

Classical Political Economy and Secular Stagnation

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This Version: September 16, 2021

Abstract

This paper presents a model of secular stagnation, income and wealth distribution, and employment in the Classical Political Economy tradition. The model can be contrasted with established neoclassical accounts of secular stagnation (Piketty, 2014; Gordon, 2015). In these explanations, an exogenous reduction in the growth rate g —be that because of declining fertility or the exhaustion of path-breaking scientific discoveries—increases the difference with the endogenous rate of return to capital r . The capital-income ratio rises, and if the elasticity of substitution is higher than one, the wage share falls. Importantly, both Piketty and Gordon assume full employment at all times. In our explanation, which does not presuppose full employment, the key tension is between profit-driven capital accumulation and wage-driven labor-augmenting technical change: both these features are defining for Classical Political Economy and have been emphasized in recent heterodox macro literature. Institutional or technological shocks to income distribution that lower the wage share initially foster capital accumulation—which is profit-driven—and increase wealth inequality. However, the effect on long-run growth is negative, because a reduction in the wage share lessens the incentives by firms to introduce labor-saving innovation, which is wage-driven. The capital/income ratio must rise in order to restore balanced growth and

stabilize the labor market in the long run; and the increase in wealth inequality is permanent. The ultimate effect on long-run employment depends on the relative strength of the response of labor-augmenting technical change vis-a-vis the response of real wage growth to labor market institutions: we identify a simple condition that delivers either a *wage-led* long-run employment regime or a *profit-led* long-run employment regime. We then test the model using time-series data for the US (1990-2019): the empirical analysis offers support to the main predictions of our model, and to the employment-population ratio being wage-led.

JEL Classifications: D31, D33, E11, E24, E25.

Keywords: Secular Stagnation, Factor Shares, Wealth Inequality, Employment.

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1 Introduction

The publication of Thomas Piketty’s *Capital in the XXI Century* (Piketty, 2014) revived the interest of the economics profession in questions of distribution of income and wealth. The combination of breath-taking data work on the historical increase in the capital-income ratio and the top wealth share—the latter occurring after the 1980s—and the use of the familiar Solow (1956) growth model to provide a comprehensive understanding of rising inequality and stagnation made a lasting mark in the profession. A complementary argument is made in Gordon (2015), who used basically the same modeling framework but emphasized forces at play that may have contributed to the growth slowdown, i.e., a reduction in g .

The combined neoclassical argument is now part of the standard economics toolbox: an exogenous reduction in the growth rate of the economy, be that because of declining fertility or the exhaustion of path-breaking scientific discoveries, increases the difference between the rate of return to wealth and the growth rate of income $r - g$: the implication is that the capital-income ratio rises. Factor substitution along a neoclassical production function provides a link from the capital-income ratio to the functional distribution of income: in particular, provided that the elasticity of substitution between capital and labor is higher than one, a rise in the capital-income ratio determines an increase in the share of profits and consequently a reduction in the wage share. These “Piketty-Gordon” facts have been widely documented in the literature: see Petach and Tavani (2021) for a recent illustration and discussion.

The purpose of this paper is to present an alternative viewpoint that builds on decades of work and debates between heterodox economists. Both scholars working in the Classical Political Economy (CPE) tradition and in the post-Keynesian tradition have been concerned primarily with questions of distribution for a long time before Piketty’s blockbuster tome. Post-Keynesians especially have produced important work that challenges the causal account by neoclassical economists introducing the question of the distribution of wealth (Pasinetti, 1962) in established demand-driven growth models in the neo-Kaleckian tradition (Taylor et al., 2020; Ederer and Rehm, 2020a; b). The key issue in these contributions is the relationship between the distribution of wealth and distributional features of aggregate demand.

Our goal is to present a complementary argument that, without denying the importance of aggregate demand for growth and distribution, operates at the same level of abstraction of Piketty and Gordon, namely that the economy is constrained by supply forces and profit-driven accumulation in the long run. This is a first reason why our approach is grounded in the CPE tradition: the assumption of Say’s law holding in the long run and the notion that capital accumulation is ultimately constrained by profits are common in Ricardo and Smith, and in contemporary work in the Marxian tradition (see for example Dumenil and Levy, 1999). Differently from neoclassical economics, we take seriously the Cambridge critique of capital theory that refuted the notion of instantaneous factor substitution along an aggregate production function (Harcourt, 2003; Felipe and McCombie, 2015) and focus instead of the Classical viewpoint that “capital-labor substitution” is in fact *biased technological change* (Foley et al., 2019, Ch. 6-8) that is driven by the firm-level incentives to introduce labor-augmenting innovations to respond to increases in the wage share (Hicks, 1932; Kennedy, 1964; Drandakis and Phelps, 1965; Foley, 2003; Zamparelli, 2015). This is another reason why our contribution is rooted in CPE: the notion that labor-augmenting technical change is a “weapon” in the capital-labor conflict, and that therefore the distribution of income between wages and profits influences and responds to technological progress is already present in Marx (citation needed: Capital III) but features prominently in more recent work such as Shah and Desai (1981) and Julius (2005). Finally, and differently from neoclassical economics and despite our acceptance of Say’s law, we do not presuppose that the economy always operates at full employment in the long run. This is another element that our argument has in common with CPE where, unlike the Solow-Piketty-Gordon framework, there is no mechanism guaranteeing that wage flexibility will clear the labor market in the long run.

Our argument goes as follows. Consider the long-run balanced growth condition $g = \gamma + n$ where g is the accumulation rate and γ, n the growth rate of labor productivity and the growth rate of the labor force. The accumulation rate in the long run depends on the profit rate r , equal to the income-capital ratio u times the profit share π . Institutional changes that occurred during the so-called Neoliberal Era (globalization, declining unionization, financialization) have put

downward pression on the labor share and upward pressure on the profit share.³ This lessened the incentives on behalf of firms to introduce labor-augmenting innovations, which in turn depressed the long-run growth rate of the economy. Moreover, it increased the wealth share of households whose incomes are mostly made up of profits (“capitalists”), given the reduction in the funds available to “workers” to save and accumulate wealth. On the other hand, the decline in the share of wages puts pressure on capital accumulation, which is profit-driven: but the long-run growth rate, which is tied up to labor productivity growth, has fallen. Restoring balanced growth requires a decline in the income-capital ratio u (or equivalently an increase in the capital-income ratio). The final portion of the argument concerns the economy’s long run employment rate. The balanced growth condition guarantees that the economy operates with constant unemployment in the long run, but the forces at play both in the labor market and the bias of technological progress produce an ambiguous response of long-run employment to changes in income shares. In our theoretical model, and with a nod to the familiar terminology in Kaleckian economics, we identify both the possibility of a wage-led and a profit-led employment regime, depending on a simple condition on two parameters representing the response of technological change vs. real wage growth to labor market institutions.

Our theoretical argument is thus that the distribution of income comes first, and not last, in the causal links between the forces producing stagnation and inequality in recent decades. Given the ambiguity in the response of the long-run employment rate to labor-crushing Neoliberal institutions, we finally present an empirical test of our argument using time-series data for the United States (1990-2019). First, we estimate a Vector-Error Correction (VEC) model using the income-capital ratio, the top 1% wealth share, the wage share and the employment/population ratio to account for endogeneity of all four variables of interest: and then we use impulse responses—both with the VEC specification and the Jordà (2005) local projection method—to visually inspect the response of the income-capital ratio, the top wealth share, and the employment rate to a one standard deviation shock to the wage share. We find empirical support for our theory: a positive (negative) shock to the wage

³ This is another point of contact with CPE, where income distribution is institutionally driven.

share produces a long-run increase (decrease) in the income-capital ratio, and a long-run increase (decrease) in the top wealth share. Moreover, the long-run response of the employment rate points to the corresponding US employment regime being wage-led over the period under consideration.

Summing up, the combination of the theoretical argument and the empirical test offers an explanation of secular stagnation and inequality according to which labor-crushing institutions over the last few decades have produced a decline in the growth rate, an increase in wealth inequality and an increase in the capital-income ratio. That the US employment-population ratio empirically appears to be wage-led in the long run points to the absence of tradeoffs between labor-friendly institutional changes and long-run employment.

The paper is organized as follows. Section 2 provides a brief literature review on the different theories about secular stagnation, both mainstream and heterodox. Section 3-5 outline the model and its comparative statics. Section 6 shows numerical simulations that qualitatively illustrate the transitional dynamics. Section 7 provides a simple tax policy exercise. Section 8 provides a time-series test of the model for the US. Section 9 extends the model by adding a Kaldor-Verdoorn term to reflect the positive effect of capital accumulation on labor productivity growth. Section 10 concludes.

