Empirical analysis of a debt-augmented Goodwin model for the United States

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Abstract

The Goodwin-Keen model was introduced to reflect the structural instability of debt-financed economies. The appeal of the model lies in its ability to reflect an economy that can either converge towards a Solow-like trajectory or towards a debt crisis. However, no empirical study has focused on the model up to now. Using U.S. data for non-financial firms over the period 1959-2019, this paper tests the empirical validity of an extended Goodwin-Keen model taking dividend payments into account. We propose an original two-step estimation procedure to simultaneously estimate parameters and quantify their uncertainty. We show that the model captures the historical cycles in the wage share and employment rate, while reflecting the trend growth in the debt-to-output ratio. This relatively good fit is achieved with sensible parameter estimates but a large uncertainty, indicating notably that the model fails to fully capture the debt dynamics. Finally, we show that, according to the estimated model projections, the probability of occurrence

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of a corporate debt crisis in the next century is less than 1%. Although the Goodwin-Keen model is too simplistic to reflect financial instability as a whole, our results show that it could be a useful framework for the development of larger macroeconomic models.
Keywords: Goodwin-Keen model, Macroeconometrics, Dynamical systems in macroeconomics, Corporate debt, Financial instability

1. Introduction

The ability of the Goodwin-Keen model (Goodwin, 1967; Keen, 1995) to account for the endogenous emergence of deep imbalances in the economy has made it the basic component of a growing class of macroeconomic models. It has for instance been enhanced to incorporate price dynamics (Grasselli and Nguyen Huu, 2015), inventories (Grasselli and Nguyen-Huu, 2018) or consumption-led behaviors and income inequalities (Giraud and Grasselli, 2021). It has also recently become the core element of a model studying the interaction between climate and financial risks (Bovari et al., 2018a,b, 2020).

The original Goodwin model (GM) describes the antagonistic evolution of the share of wages in national income and the employment rate in the economy (Goodwin, 1967). More specifically, a short-run Phillips curve enables to capture the interaction between the wage share and the employment rate through a Lotka-Voltera dynamics. By providing an endogenous explanation to macroeconomic cycles, the GM departs from the real business cycle (RBC) tradition in which fluctuations are caused by unexplained exogeneous shocks (Romer, 2016). The Goodwin-Keen model (GKM) extends the original GM by incorporating corporate loans (Keen, 1995). In particular, firms can take on debt to invest more than their current profits.

The GKM behavior is summarized by an ordinary differential equation system of three state variables: the wage share, the employment rate and the ratio of corporate debt over the total production. The joint dynamics of Goodwin cycles and debt accumulation lead to the emergence of two types of meaningful long-run equilibria. A "good" equilibrium in which both the employment and the wage share stabilize around non-zero long-term levels while debt remains finite, as well as a "bad" equilibrium in which the debt ratio becomes infinite while employment and the wage share collapse (Grasselli and Costa Lima, 2012). The latter equilibrium has been interpreted by Keen (1995) as an illustration of the "financial instability hypothesis" put forward by Minsky (1982), and as a debt-deflationary equilibrium by Giraud and Grasselli (2021). In any case, the impact of the corporate debt dynamics found out in (Keen, 1995) stood in sharp contrast with the macro-economic literature preceding the Global Financial Crisis, where debts and liabilities were often overlooked at the aggregate level.

While the interpretation of the GKM in terms of Minskian instability has been discussed (Pottier and Nguyen-Huu, 2017), the empirical relevance of the model has not been assessed. In the GM context, two main strategies have been proposed to test the model reliability. The first aims to highlight the very existence of distributive cycles using visual-based evidence (Mohun and Veneziani, 2006), vector auto-regression (Tarassow, 2010; Barrales-Ruiz et al., 2022) or wavelet transform (Barrales-Ruiz et al., 2022). The second approach aims at assessing the goodness-of-fit (Gof) of the GM equations by estimating the model parameters and comparing simulated trajectories and equilibria with the observed data series (Harvie, 2000; McIsaac, 2021).

However, estimating the GM parameters is made difficult by the non-linearity and the continuous-time feature of the model. Two main techniques have been adopted based either on an equation-by-equation estimation (Harvie, 2000; Grasselli and Maheshwari, 2018) or on the simultaneous inference of all parameters (Dibeh et al., 2007; Massy et al., 2013; McIsaac, 2021). Whatever the method used, results tend to corroborate the existence of distributive cycles as described in the Goodwin model. Recently, Grasselli and Maheshwari (2018) found a close agreement between the equilibrium employment rate of a calibrated GM with general capital accumulation rate and the corresponding empirical averages for nine OECD countries from 1960 to 2010. Applying a simulated maximum likelihood approach to U.S. data, McIsaac (2021) also highlighted the better forecasting performance of a stochastic version of the Goodwin model compared with a VAR, for horizons of up to 8 quarters.

This paper aims at contributing to the renewal of an empirically sound macro-economic theory of real business cycles by testing the statistical relevance of the Goodwin-Keen model (Keen, 1995). An original estimation strategy is proposed to simultaneously estimate the parameters of the model while accounting for the non-linearity of the dynamical system. It is applied to US nonfinancial business data series, the scope of which is consistent with the theoretical foundations of the model. More precisely, we estimate an extended GKM including dividend payments to shareholders. Functional forms for the Phillips curve and the investment function, as well as constraints on parameters are chosen according to the AIC criteria. We derive confidence intervals for parameters and model simulations using a boostrapping strategy (Efron and Tibshirani, 1993). As correlation across observations can undermine the performances of standard bootstrapping methods, we implement a sieve bootstrapping approach (Bühlmann, 1997). The probability that the model equilibria are locally stable is also derived from the uncertainty in the parameter values, as well as the probability of a corporate debt crisis occurring in the next century.

The article is organized as follows. Section 2 describes the data used for the estimation and justifies the inclusion of dividends in the model. Section 3 presents the theoretical model equations and long-run dynamics, while the statistical approach to the model inference is described in section 4. Results are presented in section 5. The specification selected in subsection 5.1, consisting of non-linear Phillips curve and investment functions, is then used to analyze the model fit in subsection 5.2. Over the entire period 1959-2019, the model manages to capture 25 to 30-year cycles in the employment rate as well as the trend in the debt ratio. This relatively good fit is achieved with meaningful values for the main macroeconomic parameters, although the uncertainty over several parameters remains high. Section 5.3 presents the analysis of the GKM long-run dynamics. The model almost never predicts a corporate debt crisis in the next century, although the "good" equilibrium is unstable for around 45% of the bootstrap parameter values. Section 6 concludes.

2. Non-financial businesses US database

The Goodwin-Keen model describes an economy consisting solely of households and private companies; neither taxes nor public investments are modeled. Thus, the public sector is excluded from our data series[1]. Furthermore, financial activities only affect the model dynamics through loan provisions and interest payments incurred by the non-financial sector. As neither employment, investment nor value added are directly related to banking activities in the model, this sector is also discarded from the time series considered. Therefore, in the following, the final scope of the dataset corresponds to the non-financial private sector.

In line with this scope, most empirical variables are constructed as the sum of corporate and non-corporate non-financial businesses data. These series, denoted as "*nonfi*" in the following, were mainly collected from the S4 and S5 tables of the Z1 financial accounts of the United States[2]. In the sequel, all monetary variables are in real terms, nominal series being deflated using the GDP deflator. Details on the data sources can be found in Appendix A.

The observed real output of the economy at time t, Y_t^o, is built as the net value added of the nonfinancial private sector minus net taxes on production:

$$Y_t^o = GVA_t^{nonfi} - Conso\ fixed\ capital_t^{nonfi} - Net\ taxes\ prod\ imp_t^{nonfi}.$$

The observed wage share at time t, ω_t^o, is the ratio of the real compensation of

[1] Government enterprises are considered negligeable, consistent with Mohun and Veneziani (2006)

[2] `https://www.federalreserve.gov/apps/fof/FOFTables.aspx`

employees for the non-financial private sector over the total real production:

$$\omega_t^o = \frac{Compensation\ of\ employees_t^{nonfi}}{Y_t^o}.$$

Consistently with Mohun and Veneziani (2006), the observed employment rate at time t, λ_t^o, is computed from the employment and unemployment series of the “Nonagriculture, Private Industries wage and salary workers”, from the Bureau of Labor statistics. Quarterly employment and unemployment data were recovered from monthly data, and the seasonality of both series were removed thanks to a Seasonal-Trend decomposition using LOESS procedure (Cleveland et al., 1990). The observed employment rate is therefore:

$$\lambda_t^o = \frac{Employment_t^{private\ nonagri}}{N_t^o}, \tag{1}$$

where $N_t^o = Employment_t^{private\ nonagri} + Unemployment_t^{private\ nonagri}$. Finally, the observed labor productivity at time t, a_t^o, is computed as the real output per employee in the private sector:

$$a_t^o = \frac{Y_t^o}{Employment_t^{Total\ private\ sector}}. \tag{2}$$

The employment series used to compute the employment rate (eq. 1) and the labor productivity (eq. 2) slightly differ. Unlike $Employment_t^{private\ nonagri}$, $Employment_t^{Total\ private\ sector}$ includes agricultural employees, to better match the scope of the total production Y_t^o in equation 2. It should be acknowledged that the scope of the numerator and denominator of the labor productivity ratio may still slightly differ, as the employment series include employees of the financial sector and exclude the self-employed.

