mass of investments in SOC would increase investment in DPA. In turn, investments in DPA would press for investment in SOC. It is through these chains of effects that generate economic growth towards mature phase. In the present model, however, a continuous unbalanced growth strategy accelerates growth only if the economy happens to start to the right of the development trap. Both forms of capital accumulation are connected through fiscal policy (tax revenues) and external productivity effects, so that it turns out to be necessary a minimum initial level of social overhead public infrastructure or private physical capital per unit of labor even in order to promote an unbalanced growth strategy.

Meanwhile, the model allows for the possibility that the development trap can be overcome by a sufficient combination of both private capital and public infrastructure investments per unit of labor, a balanced growth strategy[1]. Interestingly, in all the development strategies, overcoming the development

[1] However, the literature defends that a big push in social overhead capital would be better. As says Rosenstein-Rodan (1943 p.208) "*Let us build railways, roads, canals, hydro-electric power-stations, the rest will follow automatically*".

trap, involves a phase of strategic complementarity, or cumulative causation, between public infrastructure and private physical capital investments in aggregate capital formation, which is represented by the region around point *J*. There is a balanced combination of g and k that promotes crowding-in effects on economic growth. This theoretical result is empirically supported by Pereira (2000) that find for United States that all types of public investment crowds in private investment and that aggregate public investment has a positive effect on private output, being that the core infrastructure investment in electric and gas facilities, transit systems, and airfields, as well as in sewage and water supply systems, display the highest marginal returns. Singh (2012) finds a similar result for India and that the causality is from public to private capital.

The region of strategic complementarity of g and k, *J*, also exerts influence on the mature phase, when $L_S = 0$. Consider that the initial conditions of the system are such that the economy starts in the mature phase at, say around point *G*, with a relatively high level of private physical capital per unit of labor. In this region, the system will experience a fall in k towards the region of cumulative causation that will eventually take the economy to the steady state. Interestingly, however, the economy eventually can achieve the mature steady-state by a relatively high initial level of social overhead public infrastructure, throughout the trajectories illustrated around point *I*.

4. Conclusion

Physical capital accumulation is central for economic growth. In the literature many studies show that the interaction between private and public physical capital has a positive effect on well-being and on reducing poverty or income inequality (Calderón and Servén, 2010; Makmuri, 2017; Medeiros and Ribeiro, 2020; Bancarali, 2020; Erdogan et al., 2020 and Medeiros, Ribeiro, and Amaral, 2021). Furthermore, the literature (Aschauer, 1989; Barro, 1990; Easterly and Rebelo, 1993; Pereira, 2000; Singh, 2012) highlights a positive impact of public investment in infrastructure on the productivity of the private sector private and then, that public investments have a complementarity effect on capital accumulation or that public investment crowds in private investment.

This paper explores the interaction between public and private physical capital accumulation and its implications for economic growth in a simple Lewis development macromodel (Lewis, 1954; Ros, 2013). We extend the Lewis framework by including social overhead public infrastructure. The paper contributes to a literature that investigates the role of public infrastructure within Lewis's development framework, for example, Martins-Neto and Lima (2017). The innovations are: a) we allow for public infrastructure congestion, in which public capital services has a growth-promoting effect on the accumulation of private capital, but its productivity spillover effects are diminished by excessive use, as measured by the size of population; b) we model public infrastructure as an accumulative factor.

When both levels of public and private capital-labor ratios are relatively low, the model shows that the crowd out effect on private investment creates the necessary conditions for the emergence of a development trap, from which a surplus labor economy, if left to the free play of its structural forces, may never escape. Once caught in such a trap, we show that this theoretical economy can be potentially released from it through a Big Push of public or private capital, or a sufficient combination of both. Innovatively, our contribution shows that shows that overcoming underdevelopment inevitably involves a phase of strategic complementarity, or cumulative causation, between public and private investment in capital formation.

The present simple new extension of a Lewis development macroeconomic framework may be an useful baseline model to explore more complex fiscal policy designs, as for example, the examination of effects of wealth and capital taxation on the accumulation of private physical capital, the dynamics of public debt in the early stages of development, as well open economy complex interactions.

References

Agénor, P. R. (2012). Infrastructure, public education, and growth with congestion costs. *Bulletin of Economic Research*, *64*(4), 449-469.

Agénor, P. R., & Moreno-Dodson, B. (2006). *Public infrastructure and growth: New channels and policy implications* (Vol. 4064). World Bank Publications.

Aschauer, D. A. (1989). Is public expenditure productive? *Journal of monetary economics*, *23*(2), 177-200.

Bancalari, A. (2020). *Can white elephants kill? Unintended consequences of infrastructure development in Peru* (No. W20/32). IFS working paper.

Banerjee, A., Duflo, E., & Qian, N. (2020). On the road: Access to transportation infrastructure and economic growth in China. *Journal of Development Economics*, 145.

Barro, R. J. (1990). Government spending in a simple model of endogenous growth. *Journal of political economy*, *98*(5, Part 2), S103-S125.

Benos, N. (2009). *Fiscal policy and economic growth: empirical evidence from EU countries*. University

Library of Munich, Germany.

Calderón, C., & Servén, L. (2010). Infrastructure and economic development in Sub-Saharan Africa. *Journal of African Economies*, *19*(suppl_1), i13-i87.

Calderón, C., Moral‐Benito, E., & Servén, L. (2015). Is infrastructure capital productive? A dynamic heterogeneous approach. *Journal of Applied Econometrics*, *30*(2), 177-198.

Carboni, O. A., & Medda, G. (2011). Size and composition of public spending in a neoclassical growth model. *Metroeconomica*, *62*(1), 150-170.