2 Related Literature: Secular Stagnation

The observation that an economy may experience low economic growth and high unemployment for long periods is well-routed in the history of economics. Alvin Hansen (Hansen, 1939) coined the concept of secular stagnation to express his preoccupation with grim U.S. growth prospects and slow recovery after the Great Depression due to, among other reasons, a shortage of impetus of opportunities and new investments. Since then, the world has changed significantly. Changes in demography, financial development, changes in income distribution between labor income and capital income, and technology have played a crucial role in the composition of employment, real wages, and productivity, impacting economic growth and unemployment.

Several theories have emerged to explain the possibility of stagnation from different viewpoints. The literature is vast, but for the purpose of this analysis we will focus on demand-side and supply-side secular stagnation from a mainstream standpoint, and the countering post-Keynesian perspective. Beginning with the demand-side mainstream explanation, Summers (2014a,b) has revamped the term secular stagnation to explain the growth slowdown in advanced economies such as the United States, Europe, and Japan during the last three decades. Summer’s account is rooted in the theory of loanable funds, and amounts to a situation where demand and supply for savings translate into a negative equilibrium real interest rate. Under this scenario, the zero lower bound on the nominal interest rate is the relevant constraint on the policy that contributes to economic stagnation. According to Summers (2015), the low equilibrium occurs due to decreased investment demand and increased supply of savings. He argues that the former is explained by slow population growth in the more developed countries, a decline in the relative price of capital goods, and the problem of cutting-edge technology companies dealing with their excess cash. The latter is explained, according to Summers, by large reserves accumulated in developing countries, increase in propensity to save due to higher inequality, more rigorous collateral requirements due to financial crisis, and the increased costs of financial intermediation. Therefore, Summers (2015) alludes to an “inverse Say’s Law,” where lack of demand leads to a lack of supply.

While Summers concludes that secular stagnation occurs when the desired level of savings exceeds the level of investment and monetary policy is constrained by the zero lower bound, both Gordon (2015) and Pagano and Sbracia (2018) discuss that secular stagnation can also be explained through the supply side. These authors focus more on potential real GDP growth, labor productivity growth, and aggregate hours of work as significant variables to explain secular stagnation. Gordon (2015) argues that slow productivity growth in the past decade is due to three main reasons: (a) the business methods and installed capacity in the “dot.com” era characterized by meaningful productivity growth have faced diminishing returns; (b) the decline in net investment ratio to capital stock and the decrease in the information and communication technologies price deflator; and (c) the fall in the rate of new business start-ups. The Gordon account of secular stagnation is complementary to that presented by Piketty (2014), where an

exogenous reduction in the growth rate g is responsible for the increase in the capital-income ratio and the falling wage share through high elasticity of substitution between capital and labor. Another supply-side argument hinges on the role played by an aging population (Hansen, 1939; Gordon, 2015, 2016). The idea is that an aging population reduces the size of labor force and productivity, and generates higher savings relative to investment. A contrary perspective can be found in Acemoglu and Restrepo (2017), who not only find no evidence of a negative association between changes in age structure and GDP per capita changes but a positive and robust relationship in some econometric specifications. They show that countries with higher shares of aging populations are the ones that adopted more industrial robots. However, they recognize that they cannot establish causality between these two variables nor that the adoption of robots is necessarily the channel that offsets the potential adverse effect of population aging on economic growth. Therefore, these authors suggest that one possible explanation for the positive relationship mentioned above is that “technology adjusts so as to undo this potential negative effect.” Indeed, Acemoglu (2010) shows that labor scarcity leads to the adoption of automation processes that increase aggregate output when technology is strongly labor-saving but discourages technological advances when technology is strongly labor complementary.

Heterodox perspectives on the issue have questioned both the notion of a “natural” interest rate that equalizes investment and savings in the loanable funds market at full employment levels (Wicksell, 1898; Keynes, 1930), as well as the money neutrality proposition. In this vein, Palley (2018, 2019) presents the *investment saturation* hypothesis as a critique of the zero lower bound economics. Palley asserts that negative nominal interest rates, even if feasible, might be unable to resolve the problem of unemployment due to demand shortages. If a negative nominal interest rate does not consistently achieve full employment, the zero lower bound will not cause Keynesian unemployment and stagnation. Palley’s main point is that investment could become unresponsive to a lower interest rate if the returns on non-reproduced assets—fiat money, land, minerals, precious metals, rent streams from firms, and intellectual property—dominate the return to investment. The reason is that lower interest rates may result in bidding up the price of non-reproduced assets rather than increasing investment. In other words, since non-reproduced assets compete with investment projects, the interest

rate is not necessarily set by the demand and supply forces in a loanable funds market but by the Keynesian liquidity preference. Thus, the link between the interest rate and the savings-investment equilibrium could be broken.

A different branch of heterodox explanations draws from the Kaleckian and Steindlian tradition, according to which the rise of *financialization* is a significant factor in depressing the economy (Hein, 2013; Hein & Dodig, 2014; Hein, 2019). The first main channel found empirically in this literature is the decline in the labor share of income, and especially that of low-income households which directly affects overall consumption. The second one is the bias to favor short-run profits in the non-financial sector (see also Davis 2016; 2017; 2018) that incentivizes financial investment at the expenses of real investments. Hein rejects the idea of a single interest downward-sloping capital demand curve that, together with a supply of loanable funds curve in a more-than-one-good economy, clears the capital market at full employment level through a natural interest rate. Additionally, he argues that savings adjust to investment through income growth and changes in capacity utilization. This author also emphasizes the importance of social classes and institutions' role in understanding the secular stagnation phenomenon. In this latter respect, our perspective is similar.

The Harrodian approach has also been used to analyze the factors behind secular stagnation from a post-Keynesian viewpoint. Skott (2016) argues that public debt is not a problem in itself; however, it can be a significant burden for the economy when it reaches excessive levels. In that sense, one of the main of Skott's contributions is to show a causal relation running from low economic growth to high public debt. Hence, fiscal policy must play an essential role in making the economy tend to full employment and avoid secular stagnation. Also, Skott supports the importance of more equitable income distribution since the higher the degree of inequality, the more elevated the public debt ratio.

Our contribution draws from recent developments in the Classical Political Economy tradition to offer an account of secular stagnation that, like Piketty and Gordon, emphasizes the role played by real forces. This is of course not to deny the importance of financial factors: our goal is to present a perspective on the problem that operates at the same level of abstraction of the neoclassical explanations but is complementary to the post-Keynesian view.

3 Model: Setup

We consider a one-sector closed economy without government. Time is continuous, and the total population is assumed to be constant and normalized to one for simplicity. We also assume away capital depreciation, as well as household debt, to rule out unnecessary complications. Finally, given that the model is one-sector, we normalize the price of the single good produced in the economy to one throughout. Without denying the importance of inflation in macroeconomics, this modeling choice is meant to focus on real forces only.

3.1 Production, Income Distribution, and Wealth Accumulation

The economy is populated by two types of households. ‘Workers’ (denoted by the superscript w in what follows) supply labor services inelastically to firms, earn both labor and capital income, consume and save. ‘Capitalists’ (denoted by the superscript c) own capital stock, earn only profit incomes, consume and save. For the sake of simplicity, assume that neither type of capital depreciates. Output per worker y , homogeneous with capital stock, is produced using fixed proportions of capital per-worker $k \equiv k^c + k^w$ and labor: $y = \min\{A, uk\}$ where u denotes the output-capital ratio, endogenous to the model, and A is the stock of labor-augmenting technology, also endogenously growing over time. Let r be the uniform rate of return on capital, endogenous to the model but given to each household: both types of households are price-taking in goods and factor markets.