In addition to the series needed for the GM estimation presented so far, the analysis of the GKM requires net debt data series. The observed net debt level at time t, D_t^o, is the sum of the loans and debt securities less time and saving deposits of the non-financial businesses:

$$D_t^o = Debt\ security_t^{corpo} + loans_t^{nonfi} - Saving\ deposits_t^{nonfi}.$$

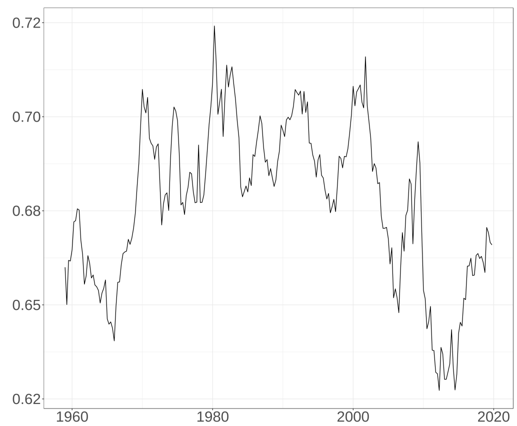
The observed real debt ratio, d_t^o, is then derived as:

$$d_t^o = \frac{D_t^o}{Y_t^o}.$$

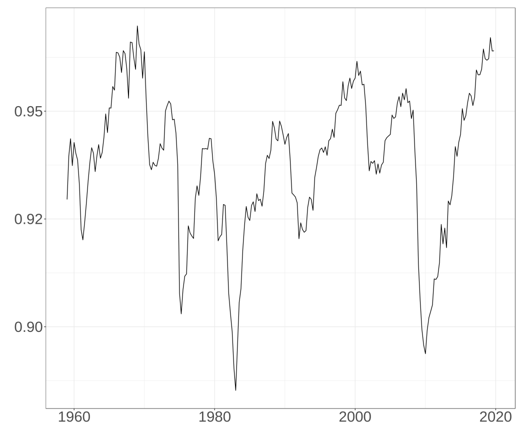
Series for ω_t^o, λ_t^o and d_t^o are displayed in figure 1.

In the original GKM, the evolution of the stock of private debt directly corresponds to the difference between investments and profits. This hypothesis is tested empirically by comparing the observed change in real debt with the gap between investments and profits (see figure 2). Real gross fixed capital formation ($gfcf_t^{nonfi}$) is used as a proxy for new investments, while profits are computed using the fourth quarter values of the state variables series presented above. Although the interest rate is often assumed to be a short-run interest rate in the GKM, the average maturity of firms' borrowings is around a decade (Çelik et al., 2020). Hence, both 3-month and 8-year real interest rates[3] are used to build the investments-profits gap in figure 2.

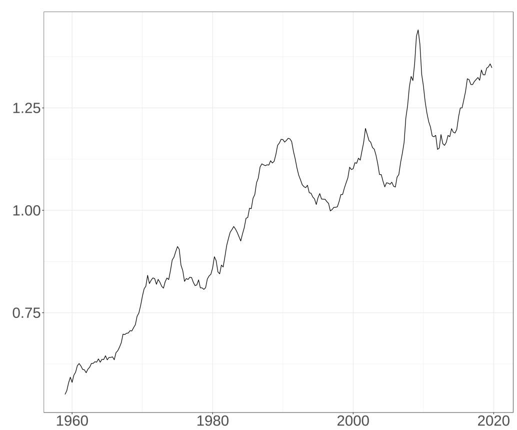
[3]The real short-term interest rate corresponds to the rate on the 3-months money market in the United-States, deflated using a GDP deflator. It is provided in the AMECO database. The 8-year real interest rate is approximated using the following formula: 0.2*short-run-int.-rate + 0.8*real-10-year-int-rate, were the 10-year interest rate corresponds to the yearly average of the monthly rate provided by the Federal Reserve Bank of Cleveland.



(a) Wage share



(b) Employment rate



(c) Debt ratio

Figure 1: Empirical wage share, employment rate and debt ratio - 1959:Q1-2019:Q4

The gap between investments and profits is always negative when dividends are not accounted for, regardless the interest rate chosen (blue lines). Thus, following the model predictions, debt should have decreased all along the 1960-2019 period, which is at odds with the overall positive growth in real debt (red lines). Estimating a model based on the original model hypothesis would necessarily lead to inaccurate estimates.

Taking into account the fact that firms take on debt to finance dividends on the top of investments leads to more realistic debt trajectories (green lines)[4]. In the following, dividends are included in the model as a fixed share of profits distributed to shareholders. Note that data series on investments, interest rates and dividends are only used for the qualitative check of the debt equation and are not involved in the estimation of the model.

3. Dividend-debt-augmented Goodwin model

3.1. Theoretical model

The initial GKM can be summarized as a dynamical system of three state variables: the wage share ω_t, defined as the ratio of the compensation of employees over the total real yearly output, the employment rate λ_t, and the debt-to-output ratio d_t. The link between employment and wages per unit of labour, w_t, is provided by a short-run Phillips curve of the form

[4]Dividends series are derived as the sum of the net dividends paid by non-financial business. Dividends received are subtracted to dividends paid by nonfinancial corporations. Dividends for noncorporate business correspond to the "withdrawals from income of quasi-corporations".

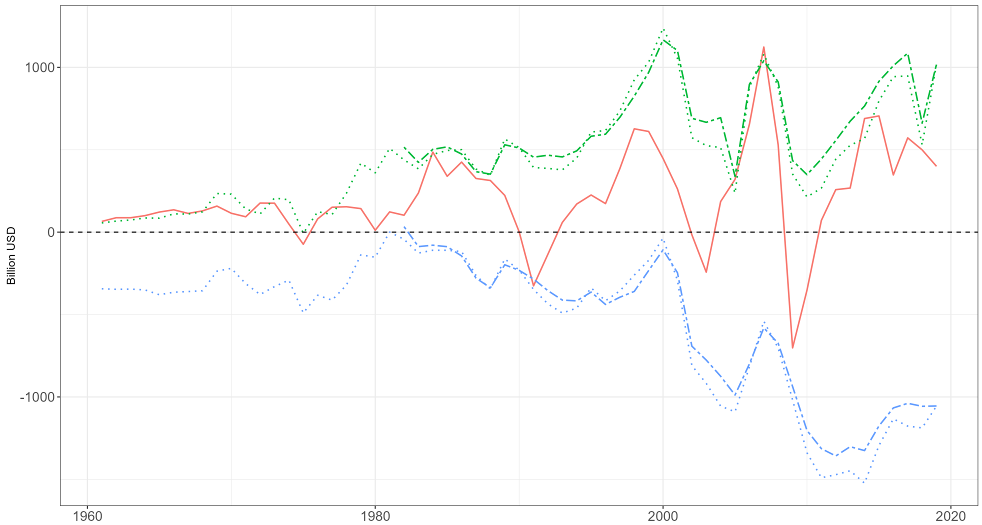


Figure 2: Comparison of the yearly debt evolution (red line) and the gap between revenues and investments when dividends are not accounted for (blue lines) or taken into account (green lines). The gap between revenues and investments can either be computed using a real short-run interest rate (dotted line) or a 8-year real interest rate (two-dash line).

$$\dot{w}_t = w_t \phi(\lambda_t),$$

where ϕ is a smooth, increasing function.

The inclusion of dividends proposed in this paper only affects the structural equation of the real debt dynamics:

$$\dot{D}_t = \kappa(1 - \omega_t - rd_t)Y_t - (1 - \omega_t - rd_t)Y_t + \Delta(1 - \omega_t - rd_t)Y_t, \qquad (3)$$

where Y_t is the real output of the economy, κ a smooth, increasing investment function, r the real interest rate and Δ the new-defined share of profits distributed to shareholders.

This modification alters the last equation of the original GKM reduced form system, leading to the dividend and debt-augmented Goodwin model (DDAGM):

$$
\begin{cases}
\dot{\omega}_t &= \omega_t \left[\phi(\lambda_t) - \alpha\right] \\
\dot{\lambda}_t &= \lambda_t \left[\frac{\kappa(1-\omega_t - rd_t)}{\nu} - \alpha - \beta - \delta\right] \\
\dot{d}_t &= d_t \left[r(1-\Delta) - \frac{\kappa(1-\omega_t - rd_t)}{\nu} + \delta\right] + \\
 & \quad \kappa(1-\omega_t - rd_t) - (1-\omega_t)(1-\Delta),
\end{cases}
\tag{4}
$$

where α and β are the constant growth rates of the labor productivity and labor force; δ, ν and r are the depreciation rate, the capital-to-output ratio and the real interest rate respectively. Function ϕ is the real short-run Phillips curve and κ an investment function. Note that if $\Delta := 0$, the model boils down to the initial GKM (Keen, 1995). Details on the calculations leading to the reduced-form model can be found in Appendix C.