Dinkelman, T., & Schulhofer-Wohl, S. (2015). Migration, congestion externalities, and the evaluation of spatial investments. *Journal of Development Economics*, *114*, 189-202.

Duflo, E. (2004). The medium run effects of educational expansion: Evidence from a large school construction program in Indonesia. *Journal of Development Economics*, 74(1).

Easterly, W., & Rebelo, S. (1993). Fiscal policy and economic growth. *Journal of monetary economics*, *32*(3), 417-458.

Erdogan, S., Adedoyin, F. F., Bekun, F. V., & Sarkodie, S. A. (2020). Testing the transport-induced environmental Kuznets curve hypothesis: The role of air and railway transport. *Journal of Air Transport Management*, *89*, 101935.

Fisher, W. H., & Turnovsky, S. J. (1998). Public investment, congestion, and private capital accumulation. *The Economic Journal*, *108*(447), 399-413.

Hansen, N. M. (1965). The structure and determinants of local public investment expenditures. *The Review of Economics and Statistics*, 150-162.

Hirschman, A. O. (1958). *The strategy of economic development*. New Haven: Yale University Press.

Lewis, W. A. (1954). Economic Development with Unlimited Supplies of Labour. *The Manchester School*, *22*(2), 139-191.

Lewis, W. A. (1960). Some reflections on economic development. *Economic Digest*, *3*(4), 3-8.

Makmuri, A. (2017). Infrastructure and inequality: An empirical evidence from Indonesia. *Economic Journal of Emerging Markets*, *9*(1), 29.

Medeiros, V., Ribeiro, R. S. M., & do Amaral, P. V. M. (2021). Infrastructure and household poverty in Brazil: A regional approach using multilevel models. *World Development*, *137*, 105118.

Munnell, A. H. (1990). Is there a shortfall in public capital investment? An overview. *Is there a shortfall in public capital investment*, 1-20.

Munnell, A. H. (1992). Taxation of capital income in a global economy: an overview. *New England Economic Review*, (Sep), 33-52.

Neto, A. S. M., & Lima, G. T. (2017). Competitive exchange rate and public infrastructure in a macrodynamic of economic growth. *Metroeconomica*, *68*(4), 792-815.

Pereira, A. M. (2000). Is all public capital created equal? *Review of Economics and Statistics*, *82*(3), 513-518.

Rioja, F. K. (1999). Productiveness and welfare implications of public infrastructure: a dynamic two-sector general equilibrium analysis. *Journal of Development Economics*, *58*(2), 387-404.

Romer, P. M. (1986). Increasing Returns and Long-Run Growth. *Journal of Political Economy*, 94(5).

Ros, J. (2013). *Rethinking economic development, growth, and institutions*. Oxford University Press.

Rostow, W. W. (1959). The stages of economic growth. *The economic history review*, *12*(1), 1-16.

Sabir, S., Shamshir, M. (2020). Impact of economic and social infrastructure on the long-run economic growth of Pakistan. Sustain. *Water Resource. Management*. 6, 10 (2020).

Singh, T. (2012). Does public capital crowd-out or crowd-in private capital in India? *Journal of Economic Policy Reform*, *15*(2), 109-133.

Uzawa, H. (1975). Optimum investment in social overhead capital. In *Economic Analysis of Environmental Problems* (pp. 9-26). NBER.

Uzawa, H. (2005). *Economic analysis of social common capital*. Cambridge University Press. World Bank, 2019

World Bank (2019), *World development indicators*, https://databank.worldbank.org/source/world-development-indicators, acess in jan. 10, 2022.

APPENDIX – Isoclines and long run equilibrium values in both development phases.

Let us set (12) and (13) to zero under the parametric assumption defined above to obtain the pair of isoclines in the surplus labor phase, illustrated in Figure 1:

$$g^g = \left[\frac{\tau\theta k}{n+\delta}\right]^{\frac{\alpha}{\alpha-\mu}}, \tag{A1}$$

$$g^k = \left[\frac{n+\delta}{s(1-\tau)\theta}\right]^{\frac{\alpha}{\mu}}. \tag{A2}$$

Hence, the values of g and k satisfying $\hat{g} = \hat{k} = 0$ are respectively:

$$g^* = \left(\frac{n+\delta}{s(1-\tau)\theta}\right)^{\frac{\alpha}{\mu}}. \tag{A3}$$

$$k^* = \frac{1}{\tau}\left(\frac{n+\delta}{\theta}\right)^{\frac{\alpha}{\mu}}\left(\frac{1}{s(1-\tau)}\right)^{\frac{\alpha-\mu}{\mu}}. \tag{A4}$$

In the mature phase, let us set (14) and (15) to zero under the parametric restriction derived along the text to obtain the pair of isoclines in the mature phase illustrated in Figure 2

$$g^g = \left(\frac{\tau\alpha}{n+\delta}\right)^{\frac{1}{1-\mu}} k^{\frac{\alpha}{1-\mu}}, \tag{A5}$$

$$g^k = \left(\frac{n+\delta}{s(1-\tau)\alpha}\right)^{\frac{1}{\mu}} k^{\frac{1-\alpha}{\mu}}. \tag{A6}$$

Similarly, the values of g and k satisfying $\hat{g} = \hat{k} = 0$ in the mature phase re respectively:

$$g^* = \left[\tau\left(\frac{\alpha(s(1-\tau))^{\alpha}}{n+\delta}\right)^{\frac{1}{1-\alpha}}\right]^{\frac{1}{1-\alpha-\mu}}. \tag{A7}$$

$$k^* = \left[s(1-\tau)\left(\frac{\alpha\tau^{\mu}}{n+\delta}\right)^{\frac{1}{1-\mu}}\right]^{\frac{1}{1-\alpha-\mu}}. \tag{A8}$$