Through their savings, workers participate in the accumulation of capital in the economy. Let their propensity to save, constant throughout, be denoted by $s^w \in (0, 1)$. Importantly for the analysis, not all workers will be employed at any given point in time: the number of employed workers is equal to labor demand by firms which, due to the Leontief production technology, is not elastic to the wage. This implies that the labor market does not clear. The employment rate in the economy is $e \equiv \frac{uk}{A}$ (recall that the labor force is normalized to one). Therefore, total workers' saving, in turn equal to workers' investment in new capital stock k^w , is:

$$\dot{k}^w = s^w \left[\frac{w}{A} u k + r k^w \right] \quad (1)$$

Next, denote the capitalists' share of wealth by $\phi \equiv \frac{k^c}{(k^c + k^w)} \in [0, 1]$ so that $k^c = \phi k$. Letting the labor share (endogenous in the model) be denoted by $\omega \equiv w/A$, simple algebra delivers the workers' accumulation rate as:

$$g^w \equiv \frac{\dot{k}^w}{k^w} = s^w u \left[\frac{\omega}{1 - \phi} + (1 - \omega) \right] \quad (2)$$

On the other hand, capitalist households only earn profit income out of the capital they own. With a constant propensity to save $s^c \in (0, 1)$, the capitalists' accumulation rate is:

$$g^c = s^c r = s^c u (1 - \omega) \quad (3)$$

Using equations (2) and (3), the economy-wide accumulation rate will be a weighted average of the growth rates of capital stock of the two types of households, the weight being the share of wealth owned by each class:

$$\begin{aligned} g &= \phi g^c + (1 - \phi) g^w \\ &= u [s^w + \phi(1 - \omega)(s^c - s^w)] \end{aligned} \quad (4)$$

Equation (4) emphasizes the profit-driven nature of capital accumulation, even with worker saving. Indeed, the economy's accumulation rate decreases in the wage share, and increases in the profit share everything else equal.

3.2 Technical Change: Induced Invention

We turn now to specify the evolution of technology through induced invention. Following Kennedy (1964); Drandakis and Phelps (1965); Funk (2002); Julius (2005), we suppose that firms have access to a menu of technological improvements

that potentially can increase both the output-capital ratio (at a rate χ) and labor productivity (at a rate γ). However, there are trade-offs between improving along one technological dimension versus the other. Such trade-offs are summarized by a twice-continuously differentiable, strictly decreasing, strictly concave *invention possibility frontier* (Kennedy, 1964, IPF henceforth) which can be written in explicit form as:

$$\gamma = f(\chi), f' < 0, f'' < 0 \tag{5}$$

Firms choose a profile of technological improvements to maximize the rate of reduction in unit costs, or equivalently the rate of change in the profit rate, subject to the constraint given by the IPF (see Julius, 2005). The solution yields a dependence on the direction of technical change— i.e., the relative growth rates of capital- and labor-augmenting technologies— on factor shares, and in particular a direct (inverse) relation between labor (capital) productivity growth and the labor share. We also assume an exogenous shift parameter for the IPF, denoted by z , which could be interpreted in standard fashion as either as the exogenous 'natural' growth rate or, following Petach and Tavani (2020) as any institutional variable positively affecting the labor share in the long-run. Thus, the growth rates of capital -and labor-augmenting technologies can be written as:

$$\chi = \chi(\omega; z); \gamma = f[\chi(\omega; z)] \tag{6}$$

with $\chi_\omega < 0$ —and correspondingly $\gamma_\omega > 0$. Observe that induced invention in this framework plays a similar role to what would be factor substitution with a neoclassical aggregate production function: higher labor costs induce more labor-saving and less capital-saving technologies. However, and as noted already in the literature, the process of capital-labor substitution happens through technological progress and not diminishing marginal products. Importantly, the induced

invention hypothesis does not suffer from the well-known issues with aggregating across different capital goods highlighted in the Cambridge controversy.

In what follows, we assume that an increase in z shifts the invention possibility frontier up everything else equal. Thus, $\chi_z > 0$, $\gamma_z \geq 0$. Thus, the evolution of the income-capital ratio is governed by induced invention, and satisfies:

$$\dot{u} = \chi(\omega; z)u \tag{7}$$

To sharpen our conclusions, we specify linear versions of both growth rates of factor-augmenting technologies that generalize the specifications presented in Petach and Tavani (2020):

$$\chi(\omega; z) = z - \beta\omega; \quad \gamma(\omega; z) = \alpha[z - \chi(\omega; z)] \tag{8}$$

The ‘speed-of-adjustment’ parameter $\alpha > 0$ is crucially important in what follows, as it will act in increasing long-run employment in balanced growth in response to increases in the wage share.

3.3 Dynamics: Wealth Distribution

Consider next the capitalist share of wealth ϕ . Its law of motion over time obeys the replicator-style equation:

$$\dot{\phi} = \phi(g^c - g) \tag{9}$$

which, using (3) and (2), gives after simple manipulation:

$$\dot{\phi} = \phi u [(1 - \phi)(1 - \omega)(s^c - s^w) - s^w \omega] \tag{10}$$

3.4 Dynamics: Income Shares and Employment Rate

To close the model, we follow Goodwin (1967) in specifying the interaction between the labor market and real wages. We assume that real wages follow a

Phillips-style curve: $\dot{\omega}/\omega = f(e; z)$ with $f_e > 0, f_z \geq 0$. Given induced bias in technical change, the evolution of the labor share obeys:

$$\dot{\omega} = [f(e) - \gamma(\omega; z)]\omega \tag{11}$$

To characterize the steady-state and policy implications in what follows, we assume a linear version of the Phillips curve: $f(e; z) = -\lambda + \delta e + \mu z$, with $\lambda > 0, \delta > 0, \mu > 0$.

Finally, the evolution of the employment rate is obtained by differentiation of its very definition: from $\dot{e} = (\chi + g - \gamma)e$,

$$\dot{e} = \{\chi(\omega; z) + u[s^w + (1 - \omega)\phi(s^c - s^c)] - \gamma(\omega; z)\}e \tag{12}$$

Equations (7), (10), (11), and (12) form a 4-dimensional dynamical system describing the growth and distribution path of the economy. The endogenous variables are: (i) the income-capital ratio u ; (ii) the distribution of wealth between the two classes ϕ ; (iii) the functional distribution of income ω , and (iv) the employment rate e .

We now turn to characterize the steady-state of the model as well as the main long-run policy implications. Start from equation (7). Setting $\dot{u} = 0$ delivers the long-run labor share as

$$\omega_{ss} = \frac{z}{\beta} \tag{13}$$

increasing in the policy/institutional parameter z . In a graph with the employment rate e on the horizontal axis and the wage share ω on the vertical axis, the long-run labor share is a horizontal line at z/β : a positive shift in the labor market parameter z moves the long-run labor share up. Next, setting $\dot{\omega} = 0$ in equation (11) gives the employment nullcline

$$e(\omega; z) = \frac{\lambda + \alpha\beta\omega - \mu z}{\delta} \tag{14}$$

increasing in the labor share everything else equal. A change in the policy variable z shifts the employment rate nullcline in the opposite direction, since $de/dz < 0$. In the (e, ω) plane below, an increase in z shifts the employment nullcline up left. However, the ultimate effect on equilibrium employment depends on the relative magnitude of the response of income distribution to a policy change *vs.* the employment response. This is because both curves shift following a change in the policy variable z . Once the employment equation is evaluated at the steady-state value for the labor share, it pins down the long-run employment rate as

$$e(\omega; z) = \frac{\lambda + (\alpha - \mu)z}{\delta} \tag{15}$$

and the effect of the labor market parameter z on employment depends on the sign of the difference $\alpha - \mu$. If $\alpha > \mu$, a positive shock to the wage share will increase the long-run employment rate: we will refer to this case as a "wage-led" model in what follows. Conversely, if $\alpha < \mu$, the long-run employment rate responds negatively to shocks to the wage share: we will refer to this case as "profit-led" below. The economic intuition has to do with the relative magnitude of the response of induced bias as opposed to labor market conflict to changes in labor institutions. The parameter α captures how strongly the firm's choice of the direction of technical change responds to an increase in the wage share; since the growth rate of labor productivity anchors the long-run growth rate of the economy, higher values of α imply that growth becomes wage-led to a higher extent. The parameter μ , on the other hand, captures how strong is the effect of labor market institutions on labor market conflict, as described by the real-wage Phillips curve. An increase in μ creates more pressure on real wage growth, which depresses the profit-driven accumulation rate, and employment everything else equal. The relative magnitude of the two effects determines whether the accumulation response to labor market

institutions is stronger or weaker than the technical change response, and the ultimate effect of z on long-run employment.

Further, imposing $\dot{\phi} = 0$ in equation (10), we find the nullcline relating the wealth distribution to factor shares as

$$1 - \phi(\omega) = \frac{s^w}{s^c - s^w} \left(\frac{\omega}{1 - \omega} \right) \quad (16)$$

which captures that the workers' share of wealth increases in the labor share of income as it is intuitive. Substituting from (13) we find the long-run wealth distribution as the solution of

$$1 - \phi_{ss} = \frac{s^w}{s^c - s^w} \left(\frac{z}{\beta - z} \right) \quad (17)$$

which, given that $1 - \phi_{ss}$ is the worker's wealth share, implies that a positive (negative) shock to the labor share will increase (decrease) the workers' share of wealth. This is intuitive, as an increase in the labor share increases the funds available to workers to accumulate capital stock.