Both linear and non-linear functional forms have been used for the short-run Phillips curve and the investment function in the Goodwin and Goodwin-Keen models (Goodwin, 1967; Harvie, 2000; Grasselli and Maheshwari, 2018; Desai et al., 2006; Grasselli and Costa Lima, 2012). In this article, we compare the following functional forms:

$$
\phi(\lambda_t) = \begin{cases} \gamma + \rho(1-\lambda_t)^{-2} \\ \gamma + \rho\lambda_t \end{cases}
\tag{5}
$$

and :

$$
\kappa(w_t, d_t) = \begin{cases} k_1 e^{k_2(1-w_t-rd_t)} \\ k_0 + k_1(1-w_t-rd_t). \end{cases}
\tag{6}
$$

3.2. Equilibria and local stability

As in the original GKM (Grasselli and Costa Lima, 2012), four long-run steady states are identified for the DDAGM system. For each of them, the equilibrium values of the state variables and the local stability conditions can be evaluated at any given set of parameters. The associated formulas are displayed in table 1, while details on the calculations can be found in Appendix C.

From an economic perspective, only the so-called “good” and “bad” equilibria are meaningful. The “good” equilibrium reflects a Solovian steady growth of the economy in which all state variables stabilize to a positive, finite level. Conversely, the “bad” equilibrium corresponds to a private debt crisis where the debt ratio goes to infinity while both the wage share and the employment rate fall to zero. Although they are economically meaningless, the analysis of the system requires to study two additional equilibria. The first one is characterized by null wage share and employment rate but positive finite debt level (meaningless equilibrium 1). The second one corresponds to a state where the wage share is null, while both the employment rate and the debt ratio are positive. This last equilibrium, sometimes called “slavery equilibrium”, only appears when the parameters fulfill a specific equality condition (see table 1). Unlike the other types of equilibrium for which local stability is analyzed, we simply check that the “slavery” state is not an equilibrium for each set of parameters studied.

Finally, note that several equilibria can be locally stable for a unique set of parameters. In this case, the long-run values of the state variables depend on the initialization of the system: our approach is decisively that of a multi-equilibria world.

Equilibrium	State variables at equilibrium	Local stability conditions
Good	$(\overline{\omega_g}, \overline{\lambda_g}, \overline{d_g})$, with: $\overline{\pi_g} = \kappa^{-1}(\nu(\alpha+\beta+\delta))$ $\overline{\omega_g} = 1 - \overline{\pi_g} - r\frac{\nu(\alpha+\beta+\delta)-\overline{\pi_g}(1-\Delta)}{\alpha+\beta}$ $\overline{\lambda_g} = \phi^{-1}(\alpha)$ $\overline{d_g} = \frac{\nu(\alpha+\beta+\delta)-\overline{\pi_g}(1-\Delta)}{\alpha+\beta}$	$r\frac{\kappa'(\overline{\pi_g})\left(\overline{d_g}-\nu\right)+\nu(1-\Delta)}{\nu} < 0$
Bad	$(\overline{\omega_b}, \overline{\lambda_b}, \overline{d_b}) = (0, 0, +\infty)$	$\frac{k_0}{\nu} - \delta < r(1-\Delta)$
Meaningless 1	$(\overline{\omega_f}, \overline{\lambda_f}, \overline{d_f}) = (0, 0, \overline{d_f})$, with d_f solution of: $d(r(1-\Delta) - \frac{\kappa(1-rd)}{\nu} + \delta)$ $+\kappa(1-rd) - (1-\Delta)$	$\phi(0) - \alpha < 0$, $\frac{\kappa(\pi_f)-\nu(\alpha+\beta+\delta)}{\nu} < 0$, $r(1-\Delta) + \delta - \frac{\kappa(\pi_f)}{\nu}$ $+\frac{r(d_f-\nu)\kappa'(\pi_f)}{\nu} < 0$ with $\pi_f = 1 - \overline{d_f}$
Meaningless 2	$(\overline{\omega_s}, \overline{\lambda_s}, \overline{d_s}) = (0, \overline{\lambda_s}, \overline{d_s})$, with: $d_s = \frac{\nu(\alpha+\beta+\delta)-\overline{\pi}_s(1-\Delta)}{\alpha+\beta}$ and $\overline{\pi}_s = 1 - rd_s = \kappa^{-1}(\nu(\alpha+\beta+\delta))$	Eq. existence condition: $1 - r\frac{\nu(\alpha+\beta+\delta)-\kappa^{-1}(\nu(\alpha+\beta+\delta))(1-\Delta)}{\alpha+\beta}$ $= \kappa^{-1}(\nu(\alpha+\beta+\delta))$

Table 1: Equilibrium values and stability conditions of the DDAGM

4. Statistical model and inference

The DDAGM model (4) is a deterministic system whose solutions only depend on the parameters and initial conditions. Our estimation approach relies on the hypothesis that data series are the noised observations of the state variables generated by the theoretical model with the "true" set of parameters. In what follows, ω_t^o, λ_t^o and d_t^o denote the observed wage share, employment rate and debt ratio respectively. Initial conditions of the system, w^{ini}, λ^{ini} and d^{ini}, are assumed unknown parameters to be estimated. Defining the initial conditions of the dynamical system as the first observed values would make the inference too reliant on the choice of the estimation period.

4.1. Likelihood and constraints

Employment rate and wage share are non-negative ratios bounded from above by one[5]. Therefore, we assume that ω_t^o and λ_t^o follow truncated-Gaussian distributions centered on the theoretical values of ω_t and λ_t respectively. As for d_t^o, it can take greater than one or negative values (corresponding to saving). It is therefore assumed to have a Gaussian distribution with mean equal to d_t:

$$y_t^o|\theta, y^{ini} \sim \mathcal{NT}\Big(\mu_t^y\left(\theta, y^{ini}\right), \sigma_y, \underline{b}_y, \overline{b}_y\Big). \tag{7}$$

In equation (7), y_t^o is either w_t^o, λ_t^o or d_t^o; $\mathcal{NT}$ denotes the truncated Gaussian distribution on the interval $]\underline{b}_y, \overline{b}_y[$ where $\underline{b}_y = (0, 0, -\infty)$ and $\overline{b}_y = (1, 1, +\infty)$ for the wage share, the employment rate and the debt ratio respectively. Solutions of the deterministic DDAGM model given the parameters θ and the initial conditions y^{ini} are denoted $\mu_t^y\left(\theta, y^{ini}\right)$ (see system (4)).

[5] Depending on the functional forms chosen for the Phillips curve and the investment function, both variables can exceed 1 in system 4. Yet, such extreme cases don't appear in the historical data used to fit the model.

The unknown parameters θ as well as the initial conditions $(\omega^{ini}, \lambda^{ini}, d^{ini})$ are estimated by maximizing the following log-likelihood under inequality constraints:

$$\ell\left(\theta, \omega^{ini}, \lambda^{ini}, d^{ini} | \omega_t^o, \lambda_t^o, d_t^o\right) = \sum_{y \in \{\omega, \lambda, d\}} \sum_{t=1}^{T} f\left(y_t, \mu_t^y\left(\theta, \omega^{ini}, \lambda^{ini}, d^{ini}\right), \sigma_y^2, \underline{b}_y, \bar{b}_y\right), \tag{8}$$

where f denotes the density of a truncated Gaussian distribution (Johnson et al., 1994).

Two types of constraints are set on the parameter values. The first set of bounds relates to the economic meaning of the parameters. Previous estimations indicate for instance that the capital-to-output ratio ν lies between 2 and 6 (Grasselli and Maheshwari, 2018; McIsaac, 2021). Likewise, the depreciation parameter δ is a positive rate lower than 50%. We further assume that the share of dividends in profits belongs to [0, 0.9] and we allow the real interest rate to be negative by assuming that r belongs to [-0.1, 0.3]. Second, conditions on the Phillips and investment curves have been shown to ensure that the employment rate does not exceed 100% and that the stability of the good equilibrium is possible (Desai et al., 2006; Grasselli and Costa Lima, 2012). The combination of these two types of restrictions provides constraints on both the level and the relative values of the parameters, displayed in Appendix D. Note that, whatever the parameters, the linear Phillips and investment curves do not meet the inequality constraints.

Finally, α and β are estimated separately based on the labor force and productivity data using a log-regression approach similar to Harvie (2000) and Grasselli and Maheshwari (2018). Although estimating them with the rest of the parameters would be straightforward adding two equations in system (4), the uncertainty over α and β is of less interest. In the following, α and β are set to 0.0102 and 0.0157 respectively.

4.2. Initialization of the estimation process

The optimization procedure is highly sensitive to initial parameter guess, as the likelihood function (8) may have local modes. To avoid convergence issues, we take as a starting point of the optimization the estimates provided by a simplified statistical model. Specifically, the initial values are the estimates from a non-linear regression of a discretized version of the observed processes:

$$\begin{cases} \dfrac{D\omega_t^o}{\omega_t^o} = \phi(\lambda_t) - \alpha + \varepsilon_\omega \\ \dfrac{D\lambda_t^o}{\lambda_t^o} = \dfrac{\kappa(1-\omega_t - rd_t)}{\nu} - \alpha - \beta - \delta + \varepsilon_\lambda \\ \dfrac{Dd_t^o}{d_t^o} = r(1-\Delta) - \dfrac{\kappa(1-\omega_t - rd_t)}{\nu} + \delta \\ \qquad + \dfrac{\kappa(1-\omega_t - rd_t) - (1-\omega_t)(1-\Delta)}{d_t} + \varepsilon_d, \end{cases}$$

where $Dy_t := \frac{y_{t+\tau_t} - y_t}{\tau_t}$ with $\tau_t = 1/4$ as series are observed on a quarterly basis.

The parameters are obtained by minimizing the Gaussian log-likelihood under the previously defined constraints:

$$\ell(\theta) = \sum_{y \in \{\omega, \lambda, d\}} \sum_{i=t}^{T} g\left(Dy_t, \theta, s_y^2\right), \tag{9}$$

where g denotes the Gaussian density function and s the standard deviation.

4.3. Confidence interval and general inference strategy

In order to measure the uncertainty of the parameter estimation in the non-linear context of the GKM, we implement a bootstrapping strategy in which new estimation series are built by resampling the residuals of the estimated ω_t, λ_t and d_t trajectories. Observed variables may be correlated over time and a simple sampling procedure would fail to replicate this dependence structure. To account for the possible correlation over time and across observed variables, we apply a sieve bootstrapping strategy (Bühlmann, 1997).

Let $\theta = (\gamma, \rho, k_0, k_1, k_2, \nu, r, \delta, \Delta)$ be the parameters of the theoretical model. Then $\psi^D = \left(\theta^D, s_\omega, s_\lambda, s_d\right)$ and $\psi^C = \left(\theta^C, \omega^{ini}, \lambda^{ini}, d^{ini}, \sigma_\omega, \sigma_\lambda, \sigma_d\right)$ denote the parameters associated with the discretized (see eq. 9) and continuous approaches (see eq. 8) respectively. Unlike ψ^D, ψ^C includes ω^{ini}, λ^{ini} and d^{ini}, as the continuous-time approach allows us to estimate the initialization of the state variables. The entire inference strategy can be described by the pseudo-code 1. The optimization procedure is carried out combining the global stochastic random-search algorithm ISRES (Runarsson and Yao, 2005) with local Nelder-Mead algorithms using the *NLopt* package available in the R freeware (R Core Team, 2022). Global and local convergence tolerances are set to $1e^{-6}$ and $1e^{-12}$ respectively with $1e^{8}$ maximum steps. R code is available upon request.

ML estimation

1 - Let $\widehat{\psi}^D_{i,i=1,\ldots,q}$ the q maximum log-likelihood estimates obtained from the discretized equations likelihood (9). Keep the p estimates leading to the highest likelihood value.

2 - Estimate $\widehat{\psi}^C_{j,j=1,\ldots,p}$ the p maximum log-likelihood estimates obtained from equations (8) using $\widehat{\theta}^D_{j,j=1,\ldots,p}$ as starting points. The estimate $\widehat{\psi}^C$ leading to the highest likelihood value is the ML estimator.

Bootstrap

3 - Let $\epsilon_{t,t=1,\ldots,T} = y_t - \mu_t(\widehat{\psi}^C)$ the residuals of the estimated model ($y_t = (\omega^0_t, \lambda^0_t, d^0_t)$). Estimate a VAR(p) on these residual series ($p \leq 15$ selected according to the AIC criterion). Let $\epsilon^{VAR}_{t,t=1\ldots,T}$ the residuals associated to the fitted VAR model.

for all $b \in \{1, \ldots, B\}$ **do**

Sample n ε^{VAR} elements: $\varepsilon^{VAR}_{(1)}, \ldots, \varepsilon^{VAR}_{(n)}$

Generate $\varepsilon^\star$ from a $VAR(p)$ with innovation $\varepsilon^{VAR}_{(1)}, \ldots, \varepsilon^{VAR}_{(n)}$.

Estimate $\widehat{\psi}^{\star C}$ for each $y^b = \mu^y_t(\widehat{\psi}^C) + \varepsilon^\star$ using the ML estimation approach.

end for

Algorithm 1: Inference and bootstrapping algorithm.

5. Fit and predictions of the dividend-augmented Goodwin-Keen model

5.1. Functional form and parameter selection

Functional forms and parameter constraints are compared according to the AIC criterion. The Phillips curve and the investment function are either linear or respectively power or exponential (see section 3). The parameter γ can be estimated or fixed to 0. Finally, we test empirically the inclusion of dividends by comparing models where the parameter Δ is estimated or set to 0. For each model, the continuous estimation process is implemented for 50 starting values selected among 100 estimates from the discrete estimation.

According to the median of the AIC (see table 2), the model endowed with a power Phillips curve, an exponential investment function, a fixed γ parameter and an unknown dividend share of profits Δ stands out. However, model performances are close. Furthermore, the standard deviations of the AIC across the 50 estimations are large, highlighting the potential presence of multiple modes in the likelihood functions. In this respect, the best model in terms of median AIC also has a relatively low standard deviation.

While models with a linear Phillips curve perform worse overall than models endowed with power functional forms, results are less clear for the investment function. In particular, exponential κ functions can lead to either better or worst results than linear ones, depending on the other model features. For instance, the power-exponential model with $\gamma = 0$ and an unknown dividend parameter almost has the same median AIC as the power-linear model with an unknown γ parameter and a Δ fixed to zero. Furthermore, the GoF is not clearly improved by accounting for dividends through the estimation of Δ.

Yet, estimating this parameter leads to more suitable investment estimates. The root mean square distance (RMSD) between the empirical investments-to-output ratio and its estimate, $\sqrt{\frac{1}{T}\sum_t \left(\frac{gfcf_t^{nonfi}}{y_t^o} - \hat{\kappa}(\omega_t^o, d_t^o)\right)^2}$, is always lower when Δ is estimated, whatsoever the form of the investment function. More specifically, RMSDs are equal to 0.074 and 0.145 for models assuming either an exponential investment form combined with $\gamma = 0$ and unknown dividend or a linear form with an unknwon γ and a null dividend.

To sum up, the dividend-debt-extended model assuming a power Phillips curve with a null γ parameter combined with an exponential investment function is chosen and used in the following. This model has the lowest median AIC and provides meaningful investment trajectories. Its nonlinear functional forms also satisfy mathematical conditions allowing the local stability analysis of system (4). ML parameters are provided in table 3.

5.2. Estimation, confidence intervals and goodness-of-fit

Main macroeconomic parameters estimates are broadly consistent with the literature (see table 3). The maximum likelihood estimate of the depreciation rate of capital is around 4%, lying between the 5.2% and 3.7% estimates using the AMECO data and the Penn World tables respectively (McIsaac, 2021). Likewise, the share of profits distributed as dividends amounts to 67%, in line with the 60% average rate observed in the S4 and S5 tables of the U.S. financial accounts[6]. The ML and median estimates for the interest rate are respectively 1 and 1.4%, close to the average observed short-term interest rate (around 1.55 between 1961 and 2019), but lower than the 8-year

[6] `https://www.federalreserve.gov/apps/fof/FOFTables.aspx`

ϕ	κ	γ	Δ	Median AIC	Sd AIC
power	exp	fixed	estimated	**-3203.28**	**98.23**
power	exp	estimated	estimated	**-3200.28**	**140.14**
power	linear	estimated	fixed	**-3199.27**	**101.66**
power	exp	fixed	fixed	-3190.71	140.28
power	linear	estimated	estimated	-3169.02	182.58
power	linear	fixed	fixed	-3165.52	108.06
power	exp	estimated	fixed	-3139.14	238.79
linear	exp	estimated	estimated	-3133.04	118.26
power	linear	fixed	estimated	-3126.85	169.90
linear	linear	estimated	fixed	-3117.86	226.99
linear	exp	estimated	fixed	-3110.16	85.52
linear	linear	estimated	estimated	-3102.31	NaN

Table 2: Model comparison according to median and standard deviation (Sd) of AIC, computed based on 50 initial starting points. Rows are ranked by median AIC. In bold, the three best models.

interest rate (2.49 between 1982 and 2019). As for the capital-to-output ratio ν, both the MLE and the median values revolve around 3, in line with previous estimations (Grasselli and Maheshwari, 2018; McIsaac, 2021).

However, confidence intervals (CI) based on a 1000 bootstrap samples are large for most macroeconomic parameters. The bootstrap CI of the capital-to-output ratio ν overlaps the predefined bounds while the bootstrap distribution of the Δ parameter displays two modes, close to zero and around 0.6. This can reflect identifiability issues due to a lack of information in the data or to inconsistencies between the observed series and the theoretical model. The limitation of the model to reflect data is also confirmed by the r coefficient being insignificant (0 belongs to the CI). This shows that the link between the debt equation and the rest of system (4) remains questionable from an empirical point of view.