Finally, imposing a steady-state in equation (12) gives the following nullcline relating the long-run output/capital ratio to the wage share and capitalist wealth share as follows:

$$u(\phi; \omega) = \frac{\alpha\beta\omega}{s^w + (s^c - s^w)\phi(1 - \omega)} \quad (18)$$

Notice that the long-run income-capital ratio increases in the wage share and decreases in the capitalist share of wealth. To find the long-run solution, plug in equations (13) and (17). After a bit of algebra, we find:

$$u_{ss} = \frac{\alpha\beta z}{s^c (\beta - z)} \quad (19)$$

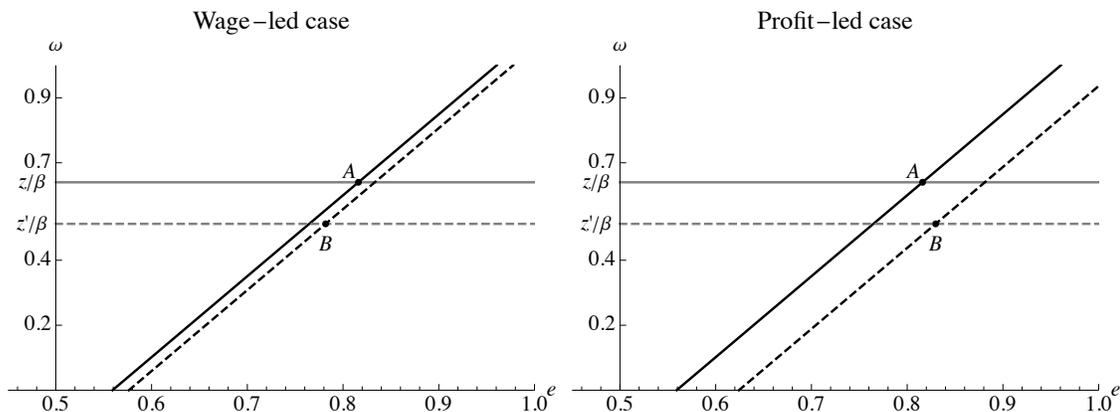
increasing in the policy variable z , while decreasing in the capitalist saving rate s^c .

4 Comparative Statics and Policy Implications

The main exogenous variables of interest in this model are the parameter describing labor market institutions z and the two classes' propensity to save, s^c and s^w . We study the comparative statics effect of each variable in turn.

Start with a change in the institutional parameter z : it will produce a change of the same sign in the long-run wage share (equation 13), workers' wealth share (equation 17), the long-run growth rate of labor productivity, and income-capital ratio (equation 19). As described above, the effect on employment is in principle ambiguous, and depends on whether the response of technical change to income distribution is stronger or weaker than the extent of labor market conflict on wage growth. Whenever $\alpha > \mu$, the long-run employment rate is *wage-led* and there is no trade-off between long-run productivity growth and employment: a labor-crushing shift in labor market institutions that lowers z reduces the long-run wage share, the growth rate of labor productivity, *and* employment. This scenario is displayed in the left panel of Figure 1. Conversely, if $\alpha < \mu$, long-run employment is *profit-led*: in this case, a capital-friendly shift in labor market institutions has a positive effect on long-run growth while a negative effect on long-run employment. This case is shown in the right panel of Figure 1.

Figure 1: The effect of a reduction in the labor market parameter z in the wage-led *vs.* profit-led steady state.



As argued in Petach and Tavani (2020), this simple model provides some interesting political economy insights on the secular stagnation and inequality that have plagued advanced economies, and especially the United States, in recent decades. A worsening of labor market institutions, which is responsible for the decline in the labor share, has also reduced the long-run growth rate of labor productivity because of the lessened incentives for firms to invest in labor-augmenting technologies. The decline in the labor share has also determined a reduction in the workers' wealth share because labor income—which is the main source of income for workers—has declined as a share of total income, thus lowering the funds available to workers for wealth accumulation. A worsening of labor market institutions has also the ultimate effect of reducing the income-capital ratio (increasing the capital-income ratio) for the following reason. The decline in the labor share puts pressure on the capitalists' accumulation rate: because of the Pasinetti theorem, the capitalist accumulation rate $s^c u(1 - \omega) = g^c$ is equal to the economy's growth rate g in balanced growth. However, the long-run growth rate of labor productivity is wage-led and has fallen. Restoring the balanced growth condition $g = \gamma(\omega)$ —which guarantees a constant employment rate in the long run—requires a decline in the long-run income-capital ratio. The ultimate effect on long-run employment can be either positive or negative, as described already and shown in Figure 1.

Next, consider the effect of the capitalist saving propensity s^c on the long run of the model. Given that the long-run wage share only depends only on labor market institutions, it will be unaffected by a change in the saving rate, and so will steady-state employment. Conversely, both the long-run capitalist share of wealth and the long-run income/capital ratio will be affected by a change in the capitalist saving rate.

Everything else equal, a higher capitalist propensity to save out of profits puts pressure on the capitalist accumulation rate, which increases their share (and reduces the workers' share) in total wealth. Given that the long-run wage share is unaffected by the change, labor productivity growth has not changed: thus, capital stock has grown more than income, which explains the reduction in the long-run income/capital ratio.

Finally, and perhaps strikingly, a change in the workers' saving rate only influences the long-run distribution of wealth while leaving the steady-state value of every other endogenous variable unaltered. This is not surprising in light of the Pasinetti theorem. Of course, there will be effects along the transitional dynamics: they are explored in the simulations below.

5 Transitional Dynamics: Numerical Simulations

Even though the model is stylized enough to be studied analytically, it is informative to carry a series of simulations exercises to showcase the transitional dynamics following a shock to the various parameters of interest, namely z , s^w , s^c .⁴ Importantly, the simulations are meant to be illustrative of the qualitative properties of the dynamics, and not to provide an accurate representation of an economy's response to a series of shocks. To calibrate the two classes' saving rates, we follow Saez and Zucman (2016) and set the capitalists' saving rate equal to 35% and the workers' saving rate at around 7.5%. We fix z at .025 and internally calibrate β at .039 to obtain a steady-state wage share of 64% in the baseline model. We then fix $\delta = .52$, which is the naïve point estimate of the slope of the Phillips curve for the United States one finds in intermediate macro textbooks such as Blanchard (2018), and $\alpha = .75$. In the wage-led model, μ is set at .25, and λ is internally calibrated to return a steady state prime- age employment/population ratio of about 80% —the pre-2008 value in the United States— in the baseline. In the profit-led model, $\mu = .95$ which requires to recalibrate λ to match the steady-

⁴ The *Mathematica* code used in these simulations rounds is available from the authors upon request.

state employment rate. All simulations assume that the economy is in steady state at time zero, when a shock to either parameter occurs.

In the first simulation, we reduce the labor market parameter z by 5% in both the wage-led model and the profit-led model. A visual representation of the simultaneous shifts in the wage share and employment nullclines (equations 13 and 14) corresponding to the two cases is already represented in Figure 1, which is obtained using the calibration described above. The actual transitional dynamics is displayed in Figure 2, where the shocked trajectories are displayed as solid lines while the baseline values are shown as dashed lines. The comparison illustrates an important implication of the model: income shares, the wealth distribution, and the income-capital ratio converge to the same values in both the profit-led and the wage-led model, while of course the trajectory and the ultimate value of the employment rate depends on whether its long run is wage-led or profit-led. The difference matters: in the wage-led case there is not trade-off between a labor-friendly change in income distribution and employment, while in the profit-led case such a trade-off does exist.

In the second set of simulations, displayed in Figure 3, we increase the capitalists' saving rate s^c by 5% at time zero (left panel); and the workers' saving rate by the same amount in the right panel. As already explained above, an increase in s^c reduces the income-capital ratio while it increases the long-run capitalist share of wealth: while an increase in s^w only affects the distribution of wealth (in favor of workers) in the long run.

Figure 2: Simulations: a 5% time-zero reduction in z .