Discussing individual parameter estimates for the Phillips and investment curves in a nonlinear context is uneasy without any value to refer to. The global quality of fit is therefore assessed by comparing observed trajectories of the state variables and investments-to-output ratio with their estimates (see figure 3).

First, the model roughly captures the around 27-year cycles in the employment rate. It also manages to replicate the non-linear growing trend in the corporate debt ratio, but not the short-term oscillations of this variable around its trend. The fit is not as good for the wage share, especially over the period 1980-2000. According to the model, the decrease in employment rate at the beginning of the 1980s should have led to a reduction in the wage share at the end of the decade. Yet, the observed wage share remained over-

	MLE	Median	Standard deviation	Ci 2.5%	Ci 97.5%
ρ	3.51e-05	3.43e-05	7.66e-06	3.9e-06	4.35e-05
k1	0.035	0.0473	0.0347	0.00485	0.122
k2	4.45	4.94	1.77	2.43	10
r	0.00968	0.0142	0.0341	-0.0528	0.0858
ν	2.26	3.53	1.36	2	6
δ	0.0359	0.0333	0.0256	9.55e-06	0.0991
Δ	0.672	0.45	0.271	2.53e-07	0.887
ω^{ini}	0.676	0.679	0.0168	0.645	0.712
λ^{ini}	0.927	0.927	0.0096	0.911	0.95
d^{ini}	0.523	0.521	0.0434	0.446	0.608
σ^{ω}	0.019	0.0151	0.00474	0.011	0.025
σ^{λ}	0.0141	0.0137	0.00494	0.0102	0.0292
σ^{d}	0.0678	0.0659	0.039	0.0477	0.207

Table 3: Maximum likelihood estimates (MLE), median and 95% bootstrap confidence intervals

all at the same level all along the 1980s and 1990s, suggesting some kind of downward stickiness of wages.

Second, estimated investments-to-output levels are consistent with observed GFCF-to-output ratios (see panel (d) of figure 3). Neither the ML nor the median trajectories manage to capture the fluctuations of the investment dynamics though. The large uncertainty over the estimated investment-to-output ratio also confirms that the model fails to precisely reflect the debt-investment dynamics. Indeed, we note that high estimated values for the dividend parameter Δ are associated with low investments-to-output trajectories.

5.3. Are the U.S. *heading towards a corporate debt crisis?*

The GKM was originally designed to study the endogeneous emergence of corporate debt crises. From a theoretical perspective, four equilibria can be reached by system 3 (see table 1). Two are economically meaningless, while the two others reflect either a debt crisis or a Solovian steady state. Stability conditions for each type of equilibrium are tested over the 1000 bootstrap parameter estimates.

The two meaningless equilibria are always respectively unstable and inexistent[7] (except for three occurrences). Conversely, the bad equilibrium is stable in around 97% of the 1000 cases, which is easily explained by the fact that $k0$ is set to 0 in our specification, making the stability condition less strict (see table 1). Interestingly, the "good" equilibrium is locally stable in

[7]The "slavery" equilibrium only emerges when the parameters meet the equality condition displayed in the bottom right cell of table 1

(a) Wage share

(b) Employment rate

(c) Debt ratio

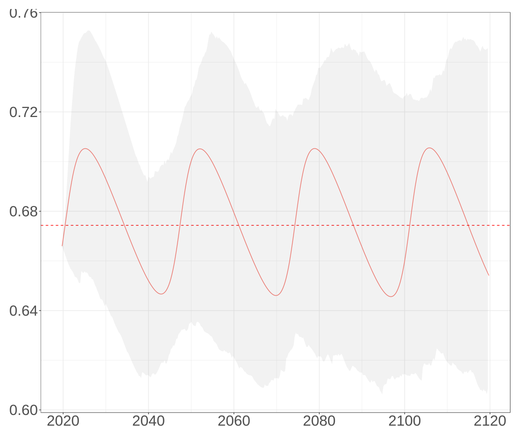
(d) Investments-to-output

Figure 3: Observed (blue-line), ML (red-line), Median (green-line) trajectories and 95% bootstrap confidence interval (shaded area).

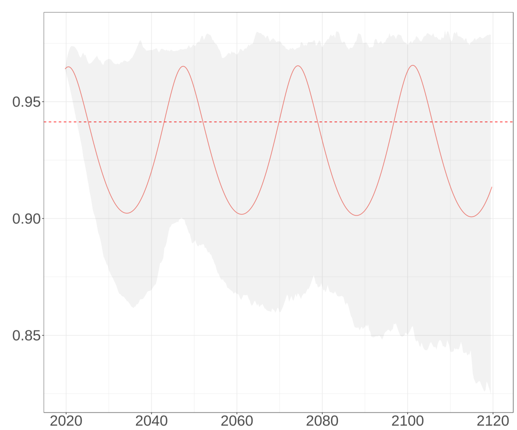
51% of the bootstrap parameter sets. Note that the sign of the interest rate r is not the only element determining the stability of the “good” equilibrium, as negative interest rates are observed for both stable and unstable "good" equilibria.

Mainly three situations are observed regarding the equilibrium behavior of the DDAGM. First, the debt-crisis steady state is the only locally stable equilibrium for 45% of the bootstrap iterations. This does not mean that the U.S. economy, as modeled by the GKM, is bound to collapse. Indeed, unless the initial conditions of (4) stand within the basin of attraction of the "bad" equilibrium, the economy may reach a limit-cycle in the long-run and oscillate forever without ever reaching any asymptotic attractor. Next, 51% of the bootstrap estimates lead the "good" and the "bad" equilibria to be simultaneously stable. Finally, none of the equilibria are locally stable for around 3% of the bootstrap iterations.

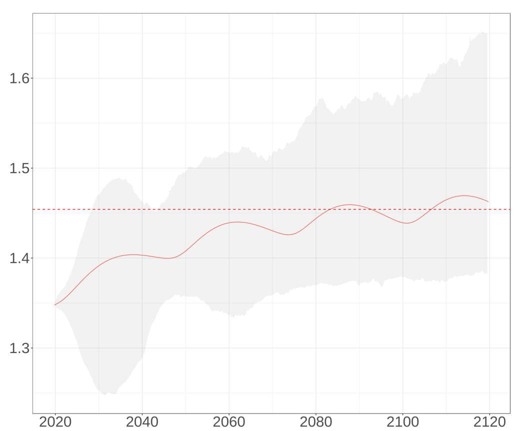
Local stability analysis does not say anything on the time required to reach equilibrium, nor on which equilibrium is reached when both the "good" and the "bad" steady states are locally stable. We address this question using numerical simulations. For the ML and each bootstrap estimates, system (4) is simulated for 100 years starting from the 2019:Q4 wage share, employment rate and debt ratio values (see figure 4). Although only the "bad" equilibrium is locally stable for the ML parameters, no corporate debt crisis appears within the next 100 years. Wage share, employment rate and debt ratio keep oscillating around their unstable "good" equilibrium values, represented by the dotted horizontal lines. More generally, only 7 out of 1000 bootstrap trajectories lead to a debt-to-output ratio higher than the estimated capital-to-output ratio — a level at which a systemic default would presumably occur if capital is used as a collateral to corporate debts, as argued by Bovari et al. (2018b). Similarly, only 4 simulated debt ratio trajectories exceed the median capital to output ratio (3.53). The probability of a corporate debt crisis occurring in the next century remains in any case below 1% when accounting for the uncertainty over the parameter values. This suggests a conclusion similar to that already obtained in Bovari et al. (2018a) at the world level: absent complications (either due to climate change or to the financial sphere), the economy under scrutiny does not wander in the basin of attraction of the debt-crisis steady state.



(a) Wage share



(b) Employment rate



(c) Debt ratio

Figure 4: Projections of the GKM for the next 100 years - ML parameters (red curve), “good” equilibrium values for the ML parameters (dotted red line) and 95% confidence intervals (shaded area).

6. Conclusion

Relying on non-financial business data series for the United States, this article provides insights on the empirical validity of the Goodwin-Keen model. We first show that the private debt equation must be modified for the model to be consistent with the observed data. Assuming that corporate debt grows only when investments exceed profits would imply an unrealistic decrease in the debt ratio. Therefore, we propose to include dividend payments in the estimated model. We also note that the interest rate in the model should be interpreted as the actual rate at which companies roll their debt over (about a decade), rather than as a short-term interest rate.

Using a statistical approach allowing to estimate simultaneously the GKM parameters, we show that the model manages to capture the antagonistic cycles in the wage share and the employment rate, while reflecting the trend growth in the debt-to-output ratio over the period 1959-2019. Moreover, parameter estimates are mostly consistent with existing literature and official statistics. However, the large uncertainty over parameter values indicates that the model only partially captures the evolution of the observed series. More specifically, the interest rate parameter being statistically insignificant indicates that the link between the debt ratio equation and the rest of the GKM system remains weak. This is confirmed by the model's inability to reproduce short-term fluctuations in the debt-to-output ratio.

Using the uncertainty bounds of the parameter estimation, we provide a statistical analysis of the model predictions for the next century. The "bad" equilibrium, associated to a corporate debt crisis, is the only stable equilibrium for 45% of bootstrap parameter values. Yet, less than 1% of the simulated trajectories lead to an unsustainable corporate debt within the next 100 years. This optimistic result should be taken *cum grano salis*, as the model remains purely real, exhibits no money, no financial sphere, and no public sector. Therefore, financial crises such as the dot.com crash of 2001, the Global Financial Crisis of 2007-2009 or the European public debt crisis of the 2010s' cannot be captured, let alone predicted, by the simplistic 3-dimensional system scrutinized in this paper.

Our results suggest that the GKM provides an interesting framework for the development of larger real business cycle models. Like the original Goodwin model, it reflects endogenous fluctuations in the wage share and the employment rate which are consistent with observed data. It also accounts for the possibility for an economy to head towards both a Solovian steady state or a debt crisis equilibrium. Yet, further work should be done to make it more realistic.

First, the modeling of debt and finance should be improved. The inclusion of dividends, as proposed in this article, is not sufficient to consistently capture the oscillations in the debt-to-output ratio observed in the data. More generally, the direct link between profits, investment and borrowings in the Goodwin-Keen model is too rough to capture the complex financing of private companies. On the one hand, firms can take on debt to finance non-productive assets, such as financial derivatives. On the other hand, alternative sources of funding, such as capital increase, can be used to finance productive investments. Moreover, a proper modelling of a financial sphere requires the explicit introduction of (non-neutral) money, as in (Dossetto, 2022). Second, our analysis suggests that some downward-stickiness of wages should be added to the short-run Phillips curve. Finally, the model's fit to observed data could be improved by relaxing the hypothesis of a constant value for the parameters. Regarding the capital-to-output ratio, this could be achieved by replacing the postulated Leontief production function by a CES technology (Bastidas et al., 2019) or a Putty-Clay structure of capital as in (Akerlof and Stiglitz, 1969). Statistical methods, relying for instance on the combination of time-varying parameters combined with regularization techniques (Tibshirani et al., 2005; Heuclin et al., 2020) could also be used to infer time-varying parameters.

Acknowledgements

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References

G. Akerlof and J. E. Stiglitz. Capital, Wages and Structural Unemployment. *Economic Journal, 1969, vol. 79, issue 314, 269-81*, 1969.

J. Barrales-Ruiz, I. Mendieta-Muñoz, C. Rada, D. Tavani, and R. Von Arnim. The distributive cycle: Evidence and current debates. *Journal of Economic Surveys*, 36(2):468–503, 2022.

D. Bastidas, A. Fabre, and F. Mc Isaac. Minskyan classical growth cycles: Stability analysis of a stock-flow consistent macrodynamic model. *Mathematics and Financial Economics*, 13:359–391, 2019.

E. Bovari, G. Giraud, and F. Mc Isaac. Coping with collapse: a stock-flow consistent monetary macrodynamics of global warming. *Ecological Economics*, 147:383–398, 2018a.

E. Bovari, O. Lecuyer, and F. Mc Isaac. Debt and damages: What are the chances of staying under the 2°c warming threshold? *International Economics*, 155:92–108, 2018b.

E. Bovari, G. Giraud, and F. McIsaac. Financial impacts of climate change mitigation policies and their macroeconomic implications: a stock-flow consistent approach. *Climate Policy*, 20(2):179–98, 2020.

P. Bühlmann. Sieve bootstrap for time series. *Bernoulli*, 3(2):123 – 148, 1997. doi: bj/1177526726. URL https://doi.org/.

R. B. Cleveland, W. S. Cleveland, J. E. McRae, and I. Terpenning. Stl: A seasonal-trend decomposition. *J. Off. Stat*, 6(1):3–73, 1990.

M. Desai, B. Henry, A. Mosley, and M. Pemberton. A clarification of the Goodwin model of the growth cycle. *Journal of Economic Dynamics and Control*, 30(12):2661–2670, December 2006. URL https://ideas.repec.org/a/eee/dyncon/v30y2006i12p2661-2670.html.

G. Dibeh, D. G. Luchinsky, D. D. Luchinskaya, and V. N. Smelyanskiy. A bayesian estimation of a stochastic predator-prey model of economic fluctuations. In *Noise and stochastics in complex systems and finance*, volume 6601, pages 266–274. SPIE, 2007.

E. Dossetto. *Macro-dynamics of the energy transition: robustness, money velocity and public policies*. PhD thesis, Université Panthéon-Sorbonne-Paris I, 2022.

B. Efron and R. J. Tibshirani. *An Introduction to the Bootstrap*. Number 57 in Monographs on Statistics and Applied Probability. Chapman & Hall/CRC, Boca Raton, Florida, USA, 1993.

G. Giraud and M. Grasselli. Household debt: The missing link between inequality and secular stagnation. *Journal of Economic Behavior & Organization*, 183:901–927, 2021.

R. Goodwin. A growth cycle. In *Socialism, Capitalism and Economic Growth*, pages 54–58. Cambridge University Press, Cambridge, 1967.

M. Grasselli and A. Maheshwari. Testing a Goodwin model with general capital accumulation rate. *Metroeconomica*, 60, 2018.

M. R. Grasselli and B. Costa Lima. An analysis of the keen model for credit expansion, asset price bubbles and financial fragility. *Mathematics and Financial Economics*, 6(3):191–210, 2012.

M. R. Grasselli and A. Nguyen Huu. Inflation and speculation in a dynamic macroeconomic model. *Journal of Risk and Financial Management*, 8(3): 285–310, 2015.

M. R. Grasselli and A. Nguyen-Huu. Inventory growth cycles with debt-financed investment. *Structural Change and Economic Dynamics*, 44:1–13, 2018.

D. Harvie. Testing Goodwin: growth cycles in ten OECD countries. *Cambridge Journal of Economics*, 24(3):349–376, 2000.

B. Heuclin, F. Mortier, C. Trottier, and M. Denis. Bayesian varying coefficient model with selection: An application to functional mapping. *Journal of the Royal Statistical Society: Series C (Applied Statistics)*, 2020.

N. L. Johnson, S. Kotz, and N. Balakrishnan. *Continuous Univariate Distributions*, volume 1. Wiley, 1994.

S. Keen. Finance and economic breakdown: modeling minsky's "financial instability hypothesis". *Journal of Post Keynesian Economics*, 17(4):607–635, 1995.

I. Massy, A. Avila, and M. Garcia-Molina. Quantitative evidence of goodwin's non-linear growth cycles. *Applied Mathematical Sciences*, 7(29):1409–1417, 2013.

F. McIsaac. Testing goodwin with a stochastic differential approach—the united states (1948–2019). *Metroeconomica*, 72(4):696–730, 2021.

H. P. Minsky. *Inflation, recession and economic policy*. Brighton, Sussex: Wheatsheaf Books, 1982.

S. Mohun and R. Veneziani. Goodwin cycles and the us economy, 1948–2004. Working paper, MPRA Working Paper No. 30444., 2006.

A. Pottier and A. Nguyen-Huu. Debt and investment in the keen model: a reappraisal of modelling minsky. *Review of Keynesian Economics*, 5(4): 631–647, 2017.

R Core Team. *R: A Language and Environment for Statistical Computing*. R Foundation for Statistical Computing, Vienna, Austria, 2022. URL https://www.R-project.org/.

P. Romer. The Trouble with Macroeconomics. https://paulromer.net/wp-content/uploads/2016/09/WP-Trouble.pdf, 2016.

T. P. Runarsson and X. Yao. Search biases in constrained evolutionary optimization. *IEEE Transactions on Systems, Man, and Cybernetics, Part C (Applications and Reviews)*, 35(2):233–243, 2005.

A. Tarassow. The empirical relevance of goodwin's business cycle model for the us economy. Working paper, MPRA Paper 21012, Munich, 2010.

R. Tibshirani, M. Saunders, S. Rosset, J. Zhu, and K. Knight. Sparsity and smoothness via the fused lasso. *Journal of the Royal Statistical Society: Series B (Statistical Methodology)*, 67(1):91–108, 2005.

S. Çelik, G. Demirtaş, and M. Isaksson. Corporate bond market trends, emerging risks and monetary policy. Technical report, OECD Capital Market Series, Paris, 2020.

Appendix A. Data construction

Data are collected from the Federal Reserve Bank of St. Louis API, using the R package *fredr*[8].

Unlike flow variables, neither stock variables nor employment and unemployment series are seasonnaly adjusted. The seasonnality of employment and unemployment series is removed using the LOESS procedure (Cleveland et al., 1990). Finally, employment data are collected on a monthly basis. Quarterly employment figures are therefore retrieved by selecting the first month's value for each quarter.