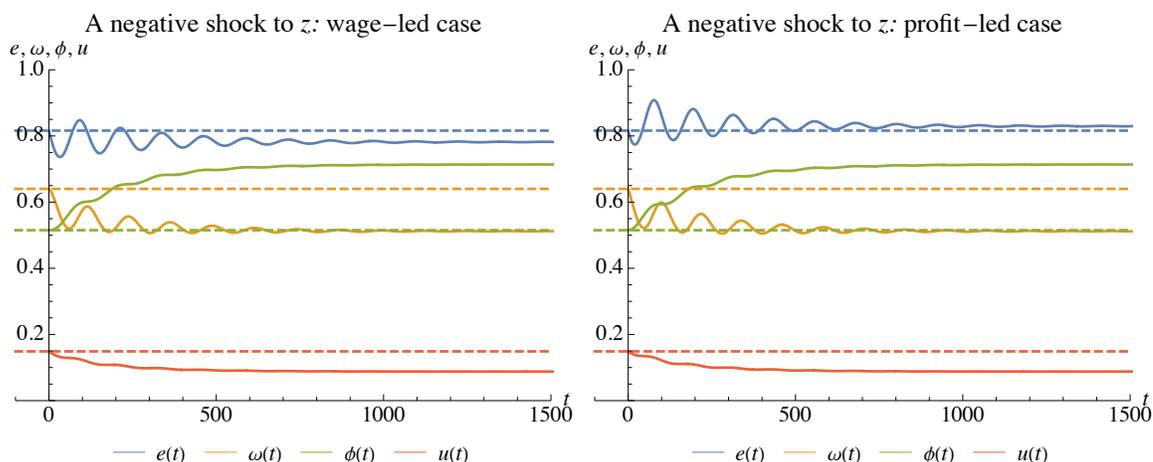
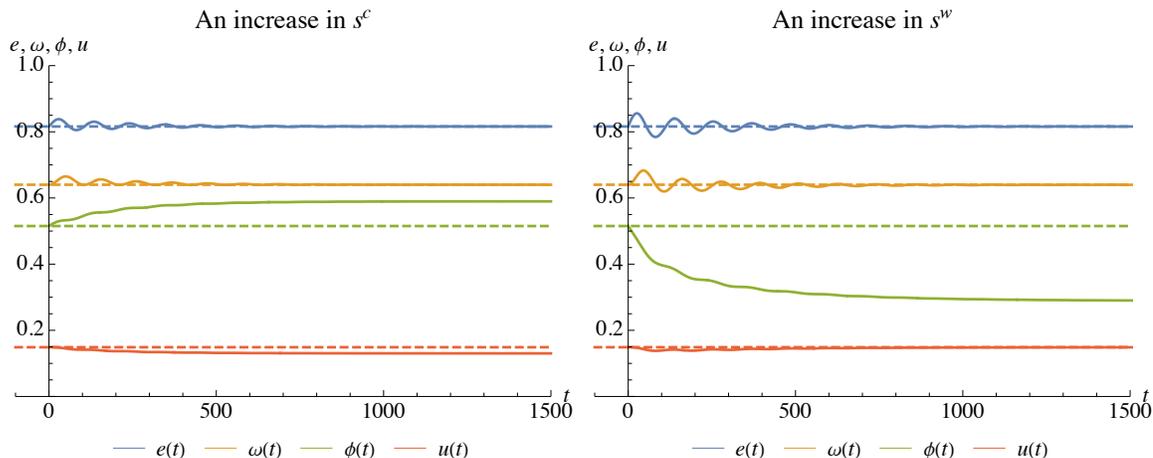


Figure 3: Simulations: a 5% time-zero increase in capitalists' and workers' saving rates.



6 Wealth Redistribution

Zamparelli (2017) has shown that, in a neoclassical economy with high elasticity of substitution between capital and labor, tax policy can be used in order to implement any wealth distribution among the two classes. The same is true here, despite the fixed-coefficients technology in production. Introduce a government that taxes capitalists' profit incomes proportionally at a rate τ and rebates the proceedings to workers in the form of subsidies. The capitalists' and workers' accumulation rates modify as follows:

$$g^c = s^c u (1 - \omega) (1 - \tau) \tag{20}$$

$$\begin{aligned} g^w &= s^w \frac{\frac{uwk}{A} + rk^w + \tau rk^c}{k^w} \\ &= s^w \frac{u}{1 - \phi} [1 - \phi(1 - \omega)(1 - \tau)] \end{aligned} \tag{21}$$

After factoring terms, the evolution of the capitalist wealth share modifies to:

$$\dot{\phi} = \phi u \{ s^c (1 - \omega) (1 - \phi) (1 - \tau) - s^w [1 - \phi(1 - \omega)(1 - \tau)] \} \tag{22}$$

Next, using the steady state wage share from equation (13), the two-class equilibrium delivers the following capitalist wealth share as a function of the tax rate:

$$\phi_{ss}(\tau) = \frac{s^c(\beta - z)(1 - \tau) - s^w\beta}{(\beta - z)(1 - \tau)(s^c - s^w)} \quad (23)$$

which reduces to (17) for $\tau = 0$ as in the baseline model. Now, the government can fix the tax rate to implement the desired distribution of wealth, given the two classes' saving propensities and the parameters determining the long-run share of labor. Differentiating with respect to τ , we find that, intuitively, the long-run capitalist wealth share decreases in the tax rate:

$$\frac{\partial \phi_{ss}}{\partial \tau} = -\frac{s^w\beta(\beta - z)(s^c - s^w)}{[(\beta - z)(1 - \tau)(s^c - s^w)]^2} < 0$$

Finally, the tax rate τ^* that implements the desired long-run wealth distribution ϕ^* is simply

$$\tau^* = 1 - \frac{s^w\beta}{\beta - z[s^c - \phi^*(s^c - s^w)]} \quad (24)$$

Petach and Tavani (2020) have provided some back-of-the-envelope calculations about various tax rates necessary to reduce wealth inequality in the United States. They have argued that both the effective corporate tax rate and the effective estate tax rate should be increased substantially in order to reduce the U.S. top wealth share to its value in 1978, the lowest since the 1920s.

7 Empirical Evidence for the United States

In addition to the numerical simulations, we performed an empirical exercise for the United States in the period 1990-2019, using the following data: the

income/capital ratio, the top 1% net personal wealth as the capitalist share of wealth, the employment-population ratio, and the labor share to form an endogenous, four-variable, time-series system.²

Appendix 1 shows that all the four variables are non-stationary in levels but stationary in first differences. Based on four out of five information criteria in Appendix 2, the optimal lag length for the vector autoregressive (VAR) model is two lags. We therefore subtract one lag from the optimal lag length for the VAR to run the Johansen cointegration test. Appendix 3 displays detailed trace and maximum eigenvalue tests for the Johansen cointegration test. We run this test allowing a linear deterministic trend, and an intercept in the cointegrating equation since it is the best specification according to the Akaike and Bayesian information criteria (not shown). The trace test confirms that one cointegrating equation exists at a 5% significance level, and the maximum eigenvalue test supports it at a 10% significance level.

Since this is a cointegrated endogenous system, we run the following vector error model (VECM) with one lag and one cointegrating equation:

$$\begin{aligned}\Delta u_t &= \sigma_{u0} + \sum_{j=1}^{k-1} \tau_{uj} \Delta u_{t-j} + \sum_{j=1}^{k-1} \zeta_{uj} \Delta \phi_{t-j} + \sum_{j=1}^{k-1} \eta_{uj} \Delta e_{t-j} + \sum_{j=1}^{k-1} \psi_{uj} \Delta \omega_{t-j} \\ &\quad + \xi_u ECT_u + v_t^u \\ \Delta \phi_t &= \sigma_{\phi 0} + \sum_{j=1}^{k-1} \tau_{\phi j} \Delta u_{t-j} + \sum_{j=1}^{k-1} \zeta_{\phi j} \Delta \phi_{t-j} + \sum_{j=1}^{k-1} \eta_{\phi j} \Delta e_{t-j} + \sum_{j=1}^{k-1} \psi_{\phi j} \Delta \omega_{t-j} \\ &\quad + \xi_{\phi} ECT_{\phi} + v_t^{\phi} \\ \Delta e_t &= \sigma_{e0} + \sum_{j=1}^{k-1} \tau_{ej} \Delta u_{t-j} + \sum_{j=1}^{k-1} \zeta_{ej} \Delta \phi_{t-j} + \sum_{j=1}^{k-1} \eta_{ej} \Delta e_{t-j} + \sum_{j=1}^{k-1} \psi_{ej} \Delta \omega_{t-j} \\ &\quad + \xi_e ECT_e + v_t^e \\ \Delta \omega_t &= \sigma_{\omega 0} + \sum_{j=1}^{k-1} \tau_{\omega j} \Delta u_{t-j} + \sum_{j=1}^{k-1} \zeta_{\omega j} \Delta \phi_{t-j} + \sum_{j=1}^{k-1} \eta_{\omega j} \Delta e_{t-j} + \sum_{j=1}^{k-1} \psi_{\omega j} \Delta \omega_{t-j} \\ &\quad + \xi_{\omega} ECT_{\omega} + v_t^{\omega}\end{aligned}$$

² The income/capital ratio is calculated dividing variable `cgdp` by `cn` from Penn World Table 10.0. Top 1% net personal wealth is obtained from the World Inequality Database. Employment-population ratio is taken from U.S. Bureau of Labor Statistics, series `EMRATIO` retrieved from Federal Reserve Economic Data (FRED). And labor share is obtained from series `labsh` from Penn World Table 10.0. All these four variables are in percentage points.