[8] `https://cran.r-project.org/web/packages/fredr/index.html`

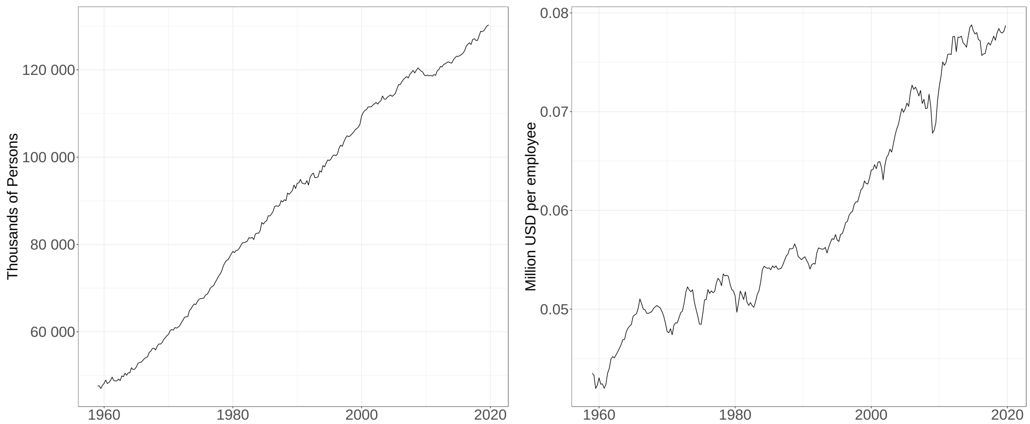
Appendix A.1. Detailled data sources - quarterly and monthly data series

Variable	Sector	FRED code	Complete series name
Gross value added	Corporate	NCBGAVQ027S	Nonfinancial Corporate Business; Gross Value Added, Transactions
Gross value added	Noncorporate	NNBGAVQ027S	Nonfinancial Noncorporate Business; Gross Value Added, Transactions
Consumption of fixed capital	Corporate	BOGZ1FA106300003Q	Nonfinancial Corporate Business; Consumption of Fixed Capital, Structures, Equipment, and Intellectual Property Products, Including Equity REIT Residential Structures (NIPA Basis), Transactions
Consumption of fixed capital	Noncorporate	NNBCCFQ027S	Nonfinancial Noncorporate Business; Consumption of Fixed Capital, Structures, Equipment, and Intellectual Property Products, Current Cost Basis, Transactions
Net taxes on production and imports	Corporate	NCBPISQ027S	Nonfinancial Corporate Business; Taxes on Production and Imports Less Subsidies, Payable, Transactions
Net taxes on production and imports	Noncorporate	NNBTPIQ027S	Nonfinancial Noncorporate Business; Taxes on Production and Imports Less Subsidies, Payable, Transactions
Compensation of employees	Corporate	NCBCEPQ027S	Nonfinancial Corporate Business; Compensation of Employees Paid, Transactions
Compensation of employees	Noncorporate	NNBCEPQ027S	Nonfinancial Noncorporate Business; Compensation of Employees Paid, Transactions
Debt securities	Corporate	NCBDBIQ027S	Nonfinancial Corporate Business; Debt Securities; Liability, Level
Loans	Corporate	NCBLILQ027S	Nonfinancial Corporate Business; Loans Including Foreign Direct Investment Intercompany Debt; Liability, Level
Loans	Noncorporate	NNBTLBQ027S	Nonfinancial Noncorporate Business; Loans Including Foreign Direct Investment Intercompany Debt; Liability, Level
Time and saving deposits	Corporate	TSDABSNNCB	Nonfinancial Corporate Business; Total Time and Savings Deposits; Asset, Level
Time and saving deposits	Noncorporate	TSDABSNNB	Nonfinancial Noncorporate Business; Total Time and Savings Deposits; Asset, Level
Employment - non agri private sector	Non agri	LNU02032189	Employment Level - Nonagriculture, Private Industries Wage and Salary Workers
Umployment - non agri private sector	Non agri	LNU03032229	Unemployment Level - Nonagriculture, Private Wage and Salary Workers
Employment - total private sector	Tot private sector	USPRIV	All Employees, Total Private
GDP deflator	All economy	GDPDEF	Gross Domestic Product: Implicit Price Deflator
10-year real interest rate	All economy	REAINTRATREARAT10Y	10-Year Real Interest Rate

Appendix A.2. Detailled data sources - Yearly data series

Variable	**Sector**	**FRED code**	**Complete series name**
Gross fixed capital formation	Corporate	NCBGCFQ027S	Nonfinancial Corporate Business; Gross Fixed Capital Formation with Equity REIT Residential Structures
Gross fixed capital formation	Noncorporate	NNBGFNQ027S	Nonfinancial Noncorporate Business; Gross Fixed Investment (IMA), Transactions
Dividends paid	Corporate	NCBDPAA027N	Nonfinancial Corporate Business; Dividends Paid, Transactions
Withdrawals from Income of Quasi-Corporations	Noncorporate	NNBICPA027N	Nonfinancial Noncorporate Business; Withdrawals from Income of Quasi-Corporations, Paid, Transactions
Dividends received	Corporate	NCBDREA027N	Nonfinancial Corporate Business; Dividends Received, Transactions
Short-term real interest rate (AMECO)	Entire economy	NA	Real short-term interest rates, deflator GDP (ISRV)

Appendix B. Labor force and productivity series



(a) Labor force (b) Labor productivity

Figure B.5: Empirical labor force and productivity - 1959:Q1-2019:Q4

Appendix C. DDAGKM derivation and equilibria

Appendix C.1. Model derivation

We rely on the approach and notations of Grasselli and Costa Lima (2012) to present the debt-augmented Goodwin model originally proposed in (Keen, 1995). As in the original Goodwin model, price dynamics are not accounted for. All variables are therefore in real terms in the sequel.

Considering a Leontief production function and assuming full capital utilization, the total real yearly output of the economy can be written as:

$$Y(t) = \frac{K(t)}{\nu} = a(t)L(t), \tag{C.1}$$

where K is the stock of capital, ν the capital-to-output ratio and L the number of employed workers. The labor productivity a, corresponding to the quantity produced by one worker each year, is assumed to grow at a constant rate α:

$$a(t) = a_0 e^{\alpha t}. \tag{C.2}$$

Likewise, the total labor force N grows at a constant rate β.

$$N(t) = N_0 e^{\beta t}. \tag{C.3}$$

The employment rate λ can then be defined as:

$$\lambda(t) = \frac{L(t)}{N(t)}.$$

Two central behavioral assumptions are made in the model. First, as in the Goodwin model, the evolution of the real wage per unit of labor $w(t)$ is governed by a real short-run Phillips curve:

$$\dot{w} = w\phi(\lambda), \tag{C.4}$$

where ϕ is an increasing function. This equation reflects the hypothesis that a higher employment rate facilitates the wage bargaining for employees and leads to increased wages. The second behavioral assumption relates to the aggregate investments. Unlike in the Goodwin model where firms invest the exact value of their profits, firms can take on debt to invest more than their profits in the Goodwin-keen model. Let $D(t)$ the stock of real debt. The net profits of firms are:

$$(1 - \omega - rd)Y,$$

where $\omega = \frac{wL}{Y}$ is the wage share, $d = \frac{D}{Y}$ the ratio of real debt over the output and r a constant real interest rate. The rate of new investments is then defined as an increasing function κ of the profits and the real capital stock evolves as follows:

$$\dot{K} = \kappa(1 - \omega - rd)Y - \delta K, \tag{C.5}$$

with δ a constant depreciation rate. While the only financial cost reflected in the original Goodwin-Keen model relates to borrowings, we postulate that firms distribute a fixed proportion Δ of their profits as dividends to their shareholders:

$$\Pi^D = \Delta(1 - \omega - rd)Y.$$

Following Bovari et al. (2018a), firms can take on debt to finance dividends. Therefore, the change in real debt corresponds to the difference between real investments and dividends on the one hand and profits on the other hand:

$$\dot{D} = \kappa(1-\omega-rd)Y - (1-\omega-rd)Y + \Pi^D. \tag{C.6}$$

Using equation C.1 and C.5, the real production growth is:

$$\frac{\dot{Y}}{Y} = \frac{\kappa(1-\omega-rd)Y - \delta K}{\nu Y} = \frac{\kappa(1-\omega-rd)}{.}\nu - \delta \tag{C.7}$$

Noticing that $\omega = \frac{wL}{Y} = \frac{w}{a}$ thanks to (C.1) and using (C.4) and (C.2), the evolution of the wage share is:

$$\frac{\dot{\omega}}{\omega} = \frac{\dot{w}}{w} - \frac{\dot{a}}{a} = \phi(\lambda) - \alpha.$$

Relying on (C.7), (C.2) and (C.3), the evolution of the employment rate is derived as follows:

$$\frac{\dot{\lambda}}{\lambda} = \frac{\dot{L}}{L} - \frac{\dot{N}}{N} = \frac{\dot{Y}}{Y} - \frac{\dot{a}}{a} - \frac{\dot{N}}{N} = \frac{\kappa(1-\omega-rd)}{\nu} - \delta - \alpha - \beta,$$

while the evolution of the debt ratio is derived using equations (C.6) and (C.7):

$$\begin{aligned}\frac{\dot{d}}{d} &= \frac{\dot{D}}{D} - \frac{\dot{Y}}{Y} \\ &= \frac{\kappa(1-\omega-rd) - (1-\omega)(1-\Delta)}{d} - \frac{\kappa(1-\omega-rd)}{\nu} + \delta + r(1-\Delta).\end{aligned}$$

Gathering the equations for ω, λ and d finally leads to the equation system (4):

$$\begin{cases} \dot{\omega} = \omega\ [\phi(\lambda) - \alpha] \\ \dot{\lambda} = \lambda\ [\frac{\kappa(1-\omega-rd)}{\nu} - \alpha - \beta - \delta)] \\ \dot{d} = d\ [r(1-\Delta) - \frac{\kappa(1-\omega-rd)}{\nu} + \delta] + \kappa(1-\omega-rd) - (1-\omega)(1-\Delta). \end{cases}$$

Appendix C.2. Equilibria analysis

Four equilibria have been identified for the original GKM (Grasselli and Costa Lima, 2012). We analyze them in the DDAGM context. Note that the existence of such equilibria depends on the functional form chosen for ϕ and κ.

"Good" equilibrium

Defining $\overline{\pi_g} = \kappa^{-1}(\nu(\alpha+\beta+\delta))$, the "good" equilibrium is characterized by the following state variable values:

$$\begin{aligned}
\overline{\omega_g} &= 1 - \overline{\pi_g} - r\frac{\nu\left(\alpha+\beta+\delta\right) - \overline{\pi_g}\left(1-\Delta\right)}{\alpha+\beta} \\
\overline{\lambda_g} &= \phi^{-1}(\alpha) \\
\overline{d_g} &= \frac{\nu\left(\alpha+\beta+\delta\right) - \overline{\pi_g}\left(1-\Delta\right)}{\alpha+\beta}.
\end{aligned}$$

When evaluated at $(\overline{\omega_g}, \overline{\lambda_g}, \overline{d_g})$, the Jacobian matrix associated to system (4) equals:

$$J^1 = \begin{pmatrix} 0 & P_0 & 0 \\ -P_1 & 0 & -rP_1 \\ P_2 & 0 & rP_2 - (\alpha+\beta) \end{pmatrix},$$

with:

$$\begin{aligned}
P_0 &= \overline{\omega_g}\phi'(\overline{\lambda_1}), \\
P_1 &= \frac{\overline{\lambda_g}\kappa'(\overline{\pi_g})}{\nu}, \\
P_2 &= \frac{\kappa'(\overline{\pi_g})\left(\overline{d_g} - \nu\right) + \nu(1-\Delta)}{\nu}.
\end{aligned}$$

The characteristic polynomial then equals:

$$X^3 + \left[(\alpha+\beta) - rP_2\right] X^2 + P_0P_1X + P_0P_1(\alpha+\beta).$$

From Routh-Hurwitz criteria and accounting for the fact that both P_0 and P_1 are greater than 0, the DDAGM is locally stable at $(\overline{w_g}, \overline{\lambda_g}, \overline{d_g})$ if and only if $\left[(\alpha+\beta) - rP_2\right] > 0$ and $\left[(\alpha+\beta) - rP_2\right] P_0P_1 > P_0P_1(\alpha+\beta)$. As α, β are positive, these conditions simplify to:

$$rP_2 < 0 \Longleftrightarrow r\frac{\kappa'(\overline{\pi_g})\left(\overline{d_g} - \nu\right) + \nu(1-\Delta)}{\nu} < 0.$$

"Bad" equilibrium

When defining $u_t := \frac{1}{d_t}$, system (4) becomes:

$$\begin{cases} \dot{\omega}_t &= \omega_t\left[\phi(\lambda_t) - \alpha\right] \\ \dot{\lambda}_t &= \lambda_t\left[\frac{\kappa(1-\omega_t-\frac{r}{u_t})}{\nu} - \alpha - \beta - \delta\right] \\ \dot{u}_t &= u_t\left[\frac{\kappa(1-\omega_t-\frac{r}{u_t})}{\nu} - r(1-\Delta) - \delta\right] + \\ & \quad -u_t^2\left[\kappa(1-\omega_t-\frac{r}{u_t}) - (1-\omega_t)(1-\Delta)\right]. \end{cases}$$

In this setting, the "bad" equilibrium $(\overline{\omega_b}, \overline{\lambda_b}, \overline{d_b}) = (0, 0, +\infty)$ is equivalent to $(\overline{\omega_b}, \overline{\lambda_b}, \overline{u_b}) = (0, 0, 0)$. The Jacobian of the modified system at this point is:

$$J^2 = \begin{pmatrix} \phi(0) - \alpha & 0 & 0 \\ 0 & \frac{k0-\nu(\alpha+\beta+\delta)}{\nu} & 0 \\ 0 & 0 & \frac{k0-\nu[r(1-\Delta)-\delta]}{\nu} \end{pmatrix}.$$

Conditions on ϕ and κ functions guaranty that $\phi(0)-\alpha$ and $\frac{k0-\nu(\alpha+\beta+\delta)}{\nu}$ are less than zero. Therefore, the equilibrium is locally stable if and only if the last eigenvalue is negative, namely if:

$$\frac{k0-\nu\left[r(1-\Delta)-\delta\right]}{\nu}<0 \Longleftrightarrow \frac{k0}{\nu}-\delta<r(1-\Delta). \tag{C.8}$$

“Meaningless” equilibrium 1

Vector $(\bar{\omega_f}, \bar{\lambda_f}, \bar{d_f}) = (0, 0, \bar{d_f})$ is an equilibrium for system (4), when $\bar{d_f}$ is a solution of:

$$d(r(1-\Delta)-\frac{\kappa(1-rd)}{\nu}+\delta)+\kappa(1-rd)-(1-\Delta)=0.$$

When evaluated at this point, the Jacobian of system (4) becomes:

$$J^f = \begin{pmatrix} \phi(0)-\alpha & 0 & 0 \\ 0 & \frac{\kappa(\pi_f)-\nu(\alpha+\beta+\delta)}{\nu} & 0 \\ \frac{(d_f-\nu)\kappa'(\pi_f)+\nu(1-\Delta)}{\nu} & 0 & \frac{\nu(r(1-\Delta)+\delta)-\kappa(\pi_f)+r(d_f-\nu)\kappa'(\pi_f)}{\nu} \end{pmatrix},$$

where $\pi_f = 1 - rd_f$. The Jacobian being lower triangular, the equilibrium is locally stable if the following conditions are met:

$$\begin{aligned} 0 &> \phi(0)-\alpha, \\ 0 &> \frac{\kappa(\pi_f)-\nu(\alpha+\beta+\delta)}{\nu}, \\ 0 &> \frac{\nu(r(1-\Delta)+\delta)-\kappa(\pi_f)+r(d_f-\nu)\kappa'(\pi_f)}{\nu}. \end{aligned}$$

“Meaningless” equilibrium 2 (“slavery” equilibrium):

When defining $d_s := \frac{\nu(\alpha+\beta+\delta)-\overline{\pi}_s(1-\Delta)}{\alpha+\beta}$ and $\overline{\pi}_s := 1-rd = \kappa^{-1}(\nu(\alpha+\beta+\delta))$, $(0, \lambda_s, d_s)$ is an equilibrium of system (4). Setting $\omega = 0$, the definition of $\overline{\pi}_s$

leads to $\dot{\lambda} = 0$, while the definition of d_s guaranties that $\dot{d} = 0$. Finally, the equilibrium exists if the following equality holds:

$$1 - r\frac{\nu(\alpha+\beta+\delta) - \kappa^{-1}(\nu(\alpha+\beta+\delta))(1-\Delta)}{\alpha+\beta} = \kappa^{-1}(\nu(\alpha+\beta+\delta)). \quad \text{(C.9)}$$

Therefore, this type of equilibrium is structurally unstable (Grasselli and Costa Lima, 2012). Hence, unlike for the other types of equilibria, the Jacobian of the system is not derived. For each set of estimated parameters, we simply ensure that the parameters do not meet equality (C.9).

Appendix D. Parameter constraints

Parameter	**Lower bound**	**Upper bound**
ν	2	6
r	-0.1	0.3
δ	0	0.5
Δ	0	0.9
γ	-1	1e-2/0.5
ρ	0	1
ξ	0	4
k1	0	1/1.5
k2	0	10/NA

Table D.4: Bounds for the estimated parameters (Non-linear/linear functional form)

Function	Linear	Non-linear
ϕ	$\gamma < \alpha$	$\gamma + \rho < \alpha$
κ	-	$k_0 < \nu(\delta + \alpha + \beta)$

Table D.5: Inequality constraints on the parameters the Phillips and investment curves