Where the notation is as follows:

Table 1: Notation for the VEC model.

| | |
|---------------|---|
| Δ | First-difference operator |
| σ_{i0} | Constant term of variable i |
| k | Optimal lag length for the VAR specification |
| ξ_i | Speed of adjustment parameter of variable i |
| ECT_i | Error correction term of variable i |
| v_t^i | Disturbance or error term of variable i |

Concerning the jointly long-run effects, the error correction terms (ECTs) for employment-population ratio and labor share are negative and statistically significant (See Table 2). The system returns to equilibrium at a speed of 31% and 24.1% annually when the employment-population ratio and labor share are the dependent variables, respectively. The ECTs for income/capital ratio and top 1% net personal wealth are statistically insignificant since their p -values are 0.21 and 0.76, respectively—not shown in the table—meaning that these two variables are weakly exogenous in the cointegration relationship.

Appendix 4 supports evidence of the absence of serial correlation, Appendix 5 shows that residuals are homoscedastic, and Appendix 6 displays three tests aimed at showing that residuals are multivariate normal. Appendix 7 demonstrates that the model is stable since the roots of the characteristic polynomial are in the unit circle. In Appendix 7, three unit-roots are expected because we have four endogenous variables ($n = 4$) and one cointegrating equation ($CE = 1$). The number of unit-roots imposed by the VECM specification is equal to $n - CE = 3$.

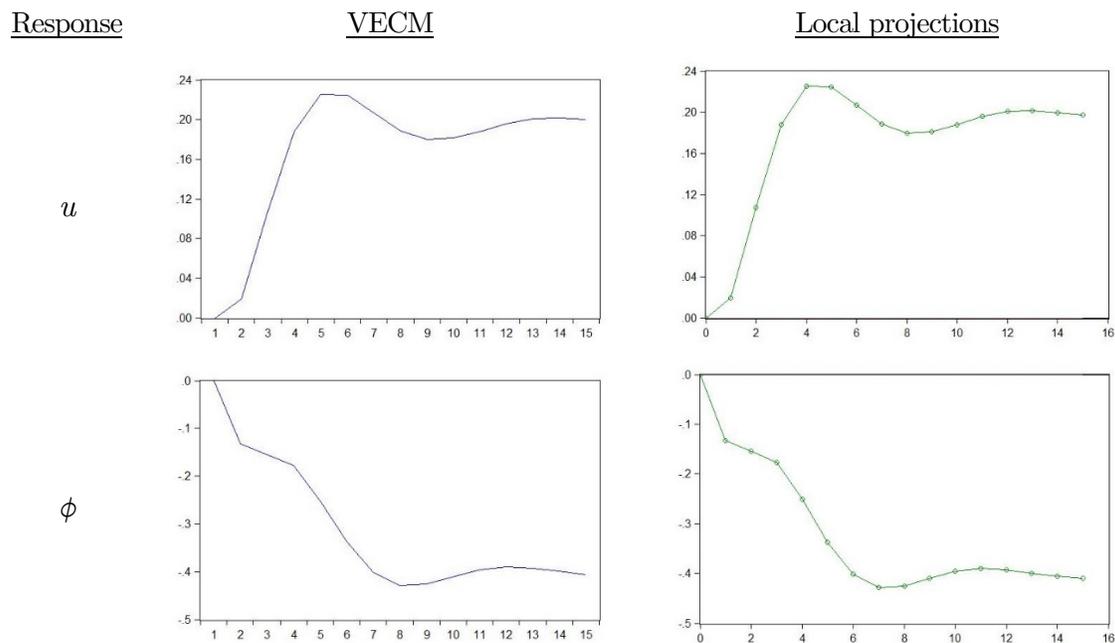
7.1 Impulse Responses

Figure 4 shows the responses of u , ϕ , and e to one standard deviation of the residuals of the labor share ω , ignoring the correlations in the VECM residuals. We also use the local projections (Jordà, 2005) method to compare our impulse

responses as a robustness check. Contrary to the VECM responses, which extrapolates into increasingly distant horizons using an iterative technique, local projections compute the responses of the three variables mentioned above at each period of interest. We find that the responses of u , ϕ , and e are robust between these two methods.

The effect on u , ϕ , e of an exogenous shock to ω , which would be equivalent to the policy parameter z in equation (13), does not die out over time but leads to new steady state values for all three variables. The impact on u is positive as expected from equation (19), and the effect on ϕ is negative, as illustrated in equation (17). The effect on e could be either positive or negative depending on the values of parameters α and μ . Importantly, the impulse-response for the employment-population ratio is initially slightly negative but becomes positive in the later periods until a higher employment rate is reached. Thus, our time-series tests appear to lend support to the wage-led employment regime in the United States over the period under consideration.

Figure 4: Impulse-response of u , ϕ , and e to nonfactorized one standard deviation innovation in ω .



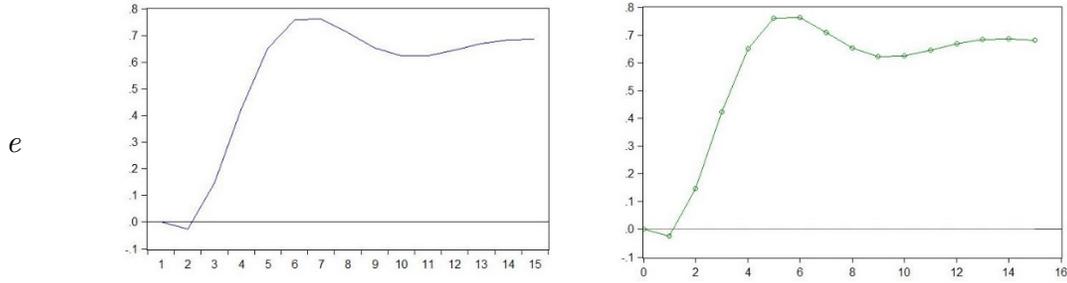


Table 2: Vector error correction model with u , ϕ , e , and ω as endogenous variables

| | | Dependent variable | | | | | | |
|-----------------------|---------------------|------------------------|---------------------|------------------------|---------------------|------------------------|----------------------|------------------------|
| | | u | ϕ | e | ω | | | |
| Long-run | ϕ | -0.5782*** (0.0673) | u | -1.7295*** (0.2477) | u | -1.0568*** (0.2983) | u | 1.6970* (0.9537) |
| | e | -0.9463*** (0.1287) | e | 1.6366*** (0.2040) | ϕ | 0.6110*** (0.1285) | ϕ | -0.9812** (0.4492) |
| | ω | 0.5893*** (0.1598) | ω | -1.0192*** (0.2770) | ω | -0.6228*** (0.1536) | e | -1.6058*** (0.3954) |
| | <i>ECT</i> | 0.1310 (0.1048) | <i>ECT</i> | 0.0349 (0.1152) | <i>ECT</i> | -0.3098** (0.1528) | <i>ECT</i> | -0.2413** (0.1059) |
| | Δu_{t-1} | 0.5923 (0.3825) | Δu_{t-1} | 0.7949 (0.7271) | Δu_{t-1} | 1.0899* (0.5894) | Δu_{t-1} | 1.6884** (0.6558) |
| $\Delta \phi_{t-1}$ | -0.1318 (0.1413) | $\Delta \phi_{t-1}$ | 0.2108 (0.2669) | $\Delta \phi_{t-1}$ | -0.1207 (0.2177) | $\Delta \phi_{t-1}$ | -0.2691 (0.2422) | |
| Δe_{t-1} | -0.1052 (0.1709) | Δe_{t-1} | -0.5226 (0.3248) | Δe_{t-1} | 0.3165 (0.2633) | Δe_{t-1} | -0.5236* (0.2929) | |
| $\Delta \omega_{t-1}$ | -0.0408 (0.1170) | $\Delta \omega_{t-1}$ | -0.2133 (0.2223) | $\Delta \omega_{t-1}$ | -0.2395 (0.1803) | $\Delta \omega_{t-1}$ | 0.2144 (0.2006) | |
| <i>constant</i> | 0.0918 (0.0754) | <i>constant</i> | 0.0562 (0.1433) | <i>constant</i> | -0.1158 (0.1161) | <i>constant</i> | -0.2183* (0.1292) | |

Note: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.10$

8 A Simple Extension: Kaldor-Verdoorn Law

A post-Keynesian economist would criticize our model on the grounds that it features no role for aggregate demand but emphasizes supply and technological forces only. One way to address this limitation without completely rethinking the whole framework is to augment the growth rate of labor-augmenting technology

by adding a Kaldor-Verdoorn term to reflect the positive effect of capital accumulation on labor productivity growth. Suppose, accordingly, that we modify the growth rate of labor-augmenting technology as follows:

$$\gamma = \theta g + \alpha\beta\omega, 0 < \theta < 1 \tag{25}$$

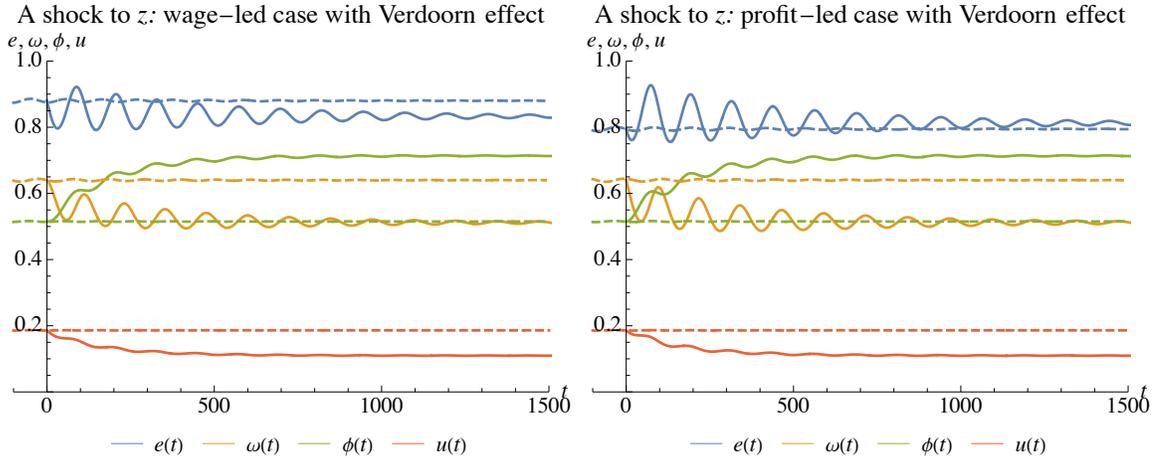
where g is defined as in equation (4). In balanced growth, $\gamma = g$ must hold. Thus, the Verdoorn effect vanishes, and the long-run growth rate is

$$\gamma_{ss} = \frac{\alpha\beta\omega_{ss}}{1-\theta} = \frac{\alpha z}{1-\theta} \tag{26}$$

which is similar to the baseline model: this is the reason why models incorporating a Verdoorn effect have been referred to as *semi-endogenous*, in that the corresponding part of the long run growth rate is determined within the model but policy-invariant (Tavani and Zamparelli, 2017). However, (i) the effect of a shock to the labor market parameter z will actually be *amplified* by the presence of the Verdoorn parameter θ , and (ii) the Kaldor-Verdoorn term matters along the transitional dynamics. The simulations below display the effect of a shock to the labor market parameter z in the extended model, both in the wage-led and profit-led case. For these simulations, we used a small value for the parameter θ , namely .015.⁴

Figure 5: Simulations – a 5% negative shock to z at time zero with a Verdoorn effect.

⁴ As verified in simulations, larger values of θ may lead to instability: this suggests that the dynamical system undergoes a Hopf-bifurcation for some threshold value for the Verdoorn parameter. An analytical study of the conditions for a Hopf-bifurcation in the model with a Verdoorn effect is outside the scope of this paper.



9 Conclusion

In this paper, we presented a comprehensive model of secular stagnation, income and wealth distribution, and employment drawing from contemporary work in the Classical Political Economy tradition, and especially Petach and Tavani (2021). Contrary to the well-known neoclassical account (Piketty, 2014; Gordon, 2015) that presupposes full employment and exogenous growth and hinges on high elasticity of substitution between capital and labor, our framework emphasizes the preeminence of the functional distribution on both capital accumulation and technological change. The former is profit-driven, while the latter is conflict- (or wage-) driven through induced bias in technology (Kennedy, 1964).

Our argument is that adverse institutional shocks to the labor share initially foster capital accumulation g , which is driven by the profit motive. However, in the long-run, the growth rate of the economy—which ultimately depends on labor-productivity growth γ —falls, due to the lessened incentive to adopt labor-augmenting technologies given that the labor share has fallen. Thus, the income-capital ratio must fall to restore the balanced growth condition $g = \gamma$.

We also emphasized the evolution of the distribution of wealth, providing a modern version of the celebrated Pasinetti (1962) theorem that ties up to the argument presented above through the inverse long-run relationship between the wage share and the top wealth share on the one hand, and between the top wealth share and the income-capital ratio on the other. Differently from previous contributions, we also explicitly studied the evolution of the employment rate and its relationship to the functional income distribution in the long run: we identified two forces at work in determining the ultimate response of employment to shocks

to the wage share, namely the strength of the induced technical change channel as opposed to the pure labor-market conflict channel. Correspondingly, depending on the relative strength of the two channels, we were able to identify a wage-led and a profit-led *employment regime* in the long run.

Finally, we ran a time-series test of the model using US data (1990-2019). Our interest is in the response of the income-capital ratio, the top wealth share, and the employment-population ratio to an exogenous shock to the wage share. Our theoretical model does quite well against the data: following a one standard-deviation shock to the wage share, the impulse-response functions display a positive long-run effect on the income-capital ratio, and a negative effect on the top wealth share. Both these effects are as expected from the theory. The long-run effect on employment appears to be positive, thus lending support to the wage-led employment regime.

Importantly, our explanation of secular stagnation and inequality does not feature a role for aggregate demand, but emphasizes technological (i.e. supply) forces. Even so, our model features a long-run where growth is wage-led through technological change; and it features the possibility of a long-run wage-led employment regime, similarly to the possibility of long-run wage-led economic activity (capacity utilization) in neo-Kaleckian economics, that appears to be supported in the empirical analysis we ran using time-series data for the United States. The main policy implication is therefore that progressive redistribution policies that increase the labor share of income can have positive effects both on economic growth and on long-run employment even in supply-constrained economies.

Appendix A: Stability Analysis

Linearization of the dynamical system around the Pasinetti steady state where $\phi_{ss} \in (0, 1)$ yields, independently of the sign of $\alpha - \mu$:

$$\begin{matrix} \dot{u} \\ \dot{\phi} \\ \dot{\omega} \\ \dot{e} \end{matrix} = \begin{bmatrix} 0 & 0 & - & 0 \\ 0 & - & - & 0 \\ 0 & 0 & - & + \\ + & + & - & 0 \end{bmatrix} \begin{bmatrix} u - u_{ss} \\ \phi - \phi_{ss} \\ \omega - \omega_{ss} \\ e - e_{ss} \end{bmatrix}$$

with the following entries in the Jacobian matrix:

$$\begin{aligned}
J_{13} &= \left. \frac{\partial \dot{u}}{\partial \omega} \right|_{ss} = -\frac{\alpha \beta^2 z}{s^c(\beta-z)} &< 0 \\
J_{22} &= \left. \frac{\partial \dot{\phi}}{\partial \phi} \right|_{ss} = -\frac{\alpha z}{s^c} \left[\frac{(s^c - s^w)(\beta - z) - s^w z}{\beta - z} \right] &< 0 \\
J_{23} &= \left. \frac{\partial \dot{\phi}}{\partial \omega} \right|_{ss} = -\frac{(s^c - s^w)(\beta - z) - s^w z}{(s^c - s^w)(\beta - z)} \frac{\alpha \beta^2 s^w z}{s^c(\beta - z)^2} &< 0 \\
J_{33} &= \left. \frac{\partial \dot{\omega}}{\partial \omega} \right|_{ss} = -\alpha z &< 0 \\
J_{34} &= \left. \frac{\partial \dot{\omega}}{\partial e} \right|_{ss} = \frac{\delta z}{\beta} &> 0 \\
J_{41} &= \left. \frac{\partial \dot{e}}{\partial u} \right|_{ss} = \left[\frac{s^c(\beta - z)}{\beta} \right] \left[\frac{\lambda + (\alpha - \mu)z}{\delta} \right] &> 0 \\
J_{42} &= \left. \frac{\partial \dot{e}}{\partial \phi} \right|_{ss} = \alpha z \left(\frac{s^c - s^w}{s^c} \right) \left[\frac{\lambda + (\alpha - \mu)z}{\delta} \right] &> 0 \\
J_{43} &= \left. \frac{\partial \dot{e}}{\partial \omega} \right|_{ss} = -\beta \left\{ (1 + \alpha) + \frac{\alpha z}{s^c(\beta - z)^2} [(s^c - s^w)(\beta - z) - s^w z] \right\} \left[\frac{\lambda + (\alpha - \mu)z}{\delta} \right] &< 0 \\
J_{11} &= J_{12} = J_{14} = J_{21} = J_{24} = J_{31} = J_{32} = J_{44} = 0
\end{aligned}$$

For local stability, the eigenvalues of J_{ss} must have uniformly negative real parts. Petach and Tavani (2020) have already proven the local stability of the 3-dimensional subsystem in (u, ω, ϕ) under $\mu = 0$. Notice however that the terms in square brackets in J_{42} and J_{43} are nothing but the steady-state employment rate, which is positive no matter whether $\alpha - \mu \gtrless 0$, with the implication that all the signs in the Jacobian are unambiguous, both in the wage-led and in the profit-led model. Thus, it is sufficient to show that the fourth eigenvalue is negative. In order to do so, consider that the equations describing the evolution of μ and ϕ do not communicate with the equation tracing the evolution of employment: the implication is that we can focus on the interaction between ω and e in the Jacobian, i.e., the submatrix formed by the entries $\begin{bmatrix} J_{33} & J_{34} \\ J_{43} & J_{44} \end{bmatrix}$ in the bottom right of the full Jacobian. Such minor has the following sign structure: $\begin{bmatrix} - & + \\ - & 0 \end{bmatrix}$ that delivers a negative trace and positive determinant, which implies that the fourth eigenvalue of J is negative as required.

Appendix B: Unit root tests

| Unit root test | Specification | u | ϕ | e | ω | Δu | $\Delta \phi$ | Δe | $\Delta \omega$ |
|----------------|---------------------|-------|--------|-------|----------|------------|---------------|------------|-----------------|
| ADF | Intercept | -1.89 | -1.08 | -1.77 | -1.02 | -4.25*** | -4.47*** | -2.98** | -3.71*** |
| | Intercept and trend | -1.40 | -2.66 | -3.10 | -1.55 | -4.15** | -4.55*** | -2.93 | -3.63** |
| PP | Intercept | -0.82 | -1.07 | -1.27 | -1.29 | -4.28*** | -4.50*** | -3.08** | -3.71*** |
| | Intercept and trend | -1.40 | -2.20 | -1.57 | -1.86 | -4.20** | -5.63*** | -3.03 | -3.63** |

Notes: ADF: Augmented Dickey-Fuller, PP: Phillips-Perron. The null hypothesis is that the variable has a unit root, while the alternative hypothesis is that the variable is stationary. *** Null hypothesis is rejected at 1% significance, ** Null hypothesis is rejected at 5% significance. The lag length for the Augmented Dickey-Fuller test is based on the Bayesian information criterion. The Bartlett kernel is selected as the spectral estimation method with a bandwidth set by the Newey-West procedure for the Phillips-Perron unit root test. Operator Δ before the name of the variables denotes that the variable is expressed in first-differences.

Appendix C: Model Selection

| Lag | LogL | LR | FPE | AIC | BIC | HQ |
|-----|---------|--------|---------|-------|-------|-------|
| 0 | -144.98 | NA | 1.1145 | 11.46 | 11.65 | 11.52 |
| 1 | -44.90 | 161.66 | 0.0017 | 4.99 | 5.96 | 5.27 |
| 2 | -9.45 | 46.36* | 0.0004* | 3.50 | 5.24* | 4.00* |
| 3 | 9.22 | 18.67 | 0.0005 | 3.29 | 5.81 | 4.01 |
| 4 | 27.96 | 12.98 | 0.0007 | 3.08* | 6.37 | 4.03 |

Notes: * Indicates lag order selected by the criterion. L.R.: sequential modified L.R. test statistic (each test at a 5% level). FPE: Final prediction error. AIC: Akaike information criterion. BIC: Bayesian information criterion. H.Q.: Hannan-Quinn information criterion. Endogenous variables: output-capital ratio, top 1% wealth share, the share of labor compensation, and the employment-population ratio. Included observations: 26.

Appendix D: Johansen cointegration test

| Hypothesized No. of CEs | Trace | | Max Eigenvalue | |
|----------------------------|-----------|---------|----------------|---------|
| | Statistic | P-value | Statistic | P-value |
| None | 64.61 | 0.0434 | 31.57 | 0.0582 |
| At most 1 | 33.04 | 0.3345 | 19.39 | 0.2799 |
| At most 2 | 13.65 | 0.6864 | 8.84 | 0.7407 |
| At most 3 | 4.81 | 0.6241 | 4.81 | 0.6241 |

Notes: The test allows a linear deterministic trend, and an intercept and trend in the cointegrating equation. Lag interval (in first differences): 1 to 1.

Appendix E: VECM residual serial correlation LM tests

Null hypothesis: No serial correlation at lag h

| Lag | LRE stat | df | Prob. | Rao F-stat | df | Prob. |
|-----|----------|----|--------|------------|------------|--------|
| 1 | 23.49 | 16 | 0.1012 | 1.60 | (16, 46.5) | 0.1061 |
| 2 | 9.62 | 16 | 0.8856 | 0.57 | (16, 46.5) | 0.8882 |

Null hypothesis: No serial correlation at lags 1 to h

| Lag | LRE stat | df | Prob. | Rao F-stat | df | Prob. |
|-----|----------|----|--------|------------|------------|--------|
| 1 | 23.49 | 16 | 0.1012 | 1.60 | (16, 46.5) | 0.1061 |
| 2 | 31.95 | 32 | 0.4694 | 0.99 | (32, 42.2) | 0.5102 |

Appendix F: VECM residual serial correlation LM tests

| White heteroskedasticity test (no cross terms) | | |
|--|--------------------|--------|
| Chi-square | Degrees of freedom | Prob. |
| 114.79 | 100 | 0.1481 |

| White heteroskedasticity test (includes cross terms) | | |
|--|--------------------|--------|
| Chi-square | Degrees of freedom | Prob. |
| 211.46 | 200 | 0.2758 |

Note: The null hypotheses are no heteroskedasticity.

Appendix G: VECM residual normality tests

| Orthogonalization: Cholesky (Lutkepohl) | | | |
|---|-----------|--------------------|---------|
| | Statistic | Degrees of freedom | P-value |
| Skewness | 2.2032 | 4 | 0.6984 |
| Kurtosis | 1.0338 | 4 | 0.9046 |
| Jarque-Bera | 3.2370 | 8 | 0.9186 |

| Orthogonalization: Residual correlation (Doornik-Hansen) | | | |
|--|-----------|--------------------|---------|
| | Statistic | Degrees of freedom | P-value |
| Skewness | 7.2204 | 4 | 0.1247 |
| Kurtosis | 4.5831 | 4 | 0.3328 |
| Jarque-Bera | 11.8036 | 8 | 0.1602 |

| Orthogonalization: residual covariance (Urzua) | | | |
|--|-----------|--------------------|---------|
| | Statistic | Degrees of freedom | P-value |
| Skewness | 7.1248 | 4 | 0.1294 |
| Kurtosis | 11.5659 | 4 | 0.0209 |
| Jarque-Bera | 64.8292 | 55 | 0.1712 |

Note: The null hypotheses are residuals are multivariate normal.

Appendix H: Roots of the characteristic polynomial of the VECM

| Root | Modulus |
|---------------------|---------|
| 1.0000 | 1.0000 |
| 1.0000 | 1.0000 |
| 1.0000 | 1.0000 |
| $0.5663 - 0.5177 i$ | 0.7673 |
| $0.5663 + 0.5177 i$ | 0.7673 |
| 0.6361 | 0.6361 |
| $0.0901 - 0.3578 i$ | 0.3690 |
| $0.0901 + 0.3578 i$ | 0.3690 |

Note: VECM specification imposes three unit-roots.

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